



THE UNIVERSITY *of York*

Discussion Papers in Economics

No. 10/17

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7 July 2010

Abstract

This paper shows that tax reforms involving an increase in consumption taxes and a decrease in income taxes cannot always be designed in a way that protects the welfare of some chosen class of consumer (e.g., low-income households), even if the government is indifferent to the welfare effects on all other consumers. This is contrary to common intuition and claims by some governments.

Keywords: Tax incidence; consumption taxes; income taxes; tax reform.

JEL classifications: D6, H22.

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1 Introduction

Over the past 20 years or so, there has been a shift in many countries away from income taxation and towards consumption taxation. For example, Canada (in 1991) and Australia (in 2000) introduced broad-based consumption taxes (the goods and services tax or GST), with offsetting reductions in income tax rates. The main arguments in favour of these reforms are: (i) the increase in post-tax income will encourage more work and employment, (ii) by raising the price of consumption relative to investment, savings and investment will increase, and (iii) tax evasion will be reduced as income earned in the underground economy (which cannot be taxed) will be taxed indirectly when it is consumed. Opponents of these reforms argue that the tax burden is shifted from higher-income to lower-income consumers, because income tax systems are typically progressive while consumption taxes are not — everyone pays the same rate irrespective of income. The government’s response to this criticism has often been to claim that it can design the tax changes in a way that protects the real income (welfare) of lower-income consumers. For example, it may increase the taxes on luxury goods by more than on necessities, or even exclude certain goods from taxation entirely. The government may also reduce the income tax rates faced by lower-income consumers by more than those faced by higher-income consumers.¹ Thus given this assumed flexibility, the expectation is that no matter the chosen class of consumer’s preferences and income, the government can always substitute consumption taxes for income taxes in a manner that does not adversely affect that class of consumer, provided it is indifferent to the welfare effects on other classes of consumers.

The purpose of this paper is to demonstrate formally that the government cannot always design the tax changes to protect the welfare of some chosen class of consumer. Demand and supply conditions in the economy, along with the requirement that the government balance its budget, impose a constraint on the number of ways that the

¹When Australia introduced the GST in 2000, the initial package proposed by the government included an across-the-board 10% consumption tax and roughly equal reductions in income tax rates along the income spectrum. The final package approved by the Senate exempted “basic foods” from GST and reduced the income tax cuts slated for the wealthy, the aim being to protect the real incomes of low-income consumers.

government can feasibly substitute consumption taxes for income taxes. And it is possible that all of these ways reduce the welfare of some consumers. We characterise this possibility in Theorem 1. In Theorem 2, we characterise the opposite possibility, i.e., when all feasible ways of substituting consumption taxes for income taxes make some class of consumer better-off. Finally, in Theorem 3, we characterise when there exist feasible ways of substituting consumption taxes for income taxes that make some class of consumer better-off and, simultaneously, there exist other feasible ways of substituting consumption taxes for income taxes that make that same class of consumer worse-off. It follows that it is only under the conditions of Theorem 3 that the government's chosen design of the tax changes becomes relevant for determining incidence.

Analytically, our methodology draws on the tax reform literature pioneered by Guesnerie [1977, 1995].² The key difference is that our methodology allows us to characterise differential incidence amongst the consumers, while the Guesnerie-style methodology can only characterise equilibrium-preserving and Pareto-improving reforms, i.e., feasible reforms that make *all* consumers better-off. Put simply, this is because the Guesnerie-style is to construct a cone from the gradients of the consumers' indirect utility functions, and then use a vector corresponding to the equilibrium conditions to determine feasibility. In order to characterise differential incidence we do the opposite, in that we construct a cone using the equilibrium conditions to determine feasibility, and then use the gradients of the consumers' indirect utility functions to determine incidence.³

The remainder of the paper is organised as follows. Section 2 describes the model, which is the general equilibrium commodity tax model of Diamond and Mirrlees [1971] extended to incorporate nonlinear income taxation. Section 3 presents the results, while Section 4 concludes. Proofs are relegated to an appendix.

²See also Diewert [1978], Weymark [1979], and for a good textbook treatment Myles [1995]. For recent applications of the Guesnerie-style tax reform methodology, see Brett [1998] and Murty and Russell [2005]. Tax reform techniques have also been used recently by Krause [2009] to examine the Laffer argument.

³Krause [2007] uses similar techniques to examine the incidence of capital taxation.

