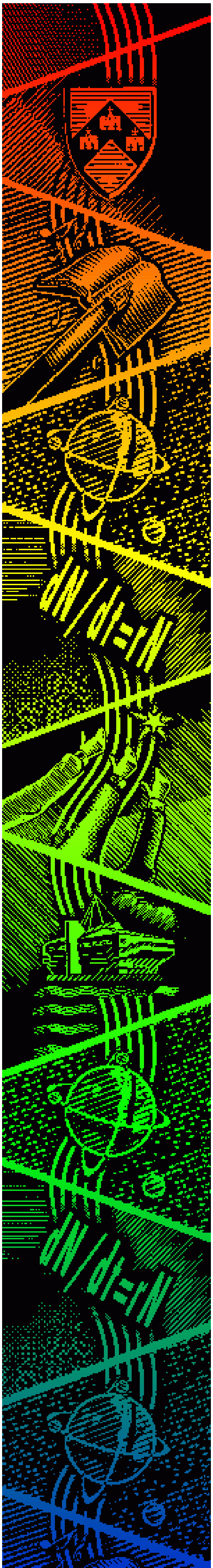

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Horizontal Mergers in the Spokes Model*

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Abstract

The theoretical analysis of merger poses a number of paradoxes. If firms compete in prices, a merger is profitable for all parties involved. Outsiders, however, free-ride and earn higher profits than insiders. The "spokes model" is a recently introduced framework to study n-firms spatial competition. This paper shows that in this model free-riding does not necessarily take place.

JEL code: D43, L11, L13.

Keywords: horizontal mergers, spokes model, mergers paradoxes.

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1 Introduction

The game theoretical analysis of mergers and coalition formation poses a number of paradoxes.

The 'merger paradox' identifies the path-breaking result by Salant-Switzer-Reynolds[14] who report that in a Cournot oligopoly with homogeneous products, linear demand and cost functions, a merger is beneficial for participating firms if more than 80 per cent of all firms merge. This is because "outsiders" enjoy more benefits than the firms participating in the merger, the "insiders". Since production costs are linear, any coalition of firms is indifferent with respect to the way of splitting its total production among the members of the coalition, so every coalition of firms behaves as if it were a single firm. Perry-Porter[10] and Farrell-Shapiro[6] challenge the view that a merged firm is no larger than any of the constituent firms. These studies introduce the existence of some crucial assets that are in limited supply in order to capture the notion that some firms are larger than others in a homogeneous product industry. This assumption implies rising marginal cost of output production and, consequently, internal cost savings from mergers could make a merger profitable.

Under price competition and differentiated products, Deneckere-Davidson[5] show that mergers are always profitable for insiders. However, according to the intuition of Stigler[13], the equilibrium displays free-riding properties: "outsiders" always earn higher profits than "insiders". A more recent literature takes into account strategic delegation (Gonzalez Maestre-Lopez Cunat[7] or Ziss[16]) to study merger profitability. Two types of competition are considered: in production and in the remuneration of managers. The effect of delegation is to increase competition between entities inside the firm. Consequently, the incentives to merge and the profitability of merger are considerably increased taking into account delegation, with respect to the standard setting. The required fraction of merger participants for a merger to be profitable is substantially smaller with delegation.

McAffee-Simons-Williams[8] suggest that the merger paradox can disappear in a context of spatial competition when market definition is considered. They show that a merger can result in a bigger firm than the two previous entities because it combines the plants of the two firms. But Norman-Pepall[9] show that internal profitability of the merger is not restored in the context of McAffee-Simons-Williams[8] and that the merging firms can lose market share, depending on the cost heterogeneity between firms. In a context of spatial competition in the circle, Brito[1] shows that even if market power is the sole motivation for merger, firms can be interested in being insiders (pre-emptive merger) and the impact of the merger on the rival firms depend on their location. Firms can prefer to be insiders even if some of the outsiders benefit more (but others less) from the merger than insiders.

The "traditional" approach to spatial competition uses the circular city model of Vickrey[15], also referred to as the Salop[12] model: one of its limit is to address only "localized" spatial competition (Rothschild[11]). Chen-Riordan[4] develop a new analytical tool to analyze spatial differentiation which naturally fits to the idea of "non-localized" competition. In the spokes model firms are located at the extreme of a market constituted of several spokes all linked at a common centre. There may be more spokes than firms (Chen-Riordan[4]) or as many spokes as firms (Caminal-Claici[2]). The model has three main properties. First, it allows multi-firm spatial competition with no neighbouring effects; second, it captures monopolistic competition *à la Chamberlin* as the number of firms and spokes tends to infinity; third, for some regions of the parameters, competition has a price increasing effect: this is due to the higher elasticity of demand in monopolistic segments as compared to the competitive ones.

The main result of this paper is to show that the price competition merger paradox, i.e. the free-riding property, does not necessarily arise as an equilibrium of the spokes model when horizontal mergers takes place. Conditions are devised for which this is the case. The spokes model, in fact, displays different types of equilibria depending on which part of the market the firms focus in supplying consumers.

The paper is organized as follows. Section 2 presents the spokes model. The effects of a merger are analyzed in section 3. In section 4, we illustrate the properties of the equilibrium, comparing the *pre-merger* with the *post-merger* equilibria. Concluding remarks follow in section 5. All proofs and mathematical derivations can be found in Appendices A, B and C.

2 The Framework

The framework chosen is the one introduced by Chen-Riordan[4]. The market has a spatial structure made up of N spokes of constant length, normalized to 1/2, with a common core. Suppose there are n firms entering the market and $n \leq N$ is exogenously given. Each firm locates at the extreme of her own spoke and supplies an homogeneous good: transportation costs are the only source of differentiation. Customers are uniformly distributed over all the N spokes. It is assumed that consumers have to travel to the firm's outlet in order to get the product. This implies that firms are no longer able to condition the price on the consumers' location. Instead of a discriminatory schedule that includes the transportation cost, a unique mill price is chosen by firms averaging between the different market segments. However, from the viewpoint of firms, this limited amount of flexibility in pricing has positive effects: price competition is less fierce as undercutting at each location is not possible in this setting. The unit transportation cost t is constant and consumers have a homogeneous valuation of the good v.

A crucial assumption is that customers only care about the brand located on the same spoke as they are and only one alternative brand. For each individual the alternative brand is extracted randomly between the N potentially available. This implies that both the favourite and the alternative brand may not be supplied and so not all consumers are served in equilibrium.

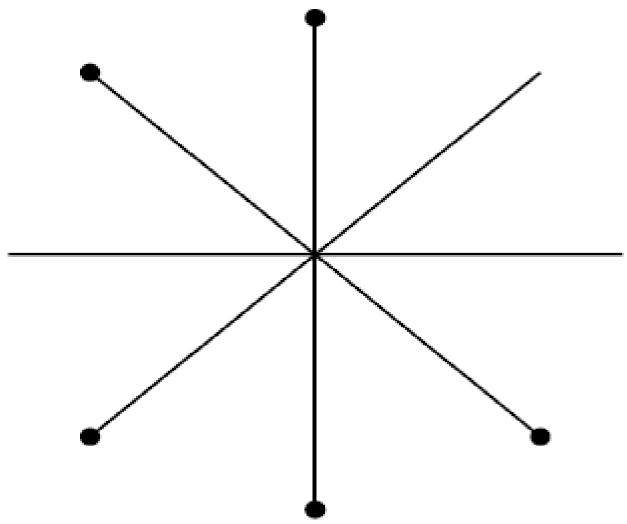

Figure 1: The Spokes Model with $n = 5$ and $N = 8$.

For example, supposing two consumers who like Fanta enter a supermarket. Fanta is not available; although not fully satisfying, they both have a second favourite brand: the assumption implies they can both like Sprite or perhaps one likes Sprite and the other likes Dr. Pepper. In the supermarket, however, there might be Dr.Pepper available but not Sprite. Although this seems a rather strong hypothesis, it is introduced by Chen-Riordan[4] to allow the tractability of the model and, in particular, the existence of a pure strategy equilibrium. A number of possible interpretations of why this could be the case are reported in their article. A convincing interpretation in the example reported above is that only the first two favourite brands give the consumers a positive utility, while all others do not. Alternatively, other example involving a spatial interpretation of the market might have to do with the consumers' imperfect knowledge of the areas where he or she is not located.

