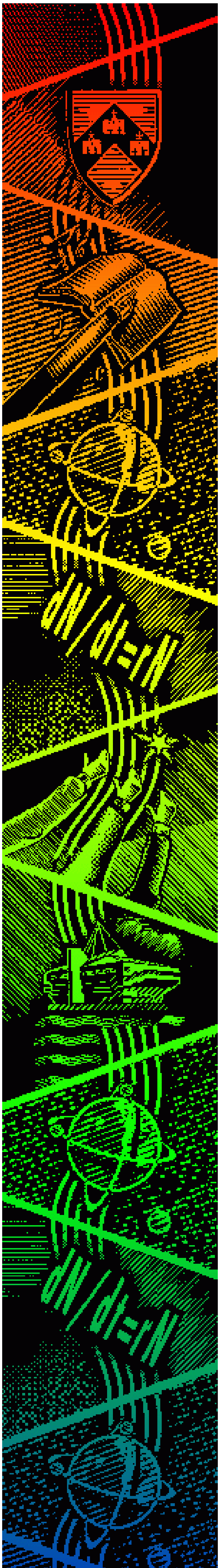

THE UNIVERSITY *of York*

Discussion Papers in Economics

No. 10/11

A General Equilibrium Corporate Finance Theorem for Incomplete Markets: A Special Case

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A General Equilibrium Corporate Finance Theorem for Incomplete Markets: A Special Case

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June 6, 2010

Abstract

This paper considers corporate equilibria in a two period endogenized asset general equilibrium model for a class of profit maximizing objective functions of the firms introduced in Stiefenhofer (2009). It shows by means of a particular case that for a fixed financial policy, every extensive form stock market equilibrium can be translated into a reduced form equilibrium. This suggests determinateness of corporate equilibria for varying financial parameters. A change in the firm's financial policy changes the production set available to it in the next period, hence real effects.

1 Introduction

The Modigliani and Miller corporate finance theorem [7] states that the value of the firm is independent of its financial policies. This result was originally derived in a series of papers in a partial equilibrium set up. The first generalization of this theorem to a general equilibrium framework is due to Stiglitz [10],[11]. Beyond a one period general equilibrium model, DeMarzo [2], Magill and Quinzii [6], and Duffie and Shafer [4], confirm the validity of this result for the case of incomplete markets. These papers have in common that they derive corporate equilibrium properties for exogenous asset structures with production sets independent of the financial activities of the firms. This dichotomy implies non-trivial economic equilibrium consequences. For example, exogenous asset formation models essentially ignore the financing of the firm since production sets are fixed, thus the Modigliani and Miller theorem holds under classical assumptions.

This paper improves on the corporate finance theory derived under exogenous asset structures considered in classical GEI models of production [3],[5]. It considers a simple version of the relevance of financial policy theorem for a two period general equilibrium model with endogenous production sets and incomplete markets [8],[9]. In this model, endogenized production sets available to firms are not independent of the financial activities of the firm. In the long run, firms issue stocks, buy capital and build up production capacity. In the short run, they choose production activities in installed production sets which maximize their profits. Inputs of production are financed with the revenue generated with the sell of production outputs. Hence, financial policies are non-neutral. This allows to consider corporate finance theorems from a new perspective.

The paper is organized in four parts. Section 2 introduces the model. Section 3 presents the result. Section 4 is a conclusion.

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2 The Model

We consider a two period $t \in T = \{0, 1\}$ model with technological uncertainty in period 1 represented by states of nature. An element in the set of mutually exclusive and exhaustive uncertain events is denoted $s \in \{1, ..., S\}$, where by convention $s = 0$ represents the certain event in period 0. We count in total $(S + 1)$ states of nature.

The economic agents are the $j \in \{1, ..., n\}$ producers and $i \in \{1, ..., m\}$ consumers which are characterized by assumptions of smooth economies. There are $k \in \{1, ..., l\}$ physical commodities and $j \in \{1, ..., n\}$ financial assets, referred to as stocks. In fact, stocks are the only financial assets considered here. Physical goods are traded on each of the $(S + 1)$ spot markets. Producers issue stocks which are traded at $s = 0$, yielding a payoff in the next period at uncertain state $s \in \{1, ..., S\}$. The quantity of stocks issued by firm $j \in \{1, ..., n\}$ is denoted $z_j \in \mathbb{R}_-$, where $\sum_{j=1}^{n} z_j = \hat{z}$.

