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Abstract

US males labour force behaviour shows lifecycle effects. We develop a lifecycle model of individual labour supply, with a single financial asset and non labour income. With widely used preferences, we derive the analytical form of the value function and optimal labour participation for any period, t. Consumption and savings switches its form as participation changes.

A spell of part time work has strong implications for earlier decisions on participation, consumption, savings and the marginal value of leisure and wealth. We apply our framework to explain the increasing prevalence of non standard retirement noted in the literature.

JEL classification: J22, J26.

Keywords: Lifecycle, Labour supply decision, Retirement, Unretirement.

1 Introduction.

In a life cycle context individuals must determine their intertemporal labour supply, consumption and savings. Here in any period an individual can choose to have zero hours of work, full time hours or, with suitable assumptions on labour demand, anything in between. Changes in participation pattern between periods can correspond to major life events like leaving education to enter employment, or retiring from the labour force. But there may also be shorter term, repeated switches over time in participation, e.g. entering part time work after the birth of a child. From the PSID there is strong evidence of life cycle effects in labour participation and hours worked, and in assets. This is so for females but also for males, even though they are less affected by fertility issues.

Analysing intertemporal choices is difficult because of the curse of dimensionality which prevents closed form solution of many or even most examples.

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If decision variables are subject to inequality constraints then it is even more complex. A prime example is intertemporal labour supply where participation can be at corners. Faced with this difficulty many researchers use numerical solution/simulation methods to try to characterise the life cycle profile of labour participation and associated decisions. For example Gustman & Steinmeier (2002) and Blau (2012) use a lifecycle model with time additive discounted preferences in which utility per period depends on a single consumption good and leisure. The per period utility function is isoelastic in consumption and quasilinear in leisure. Each period there is a time endowment which can be allocated between paid work and leisure. There is a single one period financial asset in which the individual may borrow or save to any extent desired, a wage rate per unit of work and perfect foresight.[1] A crucial aspect is that in a life cycle context the optimal participation state now depends on current assets and market conditions and also on all planned future participation states, which in turn are determined by future market conditions and preferences. The decision tree becomes formidable. For this reason there is a tendency to use simulation methods, which can represent complicated problems but at the cost of being dependent on numerical details of preferences, wage rates, etc.

The main contribution of this paper is to derive the closed form solution of this widely used model under the assumption of perfect foresight, which is also commonly made in this literature. We show that the value function at t has branches which are isoelastic in assets and other branches which are linear in assets. Which branch is optimal at t (and hence period t optimal decisions) depends in general on which branch is optimal at $t+1$ and hence on the entire future. If in any period of time t it is optimal to work part time then at all earlier periods the value function is linear in assets. This then implies that in any period prior to the last period in which part time work is optimal, the labour participation decision depends only on the wage rates, marginal utility of leisure and the elasticity of marginal utility of adjacent periods. On the other hand in any period subsequent to the latest period of part time work, labour supply is either zero or full time work and the decision depends on the whole remaining future. Given the value function, the consumption and savings functions follow. The combination of constant elasticity of consumption and quasilinearity of leisure of current period utility leads to a consumption function which is independent of wealth in periods for which the future value function is linear in wealth, or a consumption function which is linear in wealth when the future value function is isoelastic. Theoretically this is quite intuitive but it also suggests a useful empirical strategy for estimating parameters in a life cycle framework jointly studying the consumption function and intertemporal labour participation.

A useful device is to think of the optimal lifetime plan as being the union of a succession of epochs. Within an epoch the individual has a common participation state (full time, part time or zero work hours) and moving from one epoch to another corresponds to a change in participation status over time. We

[1] Blau (2012) considers uncertainty.

analyse the behaviour within and between epochs.

In this model we find that the presence of an epoch of part-time work is critical in breaking the curse of dimensionality. The role of part time work has strong implications for the form of the value function, the labour force participation decision and the marginal utility of leisure and wealth. These in turn impact on values of optimal consumption and savings. In fact prior to the final epoch of part time work, only preference and market conditions in adjacent periods matter in determining the participation decision, and so prior to this final part time epoch there is no curse of dimensionality. We also find that in any period of part time work (except if it occurs at periods characterising a change in the labour force regime or the final period), hours of work and savings are indeterminate. The individual is indifferent between lower current savings and higher leisure or higher current savings and lower current leisure.

Within our model part time work has a different economic significance to corner labour participations of full time or zero work. If part time work is optimal in a period the current period marginal utility of income (as defined by the ratio of the current marginal utility of consumption to the current wage) is equal to the expected marginal value of future wealth. Moreover if there are two adjacent periods of optimal part time work then the change in the current marginal utility of leisure and wage matches the change in the marginal value of future wealth but the former is much easier to observe and model than the latter. By contrast the corner solutions generate inequalities between the current and future marginal values of income/wealth which are harder to use empirically and less informative on how future values impact on current values and decisions.

More generally the empirical importance of flexible working practices, such as part time hours of work in recent decades has increased, particularly with the steady rise of the female labour force participation rate see Fernandez (2011). Part time work has become common practice in the majority of labour markets, approximately one in five workers in the US in 1999 were engaged in part time work, whilst in the Netherlands 38% (and 69% of women) in the workforce are engaged in part time work Houseman and Osawa (2003) Kalleberg (2000). Part time work has also become increasingly popular in smoothing the transition from full time work to full retirement, via partial retirement or a bridge job and also in terms of unretirement jobs and partial retirement (see Gustman and Steinmeier (1984,1986)). Gustman and Steinmeier (2002) and Blau (2012) investigate the labour participation of individuals, especially elderly individuals in a life cycle setting. The questions they investigate concern the response to private and public pension provisions and also the return to work decision of previously retired individuals.[2] Maestas (2010) suggests that a planned return to work post initial retirement, i.e. unretirement could be due to foreseen changes in preferences or market opportunities or could result from unplanned shocks in either of these. Kanabar (2012) looks at the specific case of unretirement in England, his findings suggest unretirement could be due to spouse, age effects

[2] For earlier studies which document unretirement or reverse flows see Rust (1989), Gustman and Steinmeier (1984, 1986).

and also unanticipated financial shocks.

The plan of the paper is as follows: section 1 presents a static illustration of key economic variables using the 2007 wave of the Panel Survey of Income Dynamics (PSID), section 2 states the general and terminal period form of the value function coming from choice of optimal labour participation, consumption and savings at each period. Section 3 characterises the labour market participation conditions which must hold within an epoch, and the nature of optimal consumption, savings and hours of work. In section 4 we show that prior to the final epoch of part-time work (if there are any) the value function is linear in assets whereas in the remaining later epochs it is isoelastic in assets. Depending on how the utility of leisure and economic variables like interest rates, wage rates and non-labour income vary over time, the patterns and lengths of epochs through time may vary widely. In section 5 we show what our framework implies for a variety of non standard retirement paths, including partial retirement and unretirement. Section 6 concludes.

2 US Male Life Cycle Labour Participation

Over the lifecycle individuals usually spend a period of time in education before subsequently entering the labour market, the amount of education consumed will likely affect the potential wage, length of time spent in the labour market, and also the amount of income which can be saved or invested in the form of assets generating future non labour income. In addition to education, heterogeneity in labour market behaviour may stem from a variety of sociodemographic, economic and institutional factors, causing individuals to differ in life cycle labour market participation. We document the lifecycle and cross section characteristics of key economic variables such as labour supply, wages, non labour income and family assets for all Head Of Household (HOH) males by their highest education level, using information from the 2007 PSID cross year index.[3] We cross tabulate each variable of interest against hours spent in paid work per week ($N = 8704$), details of sample construction and variable definitions can be found in appendix A.1.[4]

Figure 1 compares the hours spent in paid work per week versus family assets. Like Erosa et al (2010) we find a hump shaped curve in the hours worked over the lifecycle, college students tend to work lower hours initially whilst they combine work and study at younger ages, and then engage in career occupations which require higher average weekly hours in work compared to their lesser educated counterparts. Interestingly, our findings indicate a kink in the hours worked (for those in work) in later life. Such heterogeneity in the number of hours worked in later life, and more generally the fact that non standard retirement

[3] We include all family members who have subsequently left home. We do not present results for females however they are very similar to that of males, with the exception of a short dip in the hours profile due to maternity leave, results are available upon request.

[4] Note that these may not be identical samples. In each case we keep only men and create age groups for each variable of interest, we then plot the mean value for each group.

paths have become commonplace in the US since the 1970's (see Gustman and Steinmeier (1984), Rust (1989) and more recently Maestas (2010)), suggests individual responses and the precise definition of retirement which traditionally involved leisure or perhaps voluntary work, should be adjusted to include paid work post retirement. So called unretirement could be due to preference factors or unanticipated shocks to assets (Maestas (2010), Kanabar (2012)), in our model ceterus paribus a drop in assets or fall in the marginal value of leisure would serve to the increase the number of hours in work.

Indeed it is quite clear from figure 1 that as assets increase over the lifecycle, perhaps due to accumulation of savings from labour income, the average number of hours spent in work declines. As noted, in later life (for individuals between the ages of 66-70 and 71-75), assets tend to decrease whilst the number of hours for those in work increases, ceterus paribus exactly as our model would predict. Figure 1 also highlights the average difference in family assets by educational group, at younger ages families where the male HOH has a college education are slightly more wealthy than their high school or below high school counterparts, and this difference increases substantially over the lifecycle.

Figure 1 about here.

Figure 2 documents the hours spent in work for the HOH versus the HOH annual non labour income, by education level. We find that on average more educated individuals tend to start life with slightly higher annual non labour income which then increases substantially over the lifecycle, particularly between the ages of 30 and 65. Those who are less educated tend to have a flatter growth in their non labour income over the lifecycle, particularly those with a low level of education. Our model predicts that ceterus paribus as annual non labour income increases, individuals are more likely to take leisure. Indeed the shape of the curve representing the average non labour income for the 2007 cross section indicates that aside from the career periods, hours and non labour income tend to move in opposite directions as the model predicts, particularly in later life where those with below high school education tend, on average, to work more hours than their more educated counterparts.

Figure 2 about here.

Figure 3 compares weekly hours spent in the labour force versus the average hourly wage rate, by education level. We find that at younger ages there is little difference between the average reported hourly wage, irrespective of education level. However at each age the wage differential increases, particularly for those with a college education, and peaks when individuals are in their mid forties after which point it remains relatively flat. Those with the lowest wages tend to work more hours, especially given their asset and non labour income is relatively low. It is interesting to note even for the college educated, in later life as the average wage declines for those in work, average reported hours in work increases. Economic theory suggests there are income and substitution effects which arise from say a rise in the wage rate, but here this is couples with intertemporal effects. Within our model labour force participation level depends either on a comparison of the current and future or an expression denoting the contribution of the current wage to future value through raising savings and assets available next period (depending on the initial asset level). Therefore the relationship between participation and hours depends on the individual leisure preferences and wage rate they can obtain.

Figure 3 about here.