The profit function of the generic firm i is described by:

$$\pi_i(p_i, p_{-i}) = (p_i - c_i)D(p_i, p_{-i})$$

To characterize the demand function,observe that from the firm's viewpoint there are several types of customers[1]:

1. Customers on i-th firm's spoke that have one of the other firms as an alternative. The demand from this group is defined by identifying the location $\hat{x}$ of the consumers who are indifferent between buying from i or buying from the rival firm α:

$$\hat{x} = \max\left\{\min\left\{\frac{1}{2} + \frac{p_\alpha - p_i}{2t}; 1\right\} 0\right\}$$

   The constraints imposed simply require the customer to be located on either of the spokes and not outside.

2. Customers on the i-th firm's spoke who do not have an existing alternative brand *and* customers who do not have a first favorite brand but have i as a second favorite. The indifferent customer in the set of these two types is identified by:

$$\check{x} = \max\left\{\min\left\{\frac{v - p_i}{t}, 1\right\}, 0\right\}$$

The demand function faced by firm i can then be defined as:

$$D_i(p_i, p_{-i}) = \frac{2}{N}\frac{1}{N-1}\sum_{\alpha=1..n}^{\alpha\neq i} \max\left\{\min\left\{\frac{1}{2} + \frac{p_\alpha - p_i}{2t}; 1\right\} 0\right\} + \frac{2}{N}\frac{N-n}{N-1}\max\left\{\frac{v - p_i}{t}, 1\right\}$$

where $2/N$ is the density of consumers at each point of the spokes, $1/(N-1)$ is the probability of j being a customers' second favorite brand and $(N-n)/(N-1)$ is the probability of a consumer having no first or second favorite

[1] Notice that firms, despite recognizing different types of customers, are constrained to use a unique price and are not allowed any kind of price discrimination.

brand. Also, the following regularity conditions need to be satisfied: $\frac{|p_\alpha - p_i|}{2t} < \frac{1}{2}$ $\forall \alpha \neq i$ and $\frac{v-p_i}{t} \geq 1/2$.
The demand function can then be rewritten as:

$$D_i(p_i, p_{-i}) = \begin{cases} \frac{2}{N}\frac{1}{N-1}\sum_{\alpha\neq i}\left(\frac{1}{2} + \frac{p_\alpha - p_i}{2t}\right) + \frac{2}{N}\frac{N-n}{N-1}\frac{v-p_i}{t} & \text{if } \ \frac{1}{2} < \frac{v-p_i}{t} < 1 \\ \\ \frac{2}{N}\frac{1}{N-1}\sum_{\alpha\neq i}\left(\frac{1}{2} + \frac{p_\alpha - p_i}{2t}\right) + \frac{2}{N}\frac{N-n}{N-1} & \text{if } \ \frac{v-p_i}{t} > 1 \end{cases} \tag{1}$$

Consistent with Chen-Riordan[4], the attention will be focused on a situation in which firms are symmetric and have access to the same production technology, characterized by a constant marginal cost $c_i = c$ $\forall i = 1...n$. The first order conditions identifying the equilibrium prices are given by:

$$\frac{\partial \pi_i}{\partial p_i} = D_i(p_i, p_{-i}) + (p_i - c)\frac{\partial D_i(p_i, p_{-i})}{\partial p_i} = 0$$

Under symmetry, the demand function can be expressed as follows:

$$D_i(p_i, p^*) = \begin{cases} \frac{2}{N}\frac{n-1}{N-1}\left(\frac{1}{2} + \frac{p^* - p_i}{2t}\right) + \frac{2}{N}\frac{N-n}{N-1}\frac{v-p_i}{t} & \text{if } \ \frac{1}{2} < \frac{v-p_i}{t} < 1 \\ \\ \frac{2}{N}\frac{n-1}{N-1}\left(\frac{1}{2} + \frac{p^* - p_i}{2t}\right) + \frac{2}{N}\frac{N-n}{N-1} & \text{if } \ \frac{v-p_i}{t} > 1 \end{cases}$$

and the first derivative of the demand function is:

$$\frac{\partial D_i(p_i, p^*)}{\partial p_i} = \begin{cases} -\frac{2(N-n)+(n-1)}{tN(N-1)} & \text{if } \ \frac{1}{2} < \frac{v-p_i}{t} < 1 \\ \\ -\frac{n-1}{tN(N-1)} & \text{if } \ \frac{v-p_i}{t} > 1 \end{cases}$$

Given the definition of the demand and the profit functions, it can be checked that there exist four possible equilibrium regions.

The equilibrium regions can be characterized depending on the parameter v, the valuation of the good. As in Chen-Riordan[4], the Nash equilibrium prices are defined as:

$$p^*_{bm} = \begin{cases} c + t\frac{(2N-n-1)}{n-1} & \text{Region I} \quad c + \frac{2t(N-1)}{n-1} < v \leq \bar{v}_{bm} \\ v - t & \text{Region II} \quad c + 2t < v \leq c + \frac{2t(N-1)}{n-1} \\ \frac{2v(N-n)+t(n-1)+c(2N-n-1)}{4N-3n-1} & \text{Region III} \quad c + \frac{t}{2}\frac{4N-n-3}{2N-n-1} < v \leq c + 2t \\ v - \frac{t}{2} & \text{Region IV} \quad c + t < v \leq c + \frac{t}{2}\frac{4N-n-3}{2N-n-1} \end{cases}$$

and profits:

$$\pi^*_{bm} = \begin{cases} \frac{t(2N-n-1)^2}{(n-1)N(N-1)} & \text{Region I} \quad c + \frac{2t(N-1)}{n-1} < v \leq \bar{v}_{bm} \\ \frac{(v-t-c)(2N-n-1)}{N(N-1)} & \text{Region II} \quad c + 2t < v \leq c + \frac{2t(N-1)}{n-1} \\ \frac{[2(N-n)(v-c)+t(n-1)]^2(2N-n-1)}{(4N-3n-1)^2 tN(N-1)} & \text{Region III} \quad c + \frac{t}{2}\frac{4N-n-3}{2N-n-1} < v \leq c + 2t \\ \frac{2(v-c)-t}{2N} & \text{Region IV} \quad c + t < v \leq c + \frac{t}{2}\frac{4N-n-3}{2N-n-1} \end{cases}$$

where $\bar{v}_{bm} = c + t\left[2\frac{N-1}{n-1} + \frac{(2N-n-1)}{2(N-n)}\right]$. The derivation of the equilibrium expressions and the regions in which they hold follow Chen-Riordan[4] and are reproduced in Appendix A. As illustrated in Figure 2 the price is a non-decreasing function of the value of the good v. For values above $\bar{v}_{bm}$ a pure strategy equilibrium of the game does not exist. A too large valuation of the good implies firms have a unilateral incentive to raise their price to $p = v - t$ which is however not an equilibrium either. In Region I standard oligopoly competition takes place: v is large enough so that firms focus on the segment of consumers who have both a first and a second favourite brand, in other words the demand that the firm shares with all competing firms. The price is independent of v and depends only on the number of firms and spokes. All consumers who have a preference for an existing brand participate in the

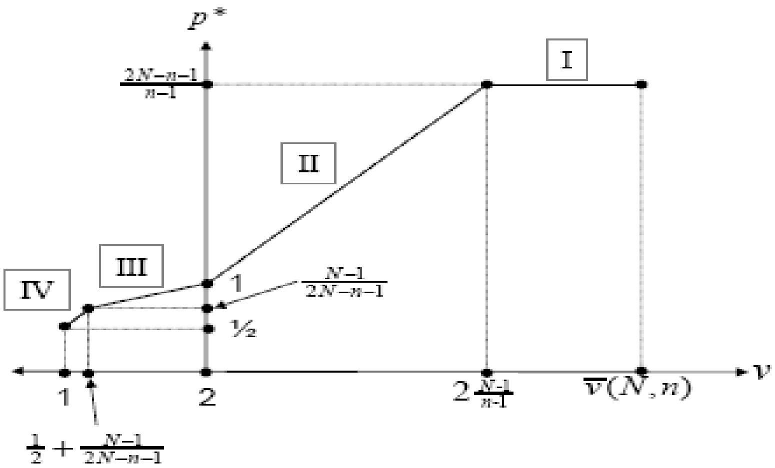

Figure 2: Nash Equilibrium prices as a function of value for $c = 0$ and $t = 1$, from Chen-Riordan[4].

