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Housing Debt and Consumption

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Abstract

The interaction between housing wealth, the financial portfolio of the consumer and consumption is a live issue. Life cycle models with closed form solutions under uncertainty are hard to find. In this paper we find analytical solutions for the effects of house price uncertainty and employment risk on consumption, savings and mortgage finance in a finite horizon life-cycle model. In each period the consumer decides whether to withdraw equity from the house or not, subject to a transaction cost and a constraint on the maximum mortgage loan to value ratio. Despite risk aversion we find that, if borrowing is allowed in the financial asset, the prime portfolio effect is the spread between the interest rate and the mortgage rate. House price uncertainty has an ambiguous effect on consumption, which depends on the interest rate differential and house price expectations since future house prices affect future remortgage possibilities. If unsecured debt is not possible, we find that the possibility of future liquidity constraints can reduce mortgage borrowing below the maximum possible.

Keywords: precautionary savings, employment risk, mortgages, housing

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1 Introduction

Housing is one of the major spending and financial decisions facing consumers. In the UK the owner occupation rate is 70%[1] and housing is one of the few spending decisions that has a specific asset to assist its financing: the mortgage. In addition, for the majority of consumers wealth largely consists of housing and pension wealth. Since the latter is usually untouchable, the flexible part of wealth is primarily housing equity. Thus, our key argument is that most cash-convertible wealth increases are in housing wealth - pension wealth is not realisable and most households have quite negligible average values of financial assets. In the context of a life cycle model the realisation of wealth accruals is driven by the desire to smooth consumption. However, since consumers need somewhere to live, increases in housing wealth can only be realised to a limited extent, mainly in three ways: either by trading down house quality, by moving into rented accommodation or by remortgaging. In this paper we focus on the last one. Our view is reinforced by the policy concern over the importance of housing equity withdrawal as a means of financing consumption sprees, which may then lead to lack of macroeconomic control of the economy. Of course the reverse side of the coin is the prospect of falls in house prices and, thus, in housing wealth, in which case housing debt can lead to consumers being locked into negative equity. Therefore, housing wealth and equity are major drivers of consumption. Figure 1 shows that in the 1990s housing borrowing secured against housing had risen by a substantially larger amount than that needed to fund new housing investment. This trend has reversed in 2008 when, due to the financial crisis, housing equity withdrawal has become negative for the first time in the last three decades, partly due to the fall in high loan-to-value new borrowing.

To analyse the effects of house price uncertainty and mortgage equity withdrawal on consumption, we use a real model with a time additive utility over a finite horizon and a positive rate of time preference. The individual has a portfolio composed of two financial assets: a mortgage and a riskless asset. The mortgage is an adjustable rate loan that is taken for the remaining life of the consumer but can be increased at any date. In the mortgage market there are two imperfections: refinancing the mortgage involves the payment of a fixed transaction cost and the new loan can never exceed the latest realised house price. In the last period of the life the house is sold and any outstanding debt must be paid off. In the model we include both

[1] Source: Eurostat SILC 2009.

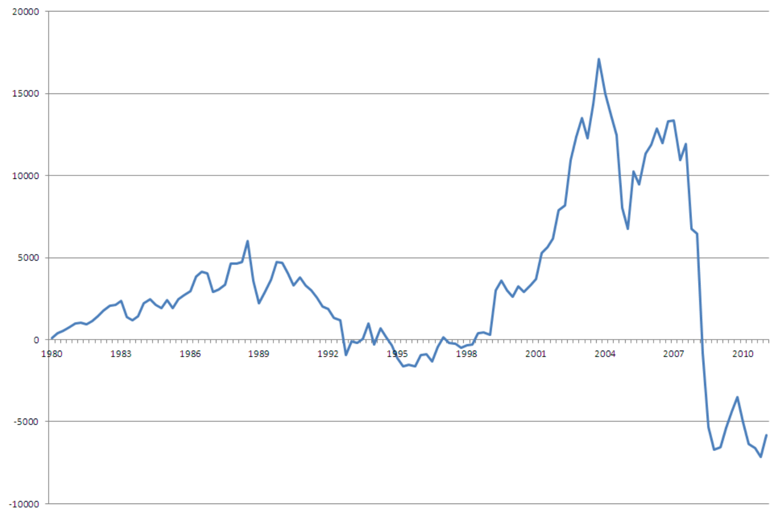

Figure 1: Housing equity withdrawal 1980-2011. Source: Bank of England.

employment and real house price risk, while wages and real interest rates are time varying and driven by stochastic processes, but they are foreseen by the individual

The new features of our framework (which in some respects is very stylised) are:

- We explicitly model housing equity withdrawal through remortgaging as house prices change in an environment in which decisions are taken under house price uncertainty and employment risk.

- There is no approximation in our approach; some variables are perfectly foreseen by the decision maker but others, such as house prices and employment status, are not, and despite this we derive a closed form analytical solution for optimal equity withdrawal and consumption.

- The result is a consumption function in which the current and random values of future house prices play a critical role. On the one hand, uncertain future house prices give the opportunity of high prices and so the chance of high future equity withdrawal, reducing the need for current buffer stock savings against future employment uncertainty. On the other hand, there is a risk that future house prices might be low, constraining the refinancing possibilities, which serves to depress current consumption. Which of these two effects dominate depends on the interest rate differential and house price expectations.

- We also find a financial portfolio decision important in financing consumption. If we have a recontractible one period mortgage and a one period bond, then with no saving or borrowing constraints in the bond, there is an issue of whether the consumer should finance consumption from bond borrowing or from remortgaging when housing wealth increases. We show that, despite any degree of risk aversion, with the preferences that we use, consumers will choose the cheapest financial source to finance consumption smoothing as housing wealth rises. The picture is more complicated in the presence of binding liquidity constraints. We show that in this case refinancing is possible even when the mortgage rate is relatively high. This result is consistent with a stream of the literature (Hurst and Stafford, 2004; Smith and Vass, 2004) that shows that the extent to which investors do refinance depends on the amount of liquid assets they hold.

- There are special end of life effects. Consumers cannot die in debt so any outstanding mortgage must be paid off by current realisable wealth. But there is a cleft stick: in the final period of life, house prices might jump to high levels leading to large unanticipated wealth increases, or they could fall leading to the need to find some financial way of paying off any existing mortgage debt outside of the value of the house.

The plan of the paper is to review the literature in section 2, give the assumptions in section 3 and derive the overall value function explicitly in sections 4. In section 5 we analytically derive the consumption function. In section 6 we present the case with liquidity constraints. We then briefly discuss extensions and conclude.

2 Literature Review

Housing impacts on individuals in various ways. One strand of the literature focuses on the role of housing on portfolio decisions. Some economies have a liquid rental market for housing and an illiquid retail market; others have a negligible rental market outside metropolitan areas but an active retail market (Chiuri and Jappelli, 2001). Flavin and Yamashita (2003) stress that with a thin rental market housing decisions have to balance financial asset portfolio considerations with the need for housing services. In the UK about 43% of homeowners bought their house outright without a mortgage and home equity accounts for about 60% of personal wealth (Banks et al., 2003). Thus owner occupiers face the risk of random house prices - both nominal and real. Several authors have argued that this risk explains why consumers do not invest in equities but in relatively safe assets: their overall portfolio including housing has the right balance of risk and return without investing in equities (Pelizzon and Weber, 2008). On the other hand, 57% of house buyers do have a mortgage and for them there are additional risks: the liquidity risk that with a bad income shock it would be difficult to maintain repayments (Fratantoni, 2001) and the risk that the mortgage interest rate might change. In addition, they have to make other decisions: when to repay the mortgage or, in common with the first group who have no mortgage, when to withdraw equity from the house. Furthermore, both of these groups face common shocks, such as wage and employment risk. The importance of these different risks varies over the life cycle; usually for highly geared young households, with housing debt a high proportion of wealth and income (Cocco, 2005), the liquidity risk is higher than for older households with, on average, more diversified wealth and more house equity. This implies that the typical life cycle portfolio composition sees systematic changes in the share of housing in wealth.

There is also a recent literature that looks at the collateral/buffer stock effect of housing investment on precautionary savings. The argument is that, although the retail housing market is illiquid, housing equity serves as a buffer stock of wealth against low probability but very negative income shocks. Thus, even if housing is rarely traded because of the high transaction costs, it allows a higher level of mean consumption since if the worst income events occur there is a buffer stock of wealth that can be realised. And, if trend real houses prices are growing then housing wealth is rising which may allow households to cut back financial asset saving (Berry *et al.*, 2009). Benito (2009) finds that housing equity acts as a buffer stock against bad shocks, e.g. in employment in the UK. However the buffer role of housing wealth must be offset against the fact that most homeowners hold mortgage debt against their housing (Campbell and Cocco, 2003), so that net housing wealth may be quite low, and also the fact that consumers need a roof over their head, so that they might not be able to sell the house where they live and, even if they can, they have to buy or rent another one. In the presence of high transaction costs in housing and a thin rental market, consumers can withdraw equity from their properties by using the buffer stock of housing wealth. This is predominantly executed by remortgaging. However, mortgage markets are imperfect in that there are costs in altering the debt position and limits on the amount that can be borrowed in a mortgage. This means that there may be inertia in the refinancing decision. For example, in the US, where most mortgages are at fixed nominal interest rates, there is active refinancing as interest rates change (Majumdar, 2004), but in Europe this is less common (Smith and Vass, 2004). Even in the US there is evidence of suboptimal timing effects (Agarwal *et al.*, 2009; Campbell, 2006).

Housing wealth interacts with unsecured borrowing. In itself housing wealth provides collateral against loans for non-housing purposes. Unsecured borrowing constraints may lead to equity withdrawal to finance consumption (Benito & Mumtaz, 2009), although the flip side of this is that negative home equity may lead to unsecured borrowing (Iacoviello, 2004). Disney *et al.* (2010) find some evidence of both in the UK.

