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Abstract

We study redistributive agreements designed collectively by individual and independent states for the joint supply of a public good. We specifically model the case of international environmental agreements but our analysis should be equally applicable to other multinational arrangements with redistributive aspects. The basic intuition of the investigated class of mechanisms is that, if part of member GDP is redistributed, then the redistributive resource has lower variance than individual income: a side effect of redistribution is risk-sharing. If, in addition, the sum of contributed parts of individual GDP forms a contest prize pool which returns the contributions as prizes to the participants depending on a relative ranking of public good provision levels, then the mechanism can also implement efficient efforts. (JEL *C7, D7.*
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1 Introduction

We study individual incentives to join an international agreement—created voluntarily among independent nations—for the purpose of jointly supplying a public good. For concreteness, we consider the case of an international environmental agreement under a specific incentive mechanism which allocates some share of the participating states' individual income (interpreted as GDP) to the members performing best among a relative ranking of all nations' emissions reduction efforts. In equilibrium, this contest takes the form of a redistributive mechanism which assigns to each participant some share of the collected revenue pool. As it turns out, for stochastic individual GDP, the variance of the collective redistribution pool is lower than that of individual income. The redistributive contest mechanism can therefore fulfill aspects of a mutual insurance agreement which can entice a country with a sufficiently strong dislike of income fluctuations to join an agreement on which, in the absence of this income smoothing argument, it would prefer to free ride. Moreover, we show that the redistributive contest can be designed such as to ensure both efficient productive as well as emissions reduction efforts.

We specifically discuss international environmental agreements in this paper. Our main arguments, however, should also apply to other redistributive agreements and, in particular, to the recent

*Thanks for helpful comments to Francesco Bravo, Subir Chattopadhyay, Pierre Fleckinger, Fabrizio Iacone, and Mike Wickens. Both authors are grateful for the hospitality of their co-authors institutions and for financial support through the University of York Super Pump Priming Fund. †LEM-Department of Economics, University Panthéon-Assas, 5/7 Avenue Vavin, 75006 Paris, France, olivier.bos@u-paris2.fr. ‡Department of Economics, University of York, Heslington, York YO10 5DD, United Kingdom, paul.schweinzer@york.ac.uk. (11-Jul-2012) debates about the stability of the European Union or the United Kingdom. Referring to the options of an independent Scotland, *The Economist* recently expressed the view that "*it remains a matter of judgement whether Scotland could go it alone. But after the banking and euro-zone crises, Scotland would be far more vulnerable to shocks as a nation of 5m people than as part of a diversified economy of 62m.*" (April 14th 2012) This is precisely the intuition this paper formalises.

Literature

There is a large and active literature on the formation of environmental agreements. We hope that the availability of comprehensive and recent surveys by, for instance, Barrett (2006), Finus (2008), and Guesnerie and Tulkens (2009) allows us to keep our overview minimal. The consensus in the literature seems to be that there is no consistent theoretical basis on which large voluntary agreements can be formed among independent nation states which do not have to revert to exogenous reward, punishment or exclusion strategies to avoid free-riding on emissions reduction.

On the subject of risk-sharing, Wilson (1968) discusses the optimal behaviour of a group of individuals, called a syndicate, who must make a common decision under uncertainty that results in a payoff to be shared jointly among them. As in our model, the strictly risk-averse players face individual payoff uncertainty which they pool in the syndicate. The paper derives an optimal investment strategy for the syndicate and is then mainly interested in the derivation of Pareto optimal sharing rules for the syndicate members characterised in terms of, e.g., risk tolerance.[1]

Deaton (1991) models the optimal intertemporal consumption behaviour of consumers whose labour income is stochastic over time. He shows that individual savings can act like a buffer stock, protecting individual consumption against shocks. The same idea has been used to model consumption-savings choices and insurance motives across consumers' life-cycles. The same variance-compression idea is present in Green (1987) who discusses tax-financed unemployment insurance and Arnott and Stiglitz (1991) who model voluntary, non-market insurance between households under moral hazard. In a macroeconomic growth model, Devereux and Smith (1994) discuss the idea of international risk diversification but find that, if used, it leads to lower growth rates than the environment in which countries retain the original risk.

In the contest literature, Green and Stokey (1983), based on work by Lazear and Rosen (1981), show that, when a common output shock affects the output of multiple agents, then rank-order tournaments may dominate individual, cardinal compensation schemes as means to motivate workers. Their reasoning is intuitive since, under a common shock, the relative rank of an agent's output is unaffected while absolute output measures expose the workers to risk. This argument is almost diametrically opposed to our idea of exploiting the pooling of individual shocks against free-riding.[2]

There are a handful of papers on contests in stochastic environments exploring, for instance, the effects of asymmetric information, an unknown number of contestants, or errors in the realisation of the strategic variables. Neither of these approaches is related to this paper since our main insurance

[1] This idea found multiple notable extensions and applications, for instance, in the work of Wildasin (1995), Banks, Blundell, and Brugiavini (2001), or Demange (2008).

[2] For a detailed survey of the contests literature see the comprehensive Konrad (2008).

argument can also be made for non-stochastic assignment probabilities, for instance, in a pure redistribution game. There is an equally small number of papers on contests with multi-dimensional efforts.[3] Nevertheless, no multi-dimensional contest model that we know of is close to our ratio of distinct efforts in a generalised Tullock/Logit form (introduced in subsection 3.3).

The basic contest model underlying our environmental agreements story is taken from Roussillon and Schweinzer (2010) who also provide a full motivation of the chosen setup. We extend this model with stochastic output and relative per-unit-GDP emissions reduction efforts. Roussillon and Schweinzer (2010) do not model stochastic output and, consequently, cannot make the case for risk-pooling as an argument for agreement formation.

After introducing the model in the following section we discuss variance compression under redistribution in various symmetric and asymmetric settings in section 3. Our main result, that players participate in the reduced variance, redistributive mechanism under sufficient risk aversion, is presented in subsection 3.4. Appendix A supplies the analytic underpinning for the asymmetric mechanism used in one of our examples. Appendix B provides analytical tools useful to determine stochastic relations dominance and arguments on stochastic dominance which we use in our main result.

2 The symmetric model

There is a set $\mathcal{N}$ of $n \geq 2$ players interpreted as independent nation states. In the basic model, these players are fully symmetric but, as we develop our arguments, we extend the model to the asymmetric case. Each player $i \in \mathcal{N}$ exerts efforts along two dimensions: productive effort $e_i \in [0, \infty]$ and reductive (abatement) effort $f_i \in [0, \infty]$. We denote the full effort vectors by $\mathbf{e} = (e_1, \ldots, e_n)$ and $\mathbf{f} = (f_1, \ldots, f_n)$, respectively. The combined effort cost $c(e_i, f_i)$ is assumed to be strictly convex and zero for zero efforts. (For the sake of simplicity we treat costs as completely separable.)

A nation's output process uses productive efforts to generate individual gains of $y(e_i, \varepsilon_i)$ – interpreted as that country's GDP. We assume that this output is stochastic, i.e., given by $y(e_i, \varepsilon_i) = \tilde{y}(e_i) + \varepsilon_i$, where $\tilde{y}(e_i)$ is weakly concave and the shocks ε_i are distributed according to the law $\mathcal{L}(\mu = 0, \sigma^2)$ characterized by the mean μ and the finite variance σ^2 of some continuously differentiable distribution F with symmetric probability density F' over (any subset) of $(-\infty, \infty)$.[4] As suggested by Chamberlain (1983), Owen and Rabinovitch (1983), or Berck (1997), any member of the class of elliptical distributions is eligible for this distribution F.[5]

Global production and abatement efforts cause strictly convex global emissions of $m(\sum_h (e_h - f_h))$ which only depend on the difference between total productive and total reductive efforts. Of this

[3] Examples of multi-dimensional effort contests are Clark and Konrad (2007) who consider a set of independent contests, one along each dimension and Arbatskaya and Mialon (2010) who analyse a model of multiple contests (conceptualised as the arms of a multi-arm bandit) and axiomatise its contest success function.

[4] While the additive structure of the shock is crucial for the arguments we develop, the zero expectation is only chosen for convenience.

[5] Elliptical distributions are a generalisation of the normal family containing, among others, the Student-t, Logistic, Laplace and symmetric stable distributions. A detailed presentation of these distributions is available in Landsman and Valdez (2003) and Fang, Kotz, and Ng (1987).

global damage, player i suffers a local share s_i. As we interpret the shares s_i as taken from physical pollution (e.g., kilograms of greenhouse gases) we assume that $\sum_h s_h = 1$.

