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Abstract

This paper develops the CCAPM model to allow for long-run risk in durable consumption. Allowing Epstein-Zin preferences to incorporate non-separability of durable and non-durable consumption in utility provides for an Euler equation which can be shown to provide a much better explanation of equity market features than either the basic CAPM or CCAPM. .The paper incorporates this discount factor into a model with long-run durable consumption risk and provides the first set of estimates of such a model. The analysis in the paper is for the UK. This is of independent interest. There is thus far no evidence for the UK on the abilities of either the durable consumption or long-run risk models. Moreover, the nature of the time series process that best explains non-durable consumption growth in the UK suggests that the standard non-durable long-run risk model is unlikely to fit the facts. In short, there is no evidence for the presence of a persistent, heteroskedastic component in non-durable consumption growth. However, there is some quite persuasive evidence that such a component exists in durable consumption growth. This paper provides positive evidence in respect of the equity premium, matching the risk-free rate and ability to explain the cross-section of equity returns.

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1 Introduction

It is a widely accepted fact that the consumption-based capital asset pricing model (CCAPM) fails to provide a good explanation of many important features of the behaviour of equity returns in a large range of countries over a long period of time. However, within a representative consumer/investor model, it is hard to see how the basic structure of the consumption based model can be safely abandoned. As a result much effort has been put into generalisations of the model which relax some of the most extreme assumptions and introduce realistic additional sources of correlation between elements of consumer choice and asset returns. Some of the most promising generalisations are those offered by the replacement of the assumption of power utility with utility of the recursive form proposed by Kreps and Porteus (1978), Epstein and Zin (1989) and Weil (1989). Initial empirical analysis of the impact of allowing attitudes to risk to differ from attitudes to time as this approach allows were not very successful (see Smith, Sorensen and Wickens (2008), for example). However, Bansal and Yaron (2004) pointed out that this distinction could be used to good effect if consumption contained a small persistent and heteroskedastic component. They also showed that this would be most effective in explaining important features of the behaviour of equity returns if the elasticity of intertemporal substitution were large enough. Empirical analysis of the long-run risk model has so far been very limited. The evidence in Bansal and Yaron (2004) has recently been questioned by Beeler and Campbell (2009) and Constantinides and Ghosh (2008) who are more sanguine. A separate generalisation of the consumption-based model is offered by Yogo (2006) who re-examines the role of durable and non-durable goods. He shows that allowing Epstein-Zin preferences to incorporate non-separability of durable and non-durable consumption in utility provides for an Euler equation which can be shown to provide a much better explanation of equity market features than either the basic CAPM or CCAPM. This analysis is at the level of the Euler equation and takes the rate of return on total wealth as given. In this paper we develop the durable consumption model to allow for long-run risk in durable consumption. The paper provides the first set of estimates of such a model and finds the initial evidence to be favourable to the model. The analysis in the paper is for the UK. There are a number of reasons why this is of independent interest. There is thus far no evidence for the UK on the ability of either the durable consumption or long-run risk models. Moreover, the nature of the time series process that best explains non-durable consumption growth in the UK suggests that the standard non-durable long-run risk model is unlikely to fit the facts. In short, there is no evidence for the presence of a persistent, heteroskedastic component in non-durable consumption growth. However, there is some quite persuasive evidence that such a component exists in durable consumption growth. Yang (2009) provides some simulation evidence for the durable long-run risk model for the US but no direct estimates. The paper is set out as follows. In Section 2 the theoretical framework for asset pricing with durable and non-durable consumption is set out. The long-run durable consumption risk model is outlined in Section 3. In Section 4 the model is generalised to allow for the elasticity of substitution of durable and non-durable consumption in utility to differ from one and for dividends and tyotal consumption to be cointegrated. This then implies that dividends and consumption cannot deviate from each other in the long run. It also means that in the short run their growth can deviate from each other, allthough only by a stationary amount. In Section 5 the data analysed in this paper are presented and, in particular, the construction of the stock of durable consumption is explained. Estimation results are presented in Section 6. Some conclusions are presented in Section 7.

2 Pricing Equity Risk with Durable and Non-Durable Consumption

In the approach followed in this paper the representative investor consumes two types of good; non-durable and durable. Non-durable goods are assumed to be consumed C_t within the single period in which they are purchased whilst durable goods provide a service flow for periods beyond the period in which they are purchased C_t^d. Consumers are assumed to consume and gain utility from these service flows. The stock of durables D_t that consumers accumulate moves through time following:

$$D_t = (1-\delta)D_{t-1} + C_t^d \tag{1}$$

for a depreciation rate δ. The service flow from the durable stock is assumed to be linear in the durable stock itself and so from hereon we refer to D_t interchangeably as the durable stock or durable consumption. Preferences follow the widely used structure proposed by Kreps and Porteus (1978), Epstein and Zin (1989) and Weil (1989). Utility in any period is a combination of non-durable and durable consumption which follows the CES formulation:

$$u_t = \left[(1-\alpha)C_t^{1-\frac{1}{\rho}} + \alpha D_t^{1-\frac{1}{\rho}}\right]^{\frac{1}{1-\frac{1}{\rho}}} \tag{2}$$

where α is a weight of durables in utility and ρ is the elasticity of substitution between durables and non-durables in utility. The marginal rate of substitution between durable and non-durable consumption is:

$$\frac{u^d}{u^c} = \frac{\alpha}{1-\alpha}\left(\frac{C}{D}\right)^{\frac{1}{\rho}} \tag{3}$$

This intraperiod utility function u_t is part of the recursive intertemporal function

$$\mathcal{U}_t = \left[(1-\beta)u_t(C_t, D_t)^{1-\frac{1}{\psi}} + \beta[E_t(\mathcal{U}_{t+1}^{1-\gamma})]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}} \tag{4}$$

where $\beta < 1$ is assumed to measure pure time preference, ψ is the elasticity of intertemporal substitution and γ the coefficient of relative risk aversion. Two restrictions on these parameters provide simplifications of this framework. If $\psi = \rho$ the elasticity of substitution between the two goods is equal to the elasticity of intertemporal substitution which make utility additively separable. If $\psi = 1/\gamma$ the elasticity of intertemporal substitution is equal to the inverse of the coefficient of relative risk aversion making utility take the expected utility form. If $\psi = 1/\gamma = \rho$ utility tales the additively separable expected utility form which underlies the consumption-based capital asset pricing model (CCAPM)

With these preferences, the pricing kernel or marginal rate of substitution is given by:

$$M_{t+1} = \left[\beta(\frac{C_{t+1}}{C_t})^{-\frac{1}{\psi}} \left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)^{\frac{1}{\rho}-\frac{1}{\psi}}\right]^{\frac{1-\gamma}{1-\frac{1}{\psi}}} (R_{h,t+1})^{\frac{1-\gamma}{1-\frac{1}{\psi}}-1} \tag{5}$$

for the gross return on total wealth $R_{h,t+1}$, where

$$v\left(\frac{D_t}{C_t}\right) = \left[1-\alpha+\alpha\left(\frac{D_t}{C_t}\right)^{1-\frac{1}{\rho}}\right]^{\frac{1}{(1-1/\rho)}} \tag{6}$$

As Bansal, Tallarini and Yaron (2008), Yogo (2006) and others show, the first-order condition for household utility maximisation and consumption and portfolio choice with durable and non-durable consumption generates an Euler equation for each risky asset with gross return $R_{i,t+1}$:

$$1 = E_t[M_{t+1}R_{i,t+1}] \tag{7}$$

or equivalently, in terms of the excess return of each asset over the gross risk-free rate $R_{f,t+1}$:

$$0 = E_t[M_{t+1}(R_{i,t+1} - R_{f,t+1})] \tag{8}$$

Similar related arguments show that an intratemporal first-order condition exists which ties together marginal utility for durable and non-durable consumption as:

$$\frac{u_t^d}{u_t^c} = Q_t = P_t - (1-\delta)E_t[M_{t+1}P_{t+1}] \tag{9}$$

for the relative price of durable to non-durable consumption P_t defining the service cost of durables Q_t. Total consumption can therefore be written as:

$$H_t = C_t + Q_tD_tP_t \tag{10}$$

As Ogaki and Reinhart (1998) and Yogo (2006) show, this intratemporal first-order condition provides an alternative source of an estimate of the elasticity of substitution ρ. Taking logs of equations (3) and (9):

$$\ln\left(\frac{\alpha}{1-\alpha}\right)+\frac{1}{\rho}(c_t-d_t)-p_t=q_t-p_t \tag{11}$$

where lower case variables are upper case variables in logarithms. So if the real value of the user cost of durables is stationary, whilst c_t, d_t and p_t are non-stationary, then cointegration between relative consumption and relative prices: (c_t-d_t) and p_t could deliver a super-consistent estimate of the elasticity of substitution ρ.

