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Abstract

This note analyzes the problem Captain MacWhirr faces in Joseph Conrad's novel *Typhoon* as an implementation problem under incomplete information. We identify a sufficient condition under which each player has a unique rationalizable strategy. In this unique rationalizable outcome, truthful revelation by each player is observed.

1 Introduction

Captain MacWhirr's problem was originally posed by Joseph Conrad in his 1902 novel Typhoon. The Siamese steamer Nan-Shan transports 200 Chinese workers, who have worked for seven years in the British tropical colonies, from Singapore to their home of Fu-chau. Each worker's accumulated savings are stored in individual camphor wood chests aboard the ship. When a typhoon strikes with ferocious force on Christmas Eve, the boxes burst open and the workers' silver dollars are scattered between decks. In the ensuing chaos, the Captain's orders result in the passengers' belongings to be amassed in a coal bunker. As soon as the storm calms down, the Captain intends to return the men's savings to their rightful owners. But the Captain faces an information revelation problem:

You couldn't tell one man's dollars from another's, he said, and if you asked each man how much money he brought on board he was afraid they would lie, and he would find himself a long way short. [Joseph Conrad, *Typhoon*]

In the first economic analysis of Captain MacWhirr's problem, Mumy (1981) claims that it is possible to motivate all passengers to truthfully report their entitlements to the Captain

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through the use of a suitable punishment strategy. The solution identified by Mumy, however, is only one of many possible Nash equilibria of the game.¹

We view Captain MacWhirr's problem as an implementation problem *under incomplete information*. Our setup is different from the standard environments discussed in the literature (e.g., Bergemann and Morris (2009) and Jackson (1991)). We assume that Captain MacWhirr has partial information regarding the true state, that is, the total amount of the passengers' endowments, which is not available to the passengers. Each passenger only knows their own entitlement (own types) and does not know the others' entitlements (others' types) or the total amount of endowments. In addition, we assume that each passenger's preferences are state independent. We also impose a financial non-negativity constraint on the passengers' final wealth.

We identify a sufficient condition under which the planner (Captain MacWhirr) can construct a direct mechanism so that each player has a unique rationalizable strategy.² In this unique outcome, each player's rationalizable strategy is to simply tell the truth. In the mechanism we provide, the planner hides the information on the total amount of money to achieve the desired outcome.

Our sufficient condition says that each player assigns a sufficiently high probability to the case that every other player's endowment is the lowest. Consider the case where everyone else's endowment is indeed the lowest. Then, irrespective of the amount she had, if a player claims more than her endowment, this would be noticed by the planner who knows the total amount. In our mechanism, this leads to a payoff below her endowment. If every player believes that this case is likely, then she will never claim more than what she had. This means that every player receives what she claims in our mechanism. Then, knowing this, no one would claim less than her endowment since doing so would lead to a lower payoff than what she could have received by telling the truth, that is, her endowment.

Note that in the argument above, we only describe a restriction on beliefs instead of specifying the players' beliefs explicitly. To incorporate such a weak condition, our analysis is based on the notion of Δ -rationalizability due to Battigalli and Siniscalchi (2003), which allows us to put a restriction on the players' (first-order) beliefs. The prefix " Δ " corresponds to such restrictions. In our setting, it is the sufficient condition mentioned above. Only assuming a common belief regarding this restriction is certainly weaker than the standard Bayesian approach. We believe that this is a more natural way to treat actual environments.³

 $^{^{1}}$ Saraydar (1983) criticizes this fact and defends MacWhirr's solution of assigning equal shares to each passenger on the grounds of feasibility and transaction costs.

 $^{^{2}}$ Bergemann, Morris, and Tercieux (2011) study implementation problems under complete information via rationalizability.

³In the Bayesian environment, we could also use the notion of *interim correlated rationalizability* by Dekel, Fudenberg, and Morris (2007). For a comparison of these notions, see Battigalli, Di Tillo, Grillo, and Penta

2 Mechanism

N is the set of players with |N| = n > 1. For each player i, x_i is the amount of money she earned, which we call *endowment*. We assume that $x_i \in \{\underline{x}, \underline{x} + 1, \dots, \overline{x} - 1, \overline{x}\} = X$ for each $i \in N$ with $\overline{x} > \underline{x} > 0$ where \underline{x} (and, hence, each element of X) is an integer. Let $y = \sum_{j \in N} x_j$ and $y^i = \sum_{j \neq i} x_j = y - x_i$.