There are in total $l(S+1)$ physical goods available for consumption. The consumption bundle of agent $i \in \{1, ..., m\}$ is denoted $x_i = (x_i(0), x_i(s), ..., x_i(S)) \in \mathbb{R}^{l(S+1)}_{++}$, with $x_i(s) = (x_i^1(s), ..., x_i^l(s)) \in \mathbb{R}^l_{++}$, and $\sum_{i=1}^{m} x_i = x$. The consumption space for each consumer $i \in \{1, ..., m\}$ is $X_i = \mathbb{R}^{l(S+1)}_{++}$, the strictly positive orthant. The associated price system is a collection of vectors represented by $p = (p(0), p(s), ..., p(S)) \in \mathbb{R}^{l(S+1)}_{++}$, with $p(s) = (p^1(s), ..., p^l(s)) \in \mathbb{R}^l_{++}$, the strictly positive orthant. Each consumer $i \in \{1, ..., m\}$ is endowed with initial resources $\omega_i \in \Omega$, where $\Omega = \mathbb{R}^{lT}_{++}$, and $\omega_i = (\omega_i(0), \omega_i(1))$ a collection of strictly positive vectors. Denote an initial resource vector at time period $t \in T = \{0, 1\}$, $\omega_i(t) = (\omega_i^1(t), ..., \omega_i^l(t)) \in \mathbb{R}^l_{++}$, and the sum of total initial resources, $\sum_{i=1}^{m} \omega_i = \omega$.

In total, there are n financial assets traded in period $t = 0$. Denote the quantity vector of stocks purchased by consumer $i \in \{1, ..., m\}$, $z_i = (z_i(1), ..., z_i(n)) \in \mathbb{R}^n_+$, a collection of quantities of stocks purchased from producers $j \in \{1, ..., n\}$, and denote $\sum_{i=1}^{m} z_i = z$, with associated stock price system $q = (q(1), ..., q(n)) \in \mathbb{R}^n_{++}$. Denote producer j's period one vector of capital purchase $y^j(0) \in \mathbb{R}^l_-$, and denote his period two state dependent net activity vector $y_j(s) = (y_j^1(s), ..., y_j^l(s)) \in \mathbb{R}^l$. Let $y_j(t = 1) = (y_j(s), ..., y_j(S)) \in \mathbb{R}^{lS}$ denote the collection of state dependent period t=1 net activity vectors. A period two input of production for every $s \in \{1, ..., S\}$ is by convention denoted $y_j^k(s) < 0$, and a production output in state $s \in \{1, ..., S\}$ satisfies $y_j^k(s) \geq 0$. For notational convenience, we treat quantity vectors as column vectors, and price vectors as row vectors, hence, we drop the notation for transposing vectors, whenever possible.

Each firm $j \in \{1, ..., n\}$ issues stocks z_j at stock price q_j in period one in order to build up production capacity. A firm's total cash acquired via stock market determines the upper bound of the total value of production capacity it can install in the same period. Denote this liquidity constraint $q_j z_j = M_j$, where $M_j \in \mathbb{R}_+$ is a non-negative real number and $z_j \in \mathbb{R}_-$ a feasible financial policy of the firm $j \in \{1, ..., n\}$. M_j constraints the quantity of capital $y(0) \in \mathbb{R}^l_-$ a producer j can purchase at spot price system $p(0) \in \mathbb{R}^l_{++}$. The quantity of intermediate goods $y_j(0)$ purchased in period one determines a correspondence $\phi_j|_Z$. This correspondence defines the technology of the firm at feasible financial policy Z. Let the production set available to each producer $j \in \{1, ..., n\}$ in period two be described by this technology, $\phi_j|_Z : \mathbb{R}^m_- \to \mathbb{R}^n_-$, a correspondence defined on the set of period two inputs, and denote it $Y_j|_z \subset \mathbb{R}^l$. Let S denote the set of all exogenously given states of nature. Then for each producer $j \in \{1, ..., n\}$ let the $t = 1$ one period production set be defined by a map $\Phi_j|_Z$ with domain $\mathbb{R}^m_- \times \mathbb{R}_{++}$ and range