2.1 The log-linear model

An alternative formulation of the pricing problem which provides a set of testing equations which are analogous to those used in many empirical exercises is the log-linear form of the model. Log-linearisation of the pricing kernel in equations (5) and (6) around $\rho = 1$, or Cobb-Douglas intraperiod utility, generates:

$$m_{t+1}\simeq\frac{1-\gamma}{1-\frac{1}{\psi}}\left[\ln\beta-(\frac{1}{\psi}+\alpha(\frac{1}{\rho}-\frac{1}{\psi}))\Delta c_{t+1}+\alpha(\frac{1}{\rho}-\frac{1}{\psi})\Delta d_{t+1}\right]-\left(1-\frac{1-\gamma}{1-\frac{1}{\psi}}\right)r_{h,t+1} \tag{12}$$

where $r_{h,t+1}$ is the log gross return. Assuming joint log-normality of all of the variables concerned, an unconditional version of equation (8) can be given a log-linear interpretation as:

$$\begin{aligned}&E\left[r_{i,t+1}-r_{f,t+1}\right]-Var\left[r_{i,t+1}-r_{f,t+1}\right] \qquad (13)\\ =\ &\alpha_1 Cov(\Delta c_{t+1},r_{i,t+1}-r_{f,t+1})+\alpha_2 Cov(\Delta d_{t+1},r_{i,t+1}-r_{f,t+1})\\ &+\alpha_3 Cov(r_{h,t+1},r_{i,t+1}-r_{f,t+1})\end{aligned}$$

where $\alpha_1=\frac{1-\gamma}{1-\frac{1}{\psi}}(\frac{1}{\psi}+\alpha(\frac{1}{\rho}-\frac{1}{\psi}))$, $\alpha_2=\frac{1-\gamma}{1-\frac{1}{\psi}}\alpha(\frac{1}{\rho}-\frac{1}{\psi})$ and $\alpha_3=1-\frac{1-\gamma}{1-\frac{1}{\psi}}$ and all rates of return are expressed as log gross rates. $Var\left[r_{i,t+1}-r_{f,t+1}\right]$ is the Jensen adjustment term which arises from the lognormal approximation.

3 A Model of Long-Run Durable Consumption Risk

The durable consumption pricing model described above provides for a set of testing equations which result from first-order conditions tying together durable and non-durable consumption growth, the rate of return on total wealth and the excess returns on any risky asset held by the consumer. They do not usually examine the properties of the general equilibrium model within which these first-order conditions might fit. Or, to put it differently, they do not model consumption growth and/or the return to total wealth. Two exceptions to this are the long-run risk model of Bansal and Yaron (2004) and the augmented CAPM described initially in Campbell (1993). In the approach of Campbell (1993), consumption growth is substituted out using the intertemporal budget constraint and the whole model is expressed in terms of financial returns. Here, we take the analysis of the first-order conditions from the durable model and use them to evaluate the validity of the long-run risk model. What we will test are joint restrictions of the long-run risk and durable models.

The structure of the long-run risk model is built upon a set of time-series processes for consumption and dividends plus some approximations of the intertemporal budget constraint:

$$\Delta d_{t+1} = \mu_d + x_t + \sigma_t z_{d,t+1} \tag{14}$$

$$\Delta c_{t+1} = \mu_c + z_{c,t+1} \tag{15}$$

$$x_{t+1} = \rho_x x_t + \varphi_x \sigma_t z_{x,t+1} \tag{16}$$

$$\sigma_{t+1}^2 = (1-\nu)\overline{\sigma}^2 + \nu\sigma_t^2 + \sigma_w z_{\sigma,t+1} \tag{17}$$

$$\Delta f_{t+1} = \mu_f + \phi x_t + \varphi_f \sigma_t z_{f,t+1} \tag{18}$$

where durable consumption growth is driven by a persistent process x_t and σ_t^2 is the conditional variance of durable consumption with a mean value $\overline{\sigma}^2$. Non-durable consumption growth, by contrast, is a homoskedastic random process with no persistence. This is in contrast to the standard version of the model in Bansal and Yaron (2004) and elsewhere but, as will be demonstrated, is more consistent with the UK data.[2] . Dividend growth Δf_t is also driven by the persistent process driving durable consumption growth with a parameter ϕ and conditional volatility of dividend growth is also proportional to the conditional volatility of durable consumption growth. The shocks to all of these processes, $(z_{d,t+1}, z_{c,t+1}, z_{x,t+1}, z_{\sigma,t+1}, z_{f,t+1})$ are distributed *iid*, $N(0,1)$. Their mutual independence focuses all of the transmission of shocks through the processes for durable consumption and dividends and their underlying persistence and heteroskedasticity. The model therefore has two state variables; the persistent process x_{t+1} and the heteroskedastic process σ_{t+1}^2, as with Bansal and Yaron. However, x_{t+1} is the persistent process in durable consumption growth whilst non-durable consumption growth is assumed to not be persistent.

We solve the model using analytical approximations following the method of Bansal and Yaron. These are built on log-linear approximations for the log return on the total consumption claim $r_{h,t}$ and on the return on the market portfolio $r_{m,t}$ following Campbell and Shiller (1988). These approximations provide a relation between the return on the consumption claim, non durable

[2] Yang (2009) also examines a long-run risk model for durable consumption but his focus is on simulations of a calibrated model and his model is a one-state variable version of.our model.

consumption growth and the price/consumption ratio[3] .

$$r_{h,t+1} = \kappa_0 + \kappa_1 z_{h,t+1} + \Delta c_{t+1} - z_{h,t+1} \tag{19}$$

$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} + \Delta f_{t+1} - z_{m,t+1} \tag{20}$$

where $z_{h,t}$ is the log price/consumption ratio (the price of the claim to future consumption divided by current consumption) and $z_{m,t}$ the log price of equity/dividend ratio and the κ's are linearisation parameters. In particular, $\kappa_1 = \frac{e^{\overline{z}}}{1+e^{\overline{z}}}$ and $\kappa_0 = \log(1 + e^{\overline{z}}) - \kappa_1 \overline{z}$ for the long-run mean of the log price/consumption ratio $\overline{z}$. In a similar way, $.\kappa_{1,m} = \frac{e^{\overline{z_m}}}{1+e^{\overline{z_m}}}$ and $\kappa_{0,m} = \log(1 + e^{\overline{z_m}}) - \kappa_1 \overline{z_m}$ for the long-run mean of the price/dividend ratio $\overline{z_m}$. Bansal and Yaron (2004) show that $z_{h,t}$ and $z_{m,t}$ can be written as affine functions of the two state variables x_t and σ_t^2

$$z_{h,t} = A_0 + A_1 x_t + A_2 \sigma_t^2 \tag{21}$$

$$z_{m,t} = A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2 \tag{22}$$

where the A coefficients are functions of the underlying preference and time-series process parameters. The appendix to this paper provides solutions to the values of $A_0, A_1, A_2, A_{0,m}, A_{1,m}$ and $A_{2,m}$ in terms of the structural parameters. These show versions of the original Bansal-Yaron results that the long-run risk component of consumption growth x_t will have a positive impact on the valuation of future consumption and the variance of that component σ_t^t a negative effect if the intertemporal elasticity of substitution $\psi > 1$. The risk-free interest rate can also be written as an affine function of the two state variables x_t and σ_t^2

$$r_{f,t} = -\ln E_t \left[\exp(m_{t+1})\right] \tag{23}$$

$$= A_{0,f} + A_{1,f} x_t + A_{2,f} \sigma_t^2 \tag{24}$$

where the appendix provides solutions to $A_{0,f}, A_{1,f}$ and $A_{2,f}$. As Constantinides and Ghosh (2009) show, the two the two state variables x_t and σ_t^2 can be solved for in terms of the observable variables, the risk free rate $r_{f,t}$ and $z_{m,t}$ the log price/dividend ratio by jointly inverting equations (24) and (22) to give

$$x_t = \alpha_0 + \alpha_1 r_{f,t} + \alpha_2 z_{m,t} \tag{25}$$

$$\sigma_t^t = \beta_0 + \beta_1 r_{f,t} + \beta_2 z_{m,t} \tag{26}$$

[3] Cobb-Douglas intraperiod utility implies that durable and non-durable consumption have constant shares of total consumption. As Yang (2009) points out, this then means that we can simplify the expressions for total consumption by not having to include the durable/non-durable consumption ratio as an additional state variable.

where the solutions to $\alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1$ and β_2 are given in the appendix. The log-linearised version of the pricing kernel in equation (12) is:

$$m_{t+1} \simeq \frac{1-\gamma}{1-\frac{1}{\psi}} \left[\ln \beta - (\frac{1}{\psi} + \alpha(\frac{1}{\rho} - \frac{1}{\psi}))\Delta c_{t+1} + \alpha(\frac{1}{\rho} - \frac{1}{\psi})\Delta d_{t+1} \right] - \left(1 - \frac{1-\gamma}{1-\frac{1}{\psi}} \right) r_{h,t+1} \quad (27)$$

Initially, we examine the model for the restriction of $\rho = 1$ which implies that:

$$m_{t+1} \simeq \frac{1-\gamma}{1-\frac{1}{\psi}} \left[\ln \beta - (\frac{1}{\psi} + \alpha(1 - \frac{1}{\psi}))\Delta c_{t+1} + \alpha(1 - \frac{1}{\psi})\Delta d_{t+1} \right] - \left(1 - \frac{1-\gamma}{1-\frac{1}{\psi}} \right) r_{h,t+1} \quad (28)$$

and substituting in the affine approximation to the log return on total wealth, $r_{h,t+1}$ from equation (19) and using equation (21) to further substitute for $z_{h,t}$

$$\begin{aligned} m_{t+1} &= (\theta \ln \delta + (\theta - 1)[\kappa_0 + (\kappa_1 - 1)A_0]) + \theta \left(1 - \frac{1}{\psi}\right) \alpha(\Delta d_{t+1} - \Delta c_{t+1}) \quad (29) \\ &\quad - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)\kappa_1 A_1 x_{t+1} + (\theta - 1)\kappa_1 A_2 \sigma_{t+1}^t - (\theta - 1)A_1 x_t - (\theta - 1)A_2 \sigma_t^t \end{aligned}$$

where $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. This can be expressed in terms of observable variables by using equations (24) and (26) to substitute for the state variables to give:

$$m_{t+1} = c_0 + c_1 \Delta c_{t+1} + c_2(\Delta d_{t+1} - \Delta c_{t+1}) + c_3 \left(r_{f,t+1} - \frac{1}{\kappa_1} r_{f,t} \right) + c_4 \left(z_{m,t+1} - \frac{1}{\kappa_1} z_{m,t} \right) \quad (30)$$

where:

$$\begin{aligned} c_0 &= (\theta \ln \delta + (\theta - 1)[\kappa_0 + (\kappa_1 - 1)A_0]) \\ c_1 &= -\frac{\theta}{\psi} \\ c_2 &= \theta \left(1 - \frac{1}{\psi}\right) \alpha \\ c_3 &= (\theta - 1)\kappa_1 [A_1 \alpha_1 + A_2 \beta_1] \\ c_4 &= (\theta - 1)\kappa_1 [A_1 \alpha_2 + A_2 \beta_2] \end{aligned}$$