The set-up we consider is different from the standard environment in the literature. That is, the planner has partial information regarding the true state which the players do not have, namely, the total amount of entitlements, y.⁴ The planner uses this information to implement the desired outcome. The planner does not make y public and, hence, this is a random variable from the players' point of view.⁵ After observing her own endowment x_i , each player announces $s_i \in X$.⁶ Given the profile of announcements, the planner distributes the collected funds according to the following rule:

- 1. If $\sum_{i \in N} s_i \leq y$, then the planner gives s_i to each player *i* and keeps the rest.
- 2. If $\sum_{j \in N} s_j > y$, then each player receives r(z) where $z = \sum_{j \in N} s_j y$.

In general, the planner constructs $r(\cdot)$ with the following properties: (i) $r(z) \in [0, \underline{x})$ for $z \in \{1, \ldots, n(\overline{x} - \underline{x})\}$, and (ii) r(z) is strictly decreasing and linear in $z \in \{1, \ldots, n(\overline{x} - \underline{x})\}$. The planner adopts a lottery rule: Each player faces a lottery with two possible outcomes $\{0, \underline{x}\}$. With the chance $\frac{z}{n(\overline{x}-\underline{x})}$, she receives 0 while she receives \underline{x} with probability $\frac{n(\overline{x}-\underline{x})-z}{n(\overline{x}-\underline{x})}$. Hence, a player's expected payoff for z > 0 is

$$r(z) = \left(\frac{n(\bar{x} - \underline{x}) - z}{n(\bar{x} - \underline{x})}\right) \underline{x}.$$

Under the specification above, given the other players' announcements, if z > 0, announcing a higher number would lead to a strictly lower payoff. In such scenarios, the marginal cost of announcing a higher number is

$$d = r(z) - r(z+1) = \frac{\underline{x}}{n(\overline{x} - \underline{x})}$$

which plays a crucial role in the sufficient condition we provide. Note (i) that d is increasing if the difference $\bar{x} - \underline{x}$ decreases, and (ii) that d is increasing if \underline{x} increases.

⁽²⁰¹¹⁾. Our sufficient condition results in uniqueness in both environments.

⁴If the planner chooses to release this information, then our mechanism would not work.

⁵We assume that the planner applies the proposed mechanism even to trivial cases, e.g., when $y = n\underline{x}$ or $y = n\overline{x}$. Otherwise, our sufficient condition does not hold.

⁶It is imperative that the set of pure actions for each player coincides with X.

For a sufficiently large \underline{x} , the planner can construct $r(\cdot)$ without having to use lotteries such that (i) $r(\cdot)$ is strictly decreasing in z, (ii) the image of r(z) for $z \in \{1, \ldots, n(\overline{x} - \underline{x})\}$ is a subset of $\{0, \ldots, \underline{x} - 1\}$, and (iii) the following expression is sufficiently large:

$$\min_{z \in \{1, \dots, n(\bar{x}-\underline{x})-1\}} \left[r(z+1) - r(z) \right] = d.$$

3 Sufficient Condition and Result

When the game described above is played, each player i knows her own endowment x_i and forms a belief about the other players' endowments $x_{-i} \in X^{n-1}$ as well as their announcements $s_{-i} \in X^{n-1}$. Let $q_{x_i} \in \Delta(X^{n-1} \times X^{n-1})$ denote player i's belief about the other players' endowments $x_{-i} \in X^{n-1}$ as well as announcements $s_{-i} \in X^{n-1}$ when her endowment is x_i . Let $p_{x_i} \in \Delta(X^{n-1})$ be player i's belief about the other players' endowments x_{-i} obtained from q_{x_i} .

We adopt the notion of Δ -rationalizability by Battigalli and Siniscalchi (2003) where Δ corresponds to the restrictions on the players' first-order beliefs.⁷ We assume that there is a common belief on Δ . Given this restriction, after the realization of her endowment, each player eliminates announcements which cannot be best responses to any of her beliefs satisfying the condition. Then, with the reduced set of endowment-announcement combinations, each player again eliminates announcements which cannot be best responses to any of her beliefs which satisfy the condition and take into account the fact that some endowment-announcement combination would not arise for the other players. This continues until there is no more announcement combinations gives the set of rationalizable announcements for each possible endowment of each player.