Another issue in the literature is related to the degree of substitutability of housing and financial wealth in determining consumption. Will two households with the same aggregate wealth, the same labour income prospects and the same preferences follow the same consumption function if one of them has a much higher proportion of housing wealth than the other? The empirical results here are mixed (Hoynes and McFadden, 1996; Bostic *et al.* 2009; Majumdar, 2004). Similarly Attanasio *et al.* (2009) and Browning et al (2010) do not find effects on consumption of housing as a special form of wealth although there is some evidence of its role as a source of collateral. The question is important since consumer spending fluctuations are often seen as an important determinant of business cycle fluctuations. The main transmission mechanism for converting changes in housing wealth into disposable resources is the mortgage; therefore, analysing how mortgage decisions are made is crucial. The paper of Campbell and Cocco (2003) is a related major study of the relative advantages of fixed and variable rate mortgages that is close to our concerns. Li and Yao (2007) also study the effects of house price changes on household consumption. However, their analysis substantially differs from ours in that all mortgage refinances in their model are for consumption purposes only and are not driven by financial efficiency considerations; this arises from the assumption that the unsecured and secured borrowing rate are set to be equal. Remortgaging and equity withdrawal is one of the main ways in which housing wealth can finance other household choices. Some empirical work reinforces our theoretical results: Canner *et al.* (2002), Schwarz *et al.* (2008) and Ebner (2010) find that financial efficiency factors are important in determining remortgaging and Agarwal *et al.* (2006) and Campbell (2006) also give these factors a major role.

With its complex features, modelling these housing choices is not straightforward. Quite special assumptions are necessary to derive analytical conclusions; for example Ortalo-Magné and Rady (2006) use utility linear in consumption, a fixed utility premium for a large house over a smaller one and no uncertainty and derive analytical results about the nature of general equilibrium house prices. Campbell and Cocco (2003), Cocco (2005), Li and Yao (2007) and Yang (2009) rely on calibrated numerical simulation involving grid search over both their state and control variables to derive the optimum. This approach allows the use of more general assumptions about the stochastic processes that drive house prices, interest rates and labour income but does not yield analytical conclusions.

3 The Model Assumptions

We consider a finite horizon T of discrete time periods t. Within a period $t < T$ the timing of the model is as follows. At the start of the period the individual has a portfolio composed of two financial assets: a riskless asset A_t with rate of return r_t and a mortgage M_t that allows the investor to borrow against the value of the house at a rate ρ_t. We assume that both r_t and ρ_t are perfectly foreseen and that a mortgage loan initiated at t matures at T. In each period, consumers decide whether to refinance or not. When refinancing occurs, they redeem the existing debt, pay a transaction cost k and choose a new mortgage size M_{t+1}, which cannot exceed a fraction θ of the latest realised price p_t. The loan to value ratio (LTV) is a key statistic used by lenders and regulators in the UK. For example the Financial Services Authority reports that in 2010 more than 70% of new mortgages had LTV below 75% and less than 2% had LTV above 90%. House prices are stationary with a deterministic trend. Then, consumption c_t is chosen and next the individual receives labour income w_t^s: if the individual has a job, this is the wage w_t, if she is unemployed, it is benefits B_t. Each period there is a constant probability a of being unemployed and wages and benefits are time dependent but perfectly foreseen. Finally, the stock of financial assets A_{t+1} to carry forward into the next period is determined. This means that assets bear all the effects of shocks in employment, but this only affects the current period and is insignificant over the lifetime[2]. Note also that as viewed from earlier periods M_{t+1} is random, since it depends on the realisation of random house prices through the constraint $M_{t+1} \leq \theta p_t$.

The general form of the budget constraint in any period before the final one is

$$A_{t+1} = (1 + r_t)A_t + w_t^s + M_{t+1} - (1 + \rho_t)M_t - k - c_t$$

[2] This is unimportant and arises from the definition of the period, but the use of this timing simplifies the analysis. You may think of shopping on Saturday and being paid the subsequent Friday.

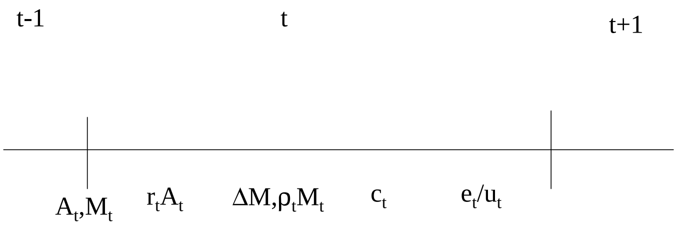

Figure 2: Timing

If there is no refinancing $M_{t+1} = M_t$ and no transaction cost has to be paid ($k = 0$):

$$A_{t+1} = (1 + r_t)A_t + w_t^s - \rho_t M_t - c_t$$

The consumer enters the final period with mortgage debt M_T, financial assets A_T and with a realised house price of p_T. At the period start the individual sells the house (but arranges to continue living in it for the duration of the period[3]), redeems any outstanding mortgage and consumes all the known cash on hand.

$$c_T = (1 + r_t)A_T + p_T + w_T^s - (1 + \rho_T)M_T - k$$

We also assume that in the last period for sure the individual is unemployed (but we could also think of this as retirement) so that $w_T^s = B_T$. This is without loss of generality since consumption is determined prior to knowledge of employment status and then in the last period it would have to be reined back to a level that will prove feasible if it turns out that the individual is unemployed, as it is impossible to die in debt.

Lifetime preferences are additive and there is a positive intertemporal discount rate ϕ:[4]

$$U_0 = \Sigma \phi^t u(c_t)$$

[3]There is a small market in which equity in the house can be realised in the last years of life eg by selling the house to a financial institution and buying back an option to live in it until death but this is not very well developed.

[4]It would be very simple to add a bequest motive, especially if the utility of bequests also has an exponential form.

The per-period utility function has a CARA form and depends only on consumption:

$$u(c_t) = 1 - \exp(-bc_t)$$

where b is the coefficient of absolute risk aversion. That is, there is a zero utility of housing and an inelastic labour supply. Since we assume that the individual keeps the house throughout her life, then omitting it from the value function is without loss of generality. Ignoring the disutility of work is more serious and is based on simplicity; we could include it assuming that jobs have fixed hours of work. Similarly, we could incorporate socio-demographic variables, such as the number of children into the analysis. CARA preferences have the advantage of exhibiting prudence and allowing us to derive exact solutions without having to approximate Euler equations. Much of the literature works with isoelastic felicity, which generally requires approximation to get solutions. On the one hand, there is some evidence (Gourinchas and Parker, 2002) that the approximation error involved can be substantial; on the other hand, since isoelastic preferences have unbounded marginal utility at zero consumption, they generally serve to keep cash on hand positive and, therefore, almost act like a liquidity constraint. With CARA, marginal utility is finite at zero consumption, so we may expect to see the consumer actively borrowing. However, the lifetime budget constraint prevents her dying in debt.

The overall optimisation problem has the form

$$\max \sum_{t=1}^{T} \phi^t \left[1 - \exp(-bc_t)\right]$$

subject to

$$\begin{aligned}
A_{t+1} &= (1+r_t)A_t + w_t^s + M_{t+1} - (1+\rho_t)M_t - k - c_t \qquad M_{t+1} > M_t, \quad t < T \\
A_{t+1} &= (1+r_t)A_t + w_t^s - \rho_t M_t - k - c_t \qquad M_{t+1} = M_t, \quad t < T \\
c_T &= (1+r_t)A_T + p_T + B_T - (1+\rho_T)M_T - k \\
M_t &\leq M_{t+1} \leq \theta p_t
\end{aligned}$$

The interesting empirical questions about mortgage refinance concern equity withdrawal and portfolio diversification among housing assets, mortgage debt and net financial assets. There are two reasons primarily for mortgage refinancing. The first motivation is that bank borrowing might get very cheap relative to mortgage debt in which case the consumer may wish to switch from secured to unsecured debt so far as this is allowed. The second reason for refinancing is that as house prices and, thus, wealth rise, the individual may want to withdraw equity from the house by increasing the mortgage. To focus on the latter we impose the constraint that $M_t \leq M_{t+1}$ and assume that the individual starts life with a given mortgage M_1. Our argument is that consumers will only wish to reduce their mortgage debt if there is an unanticipated income gain large enough so that consumption smoothing benefits overcome the transaction cost that has to be paid to reduce the mortgage. In practice in the reality this does not happen - it is confined to a tiny proportion of the population who receive large lump sum income gains, e.g. winning the national lottery or large bequests from wealthy relatives. Hence, we argue that the constraint $M_t \leq M_{t+1} \leq \theta p_t$ captures the essence of housing equity withdrawal combined with lender limits on the extent to which this is possible.

4 Value Function

In any period $t < T$ the value function is the maximum of the value function with refinancing and the value function without any mortgage refinancing:

$$V_t = \max\left\{V_t^R, V_t^{NR}\right\}$$

Based on Merton (1992), and Berloffa and Simmons (2003), we conjecture that V_t takes the form:

$$V_t(A_t, M_t, p_t) = \alpha_t - \beta_t \exp\left\{-b\delta_t\left[(1+r_t)\,A_t - (1+\rho_t)M_t\right]\right\} \tag{1}$$

The first main result of our paper is to derive this form and the functions α, β, δ. In fact, α will prove to be a discounting function depending on the rate of time preference and δ a discounting function depending on interest rates. The most interesting function is β, which depends on the expectation of future house prices and employment states and on the future optimal mortgage and saving decisions.

In periods before the final one (in which refinancing does not occur) the form of the value function depends on whether it is optimal to undertake refinancing. We derive the value functions at t with and without refinancing and then compare them to determine the optimal refinancing decision.