In this section we have described a cross section snapshot of the key economic variables, and also taken a lifecycle view on respondents average behaviour by education group. As one would expect, those with the highest education level tend to fare better in life. We have abstracted from various individual characteristics. For example individuals may be constrained in the number of hours they can supply to the market if they have care responsibilities, or health conditions. These types of effects may affect certain groups more than others. Nor have we plotted the debt characteristics by education level which may well differ. Indeed it could be the case that those with a higher earnings potentials are less credit constrained, and more willing to carry more debt today safe in the knowledge they will not default due to higher earnings in later life. Regardless, it is clear there are marked differences in the economic characteristics of male workers by their educational group, and in their lifecycle behaviour.[5] So a model which is consistent with theory and can yield explicit predictions of the life cycle pattern of labour supply will be empirically valuable. We develop this framework next.

3 The Framework and Value Function

A decision maker with known finite life T has a per period utility function which is isoelastic in consumption and quasilinear in leisure so the life cycle preferences are given by

$$U = \Sigma_t^T \delta^t [c_t^\alpha/\alpha + h_t L_t] \tag{1}$$

where c_t and L_t are respectively consumption and leisure of period t. T is the foreseen life and δ is the discount factor on preferences. There is a time endowment per period of 1 which can be allocated to work or leisure each period. Thus $0 \leq L_t \leq 1$. There is a perfect capital market with a single one period financial assets, the individual can borrow or save but cannot die in debt. The budget constraint each period is

$$A_{t+1} + c_t = r_t A_t + y_t + w_t(1 - L_t) \tag{2}$$

where A_t is the stock of financial assets at the start of the period, w_t is the wage rate per unit of work, a_t is the interest factor (1+ the interest rate) and y_t is

[5] Ideally we would have created a panel of male workers and followed them over their lifecycle and studied their behaviour and characteristics in more detail. However due to sample size and attrition effects over a large number of years, and given the main contribution of the paper is theoretical we choose not to pursue this course.

nonlabour income. The decision of the individual is then to choose a time path of consumption, net savings and leisure so as to

$$\max \Sigma_t \delta^t [\frac{c_t^\alpha}{\alpha} + h_t L_t] \tag{3}$$

$$\text{st } A_{t+1} + c_t = a_t A_t + y_t + w_t(1 - L_t)$$

$$0 \leq L_t \leq 1$$

$$A_0 \text{ given and } A_T = 0 \tag{4}$$

We start by finding the analytical value function for this problem, from which we can then deduce the consumption function, the labour participation status at each date and hours worked when these are determinate (see below). A main result is that at any time period t the value function characterising the maximum payoff for the individual is the higher of an isoelastic and a linear function:

Proposition 1 *The value function $v_t(A_t)$ at period t has the form*

$$v_t(A_t) = \max[P_t^i + M_t(N_t^i + Q_t A_t)^\alpha/\alpha, R_t^i + S_t^i A_t] \tag{5}$$

where the functions $P_t^i, N_t^i, R_t^i, S_t^i$ have alternative definitions according to current and future values of the discount rate, wage rate, non labour income and value of leisure which are denoted by the variables δ, w, y, h respectively. In particular there are two alternative forms for P_t^i, N_t^i ,$i = 0, 1$ and three alternative forms for $R_t^i, S_t^i, i = 0, I, 1$. We denote the accumulation of future interest factors by Q_t and denote the effects of future interest rates and time preference rates by M_t:[6]

$$Q_t = r_t Q_{t+1} = \Pi_t^T r_s \textit{ with } Q_{T+1} = 1, Q_t = r^{T+1-t} \textit{ if } r \textit{ is constant} \tag{6}$$

$$M_{T-t} = (\Sigma_{s=0}^t \delta^{(t-s)/(1-\alpha)} Q_{T-s+1}^{\alpha/(1-\alpha)}) \textit{ with } Q_{T+1} = 1 \tag{7}$$

This bears a family resemblance to Merton's (1971) seminal result that within the HARA class, the value function has the same functional form as the within period utility function. However it is more general given that within our framework an individual has two decision variables per period (c_t, L_t), one of which is inequality constrained, and the within period utility combines features of isoelasticity and quasilinearity. The linear branch of the value function is intimately connected with periods or phases of part time work, and involves different intertemporal tradeoffs to periods in which the value function is isoelastic.

There is a strong interpretation to the components of the value function. In the isoelastic case, N_t reflects the discounted value of future non-asset resources (non-labour income plus value of the time endowment for periods of work), discounted at the successive one period interest rates. P_t reflects the discounted

[6] We formally derive M in appendix A.2.

value of the stream of future leisure value in future periods of zero work. In the linear case R_t is a combination of $(y_t + w_t)'s$ and discounted future marginal rates of substitution (MRS) between leisure and consumption, S_t measures the discounted utility of leisure relative to the wage of one period t.

The key interest is in the optimal labour participation states over life and the associated consumption and savings paths. We describe these by piecing together different labour participation states, and note the forms of the value function to derive an overall optimal path of consumption, savings and leisure. We first show some fundamental links between the branches of the value function, and the optimal labour participation in any period.

Proposition 2 *(i) If part time work is optimal at t then whatever the form of the future value $v_{t+1,}$ the current value v_t is linear in current wealth*

(ii) If the future value v_{t+1} is linear in A_{t+1} then the current value v_t is also linear in A_t whatever the nature of current labour participation

(iii) If the future value is isoelastic and optimal current participation is at a corner then the current value is also isoelastic

To explain (i) the intuition is as follows. If at time t part time work is optimal and the future value function v_{t+1} is linear in future assets then the marginal utility of leisure at t,h_t, must equate to the marginal value of the wage w_t in raising future value S_{t+1}. Hence $\frac{h_t}{w_t}$ must equate to the marginal value of future wealth. This means that when part time work is optimal at t and the future value is linear in assets, future market conditions and leisure preferences do not affect the current marginal value of assets at t. In this sense once the value function for any period $t+1$ is linear in assets, then all earlier periods will also have a linear value function. And for all these earlier periods the choice of participation status will only depend on adjacent time period variables.

An important fact is that even if future value v_{t+1} is isoelastic in future assets A_{t+1}, if optimal current participation involves part time work, then the current value is linear in current assets A_t.This stems from the quasilinearity of utility in leisure. If part time work is optimal when v_{t+1} is isoelastic, the constant current marginal utility of leisure is equated to the future marginal value of wealth multiplied by the current wage. So the future marginal value of wealth is equated to h_t/w_t. Since $A_{t+1} = r_t A_t + y_t + w_t(1 - L_t)$ and the future marginal value of wealth is a power function of A_{t+1}, when part time work is optimal at t, current optimal leisure is a linear function of A_t. Optimal consumption equates the current marginal utility of consumption to the appropriately discounted marginal value of future wealth and so is independent of current wealth. Combining the leisure demand linear in wealth with consumption and savings being independent of current wealth yields a linear current value. Hence starting from a future value which is isoelastic, if optimal current participation involves part time work, then the current value becomes linear in A_t.[7]

[7] If the value function at $t+1$ is isoelastic and linear at t because at t part time work is optimal, then at t leisure and savings are determinate (for example in the last period), but if part time work is optimal at t and there is a linear value at $t+1$ then leisure and savings are indeterminate.

To explain (ii) if participation at period t is at a corner then within period consumption c_t is the only unknown affecting next periods assets, A_{t+1}. If we know the future value is linear in A_{t+1} then equating the current marginal utility of c_t to the discounted marginal value of wealth A_{t+1} gives an optimal level of c_t which is independent of current assets. Hence current assets A_t enter current value v_t only through their effect on the future value v_{t+1} which is linear in A_t it follows that v_t is linear in A_t.

The intuition behind (iii) is as follows, suppose that at t the future value v_{t+1} is isoelastic and optimal participation is at a corner, the current marginal utility of consumption is equated to the marginal future value of savings, both of which are isoelastic, in which case optimal consumption and savings conditional on participation are linear in current assets. Since current utility and the future value function are isoelastic respectively in consumption and assets carried forward, given that A_{t+1} is linear in current savings, this then implies that the current value is isoelastic in the starting assets A_t.Thus if future value is isoelastic and current participation is at a corner, then current value is isoelastic. The marginal value of wealth then varies with assets and so depends on the entire future profile of optimal decisions, preferences and market variables.[8]

There is a close connection between the branches of the value function and the labour participation status. It is helpful to think of an epoch as a sequence of adjacent time periods with the same choice of labour participation. Within an epoch the value function, consumption and savings functions will have the same form, but between epochs these shift between the branches of the value function. This device helps us piece together the different forms of lifetime behaviour that may be optimal, starting from the terminal period.

4 The terminal period.

In order to solve the individuals problem we start with the terminal period and use backward induction. There is no bequest motive in the model, thus $A_{T+1} = 0$. Terminal period utility is given by:

$$u_T = \frac{(rA_T + y_T + w_T(1 - L_T))^\alpha}{\alpha} + h_T L_T \tag{8}$$

Optimal leisure in the terminal period can be at a corner or interior. The individual will consume zero leisure (i.e. work 24 hours) if the marginal utility of leisure is below the wage rate multiplied by the marginal utility of consumption. Alternatively the individual will spend all their time in leisure if the marginal utility of leisure is above the wage rate multiplied by the marginal utility of consumption. If marginal utility of leisure lies between these two extremes then the individual is at an interior solution.

The saving decision at $T - 1$, is a choice variable and this in turn governs the optimal choice of leisure at T. The critical opening asset positions which govern the labour supply at T are:

[8] Proposition (i) and (ii) are derived in the text. We prove (iii) in appendix A.3.

$$A_T^0 = \frac{[(\frac{h_T}{w_T})^{1/(\alpha-1)} - w_T]}{r} \tag{9}$$

$$A_T^1 = \frac{(\frac{h_T}{w_T})^{1/(\alpha-1)}}{r} \tag{10}$$

If saving at $T-1$ results in $A_T < A_{0T}$ at T the individual works full time whereas if $A_T > A_{1T}$ the individual is retired at T. Notice that the only difference is the wage rate entering negatively in A_{0T} thus $A_{0T} < A_{1T}$.[9] Substituting in optimal leisure the terminal period value functions can be described as follows:

Proposition 3 *In the final period T*

(1) Optimally $L_T = 0$ and the value function has the form

$$v_T^0 = \frac{(y_T + w_T + r_T A_T)^\alpha}{\alpha} \textit{ if } \frac{h_T}{w_T} \le (y_T + w_T + r_T A_T)^{\alpha-1} \tag{11}$$

(2) Optimally $0 < L_T < 1$ and the value function has the form

$$v_T^I = (\frac{1}{\alpha} - 1)(\frac{h_T}{w_T})^{1/(\alpha-1)} + (\frac{h_T}{w_T})(y_T + r_T A_T + w_T)$$
$$\textit{if } w_T + y_T + r_T A_T > (\frac{h_T}{w_T})^{1/(\alpha-1)} > y_T + r_T A_T \tag{12}$$

(3) Optimally $L_T = 1$ and the value function has the form

$$v_T^1 = \frac{(y_T + r_T A_T)^\alpha}{\alpha} + h_T \textit{ if } (\frac{h_T}{w_T})^{1/(\alpha-1)} \le y_T + r_T A_T \tag{13}$$

In each of the three cases note the importance of the critical level of assets carried forward relative to the marginal rate of substitution between leisure and the wage rate in the current decision period. The intuition is that in the final period labour supply choices use just the one period comparison of the real wage with the $MRS_{h,w}$, since there is no future . From equations 11-13 it is clear assets enter the value function in either a power (at a corner) or linear form (interior).

$$v_T = \max \frac{[P_T^i + M_T(N_T^i + Q_T A_{Tt})^\alpha}{\alpha}, R_T + S_T A_T],\ i = 0, I, 1 \tag{14}$$

where

$$\begin{aligned}
P_T^1 &= h_T, N_T^1 = y_T, \\
P_T^0 &= 0, N_T^2 = y_T + w_T \\
Q_T &= r_T, M_T = 1 \\
R_T &= h_T(1 + \frac{y_T}{w_T}) + (\frac{1}{\alpha} - 1)(\frac{h_T}{w_T})^{\alpha/(\alpha-1)}, S_T = r_T \frac{h_T}{w_T}
\end{aligned}$$

[9] Derivations for terminal period assets and labour supply can be found in the appendix A.4.

