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Abstract One of the main motivations for productivity analysis is to assess the scope for overall improvements in the output possibilities of individual producers. At times of fiscal and government budgetary pressures, attention focuses particularly on the output potential of public service providers and its relationship to the inputs provided by government funding. Public services, such as education and healthcare, are themselves an important form of economic activity whose performance is of wide public interest, and which merit an adequate recognition of the richness of the additional considerations which may arise in making effectiveness assessments using frontier techniques such as Stochastic Frontier Analysis (SFA) and Data Envelopment Analysis (DEA). The interesting example of university Departments illustrates one such additional consideration, namely endogeneity of the available resource levels through their dependence on the Department's achieved outputs of teaching and research. Fortunately progress can be made in the presence of such endogeneity through the application of SFA to the assessments of the overall effectiveness and performance of the public service provider, and their decomposition into both technical and allocative components, using the notion of an Achievement Possibility Set that includes the multiplier effects which such resource endogeneity generates.

Keywords: Public services, Effectiveness, Performance measurement, Endogeneity, Stochastic frontier analysis, Data envelopment analysis

JEL Classification: C13, C18, C30, D20, I23

1 Introduction

Public services, such as education and healthcare, are an important form of economic activity whose performance is of wide public interest. Individual producers of public services, such as schools and universities, may, however, be subject to less competitive pressure to become fully efficient and effective than would occur in the standard paradigm of a perfectly competitive market for goods and services that have a competitively-determined market price. Assessing the performance of individual producers of public services is therefore an area that merits detailed attention, and an adequate recognition of the richness of the additional considerations which may arise in making such effectiveness assessments. In this paper, we examine the additional consideration that arise when the producer of a public service, such as a university, can raise additional finance to support its activities, in a way which depends upon its current level of performance in producing high quality multiple outputs.

The main context in which we will carry out our investigation is in the use of the frontier techniques of efficiency and effectiveness evaluation of Stochastic Frontier Analysis (SFA) and Data Envelopment Analysis (DEA). DEA is itself a deterministic non-parametric technique of efficiency evaluation that developed out of the seminal work of Farrell (1957) and was further extended using mathematical programming formulations by Charnes et al. (1978) and by Banker et al. (1984), and is extensively reviewed in Cooper et al. (2007) and in Thanassoulis at al. (2008), with initial applications of DEA to public services, such as education in Jesson et al. (1987), Mayston and Smith (1988) and elsewhere. Building upon the insights of Aigner et al. (1977), Battese and Corra (1977), and Meeusen and van den Broeck (1977), SFA seeks to include in the estimation of a parametric production or cost function non-deterministic stochastic variations in the performance of individual producers, based in part upon random variations across producers in their technological possibilities and in part upon their individual efficiencies. Recent theoretical developments in SFA are extensively reviewed in Greene (2008), who emphasises that "the production of public services provides one of the most interesting and important applications of the techniques discussed in this study". While SFA has been extensively applied in healthcare (see e.g. Rosko and Mutter 2008; Hollingsworth 2008), it has received less attention in the education sector, and in particular in the higher education sector, whose performance in both teaching and research is subject to increasing public interest.

One notable feature of the higher education sector is that it typically has access to additional funding sources in addition to a base funding level from central/federal or local/provincial/state government. These additional sources of funding include external research grants, and in some contexts (such as the UK, where higher education tuition fee income from non-EU domiciled students totalled £2.94 billion in 2010-11 (HESA 2012)) income from recruiting additional fee-paying students beyond those covered by the base level of government funding. In both cases, the extent of the additional funding is likely to depend upon the quality of the research and/or teaching outputs which the higher education institution (HEI) produces. In addition, the ability of any given university Department to attract able academic staff is likely to depend not only upon the wage that it offers them, but also upon its academic reputation, that in turn depends upon its performance in producing high quality teaching and research. This in turn introduces important potential sources of endogeneity into relationship between the inputs and outputs of higher education. For a discussion of the impact of multiple endogeneity relationships on the biased coefficient estimates which a standard OLS estimation of the production function produces, and of the significance of endogeneity relationships in the case of primary and secondary school education, see Mayston (2007, 2009). However, as Mutter et al. (2012) recently note, “The impact of an endogenous regressor on efficiency estimates generated by SFA is an important topic for both theory and applied research that has not been addressed in the literature”. Their own initial simulation study in the context of nursing homes suggests that “endogeneity can have a substantial impact on inefficiency estimates generated by SFA”.

In this paper we will examine the implications of endogenous resource inputs for the analysis of the productivity and overall performance of individual university Departments using frontier techniques of analysis. In doing so, we are able to nest the analysis of the efficiency of individual university Departments within a wider context of the analysis of their overall effectiveness in producing quality-adjusted outputs. One of the main motivations for productivity analysis is to assess the scope for overall improvements in the output possibilities of individual producers. At times of fiscal and government budgetary pressures, attention focuses particularly on the output potential of public service providers and its relationship to the inputs provided by government funding. Since universities provide an interesting example of such providers, we will pursue our analysis in this context while at the same time highlighting more general analytical considerations that arise for public service providers.

The remainder of the paper is organised as follows: Section 2 establishes the underlying model of quality-adjusted multiple outputs and endogenous resources for individual university Departments, Sect. 3 introduces the associated concept of the Achievement Possibility Frontier, Sect. 4 develops the effectiveness analysis and performance measurement framework for public service providers under endogenous resourcing in the context of Stochastic Frontier Analysis, Sect. 5. examines possible alternative approaches to effectiveness analysis in the presence of endogenous resources, and Sect. 5 concludes.

2 Quality-Adjusted Multiple Outputs and Endogenous Resources

We can illustrate several of the underlying issues by reference to an analytical model in which teaching quality in Department r depends firstly upon its staff-student ratio $\psi_r \equiv s_r / n_r$ ratio, where s_r is the number of academic staff in Department r and n_r is the number of students in Department r, and where for simplicity we assume that academic staff are the only resource available to the Department. For any given level of teaching ability a_{1r} in Department r and proportion θ_{1r} of the total staff time which is devoted to teaching-related activities, the maximum quality q_{ir}^o of teaching which can be attained is assumed to be given by :

$$q_{1r}^o = a_{1r}(\theta_{1r}s_r / n_r)^{\alpha_1} \quad with \;\; n_r = \gamma_0(\theta_{1r}s_r)^{\alpha_0} \tag{1}$$

where the second equation in (1) specifies the number of students who can be taught by a total of m_r staff when they devote a proportion θ_{1r} of their time to teaching activities, and where α_0, α_1 and γ_0 are positive constants. We will assume in the following analysis that for each Department r, θ_{1r} is a pre-determined variable. Using (1), the actual quality-adjusted teaching output of Department r is assumed to be given by:

$$z_{1r} \equiv q_{1r}n_j = \varepsilon_{1r} f(x_{1r}, a_{1r}) \; where \; f(x_{1r}, a_{1r}) \equiv a_{1r}\gamma_0^{1-\alpha_1}x_{1r}^{\gamma_1}, x_{1r} \equiv \theta_{1r}s_r, \gamma_1 \equiv \alpha_1 + \alpha_0(1-\alpha_1), 0 < \varepsilon_{1r} \le 1 \tag{2}$$

where ε_{1r} is a parameter that reflects the production efficiency of Department r in realising its potential in producing quality-adjusted teaching output, given its resource input of academic staff time and their teaching ability, and where we will assume that $\gamma_1 > 0$, so that additional staff time devoted to teaching is productive in boosting its quality-weighted output.

