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Does habit formation always increase the agents' desire to smooth consumption?

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Does habit formation always increase the agents' desire to smooth consumption?*

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Abstract

In the literature, habit formation has been often introduced to enhance the agents' desire to smooth consumption over time. This characteristic was found particularly useful in solving the equity premium puzzle and in matching several stylized facts in growth, and business cycles theory as, for example, the high persistence in the U.S. output volatility. In this paper we propose a definition of habit formation, which is "general" relative to the assumptions on the intensity, persistence, and lag structure, and we unveil two mechanisms which point to the opposite direction: habits may reduce the desire of smoothing consumption over time and then may potentially decrease the power of a model in explaining the previously mentioned facts. More precisely, we propose a complete taxonomy of the rich dynamics which may emerge in an AK model with external addictive habits for all the feasible combinations of the intensity, persistence and lag structure characterizing their formation and we point out to the region in the parameters' space coherent with less smoothing in consumption. An economic explanation of these mechanisms is suggested and the robustness of our results in the case of internal habits verified. Finally and crucially habit formation always reduces the desire of consumption smoothing once the model is calibrated to match the average U.S. output and utility growth rates observed in the data.

Keywords: Habit formation; endogenous fluctuations, delayed functional differential equations.
JEL Classification: E00, E30, O40.

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1 Introduction

The literature on habits is quite heterogeneous in the assumptions on the intensity, persistence and lag structure characterizing their formation. Many contributions, in line with the seminal work of Ryder and Heal [38], define the habits as the weighted average of past consumption, with the weights decreasing exponentially into the (infinite) past. Alternatively it is often assumed that the consumption pattern between today and the previous period is what really matters in term of the habit formation (see for example, Abel [1]). The relation between the persistence and intensity of the habits may also vary a lot: sometimes they are assumed to be equal, as in Ryder and Heal [38], sometimes different as emerged by the "judicious" choices in Constantinides [21], where the intensity was always assumed higher than the persistence.

Choosing among these specifications, is not obvious because there are only very few recent attempts to find a microfoundation of the internal (Rozen [37]) or external (Chetty and Szeidl [20]) habit formation while all the econometric estimates identify the habits' parameters from the Euler equation and then strictly depend on the specific functional form of the habits and on the specific model setup where they are embedded (see for example Dynan [28] and more recently Ravina [35]). As far as we know, the only exception is the article of Crawford [23] where a revealed preference approach was used to characterize the internal habits. However no sharp result emerged on the lag structure: increasing the number of period lags in the consumption of the good increases the "agreement between theory and data. However it (...) has a large negative effect on the power of the test compared with the one-lag version".

As a consequence, the choice of the habit formation, has been always done, in the literature, case by case to increase the analytical tractability of the model, and/or to enhance its explanatory power in reproducing some empirical evidences, and/or to guarantee the existence of an equilibrium path. A part from these reasons, different specifications have been used interchangeably as if their implications in term of the qualitative behavior of the main aggregate variables were the same. In particular the role of the habits in *inducing a desire to smooth consumption* is recognized by the existing literature as a common feature of the different habit's specifications; for this reason the habits have been widely used to explain several issues in growth, international and monetary economics as observed recently by Boldrin et al. [13]; for the very same reason habits have played a key role in resolving the equity premium puzzle as clearly explained by Constantinides [21]: "Habit persistence smooths consumption growth over and above the smoothing implied by the life cycle permanent income hypothesis with time separable utility. (...) This illustrate the key role of habit persistence in resolving the puzzle (...)". Last but not least, habit formation and its ability to enhance the desire to smooth consumption over time is also one of the key elements used in Boldrin et al. [13], to enhance the ability of a real business cycle model to reproduce several stylized facts, and to increase the persistence in output. [1]

In this paper we want to point out and explain two mechanisms, not yet unveiled by the existing literature, which lead to the opposite direction: introducing the habits may *reduce the desire to smooth consumption* once compared to the case without them. This means that the habits may induce the representative agent to prefer period(s) of low and high consumption to a more "flat" consumption path. Then understanding why and how these mechanisms may arise, can be useful in order to enlighten the possible weakness of the introduction of the habits in

[1] In their contribution, the other key element is a two sector technology with limited factor mobility.

explaining the previously mentioned stylized facts; this is even more relevant in front of not yet conclusive results in the identification/estimates of the habit formation parameters.

To exploit these mechanisms, we use in this paper the following "general" definition of the habits formation:

$$h(t) = \varepsilon \int_{t-\tau}^{t} \bar{c}(u) e^{\eta(u-t)} du$$

where $\varepsilon > 0$, $\eta \geq 0$, and $\tau > 0$ indicate respectively the *intensity*, *persistence* and *lag structure* of the habits, while $\bar{c}(u)$ the average consumption. The term general is quoted because it has no absolute meaning; in fact, the description of the formation of the habits is only "general" relative to the assumptions on their intensity, persistence, and lag structure but do not include, for example, the deep habits specification studied by Ravn et al. [36] because we always refer to a single consumption good. Then we embed this definition in an economy characterized by addictive habits and by a linear technology, to fully characterize the qualitative dynamics, and to find the two mechanisms which points out to the role of the habits in smoothing the consumption path less than in a model without them.[2]

The ***first mechanism*** can be observed also (but not only) in a benchmark model where the lag structure is infinite, $\tau \to \infty$. In Section 2.1 we study this model and we prove that different selections of the habits' persistence and intensity may induce two alternative regimes in our economy. In one of them, the economy converges asymptotically to a standard AK model but the presence of the habits induces a smoother consumption path. This result confirms the role of the habits as generally intended in the literature.

On the other hand, the second regime is characterized by an asymptotic growth rate of the economy equals to the difference between the intensity and persistence of the habits. This may happen because at the market equilibrium, the addiction in the habits, $c(t) \geq h(t)$, forces the habits stock to grow at least at the rate $\varepsilon - \eta$. If this difference is positive and higher than the growth rate of a standard AK model, but lower than the real interest rate, then all the variables will grow asymptotically at this rate provided a restriction on the initial condition is also respected. In fact, the initial stock of habits has to be (sufficiently) lower than the initial income, otherwise the households will be forced to choose till the very beginning a level of consumption so high that no feasible saving profile may sustain their addictive habits over time.

As a consequence a mechanism of *"save now or regret it later"* may emerge and may lead the representative agent to consume, till the very beginning, *less* than in the standard AK model in order to save enough and accumulate the capital necessary to keep over time the consumption level higher than the habits stock. The faster accumulation of capital will imply a *higher* consumption in the long run than the one in the AK model without habits. Then the habits will reduce the desire to smooth consumption once compared with the same model without them. It is also worth noting that the agents' behavior just described is consistent with their desire to smooth the difference between consumption and habit.

Moreover the mechanism just described, still holds for a finite lag structure if the parameter τ is sufficiently high to pin down an asymptotic growth rate of the habits higher than the growth rate in the standard AK model. On the other hand a too small value of τ may break down the mechanism just described by reducing the growth rate of the habits from $\varepsilon - \eta$ to a smaller,

[2] The sensitivity of the two mechanisms to different specifications of the utility function is also discussed in a section of the paper.

potentially negative value. This last consideration emphasizes the role of the lag structure in potentially undermining this first mechanism. It is also worth noting that the selection of the habit's formation parameters leading to this result is not "admissible" in an exogenous growth model since the diminishing return to scale in the accumulating factor cannot sustain forever the higher and higher level of consumption induced by the habits.

The ***second mechanism*** emerges as a consequence of a finite lag structure. Consumption smoothing is now broken down by the rising of damping fluctuations in the main aggregate variables during their transition toward the balanced growth path. In fact, a finite τ introduces a complete and periodic update of the habits which are now completely different after a while since only the average consumption between t and $t + \tau$ define them completely.[3] As a result endogenous fluctuations may arise in the habits and in aggregate consumption as well because the risk adverse agents want to smooth the path of $c(t) - h(t)$, and for this reason they adjust their consumption behavior accordingly.

Moreover we find that the endogenous fluctuations can be either small and smoothed out quickly or quite persistent and around the balanced growth path. Conditions for the two types are indeed proved analytically and reported in the parameter space (ε, η) after a normalization of the habits is done. Interestingly enough, our model with endogenous fluctuations induced by the habits is characterized, during the transition toward the balanced growth path, by a positive relation between consumption growth and income while no relation between consumption growth and the real interest rate (which is always constant): in this extent our model accounts for the *excess sensitivity puzzle* pointed out by Hall [32] among others.

The raising of endogenous fluctuations in a habits environment is not completely new to the literature as shown by Benhabib [8] who rigorously proved the possibility of Hopf bifurcation in an exogenous growth model with internal habits, or Dockner and Feichtinger [21], who confirmed this result in a model with addictive internal habits over two different consumption goods. Keeping aside the distinction between internal and external habits, three are the main differences between our paper and these contributions: the endogenous growth framework, the finite lag structure as the unique source of endogenous fluctuations and the global dynamics of the economy which may exhibit damping fluctuations around the balanced growth path but never a stable limit cycle as documented in the other two papers.

Once the two mechanisms have been unveiled, we argue in Section 4, that the region of the parameter space where habits reduce the agents' desire to smooth consumption is the empirically relevant one if the economy is calibrated to match the long run positive trend in the U.S. output growth as well as the absence of growth over time of wellbeing (or utility) as empirically found by Blanchflower and Oswald [12], and Di Tella et al. [24] among others. In these contributions, the authors test the Easterlin's hypothesis, according to which growth does not raise wellbeing, and confirm it in several developed countries, among them U.S and U.K., as well as over different time intervals. Blanchflower and Oswald observe also that comparisons with others is one of the four key factors influencing their measure of utility and then their findings. For this reason, our model seems well suited to be calibrated to match also the observations on the U.S. long-run utility's growth. Our economy calibrated to match these two targets (average U.S. output and

[3] In an overlapping generation model this revision of the habits may depend on the different generations' habits as pointed out in several articles by Caballe' and his coauthors [3] and [4]

utility growth rates) is characterized by transitional dynamics where the consumption path is less smooth than in the case without habits. On the other hand, we show that any calibration along the balanced growth path where the habits enhance the desire to smooth consumption over time leads always to violation of the Easterlin's hypothesis and that this violation is quantitatively relevant.

Finally a side contribution of the paper is the identification of the conditions for an interior solution also in the case with a finite lag structure. This technical result have relevant policy implications as well. In the recent literature, one of the exercises usually proposed (see Carroll et al. [18], [19] and Alvarez-Cuadrado et al. [5] among others) is to study the effects of the destruction of a fraction of the initial capital stock. Once embedded in our economy, this shock may lead the economy from an interior solution to a corner solution since the initial income may become so low that no feasible saving profile exists to sustain overtime the addictive habits given their initial level. Similarly income taxes, as proposed for example by Ljungqvist and Hulig [33], may lead to a corner solution if not carefully designed. The importance to identify the set of initial conditions leading to an interior solution was also pointed out recently by Alonso-Carrera et al. [2] in a different setting with multiplicative habits and a production function as in Jones and Manuelli.

The paper is organized as follows: in Section 2 we present how the first mechanism emerges in a benchmark model where the lag structure is assumed to be infinite. The results are proved analytically and a numerical exercise on fiscal policy is also proposed to better understand how the consumption dynamics may vary once the habits are introduced. Then in Section 3, we study how a more general definition of the habits may affect the qualitative dynamics of the same economy. In this context we point out and explain the second mechanism; we also investigate the different kinds of endogenous fluctuations and propose a complete taxonomy of the rich dynamics which may emerge in our economy according to the different combinations of the intensity, persistence and lag structure of the habits. In the next section, we show that the region of the parameter space where habits reduce the agents' desire to smooth consumption is the empirically relevant one if the economy is calibrated to match the average U.S. output and utility growth rates. In Section 5 we discuss how much the two mechanisms depend on our modeling choices with an emphasis to the dichotomy external vs internal habits, and subtractive vs multiplicative nonseparable utility function. Finally Section 6 concludes the paper.

2 The benchmark model

Consider a standard neoclassical growth model, where the economy consists of a continuum of identical infinitely lived atomistic households, and firms. The households' objective is to maximize over time the discounted instantaneous utility, $u(c(t), h(t))$, which is a function of current consumption, $c(t)$, and external habits $h(t)$. It is indeed assumed as in Constantinides [21], and in many others contributions in the macro-finance literature, that the instantaneous utility function has the no-separable subtractive form:

$$u(c(t), h(t)) = \frac{(c(t) - h(t))^{1-\gamma}}{1-\gamma}, \tag{1}$$

for $c(t) \geq h(t)$ and $\gamma > 0$ but different from 1. Observe that if $c(t) < h(t)$ then the utility function is not well defined in the real field for some values of γ (i.e. $\gamma = \frac{3}{2}$), and, it is never concave. For this reason, it is generally assumed that $u(c(t), h(t)) = -\infty$ as soon as $c(t) < h(t)$. The instantaneous utility function (1) is also referred to as *addictive habit formation* because current consumption is forced to remain higher than the external habits over time; alternatively it can be seen as a Stone-Geary utility function with an endogenous and time varying subsistence level of consumption, $h(t)$.[4] In a first stance, our analysis focuses on an exponentially smoothed index of the economy past average consumption rate as a mechanism of habit formation:

$$h(t) = \varepsilon \int_{-\infty}^{t} \bar{c}(u) e^{\eta(u-t)} du \qquad or \qquad \dot{h}(t) = \varepsilon \bar{c}(t) - \eta h(t) \qquad with \qquad \forall t \geq 0 \tag{2}$$

where, as mentioned in the introduction, $\eta \geq 0$ measures the persistence of habits, while $\varepsilon \geq 0$ the intensity of habits, i.e. the importance of the economy average consumption relative to current consumption. Finally h_0 is given, as well as the path of $\bar{c}(t)$ since no individual decision have an appreciable effect on the average consumption of the economy. From now on, we will call ***benchmark model*** a model where habits are defined as in (2).

