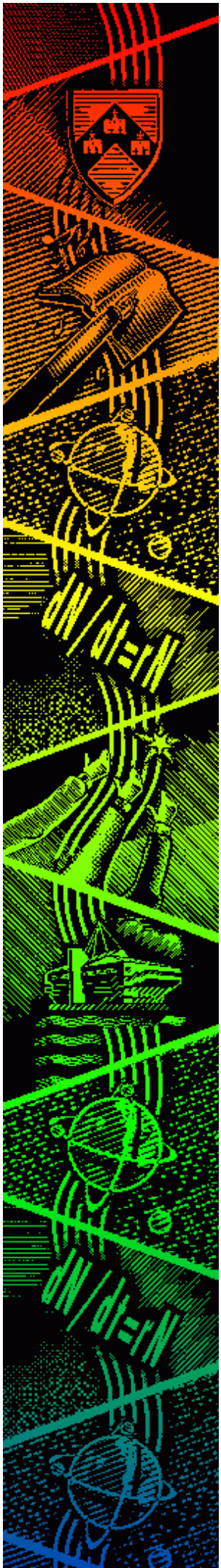

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# An OLG model of global imbalances

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# An OLG model of global imbalances

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**Abstract**

In this paper, we investigate the relationship between East Asian countries' high propensity to save and global imbalances in a two-country OLG model with production. The absence of pay-as-you-go pension systems can rationalize the saving behavior of emerging economies and capital outflows to the United States. The model predicts that the country with no pay-as-you-go system can run a trade surplus only as long as the long-run growth rate of the economy is higher than the real interest rate (capital overaccumulation case). The low real interest rates in the US is evidence in favor of the hypothesis that there is a "global saving glut" in the world economy. The model can also explain why the US current account deteriorated gradually and only in the late 1990s, although the net foreign asset position had already turned negative in the early 1980s. Finally, this analysis implies that the introduction of a pay-as-you-go system in China would have the effect of reducing the imbalances.

*Keywords:* global imbalances, capital flows, current account dynamics, OLG model, pay-as-you-go-system. *JEL Classification:* F21, F33, F34, F41.

## 1 Introduction

Not only too little capital flows from rich to poor countries - as Lucas [18] pointed out - but we have observed the reverse pattern of net capital flows for over a decade. The trade imbalances between the United States and East Asian economies, or global imbalances, do not appear to be a temporary phenomenon. Figure 1 shows that the United States' current account deficit has steadily deteriorated since the late 1990s. The recent adjustment has involved trade with Europe and oil-producing countries, but not other emerging economies. In particular, the US deficit towards China, which mirrors very closely the Chinese surplus, did not shrink with the recession.

One of the most common views on global imbalances is the "global saving glut hypothesis", due to Bernanke [3]. The core of the argument is that the high saving rates in East Asia have created an excess of savings in the world economy, which has resulted in capital flows towards the US and low real interest rates. Bernanke [3] also claimed that the understanding of global imbalances requires a "global perspective" and that they do not "primarily reflect economic policies and other economic developments within the United

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States itself". In other words, current account imbalances must be thought of as an equilibrium phenomenon.

In this paper, we provide a general equilibrium framework to discuss the global saving glut hypothesis and therefore investigate the relationship between emerging countries' high propensity to save and global imbalances. An interesting - and key, to us - aspect of the data is that while global imbalances emerged in the late 1990s, that East Asian countries save more than the United States is certainly not a new fact. Figure 2 depicts the saving rates of the US and a few East Asian countries over the last 30 years.

The heterogeneity in the pension systems is one of the plausible candidates to explain the structural difference in the countries' saving rates. In fact, pay-as-you-go social security systems are nearly absent in many emerging economies. Reforms aimed at introducing state pensions are still underway in China and other East Asian economies[1]. On the other hand, the pay-as-you go system was introduced in the United States during the Great Depression. There is a substantial body of evidence - summarized in [10] - which indicates that the pay-as-you-go system had the effect of crowding out private saving in the US. More generally, cross-sectional evidence [22] supports the idea that countries with pay-as-you-go systems tend to have lower saving rates, especially the more extensive is the coverage. Yet, the implications for global imbalances of the fact that East Asian countries need to save more to finance old age consumption are still unexplored. One of the contributions of this paper is to fill this gap in the literature.

The model that we study is a two-country OLG model with production along the lines of Diamond [9], in which the two countries are identical except that only one country has a pay-as-you-go social security system. The Diamond model is a natural framework to address the question of excess savings in an economy. In fact, the model admits the possibility that, in a perfectly competitive economy, there is capital overaccumulation. The concept of "excess savings" has a precise meaning in the OLG model as it corresponds to the notion of dynamic inefficiency, and this motivates our modeling choice.

In section 2 and 3, we present the model and characterize the direction of capital flows and trade at and outside steady states.

First, we show that the emerging country always lends to the developed country, as the young of the former country save relatively more in the absence of the pay-as-you-go system. Yet, the pattern of trade in the consumption good does depend on the long-run efficiency of the world economy. We prove that the direction of trade depends on how the population growth rate compares with the interest rate, and this is also the case outside steady states. The emerging country runs a trade surplus only as long as the world economy is beyond the golden rule level of capital (capital overaccumulation). Otherwise, the emerging country runs a trade deficit despite the fact that it's the lender country. Only in the coincidental case of the golden rule, trade happens to be balanced.

The main implication of these results is that we would not observe the current pattern of trade if there was not an excess of savings in the world

[1] On the Chinese case, see "Social Security Reform in China: Issues and Options" at Peter A. Diamond's webpage: http://econ-www.mit.edu/files/691. Diamond was one of the leading economists who participated at this study on social security reforms in China. On pension systems in Asia, see e.g. [4].

economy. In this sense, this paper provides a formal argument in favor of the "global saving glut hypothesis". Caballero et al. [6] argue that the saving glut story can be interpreted within their framework, by positively shocking the emerging country's saving parameter. This paper makes a further step as a global excess of savings arise endogenously, as a long-term consequence of the financial integration between the United States and East Asian countries.

Another interesting aspect of the trade balance result is that the developed country runs a trade deficit in the capital overaccumulation case because aggregate consumption is higher than in the other country. The reason is that pensions' growth is high enough to compensate interest payments to the emerging country. It is often claimed that global imbalances are due to the fact that emerging countries are consuming too little. This model shows that this is nothing but equilibrium behavior.

Our findings are related to two seminal papers of David Gale [11], [12]. Gale made the important point that countries can run permanent trade imbalances in general equilibrium models. His intuition was that this is especially possible in OLG economies. Gale had discovered that the sign of the balance of trade depends on efficiency properties in a Solow model with heterogenous agents and in a pure exchange OLG economy with inside money. The paper is also related to Polemarchakis and Salto [19], which found that trade is balanced at the golden rule in a pure exchange OLG economy with outside money. Previous work on international capital mobility which use the Diamond model as a framework include [5] and [14]. These papers derive conclusions on the pattern of capital flows by comparing the autarkic and the open economy steady states. On the contrary, we fully study capital accumulation, capital flows and trade dynamics as our objective is to understand the phenomenon of global imbalances through the lens of this model.

In section 4, we study the dynamics of capital flows and global imbalances for plausible initial conditions of the autarkic economies. It turns out that the model is able to account for the dynamics and the timing of global imbalances, as well as the dynamics of real interest rates and net foreign asset positions. First, the model can rationalize the fact that the US current account and real interest rates deteriorated *gradually* (Figure 1 and 4). Second, the model can explain why the accumulation of net foreign liabilities started in the early 1980s (Figure 3), well *before* the emergence of global imbalances[2].

The model provides intuitive explanations for these facts. Because of their higher saving rates, emerging countries started to lend abroad soon after they opened to trade with the US. The decline of real interest rates can be read as a consequence of capital accumulation in the world economy (Figure 4). Global imbalances arose as soon as interest rates fell below the long-run growth rate, implying that the world economy is saving too much.

Finally, we ask whether it is plausible that the economy is experiencing a global saving glut. According to the model, this requires that the long-run growth rate of the economy is higher than the real interest rate. We find evidence of this in the data. This is hardly surprising, since US real interest rates have hit a historic low in the past decade.

This paper is mainly related to the body of literature which puts emphasis

[2]See section 4 for a comparison with the literature on these stylized facts.

on differences in institutions as the main determinant of global imbalances, e.g. Caballero et al. [6], Mendoza et al. [20] and Angeletos et al. [1]. These papers' focus is on financial markets' different stages of development, and yet the sense of our analysis is very similar as the type of pension system enforced in a country surely affects saving and investment possibilities. Caballero et al. [6] explain global imbalances as the result of a negative shock to emerging countries' level of financial development, while our view is that global imbalances arose as the outcome of the financial integration between the US and emerging economies. In this respect, this paper is closer to Mendoza et al. [20] and Angeletos et al. [1].

The novel element of this model is that global imbalances are neither a temporary phenomenon, meant to disappear in the long-run (in [20] and [1]), nor a benign aspect of the world economy (as in [6]). Excess savings in an OLG economy means that there is room for policy interventions.

Hence, this paper contributes to the debate on whether and how the imbalances should be addressed from a policy point of view. While there is widespread agreement that global imbalances must be reduced, this is advocated on the basis of a variety of arguments[3].

It is often claimed that East Asian countries should introduce policies to boost domestic demand, in view of correcting the imbalances. If we accept that the world economy is overaccumulating capital, long-term policies in this direction are clearly desirable. For instance, the introduction of a pay-as-you-go system in China would not only be Pareto-improving but also have the effect of reducing the imbalances.