Here the superscript i refers to the two cases $L_T = 0$ and $L_T = 1$.This Proposition gives us the final position of the individual, working backwards we can then analyse behaviour within the final epoch terminating in v_T^i. In the next section we show the form and components of the value function for particular types of epochs which can arise, depending on which one of the two possible forms the value function takes in the future adjacent epoch.[10]

5 Epochs

We can think of epochs as successive periods of an identical labour participation state, and then find the optimal lifetime path by piecing together the optimal sequence of epochs. Within an epoch, for each period assets at the start of the period must be at a level which makes continuing the current labour participation status optimal, and, between epochs, assets at the start of the first period of the subsequent epoch determine the optimal change in participation behaviour in that period and in the new epoch.

5.1 Epochs Preceding an Epoch with a Power Function Value Function

Suppose the epoch stretches from periods t_2 to $t_1 - 1$ so that $t_1 - 1$ is the last period within the epoch and the value function at t_1 is $v_{t_1} = \frac{P_{t_1}^i + M_{t_1}(N_{t_1}^i + Q_{t_1} A_{t_1})^\alpha}{\alpha}, i = 0, I, 1$. The function $P_{t_1}^i$ measures the value arising from discounted future leisure time, whilst the function N_t^i measure the appropriately discounted values of future non-financial income.[11]

5.1.1 (i) Full Time Work t_2 to $t_1 - 1$

In this case during the epoch there is no leisure to cumulate in to values within the epoch, so the only impact of leisure on the value function within the epoch is through the discounted value of leisure which arises in future epochs. This gives the term P_s^0 at each period $s = t_2..t_1 - 1$ prior to the last in the epoch. On the other hand with full time work at every period within the epoch, nonfinancial income within the epoch is $w_t + y_t$ each period and the function N_s^0 cumulates this effect through the epoch.

$$N_s^0 = \Sigma_{\tau=s}^{t_1} Q_{\tau+1}(w_\tau + y_\tau) + N_{t_1}^i, s = t_2..t_1 - 1 \quad (15)$$

$$P_s^0 = \delta^{t_1 - s} P_{t_1}^i, s = t_2..t_1 - 1 \quad (16)$$

Period to period within this epoch, full time work must be optimal. That is the marginal value of current leisure must be no greater than the contribution

[10] We derive the value functions for each labour force state in the terminal period in appendix A4.

[11] We derive the expressions for value functions preceeding a power value function in appendix A.5.1.

of the current wage to future value through raising savings and assets available next period.

$$
\begin{aligned}
(\frac{h_s}{w_s})^{1/(\alpha-1)} &\geq B_s[y_s + w_s + r_s A_s + N^0_{s+1}], s = t_2 + 1...t_{1-2} \qquad (17)\\
\text{where } B_s &= \frac{(\delta Q_{s+1} M_{s+1})^{1/(\alpha-1)}}{1 + Q_{s+1}(\delta Q_{s+1} M_{s+1})^{1/(\alpha-1)}}
\end{aligned}
$$

The term B_s shows the combined effect of the discount and interest rate. In the final period of this epoch similarly it must be the case that

$$
(\frac{h_{t_1-1}}{w_{t_1-1}})^{1/(\alpha-1)} \geq B_{t_1-1}[y_{t_1-1} + w_{t_1-1} + r_{t_1-1} A_{t_1-1} + N^i_{t_1}], i = 1, I \qquad (18)
$$

where the subsequent epoch must display one of $L_{t_1} = 1$ or L_{t_1} interior.

5.1.2 (ii) Zero Work t_2 to $t_1 - 1$

In this case at every period within the epoch the individual is in full time leisure, therefore the term P^1_s cumulates the impacts of these leisures through the epoch, discounting them at the rate of time preference. On the other hand with zero work at every period within the epoch, nonfinancial income within the epoch is just non-labour income each period and the function N^1_s cumulates this through the epoch.

$$
\begin{aligned}
N^1_s &= \Sigma^{t_1}_{\tau=s} Q_{\tau+1} y_\tau + N^i_{t_1}, s = t_2..t_1 - 1\\
P^1_s &= \Sigma^{t_1-1}_{\tau=s} \delta^{t_1-s} h_\tau, s = t_2..t_1 - 1
\end{aligned}
$$

The marginal utility of leisure at any period within the epoch must exceed the contribution of the wage to the future value both within and importantly beyond the epoch:

$$
(\frac{h_s}{w_s})^{1/(\alpha-1)} \leq B_s[y_s + w_s + r_s A_s + N^0_{s+1}], s = t_2 + 1...t_1 - 2 \qquad (19)
$$

and in the final period of this epoch similarly it must be the case that

$$
(\frac{h_{t_1-1}}{w_{t_1-1}})^{1/(\alpha-1)} \leq B_{t_1-1}[y_{t_1-1} + w_{t_1-1} + r_{t_1-1} A_{t_1-1} + N^i_{t_1}], i = I, 0 \qquad (20)
$$

5.1.3 (iii) Part time Work t_2 to $t_1 - 1$

The value function at $t_1 - 1$ becomes linear in assets:

$$
v_{t_1-1} = R^I_{t_1-1} + S^I_{t_1-1} A_{t_1-1} \qquad (21)
$$

with

$$
\begin{aligned}
S^I_{t_1-1} &= \frac{h_{t_1-1}}{w_{t_1-1}} \\
R^I_{t_1-1} &= \frac{h_{t_1-1}}{w_{t_1-1}}(y_{t_1-1}+w_{t_1-1}) + \frac{h_{t_1-1}}{Q_{t_1}w_{t_1-1}}N^i_{t_1} + [\delta M_{t_1}\left(\frac{h_{t_1-1}}{\delta M_{t_1}Q_{t_1}w_{t_1-1}}\right)^{\alpha/(\alpha-1)} \\
&\quad -\frac{h_{t_1-1}}{Q_{t_1}w_{t_1-1}}\left(\frac{h_{t_1-1}}{\delta M_{t_1}Q_{t_1}w_{t_1-1}}\right)^{1/(\alpha-1)}](1+Q_{t_1}(\delta M_{t_1}Q_{t1})^{1/(\alpha-1)}
\end{aligned}
$$

Coming backwards in time, if part time work is optimal in all periods back to t_2 then future market conditions and leisure preferences do not affect the current marginal value of assets at any period s within the epoch. The value function is linear in assets at any $t_2 \leq s \leq t_1-1, v_s = R^I_s + S^I_s A_s$ with

$$
\begin{aligned}
S^I_s &= \frac{h_s}{w_s} r_s \\
R^I_s &= \frac{h_s}{w_s}(w_s+y_s) + \delta R^I_{s+1}, s = t_2..t_1-2 \\
&= \Sigma_{\tau=s}^{t_1-2}\delta^{t_1-1-\tau}(y_\tau + w_\tau) + \delta^{t_1-1-s}R^I_{t_1-1}
\end{aligned}
\tag{22}
$$

Here S_s is an optimally set constant marginal value of future wealth, equated to the ratio of the marginal value of leisure to the discounted wage. With optimal part time work, the period s marginal utility of labour income equates to the present value of the marginal value of future wealth. The term R_s evaluates the contribution to value of the stream of future full incomes, discounting them at the time preference rate.

Given that the value function at t_1 is a power function, for part time work to be optimal at t_1-1, the ratio of the marginal utility of leisure to the wage w_{t_1-1} at t_1-1 (which is the marginal utility of current income in t_1-1), must exceed the marginal value of wealth at t_1 evaluated at $L_{t_1-1}=0$:

$$
\frac{B_{t_1-1}\left[(y_{t_1}+w_{t_1}+r_{t_1}A_{t_1}) + N^1_{t_1+1}\right]}{(\delta M_{t_1})^{1/(\alpha-1)}Q_{t_1}} > (h_{t_1}/w_{t_1})^{1/(\alpha-1))} \tag{23}
$$

and conversely the current marginal utility of income must be lower than the marginal value of wealth evaluated at zero hours of work, $L_{t_1-1}=1$:

$$
\frac{B_{t_1-1}\left[(y_{t_1}+w_{t_1}+r_{t_1}A_{t_1}) + N^1_{t_1+1}\right]}{(\delta M_{t_1})^{1/(\alpha-1)}Q_{t_1}} < (h_{t_1}/w_{t_1})^{1/(\alpha-1))}, i=0,1 \tag{24}
$$

Within the epoch at each period s the value function becomes linear as stated in Proposition 1 , and then for part time work to be optimal at each period s within the epoch requires

$$
S^I_{s+1} = \delta r_{s+1} w_s \frac{h_{s+1}}{w_{s+1}}, s = t_2..t_1-2 \tag{25}
$$

From equation (22) within each period in the epoch the current marginal utility of income is exactly equal to the marginal value of wealth which, in this epoch, is independent of assets A_{s+1}. We next characterise epochs prior to an epoch with a linear value function.