$\mathbb{R}^n_- \times \mathbb{R}_{++}$, and denote it $Y_j|_z(s) \subset \mathbb{R}^{l(S+1)}$, where $m+n=l$. [1]

The sequential optimization structure of the firm is shown in equation (1). This objective function has the main property of being independent of any assigned utilities of the owners of the firm [9].

$$\underset{\left(\bar{y}|_{\hat{z}}(s),(\hat{z};\bar{y}(0))\right)_j}{\arg\max}\left\{\bar{q}z_j+\sum\nolimits_{s=1}^{S}\bar{p}(s)\cdot y_j|_{\hat{z}}(s):\begin{array}{ll}\bar{q}\sum\limits_{i=1}^{m}\bar{z}_i(j)\geq\bar{q}\hat{z}_j=\bar{p}(0)\cdot y_j(0) & s\in S\\ y_j|_{\hat{z}}(s)\in Y_j|_{\hat{z}}(s) & \end{array}\right\}, \tag{1}$$

Consumers play the same role in this production model as in the classical GEI model with production. They invest into firms because they want to transfer wealth between future uncertain states of nature, and to smooth out consumption across states of nature. Each consumer $i \in \{1,...,m\}$ purchases stocks z_i at stock price q in period one in return for a dividend stream in the next period. The consumer's optimization problem is to maximize utility subject to a sequence of $(S+1)$ budget constraints. Each consumer $i \in \{1,...,m\}$ is characterized by the standard assumptions for smooth economies introduced in Debreu [1].

Denote consumer i's sequence of $(S+1)$ budget constraints

$$B_{z_i}=\left\{(x_i,z_i)\in\mathbb{R}^{l(S+1)}_{++}\times\mathbb{R}^n_+\left|\begin{array}{c}p(0)\centerdot(x_i(0)-\omega_i(0))=-qz_i\\ p(s)\square(x_i(s)-\omega_i(1))=\Pi(p_1,\Phi|_Z)\theta_i(z_i)\end{array}\right.\right\}, \tag{2}$$

where $\theta_{ij} = z_i(j)[\sum_i z_i(j)]^{-1}$ is the proportion of total payoff of financial asset $j \in \{1,...,n\}$ hold by consumer $i \in \{1,...,m\}$ after trade at the stock market took place in period one. Π is the full payoff matrix of the economy of dimension $(S \times n)$.

Algebraically, each $i \in \{1,...,m\}$

$$(\overline{x}_i;\overline{z}_i)argmax\Big\{u_i(x_i;z_i):z_i\in\mathbb{R}^n_+,x_i\in B_{z_i}\Big\}. \tag{3}$$

A competitive equilibrium of the production economy defined by the initial resource vector $\omega \in \Omega$ is a price pair $(p,q) \in \mathcal{S} \times \mathbb{R}^n_{++}$ if equality between demand and supply of physical goods and financial assets is satisfied in all states of nature, $s = 0,1,...,S$. Its associated competitive equilibrium allocation is a collection of vectors $[(x,z),(y,\widetilde{z})] \in \mathbb{R}^{l(S+1)m}_{++} \times \mathbb{R}^n_+ \times \mathbb{R}^{l(S+1)n} \times \mathbb{R}^n_-$ of consumption, production and financial quantities.

$$\begin{array}{ll}\text{(i)} & \sum_{i=1}^m(\overline{x}_i(0)-\omega_i(0))=\sum_{j=1}^n\overline{y}_j(0)\\ \text{(ii)} & \sum_{i=1}^m(\overline{x}_i-\omega_i(1))=\sum_{s=1}^S\sum_{j=1}^n\overline{y}_j(s)\\ \text{(iii)} & \sum_{j=1}^n\sum_{i=1}^m(\overline{z}_i)_j=0,\ \sum_{i=1}^m\theta(\overline{z}_i)_j=1\text{ for all }j\in\{1,...,n\}\end{array} \tag{4}$$

Stiefenhofer [8] shows that equilibria for this stock market model always exist.

3 Result

We first introduce a reduced form equilibrium definition for the general equilibrium model of production with incomplete markets. This model has the property that production sets

[1] Assumptions of smooth production sets and utility functions apply. These are introduced in detail in Stiefenhofer (2009).

are independent of financial activities of the firms. The result shows that every extensive form equilibrium can be simplified to a reduced form equilibrium for any feasible fixed financial policy. The result suggest real equilibrium effects for an equilibrium parametrization over a set of feasible financial policies.

