Gaining Power through Enlargement: Strategic Foundations and Experimental Evidence

By

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Abstract

Power indices suggest that adding new members to a voting body may affect the balance of power between the original members even if their number of votes and the decision rule remain constant. Some of the original members may actually gain even if voters are bargaining over a fixed budget. We show that this phenomenon can occur as an equilibrium of a non-cooperative bargaining game based on the Baron-Ferejohn (1989) model of legislative bargaining. We implement this game in the laboratory and find that power can be gained by adding new members as the theory predicts.

Keywords: voting, non-cooperative bargaining, power indices, experiments
JEL classification: C70; C92

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1. Introduction

Power indices suggest that the addition of a new member to a voting body may increase the voting power of some original members, even if the voting body is deciding over the allocation of a fixed pie, and the relative voting weights of the original members and the decision rule remain constant. Brams and Affuso (1976) refer to this possibility as the ‘paradox of new members’. Brams and Affuso describe two instances of this phenomenon. In one, the addition of a new member causes an existing member to lose his veto power and this empowers the other existing members who can use the new member as a coalition partner. In the second instance, however, a voter gains from the addition of a new member who is of no use to him as a coalition partner. Brams and Affuso base their analysis on the application of Shapley and Banzhaf power indices to cooperative games. In this paper we show that gaining power through enlargement can occur as an equilibrium outcome of a noncooperative legislative bargaining game. We also implement this game in the laboratory, and observe comparative statics in line with equilibrium predictions.

It has been claimed that EU enlargements have increased the power of some original members. For example, in 1981 Greece was admitted to the EU. In an attempt to preserve the balance of power among existing members, the number of votes held by each country on the Council of Ministers and the percentage of the total votes required to pass a proposal was kept unchanged. Nevertheless, Brams and Affuso (1985) showed that Luxembourg’s power index increased, leading them to conclude that “if … seemingly plausible but nevertheless arbitrary voting weights and decision rules are selected - without benefit of formal voting-power analysis - then the effects may be not only unanticipated but also bizarre” (p.137). One might ask whether there is any empirical evidence of an existing member benefitting from an enlargement by enjoying a larger share of the resource being bargained over. Kauppi and Widgrén (2004) analyze data on EU budget shares from 1976-2001. According to their Figure 2, Ireland, Italy, and the UK enjoyed higher budget shares after Greece joined the EU. Of course, this may not be a manifestation of the ‘paradox of new members’. It may be that measured budget shares do not accurately measure what is being bargained over, or that the shift in budget shares reflected other factors that changed after Greece joined. These qualifying remarks illustrate the difficulties of using field data for examining the relationship between voting weights and voting power.
In our experiment participants bargain over a fixed budget, and we measure power by the average earnings of each voter type. In order to study weighted voting games in the laboratory it is necessary to impose a protocol. The bargaining game we study uses the same bargaining procedure as the Baron and Ferejohn (1989) model, which is the leading model of legislative bargaining. A player is randomly selected to submit a proposal, which is then voted on. If the proposal passes the proposal is implemented (closed rule), otherwise the game goes to the next round and the procedure is repeated. Whereas the Baron-Ferejohn model uses a one-person one-vote and majority voting rule, we examine weighted voting rules. We study three treatments, corresponding to the examples in Brams and Affuso (1976). In our VETO treatment there are three voters, one of whom is a ‘strong’ player with veto power. Our ENLARGED treatment adds a fourth voter, so that the strong player loses his veto power. Comparing these two treatments, the theory predicts that the addition of a new member empowers the non-veto players and allows them to get a higher payoff. In our SYMMETRIC treatment the three voters have the same nominal voting weights as in VETO, but the quota is modified so that any two voters can form a winning coalition. In the comparison of SYMMETRIC and ENLARGED theory predicts that the addition of a new member increases the strong player’s expected payoff. This is despite the fact that the strong player and new member never vote together.

In analyzing this bargaining procedure we add to the existing experimental literature investigating the Baron-Ferejohn procedure. Fréchette et al. (2005a, 2005b) study several treatments similar to our SYMMETRIC treatment, while Kagel et al. (2007) study treatments similar to our SYMMETRIC and VETO treatments. As we report later, our results are remarkably consistent with theirs given the wide range of procedural differences between the experiments. The ENLARGED treatment has not been previously studied, and of course it is the comparison of this treatment with the others that allows us to test whether a player can gain power after enlargement. We find that the addition of a new member does indeed cause average earnings to change in the direction implied by the equilibrium predictions.²

² Other related experimental studies of the Baron-Ferejohn procedure include McKelvey (1991), Diermeier and Morton (2005), and Fréchette et al. (2005c). Montero et al. (2008) investigate the effect of enlargement on the balance of power using a relatively unstructured bargaining procedure whereby any player can put or amend a proposal on the table at any time. They observe that earnings can increase after enlargement but, as we report later, there are some important differences between their results and those reported here.
The remainder of the paper is organized as follows. In the next section we describe our theoretical framework and apply this to three bargaining games. In section 3 we describe how we implement these games experimentally, and in section 4 we present our results. Section 5 discusses the results and offers concluding comments.

2. Strategic Foundations
2.1 Brams and Affuso’s Examples
Brams and Affuso (1976) discusses the three examples given in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Brams and Affuso’s Examples</th>
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</thead>
<tbody>
<tr>
<td>Votes controlled by player 1</td>
</tr>
<tr>
<td>VETO</td>
</tr>
<tr>
<td>SYMMETRIC</td>
</tr>
<tr>
<td>ENLARGED</td>
</tr>
</tbody>
</table>

In all examples player 1, who we will refer to as the ‘strong’ player, has three votes while players 2 and 3 (the ‘weak’ players) have two votes each. In the first example these three players comprise the voting body, and five votes are needed to pass a proposal. Here, the strong player has veto power, since no proposal can be passed without his votes, thus we refer to this as the VETO example. The second example is identical except that only four votes are needed to pass a proposal. As a consequence no player has a strategic advantage over the others, since any two members have enough votes to pass a proposal, and so we refer to this as the SYMMETRIC example. In the third example, ENLARGED, five votes are needed to pass a proposal, and there is an additional member with a single vote.

Clearly, the voting weights of strong and weak players are the same in all examples. Relative to VETO, ENLARGED holds the total number of votes required to pass a proposal constant. Brams and Affuso show that if voting power is represented by the Shapley value or Banzhaf index, the weak players have greater voting power in ENLARGED. Relative to
SYMmetric, ENLARGED holds constant a simple majority decision rule. Brams and Affuso show that the power indices give the strong player greater voting power in ENLARGED.