2 The Model

There are $k \geq 2$ types of consumers in the economy, who are distinguished by their wage rates and possibly their preferences. Without loss of generality, we assume that there is a single consumer of each type. As is true in many countries, there is linear taxation of the consumption goods and nonlinear taxation of labour income. Formally, we associate a nonlinear income tax schedule with k tax treatments $\langle y_i, m_i \rangle_{i=1}^k$, where y_i is the pre-tax income and m_i is the post-tax income of consumer i . Therefore, $y_i - m_i$ is the income tax paid by consumer i . The consumers have no profit income, as we make the common simplifying assumption that the government taxes away all pure profit.⁴

Consumer i chooses her net (of endowment) vector of consumption goods $x_i \in \mathbb{R}^n$ and self-selects her tax treatment $\langle y_i, m_i \rangle$ (which determines her labour supply) to solve the following programme:

$$\max_{x_i} \{U_i(x_i, l_i) = U_i(x_i, \frac{y_i}{w_i}) \mid qx_i \leq m_i\} \quad (2.1)$$

where $U_i(\cdot)$ is consumer i 's direct utility function, and $y_i = w_i l_i$ where w_i is consumer i 's wage rate and l_i is consumer i 's labour supply. We assume throughout that $0 < w_1 < w_2 < \dots < w_k$, so that consumer 1 is the lowest-wage type and consumer k is the highest-wage type. Following the standard practice, we assume that pre-tax income is observable (and therefore taxable) by the government, but each individual's type is private information which rules out the use of personalised lump-sum taxes. The consumer price vector corresponding to the n consumption goods is $q = p + t$, where p is the producer price vector and t is a vector of consumption taxes.

The supply side of the economy consists of a single, aggregate, profit-maximising firm. The firm's profit function is given by $\pi(p, w)$, where $w := \langle w_1, w_2, \dots, w_k \rangle$. Application of Hotelling's Lemma to the profit function yields the firm's output-supply and input-

⁴Alternatively, one could assume that the production side of the economy is characterised by constant returns to scale, which implies zero profits in equilibrium.

demand functions:

$$\nabla_p \pi(\cdot) = x(p, w) \quad \text{and} \quad \nabla_w \pi(\cdot) = -l(p, w) \quad (2.2)$$

where $x(\cdot)$ is the (net) supply vector of the n consumption goods, and $l(\cdot)$ is the demand vector for the k types of labour.

Equilibrium is obtained if and only if:

$$\sum x_i(q, \frac{y_i}{w_i}, m_i) - x(p, w) \leq 0^{(n)} \quad (2.3)$$

$$l(p, w) - \frac{y}{w} \leq 0^{(k)} \quad (2.4)$$

$$V_i(q, \frac{y_{i-1}}{w_i}, m_{i-1}) - V_i(q, \frac{y_i}{w_i}, m_i) \leq 0 \quad i = 2, \dots, k \quad (2.5)$$

where $x_i(\cdot)$ are the consumers' demand functions, and $V_i(\cdot)$ are the consumers' indirect utility functions. Equations (2.3) and (2.4) are standard market clearing conditions for the consumption goods and labour, where $\frac{y}{w} := \langle \frac{y_1}{w_1}, \frac{y_2}{w_2}, \dots, \frac{y_k}{w_k} \rangle$ is the labour supply by the k consumers. Market clearing ensures that the government's budget is exactly balanced if all the equations in (2.3) and (2.4) are satisfied as equalities, or in surplus if some of these equations are satisfied as strict inequalities.⁵ The equations in (2.5) are incentive-compatibility (or self-selection) constraints associated with nonlinear income taxation. We analyse what Stiglitz [1982] calls the “normal” case and what Guesnerie [1995] calls “redistributive equilibria”, in that the incentive-compatibility constraints may bind “downwards” but never “upwards”. This is consistent with redistributive taxation, which creates an incentive for higher-wage consumers to mimic lower-wage consumers, but not vice versa. Built into (2.5) is the simplifying assumption that only downward-adjacent incentive-compatibility constraints may bind.⁶ Finally, for analytical purposes we assume that the status quo equilibrium is “tight”, i.e., the equations in (2.3) – (2.5) all hold with equality in the initial equilibrium. This assumption allows us

⁵This follows from Walras' Law. See chapter 2 in Guesnerie [1995] for further details.

⁶This would necessarily be the case if the consumers' indifference curves satisfied the “single-crossing” property in (y_i, m_i) space.

to differentiate the system (2.3) – (2.5).