Definition 1 (RFE) *A reduced form stock market equilibrium* $(\overline{p}, \overline{q})$ *with associated equilibrium allocations* $(\overline{x}, \overline{\xi}), (\overline{y})$ *for generic initial resources* $\omega \in \Omega$*, and each producer* $j \in \{1, ..., n\}$ *maximizing long run profits satisfies:*

$$\text{(i)} \quad (\overline{x}_i; \overline{\xi}_i) argmax \Big\{ u_i(x_i; \xi_i) : \xi_i \in \mathbb{R}^n, x_i \in B_{\xi_i} \Big\} \qquad \forall i \in \{1, ..., m\}$$

$$\text{(ii)} \quad (\overline{y}_j) argmax \left\{ \overline{p}(s) \Box y_j(s) \middle| \begin{array}{c} y_j(s) \in Y_j|_z(s) \\ \overline{q}\overline{z}_j = \overline{p}(0) \centerdot \overline{y}_j(0) \end{array} \text{for all } s \in S \right\} \quad \forall\, j \in \{1, ..., n\}$$

$$\text{(iii)} \quad \textstyle\sum_{i=1}^m (\overline{x}_i - \omega_i) = \sum_j^n \overline{y}_j$$
$$\textstyle\sum_i^m \xi_i(j) = \sum_i^m \overline{z}_i(j) + \overline{z}_j, \sum_i^m \theta(\overline{\xi}_i)_j = 1, \; \forall j \in \{1, ..., n\}$$

Theorem 1 *(i) If* $(\bar{p}, \bar{q})$ *is an extensive form equilibrium with associated equilibrium allocations* $((\bar{x}|_{\hat{z}}, \bar{z}), (\bar{y}|_{\hat{z}}, \hat{z}))$ *(EFE) for generic initial resources* $\omega \in \Omega$*, then* $(\bar{p}, \bar{q})$*, is a reduced form equilibrium with associated equilibrium allocations* $((\bar{x}|_{\bar{\xi}}, \bar{\xi}), (\bar{y}|_{\bar{\xi}}))$ *(RFE) for generic initial resources* $\omega \in \Omega$ *where*

$$\sum\nolimits_{i=1}^m \bar{\xi}_{ij} = \sum\nolimits_{i=1}^m \bar{z}_i(j) + \hat{z}_j \text{ for } j = 1, ..., n \tag{5}$$

(ii) If $(\bar{p}, \bar{q})$ *is a reduced form equilibrium with associated equilibrium allocations* $((\bar{x}|_{\bar{\xi}}, \bar{\xi}), (\bar{y}|_{\bar{\xi}}))$ *(RFE) for generic initial resources* $\omega \in \Omega$*, then* $(\bar{p}, \bar{q})$*, is an extensive form equilibrium with associated equilibrium allocations* $((\bar{x}|_{\hat{z}}, \bar{z}), (\bar{y}|_{\hat{z}}, \hat{z}))$ *(EFE) for generic initial resources* $\omega \in \Omega$*, for any* $\hat{z}_j \leq \sum_{i=1}^m \bar{z}_i(j)$ *for* $j = 1, ..., n$ *satisfying*

$$\sum\nolimits_{i=1}^m \bar{z}_i(j) + \hat{z}_j = \sum\nolimits_{i=1}^m \bar{\xi}_{ij} \text{ for } j = 1, ..., n. \tag{6}$$

Lemma 1 $\bar{x}_i|_{\bar{\xi}}$ *is a solution of the reduced form problem*

$$max \Big\{ u(x; \xi)_i : x_i|_{\bar{\xi}} \in B_{\xi_i} \Big\} \tag{7}$$

if and only if, $\bar{x}_i|_{\bar{\xi}} \in B_{\xi_i}$*, and*

$$\partial u(\bar{x}_i|_{\bar{\xi}})_i \cap N_{B_{\xi_i}}(\bar{x}_i|_{\bar{\xi}}) \neq \{0\} \tag{8}$$

is satisfied.