The appendix shows that conditional on the refinancing decision, the value function with refinancing is

$$\begin{aligned} V_t^R(A_t) &= 1+\phi\alpha_{t+1}-[\phi(E\beta_{t+1})\delta_{t+1}(1+r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\ &\quad \cdot \exp\{-b\Delta_t[(1+r_t)A_t-(1+\rho_t)M_t]\} W_t \\ &\quad \cdot \exp\left\{-b\Delta_t[M_{t+1}-k-M_{t+1}(1+\rho_{t+1})/(1+r_{t+1})]\right\}/\Delta_t \end{aligned}$$

where

$$W_t = \{a\exp[-b\delta_{t+1}(1+r_{t+1})w_t]+(1-a)\exp[-b\delta_{t+1}(1+r_{t+1})B_t]\}^{1/[1+\delta_{t+1}(1+r_{t+1})]}$$

is the expected utility term corresponding to next periods labour income and:

$$\Delta_t = \frac{\delta_{t+1}(1+r_{t+1})}{1+\delta_{t+1}(1+r_{t+1})}$$

is the "discounted future interest rate".

At the start of t, given that remortgaging takes place, the mortgage refinancing decision is to choose M_{t+1} to maximise $V_t^R(A_t)$ within the constraint $M_t \leq M_{t+1} \leq \theta p_t$. Defining $\lambda_{t+1} = 1-\left(1+\rho_{t+1}\right)/(1+r_{t+1})$, this is equivalent to minimising:

$$\exp(-b\Delta_t M_{t+1}\lambda_{t+1})$$

The decision rule is then:

$$\begin{aligned} M_{t+1} &= \theta p_t \qquad \text{if } \lambda_{t+1} > 0 \\ M_{t+1} &= M_t \qquad \text{if } \lambda_{t+1} < 0 \end{aligned}$$

Therefore, refinancing occurs only when $\lambda_t > 0$ (i.e.$r_{t+1} > \rho_{t+1}$) and the individual chooses maximum equity withdrawal. Integrating this into the value function

$$\begin{aligned} V_t^R(A_t) &= 1+\phi\alpha_{t+1}-[\phi(E\beta_{t+1})\delta_{t+1}(1+r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]}\,. \\ &\quad \cdot \exp\{-b\Delta_t[(1+r_t)A_t-(1+\rho_t)M_t]\}\cdot\exp\left\{-b\Delta_t\left[\max(\lambda_{t+1},0)\left(\theta p_t-\frac{k}{\lambda_{t+1}}\right)\right]\right\} \\ &\quad \cdot \exp\left\{b\Delta_t\left[\max(-\lambda_{t+1},0)\left(M_t-\frac{k}{\lambda_{t+1}}\right)\right]\right\}\cdot W_t/\Delta_t \end{aligned}$$

Similar arguments show that without refinancing

$$\begin{aligned} V_t^{NR}(A_t) &= 1 + \phi\alpha_{t+1} - [\phi(E\beta_{t+1})\delta_{t+1}(1+r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\ &\quad \cdot \exp\{-b\Delta_t[(1+r_t)A_t - M_t]\} \cdot \exp(-b\Delta_t\lambda_{t+1}M_t) \cdot W_t/\Delta_t \end{aligned}$$

By comparing the two value functions we obtain the condition under which refinancing occurs. At t the individual chooses to refinance the mortgage if $V_t^R(A_t) > V_t^{NR}(A_t)$.

If $\lambda_{t+1} > 0$ (the mortgage rate is lower than the interest rate) then refinancing occurs when

$$\theta p_t - M_t > \frac{k}{\lambda_{t+1}}$$

In terms of the debt service costs it pays to refinance to the highest extent possible by setting $M_{t+1} = p_t$ so long as the interest gain on the sum involved more than covers the transaction cost of refinancing.

On the other hand, if $\lambda_{t+1} < 0$ (the mortgage rate is higher than the interest rate), no refinancing occurs.

We can summarise the possible remortgage actions at time t (assuming that at some past point $t-s$ a maximum mortgage had been taken) as

$\lambda_{t+1} > 0$	$M_t = \theta p_{t-s}$	$\lambda_{t+1}\theta p_t > \lambda_{t+1}\theta p_{t-s} + k$	$M_{t+1} = \theta p_t$
		$\lambda_{t+1}\theta p_t < \lambda_{t+1}\theta p_{t-s} + k$	$M_{t+1} = \theta p_{t-s}$
$\lambda_{t+1} < 0$	$M_t = \theta p_{t-s}$	$\lambda_{t+1}\theta p_{t-s} + k > 0$	$M_{t+1} = \theta p_{t-s}$

Table 1: Possible remortgage actions

4.1 Overall Value Function

The overall value function is the larger of $V_t^R(A_t, M_t)$ and $V_t^{NR}(A_t, M_t)$, which gives us the result

Proposition 1 *The value function has the form*[5]

$$V_t(A_t, M_t, p_t) = \alpha_t - \beta_t \exp\{-b\delta_t[(1+r_t)A_t - (1+\rho_t)M_t]\}$$

[5] In fact the value function has this same form (with a different definition of β) if ρ is stochastic and k is time varying or stochastic as long as the current period mortgage rate and transaction cost are realised prior to the current refinancing decision.

where $\alpha_t, \delta_t, \beta_t$ are defined recursively through

$$\alpha_t = 1 + \phi\alpha_{t+1} \tag{2}$$

$$\delta_t = \Delta_t = \frac{\delta_{t+1}\left(1+r_{t+1}\right)}{1+\delta_{t+1}\left(1+r_{t+1}\right)} \tag{3}$$

$$\begin{aligned}\beta_t &= \left[\phi(E\beta_{t+1})\frac{\delta_t}{1-\delta_t}\right]^{1-\delta_t} W_t/\delta_t \exp\left[b\Delta_t \max\left(-\lambda_{t+1},0\right)M_t\right] \\ &\quad \cdot \exp\left\{-b\delta_t\left[\max\left(\lambda_{t+1},0\right)\cdot\max\left(\theta p_t - \frac{k}{\lambda_{t+1}}, M_t\right)\right]\right\}\end{aligned} \tag{4}$$

Solving the recurrence relations for α_t and δ_t

$$\begin{aligned} a_{T-t} &= \sum_{s=0}^{t}\phi^s \\ \delta_{T-t} &= \frac{\Pi_{s=0}^{s=t-1}(1+r_{T-s})}{1+\Sigma_{\tau=0}^{t-1}\Pi_{s=0}^{s=\tau}(1+r_{T-s})}\end{aligned}$$

The recurrence relation in β, which captures the effect of future house prices and employment uncertainty, requires some careful analysis.

4.2 $\mathbf{E}\beta_{t+1}$: The Effects of Future House Price Uncertainty

Future house prices $(p_{t+2}, p_{t+3}..)$ and the current remortage decision M_{t+1} determine the opportunities and desirability of future refinancing. It is through this channel that house price uncertainty impacts on current decisions. In our model, the effects of future house price uncertainty at time t work through the expression $E_t\beta_{t+1}$. However, since the mortgage is only binding for a single period (next period there is another refinancing point) and we have assumed that any trend in p_t is not stochastic, it is only immediate future house prices that impact on the M_{t+1} decision. Therefore, $E_{t+1}\beta_{t+2}$ is not a function of p_{t+1} and at time t the random term in β_{t+1} is:

$$\begin{aligned} F_{t+1} &= \exp\left\{-b\delta_{t+1}\left[\max\left(\lambda_{t+2},0\right)\cdot\max\left(\theta p_{t+1}-\frac{k}{\lambda_{t+2}}, M_{t+1}\right)\right]\right\} \\ &= \exp(-b\delta_{t+1}G_{t+1})\end{aligned} \tag{5}$$

where

$$
\begin{aligned}
G_{t+1} &= 0 \text{ if } \lambda_{t+2} < 0 \\
&= \lambda_{t+2} M_{t+1} \text{ if } \lambda_{t+2} > 0 \text{ and } \theta p_{t+1} \lambda_{t+2} - k < \lambda_{t+2} M_{t+1} \qquad (6) \\
&= \theta p_{t+1} \lambda_{t+2} - k \text{ if } \lambda_{t+2} > 0 \text{ and } \theta p_{t+1} \lambda_{t+2} - k > \lambda_{t+2} M_{t+1} \qquad (7)
\end{aligned}
$$

We want to compute $E_t F_{t+1}$ over p_{t+1}.

If $\lambda_{t+2} < 0$, the consumer does not wish to refinance. In this case G_{t+1} is not random, so $E_t F_{t+1} = F_{t+1}$.

If $\lambda_{t+2} > 0$ it is more complex. The second case (6) holds when house prices are such that the consumer potentially wishes to refinance to the maximum permissible extent but the savings from doing so will not cover the transaction cost of refinance. This occurs when

$$\theta p_{t+1} < M_{t+1} + \frac{k}{\lambda_{t+2}}$$

On the contrary, in the third case (7), which holds for p_{t+1} above this critical value, the gains from taking out a new maximum mortgage do cover the transaction cost. Hence when $\lambda_{t+1} > 0$, if we define the probability that the maximum remortgage will not cover the transaction cost by $\gamma_t = \Pr(\theta p_{t+1} < M_{t+1} + \frac{k}{\lambda_{t+2}})$, the expression for $E_t F_{t+1}$ becomes:

$$
\begin{aligned}
E_t F_{t+1} = {} & E_t \left\{ \exp\left[-b\delta_{t+1}\left(\theta p_{t+1}\lambda_{t+2} - k\right)\right] \middle| \theta p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+2}} \right\} (1 - \gamma_t) \\
& + \exp(-b\delta_{t+1}\lambda_{t+2} M_{t+1})\gamma_t
\end{aligned}
$$

Suppose that the support of the distribution of p_{t+1} is $[\underline{p}_{t+1}, \overline{p}_{t+1}]$:

$$
\begin{array}{lrcll}
\text{if} & M_{t+1} + \dfrac{k}{\lambda_{t+2}} & < & \theta \underline{p}_{t+1} & \text{then} \quad \gamma_t = 0 \\
\text{if} & M_{t+1} + \dfrac{k}{\lambda_{t+2}} & > & \theta \overline{p}_{t+1} & \text{then} \quad \gamma_t = 1
\end{array}
$$

