On the Crowding-Out Effects of Tax-Financed Charitable Contributions by the Government

By

Alan Krause (University of York)
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Alan Krause
University of York
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Abstract

An important question in the literature on charitable contributions is the extent to which tax-financed contributions by the government crowd out private contributions. This paper examines a simple model of charitable contributions in which there exist both warm-glow and public good motives for giving, but where the warm-glow motive is competitive in the sense that individuals evaluate their own contribution relative to that of their peers. It is shown that the competitive element of the warm-glow motive may exacerbate or attenuate the crowding-out effect, depending upon certain preference and income parameters. However, as the warm-glow motive for giving becomes purely competitive, crowding out is exacerbated and is almost dollar-for-dollar.

Keywords: Charitable contributions; warm-glow; crowding out; public goods.

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†Department of Economics and Related Studies, University of York, Heslington, York, YO10 5DD, U.K. E-mail: alan.krause@york.ac.uk.
1 Introduction

The literature on charitable contributions, or more generally private contributions to a public good, has identified two key motives for giving. The first is an altruistic or public good motive, which is based on the idea that individuals care about the level of the public good. Individuals therefore give to charity in order to increase the total level of contributions. However, since individuals care only about the level of contributions, and not the source of the contributions, free-riding may become problematic. Indeed, an important result in the literature is that tax-financed contributions by the government completely crowd out private contributions (see, e.g., Sugden [1982], Warr [1982], Roberts [1984], Bergstrom, et al. [1986], and Andreoni [1988]). The second key motive put forward for giving is known as the warm-glow motive. The idea here is that individuals obtain utility or a “warm-glow” from the act of giving itself. In this case, individuals do care about the source of the contribution—they prefer, ceteris paribus, that they make the contribution. As a result, Andreoni [1989, 1990] shows that crowding out is incomplete when there exist both warm-glow and public good motives for giving.

In this paper, we extend the warm-glow motive to include a competitive element. That is, we assume that individuals evaluate the utility from their own charitable contribution relative to the contributions made by their peers. The idea is that individuals like to see themselves as being relatively generous. For example, one can imagine feeling better about making a $10 contribution to charity if others are giving $5 than when others are giving $20. Apart from being intuitively appealing, there is also empirical evidence that suggests individual contributions can be positively influenced by the contributions of others; see for example Glazer and Konrad [1996], Harbaugh [1998b], Andreoni and Petrie [2004], and Rege and Telle [2004]. This is one reason why charities often publicise the names of their major donors and the size of the contribution.

Our main finding is that the competitive element of the warm-glow motive for giving may exacerbate or attenuate the crowding-out effect, depending upon certain preference and income parameters. The intuition is that competitive warm-glow preferences make individuals more reluctant to reduce their giving in response to higher taxation, since
each individual cares about how their contribution compares with others. This acts to attenuate the crowding-out effect. However, the negative externality that individual giving imposes on others means that private contributions are higher than they otherwise would be. So suppose the government imposes a tax on individual $i$, which forces her to reduce her charitable contribution. This weakens the negative externality that her giving imposes on others, which allows the other individuals to reduce their own giving. This, in turn, weakens the negative externality that their giving imposes on individual $i$, which allows individual $i$ to further reduce her giving, and so on. If these second-round effects are strong enough, crowding out is exacerbated despite the initial reluctance individuals have to reduce their giving. Indeed, when the warm-glow motive is almost purely competitive so that the negative-externality effect is extreme, crowding out is almost dollar-for-dollar. That is, the fall in private contributions is almost equal to the tax-financed increase in the government’s contribution.

