THE UNIVERSITY of York

Discussion Papers in Economics

No. 2009/17

Myopic, Naive, Resolute or Sophisticated?
A study of how people take dynamic decisions

By

John Hey, University of York and LUISS, Rome, Italy

and

Luca Panaccione, University of Tor Vergata and LUISS, Rome, Italy

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

Myopic, Naive, Resolute or Sophisticated?

A study of how people take dynamic decisions

John D. Hey

University of York, UK, and LUISS, Rome, Italy

Luca Panaccione

University of Tor Vergata and LUISS, Rome, Italy

July 2009

Abstract

Potentially dynamically-inconsistent individuals create particular problems for economics, as their behaviour depends upon whether and how they attempt to resolve their potential inconsistency. This paper reports on the results of a new experiment designed to help us distinguish between the different types that may exist. We classify people into four types: myopic, naive, resolute and sophisticated. We implement a new and simple experimental design in which subjects are asked to take two sequential decisions (interspersed by a random move by Nature) concerning the allocation of a given sum of money. The resulting data enables us to classify the subjects. We find that the majority are resolute, a significant minority are sophisticated and rather few are naive or myopic.

JEL codes: D90, D81.

Keywords: dynamic inconsistency, sequential choice, myopic, naive, resolute, sophisticated.

Corresponding Author: John D. Hey, Department of Economics and Related Studies, University of York, Heslington, York, YO10 5DD, UK. E-mail address: jdh1@york.ac.uk

1 Introduction

Dynamic consistency is a central assumption in economics, and is essential to many of the key results and policy prescriptions in important fields, such as those of investment, saving and pensions. It implies that preferences do not change with the passage of time: the preferences an individual has over choices in some period are not dependent on the moment that the individual is asked to express these preferences. However, there is ample evidence that people are not dynamically consistent. In the context of discounting, it seems that many people do not discount exponentially; so that their relative evaluation of consumption at two given points of time depend upon the time at which the evaluation is being made. In the context of decision-making under risk, it seems that many people do not have Expected Utility preferences; so that their risk aversion about choices in some period vary depending upon the time at which their risk evaluation is being made. As Wikipedia notes:

> "In economics, dynamic inconsistency, or time inconsistency, describes a situation where a decision-maker's preferences change over time, such that what is preferred at one point in time is inconsistent with what is preferred at another point in time. It is often easiest to think about preferences over time in this context by thinking of decision-makers as being made up of many different "selves", with each self representing the decision-maker at a different point in time. So, for example, there is my today self, my tomorrow self, my next Tuesday self, my year from now self, etc. The inconsistency will occur when somehow the preferences of some of the selves are not aligned with each other."

While we are particularly concerned with decision making under risk, the issues are also important in other contexts. Of crucial interest is in how such potentially dynamically inconsistent people react to their potential inconsistency. Are they aware of it? Do they ignore it? Do they take it into account in planning their future behaviour? Do they somehow constrain themselves to act consistently?

The literature (seminal references are Machina (1989) and McClennan (1990)) discusses a number of possibilities. The key issue is that dynamically inconsistent preferences change through time. We give an example in the context of consumption, though our context is decision-making under risk. As we have already noted, the crucial problem with dynamically inconsistent preferences is that they change through time, not in the sense that preferences for period $t' > t$ consumption are different from preferences for period t consumption, but rather in the sense that preferences for period t' consumption as viewed from period t are different from preferences for period t' consumption as viewed from period $t + s$, for some $t' > t + s > t$. It is as if the individual is a different individual in different periods. In a sense it is a type of schizophrenia, but one that is familiar. The interesting issues from the point of view of economics are twofold: (1) is the individual aware of this dynamic inconsistency?; (2) if so, what does he or she do about it?

If the individual is not aware of the dynamic inconsistency, then presumably the individual works through time always choosing the best decision as viewed from the present perspective. We call such a type *naive*. This is the first of four types of decision maker we consider. If, however, the individual *is* aware of his or her dynamic inconsistency, there are various things that he or she might do about it. The individual could follow the example of Ulysses (about to be confronted by the sirens) by metaphorically binding himself to the mast – by imposing his first period preferences. Such a person has been described by McClennan (1990) and Machina (1989) as *resolute*. This is the second of our types.

Economists usually, however, adopt a different story: they assume that potentially dynamically inconsistent people realise that they will want to change their minds in the future, and anticipate this behaviour *ex ante*. Such people *backwardly induct.* Indeed this is the prevailing model used in economics. It is as if Ulysses decided to travel home by a different route. This is our third type: it is referred to in the literature as *sophisticated.*

In Hey and Lotito (2009) these three types were investigated. In this paper we add to the menu a fourth type, one which naturally arises in the context of our experiment, and for which it appears that many real-world examples can be found. This type is one who simply assumes that the decision in the present will be the last. We call this type *myopic*.

The answers to the questions posed above are important not only for economic theory but also for public policy. For example, if people are myopic, then the state might feel obliged to take action to ensure adequate pensions for the population. It is to these issues that this paper is addressed.

We adopt a particularly simple experimental design which enables us to shed light on these issues, and to discriminate between the different types. It is simpler and more informative than a design that was used in Hey and Lotito (2009). We describe the new design in the next section. We then define the various types of economic agent and discuss how they should behave in this experiment. We then describe our econometric analyses and then present our results. A final section concludes.

2 The experimental design and some theory

The design was inspired by one pioneered by Loomes (1991) and subsequently extended by Choi *et al.* (2007). This design was developed for a different decision problem in a different context. In the Loomes' design subjects were simply asked to allocate a sum of money between two risky alternatives. Choi *et al.* extended the design by endowing subjects with tokens and having different exchange rates between tokens and money for the different alternatives. We do not use that feature but extend the Loomes design in a different direction by having a dynamic allocation problem. In our design, subjects are presented with a set of N decision problems, all with the same two-stage structure. In each problem, subjects are given a sum of money, $\overline{m}$, to allocate between two probabilistic options 1 and 2, with known and stated probabilities. Suppose the allocations are x to 1 and $\overline{m} - x$ to 2. Then Nature chooses one of the options at random. If Nature chooses 1 the subject is then asked to allocate x (the amount allocated to 1 at the first stage) between two further probabilistic options, 1A and 1B, again with known and stated probabilities; call the allocations y and $x - y$. Similarly, if Nature chooses 2 the subject is then asked to allocate $\overline{m} - x$ (the amount allocated to 2 at the first stage) between two further probabilistic options, 2A and 2B, again with known and stated probabilities; call the allocations z and $\overline{m} - x - z$. Then Nature chooses one of these new options at random.

If Nature chooses 1 at the first stage and 1A at the second, then the subject earns y for that particular problem; similarly if Nature chooses 1 at the first and 1B at the second, then the subject earns $x - y$; if Nature chooses 2 at the first and 2A at the second, then the subject earns z; and finally if Nature chooses 2 at the first and 2B at the second, then the subject earns $\overline{m} - x - z$. At the end of the experiment one of the decision problems is chosen at random and the subject is paid his or her earnings on that particular problem.

The design is simple and informative. To illustrate how the design discriminates between different types of potentially dynamically inconsistent people, let us assume that the decision-maker has Rank Dependent Expected Utility (RDEU) preferences, with utility function $u(.)$ and weighting function $w(.)$. As is well-known, if $w(p) = p$ then the model reduces to that of Expected Utility (EU) theory and the decision-maker is not dynamically inconsistent. However suppose that $w(.)$ is not linear. Then potential dynamic inconsistencies arise.

Consider the problem as viewed from when the decision-maker must make the first decision. Let us denote the probabilities of Nature choosing 1 and 2 at the first stage by p and $(1 - p)$ respectively. Denote the probabilities of Nature choosing 1A and 1B by p_1 and $(1 - p_1)$ respectively, and denote the probabilities of Nature choosing 2A and 2B by p_2 and $(1 - p_2)$ respectively. The decision maker has to allocate $\overline{m}$ between 1 (x) and 2 $(\overline{m} - x)$ at the first stage and then allocate whichever of x or $\overline{m} - x$ is realised between either 1A (y) and 1B $(x - y)$ or between 2A and 2B (z) and 2 $(\overline{m} - x - z)$ at the second stage. Thus, as viewed from the first stage, the decision-maker has to choose x, y and z to maximise his or her Rank Dependent Expected Utility. The possible payoffs are y, $x - y$, z and $\overline{m} - x - z$ and the corresponding probabilities are pp_1, $p(1 - p_1)$,

$(1-p)p_2$ and $(1-p)(1-p_2)$. For the readers' ease, let us introduce the notation

$$q_1 = pp_1, \quad q_2 = p(1-p_1), \quad q_3 = (1-p)p_2, \quad q_4 = (1-p)(1-p_2),$$

together with the following one

$$m_1 = y, \quad m_2 = x - y, \quad m_3 = z, \quad m_4 = \overline{m} - x - z.$$

Naturally we have $q_1 + q_2 + q_3 + q_4 = 1$ and $m_1 + m_2 + m_3 + m_4 = \overline{m}$. The Rank Dependent EU evaluation of this risky prospect depends upon the ordering of the m_j $(j = 1, 2, 3, 4)$. Suppose we consider the following ranking[1]

$$m_1 \leq m_2 \leq m_3 \leq m_4 \tag{1}$$

Then the objective function of the individual is to maximise the following expression

$$\begin{aligned} RDEU \;=\; & u(m_1)\left[1 - w(q_2 + q_3 + q_4)\right] + \\ & u(m_2)\left[w(q_2 + q_3 + q_4) - w(q_3 + q_4)\right] + \\ & u(m_3)\left[w(q_3 + q_4) - w(q_4)\right] + \\ & u(m_4)\, w(q_4) \end{aligned} \tag{2}$$

The decision maker should choose m_1, m_2, m_3 and m_4 subject to $m_1 + m_2 + m_3 + m_4 = \overline{m}$ and to the ranking constraint (1) to maximise this expression. Denote by m_1^*, m_2^*, m_3^* and m_4^* the optimal values. The decision maker therefore allocates $m_1^* + m_2^*$ to option 1 and the residual to option 2.

The potential dynamic inconsistency arises when the decision maker gets to the second stage. Suppose that Nature chooses 1. The decision-maker has $m_1^* + m_2^*$ to allocate to 1A and 1B at this second stage. If the decision-maker reconsiders the problem at this stage, and considers how to allocate this sum between options 1A and 1B at this second stage in order to maximise his or her Rank Dependent Expected Utility as viewed

[1] Here we illustrate only one case of a possible ranking. Of course, in our analysis we consider all possible rankings.

from that point, his or her problem is to choose m_1 and m_2 to maximise the following expression subject to $m_1 + m_2 = m_1^* + m_2^*$:

$$\begin{array}{rlccc} u(m_1)\,[1 - w(1 - p_1)] + u(m_2)\,[w(1 - p_1)] & \text{if} & m_1 & \leq & m_2 \\ u(m_1)\,w(p_1) + u(m_2)\,[1 - w(p_1)] & \text{if} & m_2 & \leq & m_1 \end{array} \quad (3)$$

Let us call m_1^{**} and m_2^{**} the optimal values. We note that they both depend on $m_1^* + m_2^*$. But there is no reason why the m_1^{**} and m_2^{**} need to be equal to the m_1^* and m_2^*. This is only guaranteed to be the case if $w(.)$ is linear – that is, if the preferences are EU, and hence the individual is not potentially dynamically consistent. We will provide an explicit example of this in what follows.

Similarly if Nature had chosen option 2 at the first stage then the problem at the second stage is to choose m_3 and m_4 to maximise the following expression subject to $m_3 + m_4 = m_3^* + m_4^*$:

$$\begin{array}{rlccc} u(m_3)\,[1 - w(1 - p_2)] + u(m_4)\,[w(1 - p_2)] & \text{if} & m_3 & \leq & m_4 \\ u(m_3)\,w(p_2) + u(m_4)\,[1 - w(p_2)] & \text{if} & m_4 & \leq & m_3 \end{array} \quad (4)$$

Let us call m_3^{**} and m_4^{**} the optimal values. Again, there is no reason why these need to be equal to the m_3^* and m_4^*. As before, this is only guaranteed to be the case if $w(.)$ is linear – that is, if the preferences are EU, and hence the individual is not potentially dynamically consistent.