For any given level of research ability a_{2r} of academic staff in Department r and proportion θ_{2r} of time that members of staff in Department r devotes to research, the maximum quality $\tilde{q}_{2r}$ of its research which it could attain is assumed to be given by:

$$\tilde{q}_{2r} = a_{2r}\theta_{2r}^{\alpha_2} s_r^{\alpha_3} \ with \ \ z_{2r} \equiv q_{2r}s_r = \varepsilon_{2r}a_{2r}\theta_{2r}^{\alpha_2} s_r^{\alpha_3+1} \tag{3}$$

$\alpha_2 > 0$ is here a parameter which reflects the increase in the quality of research which could result from each member of academic staff spending a greater proportion of their time on research, and $\alpha_3 \geq 0$ is a parameter which reflects the extent to which the quality of research per member of staff can be boosted by being a member of a larger Department. The overall quality-adjusted volume of research z_{2r} that is actually achieved by Department r in (3) depends here also upon the parameter ε_{2r} where $0 < \varepsilon_{2r} \leq 1$, that reflects the production efficiency of Department r in realising its potential in producing quality-adjusted research output, given its input of academic staff time devoted to research and the research ability of its academic staff.

The number of academic staff which Department r is able to support is assumed here to depend in part upon a base level of funding which it receives from a governmental higher education funding agency. In particular we will assume that the funding agency determines exogenously a limited number n_{0r} of students which it is willing to fund for each university, with a funded unit of resource for each of these students of $\gamma_2 > 0$. Department r, however, is assumed to be able to leverage up this base level of funding $\gamma_2 n_{0r}$ by additional research funding and by income from tuition fees charged to additional students who are not financed by the government agency. Its success in attracting such additional income is assumed to depend upon both the quality of its research and teaching and upon the volume of its existing research and teaching activity. Specifically we will assume that the resultant budget constraint for Department r is given by:

$$s_r w_r = \gamma_2 n_{0r} z_{1r}^{\gamma_3} z_{2r}^{\gamma_4} \varepsilon_{3r} \tag{4}$$

where w_r is the average wage which Department r pays its academic staff, γ_3, γ_4 and n_{0r} are positive constants, and ε_{3r} is a parameter reflecting the effectiveness of Department r in securing such potential additional funding, given the level of its quality-adjusted teaching and research activity.

The average wage, w_r, which Department r must pay to attract and retain its academic staff is assumed to be an increasing function of their teaching and research abilities, to an extent that depends upon the positive coefficients η_1 and η_2 in the wage function (5) below. However, w_r is assumed to be a decreasing function of the value of Department r's existing quality-weighted teaching and research activity, so that for any given wage level Department r is able to attract and retain academic staff of a higher level of ability in research and/or teaching if it has already achieved a greater level of quality-adjusted research and/or teaching activity within the Department, with positive coefficients η_3 and η_4 in the wage function:

$$w_r = \gamma_3 a_{1r}^{\eta_1} a_{2r}^{\eta_2} z_{1r}^{-\eta_3} z_{2r}^{-\eta_4} \tag{5}$$

so that academic staff of a given ability are willing to sacrifice to some extent greater current wages for the opportunity to be member of a Department that has a higher academic status in terms of its level of quality-weighted research and teaching activity.

The budget constraint (4) and the wage function (5) imply that the level of staffing which Department r can support is given by:

$$s_r = \varepsilon_{3r} h(z_{1r}, z_{2r}, a_{1r}, a_{2r}, n_{0r}) \text{ where } h(z_{1r}, z_{2r}, a_{1r}, a_{2r}, n_{0r}) \equiv (\gamma_2 / \gamma_3) n_{0r} a_{1r}^{-\eta_1} a_{2r}^{-\eta_2} z_{1r}^{\gamma_3+\eta_3} z_{2r}^{\gamma_4+\eta_4} \tag{6}$$

where h is an increasing function of γ_2, n_{01}, z_{1r} and z_{2r}, and a decreasing function of γ_3, a_{1r} and a_{2r}. In addition to the overall Departmental budget constraint, each individual academic member of staff is assumed to face a *time-energy budget* in which the proportion θ_{2r} of each calendar year which they productively devote to research depends in a non-linear way upon the proportion θ_{1r} of each calendar year that they devote to teaching in Department r, such that:

$$\theta_{1r}^{\varsigma_1}\theta_{2r}^{\varsigma_2} = 1 \ \ where \ \ \varsigma_1 > 0 \ and \ \varsigma_2 > 0 \tag{7}$$

From equations (2), (3), (6) - (7), we have:

$$Ay_r = b_r \ where \ A \equiv \begin{bmatrix} 1 & 0 & -\gamma_1 \\ 0 & 1 & -(1+\alpha_3) \\ -(\gamma_3+\eta_3) & -(\gamma_4+\eta_4) & 1 \end{bmatrix}, \ y_r \equiv \begin{bmatrix} \ln z_{1r} \\ \ln z_{2r} \\ \ln s_r \end{bmatrix} \ and \ \ b_r \equiv \begin{bmatrix} b_{1r} \\ b_{2r} \\ b_{3r} \end{bmatrix} \tag{8}$$

$$with \ \ b_{1r} \equiv \ln\varepsilon_{1r} + \ln a_{1r} + \gamma_1 \ln\theta_{1r} + (1-\alpha_1)\ln\gamma_0, \ \ b_{2r} \equiv \ln\varepsilon_{2r} + \ln a_{2r} - \gamma_5 \ln\theta_{1r} \ \ for \ \gamma_5 \equiv \alpha_2(\varsigma_1 / \varsigma_2) \tag{9}$$

$$b_{3r} \equiv \ln\varepsilon_{3r} + \ln n_{0r} - \eta_1 \ln a_{1r} - \eta_2 \ln a_{2r} + \ln(\gamma_2 / \gamma_3) \tag{10}$$

An improvement in the production efficiency ε_{kr} of Department r in teaching or research (i.e. for $k = 1$ *or* 2) here has a *multiplier effect* on its overall effectiveness, firstly through boosting its total revenue in the budget constraint (4) by its improved quality-weighted outputs attracting additional income to the Department, and secondly by making it more attractive to able staff to work in the Department via the wage function in (5). The Department's increased ability to hire and retain more able staff can in turn further boost its quality-weighted outputs, thereby reinforcing the multiplier effect of an initial improvement in the production efficiency of the Department. The cumulative multiplier effect of an initial improvement in the Department's efficiency in producing quality-weighted teaching output is illustrated in Fig. 1 below, which includes the production function, $f(x_{1r}, a_{1r})$, for teaching in equation (1) as a function of the input of Department r's teaching staff time, x_{1r}, and the ability level, a_{1r}, of Department r's staff. It also includes the endogenous amount of teaching staff time, x_{1r}, that the Department would be able to provide as a function of z_{1r} for any given levels of $\theta_{1r}, a_{1r}, a_{2r}$ and z_{2r}, when the function h in equation (6) above indicates the associated number of number of staff that the Department could fund under the budget constraint (4) and the wage function (5) if it were fully effective in its fund-raising activities.