The papers most closely related to ours are those by Glazer and Konrad [1996], Harbaugh [1998a], Romano and Yildirim [2001], Duncan [2004], and Blumkin and Sadka [2007]. The papers by Glazer and Konrad [1996], Harbaugh [1998a], and Blumkin and Sadka [2007] all consider a prestige motive for giving. However, their focus is on the use of charitable contributions to signal social status and/or the implications for the crowding-out hypothesis are not explored.\footnote{In Glazer and Konrad [1996] charitable contributions are made to signal wealth, and they find that crowding out is incomplete. Harbaugh [1998a] and Blumkin and Sadka [2007] do not consider the crowding-out hypothesis. Harbaugh [1998a] focuses on the behaviour of charitable organisations rather than donors, while Blumkin and Sadka [2007] show that the optimal tax on charitable contributions is non-negative when it is chosen as part of an optimal redistributive tax system. Saez [2004] and Diamond [2006] also examine the taxation/subsidisation of charitable contributions as part of optimal redistributive tax systems.} Moreover, unlike the prestige motive for giving, our competitive warm-glow motive is still “warm-glow” in spirit; it is just that individuals measure their warm-glow from giving against the giving of others. Our competitive warm-glow motive is therefore an example of “keeping up with the Joneses” preferences. Romano and Yildirim [2001] also allow for a prestige-like motive for giving, but their focus is on the strategic behaviour of donors and charitable organisations. Duncan [2004] develops an “impact” theory of philanthropy, in which individuals care about...
the effectiveness of their donations. In his model, individual giving imposes a negative externality on others since under an assumption of diminishing returns, individual giving reduces the marginal effectiveness of additional contributions. Interestingly, Duncan [2004] shows, amongst other things, that tax-financed charitable contributions by the government can crowd in private contributions.

The remainder of the paper is organised as follows. Section 2 describes the model, while Section 3 examines the crowding-out hypothesis. Section 4 concludes, while proofs are relegated to an appendix.

2 A Simple Model

There are two individuals and a government. Individual $i$ chooses her own consumption $c_i$ and her contribution to charity $g_i$ to maximise the utility function:

$$u(c_i) + \lambda_i v(g_i - \alpha_i g_j) + (1 - \lambda_i) z(G + T)$$

(2.1)

subject to the budget constraint:

$$c_i + g_i \leq m_i - \tau_i$$

(2.2)

where the functions $u(\cdot)$, $v(\cdot)$ and $z(\cdot)$ are increasing and strictly concave. Total contributions to charity are denoted by $G + T$, where $G = g_i + g_j$ is total contributions by individuals $i$ and $j$, while $T$ is total contributions by the government. Following the standard practice, we assume that $T = \tau_i + \tau_j$, where $\tau_i$ and $\tau_j$ are lump-sum taxes imposed on individuals $i$ and $j$, respectively. Individual $i$’s pre-tax income is denoted by $m_i$. The parameter $\lambda_i \in (0, 1)$ represents the weight that individual $i$ places on the warm-glow motive for giving, where $\alpha_i \in [0, 1)$ measures the extent to which individual $i$ evaluates her own charitable contribution relative to that of individual $j$. The weight individual $i$ places on the public good motive for giving is $(1 - \lambda_i)$. It can be seen

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2See also Duncan [2009]. A similar idea to impact philanthropy has recently been developed by Atkinson [2009] to explain the prevalence of giving for overseas development.
that individual $j$’s charitable contribution imposes a negative externality on individual $i$ through the “competitive” warm-glow motive for giving, but it also generates a positive externality through the public good motive. It can also be seen that if $\alpha_i = 0$, our competitive warm-glow motive reduces to the standard warm-glow motive, in which utility from giving is modelled in the same manner as utility from an individual’s own consumption.

Assuming that the budget constraint (2.2) binds at an optimum, it can be solved for $c_i$ and substituted into the utility function (2.1). It is also assumed (i.e., we make the Nash conjecture) that each individual takes the charitable contribution of the other individual as given. The first-order condition on $g_i$ for an interior maximum can then be written as:

$$-u'(m_i - \tau_i - g_i) + \lambda_i v'(g_i - \alpha_i g_j) + (1 - \lambda_i)z'(G + T) = 0$$  \hspace{1cm} (2.3)

Likewise, individual $j$ will solve a programme exactly analogous to (2.1) – (2.2). The first-order condition on $g_j$ for an interior maximum is:

$$-u'(m_j - \tau_j - g_j) + \lambda_j v'(g_j - \alpha_j g_i) + (1 - \lambda_j)z'(G + T) = 0$$  \hspace{1cm} (2.4)

It is straightforward to show that the second-order conditions for a maximum corresponding to equations (2.3) and (2.4) are satisfied.