Following Roussillon and Schweinzer (2010), we assume that, as means to alleviate this problem, the players jointly—and in the absence of a supranational principal—decide to introduce a redistributive incentive system based on some ranking of individual reductive efforts and award the top-ranked players prizes. The total redistributive pool P (from which these prizes are taken) is defined as the sum of fraction $(1-\alpha)$ of each participant's individual output $y(e_i, \varepsilon_i)$. Thus, the total redistributive pool P is a function of the effort vector $\mathbf{e}$ and the noise vector $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$. The incentive mechanism awards $\beta^1 P$ to the winner, $\beta^2 P$ to the player coming second, and so on, with $\sum_h \beta^h = 1$. Thus, for feasible $\alpha \in [0, 1)$, the incentive mechanism's budget balances by definition. The mechanism is redistributive if each $\beta^h > 0$. The players' commitment is limited to fraction $1-\alpha$ of stochastic output if each $\beta^h \in [0, 1]$.[6]

We assume that some noisy (partial) ranking of the players' reductive efforts is observe- and verifiable. This ranking technology is part of the mechanism the players need to agree on. It gives player i's probability $p_i^h(\mathbf{f})$ of being awarded prize h as a function of the imperfectly monitored reductive efforts of all participants. We assume that $p_i(\mathbf{f})$ is strictly increasing in f_i, strictly decreasing in all other arguments, equal to $^1/_n$ for identical arguments, twice continuously differentiable, and zero for $f_i = 0$ with at least one $f_{j \neq i} > 0$, $j \in \mathcal{N}$. In the second, asymmetric model we extend this relative ranking technology on the basis of a ratio of abatement efforts over output $p(\mathbf{e}, \mathbf{f})$.

The full timing of the interaction is summarised as

0	1	2	3	4	$\rightarrow$ time.
Participation decision	Simultaneous choice of e, f	Shocks realise	Ranking realises	Payoffs realise	

We would like to point out that the players' efforts need not, in principle, be verifiable. As customary in the contests literature, all that is needed is some device which gathers a sufficient statistic on the individual efforts such that it can construct a relative ranking. Only the resulting ranking—the monitoring device's readings, so to speak—must be both observe- and contractible.

Given the ranking technologies $p(\mathbf{f})$ or $p(\mathbf{e}, \mathbf{f})$, a (subgame perfect) equilibrium in this game consists of two elements: a pair of sharing rules (α, β) specifying the prizes in the redistributive contest and a pair of efforts (e, f) determining output and the winning probabilities. Since we are implementing efficiency we are looking for (a)symmetric equilibrium in pure strategies.

2.1 Preferences and information

Expected utility functions are defined over a decision maker's uncertain wealth w. The idea of risk aversion is incorporated in the curvature of the Bernoulli-utility function v over certain payoffs. We

[6] A separate toy model for a currency union can easily be specified along the following lines: Leave the interpretation of e_i unchanged and think of $y(e_i, \varepsilon_i)$ as (stochastic) national income, but take $f_i \in [0, \infty)$ to be budgetary discipline (of which more is better). The linkage between the individually chosen variables is now established by some concave benefit function $g(\sum f_i)$ which models the individual benefit s_i of federal budget discipline. As before, investment into both strategic variables comes at a convex cost.

assume (in our symmetric model) that v is concave and that the player's embodied attitude towards risk is identical for all players. Recall that the only stochastic influence in our model comes from idiosyncratic shocks to output $y(e_i, \varepsilon_i) = \tilde{y}(e_i) + \varepsilon_i$, where $\tilde{y}(e_i)$ is weakly concave and the shocks ε_i are distributed according to $\varepsilon_i \sim \mathcal{L}(\mu = 0, \sigma^2)$ over $(-\infty, \infty)$. We adopt the following formulation for player $i \in \mathcal{N}$

$$\mathbb{E}[u_i(\mathbf{e}, \mathbf{f}; \varepsilon_i)] = \int_{-\infty}^{\infty} v(\underbrace{y(e_i, \varepsilon_i) - s_i m(\sum_{i \in \mathcal{N}}(e_i - f_i)) - c(e_i, f_i))}_{\text{risk-neutral wealth}})dF(\varepsilon_i) \tag{1}$$

with Bernoulli utility function $v(0) = 0$, $v' > 0$, and $v'' \leq 0$. Since the player's choice of efforts is invariant under any increasing, concave transformation of v, we can split the optimisation stage—giving the choice of efforts—from the risk-based participation decision. Hence, methodologically, we start by analysing the underbraced decision problem above in isolation and discuss the risk transformation separately.

In terms of information and as pointed out above, we adopt the customary view of a contest success function as black box translating unverifiable efforts into a contractible ranking.[7] We would like to keep the stochastic process generating income uncertainty as general as possible in terms of distribution and variance while the mean is kept at zero for modelling convenience (otherwise the riskless model could not be used as a benchmark). For a combined flow & stock interpretation of income in our one-shot model this may not be entirely unreasonable. A financial crisis, for instance, could be interpreted as a negative shock to this composite variable.

3 Analysis

The purpose of this paper is to develop the idea of the redistributive agreement formation problem. We do not attempt to provide an analytically rigourous discussion of the underlying contest theory which we only introduce in examples to make our point.[8]

We start by studying a simple redistributive mechanism based on a harmonised output tax. After showing that this mechanism cannot implement efficient efforts, we analyse a modified contest mechanism based on Roussillon and Schweinzer (2010) to solve the efficiency problem in both the symmetric and asymmetric cases. In equilibrium, these contest mechanisms also have redistributive characteristics. The equilibrium analysis for these mechanisms is done on the basis of risk-neutral preferences. In the derived equilibria, we can show that the income variance of participating players is reduced compared to their stand-alone income. We then proceed to show that, for both mechanism types, the derived equilibrium behaviour is invariant under concave transformations. This allows us

[7] Hence, the contest success function based on $x_i = f_i/\tilde{y}(e_i)$ which we adopt in subsection 3.3 could be defended on the grounds of non-contractibility of the stochastic income term.

[8] As discussed below, the contests are not necessary for the variance compression underlying our participation argument. Nevertheless, we use the contests because of their delightful efficiency properties which are, as far as the authors are aware, unique among the class of known mechanisms. For a complete derivation of the properties of underlying contests we refer to Roussillon and Schweinzer (2010) and the papers referenced there.

to apply simple but rather general arguments to show that individual players with sufficiently strong dislike of income fluctuations have incentives to join the redistributive agreement.

In our basic international emissions story, the problem of a single player $i \in \mathcal{N}$ who maximises her individual income in the absence of any agreement is to choose individual productive effort e_i and emissions reduction effort f_i in order to maximise

$$u_i(\mathbf{e}, \mathbf{f}; \varepsilon_i) = \underbrace{y(e_i, \varepsilon_i)}_{\text{output}} - \underbrace{s_i m(\sum_h (e_h - f_h))}_{\text{damage}} - \underbrace{c(e_i, f_i)}_{\text{cost}}. \tag{2}$$

Throughout the paper, we develop a canonical two-players example with quadratic costs and square-root production function to demonstrate the basic idea of our mechanism. (This example is only special in its simplicity.) In the symmetric case where all players are identical (and, therefore, $s_1 = s_2 = 1/2$), a benevolent planner who maximises the sum of social utility net of total cost would want to choose e and f in order to

$$\max_{(e,f)} 2e^{1/2} - (2e - 2f)^2 - 2(e^2 + f^2) \;\Leftrightarrow \begin{cases} e^* \approx 0.282, \\ f^* \approx 0.188. \end{cases} \tag{3}$$

The corresponding individual problem (in the absence of an incentive mechanism) leads to inefficient provision of efforts

$$\max_{(e_i,f_i)} e_i^{\frac{1}{2}} - s_i(e_i + e_j - f_i - f_j)^2 - (e_i^2 + f_i^2) \quad \Leftrightarrow \begin{cases} \hat{e}_i \approx 0.303 > e^*, \\ \hat{f}_i \approx 0.151 < f^*, \end{cases} \tag{4}$$

for any damage shares $s_1 + s_2 = 1$ (the values calculated are for $s_1 = s_2 = 1/2$). Notice that, with respect to the efficient efforts, the combined externality and free-riding inherent in the problem imply that both players simultaneously produce too much and abate too little.

3.1 The pure redistribution mechanism

In the simplest possible case of a pure redistribution agreement, each of two symmetric individuals' utility is given by

$$\alpha y(e_i, \varepsilon_i) + q_i P - s_i m(e_1 + e_2 - f_1 - f_2) - c(e_i, f_i), \text{ where } P = (1-\alpha)(y(e_1, \varepsilon_1) + y(e_2, \varepsilon_2)). \tag{5}$$

The difference to the case of no redistribution (2) is simply the presence of the predetermined subsidy share q_i of the federal budget P. Maximisation of this objective function leads—for the quadratic costs and square-root production function case of (3)—to the individually optimal effort pair which, for symmetric $s_i = q_i = 1/2$, is given as

$$\hat{e} = \frac{(1+\alpha)^{2/3}}{2^{4/3} \times 3^{2/3}}, \;\; \hat{f} = \frac{(1+\alpha)^{2/3}}{2 \times 2^{4/3} \times 3^{2/3}} \tag{6}$$

implying that $\hat{e} = e^*$ for $\alpha^* = 4/5$. Since $\hat{e} = 2\hat{f}$, however, no $\alpha \in [0, 1]$ can implement efficient abatement f^* and a simple redistributive policy can therefore never implement efficient abatement.[9]

For i.i.d. output shocks ε_i, the variance of the expected share of the pooled and redistributed resource P is given by

$$\mathbb{V}\left[\frac{1}{2}(1-\alpha)\left(y(e_1,\varepsilon_1)+y(e_2,\varepsilon_2)\right)\right] = \frac{(1-\alpha)^2}{2}\sigma^2 < \sigma^2. \tag{7}$$

The redistribution pool has therefore a lower variance than individual output σ^2. Obviously, the resulting variance is lowest if all individual income $y(e_i, \varepsilon_i)$ is pooled and redistributed (i.e., $\alpha = 0$).