Therefore, we have two boundary cases where either house prices are always so low that a maximum equity withdrawal will not cover the transaction cost or they are so high that with certainty the maximum remortgage is profitable. Table 2 summarises the various forms of

E_tF_{t+1}.Since

$$E_t\beta_{t+1} = \left[\phi(E_{t+1}\beta_{t+2})\frac{\delta_{t+1}}{1-\delta_{t+1}}\right]^{1-\delta_{t+1}} \cdot E_tF_{t+1}W_{t+1}/\delta_{t+1} \tag{8}$$

the nature of $E_t\beta_{t+1}$ follows. Thus, there are only effects of immediate future house price uncertainty on the current value function if $\lambda_{t+2} > 0$ In fact, extending this argument, the distribution of house prices at any future date τ only affects the current value function for those periods in which $\lambda_{\tau+1} > 0$. Appendix A.2 shows that[6]:

$$\begin{aligned}
\beta_t \quad = \quad & (E_{T-1}\beta_T)^{(1-\delta_t)(1-\delta_{t+1})..(1-\delta_{T-1})} \\
& \left(\frac{\phi}{1-\delta_t}\right)^{1-\delta_t}\left(\frac{\phi}{1-\delta_{t+1}}\right)^{(1-\delta_t)(1-\delta_{t+1})}...\left(\frac{\phi}{1-\delta_{T-1}}\right)^{(1-\delta_t)(1-\delta_{t+1})..(1-\delta_{T-1})} \\
& F_t\,(E_tF_{t+1})^{1-\delta_t}\,(E_{t+1}F_{t+2})^{(1-\delta_t)(1-\delta_{t+1})}...(E_{T-2}F_{T-1})^{(1-\delta_t)(1-\delta_{t+1})..(1-\delta_{T-1})} \\
& W_t\,(W_{t+1})^{1-\delta_t}...(W_{T-1})^{(1-\delta_t)(1-\delta_{t+1})..(1-\delta_{T-1})} \\
& \delta_t^{-\delta_t}\delta_{t+1}^{-\delta_{t+1}(1-\delta_t)}...\delta_{T-1}^{-\delta_{T-1}(1-\delta_t)(1-\delta_{t+1})..(1-\delta_{T-1})}
\end{aligned} \tag{9}$$

Note that there is an effect on β_t of the time to go to the horizon: the longer the remaining future, the higher the number of terms in the product for β since there are more future nodes. Therefore, in earlier periods β tends to be higher, which reflects the effect of the greater amount of uncertainty remaining. Conversely, towards the end of life there is little remaining uncertainty and so on these grounds less of a need for precautionary savings.

[6] If the probability of unemployment was either history dependent or uncertain, then so long as it is independent of house prices there is little impact on the expression for β: terms in W would become E_tW_{t+1}.

		$\mathbf{E}_t\mathbf{F}_{t+1}$
$\lambda_{t+2} < 0$		1
$\lambda_{t+2} > 0$	$M_{t+1} + \frac{k}{\lambda_{t+2}} < \theta \underline{p}_{t+1}$	$E_t \exp\left[-b\delta_{t+1}\left(\theta p_{t+1}\lambda_{t+2} - k\right)\right]$
	$M_{t+1} + \frac{k}{\lambda_{t+2}} > \theta \overline{p}_{t+1}$	$\exp\left(-b\delta_{t+1}\lambda_{t+2}M_{t+1}\right)$
	$\theta \underline{p}_{t+1} < M_{t+1} + \frac{k}{\lambda_{t+2}} < \theta \overline{p}_{t+1}$	$\exp\left(-b\delta_{t+1}\lambda_{t+2}M_{t+1}\right)\gamma_t + E_t\left\{\exp\left[-b\delta_{t+1}\left(\theta p_{t+1}\lambda_{t+2} - k\right)\right] \middle\vert \theta p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+2}}\right\}(1-\gamma_t)$

Table 2: EF_{t+1}

5 Consumption

From the relevant first order conditions for consumption we can derive the consumption function and examine its properties.

Proposition 2 *Optimal consumption is given by*

$$\begin{aligned} c_t &= -\frac{1-\delta_t}{b}\ln\left[\frac{\phi\left(E\beta_{t+1}\right)}{1-\delta_t}\right] + \delta_t\left[\left(1+r_t\right)A_t - (1+\rho_t)M_t\right] - \frac{1-\delta_t}{b}\ln(W_t) \\ &\quad +\delta_t\left[\max\left(\lambda_{t+1},0\right)\cdot\max\left(\theta p_t - \frac{k}{\lambda_{t+1}}, M_t\right) - \max\left(-\lambda_{t+1},0\right)M_t\right] \end{aligned}$$

This expression subsumes two cases: when $\lambda_{t+1} > 0$ and $\theta p_t\lambda_{t+1} - k > M_t\lambda_{t+1}$ the individual chooses to refinance at t, whereas when $\lambda_{t+1} > 0$ and $\theta p_t\lambda_{t+1} - k < M_t\lambda_{t+1}$ or $\lambda_{t+1} < 0$ optimally there is no refinance. The main features of the consumption function are summarised by proposition 3.

Proposition 3 *Uncertainty of future house prices may raise or lower current consumption. If*

(a) $\lambda_{t+2} > 0$ *and* $\theta \underline{p}_{t+1}\lambda_{t+2} - k > M_{t+1}\lambda_{t+2}$ *optimal consumption under uncertain house prices is lower than with house price certainty.*

(b) $\lambda_{t+2} > 0$ *and* $\theta E_t p_{t+1}\lambda_{t+2} - k > M_{t+1}\lambda_{t+2}$ *uncertainty in house prices may raise or lower consumption depending on the nature of the house price distribution, the interest rates and the degree of risk aversion*

(c) $\lambda_{t+2} > 0$ *and* $\theta E_t p_{t+1}\lambda_{t+2} - k < M_{t+1}\lambda_{t+2}$ *optimal consumption under uncertain house prices is higher than with house price certainty and precautionary savings are negative.*

Proof. See the appendix. ■

Both with and without refinancing the effect of future period employment risk and house price uncertainty is to shift the intercept of the consumption function by an amount that depends on the degree of risk aversion. Since it is possible that $\phi\left(E_t\beta_{t+1}\right) < 1-\delta_t$, the future uncertainty in house prices may actually increase rather than reduce consumption. Uncertain future house prices give the opportunity of high prices and so the chance of high future equity withdrawal, reducing the need for current buffer stock savings against future employment uncertainty. If future house prices are certain, then the expression for $E_t\beta_{t+1}$ changes when $\lambda_{t+2} > 0$. We can gauge the effect of this by comparing $E_t\beta_{t+1}$ with its value when house prices in the next period are constant at their mean $E_t p_{t+1}$. In (a) with both certain and uncertain house prices the consumer has the same expected refinancing possibilities; this is the standard case in which uncertainty lowers current consumption unambiguously and precautionary savings are positive. In (b) on the one hand there is a risk that future house prices might be low, constraining the refinancing possibilities, which serves to depress current consumption; on the other hand, house prices might be high, giving the opportunity of future equity withdrawal, higher than with certain house prices. The overall effect is ambiguous and depends on future house price expectations. The ambiguity exists even though we do not have any stochastic trend in house prices. If we had, this result would be even stronger. Finally in (c), when with certain future house prices no refinancing is undertaken ($\theta E_t p_{t+1} \lambda_{t+1} - k < M_{t+1} \lambda_{t+1}$), precautionary savings are negative and house price uncertainty serves to raise current consumption. The intuition is that in this case house price risk is just upside risk.

Summing up, in each of these cases (a)-(c) housing is acting like an intertemporal buffer stock of wealth with effects on the current level of savings. Knowing that in the future there will be a redeemable asset (though of uncertain value), the consumer can afford to borrow today. In addition, the composition of wealth between net housing wealth ($p_t - M_t$) and A_t has an impact on consumption. The marginal propensity to consume out of wealth is $\delta_t\,(1 + r_t)$ but out of a reduction of the current mortgage is $\delta_t(1 - \lambda_{t+1})$.

Moreover, there is an effect on consumption of the remaining length of the horizon. We have seen that $E_{t-1}\beta_t$ tends to fall through time and this serves to yield consumption growth through time *ceteris paribus*. As time passes there is less remaining uncertainty and thus less of a need for precautionary savings.

Proposition 4 *Labour income uncertainty depresses consumption and raises savings.*

Proof. See the appendix. ■

As usual with CARA preferences, consumption is linear in net wealth $(1 + r_t)\,A_t - M_t$ but is nonlinear in expected current period labour income. The latter is essentially an artifact of the assumed within period timing where consumption has to be chosen before the employment status of the current period is realised. In fact this labour income risk reduces consumption for familiar reasons due to the risk aversion of the consumer. For current labour income this works through the expected utility of current labour earnings, for future labour income risk it works through the concavity of $E_t\beta_{t+1}$ in labour incomes.

Remark 5 *When it is optimal to refinance, there is maximum equity withdrawal ($M_{t+1} = \theta p_t$)*

and consumption increases

When it is optimal to refinance, the effect of equity withdrawal on current consumption is unambiguously non-negative. However, with no refinancing, a shift in the current mortgage state may increase or reduce consumption. It will have a negative effect on consumption when $\lambda_{t+1} < 0$ and a positive one when $\lambda_{t+1} > 0$ but the cost saving from increasing the current mortgage M_t to p_t does not cover the transaction cost of doing so.

Remark 6 *Current mortgage rates influence consumption through λ_{t+1}: a fall in the mortgage rate encourages refinance which raises consumption.*

The current mortgage interest rate only has effects via λ_{t+1}. When $\lambda_{t+1} > 0$ and $p_t > M_t$ there is scope for increasing the mortgage so long as the interest gain covers the transaction cost. A fall in the current mortgage rate increases the chance of refinancing and therefore may cause a switch from no refinancing of the current mortgage to refinancing to the maximum extent possible, which causes a jump in consumption. Only a part of the current wealth change arising from remortgaging is consumed in the current period (the coefficient δ), while the other part of the wealth change is used to smooth future consumption.