market. Region II is characterized by a kink in the demand function: firms concentrate on extracting surplus on the indifferent consumers who do not have a second favourite brand. Also in this case all the consumers with at least one favourite brand are served in equilibrium. The price increases linearly with v: this is the parameter characterizing the indifferent consumers that firms consider when setting the price. In Region III firms' prices are driven by the indifferent consumer who does not have an alternative. All surplus is extracted from this type of consumer and some of them are not served in equilibrium. On the other hand, the indifferent consumer who has both the first and the second brand available maintains a positive surplus. As v increases, price increases as it is possible to extract more surplus from the consumers on the monopolistic segment of the market. This region has the interesting property that price is increasing with the number of active firms, as demand is more elastic in the monopolistic segment. Region IV is characterized by a different kink of the demand function: firms focus on the indifferent consumer who have their brand as first favourite. As in Region

III also in this region not all consumers with at least one favourite brand are served: in particular, only consumers with a first favourite brand participate on the market in this case. For even lower values of v an equilibrium would exist but all firms would be local monopolists serving only part of the consumers located on their spoke.

3 Horizontal Mergers

Once introduced the framework, the focus can shift on the effects of an horizontal merger. The question posed is: what is the effect of a merger between symmetric firms in a spatial model of non-localized competition?

Following the industrial organization literature, it is assumed that the merging firms maximize their joint profits. The profits after the merger are split in equal parts between the participating firms. In other words, the only effect of a merger is to create a multi-product firm: they supply their product independently but they adopt joint decisions on their prices to maximize the joint profits. This makes our results comparable with Caminal-Granero[3] who explicitly address the role of multi-product firms in supplying variety. As it will be clear, the results reported are consistent with theirs: as they focus on the analog of Region I of Chen-Riordan[4], the free-riding property makes a larger multi-product firm competitively disadvantaged as compared with a fringe of single product firms. However in what follows asymmetric competition is analyzed in all equilibrium regions of the spokes model.

A key-point is to recognize that in the new situation, after the merger of two symmetric firms, there is no market expansion effect: the probability for a given customer of having no liked brand available on the market does not change. The probability, in fact, depends only on the number of spokes occupied and not on the number of active firms.

3.1 The Effects of a Merger

Suppose that firm i has been formed by the merger of k of the n symmetric firms of the benchmark model. All other firms are exactly symmetric. While the number of spokes N is exogenous and not affected by the merger, the number of active firms reduces to $m = n - k + 1$.
Focus first on the merged firms who constituted i. In order to define the demand function for this new firm, the identification of indifferent customers is again the starting point. It turns out that the same expressions identifying the indifferent consumers remain valid, with a slightly different interpretation. The equation:

$$v - p_i - tx = v - p_j - t(1-x) \quad \forall i = 1...k \ \forall j \neq i$$

identifies the indifferent customer between the ones who have an alternative brand existing on the market. The set of indifferent consumers is described by:

$$\hat{x}_{ij} = \max\left\{\min\left\{\frac{1}{2} + \frac{p_j - p_i}{2t}; 1\right\} 0\right\} \quad \forall i = 1...k \ \forall j \neq i$$

There are, however, two types of indifferent consumers: consumers whose other brand is supplied by one of the other firms taking part to the merger *and* consumers whose other brand is supplied by one of the outsiders. As in most of the literature on horizontal mergers it is assumed that firms, despite maximizing joint profits, keep their price independence and set their own price. This implies that the market segment of each firm is identified as:

$$\hat{x}_{il} = \max\left\{\min\left\{\frac{1}{2} + \frac{p_l - p_i}{2t}; 1\right\} 0\right\} \quad \forall i = 1...k \ \forall j = 1..k, \ j \neq i$$

Indifferent customers with no kind of alternative brand are still identified by:

$$\check{x}_i = \max\left\{\min\left\{\frac{v - p_i}{t}, 1\right\}, 0\right\}$$

The conclusion is that, from the perspective of one of the firms who took part to the merger and constituted firm i there are three types of customers after the merger:

1. $\frac{k-1}{N-1}$ customers whose second favourite brand is supplied by factories located on other spokes but belonging to i, the firm resulting from the merger;

2. $\frac{n-k}{N-1}$ customers who have an alternative brand not supplied by other factories affiliated to i;

3. $\frac{N-n}{N-1}$ customers who do not have a second favourite brand.

A mass of $\frac{2}{N}$ customers is located at each point of the spatial market structure. There is a further class of customers which is not of interest to firms: the agents that do not have both a first and a second favourite brand existing on the market. They are excluded from participating in the market and so their existence does not affect the results: if the number of firms is exogenously given, the fraction of this type of consumers is unaffected by the merging activity so that mergers do not imply a market expansion.
The demand function of firm i, constituted by the k firms which merged, is defined by the segments served by the k firms. Each of the segments is defined as:

$$D_i(p_i, p_{-i}) = \frac{2}{N}\left\{\frac{1}{N-1}\sum_{\alpha=1..k}^{\alpha\neq i}\max\left\{0, \min\left\{\frac{1}{2}+\frac{p_\alpha - p_i}{2t}, 1\right\}\right\}+\right.$$
$$+\frac{1}{N-1}\sum_{\alpha=1}^{n-k}\max\left\{0, \min\left\{\frac{1}{2}+\frac{p_\alpha - p_i}{2t}, 1\right\}\right\}+$$
$$\left.+\frac{N-n}{N-1}\min\left\{\max\left\{\frac{v-p_i}{t}, 0\right\}, 1\right\}\right\} \quad \forall i = 1...k$$

The total demand is just the sum of the segments faced by the each of the merging firms. The first term represents consumers with both favourite brands being supplied by i. The second term consumers whose second favourite brand is supplied by one of the remaining firms. The third term the consumers whose only desired brand is supplied by the firm. The demand of each of this segment is weighted by the respective probabilities of a given consumer being one of the three possible types recalled. The demand function faced by the outsiders is:

$$D_j(p_j, p_{-j}) = \frac{2}{N}\left\{\frac{1}{N-1}\sum_{\alpha=1}^{k}\max\left\{0, \min\left\{\frac{1}{2}+\frac{p_\alpha - p_i}{2t}, 1\right\}\right\}\right.$$
$$+\frac{1}{N-1}\sum_{\alpha=1..(n-k)}^{\alpha\neq j}\max\left\{0, \min\left\{\frac{1}{2}+\frac{p_\alpha - p_i}{2t}, 1\right\}\right\}+$$
$$\left.+\frac{N-n}{N-1}\min\left\{\max\left\{\frac{v-p_j}{t}, 0\right\}, 1\right\}\right\}$$

holding for $\forall j = 1...(n-k)$. The three terms represent, respectively, the demand faced from consumers who have as other favourite a brand supplied by firm i, consumers who have as other favourite a brand supplied by another non-merged firm and consumers whose only desired brand is supplied by the firm. The weighting is given by the probabilities of a generic consumers being of a given type. The profit functions for the merged firm and for the outsiders are respectively:

$$\pi_i = \sum_{\alpha=1}^{k}(p_\alpha - c)D_\alpha(p_\alpha, p_{-\alpha})$$
$$\pi_j = (p_j - c)D_j(p_j, p_{-j})$$

The first order conditions for the merged firms are:

$$\frac{\partial \pi_i}{\partial p_i} = D_i(p_i, p_{-i}) + (p_i - c)\frac{\partial D_i(p_i, p_{-i})}{\partial p_i} + \sum_{\alpha=1..k}^{\alpha\neq i}(p_\alpha - c)\frac{\partial D_\alpha(p_\alpha, p_{-\alpha})}{\partial p_i} = 0 \quad \forall i = 1...k$$

where the first derivatives of the demand function are respectively:

$$\frac{\partial D_i(p_i, p_{-i})}{\partial p_i} = \begin{cases} -\frac{n-1}{tN(N-1)} - \frac{2(N-n)}{N(N-1)} & \text{if } \frac{1}{2} < \frac{v-p_i}{t} < 1 \\ -\frac{n-1}{tN(N-1)} & \text{if } \frac{v-p_i}{t} > 1 \end{cases}$$

$$\frac{\partial D_\alpha(p_\alpha, p_{-\alpha})}{\partial p_i} = \begin{cases} \frac{1}{tN(N-1)} & \text{if } \frac{1}{2} < \frac{v-p_i}{t} < 1 \\ \frac{1}{tN(N-1)} & \text{if } \frac{v-p_i}{t} > 1 \end{cases} \quad \forall \alpha = 1...k, \alpha \neq i$$