2.2 A Non-cooperative Bargaining Procedure

Here we study a non-cooperative game based on extending the Baron-Ferejohn model to weighted voting games. There is a budget of 120 points to be divided among \( n \) voters using weighted majority rule. Each voter is assumed to maximize the expected number of points allocated to them, i.e., they are selfish and risk neutral. The bargaining procedure starts in round one with a voter being randomly selected to make a proposal, with each voter having an equal chance of being selected. The proposer proposes a division of the budget such that each voter receives a nonnegative whole number of points, and the whole budget is distributed. The voters vote on the proposal simultaneously. If the total number of votes in favor is sufficient to pass the proposal, the proposal is implemented and each voter receives the amount of points specified in the proposal. Otherwise, round two begins. A new proposer is selected, again each voter with the same probability, and proposes a division of 120 points etc. As in our experiment, we consider a finite horizon of 20 rounds. If no agreement is reached by the end of the 20 rounds all voters get zero.\(^3\)

There is a multiplicity of subgame perfect equilibria (cf. Norman, 2002). Our goal is not to analyze them all, but to show that a player’s equilibrium payoff may increase after enlargement, and more specifically that a weak player’s equilibrium payoff can be higher in ENLARGED than VETO, and a strong player’s equilibrium payoff can be higher in ENLARGED than SYMMETRIC. In order to do this, we focus on equilibria in which strategies do not depend on past offers or voting behavior. We also assume that all voters vote as if their vote is pivotal in order to eliminate equilibria in which a proposal is accepted or rejected by all voters regardless of whether the voters actually prefer the proposal to pass, because no one of the voters can unilaterally change the outcome. We also assume that a voter accepts an offer if he is indifferent between accepting and rejecting. Furthermore, we assume that strategies are symmetric in the sense that voters of the same type play the same strategy and strategies also treat voters of the same type symmetrically.

\(^3\) Notice that, as in our experiment, points are indivisible and the game is finitely repeated with no discounting from round to round until the termination round, after which the pie disappears. We show in Appendix A of our working paper (Drouvelis et al., 2007) that the same comparative static predictions hold in the standard model with a perfectly divisible budget, an infinite horizon and discounting provided the discount factor is sufficiently large.
Denote voter $i$’s equilibrium payoff at the beginning of round $t$ by $y_i^t$. Given our assumptions, this value does not depend on past offers and counteroffers. At time $t-1$, it is optimal for a voter to accept an offer that gives him at least $y_i^t$. The proposer will then look for the cheapest group of voters that control enough votes to achieve a majority, and offer them exactly these values (rounded up to the nearest integer), keeping the residual. Following common practice, we will say that the proposer ‘proposes to’ the voters that are offered at least $y_i^t$, and, if the offer is accepted, we refer to these players as the ‘coalition partners’.

2.3 The VETO Treatment

This game is analyzed by Winter (1996) with a perfectly divisible budget. Since there is a finite horizon, the game can be solved by backward induction.

Recall voters are assumed to care only about their own material payoffs. This implies that, in a subgame perfect equilibrium, a voter must accept any offer giving him a positive number of points at round 20, and he is indifferent between accepting and rejecting offers giving him 0 points. Given our assumption that a voter must accept if he is indifferent, the proposer allocates 120 to himself and 0 to everybody else in round 20, and $y_i^{20} = 40$ for all $i$.

In round 19, suppose the strong player is the proposer. He has a choice between making the best acceptable offer (i.e., offering 40 to one of the weak players) and an offer that will be rejected. Since $120 - 40 > 40$, the strong player strictly prefers to make an acceptable proposal, and is indifferent between proposing to any of the weak players. Our symmetry assumption implies that he will propose to each of them with probability $\frac{1}{2}$. As for the weak players, they strictly prefer to offer 40 to the strong player and keep 80 rather than make an unacceptable proposal and expect 40 on average in round 20. The expected payoff for the strong player at the beginning of round 19 is given by

$$E_i^{19} = \frac{1}{2} \left[ 120 - y_1^{20} \right] + \frac{2}{3} y_i^{20}.$$ Since $y_1^{20} = y_2^{20} = 40$, $y_i^{19} = \frac{160}{3}$.

For a weak player, $y_2^{19} = \frac{1}{3} \left[ 120 - y_1^{20} \right] + \frac{1}{6} y_2^{20}$, or $y_2^{19} = \frac{100}{3}$.

In round 18, the same reasoning applies, except that since proposals must be integers player 1 must be offered at least 54, and players 2 and 3 must be offered at least 34. Denoting
rounded up expected payoffs by \( y'_i \), expected payoffs can be obtained recursively using
\[
y'_i = \frac{1}{3} \left[ 20 - y'^{i+1}_2 \right] + \frac{2}{3} y'_1 \quad \text{and} \quad y'_2 = \frac{1}{3} \left[ 20 - y'^{i+1}_1 \right] + \frac{1}{6} y'^{i+1}_2 , \quad \text{where} \ t = 1, \ldots, 19 \quad \text{and} \ y'^{20}_1 = y'^{20}_2 = 40 .
\]
Iterating this equation we obtain \( -2^2 = 118, -2^2 = 2 \). Thus, in round 1, the strong player offers 2 to one of the weak players if he is the proposer, and a weak player offers 118 to the strong player if he is the proposer. These strategies imply expected payoffs \( y'^1_1 = 118, y'^1_2 = 1 \).

2.4 The SYMMETRIC Treatment
This is the original game analyzed by Baron and Ferejohn. Under our assumption of symmetry of equilibrium strategies, the equilibrium is very simple. In round 20 the proposer will take all 120 points, and since all players are equally likely to be proposers \( y^{20}_i = 40 \) for all \( i \). In the previous round, the proposer offers 40 to another player and keeps 80; under symmetry each player is equally likely to propose to the other two and we obtain \( y^{19}_i = 40 \) for all \( i \), and indeed \( y'^i = 40 \) for all \( i \) and \( t \).

Before the game starts, the expected payoff of each player is 40. However, once the proposer for round one is selected, this player will propose that he gets 80 and another, randomly selected, player gets 40. This proposal will be passed.

2.5 The ENLARGED Treatment
If the strong player is selected to be proposer, he will seek the favorable vote of one weak player (the new member is never of use to the strong player). As for a weak player, he will seek the favorable vote of either the strong player or the other two players depending on which votes are cheaper to obtain. The new member will seek the favorable vote of two others.

Expected payoffs in round 20 are \( y^{20}_i = 30 \) for all \( i \). In round 19, the strong player will offer 30 to one weak player (the symmetry assumption implies that he will be equally likely to propose to any of the weak players). As for the weak players, they strictly prefer to offer 30 to the strong player. The situation facing the new member is more complicated. He could offer 30 to any two others. Assuming that he chooses to propose to the two weak players, expected payoffs in round 19 are \( y^{19}_1 = 37.5, y^{19}_2 = 33.75, y^{19}_4 = 15 \) and iterating this reasoning, allowing only integer offers, we obtain the minimal acceptable offers for each type in round one:
\[ y_1 = 45, y_2 = 31, y_3 = 15. \] It turns out that if the new member breaks ties differently, then expected payoffs in round 19 will be different, but by the end of the iterative process the minimal acceptable offers in round one are unaffected.

Given these minimal acceptable offers it is clear that when the strong player is proposer he will randomize between \((89,31,0,0)\) and \((89,0,31,0)\), when player 2 is the proposer he will propose \((45,75,0,0)\), when player 3 is the proposer he will propose \((45,0,75,0)\), and when the new member is the proposer he will propose \((0,31,31,58)\).

### 2.6 The Effects of Enlargement

If proposers are chosen randomly with equal probability, then it is straightforward to compute the expected payoff of each type of player from the equilibria identified above. These are presented in Table 2.