Remark 7 *The current savings interest rate has income effects on consumption, the effects of future interest rates on savings are complex.*

The current saving interest rate has obvious income effects on consumption. First, there is a consumption increasing effect through raising capital income when the consumer has positive financial assets, but a decreasing effect through raising the debt service cost when assets are negative. Future savings rates and especially that of the next period have much more complex effects: directly through altering the slope of the consumption function in most variables, indirectly through varying λ_{t+1} and affecting the discounting terms in $E_t\beta_{t+1}$. If at t the foreseen r_{t+s} increases ($s > 0$) all the terms in δ_τ for $t \leq \tau < s$ increase. In particular if $s = 1$ then δ_{t+1} is unaffected and $1/(1 + \delta_{t+1}(1 + r_{t+1}))$ falls, so that the marginal propensity to consume out of labour income expected for this period decreases.

Remark 8 *An increase in risk aversion reduces the variability of consumption over time.*

As the degree of risk aversion rises, the absolute value of the intercept of the consumption function falls in any period and so between any two periods there is less variability in consumption. In particular there are smaller jumps in consumption when refinancing occurs.

6 Liquidity constraints

We have assumed that there are no restrictions on borrowing in a one period financial asset; the only constraint is that debt must be cleared prior to death. As long as the lifetime budget constraint is respected, consumers can borrow as much as they wish. This assumption is important in making the key determinant of the refinancing decision the relative interest rate advantages of borrowing against the house or against future wealth (including future labour earnings and the future value of the house). Another effect is that despite the risk averse preferences, the special implication of CARA is that optimal financial policy is essentially bang-bang, using only either mortgage or bank finance as debt both to smooth shocks and finance the house. Empirically this is far-fetched because, according to the Family Resources Survey, in 2004 12% of British households held no savings and any unsecured debt was confined to credit card debt. What are the implications of assuming that $A_t \geq 0$ is a market constraint? The refinancing decision becomes much less transparent since it has to take account of the fact that remortgaging now influences the chance with which in the next period the consumer may end up being liquidity constrained, e.g. if she loses her job. The multiperiod decision tree increases hugely in complexity since then at any period current decisions influence the probability that in the next and further future periods it will be optimal to be credit constrained with $A_t = 0$. Even to solve for the optimal policy in this framework, let alone estimate parameters, numerical solution is necessary. And if that is the case then why use CARA or CRRA preferences? It would make more sense to use more flexible preferences allowing for a clear distinction between risk aversion and intertemporal substitution for example. What we can do however is highlight the new types of optimal solution phase that can arise by retaining our model assumptions but looking at just a three period example. The result is that when it is optimal to set $A_t = 0$ it may be optimal to have a positive level of mortgage finance below the maximum possible or even the maximum mortgage finance possible even if the mortgage rate is high. On the other hand when optimally $A_t > 0$ it is optimal to minimise on mortgage finance if the mortgage rate is higher than the interest rate.

To illustrate these effects, we take a special case of our framework in which the there is no refinancing transaction cost ($k = 0$), $\theta = 1$, there is no unemployment risk (there is a certain wage at $T-1$ and certain benefit income at T) and the mortgage interest rate ρ is always above the one period financial asset rate r, which is a reasonable assumption in this case because r

is now the saving rate. First consider the last two periods T and $T-1$. In the final period the individual consumes any savings held at the period start, cash in the house and repays any outstanding mortgage debt and receives social security benefit so utility is given by

$$u(c_T) = \exp(-b[p_T + w_T])\exp(-b((1+r_T)A_T - (1+\rho_T)M_T))$$

At period $T-1$ the value function is then

$$\begin{aligned} V_{T-1} &= 1+\phi - Z_{T-1}\exp(-b(M_T - A_T)) \\ &\quad -\phi\exp(-b((1+r_T)A_T - (1+\rho_T)M_T))X_{T-1}E\exp(-b[p_T+w_T]) \\ \text{where } Z_{T-1} &= \exp(-b[(1+r_{T-1})A_{T-1} + w_{T-1} - (1+\rho_{T-1})M_{T-1}], X_{T-1} \\ &= E\exp(-b[p_T+w_T]) \end{aligned}$$

The $T-1$ decision variables are A_T, M_T and the constraints are $A_T \geq 0, M_{T-1} \leq M_T \leq p_{T-1}$. The derivatives $\partial V_{T-1}/\partial A_T, \partial V_{T-1}/\partial M_T$ evaluated at particular points are critical in determining the optimal decisions at $T-1$. These derivatives are

$$\begin{aligned} \partial V_{T-1}/\partial A_T &= -Z_{T-1}\exp(-b(M_T - A_T) + \phi X_{T-1}(1+r_T)\exp(-b((1+r_T)A_T - (1+\rho_T)M_T)) \\ \partial V_{T-1}/\partial M_T &= Z_{T-1}\exp(-b(M_T - A_T)) - \phi X_{T-1}(1+\rho_T)\exp(-b((1+r_T)A_T - (1+\rho_T)M_T)) \end{aligned}$$

At any time t the individual can optimally choose to be in one of four states according to the values of the derivatives above (see appendix):

(1) $A_t > 0, M_t = M_{t-1}$

(2) $A_t = 0, M_t = M_{t-1}$

(3) $A_t = 0, M_{t-1} < M_t < p_{t-1}$

(4) $A_t = 0, M_t = p_{t-1}$

The values of Z_{T-1} and X_{T-1} in relation to the current mortgage size, house value and the two interest rates are critical in determining the optimal regime for the individual. Z_{T-1} reflects the utility from consuming period $T-1$ disposable resources, X_{T-1} reflects the expected utility from consuming period T real disposable resources. Figure (3)below shows how the parameters control the optimal choices at $T-1$. If Z_{T-1}/X_{T-1} is sufficiently high the individual is credit constrained and takes the maximum mortgage at $T-1$. If Z_{T-1}/X_{T-1} is sufficiently low the

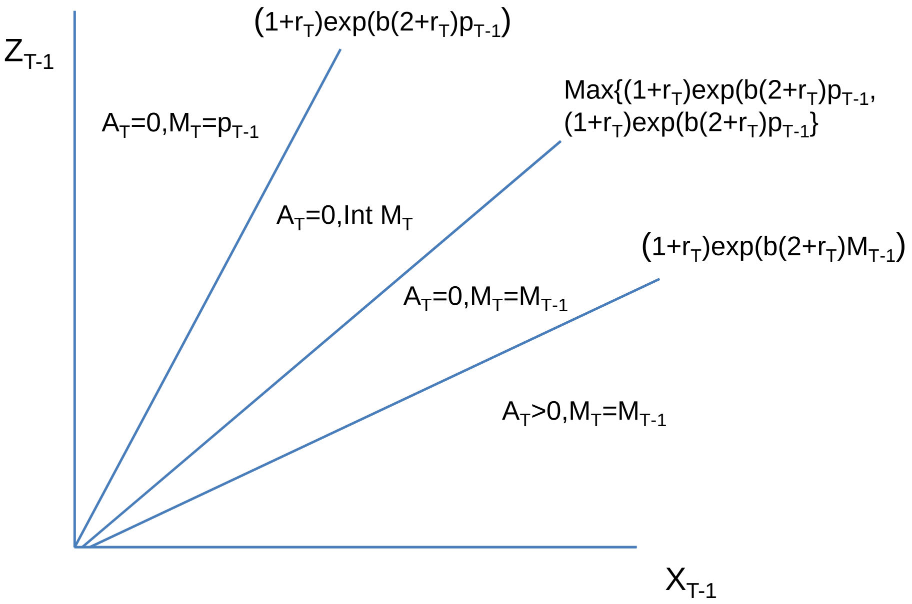

Figure 3: Optimal regimes in the presence of liquidity constraints.

individual does not refinance the mortgage but keeps it at its minimum level and also saves. Between these extremes the relation between the current house value and mortgage and the relation between the two interest rates determine whether the individual refinances with an increased mortgage but at a level below the maximum obtainable.

The value function at $T-1$ is (with $\rho^* = 1+\rho$ and $R = 1+r$) takes four possible values:

1. If $\exp(-b(R_{T-1}A_{T-1} - \rho^*_{T-1}M_{T-1})) > X_{T-1}\phi\rho^*_T \exp(b(w_{T-1} + (1+\rho^*_T)p_{T-1}))$

$$
\begin{aligned}
V_{T-1} \quad &= \quad 1 + \phi - \exp(-b\frac{R_T}{1+R_T}(R_{T-1}A_{T-1} - \rho^*_{T-1}M_{T-1}))\exp(-b(w_{T-1} + p_{T-1})) \\
&\quad -\phi X_{T-1}\exp(b\rho^*_T p_{T-1})
\end{aligned}
$$

2. If $\rho^*_T X_{T-1}\phi\exp(b(w_{T-1} + (1+\rho^*_T)p_{T-1})) > \exp(-b(R_{T-1}A_{T-1} - \rho^*_{T-1}M_{T-1}))$ $> \rho^*_T X_{T-1}\phi\exp(b(w_{T-1} + (1+\rho^*_T)M_{T-1})$

$$
\begin{aligned}
V_{T-1} \quad &= \quad 1 + \phi - \exp(-b\frac{\rho^*_T}{1+\rho^*_T}(R_{T-1}A_{T-1} - \rho^*_{T-1})M_{T-1}) \\
&\quad \cdot\exp(-b\frac{\rho^*_T}{1+\rho^*_T}w_{T-1})(\phi X_{T-1}\rho^*_T))^{\frac{1}{1+\rho^*_T}}\frac{1+\rho^*_T}{\rho^*_T}
\end{aligned}
$$

3. If $\rho_T^* X_{T-1}\phi \exp(b(w_{T-1} + (1+\rho_T^*)M_{T-1}) > \exp(-b(R_{T-1}A_{T-1} - \rho_{T-1}^* M_{T-1}))$

   $> R_T X_{T-1}\phi \exp(b(w_{T-1} + (1+\rho_T^*)M_{T-1}))$

$$\begin{aligned} V_{T-1} &= 1+\phi - \exp(-b(R_{T-1}A_{T-1} - (\rho_{T-1}^* - 1)M_{T-1}))\exp(-bw_{T-1}) \\ &\quad -\phi X_{T-1}\exp(b\rho_T^* M_{T-1}) \end{aligned}$$