3 Characterising Incidence

Starting in any tight equilibrium, we are interested in determining the incidence of a small (modelled as differential) increase in the consumption taxes and decrease in the income taxes. To this end, define a *reform* as the vector $dR := \langle dp, dt, dw, dy, dm \rangle$ where $y := \langle y_1, y_2, \dots, y_k \rangle$ and $m := \langle m_1, m_2, \dots, m_k \rangle$. The government has direct control over the consumption taxes t and the income taxes $\langle y, m \rangle$, and changes in these taxes may induce changes in p and w according to the equilibrium conditions. Specifically, a reform is *equilibrium-preserving* if and only if $\nabla Z dR \leq 0^{(n+2k-1)}$, where ∇Z is the Jacobian matrix (with respect to dR) associated with (2.3) – (2.5) and is defined as:

$$\nabla Z := \begin{bmatrix} \sum \nabla_q x_i - \nabla_p x & \sum \nabla_q x_i & \sum \nabla_w x_i - \nabla_w x & \sum \nabla_y x_i & \sum \nabla_m x_i \\ \nabla_p l & 0^{(k \times n)} & \nabla_w l - \nabla_w \frac{y}{w} & -\nabla_y \frac{y}{w} & 0^{(k \times k)} \\ \nabla_q \hat{V} - \nabla_q V & \nabla_q \hat{V} - \nabla_q V & \nabla_w \hat{V} - \nabla_w V & \nabla_y \hat{V} - \nabla_y V & \nabla_m \hat{V} - \nabla_m V \end{bmatrix}$$

where all derivatives are evaluated in the status quo equilibrium, and:

$$\hat{V} := \begin{bmatrix} V_2(q, \frac{y_1}{w_2}, m_1) \\ V_3(q, \frac{y_2}{w_3}, m_2) \\ \cdot \\ \cdot \\ \cdot \\ V_k(q, \frac{y_{k-1}}{w_k}, m_{k-1}) \end{bmatrix} \quad \text{and} \quad V := \begin{bmatrix} V_2(q, \frac{y_2}{w_2}, m_2) \\ V_3(q, \frac{y_3}{w_3}, m_3) \\ \cdot \\ \cdot \\ \cdot \\ V_k(q, \frac{y_k}{w_k}, m_k) \end{bmatrix}$$

We are interested in reforms that involve an increase in the consumption taxes and a decrease in the income taxes. That is, we are interested in reforms that satisfy:

$$\begin{bmatrix} 0^{(n \times n)} & I^{(n \times n)} & 0^{(n \times k)} & 0^{(n \times k)} & 0^{(n \times k)} \\ 0^{(k \times n)} & 0^{(k \times n)} & 0^{(k \times k)} & -I^{(k \times k)} & I^{(k \times k)} \end{bmatrix} dR \gg 0^{(n+k)}$$

For convenience, we rewrite this expression as $\nabla S dR \gg 0^{(n+k)}$.⁷

Let $\Gamma \subseteq \mathbb{R}^{2n+3k}$ be the cone generated by taking non-negative linear combinations of the rows of ∇Z . The set of equilibrium-preserving reforms is therefore the negative polar cone of Γ , i.e., $-P(\Gamma) := \{dR \in \mathbb{R}^{2n+3k} \mid \nabla Z dR \leq 0^{(n+2k-1)}\}$. Let $\Lambda \subseteq \mathbb{R}^{2n+3k}$ be the cone generated by taking non-negative linear combinations of the rows of ∇S . The positive polar cone of Λ is the set of reforms that involve an increase in the consumption taxes and a decrease in the income taxes, i.e., $P(\Lambda) := \{dR \in \mathbb{R}^{2n+3k} \mid \nabla S dR \gg 0^{(n+k)}\}$. It follows that the set of equilibrium-preserving reforms that involve an increase in the consumption taxes and a decrease in the income taxes is $-P(\Gamma) \cap P(\Lambda) =: E$. It is assumed that E is not empty; otherwise the government cannot feasibly substitute consumption taxes for income taxes. The intuitive interpretation of E is that it represents the “degrees of freedom” the government has in substituting consumption taxes for income taxes. If E is “small” in size, the government has little flexibility in how it substitutes consumption taxes for income taxes. On the other hand, if E is “large” the government has greater flexibility. For future reference, we denote the positive polar cone of E by $P(E)$ and the negative polar cone of E by $-P(E)$.

