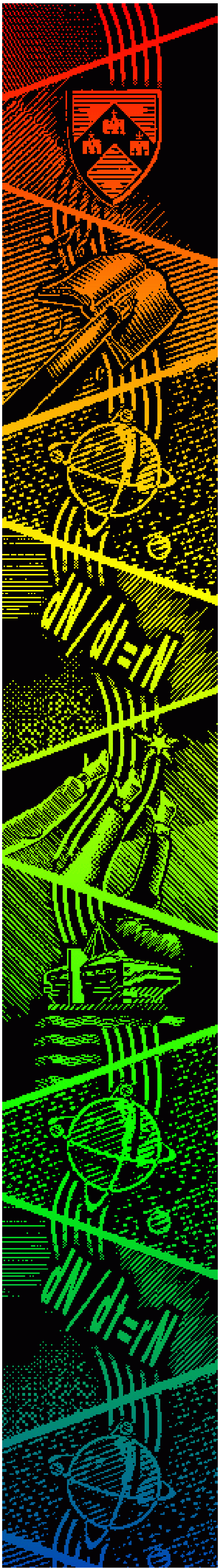

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Static Decisions May Be Optimal in a Life Cycle Setting

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Abstract

In a multicommodity life cycle setting with uncertainty and time additive expected utility, this note finds necessary and sufficient conditions on preferences for all but one optimal decision each period to be independent of the future and of uncertainty.

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The standard life cycle paradigm used in applications has expected discounted time additive utility as the payoff function and an intertemporal budget constraint that works period to period with one or more financial assets. Uncertainty can be over future preferences or prices, wages and asset returns. This approach has had well documented empirical failures and various modifications have been suggested: habit formation Constantinides, (1990)), complication of budget constraints with say liquidity constraints (Zeldes (1989)) and more fundamentally alternatives to expected utility such as ambiguity aversion (Klibanoff et al (2005)), preference for flexibility (Kreps (1979)) and behaviourally based theories (Thaler, (1990)), etc. Partly these last concerns arise from the strong information and computational demands of the standard paradigm: at any time t the set of states and the probability distribution over them must be known for all future dates. It is also true that it is extremely difficult to analytically compute an optimal time path of decisions and closed form solutions exist in only a few cases. Partly this is due to the dimensionality problem, partly it is due to the types of functions that are commonly used e.g. an intertemporal preference and diminishing marginal rates of substitution. This has led economists to use simulation methods to determine the optimal path and also the parameters of the problem (Gourinchas & Parker (2002), Campbell & Cocco (2003), Attanasio et al (2008))

In a multivariate problem both the information demands and computational complexity increase. In the standard paradigm there is an Euler equation for each decision (Meghir-Weber (1996)), and the optimal path derives from solving all these Euler equations simultaneously through time. Do decisionmakers actually struggle with the computational complexity in making decisions? Plausibly some decisions have a more obvious intertemporal impact than others e.g. it's more likely that decisions with large future effects (education, job choice/search,etc.) will be taken with the life cycle problem in mind but less likely to be so for choosing whether to have ice cream or cake today. If this is the case then cet par choices made with the life cycle model in mind should display different variability to those taken "myopically". Empirically there is some evidence that there are differences in variability of consumption on different commodities both at one age and across the life cycle. In the Appendix Table 1 shows the coefficient of variation of spending (CV) on 11 main commodity groups for households in the UK 2009 Living Costs and Food Survey Since it is a cross section, relative prices should be more or less common across observations, families differ in income and demographics. Life style differences and nonparticipation is relevant for some goods eg tobacco, fares for transport, motoring,alcohol and clothing as the number of zero expenditures shows. But still the coefficient of variation of food and fuel is about half that of other decisions. In the cross section we can see some life cycle effects by looking at the coefficient of variation by age of household head. For a few age bands and goods this is shown in the appendix in Table 2 (CV including zero expenditures). Generally the variability between households in expenditures peaks in middle age groups but more strongly with more variable goods.

Expenditure decisions made in a life cycle context should in part be determined by intertemporal smoothing of expected income and price changes, preference changes. Myopic or statically determined decisions are just subject to current resources and preferences. In principle either group could be more volatile than the other eg if expectations are more or less volatile than realisations. What is clear from the data is that there are significant differences between goods in expenditure volatility, and also some life cycle differences. Food and fuel spending is less volatile, leisure goods and services are more volatile.