The issue now is what the individual does about this potential inconsistency. One possibility is that the individual simply uses the optimal allocations derived at the first stage. We call such an individual *resolute*. A second possibility is that the individual implements, at the second stage, those allocations which appear optimal at this second stage. We call this type of individual *naive*, because at the first stage, he or she did not take into account the fact that he or she would choose differently at the second stage.

However, this latter behaviour might be considered irrational. Consider instead an individual who anticipates that, when he or she arrives at the second stage, will reoptimise at that point. In this case, the individual will solve the decision problem in two steps. First, taking as given an allocation of money between option 1 and 2, say $\overline{m}_1$ and $\overline{m}_2$, he or she chooses m_1, m_2, m_3 and m_4 to maximize his or her second stage RDEU function. Denote by $m_1^{***}(\overline{m}_1), m_2^{***}(\overline{m}_1), m_3^{***}(\overline{m}_2)$ and $m_4^{***}(\overline{m}_2)$ the optimal values, where the notation makes clear the dependence on the given allocation $(\overline{m}_1, \overline{m}_2)$. Given these values, in the second step the decision maker chooses $\overline{m}_1$ and $\overline{m}_2$ subject to $\overline{m}_1 + \overline{m}_2 = \overline{m}$ to maximize[2]

$$\begin{aligned} RDEU &= u\left(m_1^{***}\left(\overline{m}_1\right)\right)\left[1 - w(q_2 + q_3 + q_4)\right] + \\ &\quad u\left(m_2^{***}\left(\overline{m}_1\right)\right)\left[w(q_2 + q_3 + q_4) - w(q_3 + q_4)\right] + \\ &\quad u\left(m_3^{***}\left(\overline{m}_2\right)\right)\left[w(q_3 + q_4) - w(q_4)\right] + \\ &\quad u\left(m_4^{***}(\overline{m}_2)\right) w(q_4) \end{aligned}$$

Here we emphasise that the second-step decisions depend upon the values chosen in the first step. Such a decision-maker is termed in the literature *sophisticated*.

The final type of individual we consider is one who ignores the second stage decision and decides at the first assuming that the outcome at the first stage will in fact be the actual payment. Such an individual chooses $(m_1 + m_2)$ and $(m_3 + m_4)$ subject to $(m_1 + m_2) + (m_3 + m_4) = \overline{m}$ to maximise the expression below:

$$\begin{array}{rlcl} u\left(m_1 + m_2\right)\left[1 - w(1-p)\right] + u\left(m_3 + m_4\right)\left[w(1-p)\right] & \text{if} \quad m_1 + m_2 & \leq & m_3 + m_4 \\ u\left(m_1 + m_2\right) w(p) + u\left(m_3 + m_4\right)\left[1 - w(p)\right] & \text{if} \quad m_3 + m_4 & \leq & m_1 + m_2 \end{array}$$

Denote by $(m_1 + m_2)^*$ and $(m_3 + m_4)^*$ the optimal values. The decision maker therefore allocates $(m_1 + m_2)^*$ to option 1 and the residual to option 2 at the first stage. At the second stage, he or she then allocates the residual money either as in equation (3), if Nature chooses 1 at the first stage and as in equation (4) if Nature chooses 2.

The essential point is that different types – resolute, naive, sophisticated and myopic – do different things. This fact enables us to discriminate between the types and hence

[2] Here we are implicitly assuming the ranking as in (1).

identify the type of each individual. This is the purpose of the experiment. Identifying the type is important, as different types behave differently. Usually, economic theory assumes sophisticated behaviour: this requires quite elaborate planning. We test whether subjects actually do this, and if not, what they actually do.

3 The experimental implementation

The implementation was exactly as above. Subjects were given written instructions (in the Non-Mathematical Appendix) and then they were presented with 27 problems, all with the same structure, and all with the same amount of money (€40 in the experiment though £40 in the screen shots) to be allocated, but with different probabilities in the various problems. An example of the opening screen-shot of a problem is shown in Figure 1. We used the words 'Left' and 'Right', rather than Options 1 and 2, because of the physical layout of the problem on the screen. In the problem pictured in Figure 1, the probability of Nature moving Left at the first stage is 60% and that of moving Right 40%. In this particular problem, if Nature chooses Left after the first decision, then the probability of Nature moving Left (Right) after the second is 70% (30%); whereas if Nature chooses Right after the first decision, then the probability of Nature moving Left (Right) after the second is 60% (40%). We note that this, and the other screen-shots, is in English, though the experiment itself was conducted in Italian, at CESARE, the Centro di Economia Sperimentale A Roma Est, at LUISS in Rome. This first screen gives information about probabilities and the sum to allocate.

Then the subject is asked to allocate the £40 (€40 in the actual experiment) between Left and Right at this first stage. Figure 2 illustrates. The allocation when the screen is first displayed is decided at random by the computer. As will be seen, there is a slider on the screen and the subject can use this to show his or her preferred allocation. The subject then clicks on "Click to Continue" to see what Nature decides at this first stage and to proceed to the second stage. (We should note that we forced the subjects to wait 45 seconds before the "Click to Continue" button appeared on the screen.)

The random move by Nature was played out in a visually appealing and convincing way. Suppose in this problem Nature chose Left and the preferred allocation of the subject was that in Figure 2. Then the subject would have £25 (€25 in the experiment) to allocate at the second stage. The second stage screen would then open as in Figure 3. Once again, the opening allocation of the £25 (€25 in the experiment) is decided at random by the computer. Again the subject can use the slider to indicate his or her preferred allocation, and click on the "Click to Continue" button (which appeared after 15 seconds) to confirm his or her allocation. Once again the random move by Nature and the subject's payoff for that problem was displayed on the screen. This procedure was repeated for all 27 problems, which appeared in a random order, with Left and Right at both stages randomly ordered. At the end of all 27 problems, one of the problems was chosen at random and the subject paid the outcome on that particular problem.

Before we ran the experiment we carried out intensive simulations to ensure that we had a number of problems that would enable us to discriminate between the various types. The actual set of problems is listed in Table 1. As we have already noted, the order of the problems and the left/right juxtaposition were determined at random.

We should perhaps comment briefly on the simulations that we carried out and the resulting sets of problems that we used. Obviously in choosing the problem set, there is a trade-off: in general the more problems we include, the greater the discriminatory power of the experiment; however, with too many problems, there is the danger that the subjects can become tired or bored and hence lose concentration. Also there is the problem, given our incentive mechanism, that, as we increase the number of problems, the incentive for careful responses on the part of the subjects on each problem decreases. We also wanted to keep the probabilities[3] simple, and hence we decided to have all probabilities multiples of 1/10. Finally we wanted to avoid too high or too low probabilities, as the information we would gather from their choices on problems with very high or very low probabilities would be small. We therefore decided to have at each stage probabilities of 0.6, 0.7 and 0.8. We then constructed all possible combinations for these three probabilities over the values p, p_1 and p_2. Hence the 27 problems.

[3]Note that we did not use the word 'probability' in the experiment, preferring to use the more everyday word 'chance'.

At this stage we carried out simulations to see if these 27 problems would be sufficiently discriminating. Obviously the discriminating power depends upon the actual values of the parameters. We shall return to our simulations after we have discussed our parameterisation in the next section.

4 Estimation and identification of types

Our analysis is by subject, as subjects are clearly different. Our procedure was the following. For each subject, we assumed that they had Rank Dependent Expected Utility preferences.[4] We also assumed that they had a CRRA utility function, written in this particular way:[5]

$$u(m) = \begin{cases} \frac{m^{1-\frac{1}{r}}-1}{1-\frac{1}{r}} & \text{for} \quad r \neq 1 \\ \ln(m) & \text{for} \quad r = 1 \end{cases} \tag{5}$$

We note the special case when the parameter r takes the value 1. In this case the utility function becomes logarithmic.

The reason for this slightly non-standard (but equivalent to the usual) parameterisation is simply that it makes the mathematics more elegant and hence some key results more transparent. For example, the solution to the optimal allocation of an amount $\overline{m}$ between four options with probabilities p_1, p_2, p_3 and p_4 for an individual with EU preferences and the above utility function is given by:

$$m_i = \frac{\overline{m} p_i^r}{\sum_{j=1}^{4} p_j^r} \quad \text{for} \quad i = 1, 2, 3, 4 \tag{6}$$

We once again note the importance of the special case when $r = 1$ (and thus when the utility function is logarithmic). In this case optimal allocations are simply proportional to the probabilities. This special case is identical to the simple heuristic (which psychologists have noted in other contexts) by which subjects simply allocate money

[4]This is the most widely-accepted non-EU preference functional in the literature. It contains EU as a special case.

[5]See Wakker (2008) for an excellent discussion of the properties of this utility function.

proportional to the probabilities. Obviously this simple heuristic and optimising with a logarithmic utility function are observationally indistinguishable in our context.

This parameterisation also helps makes clear why an EU individual is not dynamically inconsistent. From equation (6) it follows that the optimal allocations as viewed from the first stage are proportional to p_1^r, p_2^r, p_3^r and p_4^r. Suppose the individual arrives at the second stage, for concreteness after Nature has chosen option 1. If the individual wishes to reconsider his or her choices at this stage, he or she will solve the following problem

$$\max_{m_1', m_2'} \quad p_1 u(m_1') + p_2 u(m_2') \quad s.t. \quad m_1' + m_2' = m_1 + m_2,$$

where m_1 and m_2 are given by (6). Given (5), the solution to this problem is

$$m_i' = \frac{p_i^r(m_1 + m_2)}{p_1^r + p_2^r} = \frac{\overline{m} p_i^r}{\sum_{j=1}^4 p_i^r} \quad \text{for} \quad i = 1, 2,$$

thus showing $m_i' = m_i$ for $i = 1, 2$ and similarly for $i = 3, 4$. Hence consistency is guaranteed, though this is the case for all EU decision-makers, not just those with a CRRA utility function.

The parameterisation that we have adopted implies similar notational simplifications for the RDEU model. For example, the optimal allocations for a resolute RDEU individual are given by

$$m_i = \frac{\overline{m} v_i^r}{\sum_{j=1}^4 v_j^r} \quad \text{for} \quad i = 1, 2, 3, 4 \tag{7}$$

Here the v_i are functions of the p_i which depend on the form of the weighting function $w(.)$, whose actual value depend on the ranking considered. On this latter, see below. The optimal solutions for all four types are given in Mathematical Appendix 1.

To specify fully the RDEU preferences we need also specify the weighting function. We assume that this takes the Quiggin (1982) form:[6]

$$w(p) = \frac{p^g}{(p^g + (1-p)^g)^{\frac{1}{g}}} \tag{8}$$

[6]To be strictly correct, we should attribute this to Tversy and Kahneman (1992), who proposed this variation on the original specification proposed by Quiggin, namely: $w(p) = p^g / [p^g + (1-p)^g]$.

For RDEU individuals the optimal allocations depend upon the type. Table 2 gives an example – a RDEU individual with $r = 2.0$ and $g = 0.6$. This person is moderately risk-averse and has the S-shaped weighting function shown in Figure 4; this person over-weights small decumulative probabilities and under-weights large ones. It will be seen from Table 2 that the different types do indeed take different decisions.

In order to estimate the best-fitting values of r and g for each type of subject, and hence identify the best-fitting type for each subject, we need to make some assumption about the stochastic structure of the data. This is necessary because subjects' behaviour is noisy – there is a stochastic component to the data. Because the optimal decisions (see Mathematical Appendix 1) imply given values for the *ratio* between the optimal allocation and the amount to allocate, it is natural to make some assumption about the empirical counterpart: the *ratio* between the actual amount allocated and the actual amount to allocate. Obviously this is a *proportion* and therefore lies between 0 and 1. The natural statistical distribution to assume is thus the *Beta* distribution. A random variable R with such a distribution satisfies $0 \leq R \leq 1$, and has two parameters – which we denote generically by α and β. The mean is given by $\frac{\alpha}{\alpha+\beta}$ and the variance by $\frac{\alpha\beta}{(\alpha+\beta)(\alpha+\beta+1)}$. An interesting property is that, if we want

$$E(R) = R^* \quad \text{and} \quad var(R) = \frac{R^*(1-R^*)}{s} \tag{9}$$

to be satisfied for some value of s, then we should put $\alpha = R^*(s-1)$ and $\beta = (1 - R^*)(1-s)$. Here the parameter s is an indicator of the *precision* of the distribution of R. From equation (9) it can be seen that the variance of R tends to zero as s tends to infinity or as R^* tends to either 0 or 1.