We would like to reiterate that the pure redistributive mechanism studied in this subsection cannot achieve efficiency. If, for some reason, efficiency is not an issue or is out of reach for exogenous reasons, then all variance compression results derived in what follows for the contest mechanism are also applicable for the (much simpler) pure redistributive mechanism.

3.2 The efficient symmetric mechanism

We begin the analysis of the mechanism which is capable of implementing efficient efforts in both dimensions simultaneously with a symmetric, two-players toy example.[10] Under this contest mechanism, an agreement member's problem is to choose individual productive effort e_i and emissions reduction effort f_i, $i = 1, 2$ and $j = 3 - i$, in order to maximise

$$u_i(\mathbf{e},\mathbf{f};\boldsymbol{\varepsilon})) = \underbrace{\alpha y(e_i,\varepsilon_i)}_{\text{output}} + \underbrace{p_i(\mathbf{f})\beta P}_{1^{st}\text{ prize}} + \underbrace{(1-p_i(\mathbf{f}))(1-\beta)P}_{2^{nd}\text{ prize}} - \underbrace{s_i m(\sum_h (e_h - f_h))}_{\text{damage}} - \underbrace{c(e_i,f_i)}_{\text{cost}}. \tag{8}$$

For our incentive agreement we assume in the first example that the probability of winning the reduction award is given by the generalised Tullock success function $p_i(\mathbf{f}) = \frac{f_i^r}{f_i^r+f_j^r}$ specifying a player's probability of winning as that player's effort over the total sum of efforts, all raised to some power $r > 0$. The prize pool which we collect for incentive purposes is $P = (1-\alpha)(y(e_i,\varepsilon_i) + y(e_j,\varepsilon_j))$. The corresponding example square-root/quadratic individual problem is to

$$\max_{(e_i,f_i)} \alpha e_i^{\frac{1}{2}} + \frac{f_i^r}{f_i^r+f_j^r}\beta P + \frac{f_j^r}{f_i^r+f_j^r}(1-\beta)P - s_i(e_i+e_j-f_i-f_j)^2 - (e_i^2+f_i^2) \tag{9}$$

where the exponent $r > 0$ on the efforts in the success function specifies the precision with which the ranking technology selects the highest reduction effort player among the set of competitors. We interpret this exponent as the accuracy with which the agreement monitors the emissions reduction

[9] This is a general result; for a derivation (in a different context) see, for instance, Giebe and Schweinzer (2011).

[10] As illustrated in Roussillon and Schweinzer (2010), there are simple 'take-it-or-leave-it' agreement formation games under which joining the agreement is an equilibrium in the symmetric case. We nevertheless explore this simple setup first because the novel argument presented in this paper—that the reduced variance of a pooled resource can be exploited to facilitate participation in a voluntary, multinational agreement—is easiest to make in the symmetric setup. The same argument, however, also works in the asymmetric case which is not true for the take-it-or-leave-it games. Besides this, our example closely follows Roussillon and Schweinzer (2010).

efforts of its members. Upon maximisation, this gives the set of simultaneous first-order conditions

$$e_i + 2e_i s_i - 2f_i s_i = \frac{1+\alpha}{8\sqrt{e_i}},\ \sqrt{e_i} = \frac{f_i^2(4+8s_i) - 8e_i f_i s_i}{r(1-\alpha)(2\beta-1)}. \tag{10}$$

We again invoke the fact that we assume that symmetric members are identical (as we did before for the planner) and thus set $e = e_1 = e_2$, $f = f_1 = f_2$, with $s_i = {}^1\!/_2$. We then simply force the resulting efforts in line with the efficient efforts by imposing $e = e^*$ and $f = f^*$ from (3) and solve (10) for the efficiency inducing design parameters $\langle \alpha, \beta, r \rangle$

$$\alpha^* = \frac{3}{5},\ \beta^* = \frac{1}{2} + \frac{1}{6r}. \tag{11}$$

There is a degree of freedom in the choice of r which we assume is part of the designed agreement.

Consider now the influence of stochastic output of the form $y(e_i, \varepsilon_i) = \tilde{y}(e_i) + \varepsilon_i$, i.i.d. $\varepsilon_i \sim \mathcal{L}(\mu = 0, \sigma^2)$, on the equilibrium of the contest game. In the above square-root/quadratic example $\tilde{y}(e_i) = e_i^{1/2}$ with equilibrium parameters (11) where, for $n = 2$ and symmetric $s_i = 1/n$,[11]

$$u_i(\mathbf{e}^*, \mathbf{f}^*; \boldsymbol{\varepsilon}) = \alpha^* y_i(e^*, \varepsilon_i) + \frac{1}{n}P - \frac{1}{n}(ne^* - nf^*)^2 - (e^*)^2 - (f^*)^2, \tag{12}$$

with $P = n(1-\alpha^*)\sum_{i=1}^n y(e^*, \varepsilon_i)$. The main insurance argument rests on the simple observation that the variance of individual output σ^2 is higher than that of the pooled resource $\tilde{\sigma}^2$ to which all players contribute a share. The intuition is that the risk inherent in individual shocks with zero expectation evens out among several players. This is easiest to see if we assume i.i.d. shocks ε_i for which $\mathbb{V}[\sum_i \varepsilon_i] = \sum_i \mathbb{V}[\varepsilon_i]$. In the symmetric model, players expect a prize P with probability $1/n$ in equilibrium; therefore the variance of this prize expectation is given by

$$\mathbb{V}\left[(1-\alpha^*)\frac{1}{n}\sum_{i=1}^n \left((e_i^*)^{\frac{1}{2}} + \varepsilon_i\right)\right] = (1-\alpha^*)^2 \frac{1}{n^2}\sum_{i=1}^n \sigma^2 = \frac{(1-\alpha^*)^2}{n}\sigma^2 < \sigma^2. \tag{13}$$

Therefore, the player expects a payout from the prize pool which is less risky than the own production. Therefore, any convex combination between $\alpha^* y(e^*) + (1-\alpha^*)y(e^*)$ is less risky that the stand-alone GDP. Hence, the higher the redistribution inside the agreement, the more pronounced the co-insurance effect. Using an analogous argument on the remaining term, the equilibrium variance of the complete contest is given by

$$\begin{aligned}\tilde{\sigma}^2 = \mathbb{V}\left[\left(\alpha^* + \tfrac{1-\alpha^*}{n}\right)\left((e_i^*)^{\frac{1}{2}} + \varepsilon_i\right) + \frac{1-\alpha^*}{n}\sum_{j=1,\ j\neq i}^n \left((e_j^*)^{\frac{1}{2}} + \varepsilon_j\right)\right] = \\ \left(\alpha^* + \frac{1-\alpha^*}{n}\right)^2 \sigma^2 + (1-\alpha^*)^2 \frac{n-1}{n^2}\sigma^2 = \\ \frac{(\alpha^*)^2(n-1)+1}{n}\sigma^2 < \sigma^2 \text{ for all } n \geq 2.\end{aligned} \tag{14}$$

[11] Notice that, since the efficient mechanism uses the ranking only for incentive purposes (based on expectations), the share $(1-\alpha)y(e_i, \varepsilon_i)$ that players commit to stochastic.

Thus, ceteris paribus, nations endowed with risk-averse preferences favour the redistributive contest over standing alone. In the extreme case of $\alpha^* = 0$, where all output is pooled, the pooled variance sinks to $\mathbb{V}[u_i(\mathbf{e}^*, \mathbf{f}^*; \boldsymbol{\varepsilon})] = \sigma^2/n$. Our first result relaxes the special assumptions made on output and the independence of shocks used in the above example.