The condition below states that for each $i \in N$ and $x_i \in X$, player *i* believes that the chance of everyone else's endowment being the lowest is sufficiently high. We already noted that if this corresponds to the true state (commonly known), then there is no room for each player to state anything but the truth. The condition below implies that as long as there is a common belief that each player believes it is very likely that the other players have the lowest endowment, the planner can achieve the desired outcome with the simple direct mechanism described above.

⁷See also Battigalli (2003) and Battigalli and Siniscalchi (2007).

3.1 Sufficient Condition

Take $p_{x_i} = \max_{X^{n-1}} q_{x_i}$ as the marginal on the set of endowments X^{n-1} obtained from q_{x_i} . We assume that each player *i*'s first-order belief satisfies the following condition.

Condition 1 For each $i \in N$ and $x_i \in X \setminus \{\bar{x}\}$,

$$p_{x_i}(x_j = \underline{x} \text{ for all } j \neq i) > \frac{\overline{x} - x_i}{(\overline{x} - x_i) + d}$$

We also assume that there is a common belief that the players' beliefs satisfy this sufficient condition.⁸

As a simple example, consider the case where n = 3, $\bar{x} = 220$ and $\underline{x} = 210$ (and hence $\bar{x} - \underline{x} = 10$). Then, the planner can have r(z) = 210 - 7z where $z \in \{1, \ldots, 30\}$, which implies d = 7.⁹ The sufficient condition above requires that each player *i*'s belief that $x_j = \underline{x}$, for all $j \neq i$, to be strictly higher than

$$\frac{\bar{x} - x_i}{\bar{x} - x_i + d} = \frac{220 - x_i}{227 - x_i} \le \frac{10}{17} \approx 0.588.$$

Of course, this condition can become tighter or weaker, depending on the precise set of parameters employed.

3.2 Rationalizability

Let $u_i : X^n \times X^n \to \mathbb{R}$ be player *i*'s utility function which depends on the profile of announcements $s \in X^n$ and the profile of endowments $x \in X^n$. Given $x_i \in X$ and $q_{x_i} \in \Delta(X^{n-1} \times X^{n-1})$, let

$$BR_i(x_i, q_{x_i}) = \operatorname{argmax}_{s'_i \in X} \sum_{s_{-i}, x_{-i}} u_i(s'_i, s_{-i}, x_i, x_{-i}) q_{x_i}(x_{-i}, s_{-i})$$

be the set of best responses for player i with endowment x_i .

Let Δ_{x_i} be the set of beliefs satisfying Condition 1 for each x_i and $\Delta = ((\Delta_{x_i})_{x_i \in X})_{i \in N}$.¹⁰ The following iterative procedure eliminates announcements for each x_i for each $i \in N$.

• $R_i^0 = X \times X$; that is, the procedure starts with all possible combinations of endowments and announcements for each player *i*.

^{.0}That is,
$$\Delta_{x_i} = \left\{ q_{x_i} \in \Delta(X^{n-1} \times X^{n-1}) \mid \max_{X^{n-1}} q_{x_i}(x_j = \underline{x} \text{ for all } j \neq i) > \frac{\overline{x} - x_i}{(\overline{x} - x_i) + d} \right\}.$$

 $^{^8 \}rm We$ would like to thank Pierpaolo Battigalli for pointing out imprecise statements in a previous version. $^9 \rm Here,$ the planner does not use lotteries.

• At the k-th iteration where k > 0, for each i and x_i , an announcement s_i is a best response to a belief for the player with endowment x_i which satisfies the condition above and whose support only includes the endowment-announcement combinations of the others which have survived so far;

$$R_i^k = \left\{ (x_i, s_i) \in X \times X \mid \text{ there exists } q_{x_i} \in \Delta_{x_i} \cap \Delta(R_{-i}^{k-1}) \text{ such that } s_i \in BR_i(x_i, q_{x_i}) \right\}$$

• For each $i \in N$, let $R_i = \bigcap_{k=0}^{\infty} R_i^k$.

For each $(x_i, s_i) \in R_i$, we say that an announcement s_i is *rationalizable* to player *i* with x_i .

Note that in the definition above, we only use pure strategies. In our setting, we assume that each player's utility function is independent of the state.¹¹ Moreover, we assume that each player's utility function is linear in its argument.