## 2 The World Economy

In this section, we describe the two-country model, which maintains the basic structure of Diamond (1965). We will refer to country 1 (2) as the developed (emerging) country.

Agents live for two periods and a new generation is born in each country for all $t$. The size of the population follows $L_{i,t} = L_{i,0}(1+n)^t$, where $L_{i,0}$ are the young born in country $i$ at date 0 and $n$ is the (common) population growth rate[4]. The only source of growth in the model comes from population[5].

The two countries only differ in the pension systems. Country 1 has a pay-as-you-go social security system, while the system in country 2 is fully-funded. Country 1's government levies a time-invariant lump-sum tax $\tau_1$ on the young, which is used to finance the old's pension $b_1$ at each $t$[6]. The policy is balanced so that taxes are equal to transfers at each $t$: $\tau_1 L_{1,t} = b_1 L_{1,t-1}$. It follows that the transfer which the current old receive is equal to $b_1 = (1+n)\tau_1$, i.e. each generation receives a transfer which is bigger than the tax if population is increasing.

Finally, we need to specify which markets are open for international trade. We assume that the consumption good can be costlessly traded between the

[3] See a recent collection of papers written by central bankers on the topic [15].

[4] This is realistic as the population of both China and the United States have grown at an average rate of 1% for the last 30 years (World Bank data). However, we allow for the countries' size to be different.

[5] See the Appendix for an extension of the model with labour augmenting technological progress.

[6] The results of the paper are robust to different types of taxation. In particular, the derivation of the model under proportional tax is available under request.

countries. As our focus is to analyze the pattern of trade in the good, we impose that labor is immobile.

### 2.1 Firms

Competitive firms use capital and labor to produce the consumption good by means of an identical, constant returns technology: $Y_{i,t} = F(K_{i,t}, L_{i,t})$.

As anticipated above, firms located in country $i$ can only hire workers in the domestic labor market. We consider the production function in its intensive form as the number of workers is given at each $t$: $y_{i,t} = f(k_{i,t})$. The function $f$ is strictly increasing and concave in $k_{i,t}$. Capital depreciates at the constant rate $0 \leqslant \delta \leqslant 1$ in both countries and we assume that the following boundary conditions hold:

$$\lim_{k_{i,t}\to+\infty} f'(k_{i,t}) = 0 \qquad \lim_{k_{i,t}\to 0} f'(k_{i,t}) = +\infty$$

At time 0, the two autarkic economies open to trade after production has taken place. Their "initial" level of capital will respectively be $k_{1,0}$ and $k_{2,0}$. Starting from period 1, firms' demand for capital is met in the world market and therefore they will face the same path of interest rates $\{r_t\}$. Firms solve the following maximization problem:

$$\max_{k_{i,t}} \pi_{i,t} = f(k_{i,t}) - (r_t + \delta)k_{i,t} - w_{i,t} \qquad \forall\ i, t \geqslant 1 \tag{1}$$

The necessary and sufficient conditions for a maximum are:

$$r_t = f'(k_{i,t}) - \delta \tag{2}$$
$$w_{i,t} = f(k_{i,t}) - f'(k_{i,t})k_{i,t} \tag{3}$$

Because the countries have access to the same technology, it is immediate that capital stocks per capita are equalized: because $k_{1,t} = k_{2,t} = k_t$, it is also true that $w_{1,t} = w_{2,t} = w_t$ for all $t$. While the two countries might start with different initial conditions, potential income differences vanish once the two countries open to trade.

This assumption is somewhat strong, but our theory does not aim at explaining per capita income differences between the US and East Asian countries. Moreover, it is convenient to abstract from other potential bases for trade to study how differences in pension systems have an impact on capital accumulation and trade.

### 2.2 Consumers

Agents get utility from consuming in the two periods of life. Preferences are stationary and identical both within generation and across countries. The utility function is $C^2$, strictly increasing, strictly concave and additively separable:

$$U(c^t_{i,t}, c^t_{i,t+1}) = u(c^t_{i,t}) + \beta v(c^t_{i,t+1}) \tag{4}$$

where $c^t_{i,t}$ denotes consumption when young and $c^t_{i,t+1}$ is consumption when old of the generation (born at time) $t$ in country $i$. Also:

$$\lim_{c^t_{i,t}\to 0} u'(c^t_{i,t}) = +\infty \qquad \lim_{c^t_{i,t+1}\to 0} u'(c^t_{i,t+1}) = +\infty$$

The budget constraints are:

$$c_{i,t}^t = w_t - \tau_i - s_{i,t} \quad (5)$$

$$c_{i,t+1}^t = s_{i,t}(1 + r_{t+1}) + \tau_i(1 + n) \quad (6)$$

where $\tau_2 = 0$ as there is no pay-as-you-go system in country 2. In our two-country world, the young are allowed to lend both to domestic and foreign firms. Which country is going to be the borrower (lender) will be established in equilibrium.

The maximization problems of the two consumers are the following:

$$\max_{s_{1,t}} \quad u(w_t - \tau_1 - s_{1,t}) + \beta v(s_{1,t}(1 + r_{t+1}) + \tau_1(1 + n)) \quad (7)$$

$$\max_{s_{2,t}} \quad u(w_t - s_{2,t}) + \beta v(s_{2,t}(1 + r_{t+1})) \quad (8)$$

The necessary and sufficient conditions for a maximum are:

$$u'(w_t - \tau_1 - s_{1,t}) = \beta(1 + r_{t+1})v'(s_{1,t}(1 + r_{t+1}) + \tau_1(1 + n)) \quad (9)$$

$$u'(w_t - s_{2,t}) = \beta(1 + r_{t+1})v'(s_{2,t}(1 + r_{t+1})) \quad (10)$$

The agents' optimal savings are then a function of the wage and the interest rate. In country 1, they also depend upon the taxes and transfers related to the pension system.

In the OLG model, it is well known that savings are lower in presence of a pay-as-you-go system (see e.g. [2], or [21]). Given $w_t$ and $r_{t+1}$, we have that:

$$\frac{ds_{1,t}}{d\tau_1} = -\frac{u''(c_{1,t}^t) + \beta(1 + n)(1 + r_{t+1})v''(c_{1,t+1}^t)}{u''(c_{1,t}^t) + \beta(1 + r_{t+1})^2 v''(c_{1,t+1}^t)} < 0 \quad (11)$$

At each $t$, the young in country 1 save less than in country 2 as their net wage is lower due to the tax. However, the extent of the fall in saving will depend on how $n$ and $r_{t+1}$ compares. In particular, if $n > r_{t+1} (< r_{t+1})$ then the drop in saving is larger since $\frac{ds_{1,t}}{d\tau_1} < -1 (> -1)$. In fact, the income of country 1's consumers is higher (lower) when the rate of return on the pension system is higher (lower) than the interest rate. This can be seen from the consolidated budget constraint:

$$c_{1,t}^t + \frac{c_{1,t+1}^t}{1 + r_{t+1}} = w_t - \tau_1 \frac{r_{t+1} - n}{1 + r_{t+1}} \quad (12)$$

In the Diamond model, consumption increases with income (normal goods)[7]. When $n > r_{t+1}$, $c_{1,t}^t$ increases and therefore savings will be even lower. Only when $n = r_{t+1}$, savings decrease one for one with the tax as (11) shows.

We also characterize the saving functions by the following assumption.

**Assumption 1** *Consumption when young and when old are gross substitutes:*

$$s_r > 0$$

where $s_r$ is the partial derivative of the saving function with respect to the interest rate.

[7] That savings are increasing in the wage can be derived from the first-order conditions. It can be checked that $\frac{ds_{i,t}}{dw_t} = \frac{1}{1+\beta(1+r_{t+1})^2 \frac{v''(c_{i,t+1}^t)}{u''(c_{i,t}^t)}}$, therefore $0 < s_w < 1$.

### 2.3 Equilibrium

Given $(\tau_1, k_{1,0}, k_{2,0})$, a competitive equilibrium is a sequence of capital stocks $\{k_t^*\}_{t\geqslant 1}$ and factor prices $\{r_t^*, w_t^*\}_{t\geqslant 1}$ such that:

(i) $\{c_{i,t}^{t*}, c_{i,t+1}^{t*}\}_{t\geqslant 0}$ maximize the agents' utility function (4) subject to the budget constraints (5),(6) for all $i$;

(ii) $\{k_t^*\}_{t\geqslant 1}$ maximize the firms' profit function (1);

(iii) the (world) capital market clears for $t \geqslant 0$:

$$\sum_i L_{i,t} s_{i,t}^* = \sum_i K_{i,t+1}^*$$

If the capital market clears at each $t$, the (world) market for the good will clear by Walras' Law. The good market is in equilibrium when the total resources available (after production) are equal to the consumption of the current young and old, and next period's capital stocks of the two countries.

$$\begin{aligned} \sum_i F(K_{i,t}^*, L_{i,t}) \;+\; (1-\delta)\sum_i K_{i,t}^* = \\ = \sum_i L_{i,t} c_{i,t}^{t*} + \sum_i L_{i,t-1} c_{i,t}^{t-1*} + \sum_i K_{i,t+1}^* \end{aligned} \tag{13}$$

Now, use the fact that $F(K_{i,t}^*, L_{i,t}) = L_{i,t} w_t^* + (r_t^* + \delta) K_{i,t}^*$ and $L_{1,t}\tau_1 = L_{1,t-1}\tau_1(1+n)$ to get:

$$\begin{aligned} \sum_i L_{i,t} w_t^* \;-\; \sum_i L_{i,t} c_{i,t}^{t*} - L_{1,t}\tau_1 - \sum_i K_{i,t+1}^* = \\ \sum_i L_{i,t-1} c_{i,t}^{t-1*} \;-\; L_{1,t-1}\tau_1(1+n) - \sum_i (1+r_t^*) K_{i,t}^* \end{aligned}$$

Using the budget constraints of the two agents, we obtain:

$$\left[\sum_i L_{i,t} s_{i,t}^* - \sum_i K_{i,t+1}^*\right] - (1+r_t^*)\left[\sum_i L_{i,t-1} s_{i,t-1}^* - \sum_i K_{i,t}^*\right] = 0 \tag{14}$$

Both equations (13) and (14) will be extensively used in the next section to study the pattern of trade between the two countries.