Here we find necessary and sufficient conditions for within period preferences to be such that using the standard expected utility life cycle paradigm, some or most decisions are optimally purely static whilst others must be deduced from solving the full intertemporal problem. Essentially if there are $n+1$ expenditure decisions $(x_1, x_2 ... x_n, c)$ to be made within the standard paradigm then choice of $(x_1, x_2 ... x_n)$ can be made in a purely static way iff each periods utility can be written as $u(x_1, x_2 ... x_n, c) = F(c + G(x_1, x_2 ... x_n))$ where $F(), G()$ have suitable properties to ensure monotonicity and concavity.

1 The Basic Paradigm

There are $n + 1$ consumption goods with quantities each period $(x_{1t}, x_{2t} ... x_{nt}, c_t)$. The quantities are purchased at prices $q_{it}, i = 1..n$ for the goods x and p_t for the good c. There is a single financial asset whose stock at the start of period t is A_t and which earns a certain interest rate r each period (here for simplicity of notation we take it to be constant through time). There is transfer income of m_t each period t. The effect is that disposable resources at start of t are $Z_t = (1+r)A_t + m_t$. The consumption plan is for a finite horizon with a payoff function

$$E\Sigma_{i=1}^{T}\delta^t u(c_t, x_t)$$

where δ is the per period discount factor (again assumed constant) and $u()$ is utility per period (here written as time invariant).

The value function $V_t(A_t)$ has the form

$$V_t(A_t) = \max_{c_t, x_t, A_{t+1}} \{u(c_t, x_t) + \delta E_t V(A_{t+1}) | p_t c_t = Z_t - \Sigma q_{it} x_{it} - A_{t+1}\}$$

Substituting out the constraint

$$V_t(A_t) = \max_{x_t, A_{t+1}} \{u(\frac{Z_t}{p_t} - \Sigma \frac{q_{it}}{p_t} x_{it} - \frac{A_{t+1}}{p_t}, x_t) + \delta E_t V(A_{t+1})\}$$

The first order conditions can be written as

$$\begin{aligned} u_1(t)\frac{q_{it}}{p_t} &= u_i(t) \qquad i = 1..n && (1) \\ u_1(t)\frac{1}{p_t} &= \delta E_t V'_{t+1}(A_{t+1}) = E_t u_1(t+1)\frac{(1+r)}{p_{t+1}} \\ p_t c_t &= (1+r)A_t + m_t - \Sigma q_{it} x_{it} - A_{t+1} && (2) \end{aligned}$$

Using the envelope theorem

$$V_t^{'}(A_t) = u_1(t)\frac{(1+r)}{p_t}$$

Updating this and taking expectations as at t

$$E_t V_{t+1}^{'}(A_{t+1}) = E_t u_1(t+1)\frac{(1+r)}{p_{t+1}}$$

gives

$$u_1(t) = \delta(1+r)E_t u_1(t+1)\frac{p_t}{p_{t+1}}$$

Similarly the equations for the first n goods can be written

$$\begin{aligned} u_1(t+1)\frac{q_{it+1}}{p_{t+1}} &= u_i(t+1) \\ u_i(t) &= u_1(t)\frac{q_{it}}{p_t} = \delta(1+r)E_t u_1(t+1)\frac{q_{it}}{p_{t+1}} \\ &= \delta(1+r)E_t u_i(t+1)\frac{q_{it}}{q_{it+1}} \end{aligned}$$

So equivalently in Euler equation form the first order conditions can also be written

$$\begin{aligned} u_i(t) &= \delta(1+r)E_t u_i(t+1)\frac{q_{it}}{q_{it+1}} \\ u_1(t) &= \delta(1+r)E_t u_1(t+1)\frac{p_t}{p_{t+1}} \\ p_t c_t &= (1+r)A_t + m_t - \Sigma q_{it}x_{it} - A_{t+1} \end{aligned} \tag{3}$$

2 Preferences allowing some static decisions

Suppose the equations

$$u_1(t)\frac{q_{it}}{p_t} = u_i(t) \qquad i = 1..n \tag{4}$$

are independent of c_t. Then they can depend only on x_t. The solution x_t solves

$$\max_{x_t} u(Z_t - \frac{A_{t+1}}{p_t} - \Sigma\frac{q_{it}}{p_t}x_{it}, x_t)$$

Of course optimally A_{t+1} depends on future decisions and uncertainty. But when (4) is independent of c this effect disappears. This prompts the result of this paper

Theorem 1 *The general solution $u(c_t, x_t)$ of the equations*

$$u_i(c_t, x_t) - g^i(x_t)u_1(c_t, x_t) = 0$$

is

$$u(c_t, x_t) = F(c_t - G(x_t))$$

where $F()$ is an arbitrary function and $G_i(x_t) = g^i(x_t)$.