We assume that each empirical proportion (the proportion of $\overline{m}$ allocated to 1; the proportion of x allocated to 1A if Nature chooses 1; and the proportion of $(\overline{m} - x)$ allocated to 2A if Nature chooses 2) has such a Beta distribution. Moreover we assume that the parameters of each of these three distributions are such that the means and variances are given by equation (9) where R^* is the corresponding optimal proportion. Note in passing the usefulness of the property that the variance of R tends to zero as R^* tends to either 0 or 1: this implies that if it is optimal to allocate all or nothing, then subjects do not make a mistake. Details are given in Mathematical Appendix 2.

Let us now briefly return to our pre-experimental simulations, as the properties of these simulations may qualify our results. As we have already noted, the power of our experiment in discriminating between the various types depends upon the actual parameters of the subjects. Clearly if the parameter g is close to 1, the discriminatory power is bound to be low – since with such a parameter the subject is close to being EU and hence there will be little differences between the behaviours of the various types. Instead we report here the results of a simulation with a g value of 0.8 (a moderately non-EU subject), an r value of 2.0 (a moderately risk-averse subject) and an s value of 75 (representing modest, and not untypical, precision). These are, in fact, the parameter values for the best-fitting type of subject 65, whose results we discuss in the next section and report in Table 4. We carried out 100 simulations of a subject with these parameters; on each iteration estimating all 16 combinations − each of the 4 true types combined with each of the 4 estimated types − and calculating for each of these 16 combinations the maximised log-likelihood. Ideally the highest log-likelihood would always be when the estimated type is the same as the true type, which would imply that the estimation would always identify the true type. This was not always the case as we discuss below. However, it is true that the *average* maximised log-likelihood is always greater for the true type, as Table 3.1 shows. Indeed, the highest values are along the diagonal. Table 3.2 reports the standard deviations of the log-likelihoods over the 100 iterations. With perfect discrimination these standard deviations would be small. In fact, they are reasonably so. More importantly, Table 3.3 reports the numbers of times (out of the 100 iterations) where an incorrect type has the highest maximised log-likelihood. It will be seen that incorrect identification occurs essentially when the true type is the myopic type. In this case, the 'best' type is incorrectly identified as naive 17% of the time, as resolute 12% of the time and as sophisticated just 1% of the time. However, a true naive or resolute type is always correctly identified and a true sophisticated type 97% of the time. These results should be keep in mind when interpreting the results that follow.

There is one final point about the estimation that we should mention before we proceed to the results. The above discussion has assumed that the decision variables are continuous. In the experiment, subjects were forced to choose integer values at all stages. This implies that if the optimal decision is, for example, x^*, and the decision variable including error is x, then the value indicated by the subject is the nearest integer to x. This has important consequences, in that when the subject chooses, for example to allocate nothing then the decision variable (including the error) is not necessarily zero but some number less than or equal to 0.5. The estimation program takes this into account.

We proceed subject by subject as we believe that subjects are different and because we want to see how many of each type there are. We fit the above model (RDEU with CRRA utility function and Quiggin weighting function combined with a Beta distribution stochastic specification) to each subject individually, for each of the four types of individuals. We used GAUSS's maximum likelihood procedure to estimate the parameters.[7] We thus get, for each subject and for each type estimates of the parameters r, g and s. We also obtain a maximised log-likelihood for each type. This enables us to identify, for each subject, the best-fitting type.

5 Results

There were 71 subjects in our experiment. The full estimates are available online[8], but it may be helpful to give an example here. This is subject number 65.

Type	estimate of r	estimate of g	estimate of s	log-likelihood
Myopic	1.600	1.000	26.364	-119.180
Naive	1.681	0.940	26.960	-118.426
Resolute	1.966	0.797	72.770	-91.675
Sophisticated	1.630	0.950	21.424	-123.947

[7]The program is at www-users.york.ac.uk/~jdh1/hey and panaccione/mnrs.est

[8]The full output from the estimation program can be found at www-users.york.ac.uk/~jdh1/hey and panaccione/mnrs.out

The log-likelihood is largest for the Resolute type. For this type the estimate of the parameter s, which can be interpreted as the precision of the Beta distribution generating the stochastic component of behaviour, is large – suggesting that errors had a low magnitude. The estimate of the g parameter for the Resolute is 0.797 $-$ which can be shown to be significantly different from 1[9]. Table 4 shows how well the various models fit the data.

We now concentrate on the overall results. These aggregate results are summarised in Table 5. We begin our discussion of this table with the three left-hand columns, headed "All subjects". In the first of these three columns we simply allocate the subjects to the four types on the basis of the highest maximised log-likelihood. On this basis, we classify 39 (55%) as resolute, 16 (23%) as sophisticated, 9 (13%) as naive and 7 (10%) as myopic. Hence it would appear that more than one-half are resolute, around a quarter sophisticated, and just one-fifth naive or myopic. However, one might legitimately want to ask whether the fit is *significantly* better for one of the types; that is, whether the maximised log-likelihood is *significantly* higher for the best-fitting type. To this end, we carried out *Clarke* tests[10]. The results are given in the second and third columns of Table 5. It will be seen that if we require significance at 5% then 35 (49%) of the subjects do not have any type with a log-likelihood significantly better than all the others; and if we require significance at 1% then 49 (69%) cannot be classified. However, of those classified at the 5% level, 61% are resolute, 17% are sophisticated, 17% myopic and 6% are naïve; at the 1% level the corresponding figures are 77%, 14%, 9% and 0%. There is increasing evidence of resolute behaviour amongst those classified.

[9]In order to constrain the parameters to be within the appropriate bounds, the GAUSS program transformed the parameters before estimation. The raw estimated parameters and their standard errors are given in the following table:

Parameter	r	g	s
myopic raw estimates	-1.129	-0.916	-2.710
standard errors of myopic raw estimates	0.226	0.369	0.222
naive raw estimates	-1.033	-1.138	-2.685
standard errors of naive raw estimates	0.136	0.212	0.222
resolute raw estimates	-0.727	-1.811	-1.524
standard errors of resolute raw estimates	0.064	0.089	0.255
sophisticated raw estimates	-1.093	-1.099	-2.944
standard errors of sophisticated raw estimates	0.177	0.290	0.216

[10]See Clarke (2007). An alternative test is the Vuong test, though Clarke shows that his test is more powerful.

At this stage we should remember that people with EU preferences can not be dynamically inconsistent. In a sense we have already carried out an indirect test of whether subjects are EU or not in the above analysis: if an individual is EU then the four types should fit the data approximately equally well, and thus one type should not fit significantly better than the others. However, there is an obvious direct test: whether the estimated parameter g of the weighting function is significantly different from 1[11]. Obviously this is a different test, and we cannot expect the direct and indirect tests necessarily to agree. Nevertheless, we report in the final three columns of Table 5, an analysis of the data restricted to those subjects for whom the g parameter was significantly different from 1 at the 5% level for the best-fitting type. There were 35 (49%) of such subjects. Of these, the highest maximised log-likelihood was for the resolute type for 24 (69%) of the subjects, for the sophisticated type for 7 (20%), for the naive type for 2 (6%) and for the myopic type for 2 (6%). Finally, if we carry out Clarke tests for these 35 subjects with g parameters significantly different from 1, we find that: at 5%, 11 are unclassifiable and of the 24 that are classifiable, 18 (75%) are resolute, 4 (17%) sophisticated, 2 (8%) myopic and 0 naïve; and that at 1%, 19 are unclassifiable and of the 16 that are classifiable, 14 (87%) are resolute, 2 (13%) sophisticated, and none is myopic or naïve. Once again there is increasing evidence of resolute behaviour amongst those classified.

Before concluding we should express a note of caution. Our simulation results showed that genuinely myopic subjects might be mis-classified as naive or resolute. As we have very identified very few as naive but rather a lot as resolute, it may be the case that some of those identified as resolute might in fact be myopic. However, our simulation results show that this happened rather rarely – just 12% of the time according to our simulation results[12]. So there might be slightly fewer resolute and slightly more myopic than our results show. But this does not really change the core conclusion: there seem to be a lot of resolute people out there.

[11]Recall that if $g = 1$ then the RDEU model reduces to EU.

[12]Though the precise figure does depend upon the parameter values.

6 Conclusions

This paper has been concerned with dynamic inconsistency. The issue is important in any economic analysis of behaviour through time. In a sense, this includes all types of economic behaviour, and includes particularly important examples such as saving, investment and pension decisions. Typically economists employ backward induction as their modelling of dynamic behaviour. Alternative methods include the strategy method, wherein the decision-maker is conceived as of considering all possible strategies and choosing the best one. Backward induction can be considered computationally simpler, as the dimensionality of the strategy method can be formidable, and perhaps beyond most decision-makers' capability. Nevertheless the backward induction method does implicitly assume that the decision-maker can project him or herself forward to the final decision node and then backwardly induct from there. Again this is computationally intense.

If the decision-maker is dynamically consistent, then these two methods lead to the same solution. However, for dynamically inconsistent people, the two methods may lead to different solutions. The root cause of this result is that dynamically inconsistent people have different preferences at different points of time, and hence what appears to be optimal depends upon the point from which one is viewing the problem. There seems to be no right or wrong way to decide which is the best way to solve the problem − simply because the preferences of a dynamically inconsistent person change through time. It is exactly as if the individual is schizophrenic. Who is to say which are the true preferences of the individual?

In the context of decision-making under risk, dynamic consistency is equivalent to having Expected Utility preferences, while (potential) dynamic inconsistency is equivalent to having non-EU preferences, for example Rank Dependent Expected Utility (RDEU) preferences.

For potentially dynamically inconsistent individuals, since normative analysis seems impossible, all we can do is carry out a descriptive analysis and see what such people actually do. This is the objective of this paper. We classify subjects in our experiment into different types. These different types have different ways of reacting to their dynamic inconsistency. Following the literature, we considered three types: *naive* (who simply ignore their inconsistency); *resolute* (who somehow impose their first period preferences on their future selves); and *sophisticated* (who plan in the present taking into account what they know they will do in the future). We also add a fourth type, *myopic*, who act as if each period is the last. We note that sophisticated types backwardly induct.

We consider the simplest type of dynamic decision problem − one with just two stages. In principle our method can be applied to more complex problems. We adopt an experimental method pioneered by Loomes (1991) in which subjects are asked to allocate some quantity of money between two risky alternatives. We extend the method to a dynamic decision problem in which the allocation is done in two stages, at each of which there are two risky alternatives. The subject allocates money between two alternatives. Then Nature moves and the subject then has to allocate the money that Nature's move implies between two further risky alternatives. Then Nature moves again and the subject earns the money that Nature's move implies. In our experiment, subjects were asked to repeat this decision-problem on 27 different problems, all with different probabilities for Nature's moves.

The data enables us to see which type subjects are. For each subject we assume RDEU preferences (which reduce to EU as a special case) with a CRRA utility function and with a Quiggin weighting function. Two parameters are involved to describe the preferences: the risk aversion parameter r of the CRRA utility function and the weighting parameter g of the weighting function. If g is 1 then the individual has EU preferences and thus is not dynamically inconsistent. In addition we need to estimate the precision of the probability distribution describing the noisiness of their implementation of their optimal strategy. We estimate each type separately and see which type fits best − that is, which describes best the decisions of the subject.