## 3 The pattern of trade

### 3.1 Dynamics in the capital market and capital flows

In this section, we analyze the direction of capital flows and trade in the model described above. The first step is to study how capital accumulates in this economy. The capital market is equilibrium in as long as the world demand for capital is equal to the world supply (savings):

$$\begin{aligned} K_{t+1}^* \;\equiv\; \sum_i K_{i,t+1}^* = L_{1,t} s_1(f(k_{1,t}^*) - f'(k_{1,t}^*)k_{1,t}^*, f'(k_{t+1}^*), \tau_1) + \\ +\; L_{2,t} s_2(f(k_{2,t}^*) - f'(k_{2,t}^*)k_{2,t}^*, f'(k_{t+1}^*)) \end{aligned} \tag{15}$$

where $K^*_{t+1}$ denotes the world capital stock at time $t+1$. We have already established that $k_{1,t} = k_{2,t} = k_t$ for $t \geqslant 1$, while at $t = 0$ countries might start with different levels of capital.

Before proceeding, it is convenient to introduce the following definition:

**Definition 1** *Country $i$'s size is:* $\rho_i \equiv \frac{L_{0,i}}{L_0}$.

Because the countries grow at a common rate, $\rho_i$ is constant over time and depends on the countries' initial labor forces. We can now divide (15) by the world labor supply $L_t$ and get:

$$\begin{aligned}(1+n)k^*_{t+1} &= \rho_1 s_1(f(k^*_{1,t}) - f'(k^*_{1,t})k^*_{1,t}, f'(k^*_{t+1}), \tau_1) + \\ &+ \rho_2 s_2(f(k^*_{2,t}) - f'(k^*_{2,t})k^*_{2,t}, f'(k^*_{t+1}))\end{aligned} \tag{16}$$

At each $t$, the world capital stock per capita (which is equivalent to the domestic capital stocks) is determined by the savings of country 1 and 2. Equation (16) shows that each country will contribute to the supply side of the market according to its size.

Hereafter, we study the above difference equation in the capital stock. The world economy is in steady state when $k^*_t = k^*_{t+1} = k^*$:

$$\begin{aligned}(1+n)k^* &= \rho_1 s_1[f(k^*) - f'(k^*)k^*, f'(k^*), \tau_1] + \\ &+ \rho_2 s_2[f(k^*) - f'(k^*)k^*, f'(k^*)]\end{aligned} \tag{17}$$

**Lemma 1** *(i) Given $k_{1,0} > 0$ and $k_{2,0} > 0$, there exists a unique intertemporal equilibrium as long as $\tau_1 < \bar{\tau}_1(k_{1,0})$. (ii) If $\lim_{k_t \to 0} \frac{\phi(k_t;\tau_1,\rho_1,\rho_2)}{k_t} > 1$, there exists at least a stable steady state.*

**Proof.** The proof is in the Appendix. ■

Part (i) of Lemma 1 establishes that there exists an equilibrium path only if each country's savings are positive at $t = 0$. It is intuitive that we need a condition on the tax level to avoid circumstances under which income is either zero or negative in the initial period. In other words, a perfect foresight equilibrium will exist only if the level of the tax is compatible with having positive savings in the economy[8]. Part (ii) shows that there exist paths converging to a stable steady state. This is important as the focus of the next section will be on the behavior of the economy near a stable steady state.

We can now analyze the pattern of trade between the countries[9]. We start with trade in the capital market. Given the capital market equilibrium equation, it is immediate to show which of the two countries has positive excess demand for capital.

**Definition 2** *The (per capita) excess demand function of country $i$ is:*

$$z_{i,t} \equiv (1+n)k_{t+1} - s_{i,t} \tag{18}$$

**Proposition 1 (Borrowing and lending)** *Country 1* (2) *is the borrower* (*lender*) *country for all $t \geqslant 1$.*

[8] See [8] for a detailed analysis of the (closed economy) Diamond model with lump-sum transfers.

[9] We postpone the discussion of the pattern of trade at the openness to section 4, where we study the dynamics of capital flows and global imbalances for realistic initial conditions of the autarkic economies.

**Proof.** First, substitute equation (16) into the excess demand function of country $i$. Equilibrium excess demands are:

$$z^*_{1,t} = \rho_2(s^*_{2,t} - s^*_{1,t}) \qquad z^*_{2,t} = -\rho_1(s^*_{2,t} - s^*_{1,t}) \tag{19}$$

where $\rho_1 z^*_{1,t} + \rho_2 z^*_{2,t} = 0$. From equation (11), we know that country 1 saves less than country 2 keeping factor prices as fixed. Therefore, it must be true that $s^*_{2,t} > s^*_{1,t}$ for all $k^*_t, k^*_{t+1}$. The sign of excess demand for the two countries follows:

$$z^*_{1,t} > 0 \qquad z^*_{2,t} < 0 \qquad \forall\ t \geqslant 1 \tag{20}$$

■

Proposition 1 shows that country 2 (the emerging country) will always lend to country 1, it does not matter whether the economy is in a steady state or not. The intuition behind this result is simple. We know that the equilibrium capital stock is combination of savings in the two countries and the developed country saves less than the emerging economy. Therefore, while country 1 has to borrow to sustain $k^*_{t+1}$, country 2's savings (partly) find an outlet in country 1.

It might be noted that the extent of trade will depend on how large is the difference between the two countries' savings. For instance, countries trade more the bigger is the size of the pay-as-you-go system in country 1. It is worth stressing that the direction of trade in the capital market does not depend on whether we are in the capital overaccumulation case or not. However, this becomes relevant once we consider the countries' net trade.

### 3.2 The balance of trade and efficiency

We can now study the pattern of trade in the consumption good. First, we define the balance of trade of country $i$ as the country's excess supply for the consumption good.

**Definition 3** *The (per capita) trade balance of country $i$ is:*

$$tb^*_{i,t} \equiv f(k^*_{i,t}) + (1-\delta)k^*_{i,t} - c^{t*}_{i,t} - \frac{c^{t-1*}_{i,t}}{1+n} - k^*_{i,t+1}(1+n) \tag{21}$$

If $tb^*_{i,t} > 0$ in equilibrium, then country $i$ is net exporter as output is higher than "domestic absorption".

A few words are due to explain the above definition, as it is of fundamental importance for the results of the paper. Definition 3 stems from the per capita version of (13), the consumption good's market clearing equation. Equation (13) states that the sum of the countries' balances must be zero at each $t$. While this must be true, trade imbalances between the countries are still possible in equilibrium.

It is important to stress that we have made no distinction between the current account and the balance of trade, as it is customary in international macroeconomics literature. In this model, all trade - including interest payments - takes place in the consumption good. Therefore, current account of a country is equivalent to its trade balance. In fact, note that the balance of

trade of country $i$ is also the difference between savings and investment per capita:

$$tb^*_{i,t} = s^*_{i,t} - i^*_{i,t}$$

where

$$\begin{aligned} s^*_{i,t} &\equiv f(k^*_{i,t}) - c^{t*}_{i,t} - \frac{c^{t-1*}_{i,t}}{1+n} \\ i^*_{i,t} &\equiv k^*_{i,t+1}(1+n) - (1-\delta)k^*_{i,t} \end{aligned}$$

Another way to look at the balance of trade is in terms of net capital flows. First, take equation (14) in per capita terms and obtain:

$$tb^*_{i,t} \equiv [s^*_{i,t} - k^*_{i,t+1}(1+n)] - \left(\frac{1+r^*_t}{1+n}\right)[s^*_{i,t-1} - k^*_{i,t}(1+n)] \tag{22}$$

$$\rho_1 tb_{1,t} + \rho_2 tb_{2,t} = 0 \tag{23}$$

Next, using Definition 2 rewrite (22) as follows:

$$tb^*_{i,t} = -z^*_{i,t} + \left(\frac{1+r^*_t}{1+n}\right) z^*_{i,t-1} \tag{24}$$

The above characterization shows that the balance of trade reflects trade in the capital market in period $t$ and $t-1$.

**Proposition 2 (Balance of trade and steady states)** *At the golden rule allocation ($r^* = n$), trade is balanced.*

*If the steady state is inefficient ($r^* < n$), country 2 (the emerging country) is in surplus while country 1 (the developed country) is in deficit.*

*If the steady state is efficient ($r^* > n$), the opposite is true.*

**Proof.** Consider equation (24). Imposing $z^*_{i,t} = z^*_{i,t-1} = z^*_i$ and $r^*_t = r^*$, the trade balance of country $i$ in the steady state is:

$$tb^*_i = -z^*_i \left(\frac{n - r^*}{1+n}\right) \tag{25}$$

It immediately follows that at the golden rule allocation $tb_i = 0 \ \forall \ i$. The other statements are a direct implication of our hypotheses and the sign of $z^*_i$ (Proposition 1). ■

If the world economy converges to a steady state such that $r^* = n$, not only steady state consumption will be maximized but trade will be balanced in the long-run. Yet, that trade is balanced does not imply that the two countries do not trade at all. In fact, trade in the capital market still takes place at the golden rule (by Proposition 1) but each country's capital outflows are completely offset by capital inflows.