5.2 Epochs Preceding an Epoch with a Linear Value Function

This epoch lasts from periods t_2 to $t_1 - 1$, the next adjacent epoch starts at t_1 and has a linear value function:

$$v_{t_1} = R_{t_1} + S_{t_1} A_{t_1} \tag{26}$$

The epoch t_2 to $t_1 - 1$ could for example be either the first period of part time work or any period which is followed at some point by an epoch of part time work.[12] As noted the special feature of epochs which precede an epoch with a linear value function, is the particularly simple expressions which govern the critical levels of starting assets which determine current optimal labour participation. They involve comparing the marginal utility of current income as defined by the ratio of the marginal utility of current leisure to the wage with the discounted value of the marginal value of future wealth. If the current marginal utility of income is higher then it is optimal to work full time, if lower then it is optimal not work at all, and if it is just equal part time work is optimal. Since the marginal value of future wealth is independent of assets and future income, the optimal labour participation depends only on values of exogenous variables and is independent of current hours worked or consumption. The value functions for the three possible states of labour force participation and the critical asset condition can be summarised as follows:

5.2.1 (i) Full Time Work t_2 to $t_1 - 1$

In this case the value function at $t_1 - 1$ is

$$\begin{aligned} v_{t_1-1} &= \delta S_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} + w_{t_1-1}) + \delta R_{t_1} + \delta^{\alpha/(\alpha-1)} S_{t_1}^{\alpha/(\alpha-1)}[\frac{1}{\alpha} - 1] \\ &= S^0_{t_1-1}A_{t_1-1} + R^0_{t_1-1} \end{aligned} \tag{27}$$

where the term $S^0_{t_1-1}$ captures the discounted utility of leisure relative to the wage of one period and $R^0_{t_1-1}$ is a combination of discounted future $MRS_{h,w}$ between leisure and consumption.

with

$$\begin{aligned} S^0_{t_1-1} &= \delta S_{t_1} r_{t_1-1} \\ R^0_{t_1-1} &= \delta S_{t_1}(y_{t_1-1} + w_{t_1-1}) + \delta R_{t_1} + \delta^{\alpha/(\alpha-1)} S_{t_1}^{\alpha/(\alpha-1)}[\frac{1}{\alpha} - 1] \end{aligned}$$

[12] We derive the expressions for value functions preceding a linear value function in appendix A.5.2.

The critical asset condition which governs full time work to be optimal is particularly simple

$$S_{t_1-1} < \delta S_{t_1}$$

For full time work to be optimal for each period s within the epoch, the marginal value of leisure must exceed the discounted marginal value of future wealth multiplied by the wage, formally $S_s < \delta S^0_{s+1}, s = t_2..t_1 - 2$. Optimal within period consumption is independent of wealth, $c_s = \delta^{1/(\alpha-1)} S_s^{1/(\alpha-1)}$. The period s value function for a full time work epoch is:

$$\begin{aligned} v_s &= (r_s A_s + y_s + w_s - A_{s+1})^\alpha/\alpha + \delta[S^0_{s+1}A_{s+1} + R^0_{s+1}], s = t_2..t_1 - 2 \\ &= S^0_s A_s + R^0_s \end{aligned}$$

with

$$\begin{aligned} R^0_s &= \Sigma^{t_1-2}_{\tau=s}\left((\frac{h_s}{w_s})^{\alpha/\alpha-1)}(1/\alpha - 1) + h_s + \frac{h_s}{w_s}y_s\right) + \delta R_{s+1} \\ S^0_s &= \Sigma^{t_1-2}_{\tau=s}\frac{h_s}{w_s}r_s \end{aligned} \tag{28}$$

The terms S^0_s and R^0_s denote the cumulative effect of the combination of future discounted utility of leisure relative to the wage, and discounted future $MRS_{h,w}$ between leisure and consumption.

5.2.2 (ii) Zero Work t_2 to $t_1 - 1$

The value function at $t_1 - 1$ is derived as

$$\begin{aligned} v_{t_1-1} &= h_{t_1-1} + \delta S_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1}) + \delta R_{t_1} + \delta^{\alpha/(\alpha-1)} S_{t_1}^{\alpha/(\alpha-1)}[\frac{1}{\alpha} - 1] \\ &= S^1_{t_1-1}A_{t_1-1} + R^1_{t_{1-1}} \end{aligned} \tag{29}$$

with

$$\begin{aligned} S^1_{t_1-1} &= \delta S_{t_1} r_{t_1-1} \\ R^1_{t_{1-1}} &= h_{t_1-1} + \delta S_{t_1} y_{t_1-1} + \delta R_{t_1} + \delta^{\alpha/(\alpha-1)} S_{t_1}^{\alpha/(\alpha-1)}[\frac{1}{\alpha} - 1] \end{aligned}$$

Notice that in this case $R^1_{t_{1-1}}$ contains the marginal value of leisure h_{t_1-1}, whilst in $R^0_{t_1-1}$ the contribution of working in period $t_1 - 1$ was $\delta S_{t_1} w_{t_1-1}$. The critical asset condition governing whether full time leisure is optimal is given by:

$$S_{t_1-1} > \delta S_{t_1}$$

For each period s within the epoch the individual solves

$$\begin{gathered} \max c_s^\alpha/\alpha + h_s L_s + \delta(R^1_{s+1} + S^1_{s+1}A_{s+1}) \\ c_s + A_{s+1} = rA_s + y_s \end{gathered} \tag{30}$$

For full time leisure to be optimal for each period s within the epoch, the marginal value of leisure must exceed the discounted marginal value of future wealth multiplied by the wage, formally $S_s > \delta S^1_{s+1}, s = t_2..t_1 - 2$. Optimal within period consumption is then a constant independent of assets or current income: $c_s = \delta^{1/(\alpha-1)} S_s^{1/(\alpha-1)}$. The period s value function for a linear zero work epoch is defined as:

$$\begin{aligned} v_s &= h_s + (r_s A_s + y_s - A_{s+1})^\alpha/\alpha + \delta[S^1_{s+1} A_{s+1} + R^1_{s+1}], s = t_2..t_1 - 2 \\ &= S^1_s A_s + R^1_s \end{aligned} \tag{31}$$

with

$$\begin{aligned} R^1_s &= \Sigma_{\tau=s}^{t_1-2} \left((\frac{h_s}{w_s})^{\alpha/\alpha-1)}(1/\alpha - 1) + h_s + \frac{h_s}{w_s} y_s \right) + \delta R_{s+1} \\ S^1_s &= \Sigma_{\tau=s}^{t_1-2} \frac{h_s}{w_s} r_s \end{aligned} \tag{32}$$

The interpretation of S^1_s and R^1_s is analogous to definition noted in the previous subsubsection, but for the case of zero work.

5.2.3 (iii) Part time Work t_2 to $t_1 - 1$

The value function for a switch into part time work at $t_1 - 1$ is

$$\begin{aligned} v_{t_1-1} &= \delta S_{t_1}(r_{t_1-1} A_{t_1-1} + y_{t_1-1} + w_{t_1-1}) + \delta R_{t_1} + \delta^{\alpha/(\alpha-1)} S_{t_1}^{\alpha/(\alpha-1)} [\frac{1}{\alpha} - 1] \\ &= S^I_{t-1} A_{t-1} + R^I_{t-1} \end{aligned} \tag{33}$$

with

$$\begin{aligned} S^I_{t_1-1} &= \delta S_{t_1} \\ R^I_{t_1-1} &= \delta S_{t_1}(y_{t_1-1} + w_{t_1-1}) + \delta R_{t_1} \end{aligned} \tag{34}$$

The critical asset condition which must hold for part time work to be optimal is given by

$$S_{t_1-1} = \delta S_{t_1}$$

That is, it must the case that the marginal value of income at $t_1 - 1$ must grow at the exogenous discount rate delta multiplied by the t_1 marginal utility of income, as defined by the marginal rate of substitution of leisure to the wage rate. Then for part time work to be optimal for each period s within the epoch, the this condition must hold $S_s = \delta S^I_{s+1}, s = t_2..t_1 - 2$. Optimal within period consumption is again independent of wealth and is given by $c_s =$

$\delta^{1/(\alpha-1)}S_s^{1/(\alpha-1)}$. The period s value function for a linear part time work epoch is:

$$v_s = S_s^I A_s + R_s^I, s = t_2..t_1 - 2$$

with

$$\begin{aligned} S_s &= \delta S_{s+1}^I \\ R_s^1 &= \frac{h_s}{w_s}(w_s + y_s) + \delta R_{s+1}^1, s = t_2..t_1 - 2 \\ &= \Sigma_{\tau=s}^{t_1-2}\delta^{t_1-1-\tau}(y_\tau + w_\tau) + \delta^{t_1-1-s}R_{t_1-1}^1 \end{aligned}$$

The term S_s^i $i = 0, I, 1$ measures the marginal value of wealth at period s. Starting from a linear value function at the end of the epoch, this is a constant at each period within the epoch, and grows or falls at the rate r/δ. That is, in all cases of labour participation which are determined by comparing the current and future marginal utility of income, labour supply and savings adjusts optimally whilst consumption is independent of wealth. In this case it is only at the end of the epoch when switching into the next epoch that labour supply and savings are determinate. At all periods within the epoch labour and savings are indeterminate. The terms R_s^i $i = 0, I, 1$ reflect the impacts of nonfinancial income on value during the epoch, this can vary with optimal participation status and is cumulated at the marginal value of wealth discounted by the time preference rate.

6 The Nature of the Optimal Path

We have derived the conditions on market variables and on preferences which determine the optimal participation status. From this it is possible to derive the form of consumption, savings and labour supply. This is seen more clearly in the figure 4 below:

Figure 4 about here

In figure 4 the dashed lines represent periods within an epoch where consumption is independent of wealth and the value function is linear in assets. Whilst the solid lines represent periods where consumption and savings are linear functions of assets and the value function is a power function. The dashed lines consist of periods within epochs which precede (or in the limit coincide with) the final period of part time work. We can divide the dashed lines into two subgroups: those with part time work where hours worked and savings are indeterminate and those with full time or retirement where hours worked is determinate and savings is a residual. The dashed periods have the properties in the dashed box, the solid periods have the properties in the solid box.

We know that prior to any epoch of part time employment, the value function is linear, consumption is always independent of wealth and in epochs of full time or zero work labour supply is of course constant. It is clear from the diagram the majority of the paths do in fact involve a linear value function. Savings in each period of epochs of full time or zero work before an epoch of part time work is hence a residual which fluctuates with the wage and nonlabour income. After the final period (or equivalently epoch) of part time work, consumption and savings are linear functions of current assets and the value function is always a power function, the optimal labour participation can involve either full time or zero work. An example of such a path is shown on the furthermost RHS of the diagram, note that lifecycle paths characterised by the properties in the solid box are quite rare within our model.

7 Application of theory: retirement paths.

Mandatory retirement has traditionally marked the cessation of paid work (Lazear 1979), which usually occurred when an individual reached the State Pension Age (SPA). In more recent times early retirement has also been observed, whereby individuals retire before SPA for example because they can afford to do so, or their private pension scheme allows them to draw their pension earlier than SPA. Another phenomenon which has occurred since the 1960's/1970's is that of partial retirement (Gustman and Steinmeier 1984, 1985). This can be thought of as an individual either reducing the number of hours working in their career job, or if this was not possible then changing job when they reach SPA.[13] Partial retirement can be viewed as a mechanism by which individuals can move gradually into full retirement and adjust their lifestyle and habits more smoothly.

More recently non standard retirement paths have been investigated by Maestas (2010), Kanabar (2012), Petersson (2011), Larsen and Pederson (2012) and Schlosser et al. (2012) in the form of unretirement, which is defined as a transition from retirement to partial retirement, or even back in to full time work.[14] Various reasons have been noted in the literature which may explain this: unexpected income or preference shocks, in particular it may turn out that the anticipated utility of leisure is actually lower than was expected, once it is actually experienced. Maestas (2010) finds that unretirement is an anticipated event, using expectations data, she finds that the majority of unretirees anticipated unretiring prior to initial retirement.