Proof. From (4) we have the equations

$$\frac{u_i(c,x)}{u_j(c,x)} = \frac{g_i(x)}{g_j(x)}$$

hence the mrs between any two goods in x must be independent of c and so c must be separable from x in u or $u = F(c, G(x))$. Then $u_1 = F_1 = u_i/g_i = F_2G_i/g_i = F_2G_j/g_j = u_j/g_j$ which implies that $g_i(x)$ is proportional to $g_j(x)$ say $G_i = \lambda(x)g_i$. Hence in turn $F_1 = \lambda(x)F_2$. Finally this gives F_1/F_2 independent of c which means that we must have F linear in c, G. ■

$F(), G()$ are arbitrary except for smoothness, concavity and monotonicity conditions. So for example F could be isoelastic $F = (c_t - G(x_t))^\alpha$. Two other interesting examples are

(1) $u()$ is CARA in c_t so that

$$u(c_t, x_t) = 1 - \exp(-\alpha c_t)G(x_t)$$

where $G()$ is decreasing and convex. This is used in Berloffa & Simmons (2003).

(2) $u()$ is quasilinear in c_t so that

$$u(c_t, x_t) = \alpha c_t + G(x_t)$$

This form has been used for an intertemporal labour supply problem by in which x plays the role of a single consumption good with CRRA preferences and c represents leisure (also see the interesting note by Rasmussen (2006)).

It's interesting to understand why it works in these last two examples. In the CARA case the value function problem is

$$\begin{aligned} V_t(A_t) &= \max_{x_t, A_{t+1}} \{\exp(\alpha \left(\frac{Z_t}{p_t} - \Sigma \frac{q_{it}}{p_t} x_{it} - \frac{A_{t+1}}{p_t}\right) G(x_t) + \delta E_t V(A_{t+1})\} \\ &= \max_{A_{t+1}} \{\exp(\alpha \left(\frac{Z_t}{p_t} - \frac{A_{t+1}}{p_t}\right) \max_{x_t} \left[\exp\left(-\alpha \Sigma \frac{q_{it}}{p_t} x_{it}\right) G(x_t)\right] \\ &\quad + \delta E_t V(A_{t+1})\} \\ &= \max_{A_{t+1}} \{\exp(\alpha \left(\frac{Z_t}{p_t} - \frac{A_{t+1}}{p_t}\right) H(\frac{q_t}{p_t}) + \delta E_t V(A_{t+1})\} \end{aligned}$$

The optimisation problem factors into part that involves only x_t and which is independent of the future. In the quasilinear case the same is true:

$$\begin{aligned} V_t(A_t) &= \max_{A_{t+1}} \{\alpha(\frac{Z_t}{p_t} - \frac{A_{t+1}}{p_t}) + \max_x \left(G(x_t) - \alpha \Sigma \frac{q_{it}}{p_t} x_{it}\right) + \delta E_t V(A_{t+1})\} \\ &= \max_{A_{t+1}} \{\alpha(\frac{Z_t}{p_t} - \frac{A_{t+1}}{p_t}) + H(\frac{q_t}{p_t}) + \delta E_t V(A_{t+1})\} \end{aligned}$$

Of course more than one good may bear the impact of uncertainty so that if we partition the goods into two groups (c, x) then decisions on goods x can be taken independently of the future iff within period preferences have the form $u(c_t, x_t) = F(\Sigma a_i c_{it} + G(x_t))$.

