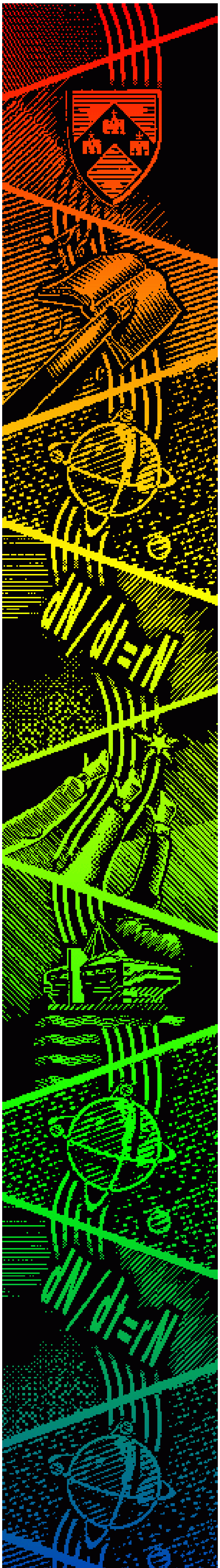

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Endogenous Incomplete Markets: A Production Model with Idiosyncratic Risk

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Endogenous Incomplete Markets: A Production Model with Idiosyncratic Risk

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Abstract

This paper considers a two period general equilibrium production model with incomplete markets (GEI). The novelty of this model is the endogenous smooth asset structure introduced in Stiefenhofer (2010). It is shown that incomplete markets is a consequence of idiosyncratic risk.

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1 Introduction

Most of the general equilibrium literature on production with incomplete markets models incomplete markets by hypothesis, and derives equilibrium properties for exogenous asset structures. These models consider aggregate risk as the only source of uncertainty. For a summary of this class of models see i.e. [1] or [2].

This paper considers an endogenous asset formation model introduced in [3],[4]. The paper shows that incomplete markets is a consequence of firm specific risk. This follows from long run market entry barriers, where firms cannot issue stocks in each state of the world.

2 The Endogenous Asset Formation Model

We consider a two period $t \in T = \{0, 1\}$ model with technological uncertainty in period 1 represented by states of nature. An element in the set of mutually exclusive and exhaustive uncertain events is denoted $s \in \{1, ..., S\}$, where by convention $s = 0$ represents the certain event in period 0. We count in total $(S + 1)$ states of nature.

The economic agents are the $j \in \{1, ..., n\}$ producers and $i \in \{1, ..., m\}$ consumers which are characterized by assumptions of smooth economies. There are $k \in \{1, ..., l\}$ physical commodities and $j \in \{1, ..., n\}$ financial assets, referred to as stocks. Physical goods are traded on each of the $(S + 1)$ spot markets. Producers issue stocks which are traded at $s = 0$, yielding a payoff in the next period at uncertain state $s \in \{1, ..., S\}$. The quantity of stocks issued by firm $j \in \{1, ..., n\}$ is denoted $z_j \in \mathbb{R}_-$, where $\sum_{j=1}^{n} z_j = \hat{z}$.

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There are in total $l(S+1)$ physical goods available for consumption. The consumption bundle of agent $i \in \{1, ..., m\}$ is denoted $x_i = (x_i(0), x_i(s), ..., x_i(S)) \in \mathbb{R}^{l(S+1)}_{++}$, with $x_i(s) = (x_i^1(s), ..., x_i^l(s)) \in \mathbb{R}^l_{++}$, and $\sum_{i=1}^m x_i = x$. The consumption space for each consumer $i \in \{1, ..., m\}$ is $X_i = \mathbb{R}^{l(S+1)}_{++}$, the strictly positive orthant. The associated price system is a collection of vectors represented by $p = (p(0), p(s), ..., p(S)) \in \mathbb{R}^{l(S+1)}_{++}$, with $p(s) = (p^1(s), ..., p^l(s)) \in \mathbb{R}^l_{++}$, the strictly positive orthant. Each consumer $i \in \{1, ..., m\}$ is endowed with initial resources $\omega_i \in \Omega$, where $\Omega = \mathbb{R}^{lT}_{++}$, and $\omega_i = (\omega_i(0), \omega_i(1))$ a collection of strictly positive vectors. Denote an initial resource vector at time period $t \in T = \{0, 1\}$, $\omega_i(t) = (\omega_i^1(t), ..., \omega_i^l(t)) \in \mathbb{R}^l_{++}$, and the sum of total initial resources, $\sum_{i=1}^m \omega_i = \omega$.