4. If $R_T X_{T-1}\phi \exp(b(w_{T-1} + (1+\rho_T^*)M_{T-1})) > \exp(-b(R_{T-1}A_{T-1} - \rho_{T-1}^* M_{T-1}))$

$$\begin{aligned} V_{T-1} &= 1+\phi - \exp(-b\frac{R_T}{1+R_T}(R_{T-1}A_{T-1} - \rho_{T-1}^*)M_{T-1})\exp(-b\frac{\rho_T^*}{1+\rho_T^*}w_{T-1}) \\ &\quad \cdot(\phi X_{T-1}\rho_T^*))^{\frac{1}{1+R_T}}\frac{1+R_T}{R_T}\exp(\frac{b(\rho_T^* - R_T)}{1+R_T}M_{T-1}) \end{aligned}$$

At $T-2$ the decision variables are M_{T-1}, A_{T-1} which are selected to maximise $u(c_{T-2}) + \phi E_{T-2}V_{T-1}$. V_{T-1} is a piecewise continuous function with four regimes or branches V_{T-1}^i, the probability of being in each branch i at $T-1$ depends on the choices made at $T-2$. So we can write

$$EV_{T-1} = \Sigma_i \Pr(i|M_{T-1}, A_{T-1})E(V_{T-1}^i|\text{ branch } i \text{ occurs})$$

For example for the first branch

$$\begin{aligned} \Pr(i &= 1|M_{T-1}, A_{T-1}) \\ &= \Pr([E_{T-1}\exp(-bp_T)]\exp(b(w_{T-1} + (1+\rho_T^*)p_{T-1})) \\ &< \exp(bB_T)\frac{\exp(-b(R_{T-1}A_{T-1} - \rho_{T-1}^* M_{T-1}))}{\phi\rho_T^*}) \end{aligned}$$

and similarly for the other branches. This demonstrates the way in which the complexity of the current decision increases with the liquidity constraints. At $T-2$ the choices of M_{T-1} and A_{T-1} will involve trade-offs between marginal value changes at $T-2$ and the expected marginal value changes at $T-1$ over the four branches, as well as shifts in the chances that each branch occurs at $T-1$.Given the strictly concave within period utility used, the marginal values are nonlinear in the decision variables and analytical solution is not possible.

To summarise adding liquidity constraints means that explicit solution of the lifetime problem requires simulation and calibration and hence the generality of a full analytic solution is lost. Nevertheless we can see that we would expect some individuals to be independent of financial markets ($M = A = 0$), others to be savers with no or a minimal mortgage and yet others to not save and have either a maximum mortgage or a mortgage but at a lower level than the maximum attainable.

7 Other extensions

Our approach suggests some obvious areas for future research and has various special assumptions whose force we try to evaluate in this section.

First, generally the amount that can be borrowed on a mortgage is limited not only by the house value but also by the current income level. The rationale for this constraint seems to be on debt service cost grounds. For example in the UK the mortgage cannot usually exceed three or four times the income. However, to incorporate the income constraint into the analytical framework would raise nothing new conceptually and it would make the algebra more complicated. Given that the individual wishes to refinance in period t, the new mortgage decision would be[7]

$$M_{t+1} = \min\left(p_t, \mu w_{t-1}\right) \qquad if \qquad \lambda_t > 0$$

Again it is optimal to remortgage only if there is a financial advantage:

$$\min\left(p_t, \mu w_{t-1}\right) \max\left(\lambda_t, 0\right) - k - M_t \lambda_t > 0$$

By substituting this condition into the value function, we obtain the term in mortgage activity:

$$\min\left\{\exp(-b\Delta_t \lambda_t M_t), \exp\left\{-b\Delta_t(1+r_{t+1})\left[\max\left(\lambda_t \min\left(p_t, \mu w_{t-1}\right), 0\right) - k\right]\right\}\right\} W_t/\Delta_t$$

Even in this case all the effects of uncertainty are captured in β, whose recurrence relation (4) becomes

$$\beta_t = \left[\phi(E\beta_{t+1})\frac{\delta_t}{1-\delta_t}\right]^{1-\delta_t} \exp\left\{-b\delta_t \max\left[\lambda_t M_t, \max\left(\lambda_t \min\left(p_t, \mu w_{t-1}\right), 0\right) - k\right]\right\} W_t/\delta_t$$

Then the time path of β can be easily deduced by following the methods of section 5.1.

Another obvious extension would be to allow for more than one type of house, e.g. a large expensive house with price p_t and a small cheaper house with price π_t, where both prices are

[7] Since the wage at t is unknown at the time of refinancing, the income constraint works on w_{t-1}.

uncertain and $\pi_t < p_t$. Consumers could then trade down from large to small houses and vice versa. In terms of housing decisions, at t there are three choices: retain the existing house and mortgage, retain the existing house but refinance, change house and refinance. In this context it makes sense to add a second transaction cost k_t^h that is incurred when changing house (in addition to the refinancing transaction cost). The effect is to add a third branch to the value function and the overall value function is then the maximum over the three branches. If we keep the other assumptions (especially that of no liquidity constraints), the refinancing decision will have the same form, once any house purchase/sale has been decided. Again all uncertainty will be channelled through β, the value function will have a similar structure and the recurrence relation for β (4) will become

$$\begin{aligned}\beta_t &= [\phi(E\beta_{t+1})\delta_{t+1}(1+r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\ &\quad \cdot \exp\{-b\delta_t \max[p_t - \pi_t - k_t^h + \pi_t \max(\lambda_t, 0) - k, \\ &\quad \lambda_t M_t, p_t \max(\lambda_t, 0) - k]\} W_t/\Delta_t\end{aligned}$$

Furthermore, in the compulsory retirement case we could make the date of retirement uncertain. This is similar to dropping the perfect foresight assumption on wages and has significant effects (see Berloffa and Simmons, 2003, and below).

The main special assumptions that we have made are:

- CARA preferences depending only on consumption and independent of housing or leisure or socio-demographics.

  A problem with CARA is that optimal consumption may turn out to be negative since marginal utility is finite at zero consumption. We could make the utility vary with housing and leisure; in the context of a model with a single indivisible house type, the former would add little, but the latter would be interesting and, although most of the structure of the value function, the refinancing decision and consumption would remain unchanged, there would be some additional preference effects (see Berloffa and Simmons, 2003).

  With general strictly concave preferences over just consumption $u(c_t)$, the refinancing decision would have a stronger interaction with other sources of disposable wealth. The problem at t can be written as

$$V_t = \max u(c_t) + EV((1 + r_{t+1})A_{t+1} - (1 + \rho_{t+1})M_{t+1})$$

or using the budget constraint with $Z_t = (1 + r_t)A_t - (1 + \rho_t)M_t + w_t^s - k$

$$\max u(c_t) + EV((1 + r_{t+1})(Z_t - c_t) + (1 + r_{t+1})M_{t+1} - (1 + \rho_{t+1})M_{t+1})$$

If c_t is set at its optimal level from the envelope theorem in c_t the marginal value of M_{t+1} is

$$\partial V_t / \partial M_{t+1} = EV'()(r_{t+1} - \rho_{t+1})$$

With CARA this generates a corner solution in M_{t+1} but with general preferences $EV'()$ varies nonlinearly with M_{t+1}.

- Perfect foresight of unemployment benefit, wage and interest rates.

  This is an important simplification with potentially large implications. If interest rates are uncertain, the consumer has a real portfolio choice, not just a choice driven by the asset with the highest return. We might then expect to get some diversification of the portfolio depending on the covariance between the interest rates and the covariance between interest rates and house prices.

  Uncertainty in the real wage when employed can readily be incorporated so long as it is uncorrelated with house prices and with the chance of having a job. The value function, the refinancing decision and consumption will have a similar functional structure where the expected labour income term W_t becomes

$$W_t = aE_t \exp(-b\delta_{t+1} (1 + r_{t+1}) w_t) + (1 - a) \exp(-b\delta_{t+1} (1 + r_{t+1}) B_t)$$

  If there is correlation between house prices and wages it is more complex. Another assumption we have made is that the real house price is stationary; we could relax this, allowing for a random walk with drift, and get similar results.

- A chance of being jobless that is constant (and foreseen) through time.

  It would be easy to allow the chance of unemployment to be time dependent - essentially it is just a notational change, the effect will be absorbed into W_t, so all the results will carry through.

- Omission of the impact of the tax system.

  The treatment of interest income and payments, capital gains on housing and of implicit user services of owned housing differs between tax systems, so it is important to interpret these variables as post-tax.

- A single one period financial asset with a perfect capital market.

  The biggest omission is the role of voluntary or involuntary contributions to pension schemes. Pension wealth can be defined by either the value of accumulated contributions to date or by the estimated pension income that will accrue at maturity. With the former approach and using the Family Resources Survey, Warren *et al.* (2001) find that individual median wealth was about £63k, which decomposed into median wealth of pensions £26k, financial wealth £1k and housing wealth £24k. The pension wealth divided into about 39% in state pensions, 53% in occupational pensions and only 8% in "discretionary" pensions. This asset structure accords with that found by others where liquid or risky financial assets are an insignificant proportion of individual wealth. Pension wealth is clearly important and serves both to remove some effects of uncertain date of death and to act as a buffer against asset shocks, e.g. falling real house prices late in life.

A further factor is that in reality there are wedges between the savings rate and the borrowing rate in bond type finance. Including this would affect the refinancing decision.