Proof 1 (Lemma 1) *(i)* $\bar{x}_i|_{\bar{\xi}}$ *is a solution of utility max (RFE) if and only if* $\bar{x}_i|_{\bar{\xi}} \in B_{\xi_i}$ *and*

$$intU_{i, \bar{x}_i|_{\bar{\xi}}} \cap B_{\xi_i} = \varnothing.$$

By the separation theorem for convex sets (appendix), there exists $P = \beta_i p \in R_{++}^{l(S+1)}, P \neq 0$ *such that*

$$H_P^- = \Big\{ x_i|_{\bar{\xi}} \in R_{++}^{l(S+1)} : P \cdot x_i|_{\bar{\xi}} \leq P \cdot x_i|'_{\bar{\xi}}, \forall\, x_i|_{\bar{\xi}} \in B_{\xi_i}, \forall\, x_i|'_{\bar{\xi}} \in intU_{i, \bar{x}_i|_{\bar{\xi}}} \Big\}$$

since $\bar{x}_i|_{\bar{\xi}} \in B_{\xi_i}$,

$$H_P^- = \left\{ x_i|_{\bar{\xi}} \in R_{++}^{l(S+1)} : P \cdot \bar{x}_i|_{\bar{\xi}} \leq P \cdot x_i|_{\bar{\xi}}', \forall\, x_i|_{\bar{\xi}}' \in intU_{i,\bar{x}_i|_{\bar{\xi}}} \right\}.$$

By continuity of utility, $intU_{i,\bar{x}_i|_{\bar{\xi}}} = U_{i,\bar{x}_i|_{\bar{\xi}}}$*, and by continuity of the scalar product,*

$$\begin{aligned} H_P^+ &= \left\{\forall\, x_i|_{\bar{\xi}}' \in U_{i,\bar{x}|_{\bar{\xi}}} : P \cdot \bar{x}_i|_{\bar{\xi}} \leq P \cdot x_i|_{\bar{\xi}}', \forall\, x_i|_{\bar{\xi}}' \in U_{i,\bar{x}_i|_{\bar{\xi}}} \right\} \quad &\Leftrightarrow \quad P \in \partial u(\bar{x}_i|_{\bar{\xi}})_i \\ H_P^- &= \left\{\forall\, x_i|_{\bar{\xi}} \in B_{\xi_i} : P \cdot x_i|_{\bar{\xi}} \leq P \cdot \bar{x}_i|_{\bar{\xi}}, \right\} \quad &\Leftrightarrow \quad P \in N_{B_{\xi_i}}(\bar{x}_i|_{\bar{\xi}})_i \end{aligned}$$

hence, there exists p such that $\partial u(\bar{x}_i|_{\bar{\xi}})_i \cap N_B(\bar{x}|_{\bar{\xi}})_i \neq \{0\}$ *is satisfied.*

(ii) Suppose that $\bar{x}_i|_{\bar{\xi}} \in B_{\xi_i}$*, and there exists* $P \in \partial u(\bar{x}_i|_{\bar{\xi}})_i \cap N_{B_{\xi_i}}(\bar{x}_i|_{\bar{\xi}})_i$, $P \neq 0$*. If* $\bar{x}_i|_{\bar{\xi}}$ *is not a solution of the (RFE) utility maximization problem, then there exists* $\bar{x}_i|_{\bar{\xi}}\prime \in intU_{i,\bar{x}_i|_{\bar{\xi}}} \cap B_{\xi_i}$*. Since* $P \in \partial u(\bar{x}_i|_{\bar{\xi}})_i$*, we have*

$$P \cdot x_i|_{\bar{\xi}}\prime > P \cdot \bar{x}_i|_{\bar{\xi}}$$

But since $P \in N_{B_{\xi_i}}(\bar{x}_i|_{\bar{\xi}})$ *and* $\bar{x}_i|_{\bar{\xi}}\prime \in B_{\xi_i}$*, it follows that* $P \cdot \bar{x}_i|_{\bar{\xi}}\prime \leq P \cdot \bar{x}_i|_{\bar{\xi}}$ *which contradicts that* $\bar{x}_i|_{\bar{\xi}}\prime$ *is preferred to* $\bar{x}_i|_{\bar{\xi}}$*.*

Lemma 2 $\bar{y}_j|_{\bar{\xi}}$ *is a solution of*

$$max \left\{ \Pi(p;\xi)_j : \bar{y}_j|_{\bar{\xi}} \in Y_j|_{\bar{\xi}} \right\} \tag{9}$$

if and only if, $\bar{y}_j|_{\bar{\xi}} \in Y_j|_{\bar{\xi}}$*, and*

$$\partial\Pi(\bar{y}|_{\bar{\xi}})_j \cap Y_j|_{\bar{\xi}}(\bar{y}_j|_{\bar{\xi}}) \neq \{0\} \tag{10}$$

is satisfied.