Suppose $\langle g_i, m_i, \tau_i, \lambda_i, \alpha_i, g_j, m_j, \tau_j, \lambda_j, \alpha_j \rangle$ is a solution to equations (2.3) and (2.4). Then by the Implicit Function Theorem there exist functions $g_i = g_i(m_i, \tau_i, \lambda_i, \alpha_i, m_j, \tau_j, \lambda_j, \alpha_j)$ and $g_j = g_j(m_i, \tau_i, \lambda_i, \alpha_i, m_j, \tau_j, \lambda_j, \alpha_j)$ that solve (2.3) and (2.4). The following proposition summarises the effect that the competitive element of the warm-glow motive for giving has on the level of charitable contributions:

**Proposition 1** An increase in $\alpha_i$ (resp. $\alpha_j$) results in an increase in $g_i$ (resp. $g_j$) and in total private contributions, but the effect on $g_j$ (resp. $g_i$) is ambiguous.

The proof of Proposition 1 is provided in the Appendix, but the intuition behind the result is fairly straightforward. An increase in $\alpha_i$ will, all else equal, increase the marginal utility individual $i$ obtains from giving. Accordingly, individual $i$ increases her charitable
contribution. This encourages individual $j$ to increase her own contribution, through the competitive warm-glow motive. But simultaneously, individual $i$’s higher contribution increases total contributions, which entices individual $j$ to lower her contribution vis-a-vis the public good motive. Therefore, the effect of an increase in $\alpha_i$ on $g_j$ is ambiguous, although the effect on total private contributions is always positive.

### 3 Crowding Out

An important question in the literature on charitable contributions is the extent to which tax-financed contributions by the government crowd out private contributions. Following the standard practice, we assume that the government increases its taxation of one of the individuals, in our case individual $i$, and we then examine the effect this has on the level of private contributions. First, it is shown in the Appendix that:

**Remark 1** $\lim_{\lambda_i, \lambda_j \to 0} \frac{\partial G(\cdot)}{\partial r_i} = -1$, i.e., crowding out is complete.

**Remark 2** If $\alpha_i = \alpha_j = 0$, then $\frac{\partial G(\cdot)}{\partial r_i} \in (-1, 0)$, i.e., crowding out is incomplete.

Remark 1 is the well-known result that crowding out is dollar-for-dollar when there is only the public good motive for giving. Remark 2 is the main contribution of Andreoni [1989, 1990] that crowding out is less than dollar-for-dollar when there are both (standard) warm-glow and public good motives for giving.

Our main interest is how the competitive element of the warm-glow motive for giving affects the extent of crowding out. To this end, we assume that the individuals’ utility functions take the Cobb-Douglas form:

\[
\ln(c_i) + \lambda_i \ln(g_i - \alpha_i g_j) + (1 - \lambda_i) \ln(G + T) \quad (3.1)
\]

\[
\ln(c_j) + \lambda_j \ln(g_j - \alpha_j g_i) + (1 - \lambda_j) \ln(G + T) \quad (3.2)
\]

for individuals $i$ and $j$, respectively. We then conduct some numerical simulations, which are illustrated in Figures 1–5.

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3 See in particular Bergstrom, et al. [1986], and Andreoni [2006] for a discussion of the literature and some related “neutrality” results.
In each figure, the horizontal axis shows the tax imposed on individual $i$, while the vertical axis shows the change in private giving (denoted $\Delta G$). The dashed line indicates complete (dollar-for-dollar) crowding out, which would be the case if there were only the public good motive for giving. Points above the dashed line correspond to incomplete crowding out. The solid lines in each figure illustrate the extent of crowding out with standard ($\alpha_i = \alpha_j = 0$) and competitive ($\alpha_i \neq 0, \alpha_j \neq 0$) warm-glow preferences. Taken together, Figures 1 – 5 show that the competitive element of the warm-glow motive for giving may exacerbate or attenuate the crowding-out effect, depending upon the preference and income parameters.