However, this can only happen by coincidence. In all other cases, there will be trade imbalances between the two countries. To comment on the result, let us consider the trade balance of country 2:

$$tb^*_2 = \underbrace{-z^*_2}_{\text{capital outflow}} + \underbrace{\left(\frac{1+r^*}{1+n}\right) z^*_2}_{\text{capital inflow}}$$

We have seen that the young in country 2 lend to firms located in country 1 as they save relatively more (capital outflow). At the same time, the old of country 1 pay the loan back, along with interest payments, to the old of country 2 (capital inflow).

The proposition states that the sign of net capital flows (or the balance of trade) will depend on how $n$ and $r^*$ compares. Indeed, notice that while $z_i^*$ is constant at the steady state, $Z_{i,t}^*$ will grow at the population growth rate. Proposition 2 then says that the lender country will have a surplus as long as the net income from abroad is not enough to compensate the increase in capital outflows induced by population growth. Instead, if the interest rate was higher than the population growth rate, country 2 should be in deficit.

Therefore, the model implies that the reason why we observe global imbalances is that there is a saving glut in the world economy. We postpone to section 4 the discussion of whether it is plausible that the world economy is on an inefficient path, with the support of some empirical evidence.

The fact that the sign of the balance of trade of a country depends on whether the world economy happens to be below or beyond the golden rule allocation is not just true at the steady state of the model. Next, we show that this holds outside stationary states too.

To this purpose, it is more convenient to work with equation (21). As technologies are identical, it is intuitive that all the action has to come from aggregate consumption. Because pension systems are different, the countries' consumption possibilities are not the same and this will explain the direction of trade in the consumption good.

**Lemma 2 (Consumption)** *For any generation $t \geqslant 1$, the agent born in country 1 consume relatively more (less) when $n > r_{t+1}$ ($< r_{t+1}$).*

**Proof.** The proof is in the Appendix. ■

In Lemma 2, we show that agents born in country 1 consume more in the capital overaccumulation case. It is interesting to note that this result supports the idea that East Asian countries are consuming too little relatively to the United States, and this has something to do with global imbalances. The reason is that country 1's generations have a higher income, despite that the United States have to pay interest rates to China. When $n > r_{t+1}^*$, there is enough growth in the economy for the pension to compensate interest payments to the foreign country. An examination of the two agents' budget constraints should convince the reader of this fact.

Given Lemma 2, we can analyze the pattern of trade in the consumption good outside steady states:

**Proposition 3 (Balance of trade outside steady states)** *Country 1 (the developed country) is in deficit at a given $t$ when $n > r_t^*$ and $n > r_{t+1}^*$, while in surplus when $r_t^* > n$ and $r_{t+1}^* > n$. If $r_t^* > n$ and $r_{t+1}^* < n$, the sign is ambiguous.*

**Proof.** We consider the developed country, the opposite is obviously true for the emerging economy. If country 1 imports, then $tb_{1,t} < tb_{2,t}$. Given Definition 3 and because $k_{1,t} = k_{2,t}\ \forall\ t \geqslant 1$, the following must hold for country 1 to be

in deficit:

$$c_{1,t}^{t*} + \frac{c_{1,t}^{t-1*}}{1+n} > c_{2,t}^{t*} + \frac{c_{2,t}^{t-1*}}{1+n}$$

Indeed, Lemma 2 showed that consumption is higher for generations in country 1 as long as next period's interest rate is lower than the population growth rate. Therefore, for $tb_{1,t} < 0$ it is sufficient that $n > r_t^*$ and $n > r_{t+1}^*$. Instead, when $r_t^* > n$ and $r_{t+1}^* > n$ generations of country 2 consume more and $tb_{2,t} < 0$.

Suppose that at a given $t$, we have that $r_t^* > n$ but next period's interest rate falls below the population growth rate. While $c_{1,t-1}^{t*} < c_{2,t-1}^{t*}$ by $r_t^* > n$, $c_{1,t}^{t*} > c_{2,t}^{t*}$ by $r_{t+1}^* < n$. The net effect will depend on other parameters of the economy (see section 6.3 for an illustration in the Cobb-Douglas case). ■

The proposition establishes that the deficit (surplus) country is the country which consumes relatively more (less) at a given $t$.

At the golden rule, it is worth noting that the consumption allocation of the two representative generations is identical despite the different pension systems (see the proof of Lemma 2 in the Appendix). This gives a different angle to the balanced trade result. Because savings decrease one for one with $\tau_1$ and consumers' wealth is not affected by the pension system when $r^* = n$, consumption choices in the two countries are the same at the golden rule. Indeed, the planner would choose such allocation if giving the same weights to the agents (in fact, we did not allow for heterogeneity in preferences).

# 4 The dynamics of net foreign assets and global imbalances

The results of section 3 imply that the dynamics of the countries' balance of trade are strongly related to the efficiency of the world economy's capital accumulation path. In particular, we have found that the lender country (the country with no pay-as-you-go pension system) runs a trade surplus only as long as the population growth rate is higher than the interest rate. Therefore, our theoretical results suggest that global imbalances are a signal that the world economy is overaccumulating capital.

In this section, we demonstrate that the model is able to qualitatively replicate the evolution of the US current account and net foreign assets' position since the early 1980s (the time of China's integration into the world economy). Second, we provide some evidence to support the claim that there is a "global saving glut" in the world economy. If we can say that the long-run growth rate of the world economy is higher than the real interest rate, it is then plausible that the world economy is on an equilibrium path characterized by an excess of savings.

To start with, we need to address the following questions. What are the conditions under which the world economy converges to an inefficient steady state? And are these reasonable enough? To make progress on these issues, we introduce some assumptions on the characteristics of the two countries in autarky. Moreover, we make a conjecture on the two countries' initial conditions at time 0, which would correspond to the financial openness of emerging

countries[10].

**Hypothesis 1 (Autarkic steady states)** *Suppose country* 1 *has a locally stable steady state such that* $r_1^{autss} = n$. *For country* 2 *instead,* $n > r_2^{aut}$.

**Hypothesis 2 (Initial conditions)** *At the time of financial integration* $t = 0$, *country* 1 *is at the autarkic steady state* $k_{1,0} \equiv k_1^{autss}$.

*Country* 2*'s initial capital stock satisfies* $k_{2,0} < k_2^{aut}$. *Moreover, it is low enough that* $k_{2,0} < k_{1,0}$ *and* $s_1(k_{1,0}, k_1^*, \tau_1) > s_2(k_{2,0}, k_1^*)$, *where* $k_1^*$ *is the equilibrium capital stock at* $t = 1$.

Our main hypothesis is that the pay-as-you-go system, which has been introduced during the Great Depression, "fixed" the long-run inefficiency of the US economy. This assumption is also consistent with the fact that the US current account was balanced before 1980. We then assume that the autarkic steady state of the emerging economy is inefficient in the absence of social security. This is coherent with our previous analysis, as we treated the two countries as identical (except for the pension systems).

That country 2 opened to trade with a relatively low capital stock and along its transition path, while country 1 was already at the autarkic steady state, should not be controversial. We will explain Hypothesis 2 in more detail in the context of Proposition 5.

We are now ready to characterize the long-run equilibrium of the world economy.

**Proposition 4 (World steady state)** *Under Hypothesis* 1, *the world economy has a locally stable steady state such that* $n > r^*$.

**Proof.** It suffices to show that the (world) interest rate is between the autarkic interest rates: $r_1^{autss} > r^* > r_2^{aut}$, because we assumed that $r_1^{autss} = n$ (see the Appendix for a proof). ■

From Hypothesis 2, it can be inferred that the initial conditions of the world economy are such that the world economy starts to the left of the steady state. Our next step is to study trade dynamics in this context. First, we analyze trade at the time of China's financial integration. For instance, $t = 0$ could correspond to 1980. That the world capital market is open means that the young can lend both to domestic and foreign firms. As it might be expected, the pattern of trade at the openness will depend on the two countries' initial conditions.

**Proposition 5 (Financial integration)** *Under Hypothesis* 2, (*i*) *the developed country is the lender and runs a trade surplus at* $t = 0$; (*ii*) *the developed country runs a trade deficit at* $t = 1$.

**Proof.** The proof is in the Appendix. ■

The proof shows that $k_1^*$ is pinned down by total savings at $t = 0$, which depend on the two countries' initial conditions. At the outset of financial integration, a realistic scenario is one in which capital flows to the capital scarce, emerging country. To impose that $k_{2,0} < k_{1,0}$ is not enough because while

[10]Lemma 1 established that there exists at least a stable steady state for the world economy. In this section, we restrict attention to those paths converging to a stable steady state.

country 1 has a higher wage, there is the negative partial equilibrium effect of the pay-as-you-go on country 1's savings to take into account. Therefore, we need more stringent conditions for country 1 to save more and therefore lend to country 2 (Hypothesis 2).

At $t = 1$, the developed country's current account position turns into deficit: the old in country 2 pay off their debt and country 1 now starts to borrow.

Next, we study the dynamics of net foreign assets and the balance of trade for $t \geqslant 2$. We restrict our analysis to the case of log-linear preferences, because it's analytically tractable[11].