The first order conditions for the non-merged firms are:

$$\frac{\partial \pi_j}{\partial p_j} = D_j(p_j, p_{-j}) + (p_j - c)\frac{\partial D_j(p_j, p_{-j})}{\partial p_j} = 0 \quad \forall j = 1...(n-k)$$

where the first derivatives of the demand function are:

$$\frac{\partial D_j(p_j, p_{-j})}{\partial p_j} = \begin{cases} -\frac{n-1}{tN(N-1)} & \text{if } \frac{v-p_j}{t} > 1 \\ -\frac{n-1}{tN(N-1)} - \frac{2(N-n)}{tN(N-1)} & \text{if } \frac{1}{2} < \frac{v-p_j}{t} < 1 \end{cases}$$

Comparing the first order conditions of the merged and the non-merged firms it is clear that the merger imposes on each firm participating to internalize the externalities imposed by one's own price choices on the demand for other brands. This property, first illustrated by Deneckere-Davidson[5], plays an important role in determining the results in some but not all regions of equilibrium and it will be discussed in detail in Section 4.

Imposing a regularity condition with the same role of the one adopted in the benchmark case, i.e. $\frac{|p_\alpha - p_i|}{2t} < \frac{1}{2}$ $\forall \alpha \neq i$, the analysis of the equilibria can be performed. In particular, the analysis of the effects of the merger on prices and profits is proposed in what follows.

3.2 The After-Merger Equilibrium

The results of the analysis proposed can be summarized reporting the after-merger equilibrium prices and profits for both merging and non-merging firms. The Nash equilibrium prices for the merged firms are:

$$p^*_m = \begin{cases} c + t\frac{(2n-1)(2N-n-1)}{2n^2-n(k+2)-k^2+2k} & \text{Region Im} \\ v - t & \text{Region IIm} \\ \frac{[2v(N-n)+t(n-1)](4N-2n-1)+c\{2N[4(N-n)-2k-1]+n(2n+3k)-k^2+2k\}}{4N(4N-5n+k+1)+6n^2+n(3k+2)-k^2+2k} & \text{Region IIIm} \\ v - \frac{t}{2} & \text{Region IVm} \end{cases}$$

where the values of v delimiting the different regions are:

Region Im $\quad c + \frac{t(4nN-3n-2N-kn-k^2-2k+1)}{(2n+k-2)(n-k)} < v \leq \bar{v}_m$

Region IIm $\quad c + t\frac{4N-2n-k-1}{2N-n-k} < v \leq c + t\frac{2N-k-1}{n-k}$

Region IIIm $\quad c + \frac{t}{2}\frac{4N(4N-3n-k-3)+n(2n+3k)+2(k+1)}{2N(4N-4n-2k-1)+n(2n+3k)-k^2+2k} < v$
$\quad v \leq c + t\frac{4N(4N-4n-k-2)+n(4n+3k+3)-k^2+2k+1}{2N(4N-4n-2k-1)+n(2n+3k)-k^2+2k}$

Region IVm $\quad c + t < v \leq c + \frac{t}{2}\frac{4N-n-k-2}{2N-n-1}$

The equilibrium prices for the non merged firms are:

$$p^*_{nm} = \begin{cases} c + t\frac{(2N-n-1)(2n-k)}{2n^2-n(k+2)-k^2+2k} & \text{Region Inm} \\ v - t & \text{Region IInm} \\ \frac{[2v(N-n)+t(n-1)](4N-2n-k)+c(2N-n-k)(4N-2n+k-2)}{4N(4N-5n-k-1)+6n^2+n(3k+2)-k^2+2k} & \text{Region IIInm} \\ v - \frac{t}{2} & \text{Region IVnm} \end{cases}$$

where the values of v delimiting the different regions are:

Region Inm $\quad c+\frac{t(4Nn-2kN-4n-k^2+3k)}{(2n+k-2)(n-k)}<v\leq\bar{v}_{nm}$

Region IInm $\quad c+2t<v\leq c+2t\frac{N-1}{n-1}$

Region IIInm $\quad \begin{array}{l} c+t\frac{4N(4N-4n-k-2)+2n(2n+k+2)-k^2+3k}{2N(4N-4n-k-2)+n(2n+k+2)-k^2+2k}<v \\ v\leq c+\frac{t}{2}\frac{4N(4N-3n-k-3)+n(2n+k+6)-k^2+4k}{2N(4N-4n-k-2)+n(2n+k+2)-k^2+2k} \end{array}$

Region IVnm $\quad c+t<v\leq c+\frac{t}{2}\frac{4N-n-3}{2N-n-1}$

The strategy followed to identify the values of parameters for which the equilibrium regions exist is analogous to the one followed in the before-merger benchmark case: all the details are reported in Appendix B.

The profits of merged firms are:

$$\pi_m^*=\begin{cases} \frac{tk[2N(2n-1)-2n^2-n+1]^2}{N(N-1)(2n+k-2)[k^2+k(n-2)-2n(n-1)]} & \text{Region Im} \\ k\frac{(v-t-c)(2N-n-1)}{N(N-1)} & \text{Region IIm} \\ \frac{(2n-k)(n-1)(2N-n-1)[4Nn+(n+1)(k-2n)]}{tN(N-1)[k^2+k(n-2)-2n(n-1)]^2} & \text{Region IIIm} \\ k\frac{2(v-c)-t}{2N} & \text{Region IVm} \end{cases}$$

while the profits of the outsiders are:

$$\pi_{nm}^*=\begin{cases} \frac{k(2N-n-k)(4N-2n-1)^2[2(N-n)(v-c)+t(n-1)]^2}{tN(N-1)[4N(4N-5n+k+1)+6n^2+n(3k+2)-k^2+2k]^2} & \text{Region Inm} \\ \frac{(v-t-c)(2N-n-1)}{N(N-1)} & \text{Region IInm} \\ \frac{(2N-n-1)(4N-2n-k)^2[2(N-n)(v-c)+t(n-1)]^2}{tN(N-1)[4N(4N-5n+k+1)+6n^2+n(3k+2)-k^2+2k]^2} & \text{Region IIInm} \\ \frac{2(v-c)-t}{2N} & \text{Region IVnm} \end{cases}$$

The next section discusses these results in detail.

4 Analysis of the Equilibrium Regions

Bringing together the results of the pre-merger benchmark situation in Section 2 and the post-merger equilibria, we identify *four equilibrium regions.* These regions can be seen as the analog of the four equilibrium regions analyzed in Chen-Riordan and they are illustrated in Figure 3 and in Example 1. The four equilibrium regions are defined as a function of v as follows:

Region 1	$v_{1D} < v \leq v_{1U}$
Region 2	$v_{2D} < v \leq v_{2U}$
Region 3	$v_{3D} < v \leq v_{3U}$
Region 4	$v_{4D} < v \leq v_{4U}$

where the limiting values identifying each region are defined as follows:

$$v_{1D} = \max\{c + 2t\tfrac{N-1}{n-1}, c + t\tfrac{(4nN-3n-2N-kn-k^2-2k+1)}{(2n+k-2)(n-k)}, c + \tfrac{t(4Nn-2kN-4n-k^2+3k)}{(2n+k-2)(n-k)}\}$$

$$v_{2D} = \max\{c + 2t, c + t\tfrac{4N-2n-k-1}{2N-n-k}, c + 2t\}$$

$$v_{3D} = \max\{c + t\tfrac{4N-n-3}{2(2N-n-1)}, c + \tfrac{t}{2}\tfrac{4N(4N-3n-k-3)+n(2n+3k)+2(k+1)}{2N(4N-4n-2k-1)+n(2n+3k)-k^2+2k}, c + t\tfrac{4N(4N-4n-k-2)+2n(2n+k+2)-k^2+3k}{2N(4N-4n-k-2)+n(2n+k+2)-k^2+2k}\}$$

$$v_{4D} = \max\{c + t, c + t, c + t\}$$

for downwards, and:

$$v_{1U} = \min\{\bar{v}_{bm}, \bar{v}_m, \bar{v}_{nm}\}$$

$$v_{2U} = \min\{c + 2t\tfrac{N-1}{n-1}, c + t\tfrac{2N-k-1}{n-k}, c + 2t\tfrac{N-1}{n-1}\}$$

$$v_{3U} = \min\{c + 2t, c + t\tfrac{4N(4N-4n-k-2)+n(4n+3k+3)-k^2+2k+1}{2N(4N-4n-2k-1)+n(2n+3k)-k^2+2k}, c + \tfrac{t}{2}\tfrac{4N(4N-3n-k-3)+n(2n+k+6)-k^2+4k}{2N(4N-4n-k-2)+n(2n+k+2)-k^2+2k}\}$$

$$v_{4U} = \min\{c + t\tfrac{4N-n-3}{2(2N-n-1)}, c + \tfrac{t}{2}\tfrac{4N-n-k-2}{2N-n-1}, c + \tfrac{t}{2}\tfrac{4N-n-3}{2N-n-1}\}$$

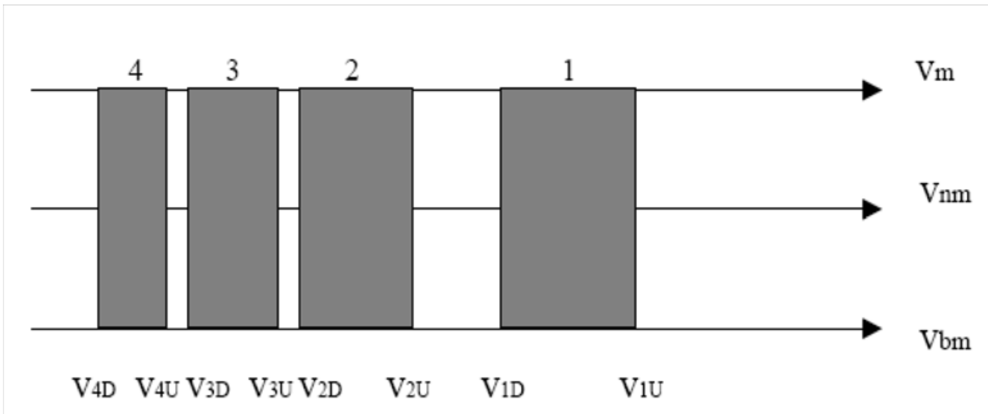

Figure 3: The Equilibrium Regions after the Merger.

for upwards boundaries. These limiting values are nothing but the collection of the boundaries for the equilibria *before merger* (identifying Region I to IV in Section 2) and *after merger*, for both *merging* (Region Im-IVm in Section 3) and *non-merging* firms (Region Inm-IVnm in Section 3). Appendix B presents the technical details of how these regions are identified.

Example 1

Consider the following situation. There are $N = 7$ spokes and $n = 5$ firms entered the market, the marginal cost c is zero and the unit transport cost t is normalized to one. Two firms decide to merge, so $k = 2$. The equilibrium regions for the spokes model with mergers are:

Region 1	$3.400 < v \leq 5.000$
Region 2	$2.140 < v \leq 3.000$
Region 3	$1.336 < v \leq 2.000$
Region 4	$1.000 < v \leq 1.250$

as it can be verified that:

$$\begin{aligned}
v_{1D} &= \max\{3.000, 3.400, 3.133\} \\
v_{2D} &= \max\{2.000, 2.140, 2.000\} \\
v_{3D} &= \max\{1.250, 1.336, 1.262\} \\
v_{4D} &= \max\{1.000, 1.000, 1.000\}
\end{aligned}$$

are the downwards and:

$$\begin{aligned}
v_{1U} &= \min\{5.000, 6.400, 5.016\} \\
v_{2U} &= \min\{3.000, 3.667, 3.000\} \\
v_{3U} &= \min\{2.000, 2.115, 2.016\} \\
v_{4U} &= \min\{1.250, 1.357, 1.2619\}
\end{aligned}$$

the upwards boundaries of the equilibrium regions before and after merger, for merging and non-merging firms respectively.

4.1 Region 1

Suppose that the parameters are such that all firms, merged and non-merged, face the duopoly segment demand while they are monopolist on the empty spokes. This is the case if, in equilibrium, prices are such that $\frac{v-p_m^*}{t} > 1$ and $\frac{v-p_{nm}^*}{t} > 1$. The equilibrium prices are:

$$p_m^* = c + t\frac{(2n-1)(2N-n-1)}{2n^2 - n(k+2) - k^2 + 2k}$$

$$p_{nm}^* = c + t\frac{(2N-n-1)(2n-k)}{2n^2 - n(k+2) - k^2 + 2k}$$

Equilibrium profits are:

$$\pi_m^* = \frac{tk[2N(2n-1) - 2n^2 - n + 1]^2}{N(N-1)(2n+k-2)[k^2 + k(n-2) - 2n(n-1)]}$$

$$\pi_{nm}^* = \frac{t(2n-k)(n-1)(2N-n-1)[4Nn + (n+1)(k-2n)]}{N(N-1)[k^2 + k(n-2) - 2n(n-1)]^2}$$

The results reported directly lead to the following Proposition:

Proposition 1 *In the spokes model, when the parameters are such that* $\frac{v-p_m^*}{t} > 1$, $\frac{v-p_{nm}^*}{t} > 1$ *and* $\frac{v-p_{bm}^*}{t} > 1$, *a merger between* k *firms displays the free-riding property: outsider firms are better off with respect to the insiders. In synthesis:* $\pi_{nm}^* > \frac{\pi_m^*}{k} > \pi_{bm}^*$.

The Proof can be found in Appendix C.

The result found confirm that the price competition mergers paradox takes place in Region 1. This is an expected result: it can be recalled from the description of Region I of the benchmark case that this region is characterized by "standard" oligopolistic competition. Under these conditions the mechanisms highlighted by Deneckere-Davidson[5] are in operation: as the best response function are upward sloping, prices of both insiders and outsiders raise their prices, earning higher profits. However, the free-riding property is also in operation. The intuition provided by Deneckere-Davidson is the following. A given outsider faces competition from both the merged entity and the other outsiders. Then, she shares with a given insider $n-2$ competitors. But they both face another competitor. For the outsider firm this competitor is a member of the merged entity, so a firm charging a higher price. The insider, on the other hand, faces competition of another outsider firm, which is charging a lower price. This implies that outsiders face less fierce competition and their profits dominate the ones of insiders. This is exactly what happens in Region 1. Finally the following example illustrates the findings reported.

Example 2

As in Example 1, assume $N = 7$, $n = 5$, $m = 4$, $k = 2$ and that $3.4 < v \leq 5.0$. As in Chen-Riordan[4] $t = 1$ and $c = 0$; the demand functions for insiders and outsiders can be then outlined following the steps in section 4.3.1. These are:

$$D_i(p_i, p_{-i}) = \frac{2}{7}\left\{\frac{1}{6}\left[\frac{1}{2} + \frac{p_m - p_i}{2}\right] + \frac{1}{6}\sum_{\alpha=1}^{3}\left[\frac{1}{2} + \frac{p_\alpha - p_i}{2}\right] + \frac{2}{6}\right\}$$

$$D_j(p_j, p_{-j}) = \frac{2}{7}\left\{\frac{1}{6}\sum_{\alpha=1}^{2}\left[\frac{1}{2}+\frac{p_\alpha - p_i}{2}\right]+\frac{1}{6}\sum_{\alpha=1..3}^{\alpha\neq j}\left[\frac{1}{2}+\frac{p_\alpha - p_i}{2}\right]+\frac{2}{6}\right\}$$

Maximizing firms' profits:

$$\pi_i = \sum_{\alpha=1}^{k} p_\alpha D_\alpha(p_\alpha, p_{-\alpha}) \qquad \pi_j = p_j D_j(p_j, p_{-j})$$

the Nash equilibrium prices are:

$$p_m = 2.400 \quad p_{nm} = 2.133$$

$$\pi_m = 0.823 \ \ \pi_{nm} = 0.433$$

as compared with the pre-merger status quo that, according to discussion in Section 2, gives the following results:

$$p_{bm} = 2 \quad \pi_{bm} = 0.381$$

4.2 Region 2

Suppose that in equilibrium $\frac{v-p_m^*}{t} = 1$ and $\frac{v-p_{nm}^*}{t} = 1$: this straightforwardly implies that $p_m^* = p_{nm}^* = v - t$.