In Figure 1, individuals $i$ and $j$ are assumed to be identical, except in the left panel they place more weight on the public good motive for giving ($\lambda_i = \lambda_j = 0.25$), while in the right panel they place more weight on the warm-glow motive ($\lambda_i = \lambda_j = 0.75$). When the public good motive dominates, it can be seen that competitive warm-glow (henceforth, CWG) preferences attenuate the crowding-out effect. This is because free-riding is relatively extensive when the public good motive dominates, but CWG preferences imply that each individual now cares about the relative size of their contribution. Hence, CWG preferences reduce free-riding and attenuate the crowding-out effect. However, when the warm-glow motive for giving dominates, CWG preferences exacerbate the crowding-out effect. In this case, free-riding is minimal and each individual’s contribution is relatively large, which is made larger still by CWG preferences. Thus the negative externality that each individual’s giving imposes on the other is substantial. A tax on individual $i$ which forces her to reduce her giving weakens the negative externality imposed on individual $j$, allowing individual $j$ to reduce her own contribution. This, in turn, weakens the negative externality imposed on individual $i$, allowing individual $i$ to further reduce her contribution, and so on. The end result is that crowding out is exacerbated.

In Figure 2, the individuals are distinguished by the weight they place on the warm-glow versus public good motives for giving. In the left panel, individual $i$ places more weight on the warm-glow motive and individual $j$ places more weight on the public good motive. In this case, CWG preferences exacerbate the crowding-out effect. Since individual $i$ is giving mainly due to the warm-glow motive, while individual $j$ is giving
mainly due to the public good motive, individual $i$’s giving is much larger than that of individual $j$. This gives individual $i$ plenty of scope to reduce her giving in response to higher taxation, since the negative externality imposed on her by individual $j$ is relatively weak. Moreover, since individual $i$’s giving is relatively large, the negative externality it imposes on individual $j$ is substantial. Thus the reduction in individual $i$’s giving weakens the (substantial) negative externality imposed on individual $j$, allowing individual $j$ to reduce her own giving. This, in turn, allows further rounds of reductions, with the end result being that crowding out is exacerbated. In the right panel of Figure 2, however, individual $i$ places more weight on the public good motive than does individual $j$, and CWG preferences attenuate the crowding-out effect. In this case, individual $j$ is giving more than individual $i$, meaning that the negative externality individual $j$’s giving imposes on individual $i$ is substantial. Accordingly, individual $i$ is more reluctant to reduce her giving in response to higher taxation, the end result being that crowding out is attenuated.

In Figure 3, the individuals are distinguished by the intensity of their CWG preferences. In the left panel, the intensity is greater for individual $i$ ($\alpha_i = 0.75, \alpha_j = 0.25$), while the right panel shows the opposite case. When individual $i$’s CWG preferences are more intense, crowding out is attenuated. This is because individual $i$ is reluctant to reduce her giving in response to higher taxation, because she cares considerably as to how her giving compares with that of individual $j$. Furthermore, the (modest) reduction in individual $i$’s giving has little effect on individual $j$, because individual $j$ cares little about how her giving compares with that of individual $i$. Therefore, the fall in total private contributions is relatively small and crowding out is attenuated. By an analogous argument, crowding out is exacerbated when individual $i$’s CWG preferences are less intense. In this case, individual $i$ is more willing to reduce her giving in response to higher taxation, since she cares relatively less about how her giving compares with that of individual $j$. The decrease in individual $i$’s giving reduces the negative externality imposed on individual $j$, which is relatively strong since individual $j$ cares considerably as to how her giving compares with that of individual $i$. This allows individual $j$ to reduce her giving, which leads to further rounds of reductions. In the end, crowding out
is exacerbated.

In Figure 4, the individuals are distinguished by their incomes. In the left panel, individual $i$ has the higher income, while in the right panel individual $j$’s income is larger. When individual $i$ is the high-income individual, CWG preferences exacerbate the crowding-out effect. In this case, since individual $j$ has a low income, her giving is low and therefore the negative externality it imposes on individual $i$ is weak. This gives individual $i$ plenty of scope to reduce her giving in response to higher taxation. Also, since individual $i$ has a high income, her giving is large and it imposes a substantial negative externality on individual $j$. The reduction in individual $i$’s giving, therefore, allows individual $j$ to make a significant reduction in her own giving. In the end, crowding out is exacerbated. When individual $j$ is the high-income individual, CWG preferences attenuate the crowding-out effect, although for the parameters chosen the attenuation is marginal.\(^4\) Since individual $j$ has the higher income, her giving is substantial and it imposes a strong negative externality on individual $i$. Accordingly, individual $i$ is reluctant to reduce her giving in response to higher taxation. Moreover, since individual $i$’s income is low, her giving is low and the negative externality it imposes on individual $j$ is weak. Therefore, individual $j$ does not make much of a reduction in her own giving in response to the (modest) decline in individual $i$’s giving. The end result is that the fall in total private giving is relatively small, and crowding out is attenuated.