Proposition 1. *Consider individual output $y(e_i, \varepsilon_i) = \tilde{y}(e_i) + \varepsilon_i$, $i \in \mathcal{N}$, for identically distributed $\varepsilon_i \sim \mathcal{L}(\mu = 0, \sigma^2)$ with covariance σ_{ij} for $(\varepsilon_i, \varepsilon_j)$. Then any balanced budget mechanism which is redistributive, i.e., $\alpha^* < 1$ and symmetric, i.e., assigns equal winning probabilities in symmetric pure strategy equilibrium, has a lower variance than individual output.*

Proof. Since each individual output $y(e_i, \varepsilon_i)$ has variance σ^2 and covariance of two distinct variables σ_{ij}, the variance of the prize pool is (for equilibrium α)

$$\begin{aligned}
\mathbb{V}[u_i(\mathbf{e}, \mathbf{f}; \boldsymbol{\varepsilon})] &= \mathbb{V}[\alpha y(e_i, \varepsilon_i)] + \mathbb{V}\left[(1-\alpha)\frac{1}{n}\sum_{i=1}^{n} y(e_i, \varepsilon_i)\right] + 2\alpha(1-\alpha)\frac{1}{n}\operatorname{Cov}\left(y(e_i, \varepsilon_i), \sum_{i=1}^{n} y(e_i, \varepsilon_i)\right) \\
&= \alpha^2\, \mathbb{V}[y(e_i, \varepsilon_i)] + (1-\alpha)^2\frac{1}{n^2}\sum_{i=1}^{n}\mathbb{V}\left[y(e_i, \varepsilon_i)\right] + 2\alpha(1-\alpha)\frac{1}{n}\operatorname{Cov}\left(y(e_i, \varepsilon_i), \sum_{i=1}^{n} y(e_i, \varepsilon_i)\right) \\
&+ 2\frac{(1-\alpha)^2}{n^2}\operatorname{Cov}\left(\sum_{i=1}^{n} y(e_i, \varepsilon_i), \sum_{j=1}^{n} y(e_j + \varepsilon_j)\right) \\
&= \sigma^2\left[\alpha^2 + (1-\alpha)^2\frac{1}{n}\right] + \alpha(1-\alpha)\frac{1}{n}\sum_{i\neq j}\sigma_{ij} + 2\frac{(1-\alpha)^2}{n}\sum_{1\leq i<j\leq n}\sigma_{ij} \\
&\leq \sigma^2\left[\alpha^2 + (1-\alpha)^2\frac{1}{n}\right] + 2\alpha(1-\alpha)\frac{n-1}{n}\sigma^2 + \frac{(1-\alpha)^2}{n}n(n-1)\sigma^2 \\
&= \frac{\sigma^2}{n}\left[2\alpha^2 - 2\alpha + n\right] \\
&< \sigma^2 \;\; \forall\alpha \in [0, 1).
\end{aligned}$$

□

The contribution of this paper is that this reduction in income risk can be used to motivate the formation of an agreement. In our leading example we study the incentives for joining an international environmental agreement. The basic variance compression argument, however, should be applicable to many other issues. (We sketch a simple example in footnote 6.) In order to show the versatility of the idea, we study a more realistic asymmetric, redistributive contest in the following subsection.

3.3 The efficient asymmetric mechanism

We now extend the basic symmetric example of the previous subsection to an asymmetric contest based on relative reductive efforts per-unit-GDP. Our motivation is to consider an equally 'fair' measure of abatement effort for all asymmetric contestants and for this 'normalisation' purpose we use individual GDP. Our main goal is again to show that, in the asymmetric, pure strategy equilibrium of this contest, the variance of the redistributive pool is lower than that of individual output. Our asymmetric abatement contest takes the form

$$\begin{aligned} u_i(\mathbf{e},\mathbf{f};\boldsymbol{\varepsilon}) \quad &= \quad \overbrace{\alpha_i y(e_i,\varepsilon_i)}^{\text{output}} + \overbrace{p_i(\mathbf{e},\mathbf{f})\beta_i P}^{\text{winning}} + \overbrace{\sum_{j\neq i\in\mathcal{N}} p_j(\mathbf{e},\mathbf{f}))\frac{1-\beta_j}{n-1}P}^{\text{losing}} - \overbrace{s_i m(\sum_h (e_h - f_h))}^{\text{damage}} - \\ &\underbrace{c_i(e_i,f_i)}_{\text{cost}}, \text{ for the redistribution pool } P = \sum_{i=1}^{n}(1-\alpha_i)y(e_i,\varepsilon_i), \end{aligned} \tag{15}$$

where the winning probability $p_i(\mathbf{e},\mathbf{f})$ is now based on the ratio x of the two strategic variables: reductive efforts over a function of productive efforts[12]

$$p_i(\mathbf{e},\mathbf{f}) = \frac{x_i^r}{x_1^r + \cdots + x_n^r}, \;\; x_i = \frac{f_i}{\tilde{y}(e_i)}. \tag{16}$$

The probabilities $p_{j\neq i}(\mathbf{e},\mathbf{f})$ are defined in the same way as player j's probability of winning in a contest not involving player i. Notice that we now use identity dependent tax rates α_i and winning shares β_i. For simplicity, we only discriminate between winners and losers, i.e., if player i wins, then we award $\beta_i P$ to the winner and $\frac{1-\beta_i}{n-1}P$ to each of the losing players.

As in the first example, output is stochastic $y(e_i,\varepsilon_i) = \tilde{y}(e_i) + \varepsilon_i$, for i.i.d. $\varepsilon_i \sim \mathcal{L}(\mu = 0, \sigma^2)$ but, in order to avoid problems with a stochastic transformation of the contest success function itself, we use deterministic, expected GDP $\tilde{y}(e_i)$ for ranking purposes. The contest success function employed in this subsection is therefore the following multi-dimensional version of the generalised Tullock success function[13]

$$p_i(\mathbf{e},\mathbf{f}) = \frac{(f_i/\tilde{y}(e_i))^r}{(f_1/\tilde{y}(e_1))^r + \cdots + (f_n/\tilde{y}(e_n))^r} = \frac{x_i^r}{x_1^r + \cdots + x_n^r}, \;\; r > 0. \tag{17}$$

As far as the authors are aware, this 'normalised,' relative efforts 'per-unit-GDP' formulation of the Tullock success function is original to this paper.[14] The general characterisation of this contest is irrelevant for the purposes of illustrating our variance compression argument and has therefore been relegated to the appendix.

In our standard square-root/quadratic example environment, efficient efforts (e^*, f^*) are independent of the contest technology and thus given by the asymmetric but otherwise unchanged

[12] For completeness, we define that $p_i(\mathbf{e},\mathbf{f}) = 1$ if $\tilde{y}(e_i) = 0$, $f_i > 0$ and all $\tilde{y}(e_{-i}) > 0$. Similarly, we let $p_i(\mathbf{e},\mathbf{f}) = 1/m$ if $m = |\tilde{y}(e_j) = 0|_{j\in\mathcal{N}}$.

[13] To exclude unbounded ratios x_i, we assume that $p_i(\mathbf{e},\mathbf{f}) = 0$ if $e_i = 0$. This discontinuity at zero plays no role in the examples discussed in this setup.

[14] This idea can be easily generalised to more than two dimensions. A simple way of achieving this is to use

$$\tilde{x}_i = \frac{f_i}{y_i^1(e_i^1) + \cdots + y_i^m(e_i^m)} \tag{18}$$

where $e_i^1, \ldots, e_i^m$ is player i's m-dimensional 'normalisation' effort transformed, if necessary, by the functions $y_i^h(\cdot)$, $h = 1, \ldots, m$.

three-players version of (3) as[15]

$$
\begin{array}{l}
(\mathbf{e}^*, \mathbf{f}^*) \in \arg\max\limits_{(\mathbf{e},\mathbf{f})} u(\mathbf{e}, \mathbf{f}, \boldsymbol{\varepsilon}) = \\
y(e_1 + \varepsilon_1) + y(e_2 + \varepsilon_2) + y(e_3 + \varepsilon_3) - \\
(e_1 + e_2 + e_3 - f_1 - f_2 - f_3)^2 - \\
\gamma_1 \left(e_1^2 + f_1^2\right) - \gamma_2 \left(e_2^2 + f_2^2\right) - \gamma_3 \left(e_3^2 + f_3^2\right)
\end{array}
\Leftrightarrow
\left\{
\begin{array}{l}
e_1^* = 0.513, f_1^* = 0.535, \\
e_2^* = 0.359, f_2^* = 0.267, \\
e_3^* = 0.288, f_3^* = 0.178.
\end{array}
\right. \tag{19}
$$

Again for simplicity, we choose a linear cost-asymmetry $\gamma_3 = 1$, $\gamma_2 = 2/3$, $\gamma_1 = 1/3$ to differentiate players but asymmetric productive capability could be modelled in exactly the same way. Finally, we use relative pollution shares of $s_1 = 4/12$, $s_2 = 3/12$, $s_3 = 5/12$. In order to focus on the intuition of the asymmetric mechanism we only present numerical results in this subsection as the corresponding analytical expressions are rather unwieldy. We hope that this is without loss of clarity as the full analysis is presented in the appendix.