If we start with all subjects, we see that the resolute type is the best for 55% of our 71 subjects, with 23% sophisticated, 13% myopic and 10% naive. If we restrict attention to those subjects for whom the best-fitting model is *significantly* better than the others at the 5% (1%) level , using the Clarke test, we find that of the 36 (22) for which this is true, 61% (77%) are resolute, 17% (14%) are sophisticated, 17% (9%) are myopic and 6% (0%) are naive. If we further confine ourselves to those 35 subjects who are significantly not EU (that is, those for whom the g parameter is significantly different from 1 at the 5% level), overall we observe 69% resolute, 20% sophisticated, 6% naive and 6% myopic.

So, the bottom line is that the majority of our dynamically inconsistent subjects are resolute, a significant minority are sophisticated; and rather few are naive or myopic. The fact that we have very few naive or myopic is good news for economic theory and policy. We are, however, rather surprised by the preponderance of resolute types. It could be argued that our experimental software is such that it encourages resolute behaviour, but we see no reason why that is so. We did not ask subjects to state, at the first stage, what amounts they wished to allocate to each of the four possible outcomes (1A, 1B, 2A and 2B), and indeed it was rather the opposite. Perhaps the statement of the probabilities in the form of Figure 1 encouraged them to think about these final outcomes, but note that the software did not tell them the probabilities of 1A, 1B, 2A and 2B. On the contrary, the problem was very much stated in a sequential way. Indeed, it could be argued that the software actually discouraged resolute play.

We would like to extend the experiment in two ways. The first is straightforward: to run the experiment with a random sample from some population, since it might be argued that student subjects are not representative of the population as a whole[13]. Second, we would like to run the same experiment with more than two stages, and perhaps with endowments of money every period. In this way, we would get closer to a savings problem. The problem then is in calculating the optimal strategies for each type of subject. Even with just two periods it is computationally difficult. However, one needs to calculate the optimal strategies in order to distinguish between the types. With just two stages, we think that we have been successful – and have a conclusion that is rather surprising. If the majority of people are resolute, then state intervention may be less necessary. That is, of course, if the first period preferences are the true ones. But who knows?

[13]Though it might be argued that non-students are likely to be even more resolute.

7 References

Choi, S., Fisman, S., Gale, D. and Kariv, S., (2007), "Consistency and Heterogeneity of Individual Behavior under Uncertainty", *American Economic Review*, 97, 1920-1938.

Clarke , K. A. (2007), "A Simple Distribution-Free Test for Non-nested Model Selection", *Political Analysis*, 15, 347-363.

Hey, J. D. and Lotito, G. (2009), "Naïve, Resolute or Sophisticated? A Study of Dynamic Decision Making", *Journal of Risk and Uncertainty*, 38, 1-25.

Loomes, G (1991), "Evidence of a new violation of the independence axiom", *Journal of Risk and Uncertainty*, 4, 91-108.

Machina, M.J. (1989), "Dynamic Consistency and Non-Expected Utility Models of Choice Under Uncertainty", *Journal of Economic Literature*, 27, 1622-1668.

McClennen, E.F. (1990), *Rationality and Dynamic Choice: Foundational Explorations*, Cambridge: Cambridge University Press.

Quiggin, J. (1982), "A Theory of Anticipated Utility", *Journal of Economic Behavior and Organization*, 3, 323-343.

Tversky, A. and Kahneman, D. (1992), "Advances in Prospect Theory: Cumulative Representation of Uncertainty", *Journal of Risk and Uncertainty*, 5, 297-323.

Wakker, P. (2008), "Explaining the Characteristics of the Power (CRRA) Utility Family", *Health Economics*, 17, 1329-1344.

Figure 1: The opening screen of a problem

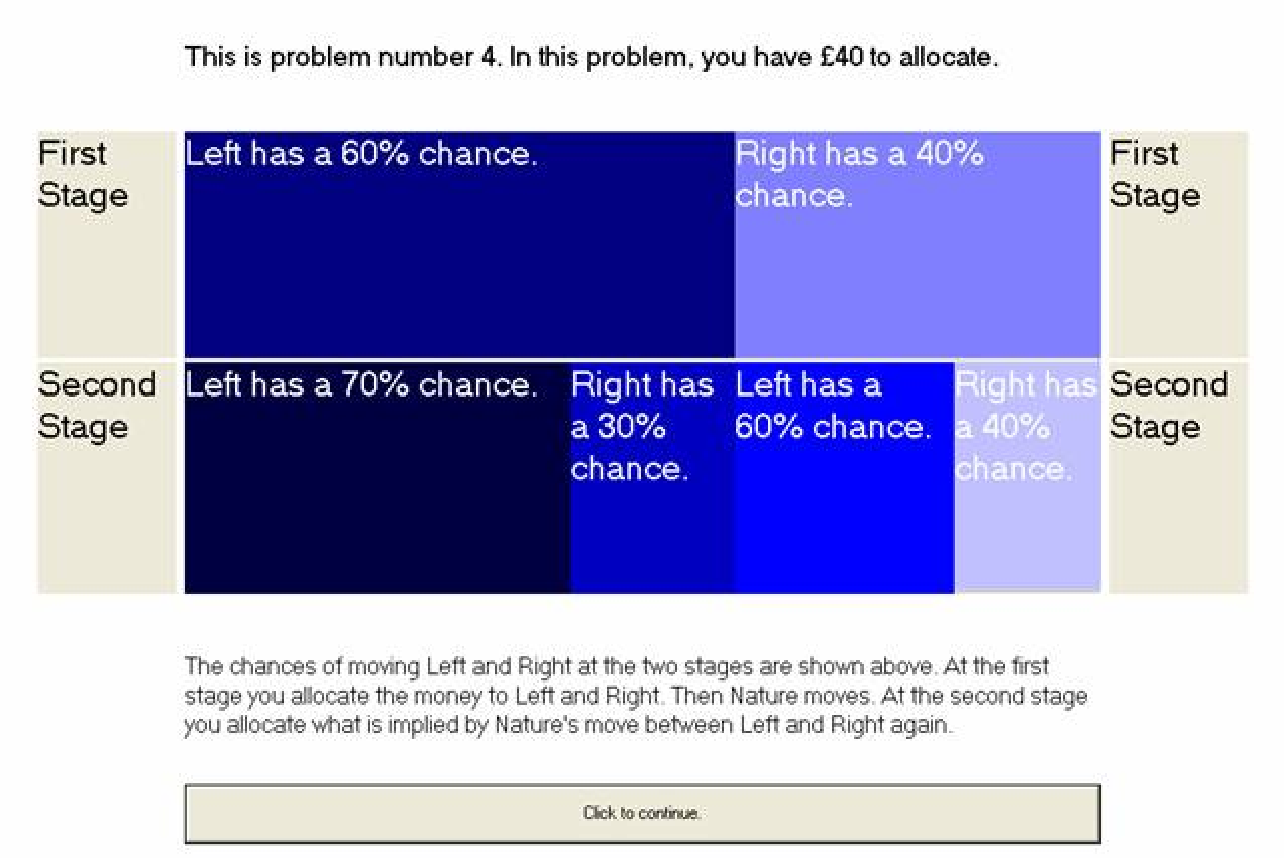

Figure 2: The first-stage decision

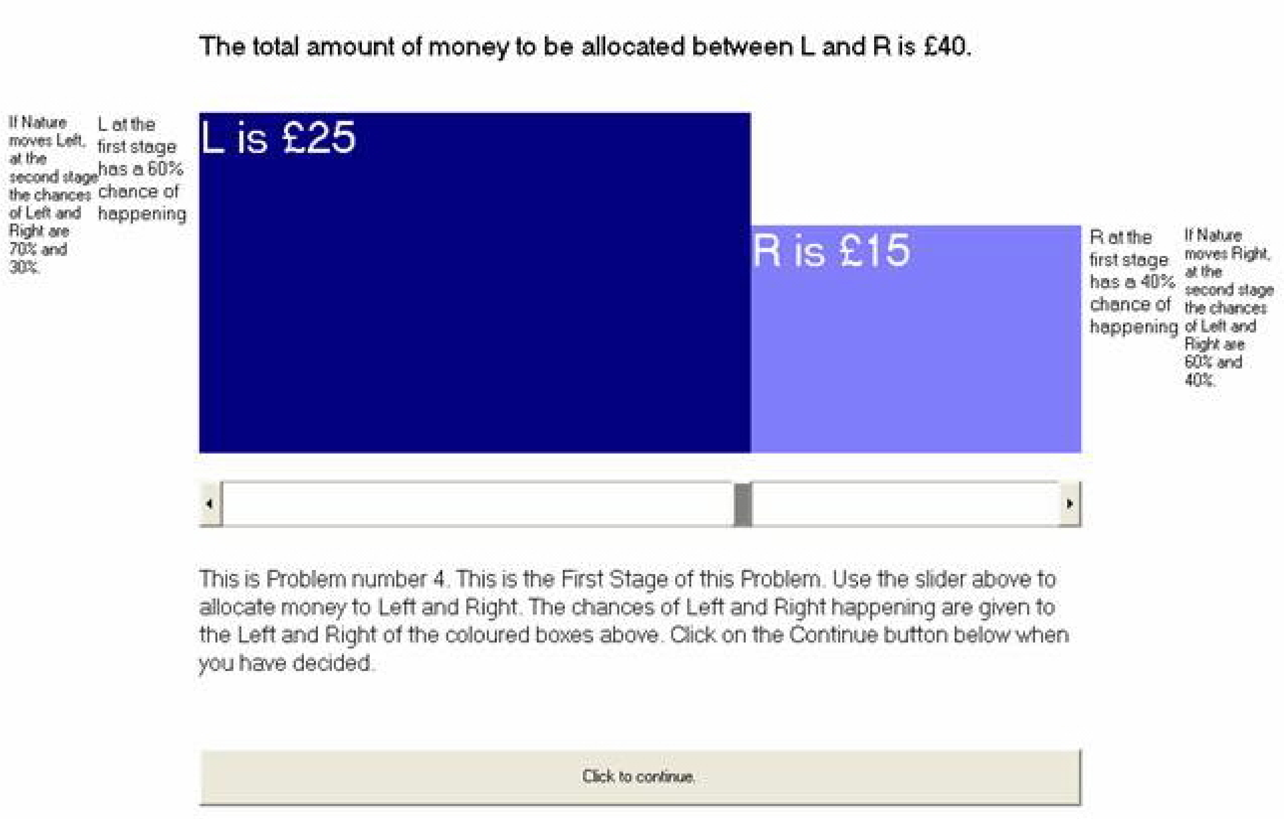

Figure 3: The second-stage decision

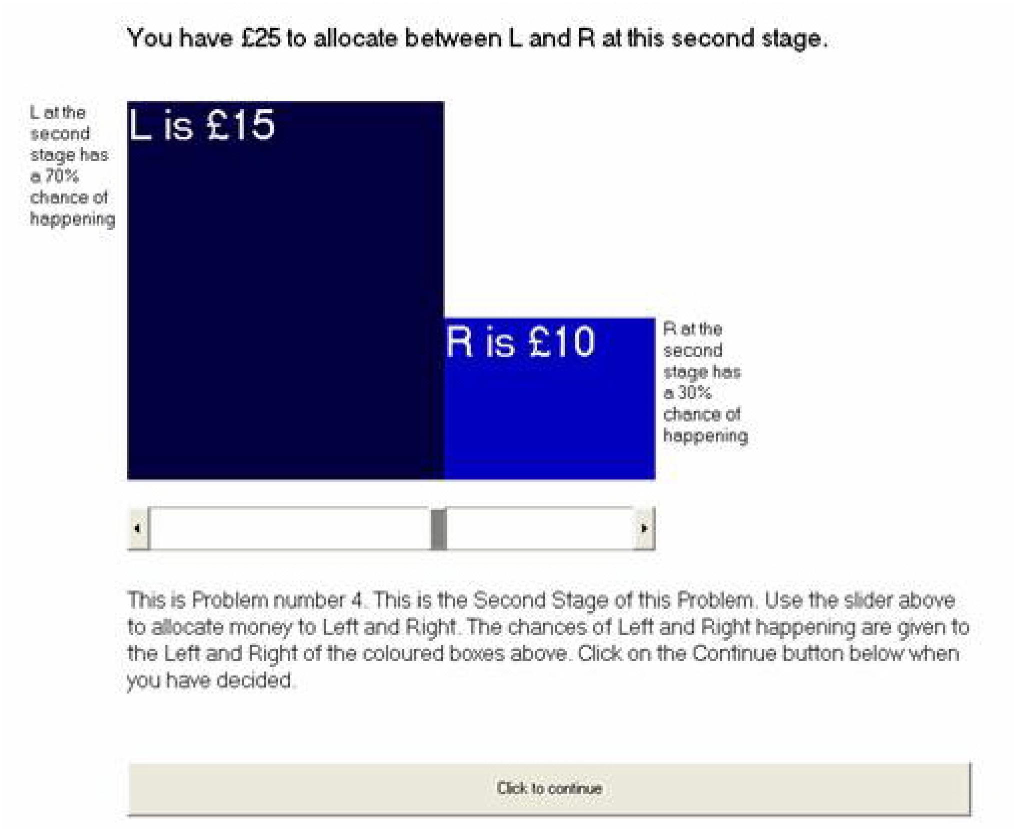

Figure 4: The weighting function behind Table 2 ($g = 0.6$)