The decentralized resources-allocation leads to a suboptimal competitive equilibrium since the habits enter as a negative externality in the instantaneous utility function. The representative household solves indeed the following problem:

$$\begin{aligned} \max & \int_0^\infty \frac{(c(t) - h(t))^{1-\gamma}}{1-\gamma} e^{-\rho t} dt \\ s.t. \;\; & \dot{a}(t) = r(t)a(t) - c(t) + w(t) \\ & a(t) \geq 0, \;\; c(t) \geq h(t) \geq 0 \\ & a(0) = a_0 > 0 \end{aligned}$$

where $a(t)$, $w(t)$ and $r(t)$ stand respectively for asset per capita, real wage and real interest rate. From now on we will assume a priori that the control is interior and the state constraint is satisfied. Then we will identify a posteriori what are the restrictions to be imposed so that all the inequalities constraints will hold. Following this strategy, we may easily write the first order conditions from the present value Hamiltonian

$$c(t) - h(t) = \varphi(t) \tag{3}$$

$$\frac{\dot{\varphi}(t)}{\varphi(t)} = \frac{1}{\gamma}(r(t) - \rho) \tag{4}$$

where $\varphi(t) = \psi(t)^{-\frac{1}{\gamma}}$ with $\psi(t)$ the costate variable. Then the representative household's behavior is fully described by these first order conditions, the asset equation and a no-Ponzi game condition to rule out chain-letter possibilities in the credit market

$$\lim_{t \to \infty} a(t) e^{-\int_0^t r(u) du} \geq 0$$

Finally the representative firm maximizes its profit function subject to a linear production function with capital the only factor. Then the rental rate of capital is constant over time and equal to the level of technology, $R(t) = A$.

[4] For a clear characterization of the same model with $h(t) = \bar{c}$ and $\bar{c}$ the minimum subsistence level of consumption see Strulik[39]

2.1 Balanced growth and transitional dynamics

The competitive equilibrium for this economy is defined taking into account the household's intertemporal maximization problem, the firm's static problem, the no arbitrage condition, $r(t) = A - \delta$, and the market clearing conditions $a(t) = k(t)$, and $\bar{c}(t) = c(t) = y(t) - i(t)$.

Definition 1 *A market equilibrium is described by any trajectory* $\{\varphi(t), k(t), h(t)\}_{t \geq 0}$ *which solves*

$$\dot{k}(t) = (A-\delta)k(t) - \varphi(t) - h(t) \tag{5}$$
$$\dot{h}(t) = \varepsilon\varphi(t) + (\varepsilon - \eta)h(t) \tag{6}$$
$$\varphi(t) = \varphi_0 e^{\frac{1}{\gamma}(A-\delta-\rho)t} \tag{7}$$

subject to (i) the initial condition of capital, $k(0) = k_0$*, (ii) the initial condition of habit,* $h(0) = h_0$*, (iii) the transversality condition* $\lim_{t\to\infty} k(t)e^{(A-\delta)t} = 0$*, and (iv) the inequality constraints,* $k(t) \geq 0$*, and* $c(t) \geq h(t) \geq 0$*.*

This system of equations can be easily solved and a φ_0 correctly specified in order to satisfy the transversality condition stated before. These results as well as the conditions for an interior solution will be proved in the next proposition. Before moving on it, we briefly remind that in a standard AK model, without habits, all the aggregate variables "jump" immediately to the balanced growth path where they grow at the rate $\Gamma = \frac{1}{\gamma}(A - \delta - \rho)$; then the economy faces positive growth only if the level of technology, A, is higher than a threshold, $\hat{A}$:

$$A \geq \hat{A} = \delta + \rho \qquad \Leftrightarrow \qquad \Gamma \geq 0 \tag{8}$$

If this condition is not met, the economy shrinks over time.

Proposition 1 *A market economy with external addictive habits, and linear production, has:*

i) a unique positive asymptotic growth rate $g = \max(\varepsilon - \eta, \Gamma)$*,*[5] *and*

ii) a unique and interior competitive equilibrium path converging monotonically to the balanced growth path:

$$k(t) = \left(h_0 + \frac{\varepsilon\tilde{\varphi}_0}{\varepsilon - \eta - \Gamma}\right)\frac{e^{(\varepsilon-\eta)t}}{A - \delta - \varepsilon + \eta} + \tilde{\varphi}_0\left(\frac{\eta + \Gamma}{(\varepsilon - \eta - \Gamma)(\Gamma - A + \delta)}\right)e^{\Gamma t} \tag{9}$$
$$h(t) = \frac{\varepsilon\tilde{\varphi}_0}{\Gamma - \varepsilon + \eta}e^{\Gamma t} + \left(h_0 + \frac{\varepsilon\tilde{\varphi}_0}{\varepsilon - \eta - \Gamma}\right)e^{(\varepsilon-\eta)t} \tag{10}$$
$$c(t) = e^{(\varepsilon-\eta)t}\left(h_0 + \frac{\varepsilon\tilde{\varphi}_0}{\varepsilon - \eta - \Gamma}\right) + \tilde{\varphi}_0 e^{\Gamma t}\left(\frac{\eta + \Gamma}{\Gamma - \varepsilon + \eta}\right) \tag{11}$$
$$\textit{with} \qquad \tilde{\varphi}_0 = \frac{(\varepsilon - \eta - A + \delta)(\Gamma - A + \delta)}{(A - \delta + \eta)}\left(k_0 + h_0\frac{1}{\varepsilon - \eta - A + \delta}\right) \tag{12}$$

if $A \geq \hat{A}$*,* $k_0 > h_0\frac{1}{A-\delta-\varepsilon+\eta}$ *and* $g < A - \delta$*. These results still hold when* $A < \hat{A}$ *and* $\varepsilon > \eta > -\Gamma$*.*

Proof. See Appendix. ■

This proposition shows that endogenous growth is still possible in an economy characterized by external habits and a level of technology so low to prevent positive growth in a standard AK

[5] The asymptotic growth rate of a variable x is defined as $\lim_{t\to+\infty}\frac{\dot{x}}{x}$.

model. Moreover the economy with external habits may grow even faster than a standard AK economy when the technology level is high ($A \geq \hat{A}$). What does determine this departure from the standard predictions of the AK model? Intuitively these results depend on the inequality $\varepsilon > \eta$ and on the habit addiction which trigger a positive growth rate in the habit stock and in the aggregate consumption. More precisely, the representative consumer faces the two constraints

$$\dot{h}(t) = (\varepsilon - \eta)h(t) + \varepsilon(\bar{c}(t) - h(t)) \qquad and \qquad c(t) \geq h(t) \tag{13}$$

At the market equilibrium, the solution path of $c(t)$, obtained by solving the representative agent problem, coincides with the given path, $\bar{c}(t)$; when this happens, (13) implies:

$$\dot{h}(t) = (\varepsilon - \eta)h(t) + \varepsilon(c(t) - h(t)) \geq (\varepsilon - \eta)h(t) \tag{14}$$

and the habit stock, as well as aggregate consumption, grows at least at the rate $\varepsilon - \eta$ even if $c(t) - h(t)$ shrinks over time at the rate $\Gamma < 0$.

As anticipated in the introduction, an asymptotic growth rate of the economy equals to $g = \varepsilon - \eta > 0$ will be accompanied by a *save now or regret it later mechanism* according to which the representative agent decides to consume, till the very beginning, less than in the standard AK model in order to save enough and accumulate the capital necessary to keep over time the consumption level higher than the habit stock avoiding, in this way, an infinity disutility. This growth rate is sustainable when lower than the interest rate of the economy, $r = A - \delta$, and unless the initial stock of habit is too high. In fact, a too high initial level of habit forces the household to choose an initial level of consumption so high that no feasible saving profile may sustain a $c(t) > h(t)$ over time.

The restriction on the initial conditions becomes clearer when $\varepsilon \simeq \eta$; in this case, the constraint becomes $rk_0 > h_0$ and implies that the initial habits have to be lower than the household's initial net income otherwise the initial level of consumption can be higher than the initial habits, ($c_0 > h_0$), only if an initial disinvestment ($i_0 < 0$) takes place. If this happens a positive growth rate of consumption and habits is clearly not sustainable over time because financed with a continuous selling of assets which leads to zero capital stock in a finite time. This restriction on the initial condition becomes more stringent if the intensity of the habits exceed their persistence since now the habits accumulation is faster than before as emerged from equation (10).[6] Through this mechanism, a faster accumulation of capital will imply higher consumption in the long run than predicted in the standard AK model. As a consequence, the habits will reduce the desire to smooth consumption.

We want now to show through a numerical exercise the mechanism just described while a proper calibration of the model is postponed to Section 4. Let us consider three possible scenarios, each of them characterized by the same parametrization (see Table 1) but different income taxation. More precisely the three scenarios are characterized by the following income tax $\tau_y = 0$, $\tau_y = 12.5\%$, and $\tau_y = 25\%$, which pin down three different levels of $\Gamma = \frac{1}{\gamma}[(1-\tau_y)A - \delta - \rho]$, as reported in the table.[7] Given this parametrization, the growth rate of the economy is

[6] In the benchmark model the additional restriction of the parameters, $\eta > -\Gamma$ has to be considered. Substituting the equilibrium path of consumption into the habits equation, (2), it appears clear that $\eta > -\Gamma$ avoids the divergence of $h(t)$ as $u \to -\infty$.

[7] This numerical exercise is equivalent to study the effects of a change in the level of technology B set to $(1 - \tau_y)A$

sustainable as long as the real interest rate remains positive which implies an income tax lower than $\frac{A-\delta}{A} = 37.5\%$. Moreover the initial conditions, k_0 and h_0, are specified in order to have always an interior solution. In the next three figures these different scenarios are also compared with a standard AK model with no habit, and a model with habit and $\varepsilon = \eta$.

A	δ	η	ε	γ	ρ	Γ_1	Γ_2	Γ_3
0.16	0.1	0.22	0.23	2	0.03	0.015	0.005	-0.005

Table 1: Parameters' selection

The first scenario (Figure 1) describes an economy with an asymptotic growth rate equal to $\Gamma_1 > \varepsilon - \eta > 0$. Differently from the standard AK model with no habit, the economy has now transitional dynamics characterized by *overconsumption in the short run* due to a saving dynamics which converges slowly from below to the constant saving rate of the AK model. The quicker capital accumulation justifies a higher consumption over time in the case of no habit. These predictions are in line with those found when $\varepsilon = \eta$, where the habits play a role in enhancing the desire to smooth consumption.

The second scenario (Figure 2) describes an economy where the higher income tax pins down an asymptotic growth rate $\varepsilon - \eta > \Gamma_2 > 0$; this economy grows faster than in the standard AK model with no habit and the same income taxation; the faster growth is possible for the *save now or regret it later* mechanism explained before. Differently from the first scenario, a higher saving will induce *underconsumption in the short run* but overconsumption in the long run as a result of the higher capital accumulation. Then comparing the consumption path of this scenario with the standard AK model (namely blue line with the dotted black line in Figure 2), it clearly emerges the reduction in the desire of smoothing consumption induced by the presence of the (growing) habits. This result is also completely different from the transitional dynamics in the case $\varepsilon = \eta$ as shown in Figure 2 comparing the red with the blue lines.

The last scenario (Figure 3) shows how an economy with $\varepsilon - \eta > 0$ grows over time even if Γ_3 is negative. The comparison with an economy without habits is even more striking since the latter shrinks over time at a rate Γ_3. The *save now or regret it later* mechanism is again behind the positive growth in the case with habits, and the underconsumption in the short run is now even more evident given the large gap in consumption between this model and the standard AK model. In this last case the model describes a growing economy where the happiness decreases over time and then account for the empirical evidences on the happiness dynamics as pointed out by Di Tella et al. [24]. Finally in the case $\varepsilon = \eta$, all the aggregate variables shrink over time but, differently from the case with no habit, converge to positive constants as appear clear from equations (9), (10), and (11).