In total, there are n financial assets traded in period $t = 0$. Denote the quantity vector of stocks purchased by consumer $i \in \{1, ..., m\}$, $z_i = (z_i(1), ..., z_i(n)) \in \mathbb{R}^n_+$, a collection of quantities of stocks purchased from producers $j \in \{1, ..., n\}$, and denote $\sum_{i=1}^m z_i = z$, with associated stock price system $q = (q(1), ..., q(n)) \in \mathbb{R}^n_{++}$. Denote producer j's period one vector of capital purchase $y^j(0) \in \mathbb{R}^l_-$, and denote his period two state dependent net activity vector $y_j(s) = (y_j^1(s), ..., y_j^l(s)) \in \mathbb{R}^l$. Let $y_j(t = 1) = (y_j(s), ..., y_j(S)) \in \mathbb{R}^{lS}$ denote the collection of state dependent period t=1 net activity vectors. A period two input of production for every $s \in \{1, ..., S\}$ is by convention denoted $y_j^k(s) < 0$, and a production output in state $s \in \{1, ..., S\}$ satisfies $y_j^k(s) \geq 0$.

Each firm $j \in \{1, ..., n\}$ issues stocks z_j at stock price q_j in period one in order to build up production capacity. A firm's total cash acquired via stock market determines the upper bound of the total value of production capacity it can install in the same period. Denote this liquidity constraint $q_j z_j = M_j$, where $M_j \in \mathbb{R}_+$ is a non-negative real number and $z_j \in \mathbb{R}_-$ a feasible financial policy of the firm $j \in \{1, ..., n\}$. M_j constraints the quantity of capital $y(0) \in \mathbb{R}^l_-$ a producer j can purchase at spot price system $p(0) \in \mathbb{R}^l_{++}$. The quantity of intermediate goods $y_j(0)$ purchased in period one determines a correspondence $\phi_j|_Z$. This correspondence defines the technology of the firm at feasible financial policy Z. Let the production set available to each producer $j \in \{1, ..., n\}$ in period two be described by this technology, $\phi_j|_Z : \mathbb{R}^m_- \to \mathbb{R}^n_-$, a correspondence defined on the set of period two inputs, and denote it $Y_j|_z \subset \mathbb{R}^l$. Let S denote the set of all exogenously given states of nature. Then for each producer $j \in \{1, ..., n\}$ let the $t = 1$ one period production set be defined by a map $\Phi_j|_Z$ with domain $\mathbb{R}^m_- \times \mathbb{R}_{++}$ and range $\mathbb{R}^n_- \times \mathbb{R}_{++}$, and denote it $Y_j|_z(s) \subset \mathbb{R}^{l(S+1)}$, where $m + n = l$. [1]

The objective function of the firm is shown in equation (1).

$$\underset{\left(\bar{y}|_{\hat{z}}(s),(\hat{z};\bar{y}(0))\right)_j}{\arg\max} \left\{ \bar{q} z_j + \sum_{s=1}^{S} \bar{p}(s) \cdot y_j|_{\hat{z}}(s) : \begin{array}{l} \bar{q} \sum_{i=1}^{m} \bar{z}_i(j) \geq \bar{q}\hat{z}_j = \bar{p}(0) \cdot y_j(0) \\ y_j|_{\hat{z}}(s) \in Y_j|_{\hat{z}}(s) \end{array} \quad s \in S \right\}, \tag{1}$$

Consumers play the same role in this production model as in the classical GEI model with production. They invest into firms because they want to transfer wealth between future uncertain states of nature, and to smooth out consumption across states of nature. Each consumer $i \in \{1, ..., m\}$ purchases stocks z_i at stock price q in period one in return for a dividend stream in the next period. The consumer's optimization problem is to

[1] Assumptions of smooth production sets and utility functions apply. These are introduced in detail in Stiefenhofer (2009).

maximize utility subject to a sequence of $(S+1)$ budget constraints.