**Table 2. Expected Payoffs**

<table>
<thead>
<tr>
<th></th>
<th>Strong Player</th>
<th>Weak Player</th>
<th>New Player</th>
</tr>
</thead>
<tbody>
<tr>
<td>VETO</td>
<td>118</td>
<td>1</td>
<td>NA</td>
</tr>
<tr>
<td>SYMMETRIC</td>
<td>40</td>
<td>40</td>
<td>NA</td>
</tr>
<tr>
<td>ENLARGED</td>
<td>44.75</td>
<td>30.375</td>
<td>14.5</td>
</tr>
</tbody>
</table>

There are two cases in which enlargements have a positive impact on an existing member. First, beginning from the VETO treatment, the addition of the new player clearly benefits the weak player. The new player breaks the monopoly of the strong player by providing the weak player with alternative opportunities to pass proposals.\(^4\) This prediction is quite intuitive. However, note that from a behavioral point of view the outcome of a test is not obvious a priori as the intuition is based on the premise that in VETO the strong player can exploit his veto power. It is well known that responders in ultimatum games reject small offers and so proposers are unable to fully exploit their theoretical bargaining power (Forsythe et al., 1994). If

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\(^4\) This prediction can be easily generalized. Consider a model with equal recognition probabilities and an infinite horizon without discounting. Starting from a game with veto players, if the addition of a new member eliminates all veto power, any original non-veto players must gain. This is a direct consequence of non-veto players receiving a zero payoff in a game with veto players and a positive payoff in games without veto players.
non-veto players reject inequitable proposals, the veto player might similarly fail to extract his predicted share and enlargement may not have the predicted effect.

Second, beginning from SYMMETRIC the addition of the new player benefits the strong player. The mechanism at work here is less straightforward. While it is clear that enlargement breaks the original symmetry between strong and weak players so that the strong player gets more than a weak player, it is far from obvious that the strong player gains also in absolute terms. After all, the new member is of no use whatsoever to the strong player: neither ever proposes to the other and, with all players having the same chance of being proposers, the strong player expects to be excluded whenever the new member is selected, i.e., ¼ of the time. The source of the gain in power of the strong player is mainly that the weak players now propose to the strong player with certainty rather than 50% of the time.\(^5\)

Many assumptions lie behind these results. First our results are based on the analysis of a game of complete information. We assumed players are expected utility maximizers, that utility depends only on own point earnings, that players are risk neutral, and that all of this is common knowledge. Second, equilibrium analysis of this game provides many subgame perfect equilibria, and we adopt several refinement criteria in order to select one. Thus, it is not clear how empirically relevant the theoretical predictions may be.

3. Experimental Design and Procedures
The experiment was conducted at the University of Nottingham using subjects recruited from a university-wide pool of undergraduate students.\(^6\) Four sessions were conducted with each treatment, with either 12 subjects (VETO and SYMMETRIC treatments) or 16 subjects (ENLARGED treatment) per session. Thus, 160 subjects participated in total.

All sessions used an identical protocol. Upon arrival, subjects were given a written set of instructions that the experimenter read aloud. Subjects were then allowed to ask questions by raising their hands and speaking to the experimenter in private. Subjects were not allowed to

\(^5\) It turns out this prediction also generalizes. Start from a game with \(n > 2\) players in which any coalition of \(n – 1\) is minimal winning, so that each receives an expected payoff in equilibrium of \(1/n\). Now suppose adding a new member results in two classes of minimal winning coalitions: one ‘strong’ player with any \(n – 2\) ‘weak’ original members and the \(n – 1\) ‘weak’ original members with the new member. It can be shown that the strong player’s expected payoff is now \(n/(n^2 – n + 2)\). This exceeds \(1/n\); thus enlargement benefits the strong player.

\(^6\) We recruited subjects using the ORSEE software (Greiner, 2004). The experiment was programmed and conducted with the software z-Tree (Fischbacher, 2007). Instructions for the VETO treatment are included in an Appendix. Instructions for other treatments paralleled these.
communicate with one another throughout the session, except via the decisions they entered on their terminals.

The decision-making phase of the session then consisted of 10 periods, where in each period a group played a multi-round bargaining game over the division of 120 points. At the beginning of each period subjects were assigned to groups of either three or four (depending on treatment). Subjects were informed that they would not know who of the other people in the room were in their group, that group compositions would change from period to period, and that the same set of subjects would never be matched together twice. At the beginning of each period subjects were assigned an ID that changed from period to period. They were also assigned roles determining how many votes they controlled, and roles also varied across periods.

The main reason for having subjects play repeatedly was to give them experience with the strategic environment. We did not expect subjects to calculate the equilibrium and play accordingly, but nevertheless considered that if subjects learn to respond to the strategic incentives theoretical predictions may still be relevant. A potential drawback of having subjects play a repeated game is that it might lead to unintended strategic effects. For example a subject may reject offers in early periods in order to establish a tough reputation. For this reason we implemented anonymous decision making together with changing roles and group compositions in order to make it difficult for a subject to build up a reputation across periods.\(^7\)

In each period at the beginning of round one all group members submitted proposals over how to divide the 120 points. When all group members had submitted a proposal, one was selected at random and revealed to all group members. Group members then voted for or against the proposal. If the total number of votes for the proposal met the quota of 5 votes (VETO and ENLARGED) or 4 votes (SYMMETRIC) the proposal was passed and each group member received the proposed number of points. Otherwise bargaining proceeded to round two. All

\(^{7}\) There are some potential downsides from having subjects change roles from period to period. First, this procedure may impair learning because subjects may understand their role better if it is the same in every period. However, it is also possible that their understanding of the situation is enhanced by experiencing what it is like to be in another voter-type’s shoes. Second, changing roles may affect subject attitudes and expectations about what proposals are acceptable. Since changing roles randomly creates a ‘procedurally fair’ experiment, subjects may be more willing to exploit their bargaining power, or be more willing to allow others to exploit their power, than if roles were constant. As we note later, the results from our VETO treatment are qualitatively similar to a comparable treatment in Kagel et al. (2007), and the results from our SYMMETRIC treatment are qualitatively similar to a comparable treatment in Fréchette et al. (2005b). These studies used fixed roles.
rounds had the same structure up to round 20. If no agreement was reached by the end of round 20, the period would end with each group member earning zero points. An important aspect of this design is that in each round all players submit proposals, then one is randomly selected, and then all players observe and vote on the randomly selected proposal. From a theoretical point of view this is equivalent to the more common description of the game in the theoretical literature, where a player is randomly selected to be the proposer and then only the randomly selected player makes a proposal. The advantage of our version is that it allows us to observe proposals from all players in every round. As we shall see in the next section, this information is useful not only for analyzing the determinants of proposals, but also for the analysis of voting patterns.

At the end of the experiment subjects were privately paid according to their accumulated point earnings from all 10 periods, using an exchange rate of 3p per point (VETO and SYMMETRIC treatments) or 4p per point (ENLARGED treatment). Earnings averaged £12 per subject and ranged from a minimum of £3.90 to a maximum of £20.20 (at the time of the experiment £1 = $1.92). Sessions lasted, on average, 50 minutes, and at most 70 minutes.