From these results another consideration on the effects of a fiscal policy on the asymptotic growth rate and the transitional dynamics emerges. In our context any income tax in the range $0 \leq \tau_y < 6.25\%$ pins down an asymptotic growth rate Γ; then any positive (negative) change in the taxation within the range just specified has an effect both on the transitional dynamics,

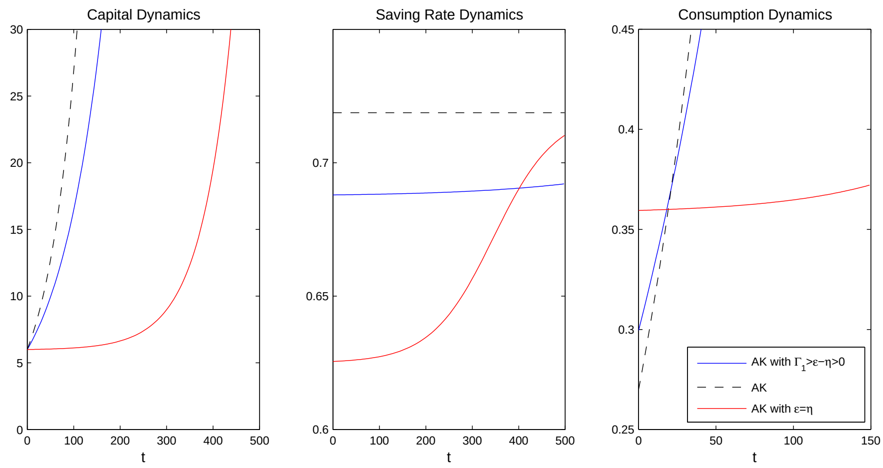

Figure 1: Capital, saving rate and consumption dynamics when no income taxation

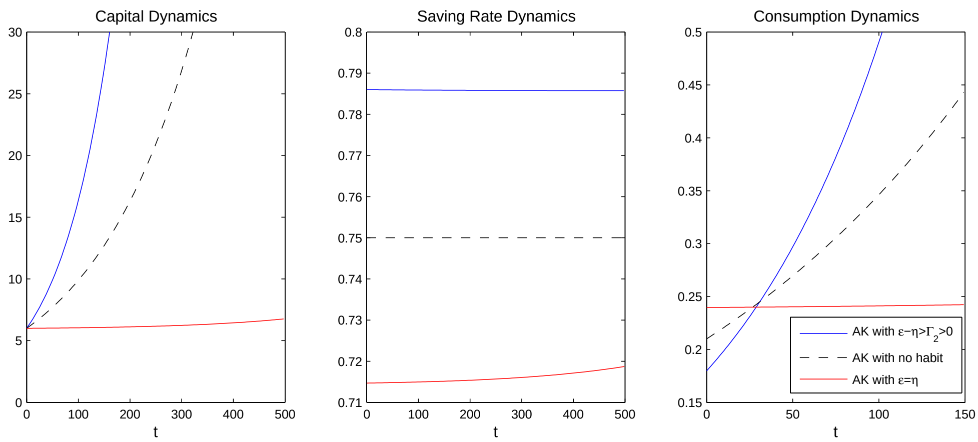

Figure 2: Capital, saving rate and consumption dynamics when $\tau_y = 12.5\%$.

for example reducing (increasing) the overconsumption in the short run, and on the asymptotic growth rate of the economy since the value of Γ depends on the tax on income. However the habits induce a higher consumption smoothing than in the standard AK model as long as the tax remains in this range. On the contrary, any income tax $6.25\% < \tau_y < 37.5\%$ will reverse this prediction.

Before concluding this section two further consideration are necessary. The first regards the condition for bounded utility which is exactly the same as in the standard AK model without habits since the difference $c(t) - h(t)$ grows at the rate Γ. The last consideration is on the corner solution; an economy which violates the habit addiction constraint, is characterized by the corner solution $c(t) = h(t)$ and the capital corner solution

$$k(t) = \left(k_0 - \frac{h_0}{A - \delta - \varepsilon + \eta}\right) e^{(A-\delta)t} + \frac{h_0}{A - \delta - \varepsilon + \eta} e^{(\varepsilon - \eta)t}.$$

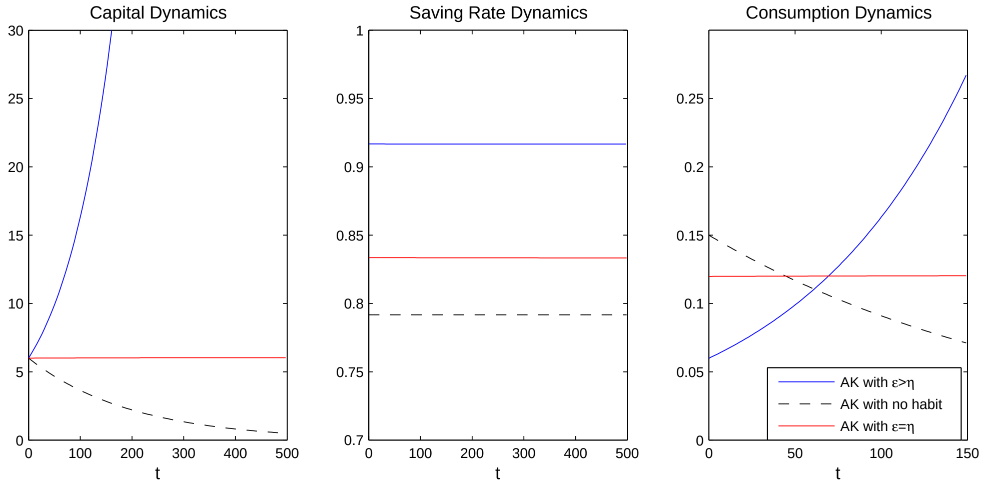

Figure 3: Capital, saving rate and consumption dynamics when $\tau_y = 25\%$

3 The "general" model

In this section, the habits equation (2) is replaced by

$$h(t) = \varepsilon \int_{t-\tau}^{t} \bar{c}(u) e^{\eta(u-t)} du \qquad or \qquad \dot{h}(t) = \varepsilon \left(\bar{c}(t) - \bar{c}(t-\tau) e^{-\eta\tau} \right) - \eta h(t) \tag{15}$$

with $t \geq 0$, and $\tau > 0$. This expression includes also the habits as intended by Abel [1], when $\tau = 1$ and by Crawford [23] who considered the possibility of a $\tau \leq 3$ in his econometric estimates.[8]

As in the benchmark model the representative agent compares his own consumption level with an exponentially smoothed index of the economy past average consumption rate but now the external habits at two sufficiently far dates may be completely unrelated since only the average consumption between t and $t - \tau$ matters: in this respect, a finite τ introduces a complete and periodic update of the habit. This new feature of the model let us describe more interesting economies. For example, we could think to an economy populated by infinitely lived identical households each of them having at any date t a single individual with a finite life length of τ periods (see for example Barro and Sala-i-Martin [9], page 86). The different generations within a household are connected through a pattern of transfers based on altruism. Then the newborn at date t will live till $t + \tau$ and will be the only member of the household during this period. Moreover, at date t, he compares his current consumption with a habit index based on the average consumption over the lifetime of his parent. As soon as he grows up he begins to look at the average consumption of his peers in the other households and his habit index will be updated accordingly. At the end of his life the habit index will be based on the average consumption of his generation only, which at the equilibrium will coincide with the consumption during his own lifetime.[9]

[8]In our continuous time framework, this specification seems preferable to the alternative $h(t) = \varepsilon \sum_{i=1}^{\tau} \bar{c}(t-i)$.

[9]In an OLG model, Alonso-Carrera et al. [3] have fully developed the effects of the different generations' habits on the bequest motive.

It is also worth noting that the habit addiction is no more enough to pin down, at the market equilibrium, an habit's growth rate greater or equal than $\varepsilon - \eta$; in fact the new more restrictive necessary condition for that is

$$c(t) \geq h(t) + c(t-\tau)e^{-\eta\tau}.$$

It will be indeed proved later that a finite lag structure in the habits may reduce the asymptotic growth rate of the economy.

Taking into account the definition of the habit stock, (15), we re-define the market equilibrium of the economy. Again we will impose a posteriori the conditions for all the inequality constraints to be satisfied.

Definition 2 *A market equilibrium is described by any trajectory* $\{\varphi(t), k(t), h(t)\}_{t\geq 0}$ *which solves*

$$\dot{k}(t) = (A-\delta)k(t) - \varphi_0 e^{\Gamma t} - h(t) \tag{16}$$

$$h(t) = \varepsilon \int_{t-\tau}^{t} \left[h(u) + \varphi_0 e^{\Gamma u}\right] e^{\eta(u-t)} du \tag{17}$$

$$\varphi(t) = \varphi_0 e^{\Gamma t} \tag{18}$$

subject to (i) the initial condition of capital, $k(0) = k_0$*, (ii) the past history of habit,* $h_0(t)$*,* $t \in [-\tau, 0]$*, and (iii) the transversality condition* $\lim_{t\to\infty} k(t)e^{(A-\delta)t} = 0$ *and (iv) satisfying* $k(t) \geq 0$ *and* $c(t) \geq h(t) \geq 0$*.*[10]

Under this more general definition of the habits, the initial continuous function, $h_0(t)$ with $t \in [-\tau, 0]$, replaces the usual initial condition, $h(0) = h_0$; as a consequence the past history of consumption, namely $c(t) \in [-2\tau, 0)$ is known, but not $c(0) = c_0$ whose value will be determined, exactly as in the case with $\tau \to +\infty$, by (univocally) choosing the initial shadow price of capital, $\psi(0)$, that rules out the transversality condition and pins down the unique balanced growth path of the economy. Observe also that equation (17) is now a non-autonomous algebraic equation in $h(.)$ only, which can be solved and used in the capital accumulation equation in order to derive a solution for the level of capital. Studying the algebraic equation (17), means, first of all, analyzing the properties of its spectrum of roots. This information will be indeed fundamental in order to find the growth rate of consumption and later the balanced growth path of the economy. Finally to present the results more clearly, we have decided to distinguish the *low technology* case, $A < \hat{A}$, from the *high technology* case, $A > \hat{A}$.

3.1 The growth rate and dynamics of consumption

Consumption behavior when the technology level is low ($\Gamma < 0$)

The first question we are going to address is wether and how the more general definition of the habit stock modifies the previous results on the consumption growth rate of the market economy. In order to do that, we begin studying the algebraic functional equation

$$z(t) = \varepsilon \int_{t-\tau}^{t} z(u)e^{\eta(u-t)} du \tag{19}$$

[10] It is worth noting that with a finite and greater than zero τ, the restriction $\Gamma + \eta > 0$ is no more necessary for a positive and finite $h(t)$ over time, because $\int_{t-\tau}^{t} e^{(\Gamma+\eta)u} du$ is always positive and finite.

obtained by equation (17) after the change of variable

$$z(t) = h(t) - e^{\Gamma t}\varphi_0\theta, \text{ with } \theta = \frac{\varepsilon\frac{1-e^{-(\Gamma+\eta)\tau}}{\Gamma+\eta}}{1-\varepsilon\left(\frac{1-e^{-(\Gamma+\eta)\tau}}{\Gamma+\eta}\right)} \tag{20}$$

(computational details are reported in the Appendix, Lemma A). Equation (19) is autonomous and then easier to study than equation (17).[11] Observe also that the growth rate of consumption depends on the growth rate of $h(t)$ and then of $z(t)$. Investigating the latter means to study the characteristic equation of (19)

$$\Delta(\lambda) = 0 \;\; with \;\; \Delta(\lambda) = -1 + \varepsilon\int_{-\tau}^{0} e^{(\lambda+\eta)u}du \tag{21}$$

and specifically to find the conditions for the existence of a positive leading real root among the *infinite number of complex conjugate complex roots* of (21) (see Diekmann et al. [25], Chapter XI).

Lemma 1 *The spectrum of roots of (21) has a leading real root, α^*, which is positive if and only if*

$$\varepsilon - \eta > 0 \;\; and \;\; \tau > \tau^* \;\; with \;\; \tau^* = -\frac{\log\left(1-\frac{\eta}{\varepsilon}\right)}{\eta} \tag{22}$$

Moreover α^* is an increasing function of τ and converges to $\varepsilon - \eta$, as soon as τ goes to infinity.

Proof. See Appendix. ■

Then $z(t)$ grows asymptotically at the positive rate α^* with damping oscillations in the transition whose presence is induced by the infinite number of complex roots. It will indeed be proved in the next proposition that the general solution of (19) is the sum of a trend component $z_0e^{\alpha^*t}$ and an oscillatory component, $\chi_1(t)$. Moreover the growth rate as well as the fluctuations of $z(t)$ will be transmitted through the relations (20), and (3) to $h(t)$ and $c(t)$.