Figure 5 examines the same situation as Figure 1, except in Figure 5 competitive warm-glow preferences are almost purely competitive in the sense that $\alpha_i = \alpha_j = 0.98$.\(^5\) For both cases considered ($\lambda_i = \lambda_j = 0.25$ and $\lambda_i = \lambda_j = 0.75$), CWG preferences now exacerbate the crowding-out effect. Indeed, crowding out is almost dollar-for-dollar, and crowding out is worse when more weight is placed on the warm-glow motive for giving. The reason is that the negative externality that each individual’s giving imposes on the other is extreme, causing each individual’s charitable contribution to be much higher (in fact, more than double) than it otherwise would be. The tax imposed on

\(^4\)It is not possible to increase the difference in the individuals’ incomes in order to make the effect of CWG preferences clearer, because this would result in $g_i - \alpha_i g_j$ being negative.

\(^5\)It is not possible to consider $\alpha_i = \alpha_j = 0.99$, because this would result in $g_i - \alpha_i g_j$ being negative.
individual $i$ which forces her to reduce her giving allows individual $j$ to make a substantial reduction in her own giving. This leads to further rounds of reductions, with total private contributions falling by almost the same amount as the tax-financed increase in contributions by the government.

4 Closing Remarks

Traditional models of private contributions to a public good (such as a charity), as exemplified by Bergstrom, et al. [1986], conclude that tax-financed contributions by the government completely crowd out private contributions. But when a warm-glow motive for giving is added, Andreoni [1989, 1990] shows that crowding out is incomplete. In this paper, we have extended the warm-glow motive to include a competitive element, in order to capture the idea that individuals feel better about giving when they give more than others. Our main finding is perhaps surprising, in that the competitive element of the warm-glow motive has an ambiguous effect on the extent of crowding out. This is because, on the one hand, CWG preferences imply that individuals are more reluctant to reduce their giving. But on the other hand, any reduction in giving weakens the negative externality that individual giving imposes on others, which allows further rounds of reductions. Accordingly, CWG preferences may exacerbate or attenuate the crowding-out effect, depending upon preferences and income.

5 Appendix

Proof of Proposition 1

The Hessian associated with equations (2.3) and (2.4) is:

$$H = \begin{bmatrix}
  u''(c_i) + \lambda_i v''(g_i - \alpha_i g_j) + (1 - \lambda_i)z''(G + T) & -\alpha_i \lambda_i v''(g_i - \alpha_i g_j) + (1 - \lambda_i)z''(G + T) \\
  -\alpha_j \lambda_j v''(g_j - \alpha_j g_i) + (1 - \lambda_j)z''(G + T) & u''(c_j) + \lambda_j v''(g_j - \alpha_j g_i) + (1 - \lambda_j)z''(G + T)
\end{bmatrix} \tag{A.1}$$
The determinant of $H$ is given by:

\[
|H| = u''(c_i) [u''(c_j) + \lambda_j v''(g_j - \alpha_j g_i) + (1 - \lambda_j) z''(G + T)] \\
+ \lambda_i v''(g_i - \alpha_i g_j) [u''(c_j) + (1 + \alpha_i) (1 - \lambda_j) z''(G + T)] \\
+ (1 - \lambda_i) z''(G + T) [u''(c_j) + (1 + \alpha_j) \lambda_j v''(g_j - \alpha_j g_i)] \\
+ (1 - \alpha_i \alpha_j) \lambda_i \lambda_j v''(g_i - \alpha_i g_j) \lambda_j v''(g_j - \alpha_j g_i) > 0
\]  

(A.2)

Since $|H| \neq 0$, it follows from the Implicit Function Theorem and Cramer’s Rule that:

\[
\frac{\partial g_i(\cdot)}{\partial \alpha_i} = \frac{\lambda_i v''(g_i - \alpha_i g_j) g_j [u''(c_j) + \lambda_j v''(g_j - \alpha_j g_i) + (1 - \lambda_j) z''(G + T)]}{|H|} > 0 \quad \text{(A.3)}
\]

\[
\frac{\partial g_j(\cdot)}{\partial \alpha_i} = \frac{\lambda_i v''(g_i - \alpha_i g_j) g_j \alpha_j \lambda_j v''(g_j - \alpha_j g_i) - (1 - \lambda_j) z''(G + T)}{|H|} < 0 \quad \text{(A.4)}
\]