A final noteworthy aspect of our design was that subjects were divided into two equally-sized matching groups at the beginning of a session, and groups were formed from within these matching groups, with no information passing across the two matching groups. This feature of our design has the disadvantage that individuals within a matching group interact more frequently than if groups were formed from all session participants, so our procedure may strengthen correlations between decisions within a matching group. Because of likely dependencies between decisions within matching groups, tests of our primary hypotheses (reported below in 4.2) take the matching group as our unit of observation. The advantage of the procedure is that it generates two observations per session. We treat these observations as independent for performing statistical tests. Our primary hypotheses refer to the impact of enlargement on existing members’ earnings. We test the hypotheses that average earnings for the

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8 Subjects had 90 seconds to submit a proposal and 30 seconds to cast their votes. If they failed to make a decision within this time constraint, the computer made a default decision. The default proposal was that each group member received zero, and the default voting decision was to reject. In fact this time constraint was rarely binding: across all sessions only 7 of 2824 proposals were made by the computer, and only 8 of 2824 voting decisions were made by the computer.

9 To ensure comparability between matching groups, we formed groups, assigned roles, and selected proposers for one matching group prior to the first session by rolling die and then used the same random draws for all matching groups.
weak player are higher in ENLARGED than in VETO, and that average earnings for the strong player are higher in ENLARGED than in SYMMETRIC. After using this conservative approach to test our primary hypotheses, we analyze individual voting decisions using disaggregated data (reported below in 4.3). The analysis of the disaggregated data uses econometric methods that adjust for intra-group correlations within matching groups.

4. Results
4.1 Overview
We observed rapid agreements in the experiment. Figure 1 shows the distribution of rounds in which agreement was reached. Out of the 480 games, 62% resulted in an agreement in the first round, and more than 90% ended within 3 rounds. Very few games got close to the 20-round deadline, and none actually reached the deadline.

Figure 1. Distribution of Rounds in which Groups reached an Agreement

In all three treatments a proposal can be implemented without unanimous support, and so within a group a subset of the players can pass a proposal that gives zero to outsiders. Thus, the

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Note that these hypotheses are derived from the expected payoffs in the equilibria identified in Section 2, which assumed players were equally likely to be the proposer in any round. We can also identify expected payoffs on the basis of the experimental realizations of who would be proposer in round one. This distinction is irrelevant for VETO, where the average earnings of a player type are independent of the proposer frequencies. In contrast, in SYMMETRIC and ENLARGED the actual frequencies matter, but the comparative statics predictions still hold. The weak player’s average earnings would change from 1 in VETO to 29.3 (rather than 30.375) in ENLARGED, and the strong player’s earnings would change from 41 (rather than 40) in SYMMETRIC to 47 (rather than 44.75) in ENLARGED.
reason for fast agreements in SYMMETRIC and ENLARGED may be that, though there are 20 rounds available for bargaining and there is no discounting from round to round, players have a strong incentive to accept a positive offer as otherwise they may end up with nothing. In VETO weak players similarly have a strong incentive to accept positive offers, but the strong player can afford to be patient since his vote is required for an agreement. Interestingly, only in VETO did we observe any trend in the duration of bargaining: we found that as the session progressed the duration of bargaining increased. Figure 2 shows how the average round of agreement changed across periods. For formal statistical tests we use the matching group as the unit of observation. In VETO the Spearman rank correlation coefficient between period and average round of agreement is positive for all eight matching groups indicating a significant trend (p = 0.008); for SYMMETRIC and ENLARGED the corresponding p-values are both 0.727.

Figure 2. Average Duration of Bargaining

In the first period agreements to divide the 120 points equally among all members were quite frequent, occurring in 17 of the 48 (≈ 35%) groups. For many subjects this must have seemed a natural and acceptable outcome. However, as shown in Figure 3, equal divisions were observed less frequently in later periods, and in the last period only 2 groups (≈ 4%) agreed upon an equal division. Equal divisions are most commonly observed in the SYMMETRIC treatment, where theoretically all players have, ex ante, equal bargaining power; even here, an equal division of the pie is quite rare, occurring in only 20/160 (≈ 13%) games.
Agreements to split the pie equally tended to be replaced by agreements that gave some players zero. Thus, while in two-person ultimatum game bargaining experiments it is well-known that subjects resist small offers, here as in other experiments on multi-person bargaining subjects are willing to propose distributions that give zero to another subject, and others are willing to vote for such a proposal (see, e.g., Güth and van Damme, 1998, Okada and Riedl, 2005). A winning coalition in which the votes of all coalition members are essential to pass a proposal and the coalition maximizes its point earnings by allocating zero points to outsiders is called a minimal winning coalition. In the equilibria discussed in Section 2, all formed coalitions are minimal. The frequency of minimal winning coalitions in our data is shown in Figure 4.
In all treatments the frequency of minimal winning coalitions increases across periods. Taking all three treatments together, minimal winning coalitions formed in 17/48 groups (≈ 35%) in the first period, compared with 43/48 (≈ 90%) in the last period. We interpret the increase in the frequency of minimal winning coalitions as reflecting a tendency toward more strategic behavior. Indeed, if one looks at all periods of data there are substantial deviations from equilibrium predictions, but if one focuses on later periods there is more conformity with equilibrium predictions. For example grand coalitions, in which all players get a strictly positive payoff, form in 124/480 games (≈26%), but in only 23/192 games (≈12%) in the last four periods. Table 3 shows the distribution of agreed coalitions for each treatment, based on both all periods and just the last four periods.

Table 3. Observed Distribution of Agreed Coalitions

<table>
<thead>
<tr>
<th></th>
<th>VETO All periods</th>
<th>VETO Last 4 periods</th>
<th>SYMMETRIC All periods</th>
<th>SYMMETRIC Last 4 periods</th>
<th>ENLARGED All periods</th>
<th>ENLARGED Last 4 periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grand</td>
<td>28</td>
<td>14</td>
<td>34</td>
<td>14</td>
<td>15</td>
<td>8</td>
</tr>
<tr>
<td>Minimal (strong included)</td>
<td>72</td>
<td>86</td>
<td>41</td>
<td>55</td>
<td>67</td>
<td>67</td>
</tr>
<tr>
<td>Minimal (strong excluded)</td>
<td>0</td>
<td>0</td>
<td>24</td>
<td>31</td>
<td>12</td>
<td>19</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

1 Each entry is the percentage of games that resulted in a type of coalition.

In VETO, minimal winning coalitions, which must include the strong player, formed in 115/160 (≈72%) of all games, and in 55/64 (≈86%) of the games in the last four periods. In SYMMETRIC, minimal winning coalitions, which can be comprised of any two players, formed in 104/160 (≈ 65%) of all games, increasing to 55/64 (≈86%) of the games in the last four periods. These coalitions could either include or exclude the strong player. In our analysis in section 2 we assumed that strong and weak players behave and are treated symmetrically, and so strong players should be included in 2/3rd of minimal winning coalitions. Our data is consistent with this symmetry assumption: 65% of the games resulted in a minimal winning coalition and
63% of these included the strong player (64% of minimal winning coalitions in the last four periods featured the strong player). In ENLARGED there can be two different types of minimal winning coalition: the strong player with one of the weak players, or the two weak players with the new member. In equilibrium the latter type of coalition occurs one quarter of the time (when the new player is the randomly selected proposer). In our data minimal winning coalitions formed in 79% of games (86% in the last four periods), and 15% of these (22% in the last four periods) excluded the strong player.