An increase in the production efficiency of Department r that raises its initial quality-adjusted teaching output from $z_{1r}^{(1)}$ to $z_{1r}^{(2)}$, and thereby places it on the production frontier for its initial level of teaching staff input $x_{1r}^{(1)}$, could generate additional income for the Department that would enable it to increase its teaching staff input to $x_{1r}^{(2)}$ in Fig. 1. The cumulative outcome of this multiplier process in Fig. 1 if the Department were fully efficient and effective in its income generation would be a funded teaching staff input of $x_{1r}^{(3)}$ and a quality-adjusted teaching output of $z_{1r}^{(3)}$, that in equilibrium both places the Department on the production frontier associated with the production function $f(x_{1r}, a_{1r})$ for any given level of ability of its staff and satisfies the budgetary constraint given by the function h in equation (6) for a fully effective Department.

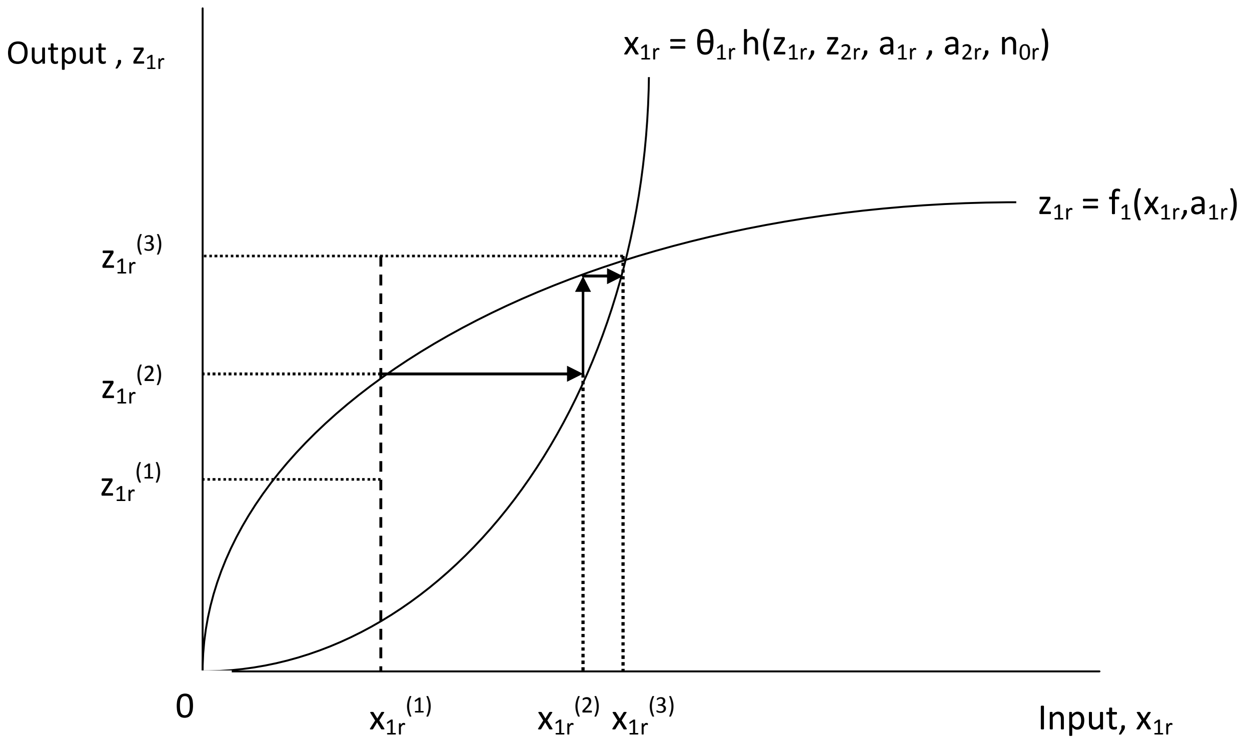

Fig. 1 Endogeous resourcing and the multiplier effect of efficiency improvements

A comparison of the actual level of Department r's quality-adjusted teaching, $z_{1r}^{(1)}$, when it is less than fully efficient and effective with simply the level of its quality-adjusted teaching, $z_{1r}^{(2)}$, it could have achieved with its existing level of staff input $x_{1r}^{(1)}$ by reference only to the production function $f(x_{1r}, a_{1r})$ would then substantially under-estimate the overall additional quality-adjusted teaching output $z_{1r}^{(3)} - z_{1r}^{(1)}$ that Department r could have achieved if it were fully efficient and effective, even given the existing abilities of its staff. If this were the only output involved, a measure of Department r's effectiveness given by $z_{1r}^{(1)} / z_{1r}^{(2)}$ would then substantially over-estimate a measure of its overall effectiveness given by $z_{1r}^{(1)} / z_{1r}^{(3)}$ that was based upon comparing the actual level $z_{1r}^{(1)}$ of its quality-adjusted output with the achievable quality-adjusted output level $z_{1r}^{(3)}$.

Stability of the feedback process associated with equations (8) – (10) above requires that:

$$(1+\alpha_3)(\gamma_4+\eta_4)+\gamma_1(\gamma_3+\eta_3)<1 \textit{ and hence } \gamma_6 \equiv [1-(1+\alpha_3)(\gamma_4+\eta_4)-\gamma_1(\gamma_3+\eta_3)]>0 \quad (11)$$

Equations (8) – (11) imply values of the *direct multiplier effects*, m_{11} and m_{22}, that exceed unity from improving the production efficiency of Department r's teaching and research activities given by:

$$\partial \ln z_{1r} / \partial \ln \varepsilon_{1r} = m_{11} \equiv [(1-(1+\alpha_3)(\gamma_4+\eta_4))/\gamma_6]>1, \partial \ln z_{2r} / \partial \ln \varepsilon_{2r} = m_{22} \equiv [(1-\gamma_1(\gamma_3+\eta_3))/\gamma_6]>1 \quad (12)$$

In addition, there are *indirect multiplier effects* given by:

$$\partial \ln z_{1r} / \partial \ln \varepsilon_{2r} = m_{12} \equiv [\gamma_1(\gamma_4+\eta_4)/\gamma_6]>0, \; \partial \ln z_{2r} / \partial \ln \varepsilon_{1r} = m_{21} \equiv [(1+\alpha_3)(\gamma_3+\eta_3))/\gamma_6]>0 \quad (13)$$

$$\partial \ln z_{1r} / \partial \ln \varepsilon_{3r} = m_{13} \equiv \gamma_1/\gamma_6>0, \; \partial \ln z_{2r} / \partial \ln \varepsilon_{3r} = m_{23} \equiv (1+\alpha_3)/\gamma_6]>0 \quad (14)$$