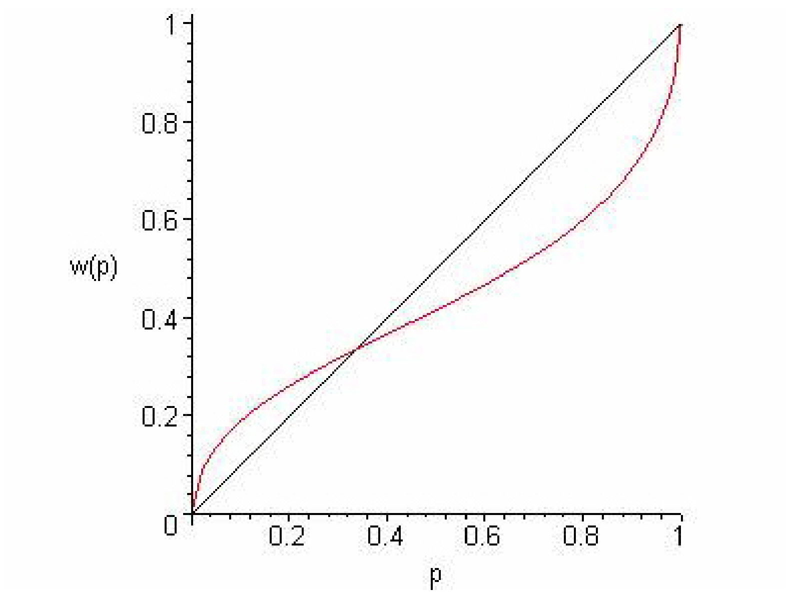

Table 1: the problem set

Probability of Left at first stage: p

Probability of Left at second stage if Nature moves Left at first stage: p_1

Probability of Left at second stage if Nature moves Right at first stage: p_2

pn	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
p	.6	.7	.8	.6	.7	.8	.6	.7	.8	.6	.7	.8	.6	.7	.8	.6	.7	.8	.6	.7	.8	.6	.7	.8	.6	.7	.8
p_1	.6	.6	.6	.7	.7	.7	.8	.8	.8	.6	.6	.6	.7	.7	.7	.8	.8	.8	.6	.6	.6	.7	.7	.7	.8	.8	.8
p_2	.6	.6	.6	.6	.6	.6	.6	.6	.6	.7	.7	.7	.7	.7	.7	.7	.7	.7	.8	.8	.8	.8	.8	.8	.8	.8	.8

Table 2: An example of choices of the different types

Parameters assumed: $r = 2.0$ and $g = 0.6$

		Probabilities			Myopic			Naive			Resolute			Sophisticated		
pn	$\overline{m}$	p	p_1	p_2	x	y	z	x	y	z	x	y	z	x	y	z
1	40	.60	.60	.60	20.00	10.00	10.00	25.55	12.78	7.22	25.55	18.33	7.22	20.00	10.00	10.00
2	40	.70	.60	.60	22.09	11.04	8.96	27.25	13.63	6.37	27.25	20.88	6.37	22.09	11.04	8.96
3	40	.80	.60	.60	27.60	13.80	6.20	28.88	14.44	5.56	28.88	23.32	5.56	27.60	13.80	6.20
4	40	.60	.70	.60	20.00	11.04	10.00	27.25	15.05	6.37	27.25	20.88	6.37	21.10	11.65	9.45
5	40	.70	.70	.60	22.09	12.19	8.96	29.14	16.09	5.43	29.14	23.72	5.43	22.63	12.49	8.69
6	40	.80	.70	.60	27.60	15.24	6.20	30.95	17.09	4.52	30.95	26.43	4.52	28.01	15.46	6.00
7	40	.60	.80	.60	20.00	13.80	10.00	28.88	19.93	5.56	28.88	23.32	5.56	24.69	17.04	7.65
8	40	.70	.80	.60	22.09	15.24	8.96	30.95	21.36	4.52	30.95	26.43	4.52	24.69	17.04	7.65
9	40	.80	.80	.60	27.60	19.04	6.20	32.92	22.71	3.54	32.92	29.38	3.54	29.23	20.17	5.38
10	40	.60	.60	.70	20.00	10.00	11.04	25.55	12.78	7.98	25.55	18.33	7.22	18.90	9.45	11.65
11	40	.70	.60	.70	22.09	11.04	9.89	27.25	13.63	7.04	27.25	20.88	6.37	18.90	9.45	11.65
12	40	.80	.60	.70	27.60	13.80	6.85	28.88	14.44	6.14	28.88	23.32	5.56	27.78	13.89	6.75
13	40	.60	.70	.70	20.00	11.04	11.04	27.25	15.05	7.04	27.25	20.88	6.37	22.09	12.19	9.89
14	40	.70	.70	.70	22.09	12.19	9.89	29.14	16.09	5.99	29.14	23.72	5.43	22.86	12.62	9.46
15	40	.80	.70	.70	27.60	15.24	6.85	30.95	17.09	5.00	30.95	26.43	4.52	28.18	15.56	6.53
16	40	.60	.80	.70	20.00	13.80	11.04	28.88	19.93	6.14	28.88	23.32	5.56	23.64	16.31	9.03
17	40	.70	.80	.70	22.09	15.24	9.89	30.95	21.36	5.00	30.95	26.43	4.52	25.62	17.68	7.94
18	40	.80	.80	.70	27.60	19.04	6.85	32.92	22.71	3.91	32.92	29.38	3.54	29.39	20.28	5.86
19	40	.60	.60	.80	20.00	10.00	13.80	25.55	12.78	9.97	25.55	18.33	7.22	15.31	7.65	17.04
20	40	.70	.60	.80	22.09	11.04	12.36	27.25	13.63	8.80	27.25	20.88	6.37	18.49	9.25	14.84
21	40	.80	.60	.80	27.60	13.80	8.56	29.02	14.51	7.58	29.02	23.32	5.70	27.80	13.90	8.42
22	40	.60	.70	.80	20.00	11.04	13.80	27.25	15.05	8.80	27.25	20.88	6.37	16.36	9.03	16.31
23	40	.70	.70	.80	22.09	12.19	12.36	29.14	16.09	7.49	29.14	23.72	5.43	24.26	13.39	10.86
24	40	.80	.70	.80	27.60	15.24	8.56	30.95	17.09	6.24	30.95	26.43	4.52	28.20	15.57	8.14
25	40	.60	.80	.80	20.00	13.80	13.80	28.88	19.93	7.67	28.88	23.32	5.56	25.49	17.59	10.01
26	40	.70	.80	.80	22.09	15.24	12.36	30.95	21.36	6.24	30.95	26.43	4.52	27.60	19.04	8.56
27	40	.80	.80	.80	27.60	19.04	8.56	32.92	22.71	4.89	32.92	29.38	3.54	29.41	20.29	7.31

Table 3: Some simulation results

In this table, the true parameters are $r = 2.0$, $g = 0.8$ and $s = 75$.

100 iterations were carried out.

Table 3.1: Average maximised log-likelihoods

	Estimated type			
True type	Myopic	Naive	Resolute	Sophisticated
Myopic	-1176.652	-1207.239	-1206.721	-1399.328
Naive	-1483.576	-1090.090	-1464.342	-1496.232
Resolute	-1625.063	-1525.753	-1127.111	-1676.821
Sophisticated	-1515.027	-1383.583	-1484.399	-1203.517

Table 3.2: Standard deviations of maximised log-likelihoods

	Estimated type			
True type	Myopic	Naive	Resolute	Sophisticated
Myopic	104.555	107.222	102.854	100.557
Naive	91.467	117.712	95.693	175.103
Resolute	56.518	65.952	101.624	53.142
Sophisticated	91.404	101.615	94.493	209.33

Table 3.3: Number of times that an incorrect type is identified

	Estimated type			
True type	Myopic	Naive	Resolute	Sophisticated
Myopic	0	17	12	1
Naive	0	0	0	0
Resolute	0	0	0	0
Sophisticated	0	3	0	0

Table 4: Fitted and actual decisions for subject 65

	Myopic			Naive			Resolute			Sophisticated			Actual[14]		
pn	*x*	*y*	*z*	*x*	*y*	*z*	*x*	*y*	*z*	*x*	*y*	*z*	*x*	*y*	*z*
1	26.27	17.25	9.02	26.49	17.29	8.81	27.01	20.51	6.50	26.16	17.01	9.00	27	*	7
2	31.80	20.88	5.38	31.74	20.71	5.39	30.97	24.18	4.52	31.55	20.52	5.49	32	27	*
3	36.07	23.69	2.58	35.98	23.48	2.62	35.37	26.70	2.32	35.82	23.30	2.72	36	*	2
4	26.27	20.88	9.02	27.25	21.54	8.32	29.52	24.27	5.24	27.00	21.24	8.45	25	21	*
5	31.80	25.28	5.38	32.17	25.42	5.11	32.08	28.12	3.96	31.95	25.13	5.23	36	31	*
6	36.07	28.68	2.58	36.21	28.62	2.47	35.77	31.09	2.11	36.04	28.34	2.58	35	32	*
7	26.27	23.69	9.02	28.72	25.79	7.36	31.40	27.59	4.78	28.29	25.30	7.62	27	24	*
8	31.80	28.68	5.38	33.06	29.69	4.53	34.23	31.35	2.88	32.76	29.30	4.71	34	32	*
9	36.07	32.53	2.58	36.54	32.81	2.26	36.27	34.29	1.87	36.35	32.51	2.37	38	35	*
10	26.27	17.25	10.92	26.42	17.24	10.73	27.61	20.46	7.14	21.89	14.24	14.24	28	23	*
11	31.80	20.88	6.52	31.38	20.48	6.81	30.82	24.07	5.92	31.21	20.30	6.91	32	*	5
12	36.07	23.69	3.12	35.78	23.35	3.33	35.27	26.62	3.08	35.63	23.17	3.44	36	31	*
13	26.27	20.88	10.92	26.95	21.30	10.31	28.70	24.18	6.79	26.69	20.99	10.46	28	23	*
14	31.80	25.28	6.52	32.11	25.37	6.24	32.53	28.06	4.47	31.62	24.87	6.59	33	28	*
15	36.07	28.68	3.12	36.03	28.47	3.14	35.68	31.01	2.82	35.86	28.20	3.26	35	30	*
16	26.27	23.69	10.92	27.96	25.10	9.52	30.36	27.31	6.59	27.39	24.50	9.91	32	30	*
17	31.80	28.68	6.52	32.86	29.50	5.65	33.90	31.31	3.52	32.55	29.11	5.86	33	*	5
18	36.07	32.53	3.12	36.37	32.66	2.87	36.45	34.23	2.22	36.18	32.36	3.00	35	31	*
19	26.27	17.25	12.38	25.82	16.85	12.73	28.04	20.00	9.07	23.16	15.06	15.06	28	*	10
20	31.80	20.88	7.39	30.72	20.05	8.33	30.87	23.52	7.35	23.16	15.06	15.06	32	*	6
21	36.07	23.69	3.54	35.41	23.10	4.12	34.82	26.29	4.28	35.27	22.94	4.23	35	25	*
22	26.27	20.88	12.38	26.32	20.79	12.29	28.42	23.74	8.86	21.28	16.74	16.74	28	24	*
23	31.80	25.28	7.39	31.95	25.25	7.23	32.99	27.64	5.35	31.65	24.89	7.46	33	*	5
24	36.07	28.68	3.54	35.67	28.19	3.89	35.27	30.65	3.91	35.52	27.93	4.01	37	33	*
25	26.27	23.69	12.38	27.02	24.26	11.66	29.01	26.70	8.67	26.74	23.92	11.86	30	28	*
26	31.80	28.68	7.39	32.41	29.10	6.82	33.83	31.01	4.67	32.10	28.71	7.06	30	*	7
27	36.07	32.53	3.54	36.25	32.56	3.36	36.61	33.96	2.66	35.87	32.08	3.70	35	32	*

[14] An asterisk indicates a missing observation (because of Nature's move).