Proposition 2 *The consumption dynamics is described by the equation*

$$c(t) = \varphi_0 e^{\Gamma t}(1+\theta) + z_0e^{\alpha^*t} + \chi_1(t), \tag{23}$$

with $\lim_{t\to\infty}\chi_1(t)e^{-\alpha^*t} = 0$*, and* $z_0 = z_0(h_0(t), \varphi_0)$ *equals to*

$$z_0 = \frac{-\varepsilon}{\Delta'(\alpha^*)}\left[\frac{1}{\eta+\alpha^*}\int_{-\tau}^{0}\left(e^{\eta\mu} - \frac{e^{-(\eta+\alpha^*)\tau}}{\eta+\alpha^*}e^{-\alpha^*\mu}\right)h(\mu)d\mu - \frac{\varphi_0\theta}{\Gamma-\alpha^*}\left(\frac{1-e^{-(\eta+\alpha^*)\tau}}{\eta+\alpha^*} - \frac{1-e^{-(\Gamma+\eta)\tau}}{\Gamma+\eta}\right)\right]$$

Then the asymptotic positive growth rate of consumption and the habit stock $g = \alpha^*$ when $\varepsilon - \eta > 0$, and $\tau > \tau^*$ and $z_0 \neq 0$.[12]

Proof. See Appendix ■

[11] Augeraud-Veron et al.[7] discuss extensively the solution and dynamics of this class of functional differential equations

[12] It will be indeed shown in subsection 4.2 that one of the conditions for an interior solution is to specify $(h_0(t), \varphi_0)$ such that $z_0 > 0$, while $z_0 = 0$ is a non generic condition.

From Lemma 1, and Proposition 2, it emerges that the introduction of a finite lag structure leads to two changes in the aggregate consumption dynamics. First, the growth rate of consumption, α^*, depends positively on τ and it will be always lower or equal than in the benchmark model; even more a choice of τ sufficiently low may imply a negative α^* breaking down the possibility of a positive growth rate even when the intensity of the habits exceeds their persistence. When the habits are specified in the same spirit as in Abel, with a $\tau = 1$, the τ^* is equal to 14.25 once the economy is parameterized as in the previously discussed numerical exercise (case income tax set to $\tau_y = 25\%$), and then the growth rate of consumption is negative and precisely equal to $\alpha^* = -0.88$. Then a finite τ may break down the mechanism of save now or regret it later, by reducing the asymptotic growth rate from $g = 0.01$, to a negative value. As anticipated at the beginning of this section, this result is not completely surprising since the representative consumer faces now the two constraints

$$\dot{h}(t) = \varepsilon\left(\bar{c}(t) - \bar{c}(t-\tau)e^{-\eta\tau}\right) - \eta h(t) \qquad \textit{and} \qquad c(t) \geq h(t) \tag{24}$$

which, at the market equilibrium, lead to the following inequality

$$\begin{aligned} \dot{h}(t) &= \varepsilon\left(c(t) - c(t-\tau)e^{-\eta\tau}\right) - \eta h(t) && (25)\\ &\geq (\varepsilon - \eta)h(t) - \varepsilon c(t-\tau)e^{-\eta\tau} && (26) \end{aligned}$$

Then the presence of a finite τ adds the last component in relation (26) which may reduce or even break down the mechanism for a positive growth rate in the habits and then in the aggregate consumption. Observe also that as $\tau \to +\infty$ this last component disappears and the consideration done in Section 2.1 for the standard definition of habit hold again.

On the other hand, the finite lag structure may reduce the desire to smooth consumption since it triggers endogenous fluctuations in the habits and in consumption. The intuition behind this result is the following: at the equilibrium the risk adverse agents rationally choose a smooth path of $c(t) - h(t)$, as described by the first order condition (18), then equation (25) can be rewritten as

$$\dot{h}(t) = (\varepsilon - \eta)h(t) + \varepsilon h(t-\tau)e^{-\eta\tau} + g(t) \tag{27}$$

with $g(t) = \varepsilon\varphi_0\left(1 - e^{-(\Gamma+\eta)\tau}\right)e^{\Gamma t}$. Then the habits are described by a delay differential equation with forcing term $g(t)$ and for this reason their dynamics can be oscillatory as soon as a finite lag structure is chosen.[13] Observe also that the fluctuations in aggregate detrended consumption are indeed "rational" because the risk adverse agents want to smooth the path of $c(t) - h(t)$, as just discussed, and then adjust their consumption behavior to offset any fluctuations in the habits.

Finally it can be also observed that after some algebra the two limits

$$\lim_{\tau\to\infty} z_0 = h_0 - \frac{\varepsilon\varphi_0}{\Gamma + \eta - \varepsilon} \quad \textit{and} \quad \lim_{\tau\to\infty} \chi_1(t) = 0$$

hold and then we may read the result on the consumption growth rate in the benchmark model as a particular case of this model.

[13] The presence of delay differential equations was found critical in different economic frameworks, as in model with vintage capital (e.g. Boucekkine et al. [14]) or with time to build (e.g. Bambi et al. [8]) , to the raising of endogenous fluctuations.

Before moving to the next section on the growth rate of consumption with a high technology level, we conclude observing that consumption and habits shrink over time to zero if condition (22) is not satisfied.

Consumption behavior when the technology level is high ($\Gamma > 0$)

In this subsection we consider the other case, $A > \hat{A}$ or $\Gamma > 0$, which implies, in a standard AK model, a positive growth of all the aggregate variables at the rate, Γ, from t greater than zero on. As explained in the benchmark model - Proposition 1, the presence of the external addictive habits may change this result when the difference between the intensity and the persistence of the habits, $\varepsilon - \eta$, is greater than Γ. If this happens the growth rate of consumption will be no more Γ but $\varepsilon - \eta$. In this subsection we want to move further and study how these findings are affected by the the finite lag structure of the habits.

In line with the previous subsection, we begin by rewriting the algebraic equation (17) in term of the new auxiliary variable $x(t) = h(t)e^{-\Gamma t} - \varphi_0 \theta$, where $\theta = \frac{\frac{\varepsilon}{\Gamma+\eta}(1-e^{-(\Gamma+\eta)\tau})}{\left(1-\varepsilon \int_{-\tau}^{0} e^{(\Gamma+\eta)u} du\right)}$:

$$x(t) = \varepsilon \int_{t-\tau}^{t} x(u) e^{(\Gamma+\eta)(u-t)} du \tag{28}$$

Computational details can be easily deduced from what previously done as $z(t) = x(t)e^{\Gamma t}$. The change of variable slightly differs from the one used for the low level of technology case, because now the difference between consumption and the habits grows at the positive rate Γ, and then we introduce a new variable which is detrended accordingly. This transformation will allow us to find more easily the growth rate of consumption and the its solution. The growth rate of consumption depends now on the growth rate of $h(t)$ and then of $x(t)$. Investigating the last one means to study the characteristic equation of (28), namely

$$\Delta(\tilde{\lambda}) = 0 \quad with \quad \Delta(\tilde{\lambda}) = -1 + \varepsilon \int_{-\tau}^{0} e^{(\Gamma+\tilde{\lambda}+\eta)u} du \tag{29}$$

Observe that the roots of $\Delta(\tilde{\lambda}) = 0$ are related to those of the corresponding characteristic equation in the low technology case through the simple linear transformation, $\Gamma + \tilde{\lambda} = \lambda$; then the spectrum of $\Delta(\tilde{\lambda}) = 0$ can be deduce by Lemma 1 in the previous section and is summarized in the next Corollary.

Corollary 1 *The spectrum of roots of (29) has a leading real root, α_*, which is positive if and only if*

$$\varepsilon - \eta > \Gamma \quad and \quad \tau > \tau_* \quad with \quad \tau_* = -\frac{\log\left(\frac{\varepsilon-\Gamma-\eta}{\varepsilon}\right)}{\Gamma+\eta}$$

Moreover the following results hold:

i) α_* is an increasing function of τ and it converges to $\varepsilon - \Gamma - \eta$, as $\tau \to \infty$;

ii) α_* is related to α^* by the relation $\alpha_* = \alpha^* - \Gamma$.

Taking into account these information on the spectrum of roots of the characteristic equation (29), the definition of $x(t)$ and relations (3), which links the variable $h(t)$ to $c(t)$, we may derived the following results on the growth rate of aggregate consumption.

Proposition 3 *The consumption dynamics is described by the equation*

$$c(t) = \sigma_0 e^{t(\Gamma+\alpha_*)} + (1+\theta)\varphi_0 e^{\Gamma t} + \chi_2(t) e^{\Gamma t} \tag{30}$$

with $\lim_{t\to\infty} \chi_2(t)\, e^{-\alpha_* t} = 0$ *and* $\sigma_0 = \sigma_0(h_0(t), \varphi_0)$ *equals to:*

$$\sigma_0 = \frac{-\varepsilon}{\Delta'(\alpha_*)}\left[\int_{-\tau}^{0}\left(\frac{e^{\eta\mu}}{\Gamma+\eta+\alpha_*} - \frac{e^{-(\Gamma+\eta+\alpha_*)\tau}}{\Gamma+\eta+\alpha_*}e^{-(\Gamma+\alpha_*)\mu}\right)h(\mu)d\mu - \theta\varphi_0\left(\frac{1-e^{-\tau(\Gamma+\eta)}}{\alpha_*(\Gamma+\eta)} - \frac{1-e^{-\tau(\Gamma+\eta+\alpha_*)}}{\alpha_*(\Gamma+\eta+\alpha_*)}\right)\right]$$

Then the asymptotic positive growth rate of consumption and the habit stock is

i) $g = \Gamma + \alpha_*$ *if* $\varepsilon - \eta > \Gamma$ *and* $\tau > \tau_*$, *and* $\sigma_0 \neq 0$, *or*

ii) $g = \Gamma$ *if* $\varepsilon - \eta < \Gamma$ *or* $\varepsilon - \eta > \Gamma$ *and* $\tau < \tau_*$, *and* $\varphi_0 \neq 0$

Proof. See Appendix. ■

Observe also that at this stage φ_0 is not yet specified and then equation (30) is not the competitive equilibrium path of consumption; it will become the competitive equilibrium path of consumption only after having determined the $\varphi_0 = \tilde{\varphi}_0$ which make the transversality condition hold. It is also worth noting that $\varphi_0 = 0$ and $\sigma_0 = 0$ are non generic conditions. Finally as $\tau \to +\infty$ we have that $\sigma_0 \to h_0 - \frac{\varepsilon\varphi_0}{\Gamma+\eta-\varepsilon}$, while the oscillatory component disappears $\lim_{\tau\to\infty} \chi_2(t) = 0$.

3.2 Balanced growth and transitional dynamics

In this section we prove the existence and uniqueness of the competitive equilibrium path for an economy characterized by a low and a high level of technology and a finite lag structure in the habits; the dynamic behavior as well as the asymptotic growth rate of all the aggregate variables will be also found. These results will be proved in Proposition 4, and 6 by substituting the solution of consumption obtained in the previous section into the capital accumulation equation, and then solving the non-autonomous equation in capital, so obtained, and checking the transversality condition explicitly. We start with the low technology case:

Proposition 4 *A market economy with a finite lag structure in the habits, and a low level of technology (*$\Gamma < 0$*) has*

i) a unique asymptotic and positive growth rate $g = \alpha^*$;

ii) a unique and interior competitive equilibrium path which oscillatory converges over time to the balanced growth path:

$$\begin{aligned} h(t) &= \theta\hat{\varphi}_0 e^{\Gamma t} + \hat{z}_0 e^{\alpha^* t} + \chi_1(t) && (31)\\ c(t) &= \hat{\varphi}_0 e^{\Gamma t}(1+\theta) + \hat{z}_0 e^{\alpha^* t} + \chi_1(t) && (32)\\ k(t) &= \frac{\hat{\varphi}_0(1+\theta)}{(A-\delta-\Gamma)}e^{\Gamma t} + \frac{\hat{z}_0}{A-\delta-\alpha^*}e^{\alpha^* t} + \int_t^\infty e^{-(A-\delta)(u-t)}\chi_1(u)du && (33) \end{aligned}$$

if $\varepsilon - \eta > 0$ *and* $\tau > \tau^*$, $A - \delta > \alpha^*$, *and* $(k_0, h_0(t))$ *such that* $\hat{z}_0 = z_0(h_0(t), \hat{\varphi}_0) > 0$ *and* $\hat{\varphi}_0 = \varphi_0(k_0, h_0(t)) > 0$. *The explicit expression of* $\hat{\varphi}_0$ *and* χ_1 *can be respectively found in the proof of this proposition and of Proposition 2.*

Proof. See Appendix. ■

Some considerations on the conditions which guarantee these results seem necessary. The first condition on the intensity, persistence and length of the habits is necessary and sufficient for a positive growth rate of consumption and the habits as shown in Proposition 2. The second condition on a real interest rate, $r = A - \delta$ higher than the asymptotic growth rate of the economy, $g = \alpha^*$, guarantees the positive sign of the consumption-output ratio in the long run; in fact, combining (32) with (33) we have that

$$\lim_{t\to\infty} \frac{c}{y} = \frac{A - \delta - \alpha^*}{A} > 0$$

Finally the last condition tells us how the initial capital, k_0, and the initial history of the habits, $h_0(t)$, have to be specified in order to have an interior solution. As in the benchmark model, $c(t) > h(t)$ holds when $\hat{\varphi}_0 > 0$ and the expression for $\hat{\varphi}_0$ is indeed found explicitly in the previous proposition. Intuitively this restriction is less stringent than in the benchmark model. Given an initial path of $\bar{c}(t)$, the initial net income required to have $c_0 > h_0$ in the latter is always enough to guarantee the same in the former. In fact

$$rk_0 > h_0 = \varepsilon \int_{-\infty}^{0} \bar{c}(u) e^{\eta u} du > \varepsilon \int_{-\tau}^{0} \bar{c}(u) e^{\eta u} du$$

Moreover the presence of the finite lag structure in the habits and then the presence of fluctuations in the aggregate variables requires an additional constrain in the specification of the initial condition, namely a $(k_0, h_0(t))$ such that $\hat{z}_0 > 0$, in order to prevent negative capital.[14]

Another feature which deserves to be investigated is the nature of the oscillatory convergence. The oscillatory behavior can arrive either through damping fluctuations around the balanced growth path or through small fluctuations outside it which are quickly smoothed out over time. In the next Proposition we give conditions for the presence of damping fluctuations because more relevant in breaking down the role of the habits in reducing the desire to smooth consumption.