Let us observe that the net and the gross foreign asset positions of a country are equivalent, because there is only one asset in the model. The stock of net foreign assets held by residents of country $i$ at the end of period $t$ can be defined as $NFA_{i,t} \equiv -L_{i,t}z_{i,t}$, which is negative for the United States starting from $t = 1$.

**Proposition 6 (The dynamics of net foreign assets)** *Under log-linear preferences, country* 1 (*the developed country*) *accumulates net foreign liabilities.*

**Proof.** Using the young's budget constraints, we write equation (19) for country 1 as follows:

$$z^*_{1,t} = \rho_2(\tau_1 + c^{t*}_{1,t} - c^{t*}_{2,t})$$

Under log-linear preferences, consumption is a constant fraction of income:

$$c^{t*}_{i,t} = \theta I^*_{i,t} \tag{26}$$

where $0 < \theta < 1$. Then, let's rearrange the above equation:

$$z^*_{1,t} = \rho_2[\tau_1 + \theta(I^*_{1,t} - I^*_{2,t})]$$

Given our definition of $I^*_{i,t}$ (see the proof of Lemma 2):

$$z^*_{1,t} = \rho_2\tau_1\left(1 - \theta\frac{r^*_{t+1} - n}{1 + r^*_{t+1}}\right)$$

It can be verified that $\frac{\partial z_{1,t}}{\partial r_{t+1}} < 0$. Since the world economy is approaching a locally stable steady state from the left, this proves that country 1's net foreign liabilities increase as the capital stock accumulates.

■

Proposition 3 established that the sign of country 1's balance of trade at a given $t$ depends on whether the current and next period's interest rates are lower or bigger than $n$. It should now be evident that trade dynamics depends both on the initial conditions and the long-run properties of the autarkic economies. By Proposition 2 and 4, we know already that country 1 will run a deficit in the long-run. In the next proposition, we study the dynamics of trade imbalances.

**Proposition 7 (The dynamics of global imbalances)** *Under log-linear preferences, the balance of trade of country* 1 *deteriorates over time.*

[11] See section 6.3 for a full derivation of the model under Cobb-Douglas utility and production functions.

**Proof.** As we did for Proposition 6, let us rewrite equation (24) for country 1 using the budget constraints of the young born at $t$ and $t-1$:

$$tb^*_{1,t} = -\rho_2(\tau_1 + c^{t*}_{1,t} - c^{t*}_{2,t}) + \left(\frac{1+r^*_t}{1+n}\right)\rho_2(\tau_1 + c^{t-1*}_{1,t-1} - c^{t-1*}_{2,t-1})$$

Using equation (26) and rearranging:

$$tb^*_{1,t} = \rho_2\tau_1\left[(1-\theta)\frac{r^*_t - n}{1+n} + \theta\frac{r^*_{t+1} - n}{1+r^*_{t+1}}\right]$$

It can be checked that each of the two terms within the brackets increases with the interest rate. As the capital stock approaches the steady state from the left, therefore each term becomes smaller. This proves that the balance of trade of country 1 deteriorates over time. ■

We can now compare the time-series of the US current account and net international position with the predictions of the model. Figure 1 shows that the sign of the US current account varied until the early 1990s, that is before the building up of global imbalances. For this period, we cannot say anything more specific as disaggregated data are not available before 1999. It is possible that China might have imported from the United States in the early stage of financial integration, as Proposition 5 suggests.

More importantly, Proposition 7 explains the widening of the United States' current account deficit versus China. Our model seems to be more successful in capturing the dynamics of global imbalances than other models, e.g. [1], [6], [20]. In these papers, the United States run a trade deficit immediately after China's financial integration (or a shock), and then the deficit gradually improves. Our framework is more consistent with the data as it predicts the *gradual* deterioration of the US deficit.

Another aspect of interest is the dynamics of US foreign assets. Proposition 6 establishes that US net foreign liabilities accumulate over time, starting from $t \geqslant 1$. Figure 3 shows this kind of pattern. In this respect, the contribution of this paper is to explain why the US net foreign assets position turned negative *before* the emergence of global imbalances.

Finally, we show that the data validate the hypothesis that there is an excess of savings in the world economy. Let us focus on the key equation of the model (equation (25)). The model requires that the interest rate is below the growth rate of the economy for the developed country to run a trade deficit.

The first variable of interest, the real interest rate, is the most controversial because the marginal product of capital and the interest rate in the international bond market are indistinguishable in the model. Figure 3 shows that the negative investment position of the US is due to net external debt (private and public), which has steadily increased and reached 40% of GDP in 2007. As it is known, the difference between NFA and net external debt is due to FDI and equity holdings, which tend to be positive for the US.

Because foreign lenders accumulate safe US assets, we take the rate of interest on the US government bonds at different maturities as a proxy for the real interest rate. Figure 4 indicates that while interest rates were quite high in the early 1980s, they have embarked on a negative trend since then.

As far as the growth part is concerned, we only allowed for population growth so far. Let us consider labor-augmenting technological progress and assume that technology grows at a common rate $g$ in the two countries[12]. We show in the Appendix that equation (25) becomes:

$$\hat{tb}_i^* \approx -\hat{z}_i^* \frac{n+g-r^*}{1+n+g} \tag{27}$$

where the hat denotes variables per effective worker. We now take $g = 0.03$ as the (conservative) growth rate of technological progress for the world economy (similarly to Caballero et al.) and $n = 0.01$ as the population growth rate (see footnote 3). Figure 4 reveals that real interest rates have been far below the combined growth rate of 4% since the 1990s. The gap between the two has particularly widened during the last decade, which saw the emergence of global imbalances.

We can conclude that there is evidence that the United States have accumulated a trade deficit because a higher saving rate in China (due to the absence of a pay-as-you-go system) has been pushing the real interest rate below the long-run growth rate of the world economy.

A final word is due about dynamic inefficiency. In our setup, we assume that the US economy was at the golden rule before integrating with inefficient (emerging) countries. We have shown that the consequence is that the integrated economy is overaccumulating capital. Part of the literature is of the view that the capital overaccumulation case is only of theoretical interest because actual economies are not dynamically inefficient (see [8] for a discussion, p. 84). These statements are often based on early tests on the dynamic efficiency of stochastic OLG economies. However, Chattopadhyay [7] has recently shown that a widely used criterion to test dynamic efficiency, the net dividend criterion, does not actually give sufficient conditions for optimality. While we are far from having an empirically implementable test, the results of this paper emphasize that the capital overaccumulation case cannot be ignored since it has something to tell us on relevant stylized facts such as global imbalances.

### 4.1 Country size

In this section, we show that country size has an impact on capital flows and current account dynamics.

First, we establish that the steady state capital stock of the world economy is increasing in country 2's size.

**Proposition 8** *Let $k^*_{\rho_2,\rho_1}$ and $k^*_{\tilde{\rho}_2,\rho_1}$ be the steady state capital stocks of two economies, for which $\tilde{\rho}_2 > \rho_2$. Then, $k^*_{\rho_2,\rho_1} < k^*_{\tilde{\rho}_2,\rho_1}$.*

**Proof.** The logic of the proof is the same as for Proposition 4. Consider equation (31) in the Appendix for the economy in which $\rho_2$ is country 2's size. In Lemma 1(ii), we have proved that there exists a stable $k^*_{\rho_2,\rho_1}$ such that $g(k^*_{\rho_2,\rho_1}) = 0$. Now consider another economy with $\tilde{\rho}_2$ as country 2's size, for which we study the function $\tilde{g}$. It is straightforward that if $k_{\tilde{\rho}_2,\rho_1} = k_{\rho_2,\rho_1}$, then

[12] As Gourinchas and Jeanne [16] observe, "that countries have the same growth rate in the long run is a standard assumption, often justified by the fact that no country should have a share of world GDP converging to 0 or 100 percent.". The same would occur in this model in the long-run.

$\tilde{g}(k_{\rho_2,\rho_1}) < 0$. Proposition 4 already showed that the function $g$ is increasing in $k$ if the steady state is stable. Hence, it must be true that $k^*_{\tilde{\rho}_2,\rho_1} > k_{\rho_2,\rho_1}$ for $\tilde{g}(k^*_{\tilde{\rho}_2,\rho_1}) = 0$. ■

This result shows that capital overaccumulation in the world economy is intensified if country 2 has a bigger size. The implications for trade are the following. First, the higher is $\rho_2$ the larger is $z^*_{1,t}$ or country 1's net foreign assets per capita (equation 19). Together with the fact that $\frac{n-r^*}{1+n}$ is also bigger, global imbalances are also larger (equations 25).

It might be argued that $\hat{\rho}_2$ is a better measure for country size (see Appendix 6.2). Under technological progress, country $i$'s share of world savings depends on country $i$'s share of total labour productivity, as well as on population size. While China has a bigger population, it is also true that its income per capita is lower than the US. A simple way to account for the fact that China is poorer than the US is to assume that $A_{2,0} < A_{1,0}$[13]. As a matter of fact, the "productivity gap" compensates for China's bigger population. Using the fact that $\frac{Y^*_t}{L_t A_t} \equiv \hat{y}^*_t = \hat{y}^*_{i,t} \equiv \frac{Y^*_{i,t}}{L_{i,t} A_{i,t}}$, we can rewrite $\hat{\rho}_i$ as follows:

$$\hat{\rho}_i = \frac{L_{i,t} A_{i,t}}{L_t A_t} = \frac{Y^*_{i,t}}{Y^*_t}$$

We compute East Asian countries' share of total GDP, where total GDP is computed as the sum of the US and East Asian countries GDP[14]. The model cannot account for the fact that East Asia's share has increased over time due to its spectacular economic growth, from 21% in 1980 to 50% in 2010, because $\hat{\rho}_2$ is constant in the model[15]. A constant $\hat{\rho}_i$ is in fact the consequence of assuming identical growth rates for the two countries[16]. Yet, Proposition 8 can explain why capital flows and current account imbalances towards East Asian countries have a huge impact on the US economy: if China was a small country, the US current account deficit and net foreign asset liabilities would be negligible.