3.3 Main Result

Under Condition 1, given the mechanism specified above, we have the following result:

Proposition 1 Given Condition 1, for each $i \in N$ and each $x_i \in X$, the only rationalizable strategy is $s_i = x_i$.

The proof for the result is given in Appendix A. Here, we outline the proof for the result. Compare two strategies $s_i = x_i$ and $s'_i > x_i$. The net *loss* from announcing s_i compared to s'_i is simply $s'_i - x_i$ if $y \ge \sum_{j \in N} s_j$. However, there are cases where $y < \sum_{j \in N} s_j$ for which there are two possible reasons; (a) $\sum_{j \neq i} s_j$ is too high, or (b) y is too low. There is at least one scenario to which (b) applies. If $x_j = \underline{x}$ for each $j \neq i$, independent of s_{-i} , s'_i leads to an expected payoff which is strictly lower than \underline{x} . In this scenario, the net gain from announcing s_i compared to s'_i is at least $(s'_i - x_i) + d$. If Condition 1 holds, this net expected gain always exceeds the net expected loss for any $s'_i > x_i$ independent of s_{-i} . Hence, $s_i = x_i$ strictly dominates $s'_i > x_i$ for each $i \in N$ and $x_i \in X \setminus \{\overline{x}\}$. This means that no one claims more than her endowment, and, hence, each player receives what she claims. Since claiming less than her endowment simply lowers a player's payoff, $s_i = x_i$ strictly dominates $s''_i < x_i$ for each $x_i \in X \setminus \{\underline{x}\}$. Hence, $s_i = x_i$ is a unique rationalizable strategy for each $i \in N$ and $x_i \in X$.

¹¹See Ben-Porath and Lipman (2011) for implementation problems when preferences are state independent.

4 Conclusion

In this note, we present a sufficient condition under which it is each player's unique rationalizable strategy to reveal the actual amount of her endowment. If, on the one hand, x_i is i.i.d. for each player $i \in N$ or if n is sufficiently large (i.e., d is small), or both, it is harder for Condition 1 to be satisfied compared to cases where correlations are allowed or nis sufficiently small.

On the other hand, if $\bar{x} - \underline{x}$ is relatively small and \underline{x} is sufficiently large, the condition may be easily satisfied. In other words, uniqueness can be obtained if the endowments are sufficiently large (i.e., large \underline{x} and hence d) but the variance is small (i.e., small $\overline{x} - \underline{x}$). Even in Conrad's story this might have been the case since the workers had "worked in the same place and for the same length of time" [Joseph Conrad, *Typhoon*].

A Appendix

We first show that for each $i \in N$ and $x_i < \bar{x}$, any $s_i > x_i$ is strictly dominated by $s_i = x_i$, and hence $s_i > x_i$ is not a best response for each $i \in N$ and $x_i < \bar{x}$. Then, we show that $s_i = x_i$ is a unique rationalizable strategy for each $i \in N$ and $x_i \in X$.

Take $x_i \in X \setminus \{\overline{x}\}$. The expected payoff from announcing x_i is

$$\sum_{l=(n-1)\underline{x}}^{(n-1)\overline{x}} q_{x_i} \left(\begin{array}{c} y^i = l \\ \sum_{j\neq i} s_j \leq l \end{array} \right) x_i + \sum_{l=(n-1)\underline{x}}^{(n-1)\overline{x}} q_{x_i} \left(\begin{array}{c} y^i = l \\ \sum_{j\neq i} s_j > l \end{array} \right) r \left(\sum_{j\neq i} s_j - l \right)$$
$$= x_i - \sum_{l=(n-1)\underline{x}}^{(n-1)\overline{x}} q_{x_i} \left(\begin{array}{c} y^i = l \\ \sum_{j\neq i} s_j > l \end{array} \right) \left[x_i - r \left(\sum_{j\neq i} s_j - l \right) \right]$$
(1)

where $y^i = \sum_{j \neq i} x_j$. The expected payoff from announcing $s_i \in \{x_i + 1, \dots, \bar{x}\}$ is

$$\sum_{l=(n-1)\underline{x}}^{(n-1)\overline{x}} q_{x_i} \left(\begin{array}{c} y^i = l \\ \sum_{j\neq i} s_j + (s_i - x_i) \leq l \end{array} \right) s_i$$