The multiplier effects on Department r's teaching and research quality-weighted output generated by a proportional increase in the ability of its staff are less than those generated by the same proportional increases in its production efficiency, with

$$\partial \ln z_{1r} / \partial \ln a_{1r} = m_{14} \equiv m_{11} - \eta_1 m_{13} < m_{11}, \partial \ln z_{2r} / \partial \ln a_{2r} = m_{25} \equiv m_{22} - \eta_2 m_{23} < m_{22} \quad (15)$$

to an extent that increases with the extent to which higher ability staff cost more to hire and retain, as reflected in the coefficients η_1 and η_2 in the wage equation (5). The indirect multiplier effects of increased abilities are similarly given by:

$$\partial \ln z_{1r} / \partial \ln a_{2r} = m_{15} \equiv m_{12} - \eta_2 m_{13} < m_{12}, \partial \ln z_{2r} / \partial \ln a_{1r} = m_{24} \equiv m_{21} - \eta_1 m_{23} < m_{21} \tag{16}$$

From (8) – (10) and (12) – (14), we may define measures of the overall effectiveness and ability levels of Department r in each direction k by:

$$\xi_{kr} \equiv \sum_{j=1}^{3} m_{kj} \ln \varepsilon_{jr} \leq 0 \textit{ and } \vartheta_{kr} \equiv \sum_{j=1}^{2} m_{k,3+j} \ln \mathrm{a}_{jr} \quad \textit{for } k = 1,2 \tag{17}$$

where the multiplier effects m_{kj} place values on the underlying potential increases in production efficiency and effectiveness in income generation in terms of their impact on the overall effectiveness of Department r in generating quality-adjusted teaching and research outputs.

We may also show from (7) – (10) that:

$$\partial \ln z_{1r} / \partial \ln \theta_{1r} = (\gamma_1 m_{11} - \gamma_5 m_{12}), \partial \ln z_{2r} / \partial \ln \theta_{2r} = (\gamma_5 m_{22} - \gamma_1 m_{21})(\varsigma_1 / \varsigma_2) \tag{18}$$

We will assume that $m_{11} > (\gamma_5 / \gamma_1) m_{12}$ and $m_{22} > (\gamma_1 / \gamma_5) m_{21}$ so that increasing the proportion of time spent on teaching (research) boosts the quality of teaching (research) performance overall.

3 The Achievement Possibility Frontier

By eliminating θ_{1r} from (8) – (10), and setting $\varepsilon_{1r} = \varepsilon_{2r} = \varepsilon_{3r} = 1$, we obtain for $z_r \equiv (z_{1r}, z_{2r})$:

$$F_r(z_r; n_{0r}) \equiv \ln z_{2r} - c_0 + c_1 \ln z_{1r} - c_2 \ln n_{0r} - c_{3r} = 0 \textit{ where } c_0 \equiv (m_{21} + c_1 m_{11})(1 - \alpha_1) \ln \gamma_0 + c_2 \ln(\gamma_2 / \gamma_3) \tag{19}$$

$$c_1 \equiv [(\gamma_5 m_{22} - \gamma_1 m_{21}) / (\gamma_1 m_{11} - \gamma_5 m_{12})] > 0, c_2 \equiv (m_{23} + c_1 m_{13}) > 0, c_{3r} \equiv \vartheta_{2r} + c_1 \vartheta_{1r} \quad (20)$$

Equation (19) maps out an *Achievement Possibility Frontier (APF)*, of the maximum level of quality-adjusted research output z_{2r} that is feasible for any given level of Department r's quality adjusted teaching output z_{1r}, given the exogenous funding parameter n_{or} which it faces and the ability levels of its staff. Associated with the APF given by equation (19), we may also derive from equations (8) – (17) that the observed levels z^o_{1r} and z^o_{2r} of the quality-adjusted teaching and research output for Department r are such that:

$$\ln z^o_{2r} = c_0 - c_1 \ln z^o_{1r} + c_2 \ln n_{0r} + c_{3r} + \xi_r \quad where\ \xi_r = \xi_{2r} + c_1 \xi_{1r} \le 0 \quad (21)$$

The use of Data Envelopment Analysis (DEA) to estimate the position of the APF in (19) encounters the problem that the feasible set which the APF defines for feasible combinations of z_{1r} and z_{2r} is non-convex, contrary to one main underlying assumption of the standard model of DEA (see Cooper et al. 2007), for any given maximal finite level of the ability parameters. The use of Stochastic Frontier Analysis (SFA) to estimate (19) and (21) directly encounters the problem that the observed level of z^o_{1r} on the RHS of (21) is not independent of the individual disturbance terms c_{3r} and ξ_r in (21). Nevertheless, we may make progress in the use of SFA in this context if we assume that the individual efficiency and effectiveness terms ε_{ir} for $i = 1,..,3$ for each Department r are independently and identically distributed across all Departments according to a multivariate half-lognormal density function, defined for $0 < \varepsilon_{ir} \le 1$, $i = 1,2,3$ by:

$$\phi(\varepsilon_{1r}, \varepsilon_{2r}, \varepsilon_{3r}) \equiv \kappa_0 \varphi(\Gamma_r) \equiv \kappa_0 \left|\Omega\right|^{-1} (2\pi)^{-3/2} \exp(-0.5 \Gamma_r \Omega^{-1} \Gamma_r') \quad where\ \Gamma_r \equiv (\ln\varepsilon_{1r}, \ln\varepsilon_{2r}, \ln\varepsilon_{3r}) \quad (22)$$

but where the covariance matrix $\Omega \equiv [\sigma_{hj}]$ permits correlation between the elements of the vector Γ_r for any given Department r selected from this distribution. $\kappa_0 = 8$ is chosen here to ensure that $\Phi(1,1,1) = 1$, where Φ is the distribution function associated with ϕ. The density function $g_k(\xi_{kr})$ of the compound effectiveness term ξ_{kr} may be derived from (17) and (18) as follows:

$$g_k(\xi_{kr}) = \int_{\varepsilon_{1r}=0}^{1} \int_{\varepsilon_{2r}=0}^{1} \phi(\varepsilon_{1r}, \varepsilon_{2r}, \varepsilon_{3r}^k) d\varepsilon_{1r} d\varepsilon_{2r} \ \ where \ \ \varepsilon_{3r}^k \equiv \exp([\xi_{kr} - \sum_{j=1}^{2} m_{kj} \ln \varepsilon_{jr}] / m_{k3}) \tag{23}$$

$$= \int_{\ln\varepsilon_{1r}=-\infty}^{0} \int_{\ln\varepsilon_{2r}=-\infty}^{0} \kappa_0 \varphi(\Gamma_r^k) d\ln\varepsilon_{1r} d\ln\varepsilon_{2r} \ \ where \ \Gamma_r^k \equiv (\ln\varepsilon_{1r}, \ln\varepsilon_{2r}, \ln\varepsilon_{3r}^k) \tag{24}$$

