THE UNIVERSITY of York

Discussion Papers in Economics

No. 12/03

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16 January 2012

Abstract

This paper addresses questions of the following nature: under what conditions does a welfare-improving reform of a nonlinear income tax system necessitate a change in a particular agent's marginal tax rate or total tax burden? Our analysis is therefore a study in tax reform, rather than in optimal taxation. We consider a simple model with three types of agents (high-skill, middle-skill, and low-skill) who have preferences that are quasi-linear in labour. Under these assumptions and using our methodology, specific characteristics of the initial suboptimal tax system can be determined when all welfare-improving tax reforms require specified changes in a particular agent's tax treatment. Some other necessary features of the tax reform can also be determined. Thus, unlike many tax reform analyses in the literature, we are able to reach a number of clear-cut conclusions.

Keywords: tax reform; nonlinear income taxation.

JEL Classifications: H21, H24.

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1 Introduction

The aim of the optimal taxation literature is to determine the features of an optimal tax system. However, there are some long-standing criticisms of this approach to normative tax theory. In particular, the optimal tax approach implicitly assumes that the government is free to choose all taxes, and that it is willing and able to implement the possibly large changes in taxes required to reach an optimum.[1] The characteristics of the status quo tax system are irrelevant under the optimal tax approach. In practice, however, the government must take the existing tax system as its starting point, and actual changes in taxes tend to be "slow and piecemeal" (Feldstein [1976]). Such observations motivate the tax reform approach, pioneered by Guesnerie [1977]. Tax reform analysis takes the existing tax system as given, and then examines the conditions under which there exist small (modelled as differential) changes in taxes that are feasible (equilibrium-preserving) and desirable (welfare-improving).[2] The tax reform approach therefore comes closer to capturing the actual behaviour of governments.

If one finds the preceding arguments reasonable, the question arises as to why the optimal tax approach continues to dominate the literature, while tax reform papers are few and far between. At first thought, one may think that the tax reform approach is in some sense redundant—once the characteristics of the optimal tax system have been determined, the government should simply change taxes toward their optimal levels. However, it has been known for some time that changes "in the right direction, but stop short of attaining the full optimum, can actually reduce welfare" (Dixit [1975]). Indeed, Guesnerie's [1977] temporary inefficiency result shows that an equilibrium-preserving and Pareto-improving policy reform may require a move from a production efficient allocation to a production inefficient allocation, even though production efficiency is desirable at an optimum (Diamond and Mirrlees [1971]). In our opinion, the reason that the tax reform approach remains relatively neglected is because it is generally difficult

[1] For example, two well-known results in the optimal tax literature are that capital should not be taxed and that the highest-skilled workers should face a zero marginal tax rate on their labour income. These recommendations stand in stark contrast to the features of real-world tax systems, and implementing them would involve a major shock to the economy.

[2] For an excellent textbook treatment of the tax reform approach, see chapter 6 in Myles [1995].

to obtain clear-cut results. For example, the main result of Guesnerie [1977, Proposition 4] on the existence of equilibrium-preserving and Pareto-improving policy reforms is very technical, relating the position of a vector representing the equilibrium conditions to a cone representing Pareto improvements.[3] Diewert [1978] and Weymark [1979] use different mathematical techniques to Guesnerie,[4] but their results also tend to be quite technical. For the most part, the results of Guesnerie, Diewert, and Weymark can be interpreted as providing empirically-testable formulae for the existence or otherwise of feasible and desirable tax reforms, rather than providing a simple description of optimal and suboptimal tax systems. Other tax reform analyses, such as those by Hatta [1977], Konishi [1995], Brett [1998], Murty and Russell [2005], Krause [2007], and Duclos, et al. [2008], also tend to yield technical results that do not have a straightforward economic interpretation.[5]

The aim of this paper is to undertake a tax reform analysis, but using a model and methodology that lead to clear-cut results. We use the nonlinear income tax model of Mirrlees [1971],[6] albeit with just three types of agents, and we assume that the utility function is quasi-linear in labour. We think the assumption that there are only three types of agents is not too restrictive, since real-world income tax systems tend to be designed broadly around how low-income, middle-income, and high-income individuals should be taxed. The assumption that preferences are quasi-linear is much more troubling, but quasi-linearity seems necessary to obtain detailed and clear results.[7] On the methodological side, we analyse tax reforms of a specific nature. That is, we examine

[3] See also chapter 3 in Guesnerie [1995].

[4] In particular, they use Motzkin's Theorem of the Alternative to analyse tax reforms, as we do in this paper.

[5] Tax reform techniques have also been used to revisit specific issues in optimal taxation, and in this case some clear conclusions can be reached. For example, Blackorby and Brett [2000] use tax reform techniques to examine the Diamond-Mirrlees production efficiency theorem. Fleurbaey [2006] takes a tax reform approach to examine the desirability of consumption taxation versus income taxation, while Krause [2009] undertakes a tax reform analysis of the Laffer argument.

[6] It should be noted that most of the tax reform literature examines linear commodity taxation rather than nonlinear income taxation, although Konishi [1995] is an exception. He examines a model with linear commodity taxation and nonlinear income taxation.

[7] This is partly because some of our results make use of comparative statics methods, which require the assumption of quasi-linear utility. The literature which examines the comparative statics of optimal nonlinear income taxes also assumes quasi-linearity. See for example Hamilton and Pestieau [2005], Simula [2010], and Brett and Weymark [2011].

the conditions under which a feasible welfare-improving tax reform requires a change in a particular agent's marginal tax rate or total tax burden. While this approach is less general than that typically taken in the tax reform literature, it does have a real-world counterpart. For example, in the U.K. recently there has been much discussion over whether the top marginal income tax rate should be reduced. In our model, this corresponds to asking under what conditions does an equilibrium-preserving and welfare-improving tax reform require a reduction in the marginal tax rate faced by high-skill individuals. Our answer, given in further detail in part (a) of Proposition 2, is that the marginal tax rates faced by low-skill and middle-skill individuals must already be optimal, but they must be paying too much tax under the current (suboptimal) tax system. Also, the marginal tax rate faced by high-skill individuals must be too high, and their tax payments too low, relative to their optimal levels. As can be seen, we are able to provide a relatively simple and clear description of the initial suboptimal tax system when such a tax reform is required. Some other features of the tax reform necessary to move towards optimality can also be determined.

The remainder of the paper is organised as follows. Section 2 describes the model we use, and defines what we mean by equilibrium-preserving and welfare-improving tax reforms. Section 3 examines the conditions under which all equilibrium-preserving and welfare-improving tax reforms require a change in a particular agent's marginal tax rate, while Section 4 examines the conditions under which all equilibrium-preserving and welfare-improving tax reforms require a change in a particular agent's total tax payments. Section 5 concludes, proofs and some other mathematical details are relegated to Appendix A, and numerical examples of our results are provided in Appendix B.