On top of this, within minimal winning coalitions we see a tendency for the division of the pie to change over the course of the session. Figure 5 shows that in the first period the pie was often split equally among members of the coalition, but that by the last period this kind of outcome was much less frequent. This pattern is particularly clear in VETO, where players quickly realized that the strong player only needed the vote of one of the weak players, and that weak players were willing to accept proposals that excluded the other weak player. In the final period minimal winning coalitions formed in 15 of 16 VETO groups, but only in one of these did the members of that coalition share the pie equally. The pattern is less pronounced in the other treatments. In the last four periods of the ENLARGED treatment about a third of the minimal winning coalitions divided the pie equally among its members, and in the last four periods of the SYMMETRIC treatment about a half of the minimal winning coalitions divided the pie equally among its members.

**Figure 5. Proportion of Minimal Winning Coalitions that Divide Equally**

![Figure 5](image_url)
The trend in Figure 5 seems to reflect a process where players learn from experience about their bargaining power and how to exploit it. At the same time, examining the proposals agreed by minimal winning coalitions suggests that some players were unable to fully extract their equilibrium shares. Table 4 reports average earnings within minimal winning coalitions. We break down earnings according to whether the player was the proposer or partner in the minimal winning coalition.

Table 4. Proposer Power in Minimal Winning Coalitions

<table>
<thead>
<tr>
<th></th>
<th>VETO</th>
<th>SYMMETRIC</th>
<th>ENLARGED</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Periods</td>
<td>Last 4 Periods</td>
<td>All Periods</td>
</tr>
<tr>
<td>Strong Proposer</td>
<td>80.60</td>
<td>83.45</td>
<td>66.76</td>
</tr>
<tr>
<td>Strong Partner</td>
<td>73.02</td>
<td>79.42</td>
<td>59.48</td>
</tr>
<tr>
<td>Weak Proposer</td>
<td>46.98</td>
<td>40.58</td>
<td>61.41</td>
</tr>
<tr>
<td>Weak Partner</td>
<td>39.40</td>
<td>36.55</td>
<td>55.68</td>
</tr>
<tr>
<td>New Proposer</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>New Partner</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

In the VETO treatment the strong player’s theoretical bargaining power is due to his voting weight and is independent of proposer power: he is predicted to get 118 points regardless of whether he is the proposer or partner in the minimal winning coalition. In the data he gets more as a proposer than as a partner, but in any case much less than 118 points. In contrast to VETO, in the SYMMETRIC treatment voting weights do not confer a theoretical advantage to any player but being proposer does: ex post equilibrium divisions give 80 points to the proposer and 40 points to the partner. In our data we find that minimal winning coalitions divide the pie much more closely to 60-60. For testing the significance of proposer power we use one-sided binomial tests. For both players and both treatments the proposer advantage is significant across all periods (largest p-value = 0.035), but in all cases declines across periods and is insignificant in the last four periods (smallest p-value = 0.363). In summary, proposer power in these two treatments is small and diminishes as subjects gain experience.
In the ENLARGED treatment players can derive a theoretical bargaining advantage either from having more votes than other players or from being the proposer. The strong player is predicted to get 89 points as a proposer and 45 points as a partner, while the weak player is predicted to get 75 as a proposer and 31 as a partner. The proposer advantage in the data is much smaller than this. Thus, while proposer power theoretically leads to highly inequitable outcomes within minimal winning coalitions, actual divisions are much more equitable.

At first sight, our observation that minimal winning coalitions form frequently in all three treatments, as predicted by standard theory, but that payoff allocations within minimal winning coalitions tend to be more equitable than predicted, would appear to be consistent with recent theories of social preferences. For example, if subjects receive disutility from earning less than others (see, for example, Fehr and Schmidt, 1999), then in a multi-player ultimatum game the proposer would still propose a minimal winning coalition, but would have to offer positive shares in order to buy the votes of coalition partners (see Okada and Riedl, 2005). However, in the multi-stage symmetric game inequality aversion results in more unequal divisions than predicted (see Montero, 2007a). Thus, these observations cannot be explained by simple models of social preferences in which agents dislike inequality, although they may be explained by a model which combines disadvantageous inequity aversion with myopic behavior.

It is interesting to compare the patterns from our SYMMETRIC and VETO treatments with those observed in previous experiments. Our SYMMETRIC treatment is similar to some of the treatments reported in Fréchette et al. (2005a, 2005b) and Kagel et al. (2007) in that, while voting weights may vary across voters, any two of the three voters can form a winning coalition. We focus on the (inexperienced, undiscounted) treatment of Fréchette et al. (2005b) as this is procedurally closest to ours. In both experiments agreements happen quickly: on average their games lasted 1.6 rounds (our games lasted 1.4 rounds), and immediate agreements occurred in 68% of their games (73% of our games). Also in both experiments egalitarian outcomes are uncommon: 7% of their games resulted in egalitarian divisions (12% of our games). In both experiments most games resulted in minimal coalitions: 69% of their games resulted in a minimal winning coalition (66% of our games). These patterns broadly support the Baron-

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11 In contrast to the other treatments this is significant for both strong and weak players whether we base tests on all or just the last four periods (largest p-value 0.0625). For the new player we have insufficient data for meaningful tests as there are too few matching groups where the new player appears in both proposer and partner roles.
Ferejohn model predictions. At the same time, there are some notable deviations from the model predictions. The model predicts ex post divisions that give the proposer 2/3 of the pie and one of the other players 1/3 of the pie. In their data the average allocation gives the proposer just 51% of the pie (in our data 47% of the pie), and even focusing on minimal winning coalitions, their proposers only get 55% of the pie (ours get 52% of the pie).

The results from our VETO treatment are also qualitatively similar to results from the (inexperienced, low delay cost) treatment of Kagel et al.’s (2007) Veto Game. In their Veto Games agreements happened quickly (although not as quickly as in a control treatment somewhat similar to our SYMMETRIC treatment): 54% of their games (and 51% of our games) ended in round one and on average their games lasted 1.95 rounds (our games lasted 2.38 rounds). As in our VETO treatment, they observe an increasing tendency toward minimal winning coalitions over the course of the session, and overall 59% of their games (72% of our games) result in minimal winning coalitions. Just like in our experiment, however, the veto player’s earnings are considerably less than predicted. He is predicted to get 92% as the proposer and 80% as the recipient of a proposal. Even within minimal winning coalitions he only gets 62% as proposer and 52% as recipient. In our VETO treatment the strong player is predicted to get 98% either as proposer or recipient, and even within minimal winning coalitions he only gets 67% as proposer and 61% as recipient.

4.2 Formal Analysis of Voting Power

In this section we look at the implication of the developments in coalition formation and payoffs within coalitions for players’ average earnings, which we use to measure voting power. Figure 6 shows how this develops across periods for each treatment and player-role.

In SYMMETRIC and ENLARGED the voting power of each player-type appears to be stable while in VETO trends in voting power appear. In VETO the Spearman rank correlation coefficients between the strong player's earnings and period are positive for each matching group, so we can reject the null hypothesis that earnings are equally likely to increase or decrease with experience (p-value = 0.008). The significant increase in the strong player's earnings is due to changes in earlier periods: there is no evidence of a relationship between earnings and period

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12 Although we emphasize that on top of the many procedural differences between experiments, their use of discounting makes us hesitant to read too much into quantitative differences between our results and theirs.
in the last four periods (the p-value based on Spearman rank correlation coefficients is 0.727). A similar analysis reveals no significant trends in SYMMETRIC or ENLARGED.