For the three-players asymmetric contest, the redistribution pool is $P = \sum_{i=1}^{3}(1-\alpha_i)(\sqrt{e_i} + \varepsilon_i)$. This setup results in the three players' objectives

$$
\begin{array}{rcl}
u_1(\mathbf{e}, \mathbf{f}; \boldsymbol{\varepsilon}) & = & \alpha_1 y(e_1, \varepsilon_1) + p_1(\mathbf{e}, \mathbf{f})\beta_1 P + (1 - p_1(\mathbf{e}, \mathbf{f})) \left(\frac{x_2^r}{x_2^r + x_3^r} \frac{1-\beta_2}{2} + \frac{x_3^r}{x_2^r + x_3^r} \frac{1-\beta_3}{2} \right) P - \\
 & & s_1(e_1 + e_2 + e_3 - f_1 - f_2 - f_3)^2 - \gamma_1 \left(e_1^2 + f_1^2\right), \\
u_2(\mathbf{e}, \mathbf{f}; \boldsymbol{\varepsilon}) & = & \alpha_2 y(e_2, \varepsilon_2) + p_2(\mathbf{e}, \mathbf{f})\beta_2 P + (1 - p_2(\mathbf{e}, \mathbf{f})) \left(\frac{x_1^r}{x_1^r + x_3^r} \frac{1-\beta_1}{2} + \frac{x_3^r}{x_1^r + x_3^r} \frac{1-\beta_3}{2} \right) P - \\
 & & s_2(e_1 + e_2 + e_3 - f_1 - f_2 - f_3)^2 - \gamma_2 \left(e_2^2 + f_2^2\right), \\
u_3(\mathbf{e}, \mathbf{f}; \boldsymbol{\varepsilon}) & = & \alpha_3 y(e_3, \varepsilon_3) + p_3(\mathbf{e}, \mathbf{f})\beta_3 P + (1 - p_3(\mathbf{e}, \mathbf{f})) \left(\frac{x_1^r}{x_1^r + x_2^r} \frac{1-\beta_1}{2} + \frac{x_2^r}{x_1^r + x_2^r} \frac{1-\beta_2}{2} \right) P - \\
 & & s_3(e_1 + e_2 + e_3 - f_1 - f_2 - f_3)^2 - \gamma_3 \left(e_3^2 + f_3^2\right).
\end{array} \tag{20}
$$

The corresponding six first-order conditions induce the set of efficient efforts (19) as individual global maxima for the following set of design parameters $\langle \alpha, \beta, r^* = 2 \rangle$[16]

$$
\begin{array}{lll}
\alpha_1^* = 0.610, & \alpha_2^* = 0.740, & \alpha_3^* = 0.810, \\
\beta_1^* = 0.767, & \beta_2^* = 0.518, & \beta_3^* = 0.460.
\end{array} \tag{22}
$$

The below figure confirms that these identify an equilibrium for the game where a ratio of asymmetric strategic variables is chosen.

The next proposition verifies the variance-compression intuition developed for the symmetric redistributive pool for the asymmetric case under both independent and non-independent shocks.

---

[15] Since the two-players asymmetric example with $\beta_1 + (1 - \beta_2) = 1 \Leftrightarrow \beta_1 = \beta_2$ is 'too symmetric' to illustrate the workings of the asymmetric mechanism we switch into a three players example in this subsection.

[16] In the setup of subsection 3.2, the efficiency inducing parameter set of the symmetric version of this 'normalised' contest based on $x_i = f_i / \tilde{y}(e_i)$ is given by

$$
\alpha^* = \frac{11}{15}, \ \beta^* = \frac{1}{2} + \frac{1}{4r}. \tag{21}
$$

This is more favourable to the participants than the corresponding parameters under the standard contest (11).

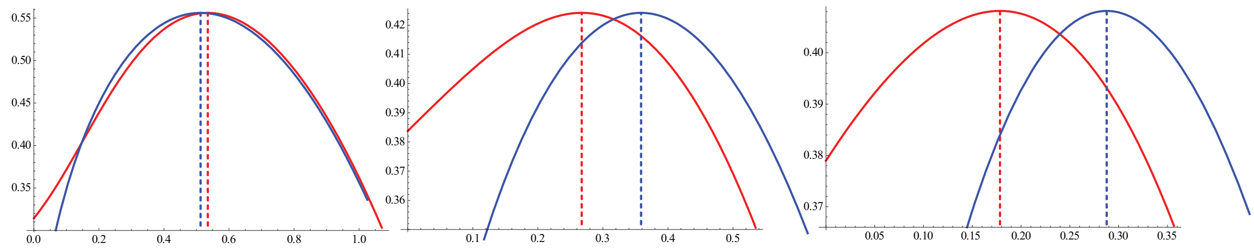

Figure 1: Efficient effort pairs as global maximisers of (20) for player 1's (left), player 2's (centre) and player 3's unilateral deviation utility (right). Red are abatement efforts, blue productive efforts. (The ranges shown contain all maxima.)

Proposition 2. *Consider a two-player, balanced budget contest mechanism which is redistributive, i.e., $\alpha < 1$ and asymmetric, i.e., assigning not necessarily equal winning probabilities $p_1(\mathbf{e}, \mathbf{f}), \ldots, p_n(\mathbf{e}, \mathbf{f})$ in asymmetric pure strategy equilibrium. The equilibrium payoff from this class of mechanisms has a lower variance than individual output if and only if the following conditions are fulfilled for all $(i, j) \in \{1, 2\}$*

$$\begin{aligned} &\alpha_i^*\beta_i^* + (1-(\beta_i^*)^2) - (\alpha_i^*)^2(1-\beta_1^*)^2 > (\beta_i^*)^2(1-\alpha_j^*)^2 \text{ for i.i.d. shocks} \\ &\beta_i^*(2\alpha_j^* - \alpha_i^*) + (1-(\beta_i^*)^2) - (\alpha_i^*)^2(1-\beta_i^*)^2 > (\beta_i^*)^2(1-\alpha_j^*)(3-2\alpha_i^*-\alpha_j^*) \text{ otherwise.} \end{aligned} \tag{23}$$

Proof. Let us first consider independent shocks. For equilibrium α_i, β_i,

$$\begin{aligned} \mathbb{V}[u_i(\mathbf{e}^*, \mathbf{f}^*); \boldsymbol{\varepsilon}] &= \mathbb{V}[\alpha_i y(e_i^*, \varepsilon_i) + p_i(\mathbf{e}^*, \mathbf{f}^*)\beta_i[(1-\alpha_i)y(e_i^*, \varepsilon_i) \\ &\quad + (1-\alpha_j)y(e_j^*, \varepsilon_j)] + (1-p_i(\mathbf{e}^*, \mathbf{f}^*))(1-\beta_j)[(1-\alpha_i)y(e_i^*, \varepsilon_i) + (1-\alpha_j)y(e_j^*, \varepsilon_j)] \\ &= [\alpha_i + (p_i(\mathbf{e}^*, \mathbf{f}^*)\beta_i + (1-p_i(\mathbf{e}^*, \mathbf{f}^*))(1-\beta_j))(1-\alpha_i)]^2\sigma^2 \\ &\quad + [p_i(\mathbf{e}^*, \mathbf{f}^*)\beta_i + (1-p_i(\mathbf{e}^*, \mathbf{f}^*))(1-\beta_j)]^2(1-\alpha_j)^2\sigma^2 \\ &= \alpha_i^2\sigma^2 + 2\alpha_i(1-\alpha_i)[p_i(\mathbf{e}^*, \mathbf{f}^*)\beta_i + (1-p_i(\mathbf{e}^*, \mathbf{f}^*))(1-\beta_j)]\sigma^2 \\ &\quad + [p_i(\mathbf{e}^*, \mathbf{f}^*)\beta_i + (1-p_i(\mathbf{e}^*, \mathbf{f}^*))(1-\beta_j)]^2((1-\alpha_i)^2 + (1-\alpha_j)^2)\sigma^2 \\ &\leq \alpha_i^2\sigma^2 + 2\alpha_i(1-\alpha_i)\beta_i\sigma^2 + ((1-\alpha_i)^2 + (1-\alpha_j)^2)[p_i(\mathbf{e}^*, \mathbf{f}^*)^2(-1+\beta_i+\beta_j)^2 \\ &\quad + 2p_i(\mathbf{e}^*, \mathbf{f}^*)(1-\beta_j)(-1+\beta_i+\beta_j) + (1-\beta_j)^2]\sigma^2 \\ &\leq \{\alpha_i^2 + 2\alpha_i(1-\alpha_i)\beta_i + ((1-\alpha_i)^2 + (1-\alpha_j)^2)\beta_i^2\}\sigma^2. \end{aligned}$$

Then, $\mathbb{V}[u_i(\mathbf{e}^*, \mathbf{f}^*); \boldsymbol{\varepsilon}] < \sigma^2$ for all i if and only if

$$\alpha_i^2(1-\beta_i)^2 + \alpha_i\beta_i + (1-\beta_i^2) > \beta_i^2(1-\alpha_j)^2. \tag{24}$$

Under non-independent shocks, the covariance term has to be added

$$\begin{aligned}
&\mathrm{Cov}([\alpha_i + A(1-\alpha_i)]y(e_i^*, \varepsilon_i), A(1-\alpha_j)y(e_j^*, \varepsilon_j)) \\
&= [\alpha_i + A(1-\alpha_i)]A(1-\alpha_j)\,\mathrm{Cov}[y(e_i^*, \varepsilon_i), y(e_j^*, \varepsilon_j)] \qquad (25)\\
&\leq [\alpha_i + \beta_i(1-\alpha_i)]\beta_i(1-\alpha_j)\sigma_{ij} \qquad (26)\\
&\leq [\alpha_i + \beta_i(1-\alpha_i)]\beta_i(1-\alpha_j)\sigma^2
\end{aligned}$$

with $A = [p_i(\mathbf{e}^*, \mathbf{f}^*)\beta_i + (1 - p_i(\mathbf{e}^*, \mathbf{f}^*))(1-\beta_j)]$. As for the independent shocks, we use the fact $A \leq \beta_i$ (equations (25) & (26)). Then, using the condition (24) for independent shocks we get

$$\beta_i(2\alpha_j - \alpha_i) + (1 - \beta_i^2) - \alpha_i^2(1-\beta_i)^2 > \beta_i^2(1-\alpha_j)(3 - 2\alpha_i - \alpha_j) \qquad (27)$$

which establishes our claim. □

The interpretation of (23) is not straightforward. The condition for variance compression depends on there being a redistribution in the first place: $\alpha_1^* + \alpha_2^* < 2$. Moreover, since we are only looking at two players, each $\alpha_i^* < 1$ in order to allow for risk pooling. From the point of view of player $i = 1, 2$, her winner's share $\beta_i^* > 1/2$ determines the income transfer in case of winning and $1 - \beta_j^*$, $j = 3 - i$, determines how much income is redistributed in case she loses. For asymmetric winning probabilities $p_i(\mathbf{e}, \mathbf{f})$, the interplay of these variables in (23) determines when risk-pooling is possible.