Table 5: A summary of the main results

	All subjects			Subjects for whom g parameter is significantly different from 1 at 5% for the best-fitting type		
Type	Number of subjects with highest log-likelihood	Number of subjects with significance on Clarke Test at 5%	Number of subjects with significance on Clarke Test at 1%	Overall	Number of subjects with significance on Clarke Test at 5%	Number of subjects with significance on Clarke Test at 1%
Myopic	7	6	2	2	2	0
Naive	9	2	0	2	0	0
Resolute	39	22	17	24	18	14
Sophisticated	16	6	3	7	4	2
None of these	0	35	49	36	47	55
Total	71	71	71	71	71	71

Supplementary material to:

Myopic, Naive, Resolute or Sophisticated?

A study of how people take dynamic decisions

John D. Hey

University of York, UK, and LUISS, Rome, Italy

Luca Panaccione

University of Tor Vergata and LUISS, Rome, Italy

July 2009

Non-Mathematical Appendix: The experimental instructions

(We omit the screen shots as they are the same as in the paper. References to figures below are to the figures in the main text of this article.)

Preamble

Welcome to this experiment. It is an experiment on the economics of dynamic decision making under risk. The Ministry for Education, University and Research of Italy (MIUR) has provided the funds to finance this research. Thank you for taking part. Please read these instructions carefully. It is important that you do so, as your payment for taking part in this experiment will depend upon the decisions that you take. The payment will be made, in cash, at the end of the experiment. The payment will consist of whatever money you earn as a result of the decisions you make during the experiment. You will be asked to sign a receipt for the payment, and to acknowledge that you participated voluntarily in the experiment. The results of the experiment will be used for the purpose of academic research and will be published in such a way that your anonymity will be preserved.

The Experiment

In the experiment you will be presented with 27 dynamic decision problems, all of the same form. Each problem has two stages. At the beginning of each of these problems you will be given an allocation of £40. At the first stage you will be asked to allocate the money between two options, which, because of the way that they are presented on the computer screen, will be called Left and Right. When you have made the allocation, a random device, which we call Nature, will determine whether you move Left or Right. The chances of each will be told to you before you make your allocation. This is the first stage of the problem. At the second stage, you will have a similar decision: to allocate the money that is implied by your first stage decision again between Left and Right. The chances of each will be told to you before you make your first allocation.

Once again, when you have made the allocation, the random device which we call Nature will determine whether you move Left or Right. The amount of money that you allocated to the realised outcome will be your payoff for that particular decision problem. At the end of all the decision problems, one will be chosen at random, and your payoff for that particular problem will be your payment for the experiment.

An Example

Look at Figure 1 below. This a screen shot of a particular decision problem. Like in every problem the amount of money with which you are initially allocated is £40. You will see that in this problem there is a 60% chance that Nature will choose Left and a 40% chance that she will choose Right after your first decision. What happens at the second decision node depends upon the move that Nature made at the first decision node. In this example, if Nature moves Left after your first decision, then there is a 70% chance that Nature will move Left and a 30% chance that she will move Right after your second decision. If instead Nature moves Right after your first decision, then there is a 60% chance that Nature will move Left and a 40% chance that she will move Right after your second decision. Note that these probabilities change from problem to problem.

Figure 1 here.

As in every problem you initially have £40 to allocate. When you click on "Click to Continue" the computer shows a random allocation. In the picture the allocation of the £40 is £25 to Left and £15 to Right. You must decide your preferred allocation by moving the slider under the boxes.

Figure 2 here.

When you have decided on and shown your preferred allocation, you should click on 'Click to Continue'. At this point the random move by Nature will be played out: the computer generates a series of random numbers, the last of which determines the move by Nature, and the stated chances are respected. Suppose that the outcome of this random process is that Nature chooses Left. Then, because £25 in this example has been allocated to Left, this is the amount of money which you are asked to allocate at the second stage. The Figure 3 below illustrates. It is restated here that there is a 70% chance that Nature will move Left and a 30% chance that she will move Right after this second decision (these are obviously the same chances that you were told when you started this problem). Once again, when the screen opens the allocation is random and you must decide and show your preferred allocation by moving the slider under the boxes. In the figure the allocation of the £25 is £15 to Left and £10 to Right.

Figure 3 here.

When you have shown your preferred allocation, you should click on 'Click to continue'. Once again, the random move by Nature will be played out: the computer generates a series of random numbers, the last of which determines the move by Nature, and the stated chances are respected. Suppose that the outcome of this random process is that Nature chooses Left. Then your payoff for that decision problem would be £15. Your payment for the experiment will be a randomly chosen one of the payoffs on all the decision problems.

Nature

Here we give some more detail about Nature and the random process that the computer uses. Nature is our word for a random process. Nature operates completely independently of your decisions. When, for example, there is a 60% chance of Nature moving Left and a 40% chance of Nature moving Right, then what Nature does depends only on these chances and not on your decision.

Nature is implemented by the computer in the following way: the computer has a routine for generating a sequence of random numbers that are equally likely to be anywhere between 0 and 1. The program generates a sequence of 10 of these and the last of these determines Nature's move: in this 60% /40% example, if the last number is less than 0.6 then Nature's move is Left and if the last number is greater than 0.6, then Nature's move is Right. In general if there is a p% chance of Left and a (100-p)% chance of Right, then Nature moves Left if the random number is less than p/100 and moves Right if the random number is greater than (100-p)/100.

Implementation

If anything is unclear after reading the Instructions, you should ask for clarification from one of the experimenters. Then you should turn to the computer. When you click on 'Click to Start' a PowerPoint presentation, which goes at a pre-determined speed, will be shown. After that, or indeed at any stage of the experiment, you can ask clarification from the experimenters. When you are ready you can start the experiment. You will see that the software forces you to wait a certain amount of time before you can confirm any decision. This is to ensure that you always state your preferred allocation. But of course you should do anyhow, as your payment depends upon your decisions. At the end of the experiment, you should call over one of the experimenters, and in front of him or her, you will randomly determine the decision problem which will determine your payoff. The experimenter will pay you in cash after you have completed a brief questionnaire and signed a receipt for the payment. You will then be free to go. We estimate that the whole experiment will last some 90 minutes.

Thank you for your participation.

Mathematical Appendix 1: The optimal strategies for the various types

Preamble

Let us consider the following utility function

$$u(x) = \begin{cases} \frac{x^{1-\frac{1}{r}}-1}{1-\frac{1}{r}} & \text{for} \quad r \neq 1 \\ \ln(x) & \text{for} \quad r = 1 \end{cases},$$

which implies that marginal utility is given by

$$u'(x) = x^{-1/r}.$$

Strict concavity requires $0 < r < \infty$. The RDEU function is

$$U = v_1 u(x_1) + v_2 u(x_2) + v_3 u(x_3) + v_4 u(x_4),$$

where x_1, x_2, x_3 and x_4 are respectively the outcomes for options 1A, 1B, 2A, and 2B. The weights attached to the different outcomes depend on actual probabilities adjusted using the weighting function w with parameter g.

As it is known, these weights depend on the ranking of the outcomes. Therefore, it is necessary to consider all possible 24 different rankings of the final outcomes x_k for $k = 1, ..., 4$. Each of these will identify a different objective function and a different admissible range for the solution to the maximization problem.

To make this point clear, we consider a simple example. Assume that $x_1 > x_2 > x_3 > x_4$. In this case, the weights are as follows

$$\begin{aligned} v_1 &= w(q_1) \\ v_2 &= w(q_1 + q_2) - w(q_1) \\ v_3 &= w(q_1 + q_2 + q_3) - w(q_1 + q_2) \\ v_4 &= 1 - w(q_1 + q_2 + q_3), \end{aligned}$$

where q_i is the compound probability of outcome i. If the ranking contains an equality, e.g. $x_1 = x_2 > x_3 > x_4$, then – letting $x_1 = x_2 = x_{21}$ – the weights are as follows

$$\begin{aligned} v_{21} &= w(q_1 + q_2) \\ v_3 &= w(q_1 + q_2 + q_3) - w(q_1 + q_2) \\ v_4 &= 1 - w(q_1 + q_2 + q_3). \end{aligned}$$

Therefore for each possible ranking of the final outcomes, there is a different set of weights, hence a different RDEU function.

Resolute

In this section we consider the choice problem for a resolute decision maker. Recall that there are $i = 1, ..., 24$ possible rankings of the final outcomes that must be taken into account. The maximization problem in this case should be written as follows

$$\max_{x_k} \quad U \quad \text{s.t.} \quad \sum_{k=1}^{4} x_k = \overline{m} \quad \text{and} \quad \{x_k\}_{k=1}^{4} \text{ respect ranking } i.$$

Since each ranking defines an admissible range for the choice variables that can be described by a system of inequalities, we should consider a different constrained maximization problem for each ranking as follows. Consider for example the following ranking

$$x_1 \geq x_2 \geq x_3 \geq x_4. \tag{1}$$

In this case, the maximization problem can be written as[1]

$$\max_{x_k} \quad v_1 u(x_1) + v_2 u(x_2) + v_3 u(x_3) + v_4 u(x_4) \tag{2a}$$
$$\text{s.t.} \quad x_2 - x_1 \leqslant 0 \tag{2b}$$
$$x_3 - x_2 \leqslant 0 \tag{2c}$$
$$x_4 - x_3 \leqslant 0 \tag{2d}$$
$$x_1 + x_2 + x_3 + x_4 = \overline{m} \tag{2e}$$

[1]We neglect the non-negativity constraints, as they are not binding for the utility function.we consider.

Let γ be the multiplier for constraint (2e) and $\lambda_1 \geq 0, \lambda_2 \geq 0$ and $\lambda_3 \geq 0$ be the multipliers respectively for constraints (2b), (2c) and (2d). The first order conditions for this problem are given by

$$
\begin{aligned}
v_1 u'(x_1) + \lambda_1 - \gamma &= 0 && \text{(3a)}\\
v_2 u'(x_2) - \lambda_1 + \lambda_2 - \gamma &= 0 && \text{(3b)}\\
v_3 u'(x_3) - \lambda_2 + \lambda_3 - \gamma &= 0 && \text{(3c)}\\
v_4 u'(x_4) - \lambda_3 - \gamma &= 0 && \text{(3d)}\\
\lambda_1 (x_2 - x_1) &= 0 && \text{(3e)}\\
\lambda_2 (x_3 - x_2) &= 0 && \text{(3f)}\\
\lambda_3 (x_4 - x_3) &= 0 && \text{(3g)}\\
x_1 + x_2 + x_3 + x_4 - \overline{m} &= 0 && \text{(3h)}
\end{aligned}
$$

Depending on which inequality constraint is binding at the solution, we have different admissible cases which varies with the parameters. While this procedure allows to completely characterize the solution set for each possible ranking and therefore to identify the global optimum by finding the highest value for the indirect expected utility, it cannot be easily coded in a program to run the necessary simulations.