$$= (0.5)^2 \kappa_0 \int_{\ln\varepsilon_{1r}=-\infty}^{\infty} \int_{\ln\varepsilon_{2r}=-\infty}^{\infty} |\Omega|^{-1} (2\pi)^{-3/2} \exp(-0.5 \Gamma_r^k \Omega^{-1} \Gamma_r^{k\prime}) \ d\ln\varepsilon_{1r} d\ln\varepsilon_{2r} \tag{25}$$

$$= 2(2\pi\sigma_k^2)^{-0.5} [\exp(-\xi_{kr}^2 / 2\sigma_k^2)] \ \ where \ \sigma_k^2 = \sum_{h=1}^{3} \sum_{j=1}^{3} m_{kh} m_{kj} \sigma_{hj} \tag{26}$$

using Mood and Graybill (1963, p. 211). The density function of the overall effectiveness term $\xi_{kr} \le 0$ in (17) in each direction k resulting from the above multivariate distribution is therefore half-normal, which we will denote by $N^-(0, \sigma_k^2)$. We can therefore generalise the concept of half-normality defined in Kumbhakar and Lovell (2000) to the multivariate case to derive an overall effectiveness term whose distribution takes into account both the multiplier effects of improvements in each relevant direction and the extent of any correlation in the possible improvements across Departments in each direction, and which is itself half-normal. The individual ability coefficients a_{1r} and a_{2r} are each assumed to be identically and independently distributed across the population of all relevant Departments according to a multivariate lognormal distribution, with $\ln a_{1r}$ and $\ln a_{2r}$ having means μ_{01} and μ_{02} respectively, and a covariance matrix of $\Omega_0 \equiv [\sigma_{0hj}]$ for $h, j = 1,2$ between the ability coefficients for any given Department in each relevant direction that is independent also of the $\varepsilon_{jr'}$ terms for all r' and $j = 1,...,3$. It then follows from Aitchison and Brown (1963, p. 12) that ϑ_{kr} in equation (17) is $N(\mu_k, \sigma_{ok}^2)$, where:

$$\mu_k = \sum_{j=1}^{2} m_{k,3+j} \mu_{oj} \ \ and \ \ \sigma_{0k}^2 = \sum_{h=1}^{3} \sum_{j=1}^{3} m_{k,3+h} m_{k,3+j} \sigma_{0hj} \ \ for \ k = 1,2 \tag{27}$$

We then have from equations (8) – (27) that for $k = 1,2$:

$$\ln z_{kr} = \beta_{k0} + \beta_{k1} \ln n_{or} + \beta_{k2} \ln \theta_{1r} + \vartheta_{kr} + \xi_{kr} \ \ where \ \vartheta_{kr} \sim iid \ N(\mu_k, \sigma_{0k}^2) \ and \ \xi_{kr} \sim iid \ N^-(0, \sigma_k^2) \tag{28}$$

with $\beta_{k0} = m_{k1}(1-\alpha_1)\ln\gamma_0 + m_{k3}\ln(\gamma_2/\gamma_3)$, $\beta_{k1} = m_{k3}, \beta_{k2} = \gamma_1 m_{k1} - \gamma_5 m_{k2}$ (29)

If observations are available on the predetermined variable θ_{1r} across Departments, Stochastic Frontier Analysis permits the consistent estimation of the coefficients β_{hk} for $h = 0,1,2$ and $k = 1,2$ in equation (28). Using equations (19), (20) and (29), estimates of the coefficients c_0, c_1 and c_2 of the APF in (19) may then be derived from the associated inter-relationship:

$$c_0 = \beta_{20} + c_1\beta_{10}, c_1 = -\beta_{22}/\beta_{12}, c_2 = \beta_{21} + c_1\beta_{11} \tag{30}$$

Since our main concern here is with the estimation of the position of the APF, the existence of resource endogeneity is incorporated within the multiplier effects that contribute to what are essentially reduced form coefficients in equations (28) and (30) that determine the position of the APF. As noted above, SFA permits a consistent estimation of the associated β_{hk} coefficients, and implied c_0, c_1 and c_2 coefficients of the APF.

One of the additional advantages of the use of SFA in this context is that for both teaching and research outcomes individually it can seek to separate out in the estimated equation (28) an estimate of ϑ_{kr} across each Departments which is due here to differences in staff ability from an estimate of ξ_{kr} that is due to differences in effectiveness within Department r. The mean (or mode) of the conditional distribution of ξ_{kr} given $\upsilon_{kr} \equiv \vartheta_{kr} + \xi_{kr}$ (see Jondrow et al. 1982) can then be used in seeking to separate out an estimate of the ability parameter ϑ_{kr} from that of the effectiveness parameter ξ_{kr} in (28). As in equation (17), these parameters reflect respectively the value of the impact of differences in the disaggregated ability parameters a_{1r} and a_{2r}, and in the disaggregated efficiency and effectiveness terms $\varepsilon_{1r}, \varepsilon_{2r}$ and ε_{3r}, on the quality-adjusted teaching and research outputs of Department r, when account is taken of the relevant multiplier effects which these individual terms have on the Department's outputs under resource endogeneity. If our prime interest is in assessing the scope for higher levels of quality-adjusted output of teaching and research from Department r, even given the current level of ability of its staff, estimation of the corresponding value of ξ_{kr} can be generated from the application of SFA to equation (28) which incorporates the multiplier effects that the above reduced form approach encompasses.

4 Effectiveness Analysis, Performance Measurement and SFA

Within our multi-output SFA framework, we can derive a coefficient of the overall technical effectiveness, φ_{Tr}, of Department r analogous to that considered by DEA. Following the seminal contribution of Farrell (1957), φ_{Tr} is equal to the inverse of the proportional increase in each output that is feasible, here within the *Achievement Possibility Set* (APS) $\Theta_r(n_{0r}) \equiv \{ z_r \mid z_r \geq 0 \,\&\, F_r(z_r; n_{0r}) \leq 0\}$ for Department r. Hence:

$$\phi_{Tr} = D_{Tr}(z_r^o, n_{0r}) \equiv \min_{\tau_T} \{ \tau_T \mid F_r(z_r^o / \tau_T; n_{0r}) \leq 0\} = OA / OB = OH / OJ \quad (31)$$

where D denotes a distance function (Shephard 1970; Fare and Primont 1990), A is the point in Fig. 2 corresponding to the quality-adjusted output vector z_r which Department r actually achieves, and B is the point on the Achievement Possibility Frontier for Department r for its given base funding level n_{or} that also lies on the ray OA from the origin, O.