$$+ \sum_{l=(n-1)\underline{x}}^{(n-1)\overline{x}} q_{x_i} \left(\begin{array}{c} y^i = l \\ \sum_{j\neq i} s_j + (s_i - x_i) > l \end{array} \right) r \left(\sum_{j\neq i} s_j - l + (s_i - x_i) \right)$$

$$= s_i - \sum_{l=(n-1)\underline{x}}^{(n-1)\overline{x}} q_{x_i} \left(\begin{array}{c} y^i = l \\ \sum_{j\neq i} s_j + (s_i - x_i) > l \end{array} \right) \left[s_i - r \left(\sum_{j\neq i} s_j - l + (s_i - x_i) \right) \right]$$

$$= s_i - \sum_{l=(n-1)\underline{x}}^{(n-1)\underline{x} + (s_i - x_i) - 1} p_{x_i} (y^i = l) \left[s_i - r \left(\sum_{j\neq i} s_j - l + (s_i - x_i) \right) \right]$$

$$- \sum_{l=(n-1)\underline{x} + (s_i - x_i)}^{(n-1)\overline{x}} q_{x_i} \left(\begin{array}{c} y^i = l \\ \sum_{j\neq i} s_j + (s_i - x_i) > l \end{array} \right) \left[s_i - r \left(\sum_{j\neq i} s_j - l + (s_i - x_i) \right) \right]$$

$$(2)$$

where the last equality comes from the observation that if $(n-1)\underline{x} + (s_i - x_i) > y^i$, it is always the case that $\sum_{j \neq i} s_j + s_i > y^i + x_i = y$. The expression (1) can be written as

$$x_{i} - \sum_{l=(n-1)\underline{x}+(s_{i}-x_{i})}^{(n-1)\underline{x}+(s_{i}-x_{i})-1} q_{x_{i}} \left(\begin{array}{c} y^{i} = l \\ \sum_{j\neq i} s_{j} > l \end{array} \right) \left[x_{i} - r \left(\sum_{j\neq i} s_{j} - l \right) \right]$$
$$- \sum_{l=(n-1)\underline{x}+(s_{i}-x_{i})}^{(n-1)\overline{x}} q_{x_{i}} \left(\begin{array}{c} y^{i} = l \\ \sum_{j\neq i} s_{j} > l \end{array} \right) \left[x_{i} - r \left(\sum_{j\neq i} s_{j} - l \right) \right].$$

Then,

$$(1) - (2) = -(s_{i} - x_{i}) + \sum_{l=(n-1)\underline{x}}^{(n-1)\underline{x}+(s_{i}-x_{i})-1} \left\{ p_{x_{i}}(y^{i} = l) \left[s_{i} - r \left(\sum_{j \neq i} s_{j} - l + (s_{i} - x_{i}) \right) \right] - q_{x_{i}} \left(\frac{y^{i} = l}{\sum_{j \neq i} s_{j} > l} \right) \left[x_{i} - r \left(\sum_{j \neq i} s_{j} - l \right) \right] \right\} + \sum_{l=(n-1)\underline{x}+(s_{i}-x_{i})}^{(n-1)\overline{x}} \left\{ q_{x_{i}} \left(\frac{y^{i} = l}{\sum_{j \neq i} s_{j} + (s_{i} - x_{i}) > l} \right) \left[s_{i} - r \left(\sum_{j \neq i} s_{j} - l + (s_{i} - x_{i}) \right) \right] - q_{x_{i}} \left(\frac{y^{i} = l}{\sum_{j \neq i} s_{j} > l} \right) \left[x_{i} - r \left(\sum_{j \neq i} s_{j} - l \right) \right] \right\}.$$

$$(3)$$

First, note that

$$p_{x_{i}}(y^{i} = l) \left[s_{i} - r \left(\sum_{j \neq i} s_{j} - l + (s_{i} - x_{i}) \right) \right]$$

$$-q_{x_{i}} \left(\begin{array}{c} y^{i} = l \\ \sum_{j \neq i} s_{j} > l \end{array} \right) \left[x_{i} - r \left(\sum_{j \neq i} s_{j} - l \right) \right]$$

$$\geq p_{x_{i}}(y^{i} = l) \left[r \left(\sum_{j \neq i} s_{j} - l \right) - r \left(\sum_{j \neq i} s_{j} - l + (s_{i} - x_{i}) \right) + (s_{i} - x_{i}) \right]$$