3.4 The main result

In this subsection, we show that our two previous variance-compression results can be used to argue that the distribution of the shock expected by a player not participating in the redistribute agreement is a mean-preserving spread of the shock expected by agreement members.[17] As a consequence, we then show that a sufficiently risk-averse player will prefer to join the agreement over staying outside.

In the simplest case of a n-member, pure redistributional contest mechanism (rm), a symmetric member's expected equilibrium payoff is

$$\mathbb{E}[u^{\mathsf{rm}}(\mathbf{e}^*, \mathbf{f}^*; \varepsilon^{\mathsf{rm}})] = \int_{-\infty}^{+\infty} (\tilde{y}(e^*) + \varepsilon^{\mathsf{rm}} - s_i m(ne^* - nf^*) - c(e^*, f^*))\, dF(\varepsilon^{\mathsf{rm}}). \qquad (28)$$

Given an existing agreement with $n-1$ participants, the equilibrium payoff of a player $i \in \mathcal{N}$ who is free-riding (fr) on the abatement efforts of the agreement is given by

$$\mathbb{E}[u^{\mathsf{fr}}(\tilde{\mathbf{e}}, \tilde{\mathbf{f}}; \varepsilon^{\mathsf{fr}})] = \int_{-\infty}^{+\infty} \Big(\tilde{y}(\tilde{e}) + \varepsilon^{\mathsf{fr}} - s_i m(\tilde{e} + (n-1)e^* - \tilde{f} - (n-1)f^*) - c(\tilde{e}^2, \tilde{f})\Big)\, dF(\varepsilon^{\mathsf{fr}}) \qquad (29)$$

where $\tilde{\mathbf{e}}, \tilde{\mathbf{f}}$ are equal to $\mathbf{e}^*, \mathbf{f}^*$ with the free-rider's positions replaced by $\tilde{e}, \tilde{f}$. We would like to show that, under a sufficiently concave transformation v of the utility implied by (28) and (29), we can ascertain agreement participation, i.e., that $\mathbb{E}[v(u^{\mathsf{rm}}(\mathbf{e}^*, \mathbf{f}^*; \varepsilon^{\mathsf{rm}}))] \geq \mathbb{E}[u^{\mathsf{fr}}(\tilde{\mathbf{e}}, \tilde{\mathbf{f}}; \varepsilon^{\mathsf{fr}})]$. Notice that, for

[17] See Rothschild and Stiglitz (1970) for the idea of mean-preserving spreads as a measure of risk.

this purpose, we can ignore the influence of the individually suffered damage share $s_i m(ne^* - nf^*)) < s_i m(\tilde{e} + (n-1)e^* - \tilde{f} - (n-1)f^*)$.

Existing results for the case of $\varepsilon^{\mathsf{rm}} = \varepsilon^{\mathsf{fr}} \equiv 0$ show,[18] however, that we cannot exclude the case where $\mathbb{E}[u^{\mathsf{rm}}(\tilde{\mathbf{e}}, \tilde{\mathbf{f}}; \varepsilon^{\mathsf{rm}})] \leq \mathbb{E}[u^{\mathsf{fr}}(\mathbf{e}^*, \mathbf{f}^*); \varepsilon^{\mathsf{fr}}]$ which implies that no agreement is formed because

$$u^{\mathsf{rm}}(\mathbf{e}^*, \mathbf{f}^*; \varepsilon^{\mathsf{rm}})\Big|_{\varepsilon^{\mathsf{rm}} \equiv 0} < u^{\mathsf{fr}}(\tilde{\mathbf{e}}, \tilde{\mathbf{f}}; \varepsilon^{\mathsf{fr}})\Big|_{\varepsilon^{\mathsf{fr}} \equiv 0}. \tag{30}$$

In our stochastic environment, remark that $\varepsilon^{\mathsf{fr}}$ and $\varepsilon^{\mathsf{rm}}$ are two random variables which follow two specific—zero mean—distributions, $\varepsilon^{\mathsf{fr}} \sim \mathcal{L}(0, \sigma^2)$ and $\varepsilon^{\mathsf{rm}} \sim \mathcal{L}\left(0, \frac{\alpha^2+1}{2}\sigma^2\right)$ with $\sigma^2 > \frac{\alpha^2+1}{2}\sigma^2$ (i.e., $\mathbb{V}[\varepsilon^{\mathsf{fr}}] > \mathbb{V}[\varepsilon^{\mathsf{rm}}]$). Notice further, that the symmetric equilibrium solutions to (28) and (29) are invariant under increasing concave transformations, i.e., that the equilibrium choice of effort does not change if the concerned decision makers change their degree of risk aversion. Hence, there exists a function $v(\cdot)$, with $v'(\cdot) > 0, v''(\cdot) \leq 0$, which leads to $\mathbb{E}[v(u^{\mathsf{rm}}(\mathbf{e}^*, \mathbf{f}^*; \varepsilon^{\mathsf{rm}}))] \geq \mathbb{E}[v(u^{\mathsf{fr}}(\tilde{\mathbf{e}}, \tilde{\mathbf{f}}; \varepsilon^{\mathsf{fr}}))]$ for any positive difference $\mathbb{V}[\varepsilon^{\mathsf{fr}}] - \mathbb{V}[\varepsilon^{\mathsf{rm}}]$. Our main result then follows immediately.

Proposition 3. *For every positive difference of the equilibrium variances between the redistributive and the free-riding mechanisms, there is a family of concave functions v which provides a higher payoff to the redistributive mechanism.*

Proof. Assume that (30) holds (otherwise we are done), where $u^{\mathsf{rm}}(\cdot)$ and $u^{\mathsf{fr}}(\cdot)$ are both linear functions in their respective arguments $\varepsilon^{\mathsf{rm}}$ and $\varepsilon^{\mathsf{fr}}$. From theorems 4 and 5 in appendix B, it follows that $\varepsilon^{\mathsf{fr}} \leq_{\mathsf{icv}} \varepsilon^{\mathsf{rm}}$ such as $\mathbb{E}[v(\varepsilon^{\mathsf{fr}})] \leq \mathbb{E}[v(\varepsilon^{\mathsf{rm}})]$ for all increasing and concave functions v. Consequently, there exists a sufficiently concave, increasing function v such that $\mathbb{E}[v(u^{\mathsf{rm}}(\mathbf{e}^*, \mathbf{f}^*; \varepsilon^{\mathsf{rm}}))] \geq \mathbb{E}[v(u^{\mathsf{fr}}(\tilde{\mathbf{e}}, \tilde{\mathbf{f}}; \varepsilon^{\mathsf{fr}}))]$. ☐

This proposition shows that, in our stochastic setup, there is a degree of risk aversion which leads to full participation in the symmetric redistribution agreement. An illustration of the intuition is attempted in figure 2. A similar but more complicated argument can be made on the basis of proposition 2 for asymmetric mechanisms.

The example with explicit preferences

We now extend our simple symmetric example from subsection 3.2 in order to explicitly make the point that risk aversion simplifies the satisfaction of a player's participation constraint. Reconsider the problem of player $i \in \{1,2,3\}$ under Bernoulli utility function $v(\cdot)$ and standard normally distributed shocks $\varepsilon \sim \mathcal{N}(0, \sigma^2 = 1)$. Since players decide their efforts on the basis of expected shocks of zero, the efficient efforts defined in (3) remain unchanged and the efficiency inducing parameters $\alpha^* = 3/5, \beta^* = 7/12, r = 2$ are still given by (11). Hence, a player's expected equilibrium utility from participating in the redistributive mechanism is given by the equivalent to (28) as

$$\mathbb{E}[u^{\mathsf{rm}}(\mathbf{e}^*, \mathbf{f}^*; \varepsilon^{\mathsf{rm}})] = \int_{-\infty}^{\infty} v\left((e^*)^{1/2} + \varepsilon^{\mathsf{rm}} - \frac{1}{3}(3e^* - 3f^*)^2 - (e^*)^2 - (f^*)^2\right) dF(\varepsilon^{\mathsf{rm}}). \tag{31}$$

[18] Solid theoretical arguments against agreement participation in the deterministic case were derived, for instance, by Diamantoudi and Sartzetakis (2006) and Guesnerie and Tulkens (2009).