Proof 2 (Lemma 2) *(i)* $\bar{y}_j|_{\bar{\xi}}$ *is a solution of the (RFE) profit max problem if and only if* $\bar{y}_j|_{\bar{\xi}} \in Y_j|_{\bar{\xi}}$ *and*

$$int\Pi_{\bar{y}_j} \cap Y_j|_{\bar{\xi}} = \varnothing.$$

By the separation theorem for convex sets (appendix), there exists $p \in \mathbb{R}_{++}^{lS}$, $p \neq 0$ *such that*

$$H_p^- = \left\{ y_j|_{\bar{\xi}} \in R^n : p \cdot y_j|_{\bar{\xi}} \leq p \cdot y_j|_{\bar{\xi}}', \forall\, y_j|_{\bar{\xi}} \in Y_j|_{\bar{\xi}}, \forall\, y_j|_{\bar{\xi}}' \in int\Pi_{j\,\bar{y}|_{\bar{\xi}}} \right\}$$

since $\bar{y}_j|_{\bar{\xi}} \in Y_j|_{\bar{\xi}}$,

$$H_p^- = \left\{ y_j|_{\bar{\xi}} \in R^n : p \cdot \bar{y}|_{\bar{\xi}} \leq p \cdot y_j|_{\bar{\xi}}', \forall\, y_j|_{\bar{\xi}}' \in int\Pi_{j\,\bar{y}|_{\bar{\xi}}} \right\}.$$

By continuity of Π_j, $int\Pi_{j,\bar{y}|_{\bar{\xi}}} = \Pi_{j,\bar{y}|_{\bar{\xi}}}$, *and by continuity of the scalar product,*

$$\begin{array}{llll} H_p^+ = \left\{\forall\, y_j|_{\bar{\xi}}' \in \Pi_{j,y_j|_{\bar{\xi}}} : p \cdot \bar{y}_j|_{\bar{\xi}} \leq p \cdot y_j|_{\bar{\xi}}', \forall\, y_j|_{\bar{\xi}}' \in \Pi_{j,\bar{y}_j|_{\bar{\xi}}}\right\} & \Leftrightarrow & p \in \partial\Pi(\bar{y}_j|_{\bar{\xi}})_j \\ H_p^- = \left\{\forall\, y_j|_{\bar{\xi}} \in Y_j|_{\bar{\xi}} : p \cdot y_j|_{\bar{\xi}} \leq p \cdot \bar{y}_j|_{\bar{\xi}}, \right\} & \Leftrightarrow & p \in N_{Y_j|_{\bar{\xi}}}(\bar{y}_j|_{\bar{\xi}})_j \end{array}$$

hence, there exists p *such that* $\partial\Pi(\bar{y}_j|_{\bar{\xi}})_j \cap N_{j,Y_j|_{\bar{\xi}}}(\bar{y}_j|_{\bar{\xi}})_j \neq \{0\}$ *is satisfied.*

(ii) Suppose that $\bar{y}_j|_{\bar{\xi}} \in Y_j|_{\bar{\xi}}$, *and there exists* $p \in \partial\Pi(\bar{y}_j|_{\bar{\xi}})_j \cap N_{Y_j|_{\bar{\xi}}}(\bar{y}_j|_{\bar{\xi}})_j, p \neq 0$. *If* $\bar{y}_j|_{\bar{\xi}}$ *is not a solution of the profit maximization problem (RFE), then there exists* $\bar{y}_j|_{\bar{\xi}}\prime \in int\Pi_{j,\bar{y}_j|_{\bar{\xi}}} \cap Y_j|_{\bar{\xi}}$. *Since* $p \in \partial\Pi(\bar{y}_j|_{\bar{\xi}})_j$, *we have*

$$p \cdot \bar{y}_j|_{\bar{\xi}}\prime > p \cdot \bar{y}|_{\bar{z}}$$

But since $p \in N_{Y_j|_{\bar{\xi}}}(\bar{y}_j|_{\bar{\xi}})$ *and* $\bar{y}_j|_{\bar{\xi}}\prime \in Y_j|_{\bar{\xi}}$, *it follows that* $p \cdot \bar{y}_j|_{\bar{\xi}}\prime \leq p \cdot \bar{y}_j|_{\bar{\xi}}$ *which contradicts that* $\bar{y}_j|_{\bar{\xi}}\prime$ *is preferred to* $\bar{y}_j|_{\bar{\xi}}$.