Figure 6. Evolution of Average Earnings

For SYMMETRIC and ENLARGED the corresponding p-values based on all periods are both 0.727; based on the last four periods they are 1.000 (SYMMETRIC) and 0.727 (ENLARGED).
Average earnings for each player type are presented in Table 5. In this table and in subsequent analysis we present results based on all periods and last four periods. For SYMMETRIC and ENLARGED this does not make much difference. Interestingly, for these treatments average earnings are quite close to the equilibrium expected payoffs given in Table 2. On the other hand in VETO, the strong player’s earnings are substantially below the equilibrium expected payoff given in Table 2, but the discrepancy is smaller in the last four periods.

<table>
<thead>
<tr>
<th></th>
<th>VETO</th>
<th></th>
<th>SYMMETRIC</th>
<th></th>
<th>ENLARGED</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All periods</td>
<td>Last 4 periods</td>
<td>All periods</td>
<td>Last 4 periods</td>
<td>All periods</td>
<td>Last 4 periods</td>
</tr>
<tr>
<td>Strong player</td>
<td>69.51</td>
<td>78.70</td>
<td>41.39</td>
<td>40.81</td>
<td>52.81</td>
<td>50.98</td>
</tr>
<tr>
<td>Weak player</td>
<td>25.24</td>
<td>20.64</td>
<td>38.92</td>
<td>39.59</td>
<td>29.78</td>
<td>30.64</td>
</tr>
<tr>
<td>New player</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>7.61</td>
<td>7.71</td>
</tr>
</tbody>
</table>

There is strong evidence that voting weights affect voting power in the VETO and ENLARGED treatments. In both treatments the strong player earns more than a weak player in every single matching group, whether we focus on all periods or just the last four periods, and thus the strong player earns significantly more than the weak player (one-sided sign-test p-value = 0.004). Similarly, the weak player earns significantly more than the new player in ENLARGED (one-sided sign-test p-value = 0.004). In the SYMMETRIC treatment we find that the earnings of the strong player are not significantly different from the earnings of the weak players (all periods: two-sided sign-test p-value = 1.000; last four periods: two-sided sign-test p-value = 0.727).

The main question motivating the design of our experiment was whether adding a new player to a weighted voting game, while retaining the relative voting weights of original players, could increase the voting power of an original player. Our first test of whether members can gain
from enlargement, as theoretically predicted, is based on a comparison of the weak player's earnings in VETO and ENLARGED, displayed in Figure 7.

**Figure 7. Effect of Enlargement on Weak Player’s Earnings**

We test the null hypothesis that the weak player's earnings are the same in VETO and ENLARGED against the alternative hypothesis that the weak player's earnings are greater in ENLARGED.\(^{14}\) We reject the null hypothesis: the weak player's earnings are significantly higher in ENLARGED (based on all periods the one-sided Wilcoxon test p-value is 0.023; based on just the last four periods the one sided Wilcoxon test p-value is 0.001). Intuitively, the reason for this effect is that the addition of a new member provides the weak players with an alternative minimal winning coalition. This possibility appears to enhance a weak player’s ability to extract larger shares of the pie from the strong player, despite the fact that the alternative coalition very seldom forms. For example, looking at winning coalitions from the last four periods consisting of the strong player and one of the weak players we see that in VETO the strong player gets about 77 points, compared with about 66 points in ENLARGED.

Our experimental design provides a second opportunity to test the prediction that enlargement benefits an existing member. Starting from the SYMMETRIC voting game, adding a new player with one vote and increasing the quota from 4 votes (out of 7) to 5 votes (out of 8) produces the ENLARGED voting game, and can in equilibrium increase the strong player’s voting power. Figure 8 displays the strong player’s voting power in these two treatments.

\(^{14}\) Thus, the null hypothesis is that there is no treatment effect, and the alternative hypothesis is that there is a treatment effect in the theoretically predicted direction.
In our experiment the strong player earns significantly more in ENLARGED than SYMMETRIC (based on all periods the one-sided Wilcoxon test p-value is 0.001; based on just the last four periods the one-sided Wilcoxon test p-value is 0.029). Here, the source of the power increase is rather different. The strong player’s bargaining power within a minimal winning coalition does not appear to be very different between SYMMETRIC and ENLARGED treatments. In a minimal winning coalition the strong player gets 64 points in SYMMETRIC (63 if we focus on the last four periods) and 66 in ENLARGED (68 if we focus on last four periods). Instead, the main reason why the strong player earns more in ENLARGED than SYMMETRIC is that he is included more often in minimal winning coalitions. He is included in 85% of minimal winning coalitions in ENLARGED (78% if we focus on last four periods), compared with 63% in SYMMETRIC (64% if we focus on last four periods). This is qualitatively consistent with the equilibrium discussed in Section 2. There, ex ante, the strong player is expected to be included in 75% of minimal winning coalitions in ENLARGED compared with 67% in SYMMETRIC, while when included in a minimal winning coalition the strong player expects to get 60 points in both ENLARGED and SYMMETRIC.

4.3 Analysis of Voting Patterns
The primary determinant of whether a given player type votes in favor of a proposal is, perhaps unsurprisingly, how much that player is offered. A player is more likely to accept higher offers.
For example, the strong player accepted 10/22 (45%) offers of 40 points in ENLARGED, compared with 37/47 (79%) offers of 60 points. However, the amount that must be offered to secure an acceptance varies considerably across player types. In ENLARGED the weak player accepted 39/52 (75%) of offers giving him 40 points, and the new player accepted 5/5 (100%) offers giving him 40 points. Also, the propensity for a given type to accept a given proposal varies across treatments. Looking again at offers of 40 points we see that the strong player accepted 12/22 (55%) of these in SYMMETRIC and 7/27 (26%) in VETO, while the weak player accepted 27/47 (57%) in SYMMETRIC and 39/58 (67%) in VETO.\footnote{We focus on offers of 40 points since, pooling over all treatments, this is the most common offer made (other than offers of 0 points).}

While these patterns are very stable in ENLARGED, in SYMMETRIC and VETO we observe a decrease in the propensity to accept offers of 40 points. Based on the last four periods the strong player accepts 6/18 (33%) and the weak players accept 2/6 (33%) of such offers in SYMMETRIC, while the strong player accepted 0/5 and the weak player accepted 8/19 (42%) of such offers in VETO.\footnote{The functions are quite stable across periods in ENLARGED, and are stable across the last four periods in SYMMETRIC and VETO. The functions in SYMMETRIC tend to be further to the left in earlier periods, and those in VETO tend to be closer together in earlier periods.}

Figure 9 displays the empirical cumulative distribution functions of accepted offers. Votes of proposers are excluded from the analysis, and we focus on the last four periods. In VETO it is clear that accepted offers tend to be lower for weak players than for strong players. Half of the acceptances by weak players were for offers less than or equal to 40, whereas for the strong player half of acceptances were for offers less than or equal to 80. Likewise in ENLARGED there is a natural ordering whereby accepted offers by strong players tend to be higher than the accepted offers of weak players, which in turn tend to be higher than the accepted offers of new players. In SYMMETRIC we present separate functions for strong and weak players. However, the functions are not very different and as we report below differences in the acceptance behavior of the different types are insignificant. Thus, in SYMMETRIC the strong player with three votes behaves no differently from a weak player with two votes in terms of voting behavior.
For a formal statistical analysis of voting behavior we estimated probit models where the dependent variable is whether or not a player voted for a proposal. Again, we excluded proposers
from the analysis and focus on the last four periods. As explanatory variables we included the number of points offered to the player and the number of points the player demanded in his own proposal. Since we treat observations as independent across, but not within, matching groups, we cluster on independent matching groups to obtain robust standard errors. The results are presented in Table 6.