8 Conclusions

This paper uses a framework that allows us to analytically solve for the value function and the optimal lifecycle policies for consumption, saving in financial assets and mortgage debt when a finitely lived, risk averse individual faces employment risk and uncertainty of house prices. This approach gives the advantage of being able to derive general propositions as opposed to specific simulation results without resorting to approximations which may have substantial inaccuracy. The financial asset market is perfect and variable rate mortgages up to a fraction of the value of the house are allowed. Using CARA preferences and the assumption that house prices do not have stochastic trends while interest rates are certain and wages are foreseen facilitate explicit solution. However, the form of the value function and optimal policy that we find is generally robust to relaxation of these special assumptions, similar results would follow if we had more constraints on available mortgages, uncertain wages, preferences depending not only on consumption but also on housing services, more than one type of house. One main result in all these cases is that there is a single "sufficient statistic" through which the effects of uncertainty on the value function and the optimal policies are channelled. This is due to the CARA form of preferences.

In terms of detail, we find that consumption is linear in wealth with an intercept that depends on future employment and house price risk, and a slope that depends on risk aversion and interest rates. The effects of housing wealth on consumption and saving/borrowing decisions primarily work through the mortgage. Consequently, the analysis of the refinancing of mortgages is important to understand how housing wealth can act as a buffer stock against bad shocks, e.g. in employment. We find that without liquidity constraints and with foreseeable interest rates, the refinancing decision is driven by financial efficiency considerations. The individual will refinance to the maximum extent possible in those periods in which the financial gains from doing so cover the transaction cost. However, if liquidity constraints are binding, then the individual might choose mortgage refinance even if the mortgage rate is high. Housing wealth and mortgage finance impact on consumption so that in periods when it is optimal to refinance consumption jumps due to equity withdrawal. The financial gains from refinance are used partly to finance present and partly future consumption. Therefore, sometimes consumption tracks cash on hand and is not fully smoothed. In other periods there is an ambiguous effect of mortgage debt on consumption.

House price uncertainty may raise or reduce current consumption. On the one hand, the opportunity of high house prices gives the chance of high equity withdrawal. On the other hand, there is a risk that house prices might fall, constraining the refinancing possibilities, which serves to depress consumption. The overall effect is ambiguous and depends on the interest rate differential and house price expectations.

A Appendix

A.1 Value function

At T all available wealth is consumed and no mortgage interest is paid since no new mortgage debt is contracted so

$$0 = (1 + r_t)\, A_T + p_T - (1 + \rho_T) M_T - c_T + B_T - k$$

In the last period the value function is:

$$V_T = 1 - \exp\left\{-b\left[(1 + r_T)\, A_T - (1 + \rho_T) M_T\right]\right\} \exp(-bB_T) \exp\left[-b(p_T - k)\right]$$

This result is obtained simply substituting the budget constraint at T into the instantaneous CARA utility function. Hence, at T

$$\begin{aligned}
\alpha_T &= 1 \\
\beta_T &= \exp(-bB_T) \exp\left[-b(p_T - k)\right] \\
\delta_T &= 1
\end{aligned}$$

In any period before the final one, with refinancing the Bellman's equation says that c_t is determined to :

$$\max_{c_t}\{u(c_t) + \phi EV_{t+1}(A_{t+1})\, | A_{t+1} = (1 + r_t)\, A_t + M_{t+1} - (1 + \rho_t) M_t - c_t + w_t^s - k\}$$

That is equivalent to:

$$\max_{c_t}\{u(c_t) + \phi E\left\{\alpha_{t+1} - \beta_{t+1} \exp\left[-b\delta_{t+1}\left((1 + r_{t+1})\, A_{t+1} - (1 + \rho_{t+1}) M_{t+1}\right)\right]\right\}$$
$$| A_{t+1} = (1 + r_t)\, A_t + M_{t+1} - (1 + \rho_t) M_t - c_t + w_t^s - k\}$$

or

$$\max_{c_t}\{1 - \exp(-bc_t) + \phi\{\alpha_{t+1} - E\{\beta_{t+1} \exp[-b\delta_{t+1}\, (1 + r_{t+1})\, ((1 + r_t)\, A_t$$
$$+ M_{t+1} - (1 + \rho_t) M_t - c_t + w_t^s - k) - (1 + \rho_{t+1}) M_{t+1}]\}\}\}$$

Since we are assuming that the risk of unemployment is foreseen and is independent of the uncertain house prices:

$$
\begin{aligned}
& E\left\{\beta_{t+1}\exp\left[-b\delta_{t+1}\left(1+r_{t+1}\right)w_t^s\right]\right\} \\
= \; & E\beta_{t+1}\left\{a\exp\left[-b\delta_{t+1}\left(1+r_{t+1}\right)w_t\right]+(1-a)\exp\left[-b\delta_{t+1}\left(1+r_{t+1}\right)B_t\right]\right\}
\end{aligned}
$$

The first order condition gives:

$$
\begin{aligned}
\exp(-bc_t) \;=\; & \left[\phi(E\beta_{t+1})\delta_{t+1}\left(1+r_{t+1}\right)\right]\exp\left[b\delta_{t+1}\left(1+r_{t+1}\right)c_t\right]\exp(b\delta_{t+1}(1+\rho_{t+1})M_{t+1}) \\
& \cdot\exp\left\{-b\delta_{t+1}\left(1+r_{t+1}\right)\left[\left(1+r_t\right)A_t+M_{t+1}-(1+\rho_t)M_t-k\right]\right\} \\
& \cdot\left\{a\exp\left[-b\delta_{t+1}\left(1+r_{t+1}\right)w_t\right]+(1-a)\exp\left[-b\delta_{t+1}\left(1+r_{t+1}\right)B_t\right]\right\}
\end{aligned}
$$

that implies

$$
\begin{aligned}
\exp(-bc_t) \;=\; & \left[\phi(E\beta_{t+1})\delta_{t+1}\left(1+r_{t+1}\right)\right]\exp\left\{-b\delta_{t+1}\left(1+r_{t+1}\right)\left[\left(1+r_t\right)A_t-(1+\rho_t)M_t\right]\right\} \\
& \cdot\exp\left[b\delta_{t+1}\left(1+r_{t+1}\right)c_t\right]\exp(b\delta_{t+1}(1+\rho_{t+1})M_{t+1}) \\
& \cdot\exp\left\{-b\delta_{t+1}\left(1+r_{t+1}\right)\left[M_{t+1}-k\right]\right\} \\
& \cdot\left\{a\exp\left[-b\delta_{t+1}\left(1+r_{t+1}\right)w_t\right]+(1-a)\exp\left[-b\delta_{t+1}\left(1+r_{t+1}\right)B_t\right]\right\}
\end{aligned}
$$

Hence:

$$
\begin{aligned}
\exp(-bc_t) \;=\; & \left[\phi(E\beta_{t+1})\delta_{t+1}\left(1+r_{t+1}\right)\right]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\
& \cdot\exp\left\{b\frac{\delta_{t+1}\left(1+r_{t+1}\right)}{1+\delta_{t+1}\left(1+r_{t+1}\right)}\left[\left(1+r_t\right)A_t-M_t\right]\right\} \\
& \cdot\exp\left\{b\frac{\delta_{t+1}\left(1+r_{t+1}\right)}{1+\delta_{t+1}\left(1+r_{t+1}\right)}\left[M_{t+1}-k-\frac{(1+\rho_{t+1})M_{t+1}}{1+r_{t+1}}\right]\right\} \\
& \cdot\{a\exp\left[-b\delta_{t+1}\left(1+r_{t+1}\right)w_t\right] \\
& +(1-a)\exp\left[-b\delta_{t+1}\left(1+r_{t+1}\right)B_t\right]\}^{1/[1+\delta_{t+1}(1+r_{t+1})]}
\end{aligned}
\tag{10}
$$

and

$$
\begin{aligned}
\exp\left[b\delta_{t+1}\left(1+r_{t+1}\right)c_t\right] = {} & \left[\phi(E\beta_{t+1})\delta_{t+1}\left(1+r_{t+1}\right)\right]^{-\delta_{t+1}(1+r_{t+1})/[1+\delta_{t+1}(1+r_{t+1})]} \\
& \cdot \exp\left\{b\frac{\left[\delta_{t+1}\left(1+r_{t+1}\right)\right]^2}{1+\delta_{t+1}\left(1+r_{t+1}\right)}\left[\left(1+r_t\right)A_t-(1+\rho_t)M_t\right]\right\} \\
& \cdot \exp\left\{b\frac{\left[\delta_{t+1}\left(1+r_{t+1}\right)\right]^2}{1+\delta_{t+1}\left(1+r_{t+1}\right)}\left[M_{t+1}-k-\frac{(1+\rho_{t+1})M_{t+1}}{1+r_{t+1}}\right]\right\} \\
& \cdot \left\{a\exp\left[-b\delta_{t+1}\left(1+r_{t+1}\right)w_t\right]\right. \\
& \left.+\left(1-a\right)\exp\left[-b\delta_{t+1}\left(1+r_{t+1}\right)B_t\right]\right\}^{-\frac{\delta_{t+1}(1+r_{t+1})}{1+\delta_{t+1}(1+r_{t+1})}}
\end{aligned}
$$

Therefore, conditional on the refinancing decision, taking expectations over the employment status at t the value function with refinancing is

$$
\begin{aligned}
V_t^R(A_t) = {} & 1+\phi\alpha_{t+1}-\left[\phi(E\beta_{t+1})\delta_{t+1}\left(1+r_{t+1}\right)\right]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\
& \cdot \exp\left\{-b\frac{\delta_{t+1}\left(1+r_{t+1}\right)}{1+\delta_{t+1}\left(1+r_{t+1}\right)}\left[\left(1+r_t\right)A_t-(1+\rho_t)M_t\right]\right\} \\
& \cdot \left\{a\exp\left[-b\delta_{t+1}\left(1+r_{t+1}\right)w_t\right]\right. \\
& \left.+\left(1-a\right)\exp\left[-b\delta_{t+1}\left(1+r_{t+1}\right)B_t\right]\right\}^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\
& \cdot \exp\left\{-b\frac{\delta_{t+1}\left(1+r_{t+1}\right)}{1+\delta_{t+1}\left(1+r_{t+1}\right)}\left[M_{t+1}-k-\frac{(1+\rho_{t+1})M_{t+1}}{1+r_{t+1}}\right]\right\} \\
& \cdot \left[\frac{1+\delta_{t+1}\left(1+r_{t+1}\right)}{\delta_{t+1}\left(1+r_{t+1}\right)}\right]
\end{aligned}
$$