# 5 Conclusions and policy implications

This paper takes seriously Bernanke's hypothesis that global imbalances might be due to a global saving glut. We have constructed a model in which a global excess of savings arises because of the financial integration between the United States and dynamically inefficient economies, which have a higher propensity to save than the US because they do not have a pay-as-you-go pension system. The increase in world savings had as long-run effects the drop of real interest rates and the emergence of global imbalances. These and other empirical evidences can be read through the lens of this model.

[13] We thank Antonia Díaz and Timothy Kehoe for having raised this point.

[14] In particular, East Asian countries include China, Taiwan, South Korea, Hong Kong and Singapore. We take the countries' PPP-converted GDP, at current prices from Heston A., Summers R., Aten B., Penn World Table Version 7.1, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, July 2012.

[15] In the Penn World Tables, there are two sets of data for China due to measurement problems. The above numbers are for China's version 2. For China version 1, the shares would be 15% in 1980 and 48% in 2010.

[16] See footnote 12 for a comment on this.

The model indicates that both the current direction of trade and the low real interest rates are signals that the world economy is on an inefficient path. If that was not the case, United States' current account should be zero or in surplus and we should also observe much higher interest rates. Pension reforms in China in the direction of introducing a pay-as-you-go system would increase domestic demand and therefore reduce world savings. The US deficit towards China would shrink, which is the outcome that many politicians and economists seem to hope for.

This paper clearly abstracts from two important, possibly related, facts: (1) China has a higher investment ratio; (2) while the United States have a negative net international position overall, they have a positive position in foreign direct investments. The key step to understand these facts could be to introduce more assets, which would require the introduction of uncertainty in the model. We leave this for future research.

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# 6 Appendix

## 6.1 Proofs

*Proof of Lemma 1*

(*i*) Take equation (16) for any $t \geqslant 1$ and define the function $g$ as follows:

$$\begin{aligned} g(k_{t+1}; k_t, \tau_1, \rho_1, \rho_2) &\equiv (1+n)k_{t+1} - [\rho_1 s_1(f(k_t) - f'(k_t)k_t, f'(k_{t+1}), \tau_1) + \\ &+ \rho_2 s_2(f(k_t) - f'(k_t)k_t, f'(k_{t+1}))] \end{aligned}$$

We want to establish the existence of $k_{t+1} > 0$ given $k_t > 0$, such that $g(k_{t+1}; k_t, \tau_1, \rho_1, \rho_2) = 0$. To do that, we study the sign of $g$ as $k_{t+1}$ tends to infinity and zero. The first limit tells us that $g$ is positive for $k_{t+1}$ approaching infinity:

$$\lim_{k_{t+1} \to +\infty} g(k_{t+1}; k_t, \tau_1, \rho_1, \rho_2) = +\infty \tag{28}$$

(savings are always bounded above by $w_t$). Therefore, for at least a $k_{t+1} > 0$ to exist we need:

$$\lim_{k_{t+1} \to 0} g(k_{t+1}; k_t, \tau_1, \rho_1, \rho_2) < 0 \tag{29}$$

When $\rho_1 = 1$ (closed economy), [8] show that it is enough that the young's income after tax is strictly positive for savings to be positive, as savings are increasing in income. In particular, the following condition must hold : $w_t > \tau_1$. It turns out that the same condition is valid in a two-country economy. It is not sufficient that aggregate savings are positive, since we only allow for strictly positive consumption. Therefore, for an equilibrium to exist we need both countries' savings to be positive.

Now, define $\bar{\tau}_1(k_t)$ as the level of tax for which savings are zero in country 1 (it is obvious that $\bar{\tau}_1$ is increasing in $k_t$). Therefore, as long as $\tau_1 < \bar{\tau}_1(k_t)$, equation (29) is satisfied and therefore $k_{t+1}$ exists.
We now prove that $k_{t+1}$ is unique given $k_t$. By Assumption 1, $g$ is increasing in $k_{t+1}$:

$$g'(k_{t+1}) = 1 + n - s_r f''(k_{t+1}) > 0 \qquad \forall\ k_{t+1}$$

This is enough to ensure uniqueness. We can then write

$$k_{t+1} = \phi(k_t; \tau_1, \rho_1, \rho_2)$$

which is a single-valued, strictly increasing function in $k_t$[17].

The above discussion is also valid at $t = 0$. It follows that if $\tau_1 < \bar{\tau}_1(k_{1,0})$ at time 0, $k_1 > 0$ exists given $(k_{1,0}, k_{2,0})$ and is unique. A unique intertemporal equilibrium will exist by induction.

(*ii*) We know already that the saving locus of the economy is increasing. Suppose that

$$\lim_{k_t \to 0} \frac{\phi(k_t; \tau_1, \rho_1, \rho_2)}{k_t} > 1$$

[17] See [13] for a throughout study of the function $\phi$.

For the saving locus to cross the 45 degree line from above at least once, we need to show that the following is true:

$$\lim_{k_t \to +\infty} \frac{\phi(k_t; \tau_1, \rho_1, \rho_2)}{k_t} < 1 \tag{30}$$

The argument is the same as for closed economies and relies on the fact that savings can never exceed the wage (see [2], p. 84). Since

$$(1+n)k_{t+1} = \rho_1 s_{1,t} + \rho_2 s_{2,t} \leqslant w_t$$

that condition (30) is satisfied can be shown by dividing both sides of the inequality by $k_t$ and then taking the limit:

$$\lim_{k_t \to +\infty} \left[ \frac{\phi(k_t; \tau_1, \rho_1, \rho_2)}{k_t} \right] \leqslant \frac{1}{1+n} \lim_{k_t \to +\infty} \left[ \frac{f(k_t)}{k_t} - f'(k_t) \right] = 0$$

This proves the existence of at least one locally stable steady state.

*Proof of Lemma 2*

Consider the budget constraints of the agents born at $t$ at equilibrium:

$$\begin{aligned} c_{1,t}^{t*} + \frac{c_{1,t+1}^{t*}}{1+r_{t+1}^*} &= w_t^* - \tau_1 \frac{r_{t+1}^* - n}{1+r_{t+1}^*} \equiv I_{1,t}^* \\ c_{2,t}^{t*} + \frac{c_{2,t+1}^{t*}}{1+r_{t+1}^*} &= w_t^* \equiv I_{2,t}^* \end{aligned}$$

It is easy to see that the two agents will always have different budget sets, except in the case $r^* = n$ where $I_{1,t}^* = I_{2,t}^*$. Iff $n > r_{t+1}^*$, $I_{1,t}^* > I_{2,t}^*$ . Because of that, note that the budget line of agent 1 is to the right of agent 2's budget line. It is parallel as they face the same interest rate $r_{t+1}^*$. Marginal rates of substitutions of the two agents are obviously equalized:

$$1 + r_{t+1}^* = \frac{u'(c_{1,t}^{t*})}{\beta v'(c_{1,t+1}^{t*})} = \frac{u'(c_{2,t}^{t*})}{\beta v'(c_{2,t+1}^{t*})}$$

Because utility functions are identical across agents and consumption goods are normal, we can conclude that $c_{1,t}^{t*} > c_{2,t}^{t*}$ and $c_{1,t+1}^{t*} > c_{2,t+1}^{t*}$. If $r_{t+1}^* > n$, the opposite is true.

*Proof of Proposition 4*

Let $k_2^{aut}$ be the level of capital such that country 2 is at the autarkic steady state, and define the function $g_2$ as follows:

$$g_2(k_2^{aut}) \equiv (1+n)k_2^{aut} - s_2(f(k_2^{aut}) - f'(k_2^{aut})k_2^{aut}, f'(k_2^{aut})) = 0$$

where

$$g_2'(k_2^{aut}) = 1 + n + s_w f''(k_2^{aut})k_2^{aut} - s_r f''(k_2^{aut})$$

When the steady state is stable, $g_2'(k_2^{aut}) > 0$ as

$$\frac{dk_{2,t+1}}{dk_{2,t}}(k_2^{aut}) = \frac{-f''(k_2^{aut})k_2^{aut} s_w}{1 + n - s_r f''(k_2^{aut})} < 1$$

Similarly, let $k^*$ be the steady state world capital stock and define the function $g$ for the world economy:

$$\begin{aligned} g(k^*;\tau_1,\rho_1,\rho_2) &\equiv (1+n)k^* - [\rho_1 s_1(f(k^*)-f'(k^*)k^*, f'(k^*),\tau_1) + \\ &+ \rho_2 s_2(f(k^*)-f'(k^*)k^*, f'(k^*))] = 0 \end{aligned} \tag{31}$$

Now suppose that $k^* = k_2^{aut}$. From equation (11), we know that country 1 saves less than country 2 for any $k$, then $g(k_2^{aut};\tau_1,\rho_1,\rho_2) > 0$. Note that $g'(k_2^{aut};\tau_1,\rho_1,\rho_2) = g_2'(k_2^{aut})$, and therefore for $g$ to be zero $k$ must fall. It follows that $k^* < k_2^{aut}$.

Similarly, it can be shown that $k_1^{autss} < k^*$. Diminishing returns to capital implies that $r_1^{autss} > r^* > r_2^{aut}$.