### Table 6. Probit Analysis of Voting Behavior

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>Dependent Variable: Accept=1, Reject=0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VETO</td>
</tr>
<tr>
<td></td>
<td>Strong Player</td>
</tr>
<tr>
<td>Offered Share</td>
<td>0.009***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
</tr>
<tr>
<td>Offered Share</td>
<td>-0.008**</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
</tr>
<tr>
<td>Obs.</td>
<td>115</td>
</tr>
<tr>
<td>Pseudo R$^2$</td>
<td>0.3624</td>
</tr>
</tbody>
</table>

1 Based on data from last four periods. Table lists marginal effects, evaluated at the means of the independent variables, and standard errors, clustered on independent matching groups, in parentheses. * denotes significance at the 10-percent level, ** at the 5-percent level, and *** at the 1-percent level.

As expected, the share offered to a player is a strong explanatory factor for whether a player will accept an offer. For all player types in all treatments, the more a player is offered the more likely he is to accept. Offering one additional point increases the probability of acceptance by between 0.9 percentage points (strong player in VETO treatment) and 8.8 percentage points (new player in ENLARGED). We also included the share that a player demanded in their own proposal as an explanatory variable – recall that all subjects, not just the randomly selected

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17 We also conducted probit estimations on all periods including period dummies, and found significant period effects in the VETO and SYMMETRIC treatments. For this reason we restrict attention to the last four periods. For the last four periods we also conducted estimations including period dummies, and found these to be jointly insignificant in all cases.
proposer, completed a proposal in every round. We expected that the share demanded would reveal an aspiration level of a subject, and so a given offer would more likely be accepted if the share demanded is lower. As can be seen in Table 6, the marginal effect of this variable is indeed negative in all cases, although it is not significant for the weak player in the VETO treatment, or for either player type in the SYMMETRIC treatment.

We also examined whether the voting behavior of different types within a treatment differ. Based on Wald tests the behavior of strong and weak players in the VETO treatment differ significantly (two-sided p-value = 0.000), as does behavior of strong, weak, and new players in ENLARGED (two-sided p-value = 0.000). However, in SYMMETRIC there is no significant difference between the voting behavior of strong and weak players (two-sided p-value = 0.968). Finally, we examined whether the voting behavior of a given player type varied across treatments. Again using Wald tests, we find voting behavior differs significantly across treatments at the 10% level for strong players (two-sided p-value = 0.082) and at the 1% level for weak players (two-sided p-value = 0.000).

5. Discussion and Conclusion
There are two approaches in the theoretical literature for solving games: the axiomatic approach, which is based on specifying some desirable properties for the solution and the strategic approach, which is based on the equilibrium of an explicit bargaining game. The ‘paradox of new members’ was introduced into the literature in terms of power indices grounded in the axiomatic approach. In this paper we follow the strategic approach and show that the addition of a new member to a voting body can increase the equilibrium payoffs of original members in the context of a specific bargaining procedure.

The bargaining procedure we use was introduced by Baron and Ferejohn and is widely used in political science. Other theoretical analyses of the Baron-Ferejohn procedure with weighted voting include Banks and Duggan (2000), Montero (2002), Diermeier et al. (2003), Snyder et al. (2005), Montero (2006), Kalandrakis (2006) and Eraslan and McLennan (2006). There are other bargaining procedures we could have chosen. In particular, since it is possible to

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18 To conduct Wald tests for differences between the voting behavior of voter types within a treatment we ran a probit for that treatment, including as explanatory variables voter type dummy variables and interactions between the dummy variables and the share offered and share demanded variables. We then tested for the joint significance of the coefficients on the dummy and interaction terms. An analogous approach was used for testing for differences in behavior across treatments for a given voter type.
devise a bargaining procedure to make equilibrium payoffs coincide with the Shapley value, and the Shapley value displays this result, the possibility of a positive impact of enlargement in a non-cooperative setting was already implicitly established. However, while the Baron-Ferejohn procedure seems a natural abstraction of bargaining under majority rule, these other bargaining procedures seem more contrived. In particular, these other procedures require unanimity rather than majority in order to pass a proposal: Hart and Mas-Colell (1996), Pérez-Castrillo and Wettstein (2001) and Laruelle and Valenciano (2008) require the unanimous consent of all players; in Vidal-Puga (2008) a coalition is formed gradually, with players having the option to join it or leave the game in a random order, and the final allocation must be accepted unanimously by all members of the coalition. Other implementations of the Shapley value, such as Gul (1989), apply only to particular classes of games that do not include majority games. Our theoretical contribution is to demonstrate that gaining power through enlargement is possible in the leading model of legislative bargaining.

The theoretical possibility of gaining power through enlargement motivated us to ask whether this phenomenon can be observed in the laboratory. Our answer is affirmative. In all of our treatments we find that as sessions progress subjects become less and less willing to propose and vote for equal divisions of the pie and more and more willing to propose and vote for proposals that give zero to other subjects. Although these trends lead to minimal winning coalitions, as predicted in equilibrium, we see some substantial deviations from equilibrium, even in later periods. In particular, players are generally unable to exploit either proposer power or advantageous voting weights to the full extent predicted by equilibrium. Despite these deviations we find that average earnings of each player-type are quite stable in our SYMMETRIC and ENLARGED treatments, and not far from equilibrium expected payoffs. Consequently, when comparing these treatments the theoretical comparative static prediction is observed. In our VETO treatment, although the weak players’ earnings move closer to equilibrium with experience, even by the end of the experiment the weak player’s earnings are far above equilibrium. This makes it harder to observe a treatment effect in the comparison of the VETO and ENLARGED treatments, but we do observe it in this case as well: the weak players’ earnings are higher in the enlarged voting body.

Previously, Montero et al. (2008) studied the effect of enlargement on the balance of power in a less structured environment where any player could place (or amend) a proposal on
the table at any time, any player could vote for or against any proposal on the table at any time, and the first proposal on the table to achieve the quota was implemented. They found that voting powers are quite close to Shapley values and so Brams and Affuso’s ‘paradox’ is also observed in their experiment. This environment seemed natural for testing hypotheses based on cooperative game theory, but of course it is difficult to compare behavior with equilibria. Nevertheless, it is interesting to note that the voting powers associated with a given player role differ across the experiments. For example, in the VETO treatment the strong player earns more in the less structured procedure than in the Baron-Ferejohn procedure. Similarly, the strong player in the ENLARGED treatment earns more in the less structured environment. A similar result, albeit in a different context, is observed in de Groot Ruiz et al. (2007). They compare a non-cooperative voting game with a more natural, but less structured, voting game and find that a median voter (the ‘strong player’ in their context) is able to secure a higher payoff in a less structured voting game. Together these results suggest that voting powers depend not only on voting weights and quotas, but also on specific features of the agenda.