This log discount factor is observable and can thus be used in a test of the durable long-run consumption risk model. Thus the model to be estimated is a version of the linear model in equation (**??**):

$$\begin{aligned} & E[r_{i,t+1} - r_{f,t+1}] - Var[r_{i,t+1} - r_{f,t+1}] \quad (31) \\ = \; & c_1 Cov(\Delta c_{t+1}, r_{i,t+1} - r_{f,t+1}) + c_2 Cov((\Delta d_{t+1} - \Delta c_{t+1}), r_{i,t+1} - r_{f,t+1}) \\ & + c_3 Cov(\left(r_{f,t+1} - \frac{1}{\kappa_1} r_{f,t} \right), r_{i,t+1} - r_{f,t+1}) + c_4 Cov(\left(z_{m,t+1} - \frac{1}{\kappa_1} z_{m,t} \right), r_{i,t+1} - r_{f,t+1}) \end{aligned}$$

4 Long-Run Durable Consumption Risk when Consumption and Dividends are Cointegrated

The specification above ties the dynamics of consumption and dividends closely together in that both the mean and variance of dividends are driven by the behaviour of the persistent process driving durable consumption.Thereis, however, nothing tying the levels of consumption and dividends together. In particular, there is no restriction to ensure that their levels don't deviate from each other in the long run. As Bansal, Gallant and Tauchen (2007) show, this can be acheived by having consumption and dividends be cointegrated. The model therefore becomes:

$$\Delta d_{t+1} = \mu_d + x_t + \sigma_t z_{d,t+1} \tag{32}$$

$$\Delta c_{t+1} = \mu_c + z_{c,t+1} \tag{33}$$

$$x_{t+1} = \rho_x x_t + \varphi_x \sigma_t z_{x,t+1} \tag{34}$$

$$\sigma^2_{t+1} = (1-\nu)\overline{\sigma}^2 + \nu\sigma^2_t + \sigma_w z_{\sigma,t+1} \tag{35}$$

$$d_t - g_t = \mu_{dg} + s_t \tag{36}$$

$$g_t = a_0 + (1-a_1)c_t + a_1 c d_t \tag{37}$$

$$s_{t+1} = \lambda_{sx} x_t + \rho_s s_t + \varphi_s \sigma_t z_{s,t+1} \tag{38}$$

$$\Delta f_{t+1} = (1-a1) * \Delta c_{t+1} + a_1 * \Delta d_{t+1} + \Delta s_{t+1} \tag{39}$$

where g_t is total consumption and s_t is the dividend/consumption ratio. s_t is assumed to be stationary but time-varying, driven by the persistent durable consumption process x_t and subject to a shock process made up of an indepenent error $z_{s,t}$ and the variance process σ_t^2 in equation (38). The shock $z_{s,t+1}$ is also distributed iid, $N(0,1)$. Log total consumption is a linear combination of log durable and non-durable consumption using the definition in equation (10) above. This specification implies that the growth of dividends follows the process in (39). In this version of the model we also allow for the elasticity of substitution between durables and non-durables in utility ρ to differ from one. It can be shown that the parameter a_1 is equal to zero when $\rho = 1$ as in the version of model presented in Section 3.

Solution of this version of the model follows that for the first version. The main difference is that an additional state variable s_t has been introduced and that the model implies that this is a further priced source of risk.Analagous relations for the log return on the total consumption claim $r_{h,t}$,the return on the market portfolio $r_{m,t}$, $z_{h,t}$ the log price/consumption ratio and $z_{m,t}$ the log price of equity/dividend ratio are:

$$
\begin{aligned}
z_{h,t} &\backsimeq A_0 + A_1 x_t + A_2 \sigma_t^2 + A_3 s_t \\
r_{h,t+1} &= \kappa_0 + \kappa_1 z_{g,t+1} + \Delta g_{t+1} - z_{g,t}
\end{aligned}
$$

$$
\begin{aligned}
z_{m,t} &\backsimeq A_{0m} + A_{1m} x_t + A_{2m} \sigma_t^2 + A_{3m} s_t \\
r_{m,t+1} &= \kappa_{0m} + \kappa_{1m} z_{m,t+1} - z_{m,t} + \Delta f_{t+1}
\end{aligned}
\tag{40}
$$

where the solutions to the parameters are given in the appendix. The expression for the risk-free rate is:

$$
r_{f,t} = A_{0f} + A_{1f} x_t + A_{2f}(\sigma_t^2 - \sigma^2) + A_{3f} s_t \tag{41}
$$

again, the two the two state variables x_t and σ_t^2 can be solved for in terms of the observable variables, the risk free rate $r_{f,t}$ and $z_{m,t}$ the log price/dividend ratio and now the dividend/consumption ratio s_t by jointly inverting equations (41) and (40) to give

$$
x_t = \alpha_0 + \alpha_1 r_{f,t} + \alpha_2 z_{m,t} + \alpha_3 s_t \tag{42}
$$

$$
\sigma_t^2 = \beta_0 + \beta_1 r_{f,t} + \beta_2 z_{m,t} + \beta_3 s_t \tag{43}
$$

The log discount factor is:

$$
m_{t+1} \simeq \theta \ln \beta - \frac{\theta}{\varphi} \Delta c_{t+1} + \theta \alpha (\frac{1}{\rho} - \frac{1}{\varphi})(\Delta d_{t+1} - \Delta c_{t+1}) + (\theta - 1) r_{h,t+1} \tag{44}
$$

$$
\text{where } \theta = \frac{1-\gamma}{1-\frac{1}{\varphi}} \tag{45}
$$

substituting for the return on total wealth, it becomes:

$$
\begin{aligned}
m_{t+1} \simeq\ & \theta \ln \beta - \frac{\theta}{\varphi} \Delta c_{t+1} + \theta \alpha (\frac{1}{\rho} - \frac{1}{\varphi})(\Delta d_{t+1} - \Delta c_{t+1}) + (\theta - 1) r_{g,t+1} \\
=\ & \theta \ln \beta + [\theta \alpha (\frac{1}{\rho} - \frac{1}{\varphi}) + (\theta - 1)(\kappa_1 A_1 \rho_x + \kappa_1 A_3 \lambda_{sx} + a_1 - A_1)] x_t + (\theta - 1)(\kappa_1 \rho_s - 1) A_3 s_t \\
& + (\theta - 1)(\kappa_1 \nu - 1) A_2 (\sigma_t^2 - \sigma^2) + (\theta - 1)[(\kappa_1 A_2 - A_2)\sigma^2 + \kappa_0 + \kappa_1 A_0 - A_0] \\
& + [(\theta - 1)(1 - a_1) - \frac{\theta}{\varphi} - \theta \alpha (\frac{1}{\rho} - \frac{1}{\varphi})] \mu_c + [\theta \alpha (\frac{1}{\rho} - \frac{1}{\varphi}) + a_1(\theta - 1)] \mu_d \\
& + (\theta - 1)(\kappa_1 A_1 \varphi_x \sigma_t z_{x,t+1} + \kappa_1 A_2 \sigma_w z_{\sigma,t+1} + \kappa_1 A_3 \varphi_s \sigma_t z_{s,t+1}) \\
& + [(\theta - 1)(1 - a_1) - \frac{\theta}{\varphi} - \theta \alpha (\frac{1}{\rho} - \frac{1}{\varphi})] z_{c,t+1} + [(\theta - 1) a_1 \\
& + \theta \alpha (\frac{1}{\rho} - \frac{1}{\varphi})] \sigma_t z_{d,t+1}
\end{aligned}
$$

and, by simplifying, then:

$$\begin{aligned} m_{t+1} &= S_0 x_t + S_1(\sigma_t^2 - \sigma^2) + C + (\theta - 1)(\kappa_1 A_1 \varphi_x \sigma_t z_{x,t+1} + \kappa_1 A_2 \sigma_w z_{\sigma,t+1} + \kappa_1 A_3 \varphi_s \sigma_t z_{s,t+1}) \\ &\quad + [(\theta - 1)(1 - a_1) - \frac{\theta}{\varphi} - \theta\alpha(\frac{1}{\rho} - \frac{1}{\varphi})] z_{c,t+1} + [(\theta - 1)a_1 + \theta\alpha(\frac{1}{\rho} - \frac{1}{\varphi})]\sigma_t z_{d,t+1} \end{aligned}$$

where,

$$\begin{aligned} S_0 &= \alpha(\frac{1}{\rho} - \frac{1}{\varphi}) \\ S_1 &= (\theta - 1)(\kappa_1 \nu - 1)A_2 \\ C &= (\theta - 1)[(\kappa_1 A_2 - A_2)\sigma^2 + \kappa_0 + \kappa_1 A_0 - A_0] \\ &\quad + [(\theta - 1)(1 - a_1) - \frac{\theta}{\varphi} - \theta\alpha(\frac{1}{\rho} - \frac{1}{\varphi})]\mu_c + [\theta\alpha(\frac{1}{\rho} - \frac{1}{\varphi}) + a_1(\theta - 1)]\mu_d + \theta \ln \beta \end{aligned}$$