2 The Model

There are three types of individual, and individuals are distinguished by their skill levels in employment or, equivalently, their wage rates. Type i's wage is denoted by w_i, where $w_3 > w_2 > w_1$ so that type 3 individuals are high-skill, type 2 individuals are middle-skill, and type 1 individuals are low-skill. We make the standard assumption that the economy's technology is linear, which implies that wages are fixed. Individuals have the same preferences, which are representable by the quasi-linear utility function:

$$u(x_i) - \alpha l_i \tag{2.1}$$

where x_i is type i's consumption and l_i is type i's labour supply. The function $u(\cdot)$ is increasing and strictly concave, while $\alpha > 0$ is a preference parameter that captures the disutility of labour. Social welfare is assumed to be measurable by the utilitarian social welfare function:

$$\sum_{i=1}^{3} n_i \left[u(x_i) - \alpha l_i \right] \tag{2.2}$$

where n_i represents the population of type i individuals.

The government imposes nonlinear taxation on labour income, where $y_i = w_i l_i$ denotes the pre-tax income of a type i individual.[8] Formally, we associate a nonlinear income tax schedule with three tax contracts: $\langle y_1, x_1 \rangle$, $\langle y_2, x_2 \rangle$, and $\langle y_3, x_3 \rangle$. Therefore, $y_i - x_i$ is taxes paid (or, if negative, transfers received) by a type i individual.

An equilibrium of our model is obtained if and only if:

$$\sum_{i=1}^{3} n_i \left[y_i - x_i \right] - G \geq 0 \tag{2.3}$$

$$u(x_2) - \alpha \frac{y_2}{w_2} - u(x_1) + \alpha \frac{y_1}{w_2} \geq 0 \tag{2.4}$$

$$u(x_3) - \alpha \frac{y_3}{w_3} - u(x_2) + \alpha \frac{y_2}{w_3} \geq 0 \tag{2.5}$$

where G is the government's exogenously determined revenue requirement. Equation (2.3) is the government's budget constraint, while equations (2.4) and (2.5) are incentive-compatibility constraints associated with nonlinear income taxation. We analyse what Stiglitz [1982] calls the "normal" case and what Guesnerie [1995] calls "redistributive equilibria", in that the incentive-compatibility constraints may bind "downwards" but never "upwards". This is consistent with redistributive taxation, which creates an incen-

[8] As in Mirrlees [1971], it is assumed that the government cannot observe an individual's skill type, and therefore it cannot implement (the first-best) personalised lump-sum taxes.

tive for higher-skill individuals to mimic lower-skill individuals, but not vice versa. Built into equations (2.4) and (2.5) is the simplifying assumption that only the downward-adjacent incentive-compatibility constraints may bind, i.e., low-skill and high-skill individuals are not directly linked through the incentive-compatibility constraints. For analytical purposes, we assume that the status quo equilibrium is "tight", i.e., equations $(2.3)-(2.5)$ all hold with equality. This assumption allows us to differentiate the system of equations $(2.3)-(2.5)$. We also assume that each type of individual has positive levels of consumption and labour in the initial equilibrium.

We define a *tax reform* as the vector $\mathrm{d}R := \langle \mathrm{d}y_1, \mathrm{d}x_1, \mathrm{d}y_2, \mathrm{d}x_2, \mathrm{d}y_3, \mathrm{d}x_3 \rangle$, which can be interpreted as the government implementing a small change in the nonlinear income tax system. Starting in an initial tight equilibrium, a tax reform is said to be *equilibrium-preserving* if and only if:

$$\nabla Z \mathrm{d}R \geq 0^{(3)} \tag{2.6}$$

where ∇Z is the Jacobian matrix (with respect to $\mathrm{d}R$) associated with equations $(2.3)-(2.5)$ and is defined as:

$$\nabla Z := \begin{bmatrix} n_1 & -n_1 & n_2 & -n_2 & n_3 & -n_3 \\ \frac{\alpha}{w_2} & -u'(x_1) & -\frac{\alpha}{w_2} & u'(x_2) & 0 & 0 \\ 0 & 0 & \frac{\alpha}{w_3} & -u'(x_2) & -\frac{\alpha}{w_3} & u'(x_3) \end{bmatrix} \tag{2.7}$$

where all derivatives are evaluated in the status quo equilibrium. An equilibrium-preserving tax reform is a tax reform that moves the economy to a neighbouring equilibrium.

A tax reform is said to be *welfare-improving* if and only if:

$$\nabla W \mathrm{d}R > 0 \tag{2.8}$$

where $\nabla W := \langle -n_1 \frac{\alpha}{w_1}, n_1 u'(x_1), -n_2 \frac{\alpha}{w_2}, n_2 u'(x_2), -n_3 \frac{\alpha}{w_3}, n_3 u'(x_3) \rangle$ is the gradient (with respect to $\mathrm{d}R$) of the utilitarian social welfare function. A welfare-improving tax reform is a tax reform that increases social welfare.

3 Reforming Marginal Tax Rates

It is shown in Appendix A that the marginal tax rate applicable to the income of type i individuals can be written as:

$$MTR_i = 1 - \frac{\alpha}{u'(x_i)w_i} \tag{3.1}$$

where MTR_i denotes the marginal tax rate faced by type i individuals. Therefore:

$$\mathrm{d}MTR_i = \frac{u''(x_i)\alpha}{u'(x_i)u'(x_i)w_i}\mathrm{d}x_i \qquad \Longleftrightarrow \qquad \frac{-u'(x_i)u'(x_i)w_i}{u''(x_i)\alpha}\mathrm{d}MTR_i = -\mathrm{d}x_i \tag{3.2}$$

It follows that $\mathrm{d}MTR_i \gtreqless 0$ if and only if $\nabla M_i \mathrm{d}R \gtreqless 0$, where $\nabla M_1 := \langle 0,\ -1,\ 0^{(4)}\rangle$, $\nabla M_2 := \langle 0^{(3)},\ -1,\ 0^{(2)}\rangle$, and $\nabla M_3 := \langle 0^{(5)},\ -1\rangle$.

Starting in an initial tight equilibrium of our model, if there does not exist a tax reform such that:

$$\nabla Z \mathrm{d}R \geq 0^{(3)} \tag{3.3}$$

$$\nabla W \mathrm{d}R > 0 \tag{3.4}$$

$$\nabla M_i \mathrm{d}R \leq 0 \tag{3.5}$$

then there are two possibilities: (i) There does not exist a tax reform that satisfies equations (3.3) and (3.4). In this case, there do not exist any equilibrium-preserving and welfare-improving tax reforms, so the status quo tax system is already optimal and equation (3.5) is redundant. (ii) There do exist tax reforms that satisfy equations (3.3) and (3.4), but all such reforms violate equation (3.5). In this case, the status quo tax system is suboptimal, and any move towards optimality requires an increase in the marginal tax rate faced by type i individuals (i.e., a violation of equation (3.5)). As we are interested in examining moves from a suboptimal towards an optimal tax system, we focus on this second possibility.