Another possibility concerns the impact of pension schemes. Many pension schemes even of the funded variety (which involve purchase of an annuity from a personal pension fund) yield a stream of nonlabour income y_t that is fixed in nominal terms but not in real terms. An individual may choose to retire at t having previously been in full or part time work because y_t jumps upwards when retirement coincides with eligibility for receipt of an income stream from a pension. But if the income flow is not indexed fully, in real terms the income

[13] This is due to the labour market regulation in the US.

[14] The exact definition of retirement is difficult to pin down precisely, given that it could be based on a self reported definition or on the number of hours reported in paid work. Unretirement is not a new phenomenon, it was observed in the US during the 1980's and was coined as 'reverse flow'. See Gustman and Steinmeier (1984) and Rust (1989,1990).

stream starts falling. Then at some date $s > t$ it could be optimal to return to work since the fall in present and future nonlabour income reduces the marginal value of future wealth. Our framework can account for the heterogeneity in retirement paths, for example: with $t_1 > t_2$

(1) Full time$_{t_1}$ $\longrightarrow$Retired$_{t_2}$

(2) Full time$_{t_1}$ $\longrightarrow$Part time$_{t_2}$ $\longrightarrow$Retired$_{t_3}$

(3) Full time$_{t_1}$ $\longrightarrow$Retired$_{t_3}$ $\longrightarrow$Full time$_{t_4}$

(4) Full time$_{t_1}$ $\longrightarrow$Retired$_{t2}$ $\longrightarrow$Part time$_{t_3}$.

After the last period of part time work (if any such exists) participation decisions depend on the whole future evolution of the h, w,y (as in cases (1)-(3) above). But in case (4) for any t before the final part time epoch the participation decision at t depends only on comparing variables at t with variables at $t+1$. Retirement paths (1) and (2) have been studied extensively (Gustman and Steinmeier (1984,1985)), within our framework these paths and also path (3) will involve future market conditions and preferences. Only path (4) has a simple structure and does not involve the future. We show the implications of our general framework for each category.

7.1 Periods for which the whole future matters

Retirement paths in which the whole future matters involve full time work or full retirement, due to the role of future non financial wealth, N, in the critical asset conditions. The critical level of assets in these cases have the general form

$$\frac{h}{w} \gtrless B(M,Q)(N^i + Q(rA + (w) + y)^{\alpha-1} \tag{35}$$

Thus the more value an individual places on the marginal value of leisure, the less likely they are to work now. Higher opening and current financial wealth A and future nonfinancial wealth, N serve to reduce the desire to work. However future non financial wealth can increase if either future w or y increase or if the balance of r, δ changes. Note that the future marginal value of leisure does not affect current participation but does affect the current value of the future.

To demonstrate the importance of the marginal value of leisure, suppose an individual follows a retirement path defined by (1) above with complete withdrawal from the labour market forever from that point on. Take two individuals with identical w, y streams and starting assets but different h paths. Then to retire at $T-2$ needs:

$$\begin{aligned} h_T/w_T &> (y_T + rA_T)^{\alpha-1} \\ h_{T-1}/w_{T-1} &> (Q_{T-1}(y_{T-1} + rA_{T-1}) + y_T)^{\alpha-1} \\ h_{T-2}/w_{T-2} &> (Q_{T-2}(y_{T-2} + rA_{T-2}) + Q_{T-1}Q_{T-2}y_{T-1} + y_T)^{\alpha-1} \end{aligned} \tag{36}$$

Therefore whilst the right hand side is the same for the 2 individuals, one may retire at $T-2$ but the other may not only if h_{T-2} is suitably different, and then subsequent assets may be different between them. A chronic health shock, retirement of spouse or existence of grandchildren can all raise h permanently into the future and lead to retirement.

Whilst retirement paths (1) and (2) have been investigated in some detail, the retirement path in (3) features full unretirement. That is to say an individual may find it optimal to fully retire at t and then subsequently return to full time work at $t+1$. For this unretirement to be optimal we must have

$$\begin{aligned}
h_t/w_t &> B_t(M_t,Q_t)(N^0_{t+1}+Q_t(rA_t+y_t))^{\alpha-1} && (37)\\
h_{t+1}/w_{t+1} &< B_t(M_{t+1},Q_{t+1})(N^i_{t+2}+Q_{t+1}(rA_{t+1}+w_{t+1}+y_{t+1}))^{\alpha-1} && (38)\\
h_t/w_t &> B_t(M_t,Q_t)(N^i_{t+2}+Q_ty_t+Q_t(rA_t+y_t))^{\alpha-1} && (39)\\
\text{where } N^0_{t+1} &= N^i_{t+2}+Q_ty_t && (40)
\end{aligned}$$

It is clear that the future plays an important role for non standard retirement paths of this type. Indeed Maestas (2010) suggests that unretirement is a planned event, individuals anticipate retiring and subsequently returning to the labour force, and it is not due to an unexpected shock to income or wealth.[15] Recent work demonstrating the empirical importance of unretirement in the case of the US and England can be found in Gustman and Steinmeier (2002), Maestas and Li (2007), Maestas (2010) and Kanabar (2012).[16] It is important to note that in the majority of these studies (with the exception of Gustman and Steinmeier (2002)) unretirement flows are defined such that they include a transition from retirement to part time work, that is to say they follow a retirement path defined by (4) above.[17] As Gustman and Steinmeier (2002) show 'full unretirement', i.e. retirement path (3) does occur, but less frequently. In the US for example both Maestas (2010) and Gustman and Steinmeier (2002) use HRS data and find unretirement rates of 26% and 15% respectively.[18] The main reason for this difference is due to the majority of the flows going from retirement into part time work (retirement path (4)), which the former paper includes in their definition of unretirement whilst the latter does not.[19]

Our framework has clear implications for retirement policy. Take for example paths (2), (3) and (4), despite the latter two of these retirement paths featuring

[15] Maestas (2010) also notes that retirees preretirement leisure expectations do not coincide with the actual retirement experience, and this difference may induce individuals to return to the labour force.

[16] Recent papers include Schlosser et al (2012) who consider unretirement in Canada. Whilst Larsen and Pederson (2012) and Petersson (2011) consider the case of Denmark and Sweden respectively.

[17] Note that a fraction of the individuals who follow a retirement path defined by (4), subsequently go on to enter full time work.

[18] Gustman and Steinmeier (2002) pp.21 and pp.37. In their model they only consider two labour force states, namely full time work and full time retirement.

[19] Note there are some differences in the way these authors construct their samples. Maestas (2010) uses the first six waves of HRS whilst Gustman and Steinmeier (2002) use the first five waves.

unretirement, the properties of the value function in each case are quite different. In retirement path (3) optimal participation changes can only be from corner to corner, if for example in the retirement phase t_2 a policy was announced which affected future non labour income equations (17) and (19) show this would affect participation at t_2 by altering the size of N. An identical policy would not affect an individual following retirement path (2) or (4) prior to the final period of part time work, equations (28) and (32) show that the conditions which determine the optimal labour force regime are not affected by current or future non labour income. However equations (17) and (19) and (28) and (32) show any policy which affected future wages, w, would impact on current optimal labour supply decision irrespective of which retirement path the individual was on.

7.2 Periods for which only adjacent periods matter

An example of retirement paths where participation decisions only depend on adjacent period variables are those such as (4) above. Appendix A.5.2 derives the particularly simple conditions governing optimal labour supply, prior to epoch with a linear value function. If we make the additional assumption that the $MRS_{h,w}$ is constant at every period and grows at the (exogenous) discount rate δ, in this case the conditions governing labour force participation are

$$\begin{aligned} L_t &= 1 \text{ if } h_t > \delta h_t \\ 0 &< L_t < 1 \text{ if } h_t = \delta h_t \\ L_t &= 0 \text{ if } h_t < \delta h_t \end{aligned} \tag{41}$$

Therefore within our framework not only can we model a variety of non standard retirement paths, in those cases where only adjacent periods matter (40) shows the optimal conditions governing labour supply depend on only the marginal value of leisure and the discount rate. An individuals tastes and preferences may display a marginal value of leisure that increases at older ages, for example to spend more time with family or phase into full retirement gradually, and they choose to reduce their working hours to part time. There are a variety of reasons which may mean it is optimal for an individual to remain engaged in the labour force permanently or even unretire, for example due to changes in their leisure preferences (Maestas (2010)).

8 Conclusion.

In this paper we have taken a common form of the utility function used in the lifecycle literature, and found a number of its general properties. In particular, we have shown that whilst previous papers have used computational methods to find a tractable solution, we are able to break the curse of dimensionality through emphasising the role of part time work. After deriving the nature of life cycle epochs of consumption, savings and labour participation, we apply our model to the increasing prevalence of non standard retirement paths. In doing so we note (1) the increasing importance of flexible working regulations in order to ensure individual's can supply an optimal level of labour over the lifecycle and (2) the framework captures several features of the 2007 cross section PSID data.

To show this we derive the explicit functional form of the value function, with its switches as different forms of labour participation become optimal. These switches occur at critical values of current assets in relation to future market and preference parameters. At such a switch generally the consumption function switches between being a linear function of assets and being independent of assets. Similarly savings switches between being a linear function of assets and being jointly indeterminate with hours worked. Knowing the value function, we are able to characterise the entire lifecycle of an individual through the use of epochs, in particular we show the way in which the future can play an important role in the participation decision. This has implications on the forces which govern the within and intertemporal marginal rate of substitution between leisure and work, and also the marginal utility of wealth.

The framework excludes various potentially important effects. We have taken the individual as the decision making unit but most individuals live in families where the family decision rules, and also externalities in the preferences of different family members are important. For example partners may coordinate retirement decisions due to a preference for shared leisure, or the need for labour income from one family member may depend on the participation status of other family members. Even in a one period world the analysis of family labour market participation decisions is not straightforward Donni (2005) Blundell et al (2007). In a multiperiod world it is obviously more complicated. We have neglected preference or budget constraint uncertainty and all employment decisions are voluntary, anyone wanting to work can find a job and wages, non-labour income and interest rates are perfectly foreseen. This is in common with many papers in the literature , although its justification has to be on grounds of imposing sufficient simplicity to be able to derive analytical solutions which will be valid in any data set. The alternative would be to numerically determine the optimal lifetime path in the context of a specific random process for preferences or elements of the budget constraint. If the purpose is to determine qualitative properties of the optimal path, analytical solution is much more useful.

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9

A Appendix

A.1 Data and sample construction

For each variable of interest we construct our sample by restricting attention to all males whom satisfy the age band condition, and compare this with the information reported by the HOH for the particular variable of interest, It means there are sightly different sample sizes in figures 1-3. We only look at males, since the number of female HOH is much lower, however at the individual level females have similar distributions of work and wages to males except for during the cihild bearing age bands.