A reform makes consumer i better-off if and only if $dV_i(\cdot) = \nabla V_i dR > 0$, where ∇V_i is the gradient of consumer i ’s indirect utility function (with respect to dR) and is defined as $\nabla V_i := \langle \nabla_q V_i, \nabla_q V_i, \nabla_w V_i, \nabla_y V_i, \nabla_m V_i \rangle$. Similarly, a reform makes consumer i worse-off if and only if $dV_i(\cdot) = \nabla V_i dR < 0$. We can now state the following theorems (proofs are in the appendix).

Theorem 1 *Substituting consumption taxes for income taxes necessarily makes consumer i worse-off if and only if $\nabla V_i \in -P(E)$. Moreover, $\nabla V_i \in -P(E)$ if and only if there exist real numbers $\langle \mu_1, \dots, \mu_n \rangle \geq 0^{(n)}$, $\langle \sigma_1, \dots, \sigma_k \rangle \geq 0^{(k)}$, $\langle \theta_2, \dots, \theta_k \rangle \geq 0^{(k-1)}$, $\lambda \geq 0$, and $\langle \gamma_1, \dots, \gamma_n, \xi_1, \dots, \xi_k \rangle > 0^{(n+k)}$ such that:*

$$\lambda \nabla V_i + \langle \gamma_1, \dots, \gamma_n, \xi_1, \dots, \xi_k \rangle \nabla S = \langle \mu_1, \dots, \mu_n, \sigma_1, \dots, \sigma_k, \theta_2, \dots, \theta_k \rangle \nabla Z$$

where all derivatives are evaluated in the status quo equilibrium.

⁷Vector notation: $z \geq \bar{z} \iff z_j \geq \bar{z}_j \forall j$, $z > \bar{z} \iff z_j \geq \bar{z}_j \forall j \wedge z \neq \bar{z}$, $z \gg \bar{z} \iff z_j > \bar{z}_j \forall j$.

Theorem 2 *Substituting consumption taxes for income taxes necessarily makes consumer i better-off if and only if $\nabla V_i \in P(E)$. Moreover, $\nabla V_i \in P(E)$ if and only if there exist real numbers $\langle \bar{\mu}_1, \dots, \bar{\mu}_n \rangle \geq 0^{(n)}$, $\langle \bar{\sigma}_1, \dots, \bar{\sigma}_k \rangle \geq 0^{(k)}$, $\langle \bar{\theta}_2, \dots, \bar{\theta}_k \rangle \geq 0^{(k-1)}$, $\bar{\lambda} \geq 0$, and $\langle \bar{\gamma}_1, \dots, \bar{\gamma}_n, \bar{\xi}_1, \dots, \bar{\xi}_k \rangle > 0^{(n+k)}$ such that:*

$$-\bar{\lambda} \nabla V_i + \langle \bar{\gamma}_1, \dots, \bar{\gamma}_n, \bar{\xi}_1, \dots, \bar{\xi}_k \rangle \nabla S = \langle \bar{\mu}_1, \dots, \bar{\mu}_n, \bar{\sigma}_1, \dots, \bar{\sigma}_k, \bar{\theta}_2, \dots, \bar{\theta}_k \rangle \nabla Z$$

where all derivatives are evaluated in the status quo equilibrium.

Theorems 1 and 2 formalise the intuition that there are only a limited number of ways in which the government can feasibly substitute consumption taxes for income taxes, and it is possible that all of these ways make some consumers worse-off (Theorem 1) and other consumers better-off (Theorem 2). In particular, Theorem 1 implies that the government cannot always design the tax changes in a way that protects the welfare of some chosen class of consumer.

The sets in Theorems 1 and 2 do not, generally, form a partition of \mathbb{R}^{2n+3k} . Theorem 3 addresses the possibility that $\nabla V_i \notin -P(E)$ and $\nabla V_i \notin P(E)$.

Theorem 3 *There exists a feasible way of substituting consumption taxes for income taxes that makes consumer i better-off and, simultaneously, there exists another feasible way of substituting consumption taxes for income taxes that makes consumer i worse-off, if and only if $\nabla V_i \notin -P(E)$ and $\nabla V_i \notin P(E)$. Moreover, $\nabla V_i \notin -P(E)$ and $\nabla V_i \notin P(E)$ if and only if there do not exist real numbers $\langle \mu_1, \dots, \mu_n \rangle \geq 0^{(n)}$, $\langle \sigma_1, \dots, \sigma_k \rangle \geq 0^{(k)}$, $\langle \theta_2, \dots, \theta_k \rangle \geq 0^{(k-1)}$, $\lambda \geq 0$, and $\langle \gamma_1, \dots, \gamma_n, \xi_1, \dots, \xi_k \rangle > 0^{(n+k)}$ such that:*