Using the definitions of the state variables, the discount factor then becomes

$$\begin{aligned} m_{t+1} &= c_0 + c_1 \Delta c_{t+1} + c_2(\Delta d_{t+1} - \Delta c_{t+1}) + c_3 \left(r_{f,t+1} - \frac{1}{\kappa_1} r_{f,t} \right) + c_4 \left(z_{m,t+1} - \frac{1}{\kappa_1} z_{m,t} \right) \\ &\quad + c_5(s_{t+1} - \frac{1}{k_1} s_t) \end{aligned} \tag{46}$$

where:

$$\begin{aligned} c_0 &= \theta \ln \beta + (\theta - 1)\kappa_0 + (\theta - 1)A_0(\kappa_1 - 1) + (\theta - 1)A_1(\kappa_1 - 1)\alpha_0 \\ &\quad + (\theta - 1)A_t(\kappa_1 - 1)\beta_0 \\ c_1 &= (\theta - 1)(1 - a_1) - \frac{\theta}{\varphi} - \theta\alpha(\frac{1}{\rho} - \frac{1}{\varphi}) \\ c_2 &= \theta\alpha(\frac{1}{\rho} - \frac{1}{\varphi}) + (\theta - 1)a_1 \\ c_3 &= (\theta - 1)\kappa_1 [A_1 \alpha_1 + A_2 \beta_1] \\ c_4 &= (\theta - 1)\kappa_1 [A_1 \alpha_2 + A_2 \beta_2] \\ c_5 &= (\theta - 1)\kappa_1 [A_1 \alpha_3 + A_2 \beta_3 + A_3] \end{aligned}$$

This log discount factor is observable and can thus be used in a further test of the durable long-run consumption risk model. Thus the model to be estimated is a further version of the linear model in equation (??):

$$
\begin{aligned}
& E\left[r_{i,t+1}-r_{f,t+1}\right]-Var\left[r_{i,t+1}-r_{f,t+1}\right] \qquad (47)\\
= \; & c_1 Cov(\Delta c_{t+1},(r_{i,t+1}-r_{f,t+1}))+c_2 Cov((\Delta d_{t+1}-\Delta c_{t+1}),(r_{i,t+1}-r_{f,t+1}))\\
& +c_3 Cov(\left(r_{f,t+1}-\frac{1}{\kappa_1}r_{f,t}\right),(r_{i,t+1}-r_{f,t+1}))+c_4 Cov(\left(z_{m,t+1}-\frac{1}{\kappa_1}z_{m,t}\right),(r_{i,t+1}-r_{f,t+1}))\\
& +c_5 Cov(\left(s_{t+1}-\frac{1}{\kappa_1}s_t\right),(r_{i,t+1}-r_{f,t+1}))
\end{aligned}
$$

5 Data

5.1 Consumption Data

All of the consumption data comes from the ONS (Office for National Statistics) publication *Consumer Trends* and it's associated databases. The timing convention that is used is that consumption is measured at the end of any quarter (i.e. it measures the flow over the quarter). Likewise, all other variables are measured at the end of the quarter. The measure of non-durable consumers expenditure is spending on non and semi-durable goods and services. The remainder of total expenditure is spending on durable goods. This spending is used to construct a measure of the total net stock of consumer durables for the UK using equation (1). Unlike for the United States, no regularly published official series exist for the stock of consumer durables. Recent calculations of the stock of durables include Hamilton and Morris (2002), Solomou and Weale (1995) and Williams (1997). The details of the calculations used to construct the durable stock measure used in this paper are described in Guo (2010). The durable stock measure is constructed from applying a current widely used version of the Perpetual Inventory Method (PIM) as described in Bureau for Business, Enterprise and Regulatory Reform (2008) along with recent service life, retirement distribution and depreciation assumptions. Expenditures on 15 sub-groups of durable spending are used to construct the total measure. Comparison between the new series and that proposed by Williams (1997) presented in Guo (2010) shows a close association but evidence that the new measure behaves in a more intuitive way in some periods of turbulence such as the 1970's. The implicit price indices for non-durable and durable expenditure are used as deflators in this paper. The consumption series are all scaled by the size of the adult population. The data are summarised in Table 1.

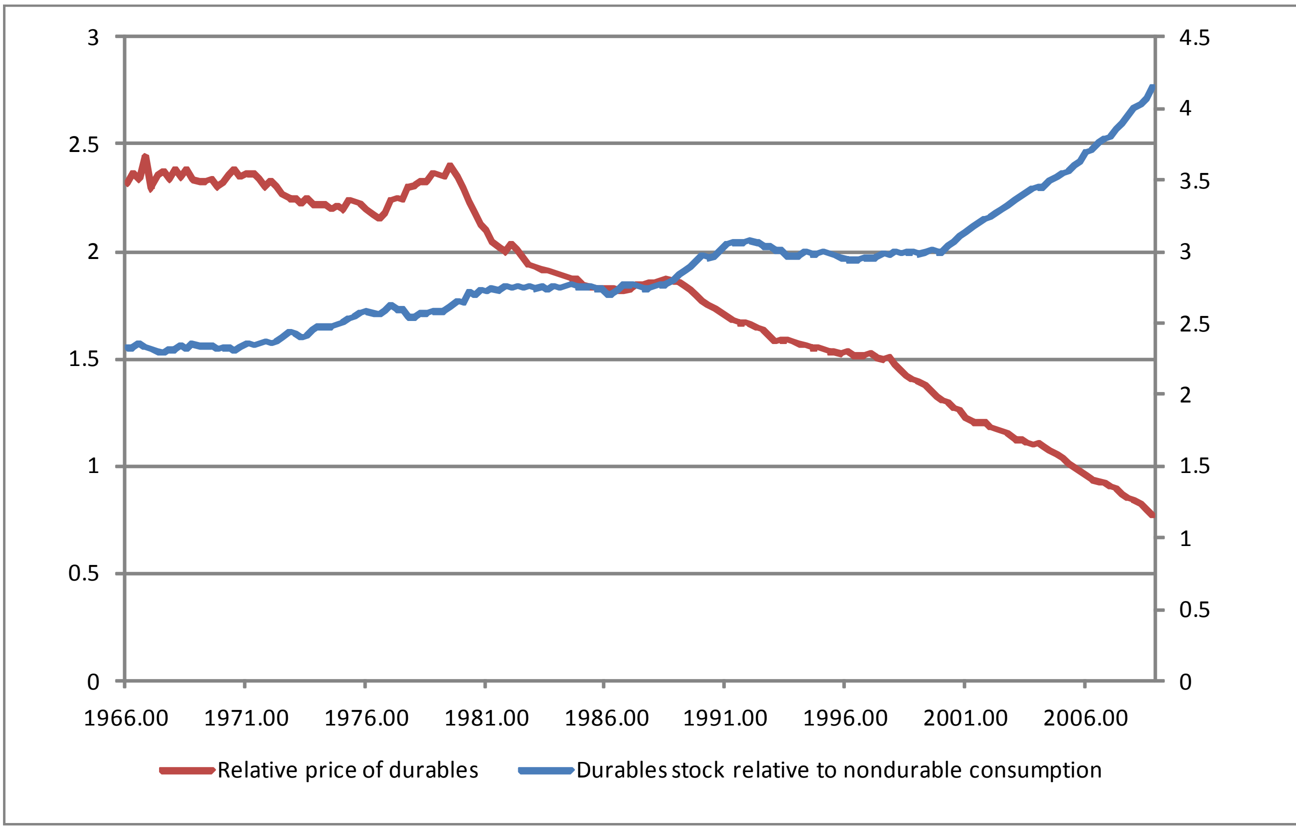

Figure 1 plots durable relative to nondurable consumption and the relative price of durable and nondurable expenditure for the period 1966 - 2008. Durable relative to non-durable consumption has tended to drift upward over time whilst the relative price of durable versus non-durable expenditure has generally trended downwards. These two trends are consistent with each other.

As we showed above, a super-consistent estimate of the elasticity of substitution between the durable and nondurable consumption, ρ can be obtained by cointegration methods. According to both ADF and KPSS tests, in our current data, $c_t - d_t$ and p_t have unit roots. We also find from the Engle-Granger two-step method an estimate of $\widehat{\rho} = 0.538$ with standard error of 0.01103. The cointegration ADF test statistic is 4.35 with a marginal significance level of 0.1136. Whilst not very decisive, this approach provides an estimate of ρ which we can employ in the estimation of the asset pricing Euler equations.

5.2 Test Asset Returns Data

Testing the cross-section and time-series abilities of US equity pricing models is routinely carried out on the portfolio return series based on the CRSP database which have been computed by Kenneth French and published on his web site. The basis for the construction of the portfolios are the size and book to market value characteristics found to be empirically important by Fama and French (1993, for example). No comprehensive and comparable set of portfolio returns has been available for the UK until recently. The first such set of portfolio returns was computed by Dimson, Nagel and Quigley (2003) for the period to 2001. More recently, Gregory, Tharyan and Huang (2009) have produced a more comprehensive and updated set of portfolio returns from 1980 to 2008. In the current study we focus on the most recent period offered by Gregory, Tharyan and Huang (2009) and provide some analysis of the stability and robustness of the results by analysing earlier periods of data. The original Gregory, Tharyan and Huang (2009) data is monthly and this is accumulated into quarterly end of quarter returns consistent with the rest of the data.