Each of the key economic variables are defined as:

1. Assets = value of all car(s) in the residence + value of all family members AMT balances + value of all bonds/insurance and excludes all residential wealth.[20]

2. Annual non labour income = HOH annual dividend income + HOH annual interest income + HOH annual rental income.

3. The hourly wage rate = wage rate reported in main job (2006 \$).

[20] We do not include individuals who report constituent asset component values in excess of 2006 US \$1 million.

Individuals education level is ‘college’ if they have spend at least 16 years in full time education, ‘high school’ if they spend between 12 and 15 years in full time education, and ‘below high school’ if they spend strictly less than 12 years in full time education.

A.2 M recursion

To show this take the three periods prior to the terminal period as an example, starting from the end substitute back into the relevant expression at each period to derive the expression for M_{T-t}.

$$\begin{aligned}M_{T-1} &= \delta[1+Q_T(\delta Q_T)^{1/(\alpha-1)}]^{1-\alpha}\\ &= \delta[\delta^{1/(\alpha-1)}(\delta^{1/(1-\alpha)}+Q_T^{\alpha/(\alpha-1)}]^{1-\alpha}\\ &= \delta\delta^{-1}[\delta^{1/(1-\alpha)}+Q_T^{\alpha/(\alpha-1)}]^{1-\alpha}\end{aligned}$$

$$\begin{aligned}M_{T-2} &= \delta M_{T-1}[1+Q_{T-1}(\delta Q_{T-1}M_{T-1})^{1/(\alpha-1)}]^{1-\alpha}\\ &= \delta M_{T-1}[(\delta M_{T-1})^{1/(\alpha-1)}((\delta M_{T-1})^{1/(1-\alpha)}+Q_{T-1}^{\alpha/(\alpha-1)})]^{1-\alpha}\\ &= [(\delta M_{T-1})^{1/(1-\alpha)}+Q_{T-1}^{\alpha/(\alpha-1)}]^{1-\alpha}\\ &= [(\delta[\delta^{1/(1-\alpha)}+Q_T^{\alpha/(\alpha-1)}]^{1-\alpha})^{1/(1-\alpha)}+Q_{T-1}^{\alpha/(\alpha-1)}]^{1-\alpha}\\ &= [\delta^{1/(1-\alpha)}[\delta^{1/(1-\alpha)}+Q_T^{\alpha/(\alpha-1)}]+Q_{T-1}^{\alpha/(\alpha-1)}]^{1-\alpha}\\ &= [\delta^{2/(1-\alpha)}+\delta^{1/(1-\alpha)}Q_T^{\alpha/(\alpha-1)}+Q_{T-1}^{\alpha/(\alpha-1)}]^{1-\alpha}\end{aligned}$$

$$\begin{aligned}M_{T-3} &= [(\delta M_{T-2})^{1/(1-\alpha)}+Q_{T-2}^{\alpha/(\alpha-1)}]^{1-\alpha}\\ &= [(\delta[\delta^{2/(1-\alpha)}+\delta^{1/(1-\alpha)}Q_T^{\alpha/(\alpha-1)}+Q_{T-1}^{\alpha/(\alpha-1)}]^{1-\alpha})^{1/(1-\alpha)}+Q_{T-2}^{\alpha/(\alpha-1)}]^{1-\alpha}\\ &= (\delta^{1/(1-\alpha)}[\delta^{2/(1-\alpha)}+\delta^{1/(1-\alpha)}Q_T^{\alpha/(\alpha-1)}+Q_{T-1}^{\alpha/(\alpha-1)}]+Q_{T-2}^{\alpha/(\alpha-1)}]^{1-\alpha}\\ &= [\delta^{3/(1-\alpha)}+\delta^{2/(1-\alpha)}Q_T^{\alpha/(\alpha-1)}+\delta^{1/(1-\alpha)}Q_{T-1}^{\alpha/(\alpha-1)}+Q_{T-2}^{\alpha/(\alpha-1)}]^{1-\alpha}\end{aligned}$$

Thus for period $T-t$ we have

$$\begin{aligned}M_{T-t} &= (\delta^{t/(1-\alpha)}+\delta^{(t-1)/(1-\alpha)}Q_T^{\alpha/(1-\alpha)}+\delta^{(t-2)/(1-\alpha)}Q_{T-1}^{\alpha/(1-\alpha)}+\delta^{(t-3)/(1-\alpha)}Q_{T-2}^{\alpha/(1-\alpha)}\\ &\quad +...Q_{T-t+1}^{\alpha/(1-\alpha)})^{1-\alpha}\\ &= (\Sigma_{s=0}^{t}\delta^{(t-s)/(1-\alpha)}Q_{T-s+1}^{\alpha/(1-\alpha)}) \text{ with } Q_{T+1}=1\end{aligned}$$

A.3 Proofs for proposition 1 (iii)

If the future value is isoelastic and optimal current participation is at a corner then the current value is also isoelastic:

$$V_t|L_t = \max h_tL_t + c_t^{\alpha}/\alpha + \delta M_{t+1}(N_{t+1}^i + Q_{t+1}(r_tA_t + y_t - c_t + w_t(1-L_t))^{\alpha}/\alpha$$

$$
\begin{aligned}
\frac{\partial V_t}{\partial c_t} &= c_t^{\alpha-1} - \delta M_t Q_t (N_t^i + Q_t(r_t A_t + y_t - c_t + w_t(1-L_t))^{\alpha-1} = 0 \\
c_t &= \frac{\delta M_t Q_t^{\frac{1}{\alpha-1}} \left(N_t^i + Q_t \left(r_t A_t + y_t + w_t \left(1-L_t\right)\right)\right)}{\left(1 + Q_t \left(\delta M_t Q_t\right)^{\frac{1}{\alpha-1}}\right)}
\end{aligned}
$$

This implies

$$
A_{t+1} = r_t A_t + y_t - \frac{\delta M_t Q_t^{\frac{1}{\alpha-1}} \left(N_t^i + Q_t \left(r_t A_t + y_t + w_t \left(1-L_t\right)\right)\right)}{\left(1 + Q_t \left(\delta M_t Q_t\right)^{\frac{1}{\alpha-1}}\right)} + w_t(1-L_t)
$$

and

$$
V_t | L_t = M_t (N_t^i + Q_t A_t)^{\alpha} / \alpha
$$

A.4 Final period

At T choose L_T to

$$
\max[y_T + w_T(1-L_T) + r_T A_T]^{\alpha}/\alpha + h_T L_T \text{ st } L_T \leq 1
$$

$$
\frac{d}{dL_T} = -w_T[y_T + w_T(1-L_T) + r_T A_T]^{\alpha-1} + h_T
$$

If this is < 0 at $L_T = 1$ have optimal $L_T < 1$, if ≥ 0 have corner $L_T = 1$, if ≤ 0 at $L_T = 0$ have corner $L_T = 0$ i.e.

$$
\begin{aligned}
L_T &= 0 \text{ if } \frac{h_T}{w_T} \leq (y_T + w_T + r_T A_T)^{\alpha-1} \\
&= [y_T + r_T A_T + w_T - (\frac{h_T}{w_T})^{1/(\alpha-1)}]/w_T \text{ if } w_T + y_T + r_T A_T > (\frac{h_T}{w_T})^{1/(\alpha-1)} > y_T + r_T A_T \\
&= 1 \text{ if } (\frac{h_T}{w_T})^{1/(\alpha-1)} \leq y_T + r_T A_T
\end{aligned}
$$

which gives

$$
\begin{aligned}
v_T &= (y_T + w_T + r_T A_T)^{\alpha}/\alpha \text{ if } \frac{h_T}{w_T} \leq (y_T + w_T + r_T A_T)^{\alpha-1} (L_T = 0) \\
&= (\frac{1}{\alpha} - 1)(\frac{h_T}{w_T})^{1/(\alpha-1)} + (\frac{h_T}{w_T})(y_T + r_T A_T + w_T) \\
&\quad \text{if } w_T + y_T + r_T A_T > (\frac{h_T}{w_T})^{1/(\alpha-1)} > y_T + r_T A_T (L_T \text{ interior}) \\
&= (y_T + r_T A_T)^{\alpha}/\alpha + h_T \text{ if } (\frac{h_T}{w_T})^{1/(\alpha-1)} \leq y_T + r_T A_T \ (L_T = 1)
\end{aligned}
$$

Using this at T

$$v_T(A_T) = \max[P^i_T + M^i_T(N^i_T + Q^i_T A_{Tt})^\alpha/\alpha, R_T + S_T A_T]$$
$$= P^2_T + M^2_T(N^2_T + Q^2_T A_{Tt})^\alpha/\alpha \text{ if } \frac{h_T}{w_T} \leq (y_T + w_T + r_T A_T)^{\alpha-1}$$
$$= R_T + S_T A_T \text{ if } (w_T + r_T A_T)^{\alpha-1} < \frac{h_T}{w_T} < (y_T + r_T A_T)^{\alpha-1}$$
$$= P^1_T + M^1_T(N^1_T + Q^1_T A_{Tt})^\alpha/\alpha \text{ if } \frac{h_T}{w_T} \geq (y_T + r_T A_T)^{\alpha-1}$$

where

$$P^1_T = h_T, N^1_T = y_T, Q^1_T = r_T, M^1_T = 1$$
$$P^2_T = 0, N^2_T = y_T + w_T, Q^2_T = r_T, M^2_T = 1$$
$$R_T = h_T(1 + \frac{y_T}{w_T}) + (\frac{1}{\alpha} - 1)(\frac{h_T}{w_T})^{\alpha/(\alpha-1)}, S_T = r_T\frac{h_T}{w_T}$$

(R^1_t, R^2_t,R^3_t and S^1_t, S^2_t,S^3_t coincide)

Then

$$v_T = \max[P_T + \frac{M_T(N_T + Q_T A_T)^\alpha}{\alpha}, R_T + S_T A_T]$$

A.5 Epochs

A.5.1 Epochs Preceding an Epoch with a Power Function Value Function

Consider the epoch lasting for periods from t_2 to $t_1 - 1$ where the value function at t_1 is a power function.

$$v_{t_1-1} = \max_{c,L} c^\alpha_{t_1-1}/\alpha + h_{t_1-1}L_{t_1-1} + \delta[P^i_{t_1} + M_{t_1}(N^i_{t_1} + Q_{t1}(r_{t_1-1}A_{t_1-1} + y_{t1} - c_{t_1} + w_{t_1}(1 - L_{t_1}))^\alpha/\alpha]$$

$$\frac{\partial}{\partial c_{t_1-1}} = c^{\alpha-1}_{t_1-1} - \delta M_{t_1}Q_{t_1}(N^i_{t_1} + Q_{t_1}(r_{t_1}A_{t_1-1} + y_{t_1-1} - c_{t_1-1} + w_{t_1-1}(1 - L_{t_1-1}))^{\alpha-1} = 0$$

Optimal consumption at t_3 is then:

$$c_{t_1-1} = \frac{(\delta M_{t_1}Q_{t_1})^{1/(\alpha-1)}}{1 + Q_{t_1}(\delta M_{t_{t1}}Q_{t_1})^{1/(\alpha-1)}}(N^i_{t_1} + Q_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} - c_{t_1-1} + w_{t_1-1}(1 - L_{t_1-1}))$$