It is extremely simple to show that in equilibrium the profits of merged and non-merged firms are exactly the same. Given that the equilibrium prices are identical, also the demand is identical both for merging and non-merging firm. The expression for profits is:

$$\frac{\pi_m^*}{2} = \pi_{nm}^* = \frac{(v-t-c)(2N-n-1)}{N(N-1)}$$

However, it is easy to check that these expressions are equivalent to the one of profits before merger.

Proposition 2 *In the spokes model, when the parameters are such that* $\frac{v-p_m^*}{t} = 1$, $\frac{v-p_{nm}^*}{t} = 1$ *and* $\frac{v-p_{bm}^*}{t} = 1$, *a merger between k firms does not display the free-riding property: outsider firms are not better off with respect to the insiders. In synthesis:* $\pi_{nm}^* = \frac{\pi_m^*}{k} = \pi_{bm}^*$.

The conclusion is that the merger paradox does not arise for this region of parameters in the spokes model: the free-riding property characterizing the equilibrium of Bertrand-like models is not operating in this equilibrium. In fact, for this region of the parameters, a merger is perfectly rational from the perspective of insiders as soon as there is an $\epsilon > 0$ cost synergy resulting from the merger.

This is one of the main results of the paper and can be interpreted as follows: in Region 2 firms focus on the kink of their demand taking place in presence of consumers who lack a second available brand. As underlined above, all consumers with at least one favourite brand available participate in the market. This implies that the size of the market does not shrink: exactly the same share of consumers is served as in the case of Region 1. However, in this case the merger does not have an effect on any kind of firm as the type of consumers that the firms find optimal to focus on is unaffected by the new market configuration. This is, in fact, driving to the conclusion that in a spatial structure like the spokes model, the free-riding merger paradox does not take place for a non-negligible subset of the parameter space.

The indifference result obtained is not new in the literature: Brito[1] has proved that non-neighbouring firms in the circle model are completely unaffected by an eventual merger. This is due to the lack of interdependence of the prices of the firms merged: when competition is limited to neighbouring firms, firms joining forces does not have implications on the demand faced. However, this is not the case in the spokes model. In principle, competition takes place between all firms, no matter their relative location. In this region, nevertheless, the firm has an incentive to focus on a particular group of consumers: it is only competition and profit incentives that determine the indifference result and not the topological structure of the market.

It is worth noting that a corollary of the results obtained above is that the profitability of the merger is completely independent of the number of firms taking part in the coalition.

4.3 Region 3

Suppose instead that all firms after merger, both merged and not, do not satisfy completely the segment of the market for which they are monopolist. This is the case for $\frac{1}{2} < \frac{v-p^*_m}{t} < 1$ and $\frac{1}{2} < \frac{v-p^*_{nm}}{t} < 1$.
The equilibrium prices are:

$$\begin{aligned} p^*_m &= \frac{[2v(N-n)+t(n-1)](4N-2n-1)}{4N(4N-5n+k+1)+6n^2+n(3k+2)-k^2+2k} + \\ &+\frac{c\{2N[4N-4n-2k-1]+n(2n+3k)-k^2+2k\}}{4N(4N-5n+k+1)+6n^2+n(3k+2)-k^2+2k} \end{aligned}$$

$$p^*_{nm} = \frac{[2v(N-n)+t(n-1)](4N-2n-k)+c(2N-n-k)(4N-2n+k-2)}{4N(4N-5n-k-1)+6n^2+n(3k+2)-k^2+2k}$$

while equilibrium profits are, respectively:

$$\pi^*_m = \frac{k(2N-n-k)(4N-2n-1)^2[2(N-n)(v-c)+t(n-1)]^2}{tN(N-1)[4N(4N-5n+k+1)+6n^2+n(3k+2)-k^2+2k]^2}$$

$$\pi^*_{nm} = \frac{(2N-n-1)(4N-2n-k)^2[2(N-n)(v-c)+t(n-1)]^2}{tN(N-1)[4N(4N-5n+k+1)+6n^2+n(3k+2)-k^2+2k]^2}$$

These expressions drive directly to the following result:

Proposition 3 *In the spokes model, when the parameters are such that* $\frac{1}{2} < \frac{v-p^*_m}{t} < 1$, $\frac{1}{2} < \frac{v-p^*_{nm}}{t} < 1$ *and* $\frac{1}{2} < \frac{v-p^*_{bm}}{t} < 1$ *, a merger between k firms is profitable with respect to the pre-merger situation. However, the merged firms are worse off with respect to the non-merged ones. In synthesis:* $\pi^*_{nm} > \frac{\pi^*_m}{k} > \pi^*_{bm}$.

The Proof can be found in Appendix C.

The results just stated confirm that the price competition mergers paradox also takes place in Region 3. This might be less intuitive than what found in Region 1: competition in this region implies extracting all surplus from the consumers who lack a second alternative brand. In this region the elasticity of demand is larger on the monopolistic segment implying price increasing competition. However, also in this case the mechanisms highlighted by Deneckere-Davidson[5] are in operation. The best response functions are still upward sloping so that prices of both insiders and outsiders increase. As both types are better off, the free-riding property is still in operation. The fact that the focus of firms is shifted from the duopolistic region does not harm the validity of the intuition provided by Deneckere-Davidson: as outsiders face competition from both the merged entity and the other outsiders, they share with a given insider $n-2$ competitors. But the remaining competitor of the outsider firm is a firm who is member of the merged entity, charging a higher price; the insider firm, instead, faces competition by another outsider, charging a lower price. This implies that the profits of the outsiders dominate the ones of insiders as in Region 3. The following example illustrates the findings reported.

Example 3

As in Example 1, assume $N = 7$, $n = 5$, $m = 4$, $k = 2$, $v = 2$ and, as in Chen-Riordan[4], $t = 1$ and $c = 0$; the demand functions for insiders and outsiders can be then outlined following the steps in section 4.3.1. These are:

$$D_i(p_i, p_{-i}) = \frac{2}{7}\left\{\frac{1}{6}\left[\frac{1}{2} + \frac{p_m - p_i}{2}\right] + \frac{1}{6}\sum_{\alpha=1}^{3}\left[\frac{1}{2} + \frac{p_\alpha - p_i}{2}\right] + \frac{2}{6}(2 - p_i)\right\}$$

$$D_j(p_j, p_{-j}) = \frac{2}{7}\left\{\frac{1}{6}\sum_{\alpha=1}^{2}\left[\frac{1}{2} + \frac{p_\alpha - p_i}{2}\right] + \frac{1}{6}\sum_{\alpha=1..3}^{\alpha\neq j}\left[\frac{1}{2} + \frac{p_\alpha - p_i}{2}\right] + \frac{2}{6}(2 - p_i)\right\}$$

Maximizing firms' profits:

$$\pi_i = \sum_{\alpha=1}^{k} p_\alpha D_\alpha(p_\alpha, p_{-\alpha}) \qquad \pi_j = p_j D_j(p_j, p_{-j})$$

the Nash equilibrium prices are:

$$p_m = 1.074 \quad p_{nm} = 1.011$$

$$\pi_m = 0.384 \quad \pi_{nm} = 0.195$$

as compared with the pre-merger status quo, that as discussed in Section 4.2, giving the following results:

$$p_{bm} = \ 1.000 \ \ \pi_{bm} = 0.190$$

4.4 Region 4

Suppose that in equilibrium $\frac{v-p_m^*}{t} = \frac{1}{2}$ and $\frac{v-p_{nm}^*}{t} = \frac{1}{2}$: this straightforwardly implies that $p_m^* = p_{nm}^* = v - \frac{t}{2}$.

It is extremely simple to show that in equilibrium the profits of merged and non-merged firms are exactly the same. Given that equilibrium prices are identical, also the demand is identical both for merging and non-merging firm. The expression for profits is:

$$\frac{\pi_m^*}{k} = \pi_{nm}^* = \frac{2(v-c)-t}{2N}$$

This expression corresponds to the one of profits before merger.