There are four sets of test assets: A) The market return and risk-free rate of return measured by the Datastream UK total market returns and 3 month Treasury bill interest rate; B) Returns for the UK Fama-French factors. Following Fama and French (1993), SMB is the average return on the three smallest-cap portfolios, minus the average return on the three largest-cap portfolios in a set of six size portfolios; HML is the average of the returns on the two highest book-to-market (value) portfolios minus the average of the returns on the two lowest book-to-market (growth) portfolios in a set of six book-to-market portfolios. Gregory, Tharyan and Huang (2009) use 70% up the gradient of market value as the breakpoint for size instead of the 50% used by Fama and French (1993). This is due to the negative correlation between size and value in the UK. C) Fama-French portfolio returns and the risk free rates. D) Returns on the 6 UK size and book-to-market portfolios that were used to construct the SMB and HML factors in B, above.

The nominal quarterly risk-free rate is the 3-month Treasury Bill interest rate. The real risk-free rate is calculated as the nominal T-bill rate divided by the inflation rate, calculated as the growth rate of the nondurable consumer expenditure deflator. Data for 1980 Q4 to 2008 Q4 is from Gregory, Tharyan and Huang (2009). For robustness checking, data from 1966 Q1 to 1980 Q3 from Dimson, Nagel and Quigley (2003) is employed.

Table 4 reports the summary statistics for portfolio returns and their correlations. Panel A presents the Mean, Standard Error, Kurtosis, Skewness and Range seperately for two periods.Some similarities and differences occur due to two data sources. (Detailed comparison can be found in Gregory, Tharyan and Huang (2009)). The SMB are insiginificantly different from 0 in both periods.By contrast, both versions of HML factors are more significant. The value premium is 1.92% in the first period; it is 1.5% in the latest period. The kurtosis and skewness differ from the two datasets for HML returns. The HML returns have a 0.1736 kurtosis and 0.463 skewness ended in 1980 Q3. From 1980 Q4 to 2008 Q4, the kurtosis is 4.301 and the skewness is -0.297 for the HML.These differences in the test assets may lead to minor differences in the empirical estimations. Panel B shows the correlation among SMB, HML, risk free rates, market returns, nondurable and durable growth rates. Noticablly, the consumption growth rates have low correlations with Fama-French portfolio returns, especially for durable growth rates.

5.3 Instruments

For the conditional moment estimation, we use five instruments. They are: constant, risk free interest rate, GDP per capita (using the adult population) and nondurable and durable consumption growth rates. In the estimates of the long-run durable risk model , we use the constant, price-dividend ratio, risk free interest rate and GDP per capita as instruments. The price/dividend ratio is for the UK equity market as a whole as computed by Datastream.

6 Estimation

6.1 Non-linear Durable Model

First we examine the ability of the durable model in equations (7) and (5) to explain both time-series and cross-sections of returns data. This is a non-linear model in the levels of the variables concerned. .The estimation is by two-step GMM with the identity matrix used as the first step weighting matrix. The standard errors are computed using the VARHAC approach to be robust to heteroskedasticity and autocorrelation. The moment conditions to be satisfied are:

$$\begin{aligned} 0 &= E_t[M_{t+1}R_{f,t+1} - 1)z_t] \\ 0 &= E_t[M_{t+1}(R_{i,t+1} - R_{f,t+1})z_t] \\ 0 &= E_t\left[\left(1 - \frac{\alpha}{1-\alpha}\left(\frac{C_t}{D_t}\right)^{\frac{1}{\rho}} - (1-\delta)M_{t+1}\frac{P_{t+1}}{P_t}\right)z_t\right] \end{aligned} \tag{48}$$

for instruments z_t. In the case of these models the set of instruments is: constant, second lags of durable and non-durable consumption growth, gdp growth and risk-free interest rate. Table 3 presents estimates for four sets of test assets: the market excess return and risk-free rate, the three Fama-French returns, namely market return and the returns on the HML and SMB mimicking portfolios, and finally, six and sixteen size and book to market portfolios. The estimates are consistent in choosing a high value for the coefficient of relative risk aversion γ and a low value for the elasticity of intertemporal substitution ψ. Whilst this is consistent with results for some non-durable models, the fact that estimate of the subjective discount factor β is significantly below 1 shows that the high value of γ required to match average excess returns does not lead to a failure to match the low risk-free rate, unlike non-durable models. The utility weight of durables α is estimated to be at least 0.74 which serves to emphasise the importance of durable consumption to consumers. The test of the hypothesis $\psi = 1/\gamma$ rejects the restriction that utility is time separable for all sets of test assets at a high level of significance supporting the Epstein-Zin form of preferences against the traditional time separable form Additive time separability is further tested by examination of the restriction $\psi = \rho$. This restriction is also rejected with a high degree of certainty for all sets of test assets. The overall J test of the specification of this model for each set of test assets fails to reject the model at usual levels of significance. These results are analogous to those for the United States presented in Yogo (2006). Here we provide a wider set of test assets for the conditional model to attempt to price.

6.2 Log-Linear Durable Model

The second version of the durable model that is analysed is the log-linear version of the model. This form allows more direct comparison with a number of other models and provides a step towards examining the long-run risk model. The moment conditions which are employed in the two-step GMM estimation of the log-linear model are those given by equation (13) and three further conditions for the three log-linear factors $x_t = (\Delta c_{t+1}, \Delta d_{t+1}, r_{h,t+1})$ whereby $E[x_t - \overline{x}] = 0$. Again the standard errors are computed using the VARHAC approach to be robust to heteroskedasticity and autocorrelation. The estimates of the unconditional log-linear durable model are given in the final columns of Table 4. The coefficient estimates are, in general, less precisely estimated than in the non-linear model. Not all of the structural coefficients can be identified but comparing the estimates of the coefficient of relative risk aversion γ and elasticity of intertemporal substitution ψ with those for the non-linear model in Table3, they can be seen to both be somewhat larger. The J test provides no evidence against the model when either the six or sixteen portfolio test asset returns are used. The log-linear form of this version of the model provides an opportunity to compare the estimates of the durable consumption model against some other, more traditional, asset pricing models. Estimates for the CAPM, CCAPM and Fama French three factor model are presented in the first six columns of Table 4. In both cases the price of risk is positive but only significantly so for the larger number of test assets. However, in both cases the J test of the overall specification resoundingly rejects the models. In the case of the Fama French model significant, positive prices of risk are estimated for all three factors with some ability to no be rejected by the J test. The overall abilities of these models to fit the cross-section of asset returns is demonstrated clearly by plots of average returns implied by the models and the actual average portfolio returns. These are shown for the 16 portfolios in Figures 2-5. It is clear from these that the CAPM has almost no ability to match average returns implying that average returns should be almost the same for all portfolios despite the distribution in the data. The association of average returns is almost as bad for the CCAPM where little predictable pattern can be seen. The

Fama French 3 factor model does a much better job of matching average returns; the association between the model predictions and the actual data is quite close. This ability is at least matched by the durable consumption model which has an even tighter association.

6.3 Durable Long-Run Consumption Risk Models

Initial estimates of the parameters of the long run durable model are presented in Table 5. These are estimates of the quasi-reduced form parameters in equation (31) i.e. $c_0 - c_4$ without the restrictions implied by the structural model applied. The moment conditions which are employed in the two-step GMM estimation of this log-linear model are those given by equation (31) and three further conditions for the four log-linear factors $x_t = (\Delta c_{t+1}, (\Delta d_{t+1} - \Delta c_{t+1}), \left(r_{f,t+1} - \frac{1}{\kappa_1} r_{f,t}\right), \left(z_{m,t+1} - \frac{1}{\kappa_1} z_{m,t}\right))$ whereby $E[x_t - \overline{x}] = 0$. Again the standard errors are computed using the VARHAC approach to be robust to heteroskedasticity and autocorrelation. Two broad results emerge from these estimates. First, the model is not rejected by the overall specification J test and second, at least some of the parameter estimates are consistent with those of the durable consumption model discussed above. For example, the large negative estimates of c_1 and c_2 are only consistent with very large values of γ, the coefficient of relative risk aversion and very low values of ψ, the elasticity of intertemporal substitution., in particular $\psi < 1$ These results are therefore consistent with the estimates from the conditional version of the durable consumption model in Table 3 above which does not apply the restrictions of the long-run risk model. These values are somewhat different from those assumed in the quantitative simulation analysis of the long-run risk model by Bansal and Yaron (2004) and Yang (2009) and potentially undermine the ability of the long-run risk model to explain the equity premium. Further analysis of the estimates in Table 5 and the implied behaviour of the risk premium is required to make much more progress on this point. The negative values of the estimates of c_3 and c_4 are similar to those estimated by Constantanides and Ghosh (2008) for the non-durable long-run consumption risk model. They, alternatively, mostly find estimates of γ closer to 10 and estimates of ψ below one, having applied the full set of restrictions.

Estimates of the cointegrated model are prersented in the next table. These show good support for the significance of the observable fstate variables as sources of risk in terms of all of the cross-sections of assets that we examine. In fact the most positive results are for the six portfolio case in the final column. It can be shown that, under the structural model, the sum of the coefficients c_1 and c_2 is equal to the coefficient of relative risk aversion. This estimate is in excess of 300 for all estimates of the model, large by any standard. The over-identifying restrictions implied by the reduced form of the model are not rejected by the J tests in this final table of estimates.