Substituting in back into the value function gives:

$$v_{t_1-1} = \max_L h_{t_1-1}L_{t_1-1} + \frac{\delta M_{t_1}(N^i_{t_1} + Q_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} + w_{t_1-1}(1 - L_{t_1-1}))^\alpha}{(1 + Q_{t_1}(\delta M_{t_1}Q_{t_1})^{1/(\alpha-1)})^{\alpha-1}}$$

$$\frac{\partial v_{t_1-1}}{\partial L_{t_1-1}} = h_{t_1-1} - \frac{\delta M_{t_1}Q_{t_1}w_{t_1-1}(N^i_{t_1} + Q_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} + w_{t_1-1}(1 - L_{t_1-1}))^{\alpha-1}}{(1 + Q_{t_1}(\delta M_{t_1}Q_{t_1})^{1/(\alpha-1)})^{\alpha-1}}$$

Optimal labour supply at t_3 is either; full time work, part time work or zero work according to the sign of the marginal value of leisure evaluated at $L_{t_1-1} = 0, 1$. In particular:

$$L_{t_1-1} = 0 \text{ if } \frac{\partial v_{t_1-1}}{\partial L_{t_1-1}}|_{L_{t_1-1}=0} < 0$$

$$\text{i.e. if } h_{t_1-1} - \frac{\delta M_{t_1} Q_{t_1} w_{t_1-1}(N^i_{t_1} + Q_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} + w_{t_1-1})^{\alpha-1}}{(1 + Q_{t_1}(\delta M_{t_1} Q_{t_1})^{1/(\alpha-1)})^{\alpha-1}} < 0$$

$$L_{t_1-1} = 1 \text{ if } \frac{\partial v_{t_1-1}}{\partial L_{t_1-1}}|_{L_{t_1-1}=1} > 0$$

$$\text{i.e. if } h_{t_1-1} - \frac{\delta M_{t_1} Q_{t_1} w_{t_1-1}(N^i_{t_1} + Q_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1})^{\alpha-1}}{(1 + Q_{t_1}(\delta M_{t_1} Q_{t_1})^{1/(\alpha-1)})^{\alpha-1}} > 0$$

$$0 < L_{t_1-1} < 1 \text{ if } \frac{\partial v_{t_1-1}}{\partial L_{t_1-1}}|_{L_{t_1-1}=0} > 0, \frac{\partial v_{t_1-1}}{\partial L_{t_1-1}}|_{L_{t_1-1}=1} < 0$$

$$\text{i.e. if } h_{t_1-1} - \frac{\delta M_{t_1} Q_{t_1} w_{t_1-1}(N^i_{t_1} + Q_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1})^{\alpha-1}}{(1 + Q_{t_1}(\delta M_{t_1} Q_{t_1})^{1/(\alpha-1)})^{\alpha-1}} < 0$$

$$\text{and } h_{t_1-1} - \frac{\delta M_{t_1} Q_{t_1} w_{t_1-1}(N^i_{t_1} + Q_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} + w_{t_1-1})^{\alpha-1}}{(1 + Q_{t_1}(\delta M_{t_1} Q_{t_1})^{1/(\alpha-1)})^{\alpha-1}} > 0$$

The value function at $t_1 - 1$ remains a power function if there is either full time work or zero work at $t_1 - 1$ but becomes linear if there is optimally part time work at $t_1 - 1$. Optimal part time work solves

$$h_{t_1-1} = \frac{\delta M_{t_1} Q_{t_1} w_{t_{1-1}}(N^i_{t_1} + Q_{t_1} - 1(r_{t_1-1}A_{t_1-1} + y_{t_1-1} + w_{t_1-1}(1 - L_{t_1-1}))^{\alpha-1}}{(1 + Q_{t_1}(\delta M_{t_1} Q_{t_1})^{1/(\alpha-1)})^{\alpha-1}}$$

which gives

$$N^i_{t_1} + Q_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} + w_{t_1-1}(1 - L_{t_1-1}) = \left(\frac{h_{t_1-1}}{\delta M_{t_1} Q_{t_1} w_{t_1-1}}\right)^{1/(\alpha-1)} (1 + Q_{t_1}(\delta M_{t_1} Q_{t_1})^{1/(\alpha-1)})$$

Effectively L is linear in wealth and labour income and this makes c linear in wealth. Moreover the resources carried forward term $N^i_{t_1} + Q_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} + w_{t_1-1}(1 - L_{t_1-1})$ is independent of current wealth.

$$Q_{t_1} w_{t_1-1} L_{t_1-1} = N^i_{t_1} + Q^i_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} + w_{t_1-1}) - \left(\frac{h_{t_1-1}}{\delta M_{t_1} Q_{t_1} w_{t_1-1}}\right)^{1/(\alpha-1)} (1 + Q_{t_1}(\delta M_{t_1} Q_{t_1})^{1/(\alpha-1)})$$

$$L_{t_1-1} = \frac{\left[N^i_{t_1} + Q_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} + w_{t_1-1}) - \left(\frac{h_{t_1-1}}{\delta M_{t_1} Q_{t_1} w_{t_1-1}}\right)^{1/(\alpha-1)} (1 + Q_{t_1}(\delta M_{t_1} Q_{t_1})^{1/(\alpha-1)})\right]}{Q_{t_1} w_{t_1-1}}$$

$$v_{t_1-1} = h_{t_1-1}L_{t_1-1} + \frac{\delta M_{t_1}(N^i_{t_1} + Q_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} + w_{t_1-1}(1 - L_{t_1-1}))^{\alpha}}{(1 + Q_{t_1}(\delta M_{t_1}Q_{t_1})^{1/(\alpha-1)})^{\alpha-1}\alpha}$$

$$= \frac{h_{t_1-1}}{Q_{t_1}w_{t_1-1}}(N^i_{t_1} + Q_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} + w_{t_1-1}) - \left(\frac{h_{t_1-1}}{\delta M_{t_1}Q_{t_1}w_{t_1-1}}\right)^{1/(\alpha-1)}(1 + Q_{t_1}(\delta M_{t_1}Q_{t_1})^{1/(\alpha-1)}))$$

$$+\frac{\delta M_{t_1-1}(\left(\frac{h_{t_1-1}}{\delta M_{t_1}Q_{t_1}w_{t_1-1}}\right)^{1/(\alpha-1)}(1 + Q_{t_1}(\delta M_{t_1}Q_{t_1})^{1/(\alpha-1)})^{\alpha}}{(1 + Q_{t_1}(\delta M_{t_1}Q_{t_1})^{1/(\alpha-1)})^{\alpha-1}}$$

$$= \frac{h_{t_1-1}}{w_{t_1-1}}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} + w_{t_1-1}) +$$

$$\frac{h_{t_1-1}}{Q_{t_1}w_{t_1-1}}(N^i_{t_1} - \left(\frac{h_{t_1-1}}{\delta M_{t_1}Q_{t_1}w_{t_1-1}}\right)^{1/(\alpha-1)}(1 + Q_{t_1}(\delta M_{t_1}Q_{t_1})^{1/(\alpha-1)})) +$$

$$\delta M_{t_1}\left(\frac{h_{t_1-1}}{\delta M_{t_1}Q_{t_1}w_{t_1-1}}\right)^{\alpha/(\alpha-1)}(1 + Q_{t_1}(\delta M_{t_1}Q_{t_1})^{1/(\alpha-1)})$$

$$= \frac{h_{t_1-1}}{w_{t_1-1}}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} + w_{t_1-1}) + \frac{h_{t_1-1}}{Q_{t_1}w_{t_1-1}}N^i_{t_1}$$

$$+[\delta M_{t_1}\left(\frac{h_{t_1-1}}{\delta M_{t_1}Q_{t_1}w_{t_1-1}}\right)^{\alpha/(\alpha-1)} - \frac{h_{t_1-1}}{Q_{t_1}w_{t_1-1}}\left(\frac{h_{t_1-1}}{\delta M_{t_1}Q_{t_1}w_{t_1-1}}\right)^{1/(\alpha-1)}](1 + Q_{t_1}(\delta M_{t_1}Q_{t_1})^{1/(\alpha-1)})$$

$$v_{t_1-1} = R_{t_1} + S_{t_1}A_{t_1}$$

A.5.2 Epochs Preceding an Epoch with a Linear Value Function

From proposition 1 we know the value function at t_1 is linear in assets, substituting in the $t_1 - 1$ budget constraint we have:

$$S_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} - c_{t_1-1} + w_{t_1-1}(1 - L_{t_1-1}) + R_{t_1}$$

This could be either the first period of part time work or any period which is followed at some point by a period of part time work. Optimal choice at $t_1 - 1$

$$\max u(c_{t_1-1}) + h_{t_1-1}L_{t_1-1} + \delta[S_{t_1}(r_{t_1-1}A_{t_1-1} + y_{t_1-1} - c_{t_1-1} + w_{t_1-1}(1 - L_{t_1-1}) + R_{t_1}]$$

Optimal labour at $t_1 - 1$ is then either; full time leisure, part time or full time work:

$$\begin{aligned} L_{t_1-1} &= 1 \text{ if } h_{t_1-1} > \delta S_{t_1}w_{t_1-1} \\ 0 &< L_{t_1-1} < 1 \text{ if } h_{t_1-1} = \delta S_{t_1}w_{t_1-1} \\ L_{t_1-1} &= 0 \text{ if } h_{t_1-1} < \delta S_{t_1}w_{t_1-1} \end{aligned}$$

Optimal consumption c_{t_1-1} is equal to $c_{t_1-1} = \delta^{1/(\alpha-1)}S_{t_1}^{1/(\alpha-1)}$.The value function at $t_1 - 1$ is then

(i) if $L_{t_1-1}=1$

$$\frac{c^{\alpha}_{t_1-1}}{\alpha}+h_{t_1-1}L_{t_1-1}+\delta[S_{t_1}(r_{t_1-1}A_{t_1-1}+y_{t_1-1}+w_{t_1-1}(1-L_{t_1-1})-c_{t_1-1})+R_{t_1}]$$
$$=c^{\alpha}_{t_1-1}/\alpha+h_{t_1-1}+\delta[S_{t_1}(r_{t_1-1}A_{t_1-1}+y_{t_1-1}-c_{t_1-1})+R_{t_1}]$$
$$=\frac{\delta^{\alpha/(\alpha-1)}S^{\alpha/(\alpha-1)}_{t_1}}{\alpha}+h_{t_1-1}+\delta S_{t_1}(r_{t_1-1}A_{t_1-1}+y_{t_1-1})-\delta S_{t_1}\delta^{1/(\alpha-1)}S^{1/(\alpha-1)}_{t_1})+\delta R_{t_1}$$
$$=h_{t_1-1}+\delta S_{t_1}(r_{t_1-1}A_{t_1-1}+y_{t_1-1})+\delta R_{t_1}+\delta^{\alpha/(\alpha-1)}S^{\alpha/(\alpha-1)}_{t_1}[1/\alpha-1]$$
$$v_{t_1-1}=h_{t_1-1}+\delta S_{t_1}[r_{t_1-1}A_{t_1-1}+y_{t_1-1}]+\delta R_{t_1}=S^1_{t_1-1}[r_{t_1-1}A_{t_1-1}+y_{t_1-1}]+R^1_{t_1-1}$$

where $S^1_{t_1-1}=\delta S_{t_1}$ and $R^1_{t_1-1}=h_{t_1-1}+\delta R_{t_1}+\delta^{\alpha/(\alpha-1)}S^{\alpha/(\alpha-1)}_{t_1}[1/\alpha-1]$