By Motzkin's Theorem of the Alternative,[9] if there does not exist a tax reform $\mathrm{d}R$ that satisfies equations (3.3) $-$ (3.5), then there exist real numbers $\langle\theta_1, \theta_2, \theta_3\rangle \geq 0^{(3)}$, $\theta_4 > 0$, and $\theta_5 \geq 0$ such that:[10]

$$\langle\theta_1, \theta_2, \theta_3\rangle \nabla Z + \theta_4 \nabla W - \theta_5 \nabla M_i = 0^{(6)} \tag{3.6}$$

The system of equations (3.6) characterises what the initial suboptimal tax system "looks like" when all equilibrium-preserving and welfare-improving tax reforms require an increase in the marginal tax rate faced by type i individuals.

Let $\overline{z}$ denote the level of variable z when the tax system is optimal, and let T_i denote type i's tax payments. Using equation (3.6) we obtain the following proposition (all proofs are provided in Appendix A):

Proposition 1: *Consider an initial tight equilibrium of our model in which the nonlinear income tax system is suboptimal*:

(a) *If all equilibrium-preserving and welfare-improving tax reforms require an increase in the marginal tax rate faced by high-skill* $(type\ 3)$ *individuals, then*: (i) *in the initial equilibrium* $MTR_1 = \overline{MTR}_1$, $MTR_2 = \overline{MTR}_2$, $MTR_3 < \overline{MTR}_3$, $T_1 > \overline{T}_1$, $T_2 > \overline{T}_2$, *and* $T_3 < \overline{T}_3$, *and* (ii) *the move towards the optimal tax system requires* $\mathrm{d}x_1 = 0$, $\mathrm{d}y_1 < 0$, $\mathrm{d}x_2 = 0$, $\mathrm{d}y_2 < 0$, $\mathrm{d}x_3 < 0$, *and* $\mathrm{d}y_3 < 0$.

(b) *If all equilibrium-preserving and welfare-improving tax reforms require an increase in the marginal tax rate faced by middle-skill* $(type\ 2)$ *individuals, then*: (i) *in the initial equilibrium* $MTR_1 = \overline{MTR}_1$, $MTR_2 < \overline{MTR}_2$, *and* $MTR_3 = \overline{MTR}_3$, *and* (ii) *the move towards the optimal tax system requires* $\mathrm{d}x_1 = 0$, $\mathrm{d}x_2 < 0$, $\mathrm{d}y_2 < 0$, *and* $\mathrm{d}x_3 = 0$.

(c) *If all equilibrium-preserving and welfare-improving tax reforms require an increase in the marginal tax rate faced by low-skill* $(type\ 1)$ *individuals, then*: (i) *in the initial equilibrium* $MTR_1 < \overline{MTR}_1$, $MTR_2 = \overline{MTR}_2$, *and* $MTR_3 = \overline{MTR}_3$, *and* (ii) *the move towards the optimal tax system requires* $\mathrm{d}x_1 < 0$, $\mathrm{d}y_1 < 0$, $\mathrm{d}x_2 = 0$, *and* $\mathrm{d}x_3 = 0$.

By reversing the inequality in equation (3.5), one can examine the conditions under

[9] A statement of Motzkin's Theorem is provided in Appendix A.

[10] Vector notation: $z \geq \widetilde{z} \iff z_j \geq \widetilde{z}_j\ \forall\ j$, $z > \widetilde{z} \iff z_j \geq \widetilde{z}_j\ \forall\ j \wedge z \neq \widetilde{z}$, $z \gg \widetilde{z} \iff z_j > \widetilde{z}_j\ \forall\ j$.

which all equilibrium-preserving and welfare-improving tax reforms require a decrease in the marginal tax rate applicable to type i individuals. This leads to:

Proposition 2: *Consider an initial tight equilibrium of our model in which the nonlinear income tax system is suboptimal*:

(a) *If all equilibrium-preserving and welfare-improving tax reforms require a decrease in the marginal tax rate faced by high-skill* $($*type 3*$)$ *individuals, then*: (i) *in the initial equilibrium* $MTR_1 = \overline{MTR}_1$, $MTR_2 = \overline{MTR}_2$, $MTR_3 > \overline{MTR}_3$, $T_1 > \overline{T}_1$, $T_2 > \overline{T}_2$, *and* $T_3 < \overline{T}_3$, *and* (ii) *the move towards the optimal tax system requires* $\mathrm{d}x_1 = 0$, $\mathrm{d}y_1 < 0$, $\mathrm{d}x_2 = 0$, $\mathrm{d}y_2 < 0$, $\mathrm{d}x_3 > 0$, *and* $\mathrm{d}y_3 > 0$.

(b) *If all equilibrium-preserving and welfare-improving tax reforms require a decrease in the marginal tax rate faced by middle-skill* $($*type 2*$)$ *individuals, then*: (i) *in the initial equilibrium* $MTR_1 = \overline{MTR}_1$, $MTR_2 > \overline{MTR}_2$, $MTR_3 = \overline{MTR}_3$, *and* $T_3 > \overline{T}_3$, *and* (ii) *the move towards the optimal tax system requires* $\mathrm{d}x_1 = 0$, $\mathrm{d}x_2 > 0$, $\mathrm{d}y_2 > 0$, $\mathrm{d}x_3 = 0$, *and* $\mathrm{d}y_3 < 0$.

(c) *If all equilibrium-preserving and welfare-improving tax reforms require a decrease in the marginal tax rate faced by low-skill* $($*type 1*$)$ *individuals, then*: (i) *in the initial equilibrium* $MTR_1 > \overline{MTR}_1$, $MTR_2 = \overline{MTR}_2$, $MTR_3 = \overline{MTR}_3$, $T_1 < \overline{T}_1$, $T_2 > \overline{T}_2$, *and* $T_3 > \overline{T}_3$, *and* (ii) *the move towards the optimal tax system requires* $\mathrm{d}x_1 > 0$, $\mathrm{d}y_1 > 0$, $\mathrm{d}x_2 = 0$, $\mathrm{d}y_2 < 0$, $\mathrm{d}x_3 = 0$, *and* $\mathrm{d}y_3 < 0$.