3 Non-Expected Utility

We do not actually even need time additive expected utility to generate the result. To illustrate consider a two period example of Epstein-Zin preferences in the form

$$U(c_t, x_t) = \left[u_t^\rho + \delta \left(E_t(u_{t+1}^\gamma)\right)^{\rho/\gamma}\right]^{1/\rho}$$

where $u_t = (c_t + G(x_t))^\alpha / \alpha$. Setting the price of $c_t = 1$ the budget constraints are

$$\begin{aligned} c_t &= Z_t - \Sigma q_{it} x_{it} - A_{t+1} \\ c_{t+1} &= (1+r) A_{t+1} + y_{t+1} - \Sigma q_{it+1} x_{it+1} \end{aligned}$$

Substituting out c each period, the choice variables are x_{t+1}, x_t, A_{t+1}.

$$\max_{x_t A_{t+1}} \left[(c_t + G(x_t))^{\alpha\rho}/\alpha + \delta \left(E_t(\max_{x_{t+1}}(c_{t+1} + G(x_{t+1}))^{\gamma\alpha}/\alpha \right))^{\rho/\gamma} \right]^{1/\rho}$$

so x_{t+1} is purely a within-state $t+1$ decision and solves $G_i(x_{t+1}) = q_{it+1}$ thus giving

$$u_{t+1} = ((1+r)A_{t+1} + y_{t+1} + H(q_{t+1}))^{\alpha}/\alpha$$

where $H()$ is the optimal value $G(x_{t+1}) - \Sigma q_{it+1} x_{it+1}$.

The problem becomes

$$\max_{x_t A_{t+1}} \left[(c_t + G(x_t))^{\alpha\rho}/\alpha + \delta \left(E_t(((1+r)A_{t+1} + y_{t+1} + H(q_{t+1}))^{\gamma\alpha}/\alpha) \right)^{\rho/\gamma} \right]^{1/\rho}$$

$$\text{subject to } c_t = Z_t - \Sigma q_{it} x_{it} - A_{t+1}$$

Optimising over x_t, the necessary conditions have the form

$$U_t'()[G_i(x_t) - q_{it}] = 0$$

where $U_t'()$ is the marginal welfare of lifetime utility from (**??**). These equations imply $G_i(x_t) = q_{it}$ for each i and so x_t as well as x_{t+1} solve purely static optimisation problems. Of course that still leaves the choice of A_{t+1} (or equivalently c_{t+1}) which carries all the intertemporal influence.

4 Conclusions

We have found necessary and sufficient conditions on preferences for all but one intertemporal decisions to reduce to just static decisions in an intertemporal decision problem with uncertainty when there is a single financial asset. Some aspects of the preferences are general (the form of the functions F, G). We have not explicitly allowed for random future preference shocks but from the Epstein-Zin example above it's clear that these can be included (think of random q_{t+1} as playing the role of preference shocks). Multiple financial assets can also be included: each additional asset will add one set of Euler equations which must be satisfied by the optimal choice of c_t.

The intertemporal choice problem in general is very complex, both information and computation demands on decisionmakers are very heavy. Given this what do decisionmakers actually do? If they happen to have preferences like these the problem is hugely simplified. If they don't then attention has focussed on alternatives to fully fledged backward induction like preference for future flexibility (Kreps (1979)), scenario planning (Rockafellar & Wets (1991)).

A Appendix

Age	CV, positives	CV all obs	Zero Expenditure
n		5288	
Food	0.67	0.67	9
Alcohol	1.13	1.55	1940
Tobacco	0.95	2.62	4430
Clothing, footwear	1.28	1.74	1981
Household goods	2.09	2.15	297
Household services	2.54	2.58	164
Personal goods	2.08	2.25	712
Motoring	1.14	1.37	1160
Fares	2.12	3.58	3505
Leisure goods	2.27	2.43	627
Leisure services	2.58	2.64	206
Fuel	0.62	0.68	312

Table 1

Age	$25-29$	$40-44$	$55-59$	$70-75$
n	332	588	577	395
Food	0.57	0.61	.64	0.66
Alcohol	1.60	1.30	1.73	1.81
Clothing, footwear	1.35	1.48	1.64	1.82
Household goods	1.41	2.92	1.79	1.71
Household services	1.77	4.13	1.35	1.22
Personal goods	1.97	2.90	2.18	1.76
Fares	2.16	4.06	3.20	2.98
Leisure goods	1.85	2.15	2.30	1.78
Leisure services	2.23	4.32	1.93	1.98
Fuel	0.64	0.62	0.67	0.61

Table 2, CV, all observations

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