Therefore, we choose to tackle the problem in a different way, to be explained in what follows. For concreteness, assume the ranking is given by (1), as all the other rankings can be treated in the same way. The first step is to realize that for each possible ranking of the final outcomes there are four possible configurations that can arise: no equality among outcomes, equality among two outcomes (with two sub-cases to be described in what follows), equality among three outcomes and equality among all outcomes.

Let us refer to these cases as case $1, 2, 3, 4$ and 5 respectively. In case 1, none of the inequality constraints (2b)$-$(2d) is binding and therefore $\lambda_j = 0$ for $j = 1, 2, 3$. In case 2, only one inequality constraint is binding and therefore $\lambda_j > 0$ for some j and $\lambda_i = 0$ for $i \neq j$. In case 3 and 4, two inequality constraints are binding and therefore $\lambda_i = 0$ for some $i = 1, 2, 3$ and $\lambda_j > 0$ for $j \neq i$, the difference between the two sub-cases being whether the binding constraints alternate or not. Finally in case 5, all constraints are binding and therefore the solution is $x_k = \overline{m}/4$ for $k = 1, ..., 4$.

In each of the remaining cases, the solution can be explicitly computed using (3a)−(3h). If case 1 holds, the solution is

$$x_k = \left(\frac{v_k^r}{\sum_{l=1}^{4} v_k^r} \right) \overline{m}. \tag{4}$$

If case 2 holds (suppose again for concreteness that $x_1 = x_2$, the other cases being symmetric), the solution is

$$x_1 = x_2 = \left(\frac{\left(\frac{v_1+v_2}{2}\right)^r}{2\left(\frac{v_1+v_2}{2}\right)^r + v_3^r + v_4^r} \right) \overline{m}, \tag{5a}$$

$$x_k = \left(\frac{v_k^r}{2\left(\frac{v_1+v_2}{2}\right)^r + v_3^r + v_4^r} \right) \overline{m} \quad \text{for} \quad k = 3, 4. \tag{5b}$$

If case 3 holds (suppose again for concreteness that $x_1 = x_2 = x_3$, the other case being symmetric), the solution is

$$x_1 = x_2 = x_3 = \left(\frac{\left(\frac{v_1+v_2+v_3}{3}\right)^r}{3\left(\frac{v_1+v_2+v_3}{3}\right)^r + v_4^r} \right) \overline{m}, \tag{6a}$$

$$x_4 = \left(\frac{v_4^r}{3\left(\frac{v_1+v_2+v_3}{3}\right)^r + v_4^r} \right) \overline{m}. \tag{6b}$$

Finally if case 4 holds, the solution is

$$x_1 = x_2 = \left(\frac{\left(\frac{v_1+v_2}{2}\right)^r}{2\left(\frac{v_1+v_2}{2}\right)^r + 2\left(\frac{v_3+v_4}{2}\right)^r} \right) \overline{m}, \tag{7a}$$

$$x_3 = x_4 = \left(\frac{\left(\frac{v_3+v_4}{2}\right)^r}{2\left(\frac{v_1+v_2}{2}\right)^r + 2\left(\frac{v_3+v_4}{2}\right)^r} \right) \overline{m}. \tag{7b}$$

The second step in the procedure requires to keep track of all possible configurations of cases 2, 3 and 4 that can arise. Depending on the various rankings considered, case 2 can arise in six possible configurations

$$x_1 = x_2 \quad x_1 = x_3 \quad x_1 = x_4$$
$$x_2 = x_3 \quad x_2 = x_4 \quad x_3 = x_4$$

On the other hand, case 3 can arise in four possible configurations

$$x_1 = x_2 = x_3 \quad x_1 = x_2 = x_4$$
$$x_1 = x_3 = x_4 \quad x_2 = x_3 = x_4$$

Finally, case 4 can arise in three possible configurations

$$x_1 = x_2 \quad \text{and} \quad x_3 = x_4$$
$$x_1 = x_3 \quad \text{and} \quad x_2 = x_4$$
$$x_1 = x_4 \quad \text{and} \quad x_2 = x_3$$

Therefore, for each ranking i we can compute the solution set $\{x_k\}_{k=1}^4$ for each possible case and verify if it indeed satisfied the given ranking. If it does, it is considered an admissible solution set, otherwise it is discarded. Finally, the third step of the procedure consists in computing the utility level corresponding to each admissible solution set and then picking as optimal choice the one that gives the highest value.

Sophisticated

In this section we consider the choice problem for a sophisticated decision maker. The individual who acts in a sophisticated way solves the problem in two steps. First, given an allocation m_1 and m_2 such that $m_1 + m_2 = \overline{m}$, he or she solves the maximization problem at the final decision nodes, that is those at the second stage.[2] Denote the solution by

$$(x_1^*, x_2^*) = (x_1(m_1), x_2(m_1)) \quad \text{and} \quad (x_3^*, x_4^*) = (x_3(m_2), x_4(m_2)),$$

as it depends on m_1 and m_2. In the second stage, he or she solves for the optimal value of m_1 and m_2, taking into account the optimal choices in the final node.

[2] In this Appendix, we replace the notation $\overline{m}_1$ and $\overline{m}_2$ with m_1 and m_2 as no confusion should arise.

Let w_j denote the weight for outcome j. Given (m_1, m_2), the sophisticated individual solves

$$\max_{x_1, x_2} \; w_1 u(x_1) + w_2 u(x_2) \quad \text{s.t.} \quad x_2 - x_1 \leqslant 0 \quad \text{and} \quad x_1 + x_2 = m_1 \tag{8}$$

$$\max_{x_3, x_4} \; w_3 u(x_3) + w_4 u(x_4) \quad \text{s.t.} \quad x_4 - x_3 \leqslant 0 \quad \text{and} \quad x_3 + x_4 = m_2 \tag{9}$$

Let (γ_1, μ_1) and (γ_2, μ_2) be the multipliers for the first and second constraint respectively in problem (8) and (9), and consider the first order conditions for problem (8), the others being analogous:

$$\begin{aligned}
w_1 u'(x_1) + \gamma_1 - \mu_1 &= 0 \\
w_2 u'(x_2) - \gamma_1 - \mu_1 &= 0 \\
\gamma_1 (x_2 - x_1) &= 0 \\
x_1 + x_2 - m_1 &= 0
\end{aligned}$$

To compute the solution of the first stage problem, which in turn will be used in the second stage problem, we have to consider all possible combinations of equality and inequalities. Using the same classification as in the previous section, we to consider the single configuration for both case 1 and case 5, six configurations for case 2, four configurations for case 3 and finally three configurations for case 4.

Consider first case 1, so that $x_1 > x_2$ and $x_3 > x_4$. As in this case $\gamma_1 = 0$, the solution to problem (8) is

$$x_i^* = x_i(m_1) = \frac{w_i^r m_1}{w_1^r + w_2^r} \quad \text{for} \quad i = 1, 2. \tag{10}$$

By symmetry, the solution to problem (9) is

$$x_i^* = x_i(m_2) = \frac{w_i^r m_2}{w_3^r + w_4^r} \quad \text{for} \quad i = 3, 4. \tag{11}$$

In the second stage, given x_i^* for $i = 1,2,3,4$, the sophisticated individual solves

$$\max_{m_1,m_2} \quad v_1 u(x_1^*) + v_2 u(x_2^*) + v_3 u(x_3^*) + v_4 u(x_4^*) \tag{12}$$

$$\text{s.t.} \quad m_1 + m_2 = \overline{m}, \tag{13}$$

where the weights depend on the actual ranking. The first order conditions for this problem are

$$v_1 u'(x_1^*)\frac{\mathrm{d}x_1}{\mathrm{d}m_1} + v_2 u'(x_2^*)\frac{\mathrm{d}x_2}{\mathrm{d}m_1} - \gamma = 0 \tag{14}$$

$$v_3 u'(x_3^*)\frac{\mathrm{d}x_3}{\mathrm{d}m_2} + v_4 u'(x_4^*)\frac{\mathrm{d}x_4}{\mathrm{d}m_2} - \gamma = 0 \tag{15}$$

$$m_1 + m_2 - \overline{m} = 0, \tag{16}$$

where γ is the multiplier for (13). From the previous stage, we know that for $i = 1,2$

$$u'(x_i^*) = \left(\frac{w_i^r m_1}{w_1^r + w_2^r}\right)^{-1/r}$$

and

$$\frac{\mathrm{d}x_i}{\mathrm{d}m_1} = \frac{w_i^r}{w_1^r + w_2^r},$$

and similarly for $i = 3,4$. Therefore, from (14) we get

$$v_1\left(\frac{w_1^r m_1}{w_1^r + w_2^r}\right)^{-1/r}\left(\frac{w_1^r}{w_1^r + w_2^r}\right) + v_2\left(\frac{w_2^r m_1}{w_1^r + w_2^r}\right)^{-1/r}\left(\frac{w_2^r}{w_1^r + w_2^r}\right) = \gamma,$$

hence

$$m_1 = \left[v_1\left(\frac{w_1^r}{w_1^r + w_2^r}\right)^{1-1/r} + v_2\left(\frac{w_2^r}{w_1^r + w_2^r}\right)^{1-1/r}\right]^r \gamma^{-r} = A\gamma^{-r}$$

and

$$m_2 = \left[v_3\left(\frac{w_3^r}{w_3^r + w_3^r}\right)^{1-1/r} + v_4\left(\frac{w_4^r}{w_3^r + w_4^r}\right)^{1-1/r}\right]^{\frac{1}{r}} \gamma^{-r} = B\gamma^{-r},$$

where

$$A = \left[v_1\left(\frac{w_1^r}{w_1^r + w_2^r}\right)^{1-1/r} + v_2\left(\frac{w_2^r}{w_1^r + w_2^r}\right)^{1-1/r}\right]^r \tag{17}$$

and

$$B = \left[v_3 \left(\frac{w_3^r}{w_3^r + w_3^r} \right)^{1-1/r} + v_4 \left(\frac{w_4^r}{w_3^r + w_4^r} \right)^{1-1/r} \right]^r. \tag{18}$$

Using (16), we get

$$\gamma^{-r} = \frac{\overline{m}}{A+B},$$

and therefore we conclude that

$$m_1^* = \left(\frac{A}{A+B} \right) \overline{m} \quad \text{and} \quad m_2^* = \left(\frac{B}{A+B} \right) \overline{m}.$$

By plugging these values into (10) and (11) we obtain the desired optimal choices of x_k for $k = 1, 2, 3, 4$.

Consider now case 2, that is the case of a single equality. Three relevant subcases will be considered in what follows, namely $x_1 = x_2$ and $x_3 > x_4$ (case 21), $x_1 > x_2 = x_3 > x_4$ (case 22) and finally $x_1 > x_2$ and $x_3 = x_4$ (case 23). The other subcases can be treated in a similar fashion.