It can be seen from Propositions 1 and 2 that the results for tax reforms requiring an increase or decrease in type i's marginal tax rate are not simply mirror images of one another. If all equilibrium-preserving and welfare-improving tax reforms require a change (increase or decrease) in type i's marginal tax rate, then the tax reform must include a change in x_i (cf. equation (3.2)). As the status quo equilibrium is assumed to be tight, one can solve equations $(2.3)-(2.5)$ and obtain the functions $y_1(x_1, x_2, x_3)$, $y_2(x_1, x_2, x_3)$, and $y_3(x_1, x_2, x_3)$. In general, the signs of the comparative statics, $\partial y_j(\cdot)/\partial x_i$, are ambiguous. However, one can use the system of equations (3.6) (or the analogous system for the case of decreasing type i's marginal tax rate) to sign at least some of these comparative statics. As the sign of $\partial y_j(\cdot)/\partial x_i$ may depend upon whether the tax reform requires an increase or decrease in type i's marginal tax rate, Propositions 1 and 2 are not simply mirror images of each other.

If all equilibrium-preserving and welfare-improving tax reforms require a change in type i's marginal tax rate, then $x_i \neq \overline{x}_i$ but $x_j = \overline{x}_j$ (for all $j \neq i$). This follows from solving the system of equations (3.6) (or the analogous system for the case of decreasing type i's marginal tax rate) for x_1, x_2, and x_3. Therefore, $MTR_i \neq \overline{MTR}_i$ and $MTR_j = \overline{MTR}_j$ (for all $j \neq i$) in all parts of Propositions 1 and 2. Correspondingly, the tax reform required to move towards optimality must include a change in x_i, but no change in x_j (for all $j \neq i$).

The other features of Propositions 1 and 2 follow from the comparative statics, $\partial y_j(\cdot)/\partial x_i$. For part (a) of Proposition 1 we have $\partial y_1(\cdot)/\partial x_3 > 0$ and $\partial y_2(\cdot)/\partial x_3 > 0$. As the tax reform requires $\mathrm{d}x_3 < 0$, we must have $\mathrm{d}y_1 < 0$ and $\mathrm{d}y_2 < 0$. Moreover, since $\mathrm{d}x_1 = \mathrm{d}x_2 = 0$, the tax reform reduces tax payments by low-skill and middle-skill individuals, implying that they must have been paying too much tax in the initial equilibrium ($T_1 > \overline{T}_1$ and $T_2 > \overline{T}_2$). This in turn implies that high-skill individuals must have been paying too little tax in the initial equilibrium ($T_3 < \overline{T}_3$). Analogously, for part (a) of Proposition 2 we have $\partial y_1(\cdot)/\partial x_3 < 0$ and $\partial y_2(\cdot)/\partial x_3 < 0$. As the tax reform in this case requires $\mathrm{d}x_3 > 0$, we must have $\mathrm{d}y_1 < 0$ and $\mathrm{d}y_2 < 0$. And since $\mathrm{d}x_1 = \mathrm{d}x_2 = 0$, the tax reform reduces tax payments by low-skill and middle-skill individuals. This again implies that they were paying too much tax in the initial equilibrium, while high-skill individuals were paying too little. Taken together, part (a) of Propositions 1 and 2 show that if the high-skill type's marginal tax rate is not optimal and must be changed, then they are paying less tax than is optimal. As is well known, it is optimal for the high-skill type to face a zero marginal tax rate, at which point their tax payments are maximised for a given level of utility. The intuition behind part (a) of Propositions 1 and 2 follows from this well-known result.

Unfortunately, less can be said about parts (b) and (c) of Propositions 1 and 2, because most of the comparative statics, $\partial y_j(\cdot)/\partial x_i$, cannot be signed. The only exception is part (c) of Proposition 2, in which the full set of comparative statics is determinate and therefore a relatively complete description of the initial suboptimal tax system and the tax reform required is possible. In this case, which deals with when a decrease in the low-skill type's marginal tax rate is required, tax payments by low-skill individuals in the initial equilibrium are lower than optimal and, correspondingly, tax payments by middle-skill and high-skill individuals are higher than optimal. The intuition is that the higher-than-optimal marginal tax rate faced by low-skill individuals distorts their labour supply downwards too much, so they earn too little income and pay too little in taxes. Accordingly, a welfare-improving tax reform requires that low-skill individuals work longer and pay more in taxes, while taxes paid by middle-skill and high-skill individuals are correspondingly reduced.

4 Reforming Total Tax Payments

Tax paid by a type i individual is equal to $T_i = y_i - x_i$. Therefore, $\mathrm{d}T_i = \mathrm{d}y_i - \mathrm{d}x_i$ and $\mathrm{d}T_i \gtreqless 0$ if and only if $\nabla T_i \mathrm{d}R \gtreqless 0$, where $\nabla T_1 := \langle 1,\, -1,\, 0^{(4)} \rangle$, $\nabla T_2 := \langle 0^{(2)},\, 1,\, -1,\, 0^{(2)} \rangle$, and $\nabla T_3 := \langle 0^{(4)},\, 1,\, -1 \rangle$.

One can analyse situations in which all equilibrium-preserving and welfare-improving tax reforms require a change in type i's tax payments in a similar manner as above for marginal tax rates. Starting in an initial tight equilibrium, if there does not exist a tax reform $\mathrm{d}R$ such that:

$$\nabla Z \mathrm{d}R \geq 0^{(3)} \tag{4.1}$$

$$\nabla W \mathrm{d}R > 0 \tag{4.2}$$

$$\nabla T_i \mathrm{d}R \leq 0 \tag{4.3}$$

then all equilibrium-preserving and welfare-improving tax reforms require an increase in tax paid by type i individuals (i.e., a violation of equation (4.3) is required). By applying Motzkin's Theorem of the Alternative, if there does not exist a tax reform that satisfies equations (4.1) – (4.3), then there exist $\langle \beta_1, \beta_2, \beta_3 \rangle \geq 0^{(3)}$, $\beta_4 > 0$, and $\beta_5 \geq 0$ such that:

$$\langle \beta_1, \beta_2, \beta_3 \rangle \nabla Z + \beta_4 \nabla W - \beta_5 \nabla T_i = 0^{(6)} \tag{4.4}$$

Using the system of equations (4.4) we obtain:

Proposition 3: *Consider an initial tight equilibrium of our model in which the nonlinear income tax system is suboptimal*:

(a) *If all equilibrium-preserving and welfare-improving tax reforms require an increase in the tax paid by high-skill* $($*type 3*$)$ *individuals, then*: (i) *in the initial equilibrium* $MTR_1 < \overline{MTR}_1$, $MTR_2 < \overline{MTR}_2$, $MTR_3 = \overline{MTR}_3$, $T_1 + T_2 > \overline{T}_1 + \overline{T}_2$, *and* $T_3 < \overline{T}_3$, *and* (ii) *the move towards the optimal tax system requires* $\mathrm{d}x_1 < 0$, $\mathrm{d}x_2 < 0$, $\mathrm{d}x_3 = 0$, $\mathrm{d}y_3 > 0$, *and* $\mathrm{d}y_1 < 0$ *and/or* $\mathrm{d}y_2 < 0$.