$$\lambda \nabla V_i + \langle \gamma_1, \dots, \gamma_n, \xi_1, \dots, \xi_k \rangle \nabla S = \langle \mu_1, \dots, \mu_n, \sigma_1, \dots, \sigma_k, \theta_2, \dots, \theta_k \rangle \nabla Z$$

and there do not exist real numbers $\langle \bar{\mu}_1, \dots, \bar{\mu}_n \rangle \geq 0^{(n)}$, $\langle \bar{\sigma}_1, \dots, \bar{\sigma}_k \rangle \geq 0^{(k)}$, $\langle \bar{\theta}_2, \dots, \bar{\theta}_k \rangle \geq 0^{(k-1)}$, $\bar{\lambda} \geq 0$, and $\langle \bar{\gamma}_1, \dots, \bar{\gamma}_n, \bar{\xi}_1, \dots, \bar{\xi}_k \rangle > 0^{(n+k)}$ such that:

$$-\bar{\lambda} \nabla V_i + \langle \bar{\gamma}_1, \dots, \bar{\gamma}_n, \bar{\xi}_1, \dots, \bar{\xi}_k \rangle \nabla S = \langle \bar{\mu}_1, \dots, \bar{\mu}_n, \bar{\sigma}_1, \dots, \bar{\sigma}_k, \bar{\theta}_2, \dots, \bar{\theta}_k \rangle \nabla Z$$

where all derivatives are evaluated in the status quo equilibrium.

In summary, it is only under the conditions of Theorem 3 that the manner in which the government substitutes consumption taxes for income taxes becomes relevant for determining consumer i 's incidence.

The geometry of Theorems 1 and 2 is illustrated in Figure 1, albeit in a stylised two-dimensional manner. Equilibrium requires that $dR \in E$, where the size of E represents the “degrees of freedom” or “number of ways” in which the government can feasibly substitute consumption taxes for income taxes. Since $\nabla V_i \in P(E)$ and $\nabla V_j \in -P(E)$, it follows that dR must form an acute angle with ∇V_i (meaning $\nabla V_i dR > 0$) and an obtuse angle with ∇V_j (meaning $\nabla V_j dR < 0$). Therefore, substituting consumption taxes for income taxes necessarily makes consumer i better-off and consumer j worse-off. Figure 2 illustrates the geometry of Theorem 3. If the government substitutes consumption taxes for income taxes in a manner corresponding to dR , consumer i is made better-off since dR and ∇V_i form an acute angle. However, substituting consumption taxes for income taxes in a manner corresponding to $d\bar{R}$ makes consumer i worse-off, since $d\bar{R}$ and ∇V_i form an obtuse angle.

4 Conclusion

This paper has demonstrated formally that tax reforms involving an increase in consumption taxes and a decrease in income taxes cannot always be designed in a way that protects the welfare of some chosen class of consumer. Demand and supply conditions in the economy, along with the requirement that the government balance its budget, limit the number of ways in which the government can substitute consumption taxes for income taxes. And it is possible that all of these ways make some consumers worse-off.

5 Appendix

Proof of Theorem 1

The proof is based on an analysis of the geometry of the problem. Feasibility requires that $dR \in E$. Note that $-P(E) = \{\alpha \in \mathbb{R}^{2n+3k} \mid \alpha dR < 0 \ \forall \ dR \in E\}$. Therefore,

$\nabla V_i \in -P(E) \implies \nabla V_i dR < 0 \ \forall \ dR \in E$. And if there does not exist a $dR \in E$ such that $\nabla V_i dR \geq 0$, then $\nabla V_i \in -P(E)$. Using Motzkin's Theorem of the Alternative,⁸ there does not exist a reform dR such that:

$$\nabla Z dR \leq 0^{(n+2k-1)} \quad \nabla V_i dR \geq 0 \quad \nabla S dR \gg 0^{(n+k)}$$

if and only if there exist real numbers $\langle \mu_1, \dots, \mu_n \rangle \geq 0^{(n)}$, $\langle \sigma_1, \dots, \sigma_k \rangle \geq 0^{(k)}$, $\langle \theta_2, \dots, \theta_k \rangle \geq 0^{(k-1)}$, $\lambda \geq 0$, and $\langle \gamma_1, \dots, \gamma_n, \xi_1, \dots, \xi_k \rangle > 0^{(n+k)}$ such that:

$$\lambda \nabla V_i + \langle \gamma_1, \dots, \gamma_n, \xi_1, \dots, \xi_k \rangle \nabla S = \langle \mu_1, \dots, \mu_n, \sigma_1, \dots, \sigma_k, \theta_2, \dots, \theta_k \rangle \nabla Z$$

as stated in the theorem. ■

Proof of Theorem 2

Analogous to that of Theorem 1. ■

Proof of Theorem 3

Follows directly from Theorems 1 and 2. ■

⁸See chapter 2 in Mangasarian [1969] for a statement and proof of Motzkin's Theorem.