(ii) if $L_{t_1-1}=0$

$$\frac{c^{\alpha}_{t_1-1}}{\alpha}+\delta[S_{t_1}(r_{t_1-1}A_{t_1-1}+y_{t_1-1}+w_{t_1-1}-c_{t_1-1})+R_{t_1}]$$
$$=\frac{\delta^{\alpha/(\alpha-1)}S^{\alpha/(\alpha-1)}_{t_1}}{\alpha}+\delta S_{t_1}(r_{t_1-1}A_{t_1-1}+y_{t_1-1}+w_{t_1-1})-\delta S_{t_1}\delta^{1/(\alpha-1)}S^{1/(\alpha-1)}_{t_1})+\delta R_{t_1}$$
$$=\delta S_{t_1}(r_{t_1-1}A_{t_1-1}+y_{t_1-1}+w_{t_1-1})+\delta R_{t_1}+\delta^{\alpha/(\alpha-1)}S^{\alpha/(\alpha-1)}_{t_1}[\frac{1}{\alpha}-1]$$
$$v_{t_1-1}=S^0_{t_1-1}[r_{t_1-1}A_{t_1-1}+y_{t_1-1}+w_{t_1-1}]+R^0_{t_1-1}$$

where $S^0_{t_1-1}=\delta S_{t_1}$ and $R^0_{t_1-1}=h_{t_1-1}+\delta R_{t_1}+\delta^{\alpha/(\alpha-1)}S^{\alpha/(\alpha-1)}_{t_1}[\frac{1}{\alpha}-1]$

(iii) if $L_{t_1-1}=$ interior

$$\frac{c^{\alpha}_{t_1-1}}{\alpha}+h_{t_1-1}(1-L_{t_1-1})+\delta[S_{t_1}(r_{t_1-1}A_{t_1-1}+y_{t_1-1}+w_{t_1-1}(1-L_{t_1-1})-c_{t_1-1})+R_{t_1}]$$
$$=\frac{c^{\alpha}_{t_1-1}}{\alpha}+\delta[S_{t_1}(r_{t_1-1}A_{t_1-1}+y_{t_1-1}+w_{t_1-1})-c_{t_1-1})+R_{t_1}]$$
$$=\delta S_{t_1}(r_{t_1-1}A_{t_1-1}+y_{t_1-1}+w_{t_1-1})+\delta R_{t_1}+\delta^{\alpha/(\alpha-1)}S^{\alpha/(\alpha-1)}_{t_1}[\frac{1}{\alpha}-1]$$
$$v_{t_1-1}=S^I_{t_1-1}[r_{t_1-1}A_{t_1-1}+y_{t_1-1}+w_{t_1-1}]+R^I_{t_1}$$

where $S^I_{t_1-1}=\delta S_{t_1}$ and $R^I_{t_1-1}=\delta R_{t_1}+\delta^{\alpha/(\alpha-1)}S^{\alpha/(\alpha-1)}_{t_1}[\frac{1}{\alpha}-1]$,since $h_{t_1-1}=\delta S_{t_1}w_{t_1-1}$ and the same equation for c holds.

B Figures

Figure 1: Family assets versus weekly hours in paid work 2006.

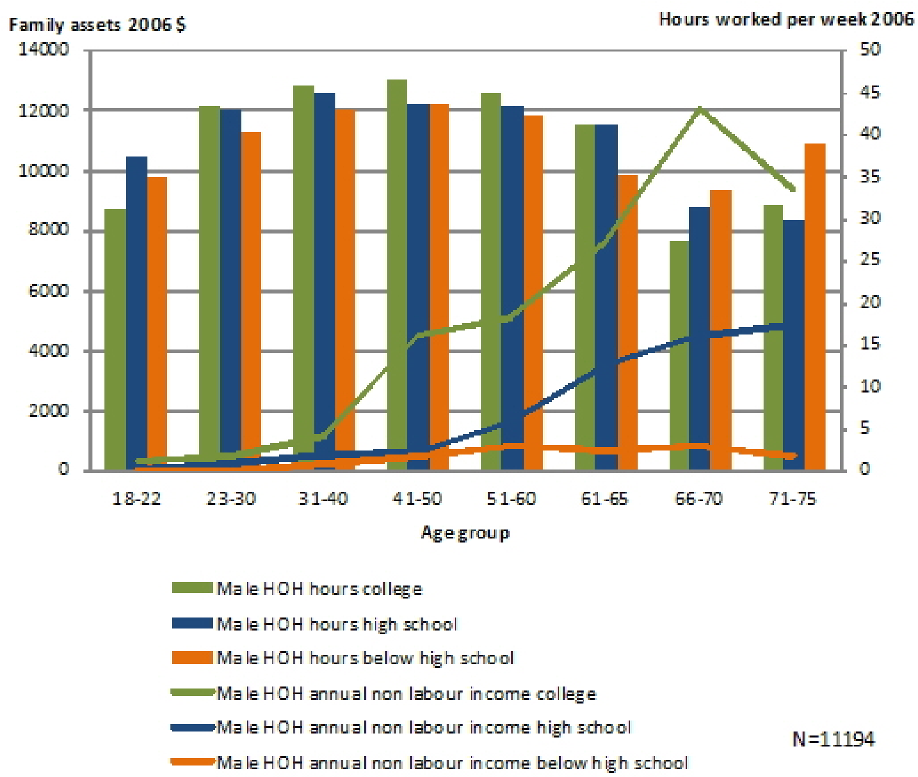

Figure 2: Annual non labour income versus weekly hours in paid work 2006.

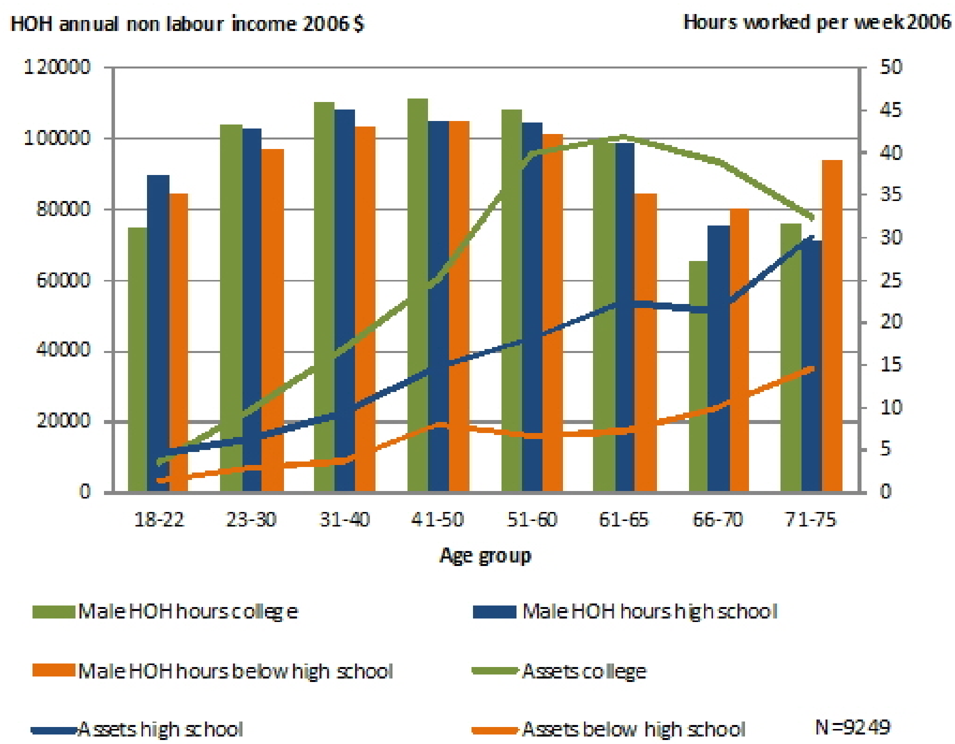

Figure 3: Hourly wage versus weekly hours in paid work 2006.

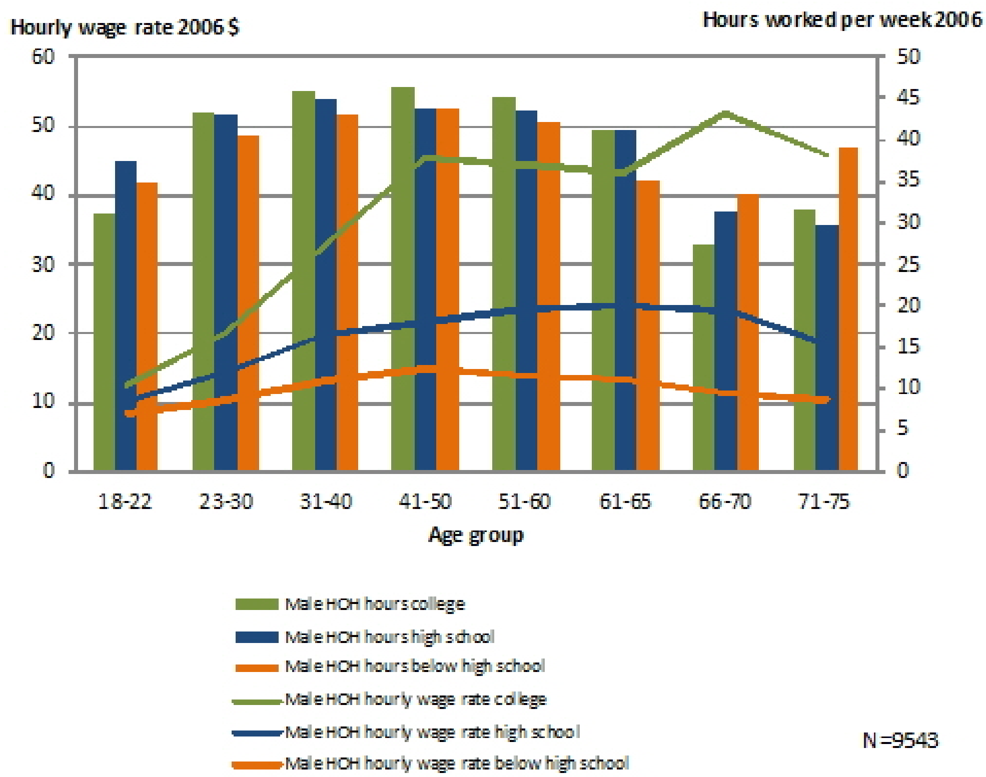

Figure 4: Stylised decision tree.