Proof 3 (Theorem 1) *Part (i). Let us first show that the reduced form equilibrium allocations* $((\bar{x}, \bar{\xi}), (\bar{y}))$ *satisfy the first order conditions (Lemma (1))* $\partial u(\bar{x}_i|_{\bar{\xi}})_i \cap N_{B_{\xi_i}}(\bar{x}_i|_{\bar{\xi}}) \neq \{0\}$ *and* $\bar{x}_i|_{\bar{\xi}} \in B_{\xi_i}$, *and (Lemma(2))* $\partial\Pi(\bar{y}|_{\bar{\xi}})_j \cap Y_j|_{\bar{\xi}}(\bar{y}_j|_{\bar{\xi}}) \neq \{0\}$ *and* $\bar{y}_j|_{\bar{\xi}} \in Y_j|_{\bar{\xi}}$, *so that the conditions (i) and (ii) in the* (RFE) *are satisfied. The first order conditions for the consumer's problem in the* (EFE) *are*

$$p \cdot x_i|_{\hat{z}_j} = p \cdot \omega_i + \Pi(\bar{y}, \bar{p}) \begin{bmatrix} \bar{z}_i \\ \theta_j\,(\bar{z}_i) \end{bmatrix}, \text{and } \beta_i \Pi(\bar{y}, \bar{p}) = 0$$

and can be rewritten as

$$p \cdot \bar{x}_i|_{\bar{\xi}} = p \cdot \omega_i + \Pi(\bar{y}, \bar{p}) \sum\nolimits_{j=1}^{n} \xi_{ij}, \text{and } \beta_i \Pi(\bar{y}, \bar{p}) = 0$$

since $\bar{\xi}_i\big|_{\hat{z}} = \sum_{i=1}^m \bar{z}_i(j) + \hat{z}_j$, *for all* $j = 1, ..., n$, *so that (lemma (1)) above holds for any feasible* $\hat{z}_j \leqslant \sum_{i=1}^m \bar{z}_i(j)$ *for all* $j = 1, ..., n$. *The firm's problem in* (EFE) *is to*

$$\underset{\left(\bar{y}|_{\hat{z}}(s),(\hat{z};\bar{y}(0))\right)_j}{\arg\max} \left\{ \bar{q}z_j + \sum\nolimits_{s=1}^{S} \bar{p}(s) \cdot y_j|_{\hat{z}}(s) : \begin{array}{ll} \bar{q}\displaystyle\sum_{i=1}^{m} \bar{z}_i(j) \geq \bar{q}z_j = \bar{p}(0) \cdot y_j(0) & s \in S \\ y_j|_{\hat{z}}(s) \in Y_j|_{\hat{z}}(s) & \end{array} \right\}.$$

For any feasible $\hat{z}_j \leqslant \sum_{i=1}^m \bar{z}_i(j)$ *the problem of the producer reduces to*

$$\underset{\left(\bar{y}|_{\bar{\xi}}(s)\right)_j}{\arg\max} \left\{ \sum\nolimits_{s=1}^{S} \bar{p}(s) \cdot y_j|_{\bar{\xi}}(s) : y_j|_{\bar{\xi}}(s) \in Y_j|_{\bar{\xi}}(s), s \in S \right\}$$

since feasible $\hat{z}_j \Rightarrow \Phi_j|_{\hat{z}}(s) \Longrightarrow Y_j|_{\hat{z}}(s)$ *for all* $s \in S$, *for which the first order conditions (lemma (2)) hold. The result follows ,since market clearing condition* $\bar{\xi}_i\big|_{\hat{z}_j} = \sum_{i=1}^m \bar{z}_i(j) + \hat{z}_j = 0$, *for all* $j = 1, ..., n$, *and* $\sum_{i=1}^m \bar{x}_i|_{\hat{z}_j}(0) = \sum_{i=1}^m \omega_i(0) + \sum_{j=1}^n \bar{y}_j(0)$, $\sum_{i=1}^m \bar{x}_i|_{\hat{z}_j}(s) = \sum_{i=1}^m \omega_i(1) + \sum_{j=1}^n \bar{y}_j|_{\hat{z}}(s)$ *for all* $s \in S$ *hold.*