7 Conclusions

The CCAPM has been shown to be an inadequate model to explain the equity premium and risk-free interest rate in the UK as in many other countries. The aim of this paper is to examine a generalisation of the consumption-based model in two directions. The first is to introduce consumption pf durable goods in a non-separable way. The second is to ally this with the insights of the long-run risk model. The clear persistence in consumption of durable goods offers the long-run risk model a more plausible role in the UK context where non-durable consumption growth shows no persistence whatever. In the paper the Euler equation for pricing equity risk in two and multiple-asset contexts is estimated. Strong evidence of the ability of the durable model to match moments from both sets of asset returns is presented. The size of average pricing errors for a log normal version of the model are small relative to those from traditional models. A characteristic of the results is that a very high coefficient of relative risk aversion is estimated. Also a very low elasticity of interntemporal substitution is estimated. There is little evidence for the restriction between them required by the standard power utility framework. The implications of the long-run risk model are also evaluated using a method which substitutes observed for unobserved state variables. The estimates imply that the extra component in the risk premium coming from long run risk may be much smaller than the proponents of the long run risk model have suggested thus far. However, ensuring that dividends and total consumption are cointegrated is supported by the estimates. Further work will seek to establish the robustness of this result.

8 Appendix

8.1 Solution of the long-run durable consumption risk model in section 3

The solution to the parameters of the equations for the model in section 3 are for equation (21):

$$
\begin{aligned}
A_1 &= \frac{\left(1-\frac{1}{\psi}\right)\alpha}{1-\kappa_1\rho_x} \\
A_2 &= \frac{0.5\left[\left(\alpha\left(\theta-\frac{\theta}{\psi}\right)\right)^2+(\theta A_1\kappa_1\varphi_x)^2\right]}{\theta\left(1-\kappa_1 v\right)} \\
A_0 &= \frac{\ln\delta+\left[-\left(1-\frac{1}{\psi}\right)\alpha+1-\frac{1}{\psi}\right]\mu_c+\frac{1}{2}\theta(\kappa_1A_2)^2\sigma_w^2}{1-\kappa_1} \\
&\quad +\frac{\kappa_0\left(1-\frac{1}{\varphi}\right)\alpha\mu_d+A_2(1-v)\sigma^2\kappa_1}{1-\kappa_1}
\end{aligned}
$$

and for equation (22):

$$
\begin{aligned}
A_{1,m} &= \frac{\phi+\left(1-\frac{1}{\psi}\right)\alpha}{1-\kappa_{1,m}\rho_x} \\
A_{2,m} &= \frac{(1-\theta)(1-\kappa_1 v)A_2+0.5\left[(\theta\left(1-\frac{1}{\psi}\right)\alpha)^2+\varphi_d^2+((\theta-1)\kappa_1A_1+\kappa_{1,m}A_{1,m})^2\varphi_x^2\right]}{1-\kappa_{1,m}v} \\
A_{0,m} &= \frac{\theta\ln\delta-\frac{\theta}{\psi}\mu_c+(\theta\left(1-\frac{1}{\psi}\right)\alpha(\mu_d-\mu_c)+(\theta-1)\kappa_0+(\theta-1)(\kappa_1-1)A_0}{1-\kappa_{1,m}} \\
&\quad \frac{+(\theta-1)\kappa_1A_2\sigma^2(1-v)+\kappa_{0,m}+\mu_d+\kappa_{1,m}A_{2,m}(1-v)\sigma^2}{1-\kappa_{1,m}} \\
&\quad \frac{+0.5\left[(1-\theta)A_2\kappa_1+\kappa_1A_{2,m}\right]^2\sigma_w^2}{1-\kappa_{1,m}}
\end{aligned}
$$

and for equation (24):

$$
\begin{aligned}
A_{1,f} &= -\left((\theta-1)(\kappa_1\rho_x-1)A_1+\theta\left(1-\frac{1}{\psi}\right)\alpha\right) \\
A_{2,f} &= -\left[(\theta-1)(\kappa_1 v-1)A_2+0.5\left((\theta-1)^2\kappa_1^2A_1^2\varphi_x^2+\left(\theta\left(1-\frac{1}{\psi}\right)\alpha\right)^2\right)\right] \\
A_{0,f} &= -\ln\delta-\left[-\theta\left(1-\frac{1}{\psi}\right)\alpha-\frac{\theta}{\psi}+(\theta-1)\right]\mu_c-\theta(1-\frac{1}{\psi})\alpha\mu_d-(\theta-1)\kappa_0-(\theta-1)(\kappa_1-1)A_0 \\
&\quad -(\theta-1)\kappa_1A_2(1-v)\sigma^2-0.5(\theta-1)^2\kappa_1^2A_2^2\sigma_w^2
\end{aligned}
$$

As Constantinides and Ghosh (2009) show, solutions to $\alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1$ and β_2 in equations (25) and (26) are computed by inverting equations (22) and (24) from the text:.

$$r_{f,t} = A_{0,f} + A_{1,f}x_t + A_{2,f}\sigma_t^2$$
$$z_{m,t} = A_{0,m} + A_{1,m}x_t + A_{2,m}\sigma_t^2$$

in terms of the unobserved persistent process in consumption and it's variance as functions of the observable risk-free rate and price dividend ratio:

$$x_t = \alpha_0 + \alpha_1 r_{f,t} + \alpha_2 z_{m,t}$$
$$\sigma_t^2 = \beta_0 + \beta_1 r_{f,t} + \beta_2 z_{m,t}$$

where, for $A = A_{1,m}A_{2,f} - A_{2,m}A_{1,f}$, $\alpha_1 = -A_{2,m}/A$, $\alpha_2 = A_{2,f}/A$, $\alpha_0 = (A_{2,m}A_{0,f} - A_{0,m}A_{2,f})/A$, and $\beta_1 = A_{1,m}/A$, $\beta_2 = -A_{1,f}/A$, $\beta_0 = (A_{0,m}A_{1,f} - A_{1,m}A_{0,f})/A$.

8.2 Solution of the cointegrated long-run durable consumption risk model in section 4

$$A_1 = \frac{\left(\frac{1}{\rho} - \frac{1}{\varphi}\right)\alpha + a_1}{1 - \kappa_1\rho_x}$$
$$A_2 = \frac{0.5\theta\left[\left(\alpha\left(\frac{1}{\rho} - \frac{1}{\varphi}\right) + a_1\right)^2 + (A_1\kappa_1\varphi_x)^2\right]}{1 - \kappa_1 v}$$
$$A_3 = 0$$
$$A_0 = \frac{\begin{array}{c} A_2(1-v)\sigma^2\kappa_1 + \ln\beta + \kappa_0 + \frac{1}{2}[(1-a_1) - \frac{1}{\varphi} - \alpha(\frac{1}{\rho} - \frac{1}{\varphi})]^2 \\ +[(1-a_1) - \frac{1}{\varphi} - \alpha(\frac{1}{\rho} - \frac{1}{\varphi})]\mu_{cn} + [\alpha(\frac{1}{\rho} - \frac{1}{\varphi}) + a_1]\mu_{cd} + \kappa_1^2 A_2^2\sigma_w^2] \end{array}}{1 - \kappa_1}$$

for equation (x):

$$
\begin{aligned}
A_{1,m} &= \frac{\alpha(\frac{1}{\rho}-\frac{1}{\varphi})+(\kappa_{1,m}A_{3,m}+1)\lambda_{sx}+a_1}{1-\kappa_{1,m}\rho_x} \\
A_{2,m} &= \frac{\begin{array}{c}(\theta-1)(\kappa_1 v-1)A_2+0.5[(\frac{1}{\rho}-\frac{1}{\varphi})\alpha+a_1]^2\theta^2+0.5[(\theta-1)\,\kappa_1 A_1+\kappa_{1,m}A_{1,m}]^2\varphi_x^2 \\ +0.5\varphi_s^2(\kappa_{1,m}A_{3m}+1)^2\end{array}}{1-\kappa_{1,m}v} \\
A_{3,m} &= \frac{\rho_s-1}{1-\kappa_{1m}\rho_s} \\
A_{0,m} &= \frac{1}{1-\kappa_{1,m}}[(\theta-1)(\kappa_1-1)A_2+0.5[\kappa_{1m}A_{1m}+(\theta-1)\kappa_1 A_1]^2\varphi_x^2 \\
&\quad +0.5\varphi_s^2[\kappa_{1m}A_{3m}+1+(\theta-1)\kappa_1 A_3]^2+0.5[a_1+\alpha(\frac{1}{\rho}-\frac{1}{\varphi})]^2\theta^2]\sigma^2 \\
&\quad +(\theta-1)(\kappa_0+\kappa_1 A_0-A_0)+\theta\ln\beta+\kappa_{0m} \\
&\quad +0.5[\theta(1-a_1)-\frac{\theta}{\varphi}-\theta\alpha(\frac{1}{\rho}-\frac{1}{\varphi})]^2+[\theta(1-a_1)-\frac{\theta}{\varphi}-\theta\alpha(\frac{1}{\rho}-\frac{1}{\varphi})]\mu_c \\
&\quad +[\theta a_1+\theta\alpha(\frac{1}{\rho}-\frac{1}{\varphi})]\mu_d+0.5(\kappa_{1m}A_{2m}\sigma_w+(\theta-1)\kappa_1 A_2\sigma_w)^2
\end{aligned}
$$