(b) *If all equilibrium-preserving and welfare-improving tax reforms require an increase in the tax paid by middle-skill* $($*type 2*$)$ *individuals, then*: (i) *in the initial equilibrium* $MTR_1 < \overline{MTR}_1$, $MTR_2 > \overline{MTR}_2$, $MTR_3 = \overline{MTR}_3$, $T_1 + T_3 > \overline{T}_1 + \overline{T}_3$, *and* $T_2 < \overline{T}_2$, *and* (ii) *the move towards the optimal tax system requires* $\mathrm{d}x_1 < 0$, $\mathrm{d}x_2 > 0$, $\mathrm{d}y_2 > 0$, $\mathrm{d}x_3 = 0$, *and* $\mathrm{d}y_1 < 0$ *and/or* $\mathrm{d}y_3 < 0$.

(c) *If all equilibrium-preserving and welfare-improving tax reforms require an increase in the tax paid by low-skill* $($*type 1*$)$ *individuals, then*: (i) *in the initial equilibrium* $MTR_1 > \overline{MTR}_1$, $MTR_2 > \overline{MTR}_2$, $MTR_3 = \overline{MTR}_3$, $T_2 + T_3 > \overline{T}_2 + \overline{T}_3$, *and* $T_1 < \overline{T}_1$, *and* (ii) *the move towards the optimal tax system requires* $\mathrm{d}x_1 > 0$, $\mathrm{d}y_1 > 0$, $\mathrm{d}x_2 > 0$, *and* $\mathrm{d}x_3 = 0$.

By reversing the inequality in equation (4.3), we obtain the results for necessitated decreases in tax payments:

Proposition 4: *Consider an initial tight equilibrium of our model in which the nonlinear income tax system is suboptimal*:

(a) *If all equilibrium-preserving and welfare-improving tax reforms require a decrease in the tax paid by high-skill* $($*type 3*$)$ *individuals, then*: (i) *in the initial equilibrium* $MTR_1 > \overline{MTR}_1$, $MTR_2 > \overline{MTR}_2$, $MTR_3 = \overline{MTR}_3$, $T_1 + T_2 < \overline{T}_1 + \overline{T}_2$, *and* $T_3 > \overline{T}_3$, *and* (ii) *the move towards the optimal tax system requires* $\mathrm{d}x_1 > 0$, $\mathrm{d}x_2 > 0$, $\mathrm{d}x_3 = 0$, $\mathrm{d}y_3 < 0$, *and* $\mathrm{d}y_1 > 0$ *and/or* $\mathrm{d}y_2 > 0$.

(b) *If all equilibrium-preserving and welfare-improving tax reforms require a decrease in the tax paid by middle-skill* $($*type 2*$)$ *individuals, then*: (i) *in the initial equilibrium* $MTR_1 > \overline{MTR}_1$, $MTR_2 < \overline{MTR}_2$, $MTR_3 = \overline{MTR}_3$, $T_1 + T_3 < \overline{T}_1 + \overline{T}_3$, *and* $T_2 > \overline{T}_2$, *and* (ii) *the move towards the optimal tax system requires* $\mathrm{d}x_1 > 0$, $\mathrm{d}x_2 < 0$, $\mathrm{d}y_2 < 0$,

$\mathrm{d}x_3 = 0$, *and* $\mathrm{d}y_1 > 0$ *and/or* $\mathrm{d}y_3 > 0$.

(*c*) *If all equilibrium-preserving and welfare-improving tax reforms require a decrease in the tax paid by low-skill* (*type 1*) *individuals, then*: (*i*) *in the initial equilibrium* $MTR_1 < \overline{MTR}_1$, $MTR_2 < \overline{MTR}_2$, $MTR_3 = \overline{MTR}_3$, $T_2 + T_3 < \overline{T}_2 + \overline{T}_3$, *and* $T_1 > \overline{T}_1$, *and* (*ii*) *the move towards the optimal tax system requires* $\mathrm{d}x_1 < 0$, $\mathrm{d}y_1 < 0$, $\mathrm{d}x_2 < 0$, *and* $\mathrm{d}x_3 = 0$.

Unlike the results for reforming marginal tax rates, the results obtained for tax reforms requiring an increase or decrease in type i's tax payments, as stated in Propositions 3 and 4, are simply mirror images of each other. These results follow from the system of equations (4.4) (or the analogous system for a decrease in type i's tax payments), rather than from the comparative statics, $\partial y_j(\cdot)/\partial x_i$. Since $T_i = y_i - x_i$, type i's tax payments can be changed without necessarily changing x_i. Thus the comparative statics, $\partial y_j(\cdot)/\partial x_i$, cannot help shed light on the characteristics of the initial suboptimal tax system, nor of the tax reform required to move towards optimality. This means that, in general, less can be said about tax reforms requiring a change in an agent's tax payments than in their marginal tax rate. Furthermore, those results that can be obtained for necessitated increases and decreases in type i's tax payments are simply mirror images of one another.

Part (a) of Proposition 3 is the case when all equilibrium-preserving and welfare-improving tax reforms require an increase in tax paid by high-skill individuals. In this case, $\nabla T_i = \nabla T_3$ in the system of equations (4.4), and these equations can be solved for x_1, x_2, and x_3. It can then be shown that $x_1 > \overline{x}_1$, $x_2 > \overline{x}_2$, and $x_3 = \overline{x}_3$, which implies that $MTR_1 < \overline{MTR}_1$, $MTR_2 < \overline{MTR}_2$, and $MTR_3 = \overline{MTR}_3$. To move towards optimality, the tax reform therefore requires $\mathrm{d}x_1 < 0$, $\mathrm{d}x_2 < 0$, and no change in x_3. As tax payments by high-skill individuals must be increased, $T_3 < \overline{T}_3$ and, correspondingly, $T_1 + T_2 > \overline{T}_1 + \overline{T}_2$ in the initial equilibrium. Therefore, the tax reform requires $\mathrm{d}y_3 > 0$, and $\mathrm{d}y_1 < 0$ and/or $\mathrm{d}y_2 < 0$ is also required to reduce aggregate tax payments by low-skill and middle-skill individuals. Finally, parts (b) and (c) of Proposition 3 can be interpreted in a similar manner to part (a), and as discussed earlier Proposition 4 is simply the reverse of Proposition 3.

5 Conclusion

We have analysed nonlinear income tax reforms using a model and methodology that lead to a relatively clear description of the initial suboptimal tax system and the tax reform required to move towards optimality. Furthermore, the types of tax reform questions addressed correspond quite closely to those actually faced by policy-makers, which typically revolve around whether a specific piecemeal reform—such as reducing the top marginal tax rate—should be implemented. The price paid for the clarity achieved in this paper is that we have used a simple model, and we have assumed that preferences are quasi-linear. That said, our model is a low-dimensional (three-type) version of the workhorse Mirrlees [1971] nonlinear income tax model, and the assumption that preferences are quasi-linear is not uncommon.