Proposition 4 *In the spokes model, when the parameters are such that* $\frac{v-p_m^*}{t} = \frac{1}{2}$, $\frac{v-p_{nm}^*}{t} = \frac{1}{2}$ *and* $\frac{v-p_{bm}^*}{t} = \frac{1}{2}$, *a merger between* k *firms does not display the free-riding property: outsider firms are not better off with respect to the insiders. In synthesis:* $\pi_{nm}^* = \frac{\pi_m^*}{k} = \pi_{bm}^*$.

The conclusion is that also for this region of parameters the merger paradox does not arise in the spokes model: the free-riding property characterizing the equilibrium of Bertrand-like models is not operating in this equilibrium. Once more, an $\epsilon > 0$ cost synergy resulting from the merger is sufficient to explain why a merger takes place.

Together with the result for Region 2, this is one of the main findings of the paper and can be interpreted as follows: in Region 4 firms focus on the kink of their demand taking place in presence of the indifferent consumers on their own spoke, who lack a second available brand. Only consumers whose first brand is available are served: this implies a restriction of the size of the market as compared to the other equilibrium regions. However, also in this case the merger does not have an effect on any of the firms as the type of consumers they find optimal to focus on is unaffected by the change in the market configuration. Once more these result do not depend on the number of firms participating in the merger.

4.5 Discussion

The analysis of the four equilibrium regions has showed how several economic mechanisms are in operation in the spokes model. These mechanisms determine the effects of an horizontal merger between firms. The results can be roughly interpreted as follows: when genuine price competition is in operation, as in Region 1 and 3, then the "free-riding" property of the equilibrium takes place and the standard classical results highlighted by the literature are confirmed. A completely new result, however, takes place in Region 2 and 4. As in these regions firms find optimal to focus on two types of indifferent consumer, then the equilibrium prices result independent of whether a subset of firms merge or they all remain independent. The same is true for profits and this implies that an infinitesimal positive further advantage of conglomeration is sufficient to explain why mergers take place in such situations. The regions just analyzed may seem to capture a rather peculiar case. However, this is not completely true. First of all, their relevance is witnessed by the extent of the sub-space of parameters for which such equilibria take place: this is not sensibly different with respect to the remaining two regions. Moreover in Region 4 firms, by focusing on the indifferent consumer who does not have an alternative, serve only their own spoke; however, in Region 2, firms serve all consumers but the ones who have preferences for non-existing brands only. In this sense competition between firms is fully in operation. Finally, it is often observed in the business world that firms target a specific

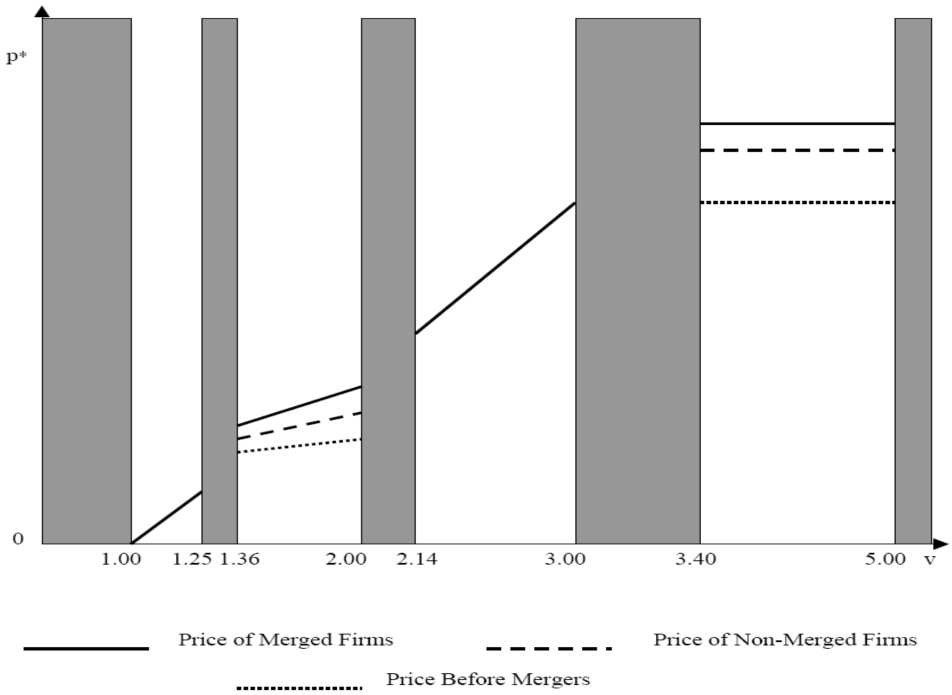

Figure 4: The effect of a merger on equilibrium *prices* for insiders and outsiders, as compared to the *status quo*.

class of indifferent consumers: this might imply that, "*mutatis mutandis*", these results may be relevant to interpret the strategic behaviour of firms in a wide set of situations.

The results just discussed are graphically summarized in Figure 4 and Figure 5. Figure 4 reports equilibrium prices for the different types of firms as a function of the valuation of the good. It is clear that both the price of merged and non-merged firms is at least as high as the pre-merger price. However, in Region 1 and Region 3 the price of merged firms exceeds the price of non-merged firms. This is the "free-riding" result, illustrated further by the comparison of profits in Figure 5: the profits after the merger are at least as high as the profits before the merger. However, in Region 1 and

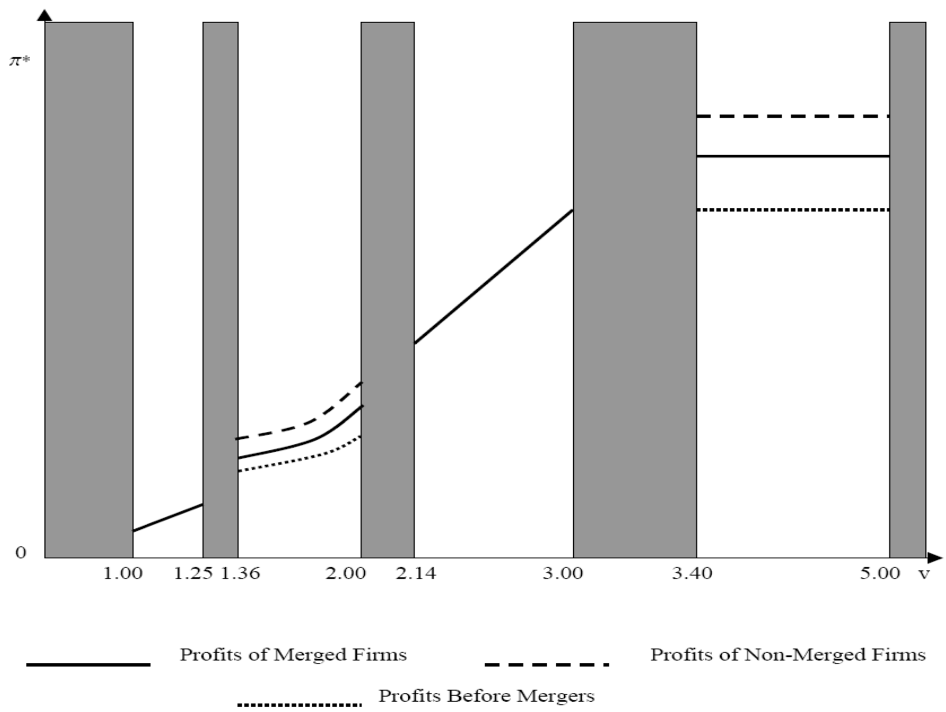

Figure 5: The effect of a merger on firms' *profits*: insiders and outsiders compared to the before merger scenario.

Region 3 the profits of non-merged firms dominate the profits of each of the firms participating in the merger.

As underlined, the equilibrium regions presented do not exhaust the space of parameters for which the equilibrium exists.We do not formally describe and analyze those regions, as in them the free-riding property is even stronger than in the regions described. The intuition is the following: in these regions, non-merged firms have an incentive to deviate to an even lower price with respect to the merged ones. This implies non-merged firms get proportionally higher profits. These regions can be identified as the shadowed areas in Figure 3, Figure 4 and Figure 5.