*Proof of Proposition 5*

(*i*) At $t = 0$, the world capital market clears if the following equation holds:

$$\begin{aligned} (1+n)k_1^* &= \rho_1 s_1(f(k_{1,0})-f'(k_{1,0})k_{1,0}, f'(k_1^*),\tau_1) + \\ &+ s_2(f(k_{2,0})-f'(k_{2,0})k_{2,0}, f'(k_1^*)) \end{aligned}$$

Under Hypothesis 2, $s_{1,0}^* > s_{2,0}^*$. By Proposition 1, it follows that:

$$z_{1,0} < 0 \qquad\qquad z_{2,0} > 0$$

Because of no trade in the previous period, the countries' trade balances will only reflect the current trade in the capital market: $tb_{i,0} = -z_{i,0}$. Hence:

$$tb_{1,0} > 0 \qquad\qquad tb_{2,0} < 0$$

(*ii*) Let us write the balance of trade of country 1 at $t = 1$:

$$tb_{1,1}^* = -z_{1,1}^* + z_{1,0}^* \frac{1+r_1^*}{1+n}$$

Because $z_{1,1}^* > 0$ (Proposition 1) and we have shown that $z_{1,0}^* < 0$, then $tb_{1,1}^* < 0$.

### 6.2 Technological progress

The aim of this section is to show how to get the condition for country 1 to run a trade deficit in the long-run under labour-augmenting technological progress (equation (27)). Under this assumption, the production function is still homogeneous of degree one in the two arguments:

$$Y_{i,t} = F(K_{i,t}, A_{i,t}L_{i,t}) \qquad\qquad A_{i,t} = (1+g)A_{i,t-1}$$

where, in principle, $A_{1,0} \neq A_{2,0}$.

We define $\hat{k}_{i,t} \equiv \frac{K_{i,t}}{A_{i,t}L_{i,t}}$ as capital per effective worker. The first-order conditions of the firms now become:

$$\begin{aligned} r_t &= f'(\hat{k}_{i,t}) - \delta \\ \hat{w}_t &= f(\hat{k}_{i,t}) - f'(\hat{k}_{i,t})\hat{k}_{i,t} \end{aligned}$$

where $\hat{w}_t \equiv \frac{w_{i,t}}{A_{i,t}}$.

Taxes must grow at the same rate of technological progress, for the tax to have an impact on savings in the long-run: $\tau_{1,t} = (1+g)\tau_{1,t-1}$. At each $t$, because $L_{1,t}\tau_{1,t} = L_{1,t-1}b_{1,t}$ must hold, $b_{1,t} = \tau_{1,t}(1+n)$. Therefore, the budget constraints become:

$$\begin{aligned} c_{i,t}^t &= w_{i,t} - \tau_{i,t} - s_{i,t} \\ c_{i,t+1}^t &= s_{i,t}(1+r_{t+1}) + \tau_{i,t}(1+n)(1+g) \end{aligned}$$

where $\tau_2 = 0$. The market clearing condition for capital expressed in capital per effective worker becomes:

$$\hat{k}_{t+1}^*(1+n)(1+g) = \hat{\rho}_1\hat{s}_{1,t}^* + \hat{\rho}_2\hat{s}_{2,t}^*$$

where $\hat{\rho}_i \equiv \frac{L_{i,t}A_{i,t}}{L_tA_t}$. Following the same steps as in section 2.3, we derive the balance of trade per effective worker for country 1:

$$\hat{tb}_{1,t}^* \equiv [\hat{s}_{1,t}^* - (1+n)(1+g)\hat{k}_{t+1}^*] - \frac{1+r_t^*}{(1+n)(1+g)}[\hat{s}_{1,t-1}^* - \hat{k}_t^*(1+n)(1+g)]$$

which at the steady state simplifies as follows:

$$\hat{tb}_1^* = -\hat{z}_1^*\frac{(1+n)(1+g)-(1+r^*)}{(1+n)(1+g)} \approx -\hat{z}_1^*\frac{(n+g)-r^*}{1+n+g}$$

where $\hat{z}_1^* \equiv \frac{Z_{1,t}^*}{A_{1,t}L_{1,t}}$.

### 6.3 A Cobb-Douglas Example

In this section, we derive the model for Cobb-Douglas utility and production functions:

$$\begin{aligned} U(c_{i,t}^t, c_{i,t+1}^t) &= \beta \log c_{i,t}^t + (1-\beta)\log c_{i,t+1}^t \quad (32) \\ f(k_t) &= k_t^\alpha \quad (33) \end{aligned}$$

We can study this example in some detail as our variables of interest have a simpler dynamics with Cobb-Douglas functions.

From profit maximization, the factor prices are:

$$\begin{aligned} r_t &= \alpha k_t^{\alpha-1} - \delta \quad (34) \\ w_t &= (1-\alpha)k_t^\alpha \quad (35) \end{aligned}$$

The saving functions in the two countries are:

$$\begin{aligned} s_{1,t} &= (1-\beta)(w_t - \tau_1) - \beta\tau_1\frac{1+n}{1+r_{t+1}} \quad (36) \\ s_{2,t} &= (1-\beta)w_t \quad (37) \end{aligned}$$

It is known that, with log-utility, savings are a constant fraction of the wage and do not depend on the rate of interest. In country 1, the young also consume a fraction of the discounted future transfer.

Overall, the impact of the pay-as-you-go system on country 1's savings is:

$$\frac{\partial s_{1,t}}{\partial \tau_1} = -(1-\beta) - \beta\frac{1+n}{1+r_{t+1}} = -1 + \beta\frac{r_{t+1}-n}{1+r_{t+1}} \tag{38}$$

The market clearing equation for capital is:

$$\begin{aligned} K^*_{t+1} &= L_{1,t}\left[(1-\beta)((1-\alpha)k_t^{*\alpha} - \tau_1) - \beta\tau_1\frac{1+n}{1+\alpha k_{t+1}^{*\alpha-1}-\delta}\right] + \\ &+ L_{2,t}(1-\beta)(1-\alpha)k_t^{*\alpha} \end{aligned} \tag{39}$$

The capital stock evolves over time as follows:

$$(1+n)k^*_{t+1} = (1-\beta)(1-\alpha)k_t^{*\alpha} - \rho_1\tau_1\left[(1-\beta) + \frac{\beta(1+n)}{1+\alpha k_{t+1}^{*\alpha-1}-\delta}\right] \tag{40}$$

while the steady state capital stock satisfies:

$$(1+n)k^* = (1-\beta)(1-\alpha)k^{*\alpha} - \rho_1\tau_1\left[(1-\beta) + \frac{\beta(1+n)}{1+\alpha k^{*\alpha-1}-\delta}\right] \tag{41}$$

For any given $k_t > 0$, it can be verified that $k_{t+1} > 0$ exists as long as $(1-\alpha)k_t^\alpha - \tau_1 > 0$ (see Lemma 1) and that the higher is $\tau_1$, the lower $k_{t+1}$ will be given $k_t$. It can also be checked that the saving locus is increasing (here, $s_r = 0$):

$$\frac{dk_{t+1}}{dk_t} = \frac{(1-\beta)\alpha(1-\alpha)k_t^{\alpha-1}}{(1+n) - \frac{\rho_1\tau_1\beta(1+n)\alpha(\alpha-1)k_{t+1}^{\alpha-2}}{(1+\alpha k_{t+1}^{\alpha-1}-\delta)^2}} > 0 \tag{42}$$

The specific feature of this example is that the saving locus is concave as $\frac{d^2k_{t+1}}{(dk_t)^2} < 0$. However, note that the saving locus of the economy does not start at the origin as in the case $\tau_1 = 0$. In fact, $(k_t, k_{t+1}) = (0,0)$ does not satisfy equation (40). When $k_t = 0$, $k_{t+1}$ must be negative.

With $\tau_1 = 0$, it is known that there exists a globally unique steady state with Cobb Douglas utility and production function. With $\tau_1 > 0$, the number of steady states depends on how big is the tax. If the tax is small enough, then there are two steady states (one unstable and one stable). At a certain threshold for the tax, the steady state is not hyperbolic and above that we have non-existence of steady states. See [8] for a detailed discussion[18].

#### 6.3.1 Trade and Consumption

We can now compute the excess demand. For instance, for country 1:

$$z^*_{1,t} = \rho_2\tau_1\left[(1-\beta) + \frac{\beta(1+n)}{1+\alpha k_{t+1}^{*\alpha-1}-\delta}\right] \tag{43}$$

It can be verified that $\frac{\partial z_{1,t}}{\partial k_{t+1}} > 0$.

At the golden rule $k^{GR}$ and other stationary allocations, $z_1$ is respectively:

$$z_1^{GR} = \rho_2\tau_1 \tag{44}$$

$$z_1^* = \rho_2\tau_1\left[(1-\beta) + \frac{\beta(1+n)}{1+\alpha k^{*\alpha-1}-\delta}\right] \tag{45}$$

[18] They discuss a closed economy, but the substance of the argument does not change.