Part (ii). If $((\bar{x},\bar{\xi}),(\bar{y}))$ is a (RFE) for implicit $\hat{z}_j$, for $j=1,...,n$, then the first order conditions are satisfied. This implies that for any feasible $\hat{z}_j \leqslant \sum_{i=1}^m \bar{z}_i(j)$

$$\underset{\left(\bar{y}|_{\bar{\xi}}(s)\right)_j}{\arg\max}\left\{\sum\nolimits_{s=1}^{S}\bar{p}(s)\cdot y_j|_{\bar{\xi}}(s) : y_j|_{\bar{\xi}}(s)\in Y_j|_{\bar{\xi}}(s),\ s\in S\right\} \tag{11}$$

expands to

$$\underset{\left(\bar{y}|_{\hat{z}}(s),(\hat{z};\bar{y}(0))\right)_j}{\arg\max}\left\{\bar{q}z_j+\sum\nolimits_{s=1}^{S}\bar{p}(s)\cdot y_j|_{\hat{z}}(s) : \begin{array}{l}\bar{q}\sum\limits_{i=1}^{m}\bar{z}_i(j)\geq\bar{q}\hat{z}_j=\bar{p}(0)\cdot y_j(0)\\ y_j|_{\hat{z}}(s)\in Y_j|_{\hat{z}}(s)\end{array}\quad s\in S\right\}, \tag{12}$$

for which the first order conditions are satisfied (Lemma (2)), hence $\bar{y}_j|_{\hat{z}}$ is a solution of (ii) in (EFE) for feasible $\hat{z}_j$. Pick any feasible $\hat{z}_j$ and define

$$\sum\nolimits_{i=1}^{m} z_i(j)+\hat{z}_j=\bar{\xi}_i,\ \text{for all } j=1,...,n \tag{13}$$

such that $\Pi z_i+\Pi\hat{z}_j=\Pi\bar{\xi}_i$ becomes $\Pi(z_i+\hat{z}_j)=\Pi\bar{\xi}_i$, then the first order conditions for the consumer of the (EFE) (Lemma (1)) are satisfied for $(\bar{x}_i|_{\hat{z}_j},z_i)$. $(\bar{x}_i|_{\hat{z}_j},z_i)$ is a solution of (EFE) (i) and $(\bar{y}_j|_{\hat{z}},\hat{z}_j)$ is a solution of (EFE) (ii). The result follows from $0=\sum_{i=1}^m\xi_i=\sum_{i=1}^m z_i+\sum_{j=1}^n\hat{z}_j$ and $\sum_{i=1}^m\bar{x}_i|_{\hat{z}_j}(0)=\sum_{i=1}^m\omega_i(0)+\sum_{j=1}^n\bar{y}_j(0)$, $\sum_{i=1}^m\bar{x}_i|_{\hat{z}_j}(s)=\sum_{i=1}^m\omega_i(1)+\sum_{j=1}^n\bar{y}_j|_{\hat{z}}(s)$ for all $s\in S$.

4 Conclusion

This paper studies a simplified version of the Modigliani and Miller theorem. It shows for a special case of a stock market economy with incomplete markets that financial policies have real equilibrium effects. The determinateness of equilibria follows from the structure imposed on the model. The endogenous asset structure considered improves on the theory of the firm in general equilibrium with incomplete markets. Firms optimize long and short run economic activities. In the long run, they issue stocks and buy capital in order to build up their production sets. In the short run, they maximize short run profits at competitive prices, taking their production sets as given in each state of nature. The sequential optimization structure of the firm links the efficient boundary of the production set of the firm with the sphere of the financial asset set. Hence, real equilibrium effects.

The paper improves on the implicit assumption present in exogenous asset formation models that production is automatically financed. This result should be generalized to more general asset structures. This research is well under progress.

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