and for equation (y):

$$
\begin{aligned}
A_{1,f} &= -[\theta\alpha(\frac{1}{\rho}-\frac{1}{\varphi})+a_1(\theta-1)+(\theta-1)(\kappa_1\rho_x-1)A_1] \\
A_{2,f} &= -(\theta-1)(\kappa_1 v-1)A_2-0.5(\theta-1)^2\kappa_1^2A_1^2\varphi_x^2-0.5((\theta-1)a_1+\theta\alpha(\frac{1}{\rho}-\frac{1}{\varphi}))^2 \\
A_{3f} &= 0 \\
A_{0,f} &= -[(\theta-1)(\kappa_1-1)A_2+0.5(\theta-1)^2\kappa_1^2A_1^2\varphi_x^2+0.5((\theta-1)a_1 \\
&\quad +\theta\alpha(\frac{1}{\rho}-\frac{1}{\varphi}))^2]\sigma^2-(\theta-1)(\kappa_0+\kappa_1 A_0-A_0)-\ln\beta-0.5((\theta-1)(1-a_1) \\
&\quad -\frac{\theta}{\varphi}-\theta\alpha(\frac{1}{\rho}-\frac{1}{\varphi}))^2-[(\theta-1)(1-a_1)-\frac{\theta}{\varphi}-\theta\alpha(\frac{1}{\rho}-\frac{1}{\varphi})]\mu_c \\
&\quad -[(\theta-1)a_1+\theta\alpha(\frac{1}{\rho}-\frac{1}{\varphi})]\mu_d-0.5(\theta-1)^2\kappa_1^2A_2^2\sigma_w^2
\end{aligned}
$$

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Table1 : Descriptive Statistics

	Quarterly, annualised				
	$Mean$	$S\tan dardDev$	$ExcessKurtosis$	$Skewness$	$AR(1)$
Δc_t	1.92%	3.30	1.14	-0.0518	0.0607
Δd_t	3.35%	2.16	-0.157	0.308	0.793
$R_{f,t}$	8.00%	3.20	-0.694	0.576	0.939
$R_{m,t}$	12.12%	39.6	5.81	0.240	0.120
SMB_t	0.243%pa	6.31	0.812	-0.308	0.207
HML_t	1.48%pa	4.87	4.39	-0.575	0.205
	$1966Q1 - 2008Q4$				

Table 2: Non-Stationarity and Cointegration Tests

	ADF	$ADF(\Delta)$	$KPSS$	$KPSS(\Delta)$
c_t	-1.59	-6.40	0.0864	0.183
d_t	-2.90	-4.02	0.218	0.0770
p_t	0.31		1.55	
$d_t - c_t$	0.86		1.54	
Critical Values	1% : -4.01 5%: -3.44		1% : 0.739 5%: 0.463	

Number of observations:

Period of estimation: 1966 Q1 - 2008 Q3

Table 3. Estimates of Non-linear Durable Model

Model	Durable Model			
Portfolios	$r_{m,t+1}, r_{f,t+1}$	SMB_{t+1} HML_{t+1}	$r_{m,t+1}, SMB_{t+1}$ $HML_{t+1},, r_{f,t+1}$	$6Port$
ψ	0.01885 (0.0050)	0.01530 (0.010)	0.01857 (0.0045)	0.01483 (0.0051)
γ	192.68 (15.0)	186.94 (18.1)	186.54 (16.5)	183.84 (21.45)
ρ	0.8028 (0.064)	0.8961 (0.082)	0.7416 (0.14)	0.7228 (0.055)
α	0.8351 (0.015)	0.8149 (0.017)	0.8482 (0.036)	0.8555 (0.014)
β	0.9394 (0.022)	0.8910 (0.053)	0.9280 (0.023)	0.9330 (0.039)
$J-test$ sig	60.27 (0.00)	51.17 (0.00)	60.33 (0.00)	55.67 (0.00)
$\psi=\rho$	138.6 (0.00)	113.54 (0.00)	27.25 (0.00)	156.52 (0.00)
$\psi=1/\gamma$	8.36 (0.004)	0.979 (0.322)	10.16 (0.001)	3.943 (0.05)

Standard errors in brackets

Period of estimation: 1980q4 - 2008q4

Table 4. Estimates of Linear Models

Model	CAPM	CCAPM	Fama French	Durable Model
6 Portfolio returns				
$r_{m,t+1}$	4.85 (1.47)		-3.82 (3.19)	1.00 (1.70)
Δc_{t+1}		76.09 (33.6)		113.74 (82.0)
SMB_{t+1}			5.13 (2.35)	
HML_{t+1}			5.93 (1.89)	
Δd_{t+1}				135.40 (78.41)
γ				250.2 (41.9)
ψ				0.001 (0.0068)
$J-test$ $_{sig}$	8.73 (0.12)	12.51 (0.029)	1.75 (0.625)	4.388 (0.22)

Standard errors in brackets

Period of estimation: 1980q4 - 2008q4

Table 5. Estimates of the Durable Long Run Risk Model

Model	Durable LRR Model				
Portfolios	$r_{m,t+1}, r_{f,t+1}$	SMB_{t+1}, HML_{t+1}	$r_{m,t+1}, SMB_{t+1}$ HML_{t+1}	FF	$6Port$
c_0	0.478 (0.623)	0.498 (0.435)	0.238 (0.352)	0.200 (0.245)	−0.902 (0.062)
c_1	−185.4 (173.0)	−184.7 (98.1)	−185.50 (99.7)	−187.0 (62.9)	−186.9 (12.9)
c_2	−148.6 (277.8)	−148.4 (159.8)	−148.2 (170.7)	−148.0 (84.9)	−148.0 (11.8)
c_3	−1.737 (2.63)	−5.26 (2.48)	−4.95 (1.58)	−4.00 (1.75)	−13.93 (0.35)
c_4	−3.09 (3.22)	−2.92 (3.56)	−2.47 (3.19)	−5.00 (1.26)	−0.721 (0.493)
$J-test$ sig	8.65 (0.799)	9.03 (0.771)	13.22 (0.827)	10.42 (0.942)	29.05 (0.821)

Standard errors in brackets

Period of estimation: 1980q4 - 2008q4

Table 6. Estimates of the Extended Durable Long Run Risk Model

Model	Durable LRR Model		
Portfolios	$r_{m,t+1}, r_{f,t+1}$	FF	$6Port$
c_0	0.518 (0.355)	0.379 (0.199)	0.597 (0.091)
c_1	-186.7 (29.8)	-178.0 (26.3)	-171.8 (6.88)
c_2	-147.8 (57.3)	-157.9 (26.0)	-152.8 (8.83)
c_3	-6.28 (1.91)	-7.47 (0.676)	-6.37 (0.428)
c_4	-5.97 (2.50)	-6.66 (1.51)	-6.29 (0.705)
c_5	-9.64 (10.1)	-10.15 (9.57)	-9.70 (2.19)
$J-test$ sig	7.43 (0.828)	9.23 (0.954)	31.2 (0.355)

Standard errors in brackets

Period of estimation: 1980q4 - 2008q4

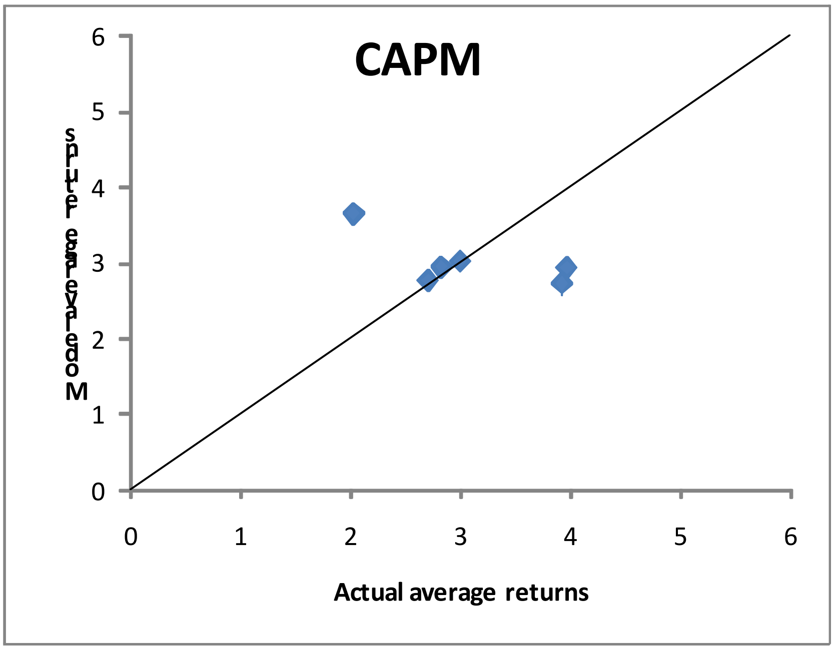

Figure 1:

9 Appendix: Further Estimation Results

9.1 (a) Coefficient Estimates for 1966q1 - 1980q3

Table A3. Estimates of Non-linear Durable Model

Model	Durable Model			
Portfolios	$r_{m,t+1}, r_{f,t+1}$	SMB_{t+1} HML_{t+1}	$r_{m,t+1}, SMB_{t+1}$ $HML_{t+1,}, r_{f,t+1}$	$6Port$
ψ	0.01307 (0.0098)	0.01807 (0.0085)	0.01675 (0.0031)	0.01883 (0.0028)
γ	164.87 (89.1)	176.99 (49.0)	166.68 (24.4)	183.84 (20.11)
ρ	0.9910 (0.292)	0.9573 (0.409)	0.8198 (0.603)	0.9890 (0.767)
α	0.7985 (0.042)	0.8001 (0.064)	0.8320 (0.111)	0.8018 (0.112)
β	0.8393 (0.131)	0.8042 (0.122)	0.8760 (0.041)	0.9035 (0.018)
$J - test$ sig	60.27 (0.00)	28.00 (0.0215)	56.11 (0.00)	52.13 (0.00)
$\psi = \rho$	11.69 (0.00)	0.0545 (0.815)	1.77 (0.183)	1.60 (0.205)
$\psi = 1/\gamma$	0.854 (0.355)	0.1697 (0.68)	17.28 (0.00)	30.75 (0.00)

Standard errors in brackets

Period of estimation: 1966q1 - 1980q3

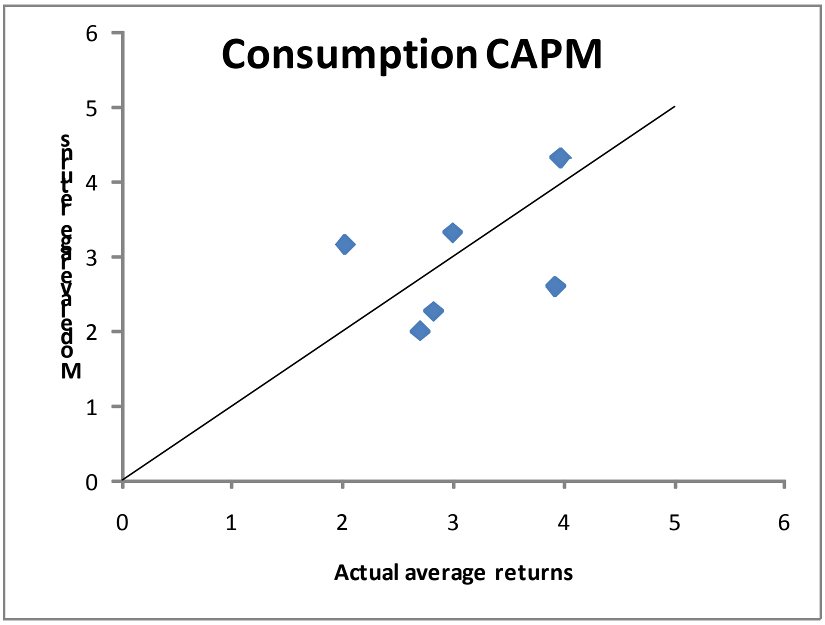

Figure 2:

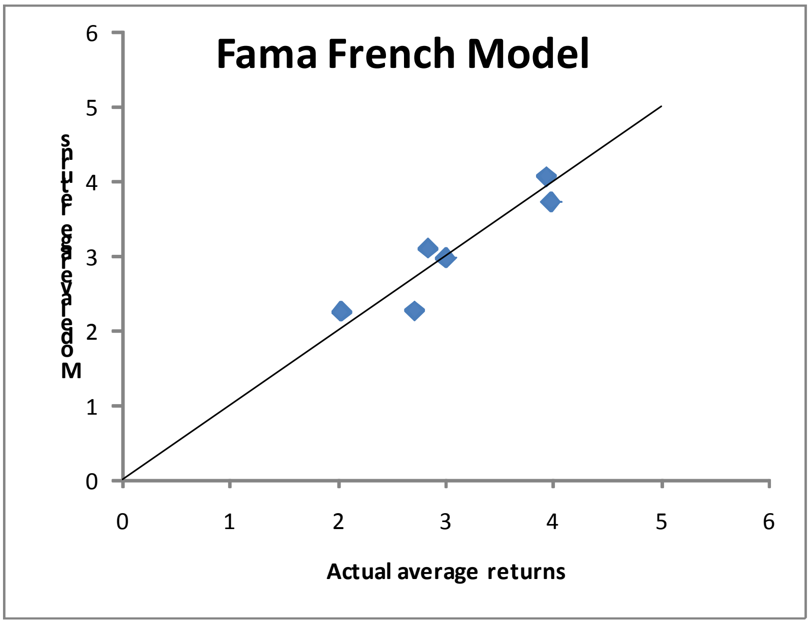

Figure 3:

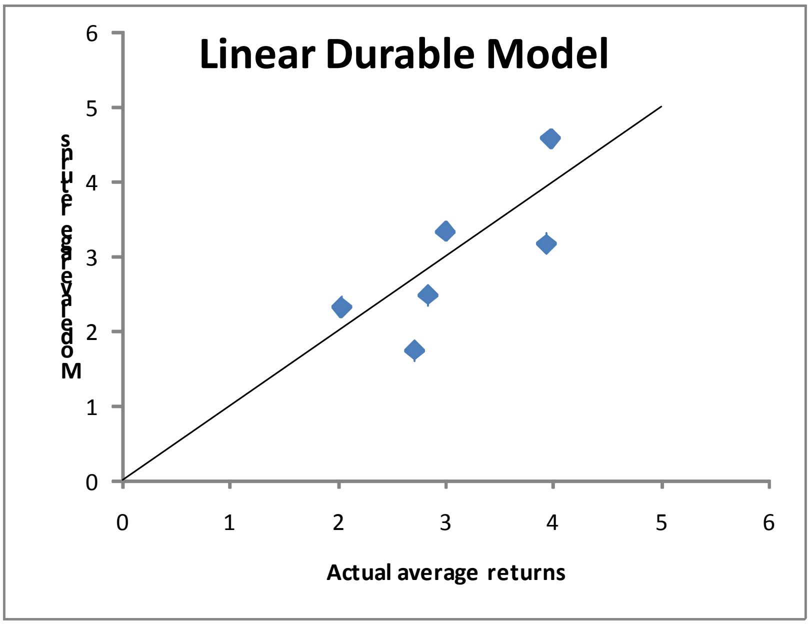

Figure 4:

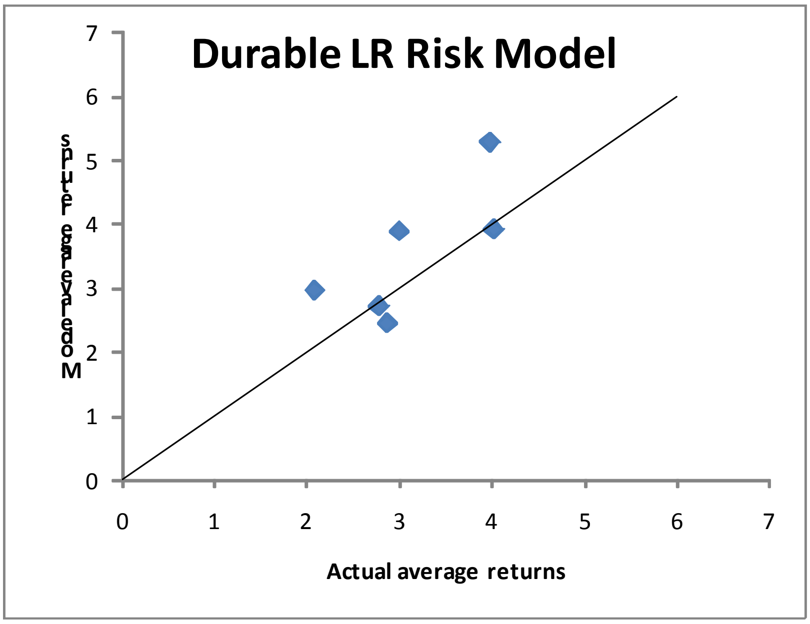

Figure 5:

Table A4. Estimates of Linear Models

Model	CAPM	CCAPM	Fama French	Durable Model
		6 Portfolio returns		
$r_{m,t+1}$	2.40 (0.76)		6.81 (2.54)	1.24 (1.70)
Δc_{t+1}		252.5 (73.6)		70.36 (82.0)
SMB_{t+1}			15.52 (5.33)	
HML_{t+1}			4.15 (1.40)	
Δd_{t+1}				651.8 (78.41)
γ				580.2 (41.9)
ψ				0.001 (0.0068)
$J - test$ $_{sig}$	5.42 (0.37)	19.3 (0.0019)	10.98 (0.012)	7.29 (0.063)

Standard errors in brackets

Period of estimation: 1966q1 - 1980q3

Table A5. Estimates of the Conditional Durable Long Run Risk Model

Model	Durable LRR Model				
Portfolios	$r_{m,t+1}, r_{f,t+1}$	SMB_{t+1}, HML_{t+1}	$r_{m,t+1}, SMB_{t+1}$ HML_{t+1}	FF	$6Port$
c_0	-0.663 (0.406)	0.780 (0.191)	0.749 (0.180)	0.708 (0.179)	0.503 (0.080)
c_1	-163.6 (123.8)	-178.7 (164.5)	-180.2 (121.3)	-171.0 (118.9)	-174.9 (36.3)
c_2	-155.4 (154.3)	-155.3 (174.5)	-153.9 (128.9)	-159.8 (124.3)	-151.9 (33.8)
c_3	-4.94 (1.67)	-0.504 (2.20)	-0.295 (1.79)	-0.364 (2.12)	-3.51 (0.58)
c_4	-3.53 (1.24)	-1.42 (1.39)	-1.36 (1.16)	-1.44 (0.88)	-2.83 (0.34)
$J-test$ sig	27.27 (0.011)	25.91 (0.017)	29.23 (0.062)	26.32 (0.121)	44.69 (0.041)

Standard errors in brackets

Period of estimation: 1966q1 - 1980q3