The existing tax reform literature typically yields results that are quite technical, and that are lacking in economic intuition. We have been able to obtain a number of clear-cut results, but it remains difficult to provide a simple economic explanation for many of our results. This may suggest that these results are heavily dependent upon the quasi-linearity assumption. In future work, it would be worth exploring the possibility of generalising the model and the utility function. We expect that such generalisations will make it more difficult to obtain clear-cut results, but those results that can be obtained are likely to have a fairly straightforward economic intuition.

6 Appendix A

Deriving the Expression for the Marginal Tax Rate

To derive equation (3.1), suppose the individuals faced a smooth nonlinear income tax function $T(y_i)$. Each individual i would solve the following programme:

$$\max_{x_i,\, l_i} \{u(x_i) - \alpha l_i \mid x_i \leq y_i - T(y_i)\} \tag{A.1}$$

The relevant first-order conditions corresponding to this programme are:

$$u'(x_i) - \lambda = 0 \tag{A.2}$$

$$-\alpha + \lambda w_i \left[1 - T'(y_i)\right] = 0 \tag{A.3}$$

where $\lambda > 0$ is the Lagrange multiplier, and $T'(y_i)$ can be interpreted as individual i's marginal tax rate. Straightforward manipulation of equations (A.2) and (A.3) leads to equation (3.1). ■

Motzkin's Theorem of the Alternative

Let A, C, and D be $a_1 \times m$, $a_2 \times m$, and $a_3 \times m$ matrices, respectively, where A is non-vacuous (not all zeros). Then *either*:

$$Az \gg 0^{(a_1)} \qquad Cz \geq 0^{(a_2)} \qquad Dz = 0^{(a_3)}$$

has a solution $z \in \mathbb{R}^m$, *or*:

$$b_1 A + b_2 C + b_3 D = 0^{(m)}$$

has a solution $b_1 > 0^{(a_1)}$, $b_2 \geq 0^{(a_2)}$, and b_3 sign unrestricted, *but never both.* A proof of Motzkin's Theorem can be found in Mangasarian [1969].

Proof of Part (a) of Proposition 1

For part (a) of Proposition 1, we have $\nabla M_i = \nabla M_3$ in the system of equations (3.6). If there exist real numbers $\langle \theta_1, \theta_2, \theta_3 \rangle \geq 0^{(3)}$, $\theta_4 > 0$, and $\theta_5 \geq 0$ such that system (3.6) is satisfied, then there must also exist real numbers under the same sign restrictions that satisfy (3.6), but with $\theta_4 = 1$. Thus, without loss of generality, we set $\theta_4 = 1$. Also, if $\theta_5 = 0$ the status quo tax system is already optimal. Therefore, we consider the case in which $\theta_5 > 0$. Expanding (3.6) now yields:

$$\theta_1 n_1 + \theta_2 \frac{\alpha}{w_2} - n_1 \frac{\alpha}{w_1} = 0 \tag{A.4}$$

$$-\theta_1 n_1 - \theta_2 u'(x_1) + n_1 u'(x_1) = 0 \tag{A.5}$$

$$\theta_1 n_2 - \theta_2 \frac{\alpha}{w_2} + \theta_3 \frac{\alpha}{w_3} - n_2 \frac{\alpha}{w_2} = 0 \tag{A.6}$$

$$-\theta_1 n_2 + \theta_2 u'(x_2) - \theta_3 u'(x_2) + n_2 u'(x_2) = 0 \tag{A.7}$$

$$\theta_1 n_3 - \theta_3 \frac{\alpha}{w_3} - n_3 \frac{\alpha}{w_3} = 0 \tag{A.8}$$

$$-\theta_1 n_3 + \theta_3 u'(x_3) + n_3 u'(x_3) = -\theta_5 \tag{A.9}$$

One can solve equations (A.4), (A.6), and (A.8) for θ_1, θ_2, and θ_3. Notice that the solution obtained will be independent of θ_5. It then follows from equations (A.5), (A.7), and (A.9), respectively, that $x_1 = \overline{x}_1$, $x_2 = \overline{x}_2$, and $x_3 > \overline{x}_3$ (since $\theta_5 > 0$ and $u(\cdot)$ is strictly concave). Using equation (3.1), this establishes that $MTR_1 = \overline{MTR}_1$, $MTR_2 = \overline{MTR}_2$, and $MTR_3 < \overline{MTR}_3$.

As the status quo equilibrium is assumed to be tight, one can solve equations (2.3) – (2.5) to obtain:

$$y_1 = \frac{w_2(n_2 + n_3)\left[u(x_1) - u(x_2)\right] + \alpha\left[\sum_i n_i x_i + G - \frac{n_3 w_3}{\alpha}\left[u(x_3) - u(x_2)\right]\right]}{\alpha \sum_i n_i} \tag{A.10}$$

$$y_2 = \frac{\sum_i n_i x_i + G - n_1 y_1 - \frac{n_3 w_3}{\alpha}\left[u(x_3) - u(x_2)\right]}{n_2 + n_3} \tag{A.11}$$

$$y_3 = \frac{w_3}{\alpha}\left[u(x_3) - u(x_2)\right] + y_2 \tag{A.12}$$

Using equation (A.10), we obtain:

$$\alpha \sum_i n_i \frac{\partial y_1(\cdot)}{\partial x_3} = n_3\left[\alpha - w_3 u'(x_3)\right] \tag{A.13}$$

From equations (A.8) and (A.9) it follows that $\alpha - w_3 u'(x_3) > 0$, which implies that $\partial y_1(\cdot)/\partial x_3 > 0$. Using equation (2.4), $\partial y_1(\cdot)/\partial x_3 > 0$ implies that $\partial y_2(\cdot)/\partial x_3 > 0$. And using equation (A.12), $\partial y_2(\cdot)/\partial x_3 > 0$ implies that $\partial y_3(\cdot)/\partial x_3 > 0$.