In case 21, we have $x_1^* = x_2^* = \frac{m_1}{2}$, while x_3^* and x_4^* are still given by equation (11). This implies that $m_2 = B\gamma^{-r}$, where B is given by equation (18), while using equation (14) we get

$$m_1 = 2 \left(\frac{v_1 + v_2}{2} \right)^r \gamma^{-r}$$

Finally, using equation (16) we get

$$m_1^* = \left(\frac{C}{C+B} \right) \overline{m} \quad \text{and} \quad m_2^* = \left(\frac{B}{C+B} \right) \overline{m},$$

where

$$C = 2 \left(\frac{v_1 + v_2}{2} \right)^r$$

By symmetry, in case 23 we get $x_3^* = x_4^* = \frac{m_2}{2}$ and

$$m_1^* = \left(\frac{A}{A+D} \right) \overline{m} \quad \text{and} \quad m_2^* = \left(\frac{D}{A+D} \right) \overline{m},$$

where A is given by (17) and

$$D = 2\left(\frac{v_3 + v_4}{2}\right)^r.$$

Finally, in case 22 we have

$$\begin{aligned} x_i^* &= x_i(m_1) = \frac{w_i^r m_1}{w_1^r + w_2^r} \quad \text{for} \quad i = 1,2 \\ x_i^* &= x_i(m_2) = \frac{w_i^r m_2}{w_3^r + w_4^r} \quad \text{for} \quad i = 3,4. \end{aligned}$$

Given that $m_1 + m_2 = m$, we get

$$\begin{aligned} m_1^* &= \frac{w_3^r(w_1^r + w_2^r)\overline{m}}{w_3^r(w_1^r + w_2^r) + w_2^r(w_3^r + w_4^r)} \\ m_2^* &= \frac{w_2^r(w_3^r + w_4^r)\overline{m}}{w_3^r(w_1^r + w_2^r) + w_2^r(w_3^r + w_4^r)} \end{aligned}$$

and hence that

$$x_1^* = \frac{w_1^r w_3^r \overline{m}}{w_3^r(w_1^r + w_2^r) + w_2^r(w_3^r + w_4^r)},$$

$$x_2^* = \frac{w_2^r w_3^r \overline{m}}{w_3^r(w_1^r + w_2^r) + w_2^r(w_3^r + w_4^r)},$$

$$x_3^* = \frac{w_2^r w_3^r \overline{m}}{w_3^r(w_1^r + w_2^r) + w_2^r(w_3^r + w_4^r)},$$

$$x_4^* = \frac{w_2^r w_4^r \overline{m}}{w_3^r(w_1^r + w_2^r) + w_2^r(w_3^r + w_4^r)}.$$

Consider now case 3, that is the case of a two equalities. Two relevant subcases will be considered in what follows, namely $x_1 = x_2 = x_3 > x_4$ (case 31) and $x_1 > x_2 = x_3 = x_4$ (case 32), as the other subcases can be treated in a similar fashion.

In case 31, it is easy to see that

$$x_1^* = x_2^* = x_3^* = \frac{m_1^*}{2} = \frac{w_3^r \overline{m}}{3w_3^r + w_4^r} \quad \text{and} \quad x_4^* = \frac{w_4^r \overline{m}}{3w_3^r + w_4^r}.$$

By symmetry, in case 32 we have that

$$x_1^* = \frac{w_1^r \overline{m}}{w_1^r + 3w_2^r} \quad \text{and} \quad x_2^* = x_3^* = x_4^* = \frac{m_2^*}{2} = \frac{w_2^r \overline{m}}{w_1^r + 3w_2^r}$$

Consider finally case 4, and in particular the subcase: $x_1 = x_2 > x_3 = x_4$. In this case, we have

$$x_1^* = x_2^* = \frac{m_1}{2} \quad \text{and} \quad x_3^* = x_4^* = \frac{m_2}{2},$$

and, using equations (14)−(16), we easily get

$$m_1^* = \left(\frac{C}{C+D}\right)\overline{m} \quad \text{and} \quad m_2^* = \left(\frac{D}{C+D}\right)\overline{m},$$

where C and D are defined above.

Naive

In this section we consider the choice problem for a naive decision maker. From the discussion in the main text, it should be clear that a naive individual will behave like a resolute decision maker in the first stage of the choice problem. However, at the second stage, he or she will solve the same first-step problem that a sophisticated individual solve. Therefore, while in the first stage the optimal choices are the same as those derived for the case of a resolute individual, in the second stage the optimal choices are solution to problems (8) and (9), where m_1 and m_2 are those implied by the first stage decision.

It follows that for this case no new computations are needed, as all the relevant optimal choices have been derived above.

Myopic

In this section we consider the choice problem for a myopic decision maker. While this type of decision maker will solve in the second stage a problem analogous to (8) and (9), in the first stage he or she will solve the decision problem actually ignoring the second stage.

Let $\hat{x} = x_1 + x_2$ and $\tilde{x} = x_1 + x_2$. Assuming $\hat{x} \geq \tilde{x}$, the myopic decision maker will solve

$$\max_{\hat{x},\tilde{x}} v_1 u(\hat{x}) + v_2 u(\tilde{x}) \quad s.t. \quad \tilde{x} + \hat{x} = \overline{m} \quad and \quad \tilde{x} - \hat{x} \leqslant 0,$$

where the weight now only involve the probabilities of the first node, namely p and $(1 - p)$ depending on the ranking. As the above problem has the same structure as problems (8) and (9), the optimal choices can be computed following an analogous procedure.

Mathematical Appendix 2: The stochastic specification

Preamble

This document discusses the stochastic specifications employed in the estimation. Throughout we assume Beta distributions. We begin with a discussion of the general properties of that distribution.

We use x^*, y^* and z^* to denote the optimal values of the decision variables (under some decision rule) and x, y and z the actual values. We also use m to denote the initial amount of money to be allocated.

Properties of Beta distribution

Suppose that x has a Beta distribution with parameters α and β. The parameters must be positive. The distribution has the following properties:

$$0 \leq x \leq 1 \tag{1}$$

The *pdf* (probability density function) $f(.)$ is given by

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \tag{2}$$

The function $\Gamma(.)$ is what is known as the GAMMA function. It is defined by

$$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt \tag{3}$$

The *cdf* (cumulative distribution function) we denote by $F(.)$ - is the integral of $f(.)$ from minus infinity to x.

The mean of x is given by

$$Ex = \frac{\alpha}{\alpha+\beta} \tag{4}$$

The variance of x is given by

$$var(x) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \tag{5}$$

We note that if we want the mean and variance to be such that they are equal to y and $y(1-y)/s$ for some y, then the following must hold

$$\alpha = y(s-1) \tag{6}$$

$$\beta = (1-y)(s-1) \tag{7}$$

Note that here the parameter s is the inverse of the variance (divided by $y(1-y)$). We note that the variance depends upon y - approaching 0 when y approaches 0 and 1. Note that s must be greater than 1. These are very nice properties.

Stochastic Specification

(1) We make the following assumption about the distribution of $\frac{x}{m}$:

A1 $\frac{x}{m}$ *has a Beta distribution with parameters α and β given by*

$$\alpha = \frac{x^*}{m}(s-1) \tag{8}$$

$$\beta = \left(1 - \frac{x^*}{m}\right)(s-1) \tag{9}$$

These restrictions on the parameters imply that the mean of $\frac{x}{m}$ is equal to $\frac{x^*}{m}$ and that the variance of $\frac{x}{m}$ is $\frac{x^*}{m}(1-\frac{x^*}{m})/s$. So the variance goes to zero at the extremes and the parameter s characterises the *precision* of the distribution. Also x is bounded to lie between 0 and m.

(2) We now make the following assumptions about the distributions of $\frac{y}{x}$ and $\frac{z}{m-x}$:

A2.1 $\frac{y}{x}$ *has a Beta distribution with parameters α and β given by*

$$\alpha = \frac{y^*}{x^*}(s-1) \tag{10}$$

$$\beta = \left(1 - \frac{y^*}{x^*}\right)(s-1) \tag{11}$$

These restrictions on the parameters imply that the mean of $\frac{y}{x}$ is equal to $\frac{y^*}{x^*}$and that the variance of $\frac{y}{x}$ is $\frac{y^*}{x^*}(1-\frac{y^*}{x^*})/s$. And that $\frac{y}{x}$ is bounded between 0 and 1; and hence that y is bounded between 0 and x.

A2.2 $\frac{z}{m-x}$ *has a Beta distribution with parameters* α *and* β *given by*

$$\alpha = \frac{z^*}{m-x^*}(s-1) \tag{12}$$

$$\beta = \left(1 - \frac{z^*}{m-x^*}\right)(s-1) \tag{13}$$

These restrictions on the parameters imply that the mean of $\frac{z}{m-x}$ is equal to $\frac{z^*}{m-x^*}$and that the variance of $\frac{z}{m-x}$ is $\frac{z^*}{m-x^*}(1-\frac{z^*}{m-x^*})/s$. And that $\frac{z}{m-x}$ is bounded between 0 and 1 - and hence that z is bounded between 0 and $m-x$.

We note that the means of x/m, y/x and $z/(m-x)$ always are between 0 and 1.

A Discretised Specification

We now note that the experimental software forces the subjects to choose integer values for the decision variables. So we should estimate a discretised version. Take x for example. This is between 0 and m. The value reported in the experiment is rounded to the nearest integer. So we get:

- $[0, 0.5]$ becomes 0
- $[0.5, 1.5]$ becomes 1
- $[1.5, 2.5]$ becomes 2

...

- $[m-1.5, m-0.5]$ becomes $m-1$
- $[m-0.5, m]$ becomes m.

In terms of proportions, this means that

- $[0, 0.5/m]$ becomes 0
- $[0.5/m, 1.5/m]$ becomes $1/m$

- $[1.5/m, 2.5/m]$ becomes $2/m$

...

- $[(m-1.5)/m, (m-0.5)/m]$ becomes $(m-1)/m$
- $[(m-0.5)/m, 1]$ becomes 1.

We can write this in general as

- If $x \leq 0.5/m$ then $x = 0$
- If $(i-0.5)/m < x \leq (i+0.5)/m$ then $x = i$ (for $i = 1, 2, ..., (m-1)$)
- If $(m-0.5)/m < x$ then $x = m$

So the probabilities are as follows (where cdf denotes the cumulative distribution function):

- $P(x = 0) = cdf[0.5/m]$
- $P(x = i) = cdf[(i+0.5)/m] - cdf[(i-0.5)/m]$ for $i = 1, 2, ..., (m-1)$
- $P(x = m) = 1 - cdf[(m-0.5)/m]$

By analogy, we have the following for y:

- $P(y = 0) = cdf[0.5/x]$
- $P(y = i) = cdf[(i+0.5)/x] - cdf[(i-0.5)/x]$ for $i = 1, 2, ..., (x-1)$
- $P(y = x) = 1 - cdf[(x-0.5)/x]$,

and for z:

- $P(z = 0) = cdf[0.5/(m-x)]$
- $P(z = i) = cdf[(i+0.5)/(m-x)] - cdf[(i-0.5)/(m-x)]$ for $i = 1, 2, ..., (m-x-1)$

- $P(z = m - x) = 1 - cdf[((m - x) - 0.5)/(m - x)]$

So the contributions to the likelihood are (where $cdfbeta(x, \alpha, \beta)$denotes the cumulative density up to x under a Beta distribution with parameters α and β):

- for x:

$$\ln\left(cdfbeta\left(\frac{0.5}{m}, \alpha, \beta\right)\right) \quad \text{if} \quad x = 0 \tag{14}$$

$$\ln\left(cdfbeta\left(\frac{x + 0.5}{m}, \alpha, \beta\right)\right) - \ln\left(cdfbeta\left(\frac{x - 0.5}{m}, \alpha, \beta\right)\right) \quad \text{if} \quad 0 < x < m$$

$$\ln\left(1 - cdfbeta\left(\frac{m - 0.5}{m}, \alpha, \beta\right)\right) \quad \text{if} \quad x = m$$

- for y (if observed):

$$\ln\left(cdfbeta\left(\frac{0.5}{x}\alpha, \beta\right)\right) \quad \text{if} \quad y = 0 \tag{15}$$

$$\ln\left(cdfbeta\left(\frac{y + 0.5}{x}, \alpha, \beta\right)\right) - \ln\left(cdfbeta\left(\frac{x - 0.5}{m}, \alpha, \beta\right)\right) \quad \text{if} \quad 0 < y < x$$

$$\ln\left(1 - cdfbeta\left(\frac{x - 0.5}{x}, \alpha, \beta\right)\right) \quad \text{if} \quad y = x$$

Rather trivially if $x = 0$ then y has to be zero and there is no contribution to the likelihood from y.

- for z (if observed):

$$\ln\left(cdfbeta\left(\frac{0.5}{m - x}, \alpha, \beta\right)\right) \quad \text{if} \quad z = 0 \tag{16}$$

$$\ln\left(cdfbeta\left(\frac{z + 0.5}{m - x}, \alpha, \beta\right)\right) - \ln\left(cdfbeta\left(\frac{z - 0.5}{m - x}, \alpha, \beta\right)\right) \quad \text{if} \quad 0 < z < m - x$$

$$\ln\left(1 - cdfbeta\left(\frac{z - 0.5}{m - z}, \alpha, \beta\right)\right) \quad \text{if} \quad z = m - x$$

Rather trivially if $x = m$ then z has to be zero and there is no contribution to the likelihood from z.