Proposition 5 *The transitional dynamics is characterized by damping fluctuations around the balanced growth path when* $0 < \eta < -\Gamma$.

Proof. See Appendix. ■

Similar results in term of transitional dynamics and the asymptotic growth rate of the economy are proved for the case with high technology, in the next two Propositions.

Proposition 6 *A market economy with a finite lag structure in the habits, and a high level of technology* ($\Gamma > 0$) *has*

i) a unique asymptotic and positive growth rate $g = \max(\Gamma, \Gamma + \alpha_*)$;

[14]The nature of this additional constraint for the positiveness of the capital stock is indeed very similar to that one investigated by Bambi et al. [8] in a time to build framework or by Fabbri and Gozzi [30] in a model with vintage capital.

ii) a unique and interior competitive equilibrium path which oscillatory converges over time to the balanced growth path:

$$
\begin{aligned}
h(t) &= \hat{\sigma}_0 e^{(\Gamma+\alpha_*)t} + \theta\hat{\varphi}_0 e^{\Gamma t} + \chi_2(t)e^{\Gamma t} & (34)\\
c(t) &= \hat{\sigma}_0 e^{(\Gamma+\alpha_*)t} + (1+\theta)\hat{\varphi}_0 e^{\Gamma t} + \chi_2(t)e^{\Gamma t} & (35)\\
k(t) &= \frac{\hat{\varphi}_0(1+\theta)}{A-\delta-\Gamma}e^{\Gamma t} + \frac{\hat{\sigma}_0}{A-\delta-\Gamma-\alpha_*}e^{(\Gamma+\alpha_*)t} + \int_t^\infty \chi_2(u)e^{-(A-\delta-\Gamma)(u-t)}du & (36)
\end{aligned}
$$

if $A-\delta > \Gamma+\alpha_*$*, and* $(k_0, h_0(t))$ *such that* $\hat{\sigma}_0 = \sigma_0(h_0(t), \hat{\varphi}_0)$ *and* $\hat{\varphi}_0 = \varphi_0(k_0, h_0(t)) > 0$*. The explicit expression of* $\hat{\varphi}_0$ *and* χ_2 *can be respectively found in the proof of this proposition and of Proposition 3.*

Proof. See Appendix. ■

As before, the inequality constraints indicate how the initial conditions have to be specified in order to have an interior solution; an explicit analysis of the corner solution is proposed in the Appendix, while the next proposition finds the conditions for damping fluctuations around the balanced growth path for the case with high technology.

Proposition 7 *The transitional dynamics is characterized by damping fluctuations around the balanced growth path when* $\Gamma > \varepsilon(e^{\tau\Gamma} - 1)$*.*

Proof. See Appendix. ■

It is again worth noting that if the conditions stated in this proposition are not satisfied then the oscillations in the aggregate variables are not around the balanced growth path and are smoothed out more quickly.

Finally to summarize all the results in term of the transitional dynamics and asymptotic growth rates, it may be useful to consider the normalization

$$\int_{t-\tau}^{t} e^{\eta(u-t)} = 1 \qquad t \geq 0 \tag{37}$$

which allows us to visualize all our findings in the parameter space (ε, η). This normalization let us to express τ in terms of the persistence of the habits; in fact, solving the last expression for τ leads to

$$\tau = -\frac{\ln(1-\eta)}{\eta}$$

which is well defined as as soon as $\eta < 1$. Observe also that equation (37) implies that $\tau \in [1, \infty)$ as $\eta \in (0, 1)$. In Figure 4 we report all the results found in term of growth rates and transitional dynamics. The red curve separates the region of parameters inducing damping fluctuation around the balanced growth path from the other region of parameters inducing fluctuations outside the balanced growth path which tend to be smoothed out more quickly during the transitional dynamics. On the other hand the blue curve separates the parameter space in two regions: one where the mechanism of save now or regret it later mechanism holds, the other where it does not.

Finally the damping fluctuations induced by the habits are characterized, during the transition toward the balanced growth path, by a positive relation between consumption growth

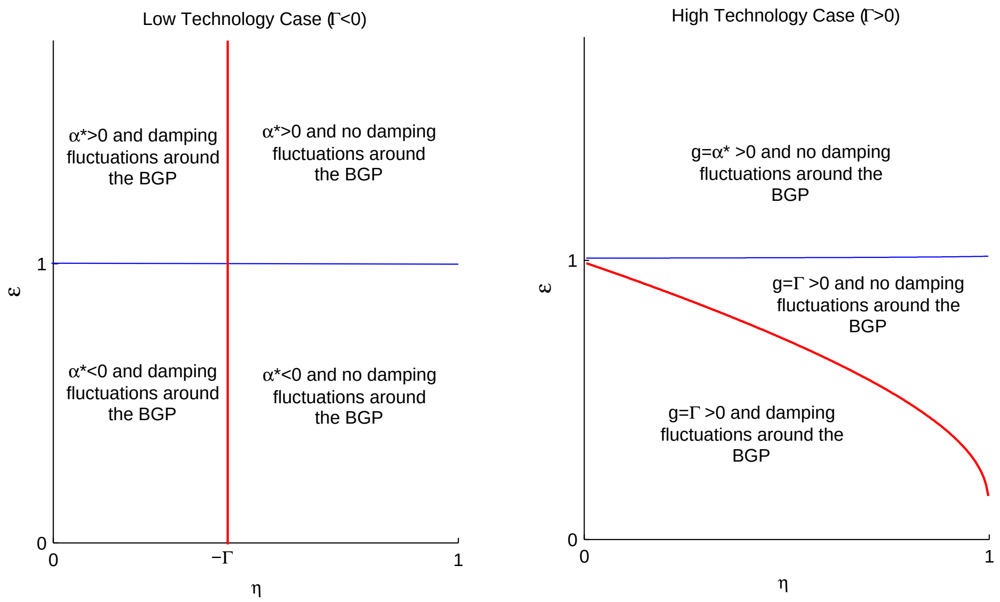

Figure 4: Taxonomy of the dynamics in the space (η, ε)

and income growth while no relation between consumption growth and the real interest rate. The latter is immediate to see since the interest rate is always constant. The positive relation between consumption growth and income growth can be seen starting from the capital accumulation equation and focusing on a sufficiently small neighborhood of the balanced growth path. In particular, from the capital accumulation equation we have that

$$g_k(t) = A - \delta - \frac{c(t)}{k(t)} \qquad or \qquad \frac{c(t)}{k(t)} = A - \delta - g_k(t) > 0$$

Now, if we move to the logarithms and differentiate both sides we get

$$g_c(t) = g_k(t) + \frac{-\dot{g}_k(t)}{A - \delta - g_k(t)}$$

Observe that at the balanced growth path $g_k(t) = g_c(t)$, then the last component in the right hand side of the previous expression describes the adjustment of capital around the trend. On the other hand, damping fluctuations imply smaller and smaller adjustments which makes this term smaller and smaller over time. Then our model accounts for the excess sensitivity puzzle provided we are close enough to the balanced growth path.

4 Model calibration and consumption smoothing

In this section, we argue that the region of the parameter space where the non-smoothing results apply is the empirically relevant one if the economy is calibrated to match the long run positive trend in the U.S. output growth as well as the absence of growth over time of wellbeing (or utility) as empirically found by Blanchflower and Oswald [12], and Di Tella et al. [24] among others. In these contributions, the authors test the Easterlin's hypothesis, according to which growth does not raise wellbeing, and confirm it in several developed countries, among them U.S.

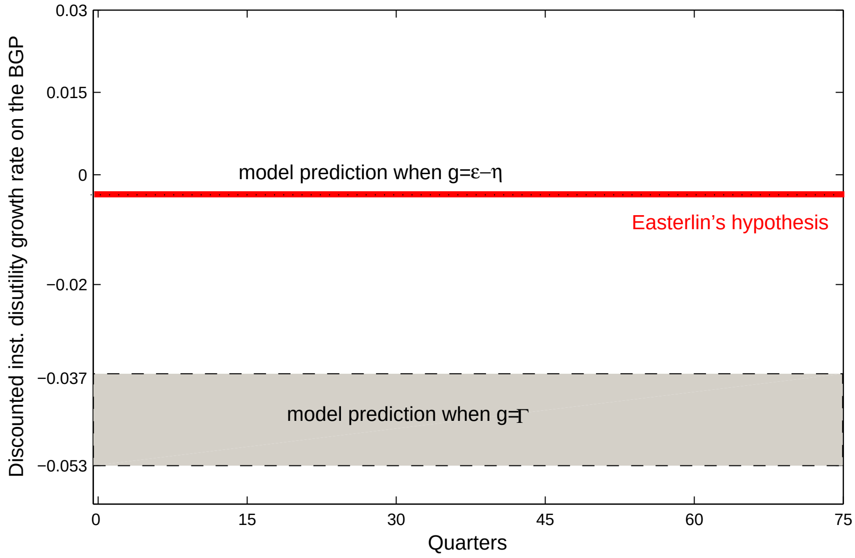

Figure 5: Model predictions and the Easterlin's hypothesis

and U.K., as well as over different time intervals. Moreover Blanchflower and Oswald observe that *comparisons with others* is one of the four key factors influencing their measure of utility and then their findings. For this reason, our model seems well suited to be calibrated to match also the observations on the U.S. long-run utility's growth. The calibration exercise is performed first when the habits are characterized by an infinite lag structure and then we move to study the case with a finite lag structure.

Infinite Lag Structure ($\tau \to +\infty$). We proceed as follows: first we show that we can calibrate the model along the balanced growth path where $g = \varepsilon - \eta$ to match the average U.S. annual output growth of 2% per year, and the absence of *utility* growth as observed in the data. Secondly that calibrations, usually suggested in the literature, along the balanced growth path with $g = \Gamma$, lead always to a violation of the Easterlin's hypothesis. The discrepancy between the predicted and actual utility growth rates is also computed.

In the case where $g = \varepsilon - \eta$, the main aggregate variables evolve along the balanced growth path as follows:

$$h(t) = h_0 e^{(\varepsilon-\eta)t}, \qquad c(t) = h(t), \qquad k(t) = \frac{c(t)}{A - \delta - \varepsilon + \eta}$$

then the parameters ε and η can be easily calibrated to match the balanced growth path target of $g = 0.02$, and at the same time the Easterlin's hypothesis since on the balanced growth path $c(t) = h(t)$ implies a zero utility growth rate. Then the model is consistent with both the empirical evidences on the positive trend of output as well as the absence of growth in utility. These results are reported in Figure 5. Observe also that this finding is robust to different choices of the habits' lag parameter, τ, provided its value is sufficiently large. In fact a too small value of τ may lead to either a negative growth rate of the economy or a growth rate equal to $g = \Gamma$ (see Proposition 4 and 6 and later in this section).

Let us now move to the case with $g = \Gamma$; in this case the main aggregate variables evolve along the balanced growth path as follows:

$$h(t) = h_0 e^{\Gamma t}, \qquad c(t) = \frac{(\Gamma + \eta)h(t)}{\varepsilon}, \qquad k(t) = \frac{c(t)}{A - \delta - \Gamma}$$

On the other hand the path of the instantaneous discounted utility, $\tilde{u}(t) = u(t)e^{-\rho t}$, is

$$\tilde{u}(t) = \frac{[(\Gamma + \eta - \varepsilon)h_0]^{1-\gamma}}{\varepsilon^{1-\gamma}(1-\gamma)} e^{(g-r)t} \tag{38}$$

The two main parameters to be calibrated are γ and ρ. We set $\rho = 0.017$ and we propose different values for γ. To explain the equity premium puzzle, γ was always chosen greater than 1; for example Constantinides' range is between 1.1 and 2.8 while Campbell and Cochrane [15] set it to 2. A value of γ equals to 1.1, 2, and 2.8 implies an annual real interest rate on the balance growth path of 3.9%, 5.7% and 7.3% respectively. Moreover the discounted utility (38) is always *negative* as well as the argument at the exponent. This means that over time the wellbeing *increases* since the discounted *dis*utility $(-\tilde{u})$ grows at the constant and negative rate of

$$g_{-\tilde{u}} = g - r_{\gamma=1.1} = -0.019 \qquad g_{-\tilde{u}} = g - r_{\gamma=2} = -0.037 \qquad g_{-\tilde{u}} = g - r_{\gamma=2.8} = -0.053$$

Then the Easterlin's hypothesis is always violated and the discrepancy with the actual utility growth rate quantitatively relevant. This result is found robust to different choices of ρ.[15] It is also worth noting that calibrating the economy along this balanced growth path implies a wellbeing growth rate generally higher than the output's growth rate.