As all equilibrium-preserving and welfare-improving tax reforms require an increase in the high-skill type's marginal tax rate, the tax reform must include $\mathrm{d}x_3 < 0$. And because $x_1 = \overline{x}_1$ and $x_2 = \overline{x}_2$, the tax reform also has $\mathrm{d}x_1 = \mathrm{d}x_2 = 0$. The comparative statics results now imply that the tax reform must include $\mathrm{d}y_j < 0$ for all j. Finally, since the tax reform reduces tax payments by low-skill and middle-skill individuals, and because the tax reform moves the tax system towards optimality, $T_1 > \overline{T}_1$, $T_2 > \overline{T}_2$, and $T_3 < \overline{T}_3$ must hold in the initial equilibrium. ■

Proofs of Parts (b) and (c) of Proposition 1, and Proof of Proposition 2

As the strategy for proving parts (b) and (c) of Proposition 1, and for proving all parts of Proposition 2, is basically the same as that for proving part (a) of Proposition 1, we omit these proofs. Details of these proofs are, however, available upon request. ■

Proof of Part (a) of Proposition 3

For part (a) of Proposition 3, we have $\nabla T_i = \nabla T_3$ in the system of equations (4.4). If there exist real numbers $\langle \beta_1, \beta_2, \beta_3 \rangle \geq 0^{(3)}$, $\beta_4 > 0$, and $\beta_5 \geq 0$ such that system (4.4) is satisfied, then there must also exist real numbers under the same sign restrictions that satisfy (4.4), but with $\beta_4 = 1$. Thus, without loss of generality, we set $\beta_4 = 1$. Also, if $\beta_5 = 0$ the status quo tax system is already optimal. Therefore, we consider the case in which $\beta_5 > 0$. Expanding (4.4) now yields:

$$\beta_1 n_1 + \beta_2 \frac{\alpha}{w_2} - n_1 \frac{\alpha}{w_1} = 0 \tag{A.14}$$

$$-\beta_1 n_1 - \beta_2 u'(x_1) + n_1 u'(x_1) = 0 \tag{A.15}$$

$$\beta_1 n_2 - \beta_2 \frac{\alpha}{w_2} + \beta_3 \frac{\alpha}{w_3} - n_2 \frac{\alpha}{w_2} = 0 \tag{A.16}$$

$$-\beta_1 n_2 + \beta_2 u'(x_2) - \beta_3 u'(x_2) + n_2 u'(x_2) = 0 \tag{A.17}$$

$$\beta_1 n_3 - \beta_3 \frac{\alpha}{w_3} - n_3 \frac{\alpha}{w_3} = \beta_5 \tag{A.18}$$

$$-\beta_1 n_3 + \beta_3 u'(x_3) + n_3 u'(x_3) = -\beta_5 \tag{A.19}$$

Solving equations (A.14), (A.16), and (A.18) for β_1, β_2, and β_3 yields:

$$\beta_1 = \frac{\alpha \sum_i \frac{n_i}{w_i} + \beta_5}{\sum_i n_i} \tag{A.20}$$

$$\beta_2 = \frac{n_1 w_2}{w_1} - \frac{n_1 w_2 \sum_i \frac{n_i}{w_i}}{\sum_i n_i} - \frac{n_1 w_2 \beta_5}{\alpha \sum_i n_i} \tag{A.21}$$

$$\beta_3 = \frac{n_3 w_3 \sum_i \frac{n_i}{w_i}}{\sum_i n_i} - n_3 - \frac{w_3(n_1+n_2)\beta_5}{\alpha \sum_i n_i} \tag{A.22}$$

Using equation (A.15), we obtain:

$$u'(x_1) = \frac{\beta_1 n_1}{n_1 - \beta_2} \tag{A.23}$$

Therefore:

$$\frac{\partial u'(x_1)}{\partial \beta_5} = \frac{\frac{\partial \beta_1}{\partial \beta_5} n_1(n_1 - \beta_2) + \frac{\partial \beta_2}{\partial \beta_5}\beta_1 n_1}{(n_1-\beta_2)^2} \tag{A.24}$$

Using equations (A.20) and (A.21), equation (A.24) simplifies to:

$$(n_1-\beta_2)^2 \frac{\partial u'(x_1)}{\partial \beta_5} = \frac{n_1^2}{\sum_i n_i}\left(1 - \frac{w_2}{w_1}\right) < 0 \tag{A.25}$$

As $u(\cdot)$ is strictly concave, from equation (A.25) we obtain $u'(x_1) < u'(\overline{x}_1) \implies x_1 > \overline{x}_1 \implies MTR_1 < \overline{MTR}_1$.

Using equation (A.17), we obtain:

$$u'(x_2) = \frac{\beta_1 n_2}{\beta_2 - \beta_3 + n_2} \tag{A.26}$$

Therefore:

$$\frac{\partial u'(x_2)}{\partial \beta_5} = \frac{\frac{\partial \beta_1}{\partial \beta_5} n_2(\beta_2 - \beta_3 + n_2) - \left(\frac{\partial \beta_2}{\partial \beta_5} - \frac{\partial \beta_3}{\partial \beta_5}\right)\beta_1 n_2}{(\beta_2-\beta_3+n_2)^2} \tag{A.27}$$

Using equations (A.20), (A.21), and (A.22), equation (A.27) simplifies to:

$$(\beta_2-\beta_3+n_2)^2 \frac{\partial u'(x_2)}{\partial \beta_5} = \frac{n_2}{\sum_i n_i}\left[\frac{n_1}{w_1}(w_2 - w_3) + n_2\left(1 - \frac{w_3}{w_2}\right)\right] < 0 \tag{A.28}$$

As $u(\cdot)$ is strictly concave, from equation (A.28) we obtain $u'(x_2) < u'(\overline{x}_2) \implies x_2 > \overline{x}_2 \implies MTR_2 < \overline{MTR}_2$.

Using equations (A.18) and (A.19), we obtain $u'(x_3) = \alpha/w_3$. Therefore, $u'(x_3) = u'(\overline{x}_3) \implies x_3 = \overline{x}_3 \implies MTR_3 = \overline{MTR}_3$.

Finally, $x_1 > \overline{x}_1$, $x_2 > \overline{x}_2$, and $x_3 = \overline{x}_3$ implies that a tax reform towards optimality requires $\mathrm{d}x_1 < 0$, $\mathrm{d}x_2 < 0$, and $\mathrm{d}x_3 = 0$. Since $\mathrm{d}x_3 = 0$ and tax payments by high-skill individuals must be increased, the tax reform also requires $\mathrm{d}y_3 > 0$. This in turn implies that aggregate tax payments by low-skill and middle-skill individuals must be reduced, hence $\mathrm{d}y_1 < 0$ and/or $\mathrm{d}y_2 < 0$, and $T_1 + T_2 > \overline{T}_1 + \overline{T}_2$ and $T_3 < \overline{T}_3$ must hold in the initial equilibrium. ■

Proofs of Parts (b) and (c) of Proposition 3, and Proof of Proposition 4

As the strategy for proving parts (b) and (c) of Proposition 3, and for proving all parts of Proposition 4, is basically the same as that for proving part (a) of Proposition 3, we omit these proofs. Details of these proofs are, however, available upon request. ■

7 Appendix B

In this appendix we provide numerical examples of our results. These present concrete examples of suboptimal tax systems in which all feasible welfare-improving tax reforms require the specified change in the particular agent's tax treatment. They also provide a useful check on the validity of each of our propositions. In the numerical examples, we assume that $u(x_i) = \ln(x_i)$ and the size of the population is normalised to unity. The model parameter values used in the examples are presented in Table A.