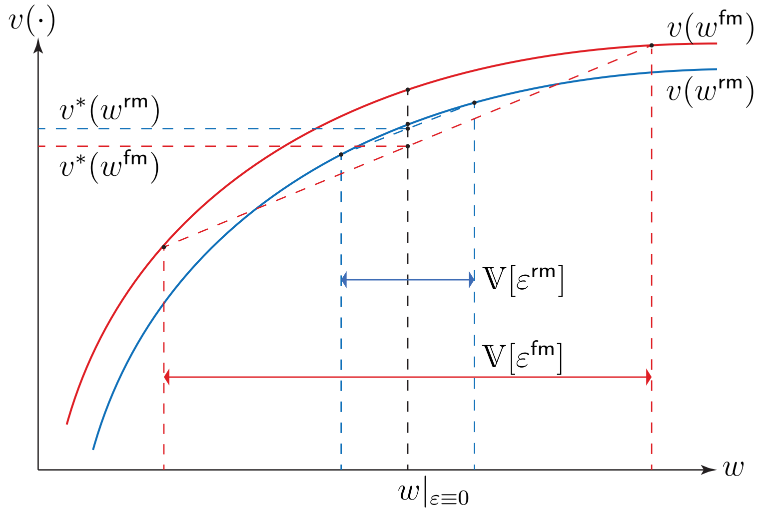

Figure 2: Variance compression leads to participation under sufficient risk aversion.

Similarly, a player's equilibrium utility from free riding on the residual two-player agreement follows from (29) as

$$\mathbb{E}[u^{\mathsf{fr}}(\tilde{\mathbf{e}},\tilde{\mathbf{f}};\varepsilon^{\mathsf{fr}})] = \int_{-\infty}^{\infty} v\left(\tilde{e}^{1/2} + \varepsilon^{\mathsf{fr}} - \frac{1}{3}(\tilde{e} + 2e^* - \tilde{f} - 2f^*)^2 - \tilde{e}^2 - \tilde{f}^2\right) dF(\varepsilon^{\mathsf{fr}}). \tag{32}$$

Solving these problems under linear $v(\cdot)$ for their symmetric equilibrium utilities gives for the agreement

$$e^* = 0.273,\ \ f^* = 0.205,\ \ \alpha^* = 0.455,\ \ \beta^* = 0.574\ \Rightarrow\ \mathbb{E}[u^{\mathsf{rm}}(\mathbf{e}^*,\mathbf{f}^*;\varepsilon^{\mathsf{rm}})] = 0.392 \tag{33}$$

and, for the free-riding player,[19]

$$\tilde{e} = 0.317,\ \ \tilde{f} = 0.126\ \Rightarrow\ \mathbb{E}[u^{\mathsf{fr}}(\tilde{\mathbf{e}},\tilde{\mathbf{f}};\varepsilon^{\mathsf{fr}})] = 0.399. \tag{34}$$

Therefore, a risk neutral player will not join the two-player agreement. Calculating the same expected equilibrium utilities under a concave Bernoulli utility function $v(w) = w^{3/4}$, however, gives

$$\mathbb{E}[v(u^{\mathsf{rm}}(\mathbf{e}^*,\mathbf{f}^*;\varepsilon^{\mathsf{rm}}))] = 0.495 > \mathbb{E}[v(u^{\mathsf{fr}}(\tilde{\mathbf{e}},\tilde{\mathbf{f}};\varepsilon^{\mathsf{fr}}))] = 0.435 \tag{35}$$

and therefore full participation in the three players agreement.

4 Concluding remarks

We present a simple redistributive international agreements model in which an individual member's variance of income is compressed through becoming a member of the agreement. This risk sharing

[19] The two-player agreement efforts and utilities required for the calculation of the free-riding utility are the same as in the example of subsection 3.2.

is an effect of the pooled risk of the redistributed income. We use this property of redistributive agreements in a contest model which implements efficient efforts choices among nations facing multiple external effects. Since the contest must redistribute wealth in order to implement efficient efforts—through awarding member states first, second, etc prizes—the efficient contest compresses the income risk of member states. This insurance aspect is a formidable reason to join international agreements which so far seems to have been overlooked in the agreement formation literature.

Appendix

A: Analytic discussion of the asymmetric mechanism

In the setup of subsection 3.3 we introduce a single cost parameter $\gamma > 0$ into a three players game. For (symmetric) individual output $y(e_i, \varepsilon_i) = e_i^{1/2} + \varepsilon_i$, $i = 1, 2, 3$, the social planner's problem is to choose the three players' efforts as

$$\begin{aligned}(\mathbf{e}^*, \mathbf{f}^*) \in \arg\max_{(\mathbf{e},\mathbf{f},\boldsymbol{\varepsilon})} u(e, f) = \;& y(e_1, \varepsilon_1) + y(e_2, \varepsilon_2) + y(e_3, \varepsilon_3) - (e_1 + e_2 + e_3 - f_1 - f_2 - f_3)^2 - \\ & \gamma \left(e_1^2 + f_1^2\right) - \left(e_2^2 + f_2^2\right) - \left(e_3^2 + f_3^2\right) - 3\rho\sigma^2\end{aligned} \tag{36}$$

resulting in the set of first-order conditions

$$\begin{gathered} e_1 = \frac{3 + \gamma^{-1}}{4\sqrt{e_2}} - \left(5 + \gamma^{-1}\right) e_2, \; e_2 = \frac{3 + \gamma^{-1}}{8\sqrt{e_1}} - \frac{8 + 12\gamma}{8} e_1 \\ f_2 = f_3 = \gamma f_1, \; e_3 = e_2, \; f_1 = \frac{e_1 + 2e_2}{1 + 3\gamma}. \end{gathered} \tag{37}$$

These result in surprisingly unappealing expressions[20] which, for a particular parameter of $\gamma = 1/3$, amount to

$$e_1^* = \frac{2 \times 2^{1/3}}{11^{2/3}}, \; e_2^* = e_3^* = \frac{9}{4 \times 22^{2/3}}, f_1^* = \frac{17}{4 \times 22^{2/3}}, \; f_2^* = f_3^* = \frac{17}{12 \times 22^{2/3}}. \tag{39}$$

[20] The explicit efficient quantities are

$$\begin{aligned} e_1^* &= \frac{3\times 3^{1/3}(1+5\gamma)^{4/3} c_3^{4/9}\left(\frac{1}{\gamma(2+5\gamma)}\left(6\times 2^{1/3}\gamma^{2/3}+22\times 2^{1/3}\gamma^{5/3}+12c_3^{1/3}+74\gamma c_3^{1/3}+90\gamma^2 c_3^{1/3}-2^{2/3}\left(\gamma c_3^2\right)^{1/3}\right)\right)^{2/3}}{\left(2^{2/3}c_3^{2/3}+4(\gamma^2 c_3)^{1/3}-2\times 2^{1/3}\gamma^{1/3}(3+11\gamma)\right)^2}, \\ e_2^* &= \frac{\left(\frac{1}{(1+5\gamma)(2+5\gamma)}\left(3\times 2^{1/3}\gamma^{2/3}+11\times 2^{1/3}\gamma^{5/3}+6c_3^{1/3}+37\gamma c_3^{1/3}+45\gamma^2 c_3^{1/3}-\left(\frac{1}{2^{1/3}}\left(\gamma c_3\sqrt{3}c_1+c_2\right)^2\right)^{1/3}\right)\right)^{2/3}}{2\times 2^{1/3}3^{2/3}c_3^{2/9}} \end{aligned} \tag{38}$$

for $c_1 = \sqrt{108 + \gamma(1336 + \gamma(5795 + 3\gamma(3362 + 2025\gamma)))}$, $c_2 = \gamma\left(603 + \gamma(2086 + 2025\gamma) + 15\sqrt{3}c_1\right)$, and $c_3 = 54 + 3\sqrt{3}c_1 + c_2$.