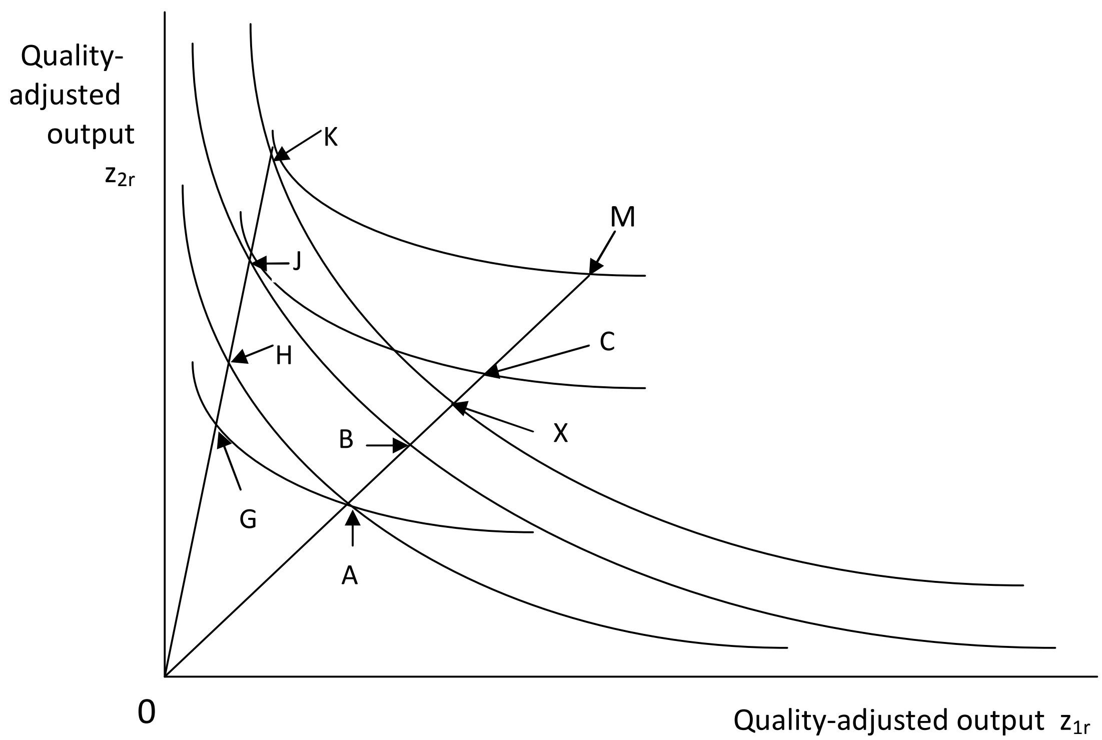

Fig. 2 Iso-valuation curves and APFs for different levels of base funding

Equations (19), (21) and (31) yield:

$$\varphi_{Tr} = \exp(\xi_r / (1 + c_1)) \text{ with } 0 \le \varphi_{Tr} \le 1 \qquad (32)$$

J in equation (31) and Fig. 2 is a point lying on the same APF that passes through B. H is the point lying on the same ray through the origin as J but on the APF that passes through point A and has a correspondingly lower level of the base funding parameter, n_{or}, equal to:

$$n_{oor} = n_{or}\varphi_{ITr} \quad where \quad \varphi_{ITr} \equiv \exp(\xi_r / c_2) \qquad (33)$$

with which Department r could achieve its existing levels of quality-adjusted outputs z_{1r}^{o} and z_{2r}^{o} in (21) if it were fully technically effective. φ_{ITr} provides here an *input-orientated measure* of technical effectiveness for our multiple-output model with endogenous inputs, of the proportional reduction in the base funding from government that would still enable Department r to produce its existing quality-adjusted outputs if it were fully effective. The last equality in (31) follows from equations (21), (32) and (33). Estimates of the overall output- and input-orientated measures of technical effectiveness, φ_{Tr} and φ_{ITr}, in the presence of resource endogeneity can then be derived from equations (21), (32) and (33) given estimates of c_1, c_2, ξ_{1r} and ξ_{2r} from the stochastic frontier estimation of equation (28).

If we introduce a *valuation function,* $V(z_r)$*,* defined upon the quality-adjusted outputs, we may define a value-based measure of the overall effectiveness, φ_{Vr}, of Department r to be given by:

$$\varphi_{Vr} = D_{or}(z_r^o, n_{0r}) \equiv \max_{\tau_o}\{\tau_o \,|\, V(z_r^o / \tau_o) \ge V_r^*(n_{0r})\} = OA/OC \;\; for\; V_r^*(n_{0r}) \equiv \max_{z_r} V(z_r) : F_r(z_r; n_{0r}) \le 0 \qquad (34)$$

where C is the point in Fig. 2 that lies both on the ray OA from the origin O and the highest possible iso-valuation curve that can be reached within the Achievement Possibility Set for Department r.

In particular, a homothetic CES valuation function of the form:

$$V(z_r) = (\mathrm{v}_1 z_{1r}^{\varpi} + \mathrm{v}_2 z_{2r}^{\varpi})^{1/\varpi} \quad where\ \varpi < 0, \partial V / \partial z_{1r} > 0\ and\ \partial V / \partial z_{2r} > 0 \tag{35}$$

achieves a maximum value, V_r^*, subject to $F_r(z_r;n_{0r}) \leq 0$ at a point such as J in Fig. 2 that is on the APF passing through point B but where the first-order conditions:

$$V_{21} \equiv -(dz_{2r} / dz_{1r})\big|_{V=V_r^*} = (\mathrm{v}_1 z_{1r}^{\varpi-1} / \mathrm{v}_2 z_{2r}^{\varpi-1}) = (c_1 z_{2r} / z_{12}) = -(dz_{2r} / dz_{1r})\big|_{F_r=0} \equiv F_{r12} \tag{36}$$

also hold, so that at the optimal point J:

$$(z_{2r}^* / z_{1r}^*) = (\mathrm{v}_2' c_1)^{-1/\omega} \quad and\ z_{1r}^* = [n_{or}^{c_2} (\mathrm{v}_2' c_1)^{1/\omega} \exp(c_1 + c_{3r})]^{1/(1+c_1)} \quad where\ \ \mathrm{v}_2' \equiv \mathrm{v}_2 / \mathrm{v}_1 \tag{37}$$

using (19) and (36). In addition, the associated second-order condition:

$$(dV_{21} / dz_{1r})\big|_{V=V_r^*} = (\varpi - 1)(z_{2r} / z_{1r}^2) c_1 (1 + c_1) < -(z_{2r} / z_{1r}^2) c_1 (1 + c_1) = (dF_{r21} / dz_{1r})\big|_{F_r=0} \tag{38}$$

is satisfied at the point of maximum effectiveness, using (35) and (36). From (19), (34) - (37), we have:

$$\phi_{Vr} = (OA / OC) = (OG / OJ) = V_A / V_J = z_{1r}^o \Psi_r^o \quad for\ \ \Psi_r^o \equiv [(1 + v_2' (z_{2r}^o / z_{1r}^o)^{\varpi}) / (1 + (1/c_1))]^{1/\varpi} / z_{1r}^* \tag{39}$$

where G is the point on the ray OJ in Fig. 2 that lies on the same iso-valuation curve as point A, so that the measure of overall effectiveness of Department r is equal to the ratio between the valuation V_A of its quality-adjusted outputs that it does actually achieve at point A and the valuation V_J which it could have achieved at point J if it were both fully technically and allocatively effective. Moreover using (38) and (40) the measure of the overall effectiveness of Department r, φ_{Vr}, can be evaluated in terms of its observed levels z_{1r}^o and z_{2r}^o of its quality-adjusted output, the relative values v_2' placed upon these outputs, the elasticity of substitution, $1/(1-\varpi)$, between these outputs in the valuation function, its base funding parameter n_{0r}, and the coefficients c_0, c_1, c_2 and c_{3r} that can be derived from the SFA estimation of the Achievement Possibility Function, F_r, for Department r.