Using the capital flows definition (24), we can plug equation (43) in and compute the balance of trade of country 1:

$$tb^*_{1,t} = \rho_2\tau_1(1-\beta)\left[\frac{(\alpha k_t^{*\alpha-1}-\delta)-n}{1+n}\right] + \rho_2\tau_1\beta\left[\frac{(\alpha k_{t+1}^{*\alpha-1}-\delta)-n}{1+\alpha k_{t+1}^{*\alpha-1}-\delta}\right] \quad (46)$$

When both interest rates are bigger than the population growth rate, it is evident that $tb^*_{1,t} > 0$. Suppose now at a given $\bar{t}$, $k^*_{\bar{t}}$ and $k^*_{\bar{t}+1}$ are such that $r^*_{\bar{t}} > n$ and $r^*_{\bar{t}+1} < n$. The first part of the equation is positive and reflects the fact that the old in country 2 are consuming more (exports). But part two is negative as the young in country 2 are now consuming less (imports). It is now clear that which of the two is bigger will also depend on $\beta$.

In the long-run, the balance of trade satisfies:

$$tb^*_1 = \frac{(\alpha k^{*\alpha-1}-\delta)-n}{1+n}\rho_2\tau_1\left[(1-\beta)+\frac{\beta(1+n)}{1+\alpha k^{*\alpha-1}-\delta}\right] \quad (47)$$

The two representative agents' consumption obeys:

$$c^{t*}_{1,t} = \beta\left[(1-\alpha)k_t^{\alpha*} - \tau_1\frac{(\alpha k_{t+1}^{\alpha-1*}-\delta)-n}{1+\alpha k_{t+1}^{\alpha-1*}-\delta}\right] \quad (48)$$

$$c^{t*}_{1,t+1} = (1+\alpha k_{t+1}^{\alpha-1*}-\delta)(1-\beta)\left[(1-\alpha)k_t^{\alpha*} - \tau_1\frac{(\alpha k_{t+1}^{\alpha-1*}-\delta)-n}{1+\alpha k_{t+1}^{\alpha-1*}-\delta}\right] \quad (49)$$

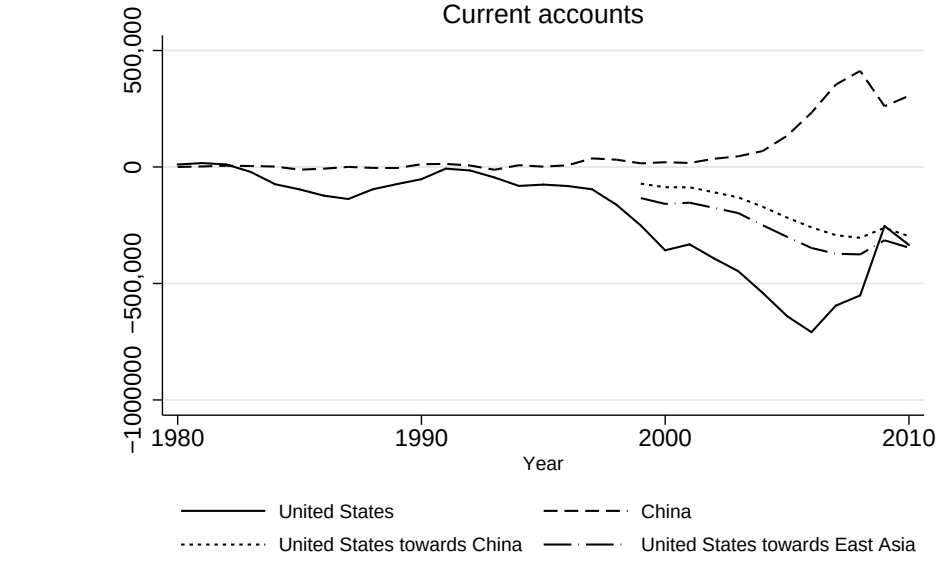
$$c^{t*}_{2,t} = \beta(1-\alpha)k_t^{\alpha*} \quad (50)$$

$$c^{t*}_{2,t+1} = (1+\alpha k_{t+1}^{\alpha-1*}-\delta)(1-\beta)(1-\alpha)k_t^{\alpha*} \quad (51)$$

As we established in Lemma 2, agents born in country 1 consumes more (less) when the world economy happens to be beyond (below) the golden rule allocation.

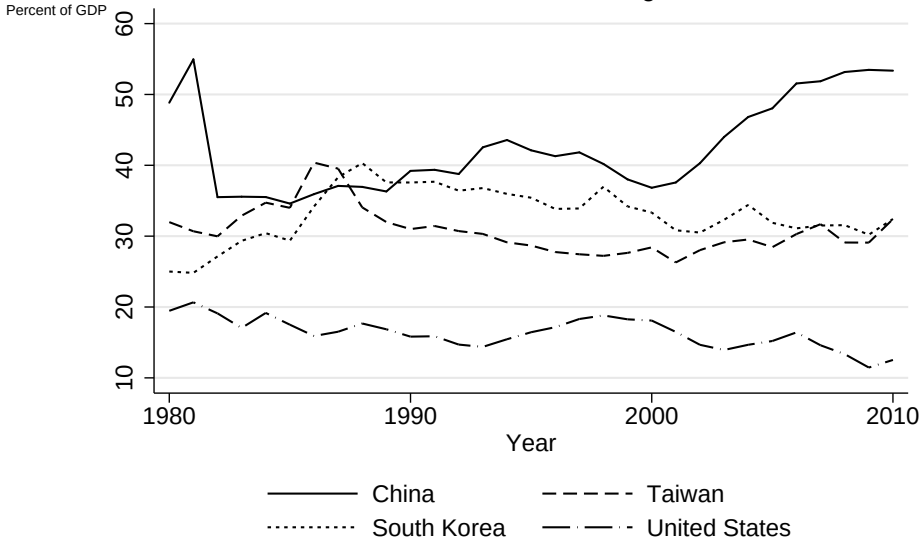
# 7 Figures

FIGURE 1
Current accounts



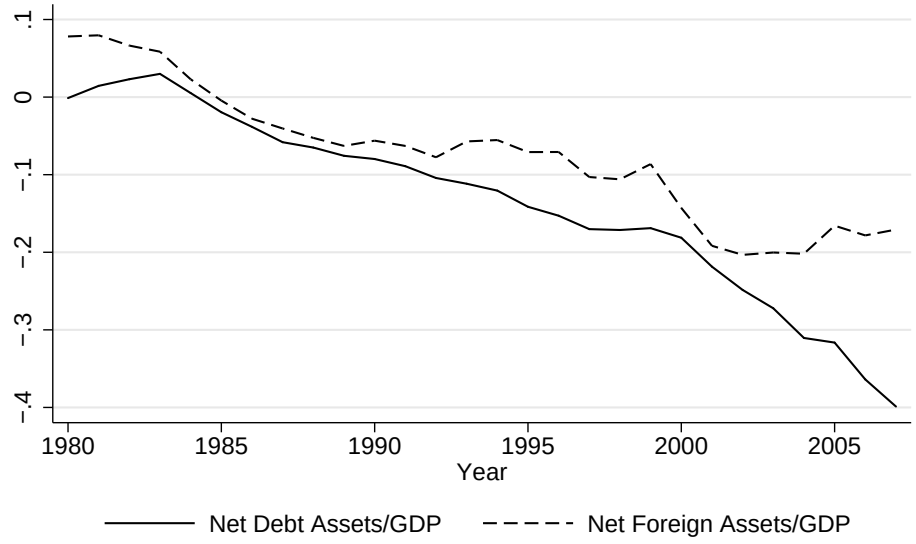

Sources: Bureau of Economic Analysis (US); World Economic Outlook database (IMF).
Notes: The category 'East Asia' includes Taiwan, South Korea, Other Asia and Pacific (BEA definition), as well as China.

FIGURE 2
Gross National Savings



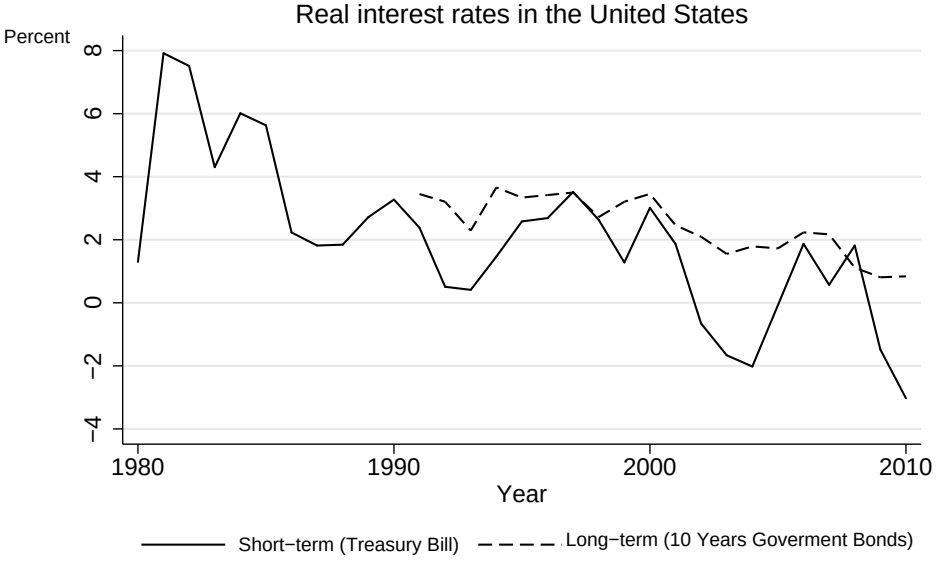

Source: World Economic Outlook database (IMF).

FIGURE 3
United States' Net International Position




Sources: Lane and Milesi−Ferretti's database (updated to 2007).

FIGURE 4
Real interest rates in the United States




Notes: IFS data, yearly rates. The Treasury Bill rate is adjusted for the actual CPI.
The long−term yield is adjusted for expected inflation from the Survey of Professional Forecasters as in Caballero et al. (2008).