$$(*)$$

for each $l \in \{(n-1)\underline{x}, \ldots, (n-1)\underline{x} + (s_i - x_i) - 1\}$ where the inequality comes from

$$p_{x_i}(y^i = l) \ge q_{x_i} \left(\begin{array}{c} y^i = l \\ \sum_{j \ne i} s_j > l \end{array} \right)$$

for each $l \in \{(n-1)\underline{x}, \ldots, (n-1)\underline{x} + (s_i - x_i) - 1\}$ and the assumption that $r(z) < \underline{x}$ for each $z \in \{1, \ldots, n(\overline{x} - \underline{x})\}$. Note also that the expression in the bracket (*) is strictly positive since r(z) is strictly decreasing in z for $z \in \{1, \ldots, n(\overline{x} - \underline{x})\}$.

Likewise, we have

$$q_{x_i} \left(\begin{array}{c} y^i = l \\ \sum_{j \neq i} s_j + (s_i - x_i) > l \end{array} \right) \left[s_i - r \left(\sum_{j \neq i} s_j - l + (s_i - x_i) \right) \right]$$
$$-q_{x_i} \left(\begin{array}{c} y^i = l \\ \sum_{j \neq i} s_j > l \end{array} \right) \left[\underline{x} - r \left(\sum_{j \neq i} s_j - l \right) \right]$$
$$\geq q_{x_i} \left(\begin{array}{c} y^i = l \\ \sum_{j \neq i} s_j + (s_i - x_i) > l \end{array} \right)$$
$$\times \left[r \left(\sum_{j \neq i} s_j - l \right) - r \left(\sum_{j \neq i} s_j - l + (s_i - x_i) \right) + (s_i - x_i) \right]$$
$$\geq 0$$

for each $l \in \{(n-1)\underline{x} + (s_i - x_i), \dots, (n-1)\overline{x}\}$ where the first inequality comes from

$$q_{x_i} \left(\begin{array}{c} y^i = l \\ \sum_{j \neq i} s_j + (s_i - x_i) > l \end{array} \right) \ge q_{x_i} \left(\begin{array}{c} y^i = l \\ \sum_{j \neq i} s_j > l \end{array} \right)$$

for each $l \in \{(n-1)\underline{x} + (s_i - x_i), \dots, (n-1)\overline{x}\}$ and the assumption $r(z) < \underline{x}$ for each $z \in \{1, \dots, n(\overline{x} - \underline{x})\}.$

Then,

$$(3) \geq -(s_{i} - x_{i}) + \sum_{l=(n-1)\underline{x}}^{(n-1)\underline{x}+(s_{i}-x_{i})-1} p_{x_{i}}(y^{i} = l) \\ \times \left[r \left(\sum_{j \neq i} s_{j} - l \right) - r \left(\sum_{j \neq i} s_{j} - l + (s_{i} - x_{i}) \right) + (s_{i} - x_{i}) \right] \\ \geq -(s_{i} - x_{i}) + p_{x_{i}}(x_{j} = \underline{x} \text{ for all } j \neq i) [(s_{i} - x_{i}) + d] \\ = [(s_{i} - x_{i}) + d] \left\{ p_{x_{i}}(x_{j} = \underline{x} \text{ for all } j \neq i) - \frac{(s_{i} - x_{i})}{(s_{i} - x_{i}) + d} \right\} \\ \geq [(s_{i} - x_{i}) + d] \left\{ p_{x_{i}}(x_{j} = \underline{x} \text{ for all } j \neq i) - \frac{(\overline{x} - x_{i})}{(\overline{x} - x_{i}) + d} \right\} \\ \geq 0$$

where the last inequality comes from Condition 1. This implies that (1)-(2) > 0 independent of s_{-i} . Hence, we have the following result.

Lemma 1 Given Condition 1, for any $i \in N$, any $s_i > x_i$ is strictly dominated by $s_i = x_i$ for each $x_i < \bar{x}$.

Given Lemma 1, each player knows that no opponent chooses $s_i > x_i$. Hence, in each possible outcome, each player with her endowment equal to x_i receives an amount at most x_i . Given this, it is easy to see that choosing $s_i < x_i$ is strictly dominated since such an announcement strictly lowers the payoff.

Lemma 2 Given Condition 1, for each $i \in N$, any $s_i < x_i$ is strictly dominated by $s_i = x_i$ for each $x_i > \underline{x}$.

The results above imply Proposition 1.

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