Finite Lag Structure. We move now to the case with a finite lag structure and our first objective is again to show that we may calibrate the model along the balanced growth path where now $g = \alpha^*$ to match the average U.S. annual output growth of 2% per year, and the absence of *utility* growth as observed in the data, while the calibrations, usually suggested in the literature, along the balanced growth path with $g = \Gamma$, lead always to a violation of the Easterlin's hypothesis. The second objective is to discuss the raising and nature of endogenous fluctuations.

In the case where $g = \alpha^*$, the main aggregate variables evolves along the balanced growth path as follows:

$$h(t) = \hat{\sigma}_0 e^{\alpha^* t} \qquad c(t) = h(t) \qquad k(t) = \frac{c(t)}{A - \delta - \alpha^*}$$

To calibrate the parameters ε and η in order to match the balanced growth path target of $g = 0.02$ we need first to set a value for τ and then use equation 21 to find the values of ε and η, if any, which solve it and respect also the other two conditions in Lemma 1. According to Crawford's test on microeconomic panel data (see Crawford [23], page 15), the choice of $\tau = 2$ improves the agreement between theory and data with respect to the one lag case without reducing too much the power of the test. Then we set $\tau = 2$ and we find numerically that $\varepsilon = 0.8129$ and $\eta = 0.5129$ solve equation 21 and respect the two conditions in Lemma 1.

[15] Introducing an asset with a random return does not alter this conclusion as it appears clear in Constantinides [21], equation (15).

This calibration matches both the average growth rate of the U.S. economy and the Easterlin's hypothesis.

Regarding the second mechanism which may emerge when τ is finite, we may set, as previously done, $\rho = 0.017$ and then calibrate the parameter γ to match the average U.S. real interest rate of around 4% per year; this implies a positive value of Γ. Then the economy is always in the region of the parameter space (see Figure 4, graph on the right) where small deviations from the balanced growth path imply transitional dynamics without oscillations.

On the other hand if we calibrate the model along the balanced growth path where $g = \Gamma$ the same conclusions as in the case of an infinite lags structure can be easily found to hold. Finally we can observe back in Figure 4, graph on the right, that the region under the blue line in the parameter space (η, ε) can be also characterized by damping fluctuations around the balanced growth path. In particular, we may use Proposition 7 to find the values of the habit lag parameter, τ, consistent with it. To do so we follow Constantinides [21] in assigning a value in the range $(0.093, 0.492)$ to the habit intensity parameter, ε. Any choice of τ lower than 9.73 when $\varepsilon = 0.093$ or lower than 1.99 when $\varepsilon = 0.492$ leads to damping fluctuations around the balanced growth path. Interestingly enough, the second mechanism responsible of a reduced desire in consumption smoothing would have always emerged if τ was set to 1 as often done in the literature.

5 Discussion on modeling choices

In this last section, we discuss how the two unveiled mechanisms may depend on the modeling choices. In this last section, we will not provide formal proofs.

Internal vs External habits: It can be proved that the internal and external addictive habits are characterized by the same policy function

$$\frac{c(t) - h(t)}{A - \delta - \Gamma} = \left[\frac{A - \delta + \eta - \varepsilon}{A - \delta + \eta}\right]\left[k(t) - \frac{h(t)}{A - \delta + \eta - \varepsilon}\right]$$

when the lag structure is infinite ($\tau = \infty$).[16] Then the first mechanism holds also in the case with internal addictive habits; this observation is indeed quite relevant because it underlines how the solution of the equity premium puzzle suggested by Constantinides [21] depends crucially on his "judicious" choices of the numerical values of the parameters. On the other hand, we have just shown that these choices lead always to a violation of the Easterlin's hypothesis and that any calibration done to match this target implies a consumption path less smooth than in a model without habits. It is also worth noting that this equilibrium path satisfies the condition on the finiteness of the value function in the case of internal habits. The comparison between the internal and external case with finite τ is left for future research.

Subtractive vs Multiplicative nonseparable utility function: The first mechanism, which we explained to reduce the willingness of smoothing consumption over time, strictly depends, in our framework, on the addiction of the habits which may pins down an asymptotic growth rate in the economy higher than Γ. Does a similar mechanism still exist if we move to an utility

[16] In a context with conspicuous consumption, Arrow and Dasgupta [6] have shown that the internal and external case may lead to the same policy function.

function having multiplicative nonseparable form? In this case the utility function writes

$$u(c(t), h(t)) = \frac{\left(\frac{c(t)}{h(t)^{\sigma}}\right)^{1-\gamma}}{1-\gamma}$$

and its concavity is guaranteed by the condition $\gamma \geq \frac{1}{1-\sigma}$ with $\sigma \geq 0$ while no constraint on addictive behavior is necessary. Under this specification, the asymptotic growth rate of an economy with linear technology is

$$g = \frac{A - \delta - \rho}{\gamma + \sigma(1-\gamma)}$$

which is greater than Γ as soon as $\sigma < 1$. If this is the case, Carroll et al. [18]) observe that *"an economy that starts off in the lucky state of having a low reference stock relative to its capital stock will spread out its good fortune by saving a lot and growing quickly, thus lengthening the time during which the level of consumption will be high relative to the reference stock."* This implies that the agents have an incentive to decide an initial level of consumption quite low to pin down this high saving. If the initial level of consumption is lower than in the standard AK model then a similar mechanism as the one described in this paper could emerge.[17]

Moreover, a finite lag structure in the habits triggers endogenous fluctuations also in the case of an utility function having multiplicative form since the habits equation takes the form of a delay differential equation exactly as happened in our framework. Moreover the risk adverse agent will try to smooth the ratio $\frac{c(t)}{h(t)^{\sigma}}$ and then they will decide an oscillatory consumption dynamics to offset the fluctuations in their habits.

Technology: The first mechanism emerges when the difference between intensity and persistence of the habits is positive. In fact, this condition together with the habits' addiction trigger a positive growth rate in the habits and in consumption (see Section 2.1). Then the assumption on linear technology is only relevant to make this growth rate sustainable. Alternative endogenous growth models would have been used to show the same results but with a probably higher difficulty in the analytical derivations. The second mechanism is even more robust to alternative choices in the production function since it does not even require a positive growth rate of the economy.

6 Conclusion

In this paper, we have shown how different selections of the key parameters describing the formation of the habits may induce different dynamic behaviors in the aggregate variables. A taxonomy of this rich dynamics has allowed to identify the regions in the parameters space where the habits *reduce the desire to smooth consumption over time.* This region has been shown to be the empirically relevant one when the economy was calibrated to match the average U.S. output growth and the Easterlin's hypothesis. Then our findings allow a full understanding of the relation between habits and consumption which is more complex than the one usually stressed by the literature where the habits were always used to increase the desire of the agents to smooth their consumption over time. Finally a quantitative analysis of the magnitude of the

[17] Again this is just an intuition in favor of the possible raising of our mechanism in this different set-up while a formal proof of this point is left for future research.

endogenous fluctuations induced by the second mechanism is not necessary when the economy is calibrated to match the previously mentioned two targets, since we have proved analytically that in that case these fluctuations are small and smoothed out quickly. On the other hand a quantitative analysis of the magnitude of these fluctuations for a larger set of parameters is left for future research.

Appendix: Proofs

Proof of Proposition 1. Solving the system (5), (6), and (7) leads to:

$$h(t) = \frac{\varepsilon\varphi_0}{\Gamma-\varepsilon+\eta}e^{\Gamma t} + \left(h_0 + \frac{\varepsilon\varphi_0}{\varepsilon-\eta-\Gamma}\right)e^{(\varepsilon-\eta)t} \quad (39)$$

$$c(t) = e^{(\varepsilon-\eta)t}\left(h_0 + \frac{\varepsilon\varphi_0}{\varepsilon-\eta-\Gamma}\right) + \varphi_0 e^{\Gamma t}\left(\frac{\eta+\Gamma}{\Gamma-\varepsilon+\eta}\right) \quad (40)$$

$$k(t) = \left[k_0 - \frac{h_0}{A-\delta-\varepsilon+\eta} - \frac{(A-\delta+\eta)\varphi_0}{(A-\delta-\varepsilon+\eta)(A-\delta-\Gamma)}\right]e^{(A-\delta)t} \quad (41)$$

$$+\frac{1}{A-\delta-\varepsilon+\eta}\left(h_0 + \frac{\varepsilon\varphi_0}{\varepsilon-\eta-\Gamma}\right)e^{(\varepsilon-\eta)t} - \frac{(\eta+\Gamma)\varphi_0}{(\varepsilon-\eta-\Gamma)(A-\delta-\Gamma)}e^{\Gamma t} \quad (42)$$

The value of φ_0 which satisfies the transversality condition can be easily computed by looking at equation (42):

$$\tilde{\varphi}_0 = \frac{(\varepsilon-\eta-A+\delta)(\Gamma-A+\delta)}{(A-\delta+\eta)}\left(k_0 + h_0\frac{1}{\varepsilon-\eta-A+\delta}\right) \quad (43)$$

and then consumption and capital grow asymptotically at the same rate $g = \max(\varepsilon-\eta, \Gamma)$. Moreover dividing both sides of the capital accumulation equation by $k(t)$, and taking the limit $t \to \infty$, it follows that a positive consumption over capital ratio in the long run is guaranteed when

$$\lim_{t\to\infty}\frac{c(t)}{k(t)} = A - \delta - \max(\varepsilon-\eta, \Gamma) > 0.$$

We have now to verify the conditions for an interior solution. Let us begin with $h(t) > 0$; taking into account the definition of $h(t)$ and relation (3) we have that

$$h(t) = \varepsilon\int_{-\infty}^{t} h(u)e^{\eta(u-t)}du + \varepsilon\tilde{\varphi}_0 e^{\Gamma t}\int_{-\infty}^{t} e^{(\Gamma+\eta)u}du$$

The last term diverges to minus infinity when $\Gamma + \eta < 0$. Then the inequality $\Gamma > -\eta$ is necessary to guarantee an interior solution.

The second condition to check is $c(t) > h(t)$; according to equation (3), this condition is satisfied when $\tilde{\varphi}_0 > 0$, which happens if and only if $k_0 > h_0\frac{1}{A-\delta-\varepsilon+\eta}$.

To check the last condition for an interior solution, namely $k(t) > 0$ for any t, is useful to observe that The sign of the competitive equilibrium path of $k(t)$ is indeed the same as the sign of $\hat{k}(t) = k(t)e^{-(\varepsilon-\eta)t}$ and since $\hat{k}(0) = k_0 > 0$ and $\frac{d\hat{k}(t)}{dt} = \tilde{\varphi}_0\frac{\eta+\Gamma}{A-\delta-\Gamma} > 0$ then $\hat{k}(t)$ is always positive which implies $k(t)$ positive. These considerations are also sufficient to prove the positiveness of the consumption over capital ratio during the transition. ■

Lemma A *The algebraic equation (17) can be rewritten as equation (19) once the new variable $z(t)$, as defined in (20), is introduced.*

Proof. Let $z(t) = h(t) - e^{\Gamma t}\varphi_0\theta$. According to equation (17), $h(t) = \varepsilon\int_{t-\tau}^{t}\varphi_0 e^{\Gamma u}e^{\eta(u-t)}du + \varepsilon\int_{t-\tau}^{t} h(u)e^{\eta(u-t)}du$, and then we have that

$$h(t) = \varepsilon\int_{t-\tau}^{t}\varphi_0 e^{\Gamma u}e^{\eta(u-t)}du + \varepsilon\int_{t-\tau}^{t}\left(z(u) + e^{\Gamma u}\varphi_0\theta\right)e^{\eta(u-t)}du$$

Choosing θ such that $\varepsilon e^{\Gamma t}\int_{t-\tau}^{t}\varphi_0 e^{(\eta+\Gamma)(u-t)}du + \varepsilon e^{\Gamma t}\int_{t-\tau} t\varphi_0\theta e^{(\eta+\Gamma)(u-t)}du = e^{\Gamma t}\varphi_0\theta$ leads to the result. ■

Proof of Lemma 1. Since $\Delta'(\lambda) = \varepsilon\int_{-\tau}^{0} ue^{(\lambda+\eta)u}du < 0$, $\lim_{\lambda\to-\infty}\Delta(\lambda) = \infty$, and $\lim_{\lambda\to\infty}\Delta(\lambda) = -1$ then a unique real root, α^*, always exists, and its sign is given by the sign of $\Delta(0) = -1 + \frac{\varepsilon}{\eta}(1 - e^{-\eta\tau})$.