Associated with $V(z_r)$, we may also define a measure of the *allocative effectiveness*, ϕ_{Lr}, of Department r to be given by:

$$\varphi_{Lr} = D_{Lr}(z_r^o, n_{0r}) \equiv D_{or}(z_r^o / \varphi_{Tr}, n_{0r}) = \max_{\tau_O}\{\tau_o \mid V(z_r^o / \varphi_{Tr}\tau_o) \geq V_r^*(n_{0r})\} = OB / OC \tag{40}$$

Department r's allocative effectiveness is in turn equal to its overall effectiveness divided by its technical effectiveness, as given by SFA from equation (32), with:

$$\varphi_{Lr} = \varphi_{Vr} / \varphi_{Tr} = (OA / OC)/(OA / OB) = (OG / OJ)/(OH / OJ) = (OB / OC) = (OG / OH) \tag{41}$$

$$= V_B / V_J = \exp[\int_J^B \sum_{j=1}^{2} (\partial V / \partial z_{jr}) V^{-1} dz_{jr}] = (z_{1r}^o / \varphi_{Tr}) \Psi_r^o \tag{42}$$

using (39) and the homotheticity of (19) and (36), and where the integration takes place along the APF between points J and B. φ_{Lr} is thus equal to the ratio between the valuation V_B it would have achieved if it were technically effective at B and the valuation it would have achieved at V_J if it were both fully technically and allocatively effective. Such a measure of allocative effectiveness thus addresses the 'Greene problem' (Greene 1993, 2008) of providing a measure of allocative effectiveness, here in the context of public services, that is derived from the extent to which the allocation of resources between the outputs z_{1r} and z_{2r} fails to satisfy the marginal equivalences given by (36) for the optimization of the output valuation function as one moves from the optimal point J to the point B, which is on the APF but no longer optimal, and in which "deviations from the optimality conditions in any direction translate" (Greene 2008, p. 188) here to a lower valuation of the outputs. Our focus here is upon the allocative effectiveness of the public sector producer in choosing its output-mix, rather than allocative efficiency of its input choices for a given level of output. Thus, rather than make use of shadow costs, as in Atkinson and Primont (2002), (42) involves the integration of the shadow marginal valuations on the quality-adjusted outputs in such a way as to permit the computation of the allocative effectiveness of Department r, using (38) and (40), from the data specified above for the computation of ϕ_{Vr} and for the computation of ϕ_{Lr}.

A further advantage of using SFA to estimate the Achievement Possibility Frontier, F_r, for Department r is its use here in seeking to identify feasible improvements in both technical and allocative effectiveness for Department r, *holding constant the current ability level of its staff*. In contrast, the improvements which DEA seeks to identify assume directly comparable labour and other inputs of the same homogeneous quality for all decision-making units (DMUs) in the sample. However, if staff ability levels vary across different Departments, it is important to be able to distinguish between the improvements in effectiveness which are feasible for Department r even given the current ability levels of its staff and the shortfalls in current performance which are associated with lower comparative levels of the underlying ability of its staff. Fortunately we may extend the use of SFA in our current multiple-output context to include an assessment of the extent of these latter shortfalls in performance. When we denote by $r*$ the Department with the highest value of the coefficient c_{3r} in the population of all Departments for which comparative data are available, we may define an index of relative ability by:

$$P_{Rr} \equiv \exp((c_{3r} - c_{3r*}) / (1 + c_1)) = \exp((\vartheta_{2r} - \vartheta_{2r*} + c_1(\vartheta_{1r} - \vartheta_{1r*})) / (1 + c_1)) \quad (43)$$

$$= D_{or*}(z_r^o, n_{0r}) / D_{or}(z_r^o, n_{0r}) = (OA / OM)/(OA / OC) = (OC / OM) = (OJ / OK) = V_J / V_K$$

using equations (19), (20) and (35). K is here the point of tangency in Fig. 2 between the Achievement Possibility Frontier that Department r would have if its ability coefficient was c_{3r^*} rather than c_{3r} and the highest iso-valuation curve that it could reach within the associated Achievement Possibility Set. M is the point in Fig. 2 on this highest attainable iso-valuation curve that is on the same ray through the origin O as point A. We may then define an *overall index of performance* for Department r by:

$$P_{Or} = P_{Rr}\varphi_{Vr} = P_{Rr}\varphi_{Lr}\varphi_{Tr} = (OC / OM)(OA / OC) = (OA / OM) = D_{or*}(z_r^o, n_{0r}) \quad (44)$$

$$= (OJ / OK)(OG / OJ) = (OG / OK) = V_A / V_K$$

that is equal to the product of the index of relative ability, the coefficient of allocative effectiveness and the coefficient of technical effectiveness for Department r, and equal to the ratio between the valuation V_A of its quality-adjusted outputs that it actually achieves at point A and the valuation V_K that it would have achieved at K had it been fully technically and allocatively effective and had the same ability level of its staff as Department $r*$.

Associated with (44) we may define the index of technical performance of Department r by:

$$P_{Tr} = P_{Br}\varphi_{Tr} = \exp[(\upsilon_r - \upsilon_{r*})/(1+c_1)] = (OJ/OK)(OH/OJ) = (OH/OK) \;\; where\, \upsilon_r \equiv \upsilon_{2r} + c_1\upsilon_{1r} \tag{45}$$

using (20), (21), (32) and (43) under the assumption that the most able Department, $r*$, is itself on its own APF and thus technically effective. In addition, we may define the index of allocative performance of Department r to be given by:

$$P_{Lr} = P_{Or}/P_{Tr} = (z^o_{1r}/P_{Tr})\Psi^o_{r*} = [n^{c_2}_{or}(\mathrm{v}'_2 c_1)^{1/\omega}\exp(c_1 + \upsilon_r)]^{1/(1+c_1)} = (OX/OM) = (OG/OH) = \varphi_{Lr} \tag{46}$$

using (38) and (40) – (44), where X is the point in Fig. 2 that is on the APF for Department $r*$ and on the ray OA. The allocative performance of Department r can thus be evaluated here independently of the choice of $r*$ in equation (40), whereas both the index of technical performance in (45) and the overall index of performance in (44) depend upon the value of υ_{r*}. All three of these performance indices are, however, independent of the division between the ξ_{kr} and ϑ_{kr} terms of the overall disturbance terms υ_{kr} for each $k = 1, 2$. Nevertheless, equations (42) – (45) enable the overall and technical performance indices to be disaggregated under SFA into different components which reflect the relative ability of Department r and its technical effectiveness in ways which do depend upon this division.