By adding (A.3) and (A.4) it can be shown that $\frac{\partial g_i(\cdot)}{\partial \alpha_i} + \frac{\partial g_j(\cdot)}{\partial \alpha_i} > 0$. The proof for an increase in $\alpha_j$ is analogous. ■

**Proof of Remarks 1 and 2**

Application of the Implicit Function Theorem and Cramer’s Rule to equations (2.3) – (2.4) yields:

\[
\frac{\partial g_i(\cdot)}{\partial \tau_i} = \frac{A}{|H|} < 0 \quad \text{and} \quad \frac{\partial g_j(\cdot)}{\partial \tau_i} = \frac{B}{|H|} < 0
\]  

(A.5)

where:

\[
A = -u''(c_i) [u''(c_j) + \lambda_j v''(g_j - \alpha_j g_i) + (1 - \lambda_j) z''(G + T)] \\
- (1 - \lambda_i) z''(G + T) [u''(c_j) + \lambda_j v''(g_j - \alpha_j g_i)] - (1 - \lambda_j) z''(G + T) \alpha_i \lambda_i v''(g_i - \alpha_i g_j) < 0
\]  

(A.6)

and:

\[
B = -(1 - \lambda_j) z''(G + T) \lambda_i v''(g_i - \alpha_i g_j - \alpha_j \lambda_j v''(g_j - \alpha_j g_i) [u''(c_i) + (1 - \lambda_i) z''(G + T)] < 0
\]  

(A.7)

Remark 1 follows from adding $\frac{\partial g_i(\cdot)}{\partial \tau_i}$ and $\frac{\partial g_j(\cdot)}{\partial \tau_i}$, with $\lambda_i \to 0$ and $\lambda_j \to 0$. Remark 2 follows from adding $\frac{\partial g_i(\cdot)}{\partial \tau_i}$ and $\frac{\partial g_j(\cdot)}{\partial \tau_i}$, with $\alpha_i = \alpha_j = 0$. ■
References


FIGURE 1
Crowding Out: Identical Individuals

\[ \lambda_i = 0.25, \ m_i = 100, \ \lambda_j = 0.25, \ m_j = 100, \ \tau_j = 0 \]

\[ \lambda_i = 0.75, \ m_i = 100, \ \lambda_j = 0.75, \ m_j = 100, \ \tau_j = 0 \]
FIGURE 2
Crowding Out: Individuals Distinguished by $\lambda_i$ and $\lambda_j$

$\lambda_i = 0.75$, $m_i = 100$, $\lambda_j = 0.25$, $m_j = 100$, $\tau_j = 0$

$\lambda_i = 0.25$, $m_i = 100$, $\lambda_j = 0.75$, $m_j = 100$, $\tau_j = 0$

$\alpha_i = \alpha_j = 0$

$\alpha_i = \alpha_j = 0.5$

$\alpha_i = \alpha_j = 0$

$\alpha_i = \alpha_j = 0.5$
FIGURE 3
Crowding Out: Individuals Distinguished by $\alpha_i$ and $\alpha_j$

$\lambda_i = 0.5, m_i = 100, \lambda_j = 0.5, m_j = 100, \tau_j = 0$

$\lambda_i = 0.5, m_i = 100, \lambda_j = 0.5, m_j = 100, \tau_j = 0$
FIGURE 4
Crowding Out: Individuals Distinguished by $m_i$ and $m_j$

$\lambda_i = 0.5, \ m_i = 150, \ \lambda_j = 0.5, \ m_j = 50, \ \tau_j = 0$

$\lambda_i = 0.5, \ m_i = 50, \ \lambda_j = 0.5, \ m_j = 150, \ \tau_j = 0$
FIGURE 5
Crowding Out: Identical Individuals

\[ \lambda_i = 0.25, \ m_i = 100, \ \lambda_j = 0.25, \ m_j = 100, \ \tau_j = 0 \]

\[ \lambda_i = 0.75, \ m_i = 100, \ \lambda_j = 0.75, \ m_j = 100, \ \tau_j = 0 \]