Denote consumer i's sequence of $(S+1)$ budget constraints

$$B_{z_i} = \left\{ (x_i, z_i) \in \mathbb{R}_{++}^{l(S+1)} \times \mathbb{R}_{+}^{n} \middle| \begin{array}{c} p(0) \centerdot (x_i(0) - \omega_i(0)) = -qz_i \\ p(s)\Box(x_i(s) - \omega_i(1)) = \Pi(p_1, \Phi|_Z)\theta_i(z_i) \end{array} \right\}, \tag{2}$$

where $\theta_{ij} = z_i(j)[\sum_i z_i(j)]^{-1}$ is the proportion of total payoff of financial asset $j \in \{1, ..., n\}$ hold by consumer $i \in \{1, ..., m\}$ after trade at the stock market took place in period one. Π is the full payoff matrix of the economy of dimension $(S \times n)$.

Algebraically, each $i \in \{1, ..., m\}$

$$(\overline{x}_i; \overline{z}_i) argmax \Big\{ u_i(x_i; z_i) : z_i \in \mathbb{R}_+^n, x_i \in B_{z_i} \Big\}. \tag{3}$$

A competitive equilibrium of the production economy defined by the initial resource vector $\omega \in \Omega$ is a price pair $(p, q) \in \mathcal{S} \times \mathbb{R}_{++}^n$ if equality between demand and supply of physical goods and financial assets is satisfied in all states of nature, $s = 0, 1, ..., S$. Its associated competitive equilibrium allocation is a collection of vectors $[(x, z), (y, \widetilde{z})] \in \mathbb{R}_{++}^{l(S+1)m} \times \mathbb{R}_+^n \times \mathbb{R}^{l(S+1)n} \times \mathbb{R}_-^n$ of consumption, production and financial quantities.

$$\begin{array}{ll} \text{(i)} & \sum_{i=1}^m (\overline{x}_i(0) - \omega_i(0)) = \sum_{j=1}^n \overline{y}_j(0) \\ \text{(ii)} & \sum_{i=1}^m (\overline{x}_i - \omega_i(1)) = \sum_{s=1}^S \sum_{j=1}^n \overline{y}_j(s) \\ \text{(iii)} & \sum_{j=1}^n \sum_{i=1}^m (\overline{z}_i)_j = 0,\ \sum_{i=1}^m \theta(\overline{z}_i)_j = 1 \text{ for all } j \in \{1, ..., n\} \end{array} \tag{4}$$

Stiefenhofer [3] shows that equilibria for this stock market model always exist.

3 Result

We formally introduce the assumption of idiosyncratic risk.

Assumption 1 (T) *For each producer $j \in \{1, ..., n\}$, technological uncertainty is represented by a technology correspondence $\phi|_Z(s)_j$, a period $t = 1$ map from $\mathbb{R}_-^m \times \mathbb{R}_{++} \to \mathbb{R}_-^n \times \mathbb{R}_{++}$ for all $s \in \{1, ..., S\}$ such that for each $j \in \{1, ..., n\}$, $S \geq 2$.*

Theorem 1 *Incomplete markets, $n < S \Longleftarrow \sum_j Y_j|_{\bar{z}}$ is a consequence of assumption 1 (T).*

Proof 1 (Theorem 1) *Let $S_j = 1$ for every $j \in \{1, ..., n\}$, and $\sum_j S_j = \mathcal{S}$. Then long run profit prospects $\pi(p) > 0$ imply long run capacity adjustment and market entrance until $n = S$. Similar for negative long run prospects, the number of firms decreases until $n = S$, and $\pi(p) = 0$ satisfied. This violates assumption (T). Let $S > 1$ for every $j \in \{1, ..., n\}$, and $\sum_j S_j = \mathcal{S}$. Then $\pi(p) > 0$ implies market entrance and the issue of new securities such that in the limit as $\pi(p) \to 0$ the number of firms increases until $j \to n < S$ by assumption (T). Similar for $\pi(p) < 0$, firms exit the market and as $\pi(p) \to 0$ the number of firms decreases until $j \to n < S$ by assumption (T), and $\pi(p) = 0$ satisfied.*

4 Conclusion

The paper considers idiosyncratic risk in an endogenized asset model of production. It shows that incomplete markets is a consequence of technological uncertainty. This improves on the literature in GEI with production which considers aggregate risk, and models incomplete markets by assumption.

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