Then after some algebra, it can be seen that $\alpha^* > 0$ if and only if $\varepsilon > \eta$ and $\tau > \tau^*$ with $\tau^* = -\frac{\log\left(1-\frac{\eta}{\varepsilon}\right)}{\eta}$. We now prove that the real root is the leading root, namely $\alpha^* > p$ with $\lambda = p + iq$ any complex root of $\Delta(\lambda) = 0$.

$$|\Delta(p+iq)| > \left|\varepsilon \int_{-\tau}^{0} e^{(\lambda+\eta)u} du - 1\right|$$

If $p > \alpha^*$, then $\Delta(\lambda) < 0$ which implies $\left|-1 + \varepsilon \int_{-\tau}^{0} e^{(\lambda+\eta)u} du\right| = 1 - \varepsilon \int_{-\tau}^{0} e^{(\lambda+\eta)u} du$ and then

$$|\Delta(p+iq)| > 1 - \varepsilon \int_{-\tau}^{0} e^{(p+\eta)u} du > 0$$

Finally the real root, α^*, solves $\varepsilon \int_{-\tau}^{0} e^{(\alpha^*+\eta)u} du = 1$. Using implicit function theorem, we have

$$\frac{d\alpha_*}{d\tau} = -\frac{e^{-(\alpha^*+\eta)\tau}}{\int_{-\tau}^{0} u e^{(\alpha^*+\eta)u} du} > 0$$

When τ tends to infinity, the real root solves $\varepsilon \int_{-\infty}^{0} e^{(\alpha^*+\eta)u} du = 1$. This implies that $\varepsilon = \eta + \alpha^*$. It is also worth noting that as α^* is defined by $-1 + \varepsilon \int_{-\tau}^{0} e^{(\alpha^*+\eta)u} du = 0$, then from the implicit functions theorems

$$\frac{d\alpha^*}{d\eta} = -1 \text{ and } \frac{d\alpha^*}{d\varepsilon} = \frac{-\int_{-\tau}^{0} e^{(\alpha^*+\eta)u} du}{\varepsilon \int_{-\tau}^{0} u e^{(\alpha^*+\eta)u} du}$$

■

Proof of Proposition 2. As $h(t) = z(t) + e^{\Gamma t}\varphi_0\theta$, and $c(t) = e^{\Gamma t}\varphi_0 + h(t)$ it follows immediately from assumption $\Gamma < 0$, that $\lim_{t\to\infty} \frac{\dot{z}}{z} = \lim_{t\to\infty} \frac{\dot{h}}{h} = \lim_{t\to\infty} \frac{\dot{c}}{c}$. In order to determine the growth rate and the solution of $z(t)$, we apply the Laplace transform $\mathtt{L}(\lambda) = \int_0^\infty e^{-\lambda t} z(t) dt$, on equation $z(t) = \varepsilon \int_{t-\tau}^{t} z(u) e^{\eta(u-t)} du$. Thus

$$\begin{aligned} \mathtt{L}(\lambda) &= \varepsilon \int_0^\infty e^{-\lambda t} \int_{-\tau}^{0} z(u+t) e^{\eta u} du dt \\ &= \varepsilon \int_{-\tau}^{0} e^{\eta u} \int_0^\infty e^{-\lambda t} z(u+t) dt du \\ &= \varepsilon \int_{-\tau}^{0} e^{(\eta+\lambda)u} \left(\int_u^0 e^{-\lambda t} z(t) dt + \mathtt{L}(\lambda) \right) du \end{aligned}$$

So $\mathtt{L}(\lambda)\left(1 - \varepsilon \int_{-\tau}^{0} e^{(\eta+\lambda)u} du\right) = \varepsilon \int_{-\tau}^{0} e^{(\eta+\lambda)u} \left(\int_u^0 e^{-\lambda t} z(t) dt\right) du$. Using inverse Laplace transform, and the result in Lemma 1, the solution admits for representation, for $t > 0$

$$z(t) = z_0 e^{\alpha^* t} + \chi_1(t) \tag{44}$$

where $\chi_1(t) = \frac{1}{2i\pi} \int_{L(\gamma)} e^{\lambda t} F(\lambda) d\lambda$ with $\gamma > \sup\{Re(\lambda) < \alpha^*, \det(\Delta(\lambda)) = 0\}$, $L(\gamma) = \{\lambda \in \mathbb{C}, Re(\lambda) = \gamma\}$ and with $F(\lambda) = -\varepsilon \Delta(\lambda)^{-1} \left(\int_{-\tau}^{0} e^{(\eta+\lambda)u} \left(\int_u^0 \left(h(\mu) - e^{\Gamma\mu}\varphi_0\theta\right) e^{-\lambda\mu} d\mu\right) du\right)$. Finally, z_0 can be obtained by using the Residue Formula $res_{\lambda=\alpha^*} e^{\lambda t} F(\lambda) = z_0 e^{\alpha^* t}$,

$$z_0 = -\frac{\varepsilon}{\Delta'(\alpha^*)} \left(\int_{-\tau}^{0} e^{(\eta+\alpha^*)u} \left(\int_u^0 \left(h(\mu) - e^{\Gamma\mu}\varphi_0\theta\right) e^{-\alpha^*\mu} d\mu \right) du \right) \tag{45}$$

Now since the arguments of the integrals are continuous functions we may use Fubini's theorem to change the order of integration, and rewriting z_0 as follows:

$$\begin{aligned} z_0 &= -\frac{\varepsilon}{\Delta'(\alpha^*)} \left[\int_{-\tau}^{0} h(\mu) e^{-\alpha^*\mu} \left(\int_{-\tau}^{\mu} e^{(\eta+\alpha^*)u} du \right) d\mu - \varphi_0 \left(\int_{-\tau}^{0} e^{(\eta+\alpha^*)u} \int_u^0 e^{(\Gamma-\alpha^*)\mu} d\mu du \right) \right] && (46) \\ &= -\frac{\varepsilon}{\Delta'(\alpha^*)} \left[\begin{array}{c} \frac{1}{\eta+\alpha^*} \int_{-\tau}^{0} h(\mu) e^{\eta\mu} d\mu - \frac{e^{-(\eta+\alpha^*)\tau}}{\eta+\alpha^*} \int_{-\tau}^{0} h(\mu) e^{-\alpha^*\mu} d\mu \\ -\frac{\varphi_0\theta}{(\Gamma-\alpha^*)} \left(\frac{1-e^{-(\eta+\alpha^*)\tau}}{(\eta+\alpha^*)} - \frac{1-e^{-(\Gamma+\eta)\tau}}{(\Gamma+\eta)} \right) \end{array} \right] = z_0(h_0(t), \varphi_0) && (47) \end{aligned}$$

As $c(t) = \varphi_0 e^{\Gamma t}(1+\theta) + z(t)$ and as $z(t)$ is defined with equation (44) with $\lim_{t\to\infty}\chi_1(t)e^{-\alpha^* t} = 0$, then we have the stated result. ■

Proof of Proposition 3. By applying the Laplace transform $\mathtt{L}(\lambda) = \int_0^\infty e^{-\lambda t} x(t)\, dt$ on $x(t) = \varepsilon \int_{t-\tau}^t x(u)\, e^{(\Gamma+\eta)(u-t)} du$, we have:

$$\mathtt{L}(\lambda) = \varepsilon \int_{-\tau}^0 e^{(\Gamma+\eta)u} \int_0^\infty x(u+t)\, e^{-\lambda t} dt du = \varepsilon \int_{-\tau}^0 e^{(\Gamma+\eta+\lambda)u} \left(\int_u^0 x(t)\, e^{-\lambda t} dt + \mathtt{L}(\lambda) \right) du$$

and then

$$\mathtt{L}(\lambda) = -\varepsilon \Delta(\lambda)^{-1} \int_{-\tau}^0 e^{(\Gamma+\eta+\lambda)u} \left(\int_u^0 x(t)\, e^{-\lambda t} dt \right) du$$

By inverting the Laplace transform, the solution of $x(t)$ for $t > 0$ is

$$x(t) = \sigma_0 e^{\alpha_* t} + \frac{-1}{2i\pi} \int_{L(\gamma')} e^{\lambda t} \Delta(\lambda)^{-1} \left(\varepsilon \int_{-\tau}^0 e^{(\Gamma+\eta+\lambda)u} \left(\int_u^0 x(\mu)\, e^{-\lambda\mu} d\mu \right) du \right) d\lambda$$

where $\gamma' > \sup\{Re(\lambda) < \alpha_*, \det(\Delta(\lambda)) = 0\}$ and $L(\gamma) = \{\lambda \in \mathbb{C}, Re(\lambda) = \gamma\}$ where α_* is the only real root of $\Delta(\lambda) = 0$

Letting $F(\lambda) = -\varepsilon\Delta(\lambda)^{-1} \left(\int_{-\tau}^0 e^{(\Gamma+\eta+\lambda)u} \left(\int_u^0 \left[h(\mu)\, e^{-\Gamma\mu} - \theta\varphi(0) \right] e^{-\lambda\mu} d\mu \right) du \right)$, projection formula enables to compute σ_0, as $res_{\lambda=\alpha_*} e^{\lambda t} F(\lambda) = \sigma_0 e^{\alpha_* t}$, that is

$$\sigma_0 = -\frac{\varepsilon}{\Delta'(\alpha_*)} \left(\int_{-\tau}^0 e^{(\Gamma+\eta+\alpha_*)u} \left(\int_u^0 \left[h(\mu)\, e^{-\Gamma\mu} - \theta\varphi(0) \right] e^{-\alpha_*\mu} d\mu \right) du \right) \tag{48}$$

Using Fubini's theorem we may rewrite σ_0 as follows:

$$\begin{aligned}
\sigma_0 &= -\frac{\varepsilon}{\Delta'(\alpha_*)} \left[\int_{-\tau}^0 h(\mu)\, e^{-(\Gamma+\alpha_*)\mu} \left(\int_{-\tau}^\mu e^{(\Gamma+\eta+\alpha_*)u} du \right) d\mu - \theta\varphi_0 \int_{-\tau}^0 e^{(\Gamma+\eta+\alpha_*)u} \left(\int_u^0 e^{-\alpha_*\mu} d\mu \right) du \right] \\
&= -\frac{\varepsilon}{\Delta'(\alpha_*)} \left[\begin{array}{c} \int_{-\tau}^0 h(\mu) \frac{e^{\eta\mu}}{\Gamma+\eta+\alpha_*} d\mu - \left(\frac{e^{-(\Gamma+\eta+\alpha_*)\tau}}{\Gamma+\eta+\alpha_*} \right) \int_{-\tau}^0 h(\mu)\, e^{-(\Gamma+\alpha_*)\mu} d\mu \\ -\theta\varphi_0 \left[\frac{1-e^{-\tau(\Gamma+\eta)}}{\alpha_*(\Gamma+\eta)} - \frac{1-e^{-\tau(\Gamma+\eta+\alpha_*)}}{\alpha_*(\Gamma+\eta+\alpha_*)} \right] \end{array} \right]
\end{aligned}$$

Finally, given that

$$h(t) = \varphi_0\theta e^{\Gamma t} + \sigma_0 e^{(\Gamma+\alpha_*)t} + \chi_2(t) e^{\Gamma t}$$

with $\chi_2(t) = \frac{1}{2i\pi}\int_{L(\gamma')} e^{\lambda t} F(\lambda)\, d\lambda$, and taking into account that $c(t) = \varphi_0 e^{\Gamma t} + h(t)$, then equation (30) follows immediately. The end of the proof is based on the fact that χ_2 has an asymptotic growth rate lower than α_*. ■

Proof of Proposition 4. By substituting the equation of consumption (23) found in proposition (2), into the capital accumulation equation and integrating it, we arrive to

$$k(t) = k_0 e^{(A-\delta)t} - \int_0^t e^{-(A-\delta)(u-t)} \left(\varphi(0)(1+\theta)\, e^{\Gamma u} + z_0 e^{\alpha^* u} + \chi_1(u) \right) du$$

Given the transversality condition $\lim_{t\to\infty} k(t)\, e^{-(A-\delta)t} = 0$, it must be that

$$k_0 = \varphi_0(1+\theta) \frac{1}{(A-\delta-\Gamma)} + \frac{z_0}{A-\delta-\alpha^*} + \int_0^\infty e^{-(A-\delta)u} \chi_1(u) du \tag{49}$$

where z_0 and $\chi_1(u)$ have been defined in proposition (2) as

$$z_0 = -\frac{\varepsilon}{\Delta'(\alpha^*)} \left(\int_{-\tau}^0 e^{(\eta+\alpha^*)u} \left(\int_u^0 \left(h(\mu) - e^{\Gamma\mu}\varphi_0\theta \right) e^{-\alpha^*\mu} d\mu \right) du \right)$$

and

$$\chi_1(t) = \frac{1}{2i\pi}\int_{L(\gamma)} e^{\lambda t}\left(-\varepsilon\Delta(\lambda)^{-1}\left(\int_{-\tau}^{0} e^{(\eta+\lambda)u}\left(\int_{u}^{0}\left(h(\mu) - e^{\Gamma\mu}\varphi_0\theta\right)e^{-\lambda\mu}d\mu\right)du\right)\right)d\lambda$$

with $\gamma > \sup\{Re(\lambda) < \alpha^*, \det(\Delta(\lambda)) = 0\}$, $L(\gamma) = \{\lambda \in \mathbb{C}, Re(\lambda) = \gamma\}$. Since $A - \delta > \alpha^*$, otherwise in the long run the positive sign of the consumption-output ratio is not guaranteed, then $\int_0^\infty e^{-(A-\delta)u}\chi_1(u)du$ exists.