A.2 $E_t\beta_{t+1}$

Since there is no intertemporal stochastic dependence in house prices, from (8) it follows that:

$$
\begin{aligned}
E_t\beta_{t+1} &= E_t\left\{(E_{t+1}\beta_{t+2})^{1-\delta_{t+1}}\left[\frac{\phi\delta_{t+1}}{1-\delta_{t+1}}\right]^{1-\delta_{t+1}}F_{t+1}W_{t+1}/\delta_{t+1}\right\} \\
&= (E_{t+1}\beta_{t+2})^{1-\delta_{t+1}}\left[\frac{\phi}{1-\delta_{t+1}}\right]^{1-\delta_{t+1}}\delta_{t+1}^{-\delta_{t+1}}(E_tF_{t+1})W_{t+1}
\end{aligned}
$$

This equation can be solved recursively to get

$$
\begin{aligned}
E_t\beta_{t+1} &= (E_{T-1}\beta_T)^{(1-\delta_{t+1})(1-\delta_{t+2})..(1-\delta_{T-1})} \\
& \left(\frac{\phi}{1-\delta_{t+1}}\right)^{1-\delta_{t+1}}\left(\frac{\phi}{1-\delta_{t+2}}\right)^{(1-\delta_{t+1})(1-\delta_{t+2})}...\left(\frac{\phi}{1-\delta_{T-1}}\right)^{(1-\delta_{t+1})..(1-\delta_{T-1})} \\
& E_tF_{t+1}(E_{t+1}F_{t+2})^{1-\delta_{t+1}}...(E_{T-2}F_{T-1})^{(1-\delta_{t+1})..(1-\delta_{T-2})} \\
& W_{t+1}(W_{t+2})^{1-\delta_{t+1}}...(W_{T-1})^{(1-\delta_{t+1})..(1-\delta_{T-2})} \\
& \delta_{t+1}^{-\delta_{t+1}}\delta_{t+2}^{-\delta_{t+2}(1-\delta_{t+1})}...\delta_{T-1}^{-\delta_{T-1}(1-\delta_{t+1})..(1-\delta_{T-2})}
\end{aligned}
$$

The future EFs only include elements of the distribution of house prices for cases in which their corresponding future λ is positive. Using this equation together with the expression for β_t and the fact that

$$
\begin{aligned}
\frac{1}{1+\delta_{t+1}(1+r_{t+1})} &= 1-\delta_t \\
\beta_t &= \left(\frac{\phi}{1-\delta_t}\right)^{1-\delta_t}\delta_t^{-\delta_t}W_t(E_t\beta_{t+1})^{1-\delta_t}F_t
\end{aligned}
$$

gives equation (9)

A.3 Proof of Proposition 2

The relevant first order conditions are

1. With refinancing ($\lambda_t > 0$ and $\theta p_t - k - M_t\lambda_{t+1} > 0$)

$$
\begin{aligned}
\exp(c_t^R) &= \left[\phi\left(E\beta_{t+1}\right)\delta_{t+1}\left(1+r_{t+1}\right)\right]^{-\frac{1}{b(1+\delta_{t+1}(1+r_{t+1}))}} \\
&\cdot\exp\left\{\delta_t\left[\left(1+r_t\right)A_t-(1+\rho_t)M_t\right]\right\} \\
&\cdot W_t^{-\frac{1}{b(1+\delta_{t+1}(1+r_{t+1}))}}\exp\left[b\delta_t\left(\lambda_{t+1}\theta p_t-k\right)\right]
\end{aligned}
$$

2. Without refinancing ($\lambda_t > 0$ and $\theta p_t - k - M_t\lambda_t < 0$ or $\lambda_{t+1} < 0$) we have:

$$
\begin{aligned}
\exp(c_t^{NR}) &= \left[\phi\left(E\beta_{t+1}\right)\delta_{t+1}\left(1+r_{t+1}\right)\right]^{-\frac{1}{b(1+\delta_{t+1}(1+r_{t+1}))}} \\
&\cdot\exp\left\{\delta_t\left[\left(1+r_t\right)A_t-(1+\rho_t)M_t\right]\right\} \\
&\cdot W_t^{-\frac{1}{b[1+\delta_{t+1}(1+r_{t+1})]}}\exp\left(\delta_t\lambda_{t+1}M_t\right)
\end{aligned}
$$

A.4 Proof of Proposition 3

There are 3 subcases to consider

A.4.1 Case 1

If $\lambda_{t+2} > 0$ and $\theta \underline{p}_{t+1}\lambda_{t+2} - k > M_{t+1}\lambda_{t+2}$, then $\theta E p_{t+1}\lambda_{t+2} - k > M_{t+1}\lambda_{t+2}$. Hence, with certain house prices:

$$EF^c_{t+1} = F^c_{t+1} = \exp\left[-b\delta_{t+1}(\lambda_{t+2}\theta E p_{t+1} - k)\right]$$

as opposed to

$$EF_{t+1} = E\exp\left[-b\delta_{t+1}\left(\theta p_{t+1}\lambda_{t+2} - k\right)\right]$$

From Jensen's inequality it follows that $EF_{t+1} = E\exp(-x) > \exp(-Ex) = EF^c_{t+1}$ since $\exp(-x)$ is convex.

$$c^C - c^U = \frac{1}{b\left[1 + \delta_{t+1}\left(1 + r_{t+1}\right)\right]}\ln\left(\frac{EF_{t+1}}{EF^c_{t+1}}\right) > 0$$

where c^C is consumption when future house prices are certain and c^U is consumption under uncertainty. Here whilst certain future house prices are going to allow mortgage refinancing for sure, this may fail to be true with uncertain house prices. Therefore, house price uncertainty raises precautionary savings.

A.4.2 Case 2

If $\lambda_{t+1} > 0$ and $Ep_{t+1}\lambda_{t+1} - k > M_{t+1}\lambda_{t+1}$ then

$$EF^c_{t+1} = F^c_{t+1} = \exp\left[-b\delta_{t+1}(\lambda_{t+2}\theta E p_{t+1} - k)\right]$$

as opposed to

$$\begin{gathered} EF_{t+1} = \exp(-b\delta_{t+1}\lambda_{t+2}M_{t+1})\gamma_t \\ +E\left\{\exp\left[-b\delta_{t+1}\left(\theta p_{t+1}\lambda_{t+2} - k\right)\right]\middle|\theta p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+2}}\right\}(1-\gamma_t) \end{gathered}$$

Since $\theta E p_{t+1}\lambda_{t+1} - k > M_{t+1}\lambda_{t+1}$ we know that

$$\exp(-b\delta_{t+1}\lambda_{t+2}M_{t+1}) > \exp\left[-b\delta_{t+1}\left(\lambda_{t+2}\theta E p_{t+1} - k\right)\right]$$

However, $E\left\{\exp\left[-b\delta_{t+1}\left(\theta p_{t+1}\lambda_{t+2} - k\right)\right] \middle| \theta p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+2}}\right\}$ might be higher or lower than $\exp\left[-b\delta_{t+1}(\lambda_{t+2}\theta E p_{t+1} - k)\right]$ depending on the nature of the house price distribution. Hence, the overall comparison is ambiguous: there is a risk that future house prices might be low, constraining the refinancing possibilities.

A.4.3 Case 3

On the other hand if $\lambda_{t+2} > 0$ and $\theta E p_{t+1}\lambda_{t+2} - k < M_{t+1}\lambda_{t+2}$ then

$$EF^c_{t+1} = F^c_{t+1} = \exp(-b\delta_{t+1}\lambda_{t+2}M_{t+1}) < \exp\left[-b\delta_{t+1}(\lambda_{t+2}\theta E p_{t+1} - k)\right]$$

Hence:

$$\frac{EF_{t+1}}{EF^c_{t+1}} = \gamma_t + \frac{E\left\{\exp\left[-b\delta_{t+1}\left(\theta p_{t+1}\lambda_{t+2} - k\right)\right] \middle| \theta p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+2}}\right\}}{\exp(-b\delta_{t+1}\lambda_{t+2}M_{t+1})}(1-\gamma_t)$$

Since

$$\frac{E\left\{\exp\left[-b\delta_{t+1}\left(\theta p_{t+1}\lambda_{t+2} - k\right)\right] \middle| \theta p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+2}}\right\}}{\exp(-b\delta_{t+1}\lambda_{t+2}M_{t+1})} < 1$$

then

$$\frac{EF_{t+1}}{EF^c_{t+1}} < 1$$

and

$$c^C - c^U = \frac{1}{b\left[1 + \delta_{t+1}\left(1 + r_{t+1}\right)\right]} \ln\left(\frac{EF_{t+1}}{EF^c_{t+1}}\right) < 0$$

A.5 Proof of Proposition 4

The effects of labour income risk: for the current period suppose that income were certain at the level $\overline{w_t} = \alpha w_t + (1-\alpha_t)B_t$. Then since the exponential is a convex function and $\delta > 0$

$$\begin{aligned} & \alpha \exp\left[-b\delta_{t+1}(1 + r_{t+1})w_t\right] + (1-\alpha)\exp\left[-b\delta_{t+1}(1 + r_{t+1})B_t\right] \\ < \quad & \exp\left[-b\delta_{t+1}(1 + r_{t+1})\overline{w}_t\right] \end{aligned}$$

and then since ln() is an increasing function, consumption is depressed by the labour income uncertainty. For future labour income risk the relevant terms are in $E\beta_{t+1}$, W_{t+s}. If labour income of some future period were certain at its mean level this would increase the term in W_{t+s} which *ceteris paribus* would raise $E\beta_{t+1}$ and tend to raise current consumption.

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