TABLE A

Model Parameter Values

α	1.00	n_1	0.25	w_1	1.00
G	2.25	n_2	0.50	w_2	2.00
		n_3	0.25	w_3	3.00

Using these parameters, the values of the endogenous variables when the tax system is optimal are presented in Table B, while the subsequent tables present examples of suboptimal tax systems for each of our propositions. For Propositions 1 and 2 we normalise $\theta_4 = 1$, and we set $\theta_5 = 0.01$. For Propositions 3 and 4 we normalise $\beta_4 = 1$, and we set $\beta_5 = 0.01$.

TABLE B

Optimal Tax System

$\bar{y}_1$	0.82632	$\bar{x}_1$	0.28571	$\overline{MTR}_1$	0.71429	$\bar{T}_1$	0.54060
$\bar{y}_2$	4.49148	$\bar{x}_2$	1.78571	$\overline{MTR}_2$	0.10714	$\bar{T}_2$	2.70577
$\bar{y}_3$	6.04786	$\bar{x}_3$	3.00000	$\overline{MTR}_3$	0.00000	$\bar{T}_3$	3.04786
Memo item: *multipliers*							
θ_1	0.58333	θ_2	0.20833	θ_3	0.18750		

TABLE 1a

Part (a) of Proposition 1: Suboptimal Tax System

y_1	0.82826	x_1	0.28571	MTR_1	0.71429	T_1	0.54254
y_2	4.49342	x_2	1.78571	MTR_2	0.10714	T_2	2.70770
y_3	6.26291	x_3	3.22086	MTR_3	−0.07362	T_3	3.04205
Memo item: *multipliers*							
θ_1	0.58333	θ_2	0.20833	θ_3	0.18750		

TABLE 1b

Part (b) of Proposition 1: Suboptimal Tax System

y_1	0.83185	x_1	0.28571	MTR_1	0.71429	T_1	0.54614
y_2	4.56679	x_2	1.84911	MTR_2	0.07544	T_2	2.71768
y_3	6.01851	x_3	3.00000	MTR_3	0.00000	T_3	3.01851
Memo item: *multipliers*							
θ_1	0.58333	θ_2	0.20833	θ_3	0.18750		

TABLE 1c

Part (c) of Proposition 1: Suboptimal Tax System

y_1	0.93813	x_1	0.30675	MTR_1	0.69325	T_1	0.63138
y_2	4.46122	x_2	1.78571	MTR_2	0.10714	T_2	2.67551
y_3	6.01760	x_3	3.00000	MTR_3	0.00000	T_3	3.01760
Memo item: *multipliers*							
θ_1	0.58333	θ_2	0.20833	θ_3	0.18750		

TABLE 2a

Part (a) of Proposition 2: Suboptimal Tax System

y_1	0.82793	x_1	0.28571	MTR_1	0.71429	T_1	0.54222
y_2	4.49309	x_2	1.78571	MTR_2	0.10714	T_2	2.70738
y_3	5.85051	x_3	2.80749	MTR_3	0.06417	T_3	3.04302
Memo item: *multipliers*							
θ_1	0.58333	θ_2	0.20833	θ_3	0.18750		

TABLE 2b

Part (b) of Proposition 2: Suboptimal Tax System

y_1	0.82200	x_1	0.28571	MTR_1	0.71429	T_1	0.53629
y_2	4.41974	x_2	1.72652	MTR_2	0.13674	T_2	2.69323
y_3	6.07726	x_3	3.00000	MTR_3	0.00000	T_3	3.07726
Memo item: *multipliers*							
θ_1	0.58333	θ_2	0.20833	θ_3	0.18750		

TABLE 2c

Part (c) of Proposition 2: Suboptimal Tax System

y_1	0.72225	x_1	0.26738	MTR_1	0.73262	T_1	0.45487
y_2	4.52006	x_2	1.78571	MTR_2	0.10714	T_2	2.73434
y_3	6.07644	x_3	3.00000	MTR_3	0.00000	T_3	3.07644
			Memo item: *multipliers*				
θ_1	0.58333	θ_2	0.20833	θ_3	0.18750		

TABLE 3a

Part (a) of Proposition 3: Suboptimal Tax System

y_1	0.98045	x_1	0.31461	MTR_1	0.68539	T_1	0.66584
y_2	4.48505	x_2	1.81461	MTR_2	0.09270	T_2	2.67044
y_3	5.99328	x_3	3.00000	MTR_3	0.00000	T_3	2.99328
			Memo item: *multipliers*				
β_1	0.59333	β_2	0.20333	β_3	0.16500		

TABLE 3b

Part (b) of Proposition 3: Suboptimal Tax System

y_1	0.97706	x_1	0.31461	MTR_1	0.68539	T_1	0.66245
y_2	4.43556	x_2	1.77326	MTR_2	0.11337	T_2	2.66230
y_3	6.01294	x_3	3.00000	MTR_3	0.00000	T_3	3.01294
			Memo item: *multipliers*				
β_1	0.59333	β_2	0.20333	β_3	0.19500		

TABLE 3c

Part (c) of Proposition 3: Suboptimal Tax System

y_1	0.21247	x_1	0.19277	MTR_1	0.80723	T_1	0.01970
y_2	4.65921	x_2	1.78090	MTR_2	0.10955	T_2	2.87831
y_3	6.22369	x_3	3.00000	MTR_3	0.00000	T_3	3.22369
			Memo item: *multipliers*				
β_1	0.59333	β_2	0.22333	β_3	0.19500		

TABLE 4a

Part (a) of Proposition 4: Suboptimal Tax System

y_1	0.65074	x_1	0.25581	MTR_1	0.74419	T_1	0.39493
y_2	4.50322	x_2	1.75581	MTR_2	0.12209	T_2	2.74741
y_3	6.11026	x_3	3.00000	MTR_3	0.00000	T_3	3.11026
			Memo item: *multipliers*				
β_1	0.57333	β_2	0.21333	β_3	0.21000		

TABLE 4b

Part (b) of Proposition 4: Suboptimal Tax System

y_1	0.65401	x_1	0.25581	MTR_1	0.74419	T_1	0.39820
y_2	4.55369	x_2	1.79775	MTR_2	0.10112	T_2	2.75594
y_3	6.08992	x_3	3.00000	MTR_3	0.00000	T_3	3.08992
			Memo item: *multipliers*				
β_1	0.57333	β_2	0.21333	β_3	0.18000		

TABLE 4c

Part (c) of Proposition 4: Suboptimal Tax System

y_1	1.23368	x_1	0.36957	MTR_1	0.63043	T_1	0.86412
y_2	4.38975	x_2	1.79070	MTR_2	0.10465	T_2	2.59905
y_3	5.93777	x_3	3.00000	MTR_3	0.00000	T_3	2.93777
			Memo item: *multipliers*				
β_1	0.57333	β_2	0.19333	β_3	0.18000		

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