This point is reinforced by considering the implications of the Baron-Ferejohn model for the first two EU enlargements. Table 7 displays the voting weights and voting powers of the various countries, where the latter are based on the stationary expected equilibrium payoffs of the infinite-horizon Baron-Ferejohn model with no discounting and where all countries have equal recognition probabilities.

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19 This is despite the fact that the strong player gets almost the whole pie in the equilibrium of the Baron-Ferejohn procedure, while in the less structured environment the Shapley Value assigns 2/3 of the pie to the strong player.

20 For detailed derivations see Montero (2007b).
### Table 7. Voting Weights and Voting Powers on the EU Council of Ministers

<table>
<thead>
<tr>
<th></th>
<th>1958 voting weights (% voting power)</th>
<th>1973 voting weights (% voting power)</th>
<th>1981 voting weights (% voting power)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W. Germany</td>
<td>4 (23.8)</td>
<td>10 (15.9)</td>
<td>10 (16)</td>
</tr>
<tr>
<td>France</td>
<td>4 (23.8)</td>
<td>10 (15.9)</td>
<td>10 (16)</td>
</tr>
<tr>
<td>Italy</td>
<td>4 (23.8)</td>
<td>10 (15.9)</td>
<td>10 (16)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td></td>
<td>10 (15.9)</td>
<td>10 (16)</td>
</tr>
<tr>
<td>Belgium</td>
<td>2 (11.9)</td>
<td>5 (7.9)</td>
<td>5 (8)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>2 (11.9)</td>
<td>5 (7.9)</td>
<td>5 (8)</td>
</tr>
<tr>
<td>Greece</td>
<td></td>
<td></td>
<td>5 (8)</td>
</tr>
<tr>
<td>Denmark</td>
<td></td>
<td>3 (7.1)</td>
<td>3 (4)</td>
</tr>
<tr>
<td>Ireland</td>
<td></td>
<td>3 (7.1)</td>
<td>3 (4)</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>1 (4.8)</td>
<td>2 (6.3)</td>
<td>2 (4)</td>
</tr>
<tr>
<td>Quota</td>
<td>12</td>
<td>41</td>
<td>45</td>
</tr>
</tbody>
</table>

In the first enlargement relative voting weights of existing members were constant (except for Luxembourg whose relative weight fell). In the second enlargement relative voting weights were held constant. This reflected a desire to maintain the balance of power among existing members. For example the European Commission communicated that in consideration of adjustments to the treaties necessitated by enlargement “the Community must also remain consistent and avoid any appreciable shift in the existing balance of power between Member States” (European Commision 1978, point 19). How did the enlargements affect the balance of power? As with cooperative power indices, the Baron-Ferejohn model predicts that Luxembourg’s power increased after the first enlargement. However the implications of the second enlargement are more sensitive to how power is measured. While power indices predict a further increase in Luxembourg’s power, the Baron-Ferejohn model predicts a different
manifestation of the ‘paradox of new members’ – it was the six largest countries that benefitted.\textsuperscript{21} Whether the cooperative power index or the noncooperative equilibrium prediction provides the better measure of theoretical power is open to question. Moreover, which (if any) country’s voting power, measured in terms of its realized share of whatever is being bargained over, really did increase as a result of the enlargement is difficult to determine empirically.

For this reason we have used laboratory experiments to study the relationship between voting weights and power, and in particular to study the effects of enlargement on the balance of power. The advantage of the experimental approach is that it can systematically examine the effects of voting weights and voting rules, including features of the extensive form governing the way in which proposals are made and votes cast, in order to enhance our understanding of voting power.

\textsuperscript{21} Note that not only do some existing members get more voting power from an enlargement, but a proposal to enlarge would obtain enough votes to be passed.
References


Montero, Maria. 2006. “Noncooperative Foundations of the Nucleolus in Majority Games.”


Appendix: Instructions for VETO treatment

Introduction

This is an experiment about group decision-making. There are other people in this room who are also participating in this experiment. You must not talk to them or communicate with them in any way during the experiment. The experiment will take no more than 90 minutes, and at the end you will be paid in private and in cash. The amount of money you earn will depend on the decisions that you and the other participants make.

In this experiment you will participate in ten periods. In each period you will be in a group with two other people, but you will not know which of the other people in this room are in your group. The people in your group will change from period to period, and in particular you will never be matched with the same set of two other people twice. The decisions made by you and the other people in your group will determine how many points you earn in that period. At the end of the experiment you will be paid according to your total point earnings from all ten periods. You will be paid 3p per point.

Description of a period

At the beginning of each period, you will be randomly allocated a subject identification number, either 1, 2, or 3. (Thus, your identification number may change from period to period.)

Within each group of three people one person will control three votes, one person will control two votes, and one person will control two votes. The assignments of votes to subject identification numbers will vary from period to period.

In each period you and the other people in your group have 120 points to divide. You and the other people in your group can make proposals about how these points are to be divided among the group members. You and the other people in your group then cast votes for or against proposals. The first proposal to receive five or more votes will be enforced. When a proposal is enforced the period ends and each person earns the number of points specified in that proposal.

How you make and vote on proposals

A period can consist of up to 20 rounds of making and voting on proposals.

At the beginning of each period your computer screen will look like the one shown in Figure 1. You must propose the number of points that each person in your group will receive. For each person you can type in any whole number between 0 and 120, but the total number of points received by the group members must add up to 120. Note that the row where you propose how much you yourself will receive is in boldface.
When you have completed a proposal you click on the "submit" button to submit it. You will have ninety seconds to make a proposal. If you do not submit a proposal in ninety seconds the computer will submit a proposal for you, and this proposal will propose that all group members get zero.

After all three group members have submitted proposals the computer will randomly choose one of them and the three group members will vote on this proposal. A sample screen is shown in Figure 2 (except that the entries marked XXX will be the numbers one of the group members entered in their proposal). Note that the row proposing how much you will receive is in boldface. You must then cast your votes either for or against the proposal by clicking accept or reject. If you accept, all the votes you control are cast in favour of the proposal, while if you reject, all the votes you control are cast against the proposal.

You will have thirty seconds to cast your votes. If you do not cast your votes in thirty seconds the computer will vote for you, and will vote against the proposal.

If the proposal receives five or more votes in favour it will be enforced. This means that the period will end and this proposal will determine the number of points you earn in that period.

If the proposal receives less than five votes, the process will be repeated: once again all group members will make proposals, one of them will be randomly chosen, and all group members will get to vote on this proposal. This process will continue until a proposal is enforced, or until 20 rounds of making and voting on proposals have passed. If no proposal has been enforced after the twentieth vote the period will end and all three group members will receive zero.

**Ending the session**

At the end of period ten your total points from all periods will be converted to cash at a rate of 3p per point and you will be paid this amount in private and in cash. The computer will keep track of your point earnings from period to period, but if you want to keep a record for yourself you can use the attached Record Sheet.

If you have any questions raise your hand and a monitor will come to your desk to answer them. Now, please begin period 1.
FIGURE 1. SCREEN FOR MAKING PROPOSALS

You can propose any numbers that add up to 120 points. The number of votes required to pass a proposal is 5.
When you are ready, please press the "Submit" button.
There may be up to 10 rounds of making and voting on proposals.

FIGURE 2: SCREEN FOR VOTING ON PROPOSALS