5 Alternative Approaches

An alternative approach to tackling the endogeneity of the overall funding for individual universities would be to adopt a fixed-effects panel data formulation, in which the endogeneity can be accommodated. However, as Greene (2008 p. 229) notes: "The fixed-effects model does carry with it the necessity that the analyst revert back, essentially, to the deterministic frontier model. The random effects model, on the other hand, has the appeal of the single-equation stochastic frontier. However, as in other settings, the drawback to this approach is that the effects must be assumed to be uncorrelated with the regressors". Moreover, as Greene (2008) also emphasises, the efficiency estimates of SFA appear to be particularly sensitive to the specification of whether or not each producer's efficiency level is assumed to be time-invariant, as in the basic fixed-effects model, or to vary over time. Estimating a specific pattern of time variation in producers' efficiency levels, as in Cornwell, Schmidt and Sickles (1990), imposes restrictions on their behaviour which may well not be justified and requires a sufficiently extensive panel dataset over time that may not be available if systematic quality surveys on both the teaching and the research outputs of universities are conducted infrequently.

A second alternative approach would be to apply SFA to each institution's costs as a function of the multiple outputs of the institution. This approach is adopted in Izadi et al. (2002) using total university expenditure as the dependent variable and the explanatory variables of undergraduate numbers in arts subjects and in science subjects, postgraduate numbers and the value of research grants and contracts received in a cross-sectional study of 99 UK universities. However, this does not avoid the endogeneity problem once research income and student recruitment depend upon the efficiency with which the university is run, and on the current quality of its teaching and research output, which themselves may be compromised by lower levels of university expenditure. While some escape from the endogeneity problem might be provided by making these inter-relationships lagged ones, persistence or correlation over time in the efficiency and effectiveness terms for a given university or Department would tend to negate this escape. Progress has been made more recently by Blank (2012) in developing a cost indirect revenue model based upon the assumption of revenue maximisation under a budget constraint for a public sector provider whose revenue does depend on their output, though with fixed input and output prices and a fixed level of subsidy, with estimation of the parameters of an associated indirect translog output value function using Seemingly Unrelated Regression methods rather than frontier techniques.

A third alternative approach is the use of DEA to estimate the production frontier, as in Johnes and Johnes (1995), who carried out a DEA estimation of the efficiency of UK Departments of Economics using six dimensions of published research output during the period 1984-88 and inputs of the numbers of staff employed on teaching and research contracts and per capita research grant income. More recently, Foltz et al. (2012) have used DEA to estimate the efficiency and rate of technological progress of 92 US research-orientated universities over the period 1981-1998. As a deterministic frontier technique without a stochastic structure, DEA might be considered to be immune from endogeneity bias, which might otherwise arise when a strong recent publication record itself helps to attract greater research grant income. However, as Orme and Smith (1999) demonstrate in another context, endogenous inputs may still lead to 'sparsity bias' when a correlation between efficiency and resource inputs leads to fewer observations being available of efficient producers at some levels of the resource inputs than others, thereby distorting DEA's estimated efficient isoquants for the underlying production function. In contrast to the case considered by Orme and Smith (1999) of negative feedback resulting from compensatory funding for adverse local socio-economic circumstances and performance, the positive feedback relationships considered above support the inclusion of the associated multiplier effects of improved efficiency upon outputs within the estimation of an Achievement Possibility Frontier rather than a conventional production frontier. As we have seen, the convexity requirement of DEA may still, however, be broken in this context. While the alternative deterministic stepwise frontier technique of fitting a Free Disposal Hull (FDH) (see Deprins et al. 1984; Mayston, 2003) to envelop the dataset avoids the convexity requirement, the use of SFA in this context still has advantages over DEA and FDH in seeking to separate out differences between Departments which are due to differences in staff ability levels from those which are due to differences in their effectiveness for existing levels of staff ability, with the efficiency estimates of both DEA and FDH potentially very sensitive to the existence of atypical outliers and extreme values in the dataset. Moreover, the welcome attempt by Simar and Wilson (2000, 2008) to introduce tests of statistical significance into DEA and FDH methodology poses the problem that endogeneity bias may be thereby introduced into resultant semi-parametric efficiency estimates. Our above focus on the Achievement Possibility Frontier, with the inputs into the analysis restricted to the exogenous funding variable rather than all endogenously determined individual inputs, nevertheless has scope for application to DEA and FDH both for tackling possible endogeneity bias and as a means of limiting the problem noted by Dyson et al. (2001) and Podinovski and Thanassoulis (2007) of a lower ability of DEA to discriminate amongst DMUs in their performance ratings when the number of inputs and outputs considered is large relative to the number of DMUs.

A further alternative approach would be the use of Instrumental Variables (IV) in place of the resource inputs in the estimation of the production functions of equations (2) and (3). Under the associated model specification given by the matrix equation (8), and with the equation for each individual production function excluding both the other output and the exogenous variable $\ln n_{0r}$, equations (2) and (3) satisfy the associated necessary and sufficient rank condition for being (asymptotically) just identified (Davidson and MacKinnon 2004). By parallel reasoning to that in equations (22) – (28) above, SFA may then be applied in conjunction with a 2SLS approach to the IV estimation of the individual production efficiency terms ε_{1r} and ε_{2r}. However, in the absence of additional pre-determined variables elsewhere in the model, the production function equation (2) would no longer be identified if it were modified to include in the production function a non-zero effect of quality-adjusted research output on quality-adjusted teaching output, such as via greater research activity itself contributing towards enhanced knowledge and teaching capability within the Department. Similarly, the production function equation (3) for research would no longer be identified here if it were modified to include a non-zero effect of teaching activity on research, such as through engagement in teaching helping to inspire new ideas for research. Nevertheless, even in the absence of such identifiability of the individual production functions, the inclusion of non-zero terms in place of the zero entries in the existing A matrix in (8) will modify the value of the relevant multiplier effects to be given by $[m_{kj}] = A_1^{-1}$ for $k, j = 1,..,3$, where A_1 is the new coefficient matrix, but will leave unchanged the form of equations (15) – (45). SFA therefore still has a valuable role to play in estimating the Achievement Possibility Frontier and each Department's distance from it under resource endogeneity, even when the underlying production function equations are not identified.

6 Conclusion

The ability of a university Department to generate additional finance to help fund increases in its quality-adjusted research and teaching illustrates important more general issues concerning the analysis of the efficiency and effectiveness of public service providers in the presence of resource endogeneity. Fortunately progress can be made through the application of Stochastic Frontier Analysis to the assessment of the overall effectiveness and performance of the public service provider, and their decomposition into both technical and allocative components, using the wider concept of an Achievement Possibility Set that includes the multiplier effects which such resource endogeneity generates, rather than the standard production possibility set. The existence of endogeneity issues in the application of SFA and other frontier techniques deserves greater recognition, with the need for adequate attention to be given to methods of making progress when endogeneity does exists.

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