Equation (49) leads to a linear equation in φ_0, or equivalently in c_0 as $c_0 = \varphi_0 + h_0$. After some calculus we may find the value of $\varphi_0 = \hat{\varphi}_0$ which make the transversality condition hold

$$\hat{\varphi}_0 = \frac{\left(\begin{array}{c} k_0 + \frac{\frac{\varepsilon}{\Delta'(\alpha^*)}\int_{-\tau}^{0} e^{(\eta+\alpha^*)u}\left(\int_u^0 h(\mu)e^{-\alpha^*\mu}d\mu\right)du}{A-\delta-\alpha^*} \\ +\int_0^\infty e^{-(A-\delta)v}\frac{1}{2i\pi}\int_{L(\gamma)} e^{\lambda v}\varepsilon\Delta(\lambda)^{-1}\int_{-\tau}^{0} e^{(\eta+\lambda)u}\int_u^0 h(\mu)e^{-\lambda\mu}d\mu du d\lambda dv \end{array}\right)}{\left[\begin{array}{c} (1+\theta)\frac{1}{(A-\delta-\Gamma)} + \frac{\frac{\varepsilon}{\Delta'(\alpha^*)}\left(\int_{-\tau}^{0} e^{(\eta+\alpha^*)u}\left(\int_u^0 e^{\Gamma\mu}\theta e^{-\alpha^*\mu}d\mu\right)du\right)}{A-\delta-\alpha^*} \\ +\int_0^\infty e^{-(A-\delta)v}\frac{1}{2i\pi}\int_{L(\gamma)} e^{\lambda v}\varepsilon\Delta(\lambda)^{-1}\left(\int_{-\tau}^{0} e^{(\eta+\lambda)u}\left(\int_u^0 e^{\Gamma\mu}\theta e^{-\lambda\mu}d\mu\right)du\right)d\lambda dv \end{array}\right]} \tag{50}$$

Observe that $\hat{\varphi}_0$ depends only on the initial conditions, k_0 and $h_0(t)$, namely $\hat{\varphi}_0 = \hat{\varphi}_0(k_0, h_0(t))$. Then the nontrivial interior solution of the problem, (31), (23) and (33), can be easily derived. ■

Proof of Proposition 5. To prove no damping fluctuations around the balanced growth path, we need to prove that

$$\Gamma \geq \sup\{Re(\lambda), \Delta(\lambda) = 0\} \setminus \{\alpha^*\}$$

In fact, we will prove that $-\eta > \sup\{Re(\lambda), \Delta(\lambda) = 0\} \setminus \{\alpha^*\}$. We first consider

$$\dot{z}(t) = \varepsilon z(t) - \varepsilon z(t-\tau)e^{-\eta\tau} - \eta z(t) \tag{51}$$

obtained by differentiating

$$z(t) = \varepsilon\int_{t-\tau}^{t} z(u)e^{\eta(u-t)}du$$

Let $\widetilde{\Delta}(\lambda) = 0$ be the characteristic equation of (51). $\widetilde{\Delta}(\lambda) = 0$ has two real roots α^* and $-\eta$, and $\widetilde{\Delta}(\lambda) > 0$ for $\lambda \in]-\eta, \alpha^*[$. We now prove that there are no complex roots in the strip $]-\eta, \alpha^*[$. Let us consider $\lambda = p + iq$

$$\begin{aligned} Re\widetilde{\Delta}(p+iq) &= -p + \varepsilon - \varepsilon e^{-(p+\eta)\tau}\cos(q\tau) - \eta \\ &\geq Re\widetilde{\Delta}(p) > 0 \end{aligned}$$

Thus all complex roots of Δ have real part smaller that $-\eta$, and thus smaller than $-\Gamma$. ■

Proof of Proposition 6. Let's call $\kappa(t) = k(t)e^{-(\Gamma+\alpha_*)t}$, then from the capital accumulation equation we have:

$$\dot{\kappa}(t) = \kappa(A - \delta - \Gamma - \alpha_*) - c(t)e^{-(\Gamma+\alpha_*)t}$$

Taking into account the equation of consumption (30) in Proposition 3 and integrating, we obtain

$$\kappa(t) = k_0 e^{(A-\delta-\Gamma-\alpha_*)t} - \int_0^t \left[\varphi_0(1+\theta) + \sigma_0 e^{\alpha_* u} + \chi_2(u)\right]e^{-\alpha_* u}e^{-(A-\delta-\Gamma-\alpha_*)(u-t)}du$$

where σ_0 and $\chi_2(t)$ were defined previously. Observe that the condition for an asymptotic positive consumption over output ratio, $A - \delta - \Gamma - \alpha_* > 0$, guarantees the existence of $\lim_{t\to\infty}\int_0^\infty \chi_2(u)e^{-(A-\delta)u}du$ since $\lim_{t\to\infty}\chi_2(t)e^{-(\Gamma+\alpha_*)t} = 0$. Applying the transversality condition leads to

$$k_0 = \varphi_0\frac{(1+\theta)}{A-\delta-\Gamma} + \frac{\sigma_0}{A-\delta-\Gamma-\alpha_*} + \int_0^\infty \chi_2(u)e^{-(A-\delta-\Gamma)u}du$$

As previously, the linearity in φ_0 enables us to write explicitly the unique $\hat{\varphi}_0 = \hat{\varphi}_0(k_0, h_0(t))$ that pinned down the unique asymptotic growth rate of the economy, $\Gamma + \alpha_*$:

$$\hat{\varphi}_0 = \frac{\left[\begin{array}{c} k_0 + \frac{\varepsilon(\Delta'(\alpha_*))^{-1}\left[\int_{-\tau}^{0} h(\mu)\frac{e^{\eta\mu}}{\Gamma+\eta+\alpha_*}d\mu - \left(\frac{e^{-(\Gamma+\eta+\alpha_*)\tau}}{\Gamma+\eta+\alpha_*}\right)\int_{-\tau}^{0} h(\mu)e^{-(\Gamma+\alpha_*)\mu}d\mu\right]}{A-\delta-\Gamma-\alpha_*} \\ +\int_0^\infty \frac{1}{2i\pi}\int_{L(\gamma')} e^{\lambda u}\left(\varepsilon\Delta(\lambda)^{-1}\left(\int_{-\tau}^{0} e^{(\Gamma+\eta+\lambda)v}\left(\int_v^0 h(\mu)\, e^{-(\Gamma+\lambda)\mu}d\mu\right)dv\right)\right)d\lambda e^{-(A-\delta-\Gamma)u}du \end{array}\right]}{\left[\frac{(1+\theta)}{A-\delta-\Gamma} + \frac{\varepsilon(\Delta'(\alpha_*))^{-1}\theta\left[\frac{1-e^{-\tau(\Gamma+\eta)}}{\alpha_*(\Gamma+\eta)} - \frac{1-e^{-\tau(\Gamma+\eta+\alpha_*)}}{\alpha_*(\Gamma+\eta+\alpha_*)}\right]}{A-\delta-\Gamma-\alpha_*} + \theta\int_0^\infty \frac{1}{2i\pi}\int_{L(\gamma')} e^{\lambda u}\left(\varepsilon\Delta(\lambda)^{-1}\int_{-\tau}^{0}\left(\frac{e^{(\Gamma+\eta)v}-e^{(\Gamma+\eta+\lambda)v}}{\lambda}\right)dv\right)d\lambda e^{-(A-\delta-\Gamma)u}du\right]}$$

Taking into account this result, the competitive equilibrium path of capital is:

$$k(t) = \hat{\varphi}_0(1+\theta)\frac{e^{\Gamma t}}{A-\delta-\Gamma} + \hat{\sigma}_0\frac{e^{(\Gamma+\alpha_*)t}}{A-\delta-\Gamma-\alpha_*} + \int_t^\infty \chi_2(u)e^{-(A-\delta-\Gamma)(u-t)}du \tag{52}$$

where $\hat{\sigma}_0 = \hat{\sigma}_0(h_0(t), \hat{\varphi}_0)$. Finally $\chi_2(t)$ is spanned by complex non real root of the characteristic equation $-1 + \varepsilon\int_{-\tau}^{0} e^{(\lambda+\eta+\Gamma)u}du = 0$. So for t not too large, this term is responsible for damped oscillations in aggregate variable. As this term goes to zero as τ tends to infinity, damped oscillations disapear for large τ. ■

Proof of Proposition 7. We know that roots $\widetilde{\lambda}$of $\widetilde{\Delta}(\lambda) = 0$ are such that $\widetilde{\lambda} = \lambda - \Gamma$, where λ is a root of $\Delta(\lambda) = 0$. According to what have been done previously, there are no roots complex root in the strip $]-\eta-\Gamma, \alpha_*[$. If $\alpha_* > 0$, as $\Gamma \in]-\eta-\Gamma, \Gamma+\alpha_*[$, there is always no damping fluctuations around the BGP. If $\alpha_* < 0$, there is no damping fluctuations around the BGP if $\alpha_* + \Gamma > -\eta - \Gamma$. A necessary condition to have damping fluctuations around the BGP is to have $\alpha_* + \Gamma + \eta < -\Gamma$. As

$$0 = -1 + \varepsilon\int_{-\tau}^{0} e^{(\alpha_*+\Gamma+\eta)u}du = -1 + \varepsilon\frac{1-e^{-\tau(\alpha_*+\Gamma+\eta)}}{\alpha_*+\Gamma+\eta}$$

we have

$$\varepsilon = \frac{\alpha_*+\Gamma+\eta}{1-e^{-\tau(\alpha_*+\Gamma+\eta)}}$$

As $f(x) = \frac{x}{1-e^{-\tau x}}$ is an increasing function of x, $\alpha_* + \Gamma + \eta > -\Gamma$, implies that $\frac{\alpha_*+\Gamma+\eta}{1-e^{-\tau(\alpha_*+\Gamma+\eta)}} > \frac{-\Gamma}{1-e^{\tau\Gamma}}$. Thus condition to have no damping fluctuations around the BGP rewrites

$$\varepsilon > \frac{-\Gamma}{1-e^{\tau\Gamma}}$$

■

Corner solutions. Let us begin with the case of low technology. In order to have a nontrivial interior solution for our problem the two inequality constraints $c(t) > h(t)$ and $k(t) > 0$ have to be satisfied. From Proposition 2 and Proposition 4, it emerges that if we specify an initial condition $(\tilde{h}_0(t), \tilde{k}_0)$ such that $\hat{\varphi}_0 = \hat{\varphi}_0(\tilde{h}_0(t), \tilde{k}_0) > 0$ then the first inequality constraint holds while $\hat{z}_0 = \hat{z}_0(\tilde{h}_0(t), \tilde{k}_0) > 0$ guarantees (at least for large t) positive capital.

On the other hand, initial conditions which implies $\hat{\varphi}_0 < 0$ leads to a corner solution characterized by $c(t) = h(t)$. In this case $\hat{\varphi}_0 = 0$, and then from Proposition 2, it follows that the solution of consumption has the form $c(t) = z_0 e^{\alpha^* t} + \chi_1(t)$ and then the capital solution is

$$k(t) = k_0 e^{(A-\delta)t} - \int_0^t e^{-(A-\delta)(u-t)}\left(\hat{z}_0 e^{\alpha^* u} + \chi_1(u)\right)du$$

with $z_0 = z_0(h_0(t), 0)$. Then the transversality condition $\lim_{t\to\infty} k(t)\, e^{-(A-\delta)t} = 0$, is only solved under the *non generic condition*:

$$k_0 = \frac{\hat{z}_0}{A - \delta - \alpha^*} + \int_0^\infty e^{-(A-\delta)u} \chi_1(u)\, du \tag{53}$$

which implies a general equilibrium path of capital

$$k(t) = \hat{z}_0 \frac{e^{\alpha^* t}}{A - \delta - \alpha^*} + \int_t^\infty e^{-(A-\delta)(u-t)} \chi_1(u)\, du \tag{54}$$

and an unique balanced growth path of the economy. For all the other choices of $(k_0, h_0(t))$ the economy will be characterized by different asymptotic growth rates of consumption and capital. In any case, the value of the instantaneous utility function at a corner solution is always zero.

Similar considerations can be done when high technology. Again a φ_0 negative implies a corner solution which will be characterized by a common growth rate of all the aggregate variables only if the following non generic condition is chosen:

$$k_0 = \frac{\hat{\sigma}_0}{A - \delta - \Gamma - \alpha_*} + M$$

where $\hat{\sigma}_0 = \sigma_0(\tilde{h}_0(t), 0; \Xi)$ and

$$M = \int_0^\infty \frac{1}{2i\pi} \int_{L(\gamma')} e^{\lambda u} \left[-\varepsilon \Delta(\lambda)^{-1} \left(\int_{-\tau}^0 e^{(\Gamma+\eta+\lambda)u} \left(\int_u^0 \tilde{h}(\mu)\, e^{-\Gamma\mu} e^{-\lambda\mu} d\mu \right) du \right) \right] d\lambda e^{-(A-\delta)u} du$$

The general equilibrium path of capital related to this initial condition is

$$k(t) = \hat{\sigma}_0 \frac{1}{A - \delta - \Gamma - \alpha_*} e^{(\Gamma+\alpha_*)t} + \int_t^\infty \chi_2(u)\, e^{-(A-\delta)(u-t)} du$$

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