Turning to the individual problems which implement these efficient efforts as individual equilibria, the three players' objectives are given by

$$\begin{aligned}
u_1(\mathbf{e},\mathbf{f};\boldsymbol{\varepsilon}) &= \alpha_1 y(e_1,\varepsilon_1) + p_1(\mathbf{e},\mathbf{f})\beta_1 P + (1-p_1(\mathbf{e},\mathbf{f}))\left(\tfrac{x_2^r}{x_2^r+x_3^r}\tfrac{1-\beta_2}{2} + \tfrac{x_3^r}{x_2^r+x_3^r}\tfrac{1-\beta_3}{2}\right)P - \\
&\quad s_1(e_1+e_2+e_3-f_1-f_2-f_3)^2 - \gamma\left(e_1^2+f_1^2\right) - \rho\tilde{\sigma}^2, \\
u_2(\mathbf{e},\mathbf{f};\boldsymbol{\varepsilon}) &= \alpha_2 y(e_2,\varepsilon_2) + p_2(\mathbf{e},\mathbf{f})\beta_2 P + (1-p_2(\mathbf{e},\mathbf{f}))\left(\tfrac{x_1^r}{x_1^r+x_3^r}\tfrac{1-\beta_1}{2} + \tfrac{x_3^r}{x_1^r+x_3^r}\tfrac{1-\beta_3}{2}\right)P - \\
&\quad s_2(e_1+e_2+e_3-f_1-f_2-f_3)^2 - \left(e_2^2+f_2^2\right) - \rho\tilde{\sigma}^2, \\
u_3(\mathbf{e},\mathbf{f};\boldsymbol{\varepsilon}) &= \alpha_3 y(e_3,\varepsilon_3) + p_3(\mathbf{e},\mathbf{f})\beta_3 P + (1-p_3(\mathbf{e},\mathbf{f}))\left(\tfrac{x_1^r}{x_1^r+x_2^r}\tfrac{1-\beta_1}{2} + \tfrac{x_2^r}{x_1^r+x_2^r}\tfrac{1-\beta_2}{2}\right)P - \\
&\quad s_3(e_1+e_2+e_3-f_1-f_2-f_3)^2 - \left(e_3^2+f_3^2\right) - \rho\tilde{\sigma}^2,
\end{aligned} \tag{40}$$

where the redistribution pool is given by $P = \sum_{i=1}^{3}(1-\alpha_i)y(e_i,\varepsilon_i)$, and the success probabilities are given by the two-dimensional, generalised Tullock success function

$$p_i(\mathbf{e},\mathbf{f}) = \frac{(f_i/\tilde{y}(e_i))^r}{(f_1/\tilde{y}(e_1))^r + \cdots + (f_n/\tilde{y}(e_n))^r} = \frac{x_i^r}{x_1^r + \cdots + x_n^r},\ r > 0. \tag{41}$$

Plugging in the efficient efforts from (39), the corresponding four (plus 2) first-order conditions are

$$\begin{aligned}
&2\times 9^r(17s_1 + 33\alpha_1(\beta_1-1) - 33\beta_1 + 48\gamma) + 4^r(68s_1 + 33(\beta_2+\beta_3-2) - 33\alpha_1(2+\beta_2+\beta_3) + 192\gamma) \\
&= \frac{11\times 3^{1+2r}4^r r(4\alpha_1+3\alpha_2+3\alpha_3-10)(4\beta_1+\beta_2+\beta_3-2)}{4\left(2^{1+2r}+9^r\right)}, \\
&289\left(2^{1+2r}+9^r\right)^2(s_1-3\gamma) = 11\times 3^{1+2r}4^{1+r}r(4\alpha_1+3\alpha_2+3\alpha_3-10)(4\beta_1+\beta_2+\beta_3-2), \\
&\frac{11\times 2^{1+2r}r(4\alpha_1+3\alpha_2+3\alpha_3-10)\left(9^r(\beta_1+2\beta_2-1)+4^r(2\beta_2+\beta_3-1)\right)}{3\left(2^{1+2r}+9^r\right)} \\
&= \left(9^r(5+17s_2+22\beta_1-22\alpha_2(1+\beta_1)) + 2^{1+2r}(16+17s_2-22\beta_2+11\alpha_2(2\beta_2-\beta_3-3)+11\beta_3)\right), \\
&(4\alpha_1+3\alpha_2+3\alpha_3-10)\left(9^r(\beta_1+2\beta_2-1)+4^r(2\beta_2+\beta_3-1)\right) = \frac{289\left(2^{1+2r}+9^r\right)^2(s_2-1)}{99\times 4^{1+r}r}.
\end{aligned}$$

By plugging in $s_1 = 1/3$, $s_2 = 1/4$, $s_3 = 5/12$, $\gamma = 1/3$ (as above), and $r = 2$, these simplify to

$$\begin{aligned}
\alpha_1^* &= 0.49384897900726243, \\
\alpha_2 &= \frac{218746118259809 + 178464\alpha_1(2408313600\alpha_1 - 3639465821)}{3685766768640}, \\
\alpha_3 &= \frac{-13952829945931 - 81312\alpha_1(344044800\alpha_1 - 521791163)}{216809809920}, \\
\beta_1 &= \frac{92650687360943 + 576576\alpha_1(344044800\alpha_1 - 493180163)}{718354050795}, \\
\beta_2 &= \frac{3(419270336285477 + 2589664\alpha_1(344044800\alpha_1 - 493180163))}{37886524753040}, \\
\beta_3 &= \frac{454544111225153 + 2757216\alpha_1(344044800\alpha_1 - 493180163)}{37886524753040}
\end{aligned} \tag{42}$$

resulting in the following efficiency inducing design parameters $\langle \alpha, \beta, r = 2 \rangle$

$$\alpha_1^* \approx 0.494,\ \alpha_2^* \approx 0.761,\ \alpha_3^* \approx 0.818,\ \beta_1^* \approx 0.837,\ \beta_2^* \approx 0.462,\ \beta_3^* \approx 0.379 \tag{43}$$

where the symmetric-cost players 2 and 3 differ because of their unequal damage shares s_i. The intuition is that larger countries (or countries suffering from a large share of global pollution) need to pay a higher share of their income into the mechanism than (pollution share) smaller countries. They need lower incentives, however: the winning prize fraction they require from the redistributive pool is lower than for smaller (pollution share) countries.

B: Stochastic Dominance Relations

This appendix provides analytical tools and new results on stochastic dominance which are useful to establish proposition 3.

Definition 1 (Shaked and Shanthikumar (1994))**.** *Let X and Y be two random variables. X is said to be smaller than Y in the increasing concave order and denoted $X \leq_{icv} Y$ if and only if*

$$\mathbb{E}[v(X)] \leq \mathbb{E}[v(Y)]$$

for all increasing, concave functions $v(\cdot)$.

In the economics literature, increasing concave orders are typically referred to as *second order stochastic dominance*. Similarly, increasing concave orders with equal means are usually called *mean preserving spreads*. An alternative, equivalent definition is:

Definition 2 (Shaked and Shanthikumar (1994))**.** *Let X and Y be two random variables with F_X and F_Y their continuous cumulative functions. $X \leq_{icv} Y$ if and only if*

$$\int_{-\infty}^{z} F_X(t)dt \geq \int_{-\infty}^{z} F_X(t)dt.$$

The following result generalises theorem 5 of Müller (2001) for univariate elliptical distributions and increasing concave orders. The proof is provided for completeness and is an adaptation of that in Müller (2001).

Theorem 4. *Let $X \sim \mathcal{L}(\mu_x, \sigma_x^2)$ and $Y \sim \mathcal{L}(\mu_y, \sigma_y^2)$, which are two elliptically distributed random variables. If $\sigma_y^2 \leq \sigma_x^2$ and $\mu_y \leq \mu_x$ then $X \leq_{icv} Y$.*

Proof. Given the properties of elliptical distributions,[21] the distribution of Y is equal to that of

[21] See Fang, Kotz, and Ng (1987) or Landsman and Valdez (2003) for details.

$X + Z$ with $Z \sim \mathcal{L}(\mu_y - \mu_x, \sigma_y^2 - \sigma_x^2)$. Then,

$$\begin{aligned} \mathbb{E}[v(Y)] &= \mathbb{E}[v(X+Z)] \\ &= \mathbb{E}[\mathbb{E}[v(X+Z)/X]] \quad (44) \\ &\leq \mathbb{E}[v(X+\mathbb{E}[Z])] \quad (45) \\ &\leq \mathbb{E}[v(X)]. \quad (46) \end{aligned}$$

The Jensen inequality for concave functions leads to (45). Then, using $\mathbb{E}[Z] \leq 0$ (46) follows. □

Theorem 5. *Let $X \sim \mathcal{L}(\mu_x, \sigma_x^2)$ and $Y \sim \mathcal{L}(\mu_y, \sigma_y^2)$, which are two elliptically distributed random variables such that $\mu_y = \mu_x = 0$. Then the following statements are equivalent: (i) $X \leq_{\mathsf{icv}} Y$, and (ii) $\sigma_y^2 \leq \sigma_x^2$.*

Proof. Every probability density $(F_i)'$ is symmetric and continuous. Zero mean implies that the symmetry of the probability density is with respect to the ordinate. Then, every cumulative distribution has an inflection point at zero. Furthermore, X has a probability density with bigger tails than Y because of the ranking of the variance. Consequently of the tails and the inflection point, the cumulative distribution of X is above (below) the one of Y for all value inferior (superior) to zero such that the surfaces between the two cumulative distributions before and after zero are equal. Then,

$$\int_{-\infty}^{z} F_X(t)dt \geq \int_{-\infty}^{z} F_Y(t)dt \quad (47)$$

which implies that $X \leq_{\mathsf{icv}} Y$. Moreover, it is well-known that if $X \leq_{\mathsf{icv}} Y$ and $\mathbb{E}[X] = \mathbb{E}[Y]$ then $\sigma_y^2 \leq \sigma_x^2$ follows (see, for instance, Rothschild and Stiglitz (1970)). □

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