FIGURE 1

The Geometry of Theorems 1 and 2

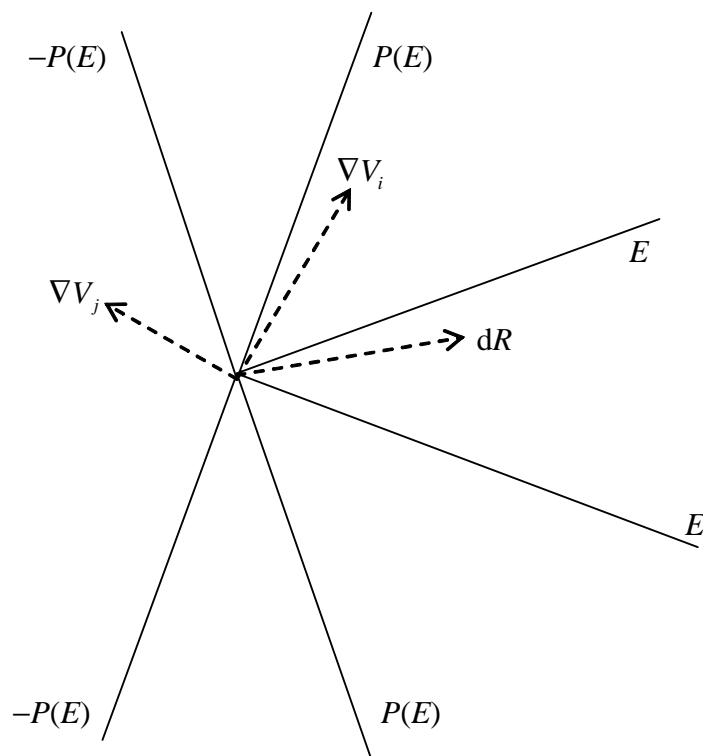
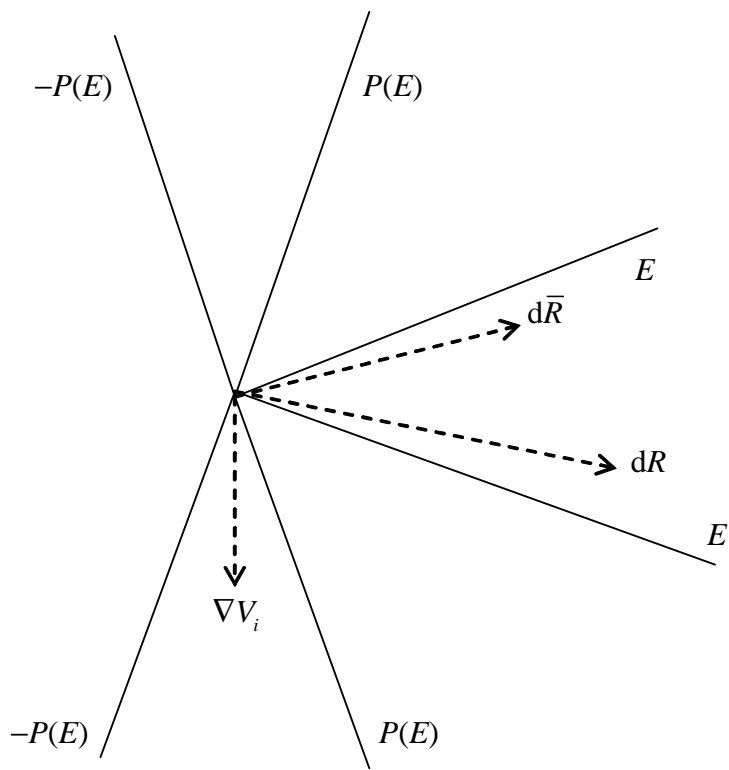


FIGURE 2

The Geometry of Theorem 3



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