5 Conclusions

This paper has shown how the mergers paradox may not take place when firms are competing in prices on a spatially differentiated market. The spokes model is based on a system of preferences representing a generalization of the Hotelling linear city framework to n firms competition. It is probably the first model of non-localized spatial competition and captures the idea of Chamberlinian competition as a limiting case. In the context of the spokes model, it is established for which combinations of the parameters firms do have an incentive to merge. These are contrasted with regions in which the standard free-riding property is in operation.

The results presented may provide a possible interpretation of mergers in several markets. The spokes model builds on a system of preferences which can be seen as well approximating the structure of real geographical markets or differentiated product markets. When the geographical structure is such that there is a centre and all firms are concentrated around that, as in a city with several industrial districts in the periphery, the spokes model may seem an appropriate description of reality. This interpretation may be even more sensible in cases of product differentiation in which consumers are interested to a specific brand and are indifferent between all other brands supplied in the market. The merger waves registered in those markets may be interpreted, in the light of the result shown, on purely competitive grounds. In particular, the results obtained suggest this might be the case when firms target a specific segment of indifferent consumers.

These findings have important policy implications too. A typical concern in presence of merging activity is the price effect: concentration may imply an increase in the price level that damages consumers' welfare. In markets whose features are well described by the spokes model, however, this is not always the case. Prices for both insiders and outsiders increase if the equilibrium implies focusing on the elastic segments of the demand: in this case a merger can be questioned on consumers' welfare ground. In case, instead, that the equilibrium involves firms focusing on the inelastic segments of the demand, an antitrust authority should not be worried as the prices are not affected by the merger and there may only be positive effects linked to synergies or other types of gains in efficiency foreseen by the companies which decided to merge.

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A Appendix: the Benchmark Case

This appendix contains the derivation of the price equilibrium in the four regions in the Chen-Riordan[4] benchmark case.

Region I

Assume that: $\frac{v-p^*}{t} > 1$. The first order condition is:

$$\left\{ \frac{n-1}{N(N-1)} \left[1 + \frac{p^* - p_i}{t} \right] + \frac{N-n}{N-1} \frac{2}{N} \right\} - (p_i - c) \frac{(n-1)}{tN(N-1)} = 0$$

The best response function is:

$$p_i = \frac{1}{2}(p^* + c) + t \frac{2N - n - 1}{2(n-1)}$$

Imposing symmetry, the equilibrium price turns out to be:

$$p^* = c + t \frac{2N - n - 1}{n - 1}$$

It can be noticed that setting $t = 1$ and $c = 0$ the symmetric price equilibrium is:

$$p^* = 2 \frac{N-1}{n-1} - 1$$

consistent with the results of Chen-Riordan[4]. Clearly, the equilibrium quantity and profits are, respectively:

$$q^* = \frac{2N - n - 1}{N(N-1)} \qquad \pi^* = \frac{t(2N - n - 1)^2}{(n-1)N(N-1)}$$

Incentives to deviate globally from the candidate equilibrium must be checked. The only potentially profitable deviation is to raise price to $p = v - t$. This would ensure the firm a profit:

$$\pi(v - t) = (v - t - c) \left(\frac{2}{N} \frac{N-n}{N-1} \right)$$

Such a deviation is not profitable if $\pi \leq \pi^*$ which implies:

$$v \leq c + t \left[2 \frac{N-1}{n-1} + \frac{(2N - n - 1)}{2(N-n)} \right] = \bar{v}(N, n)$$

Region II

Suppose: $p^* = v - t$: to show that the candidate price is an equilibrium, it has to be checked that a small increase or decrease in price does not lead to an increase in profit. A small increase in price leads to Region III. The incentive to deviate is then evaluated as:

$$\frac{2}{N}\left[\frac{n-1}{N-1}\frac{1}{2}+\frac{N-n}{N-1}\right]-(v-t-c)\left[\frac{2(N-n)+(n-1)}{tN(N-1)}\right]$$

which implies:

$$v \geq c + 2t$$

A small decrease in price leads to Region I. The incentive to deviate is then evaluated as:

$$\frac{2}{N}\left[\frac{n-1}{N-1}\frac{1}{2}+\frac{N-n}{N-1}\right]-(v-t-c)\left[\frac{n-1}{tN(N-1)}\right]$$

which implies:

$$v \leq c + 2t\frac{N-1}{n-1}$$

Region III

Assume that: $\frac{1}{2} < \frac{v-p*}{t} < 1$. The first order condition is given by:

$$\left\{\frac{n-1}{2\,(N-1)}\left[1+\frac{p^*-p_i}{t}\right]+\frac{N-n}{t(N-1)}\frac{v-p_i}{t}\right\}-(p_i-c)\left[\frac{2(N-n)+(n-1)}{t(N-1)}\right]=0$$

which implies a best response function:

$$p_i = \frac{c}{2}+\frac{(n-1)(p^*+t)+2v(N-n)}{2v(2N-n-1)}$$

and, imposing symmetry, the equilibrium price:

$$p^* = \frac{2v(N-n)+t(n-1)+c(2N-n-1)}{4N-3n-1}$$

Clearly, the equilibrium quantity and profits are, respectively:

$$q^* = \frac{(2N-n-1)[2(v-c)(N-n)-t(n-1)]}{tN(N-1)(4N-3n-1)}$$

$$\pi^* = \frac{(2N-n-1)[2(v-c)(N-n)-t(n-1)]^2}{tN(N-1)(4N-3n-1)^2}$$

Region IV

Assume $p^* = v - \frac{t}{2}$: to show that the candidate price is an equilibrium, it has to be checked that a small increase or decrease in price does not lead to an increase in profit. A small increase in price leads to an equilibrium region in which the market is not fully covered and firms behave like monopolists. The incentive to deviate is then evaluated as:

$$\frac{2}{N}\left(\frac{v-p^*}{t}\right) - (p^*-c)\left[\frac{2}{tN}\right]$$

which, around $p^* = v - \frac{t}{2}$, is:

$$\frac{1}{N} - (v - \frac{t}{2} - c)\frac{2}{tN}$$

implying:

$$v \geq c + t$$

A small decrease in price leads to Region III. The incentive to deviate is then evaluated as:

$$\frac{2}{N}\left[\frac{n-1}{N-1}\frac{1}{2} + \frac{N-n}{N-1}\frac{v-\frac{t}{2}-c}{t}\right] - (v-\frac{t}{2}-c)\left[\frac{2(N-n)+(n-1)}{tN(N-1)}\right]$$

which implies:

$$v \leq c + \frac{t}{2}\frac{4N-n-3}{2N-n-1}$$

B Appendix: Identification of Equilibrium Regions

The identification of the equilibrium regions passes through the characterization of the incentives to deviate of firms from the candidate equilibrium prices. This allows to characterize the threshold values of v reported below and in the text.

$$
\begin{array}{cc}
v_{1D} = \max\{c + \frac{2t(N-1)}{n-1}, v_{IDm}, v_{IDnm}\} & v_{1U} = \min\{\bar{v}_{bm}, \bar{v}_m, \bar{v}_{nm}\} \\
v_{2D} = \max\{c + 2t, v_{IIDm}, v_{IIDnm}\} & v_{2U} = \min\{c + 2t\frac{N-1}{n-1}, v_{IIUm}, v_{IIUnm}\} \\
v_{3D} = \max\{c + \frac{t}{2}\frac{4N-n-3}{2N-n-1}, v_{IIIDm}v_{IIIDnm}\} & v_{3U} = \min\{c + 2t, v_{IIIUm}, v_{IIIUnm}\} \\
v_{4D} = \max\{c + t, v_{IVDnm}, v_{IVDnm}\} & v_{4U} = \min\{c + \frac{t}{2}\frac{4N-n-3}{2N-n-1}, v_{IVUm}, v_{IVUnm}\}
\end{array}
$$

Region 1

In this case parameters are to such that $\frac{v-p^*_{bm}}{t} > 1$, $\frac{v-p^*_m}{t} > 1$ and $\frac{v-p^*_{nm}}{t} > 1$. In order to check this is an equilibrium, it has to be shown that both merged and non-merged firms do not have an incentive to deviate to a different price. Focus on one of the merged firms, say i. First, she must not have incentives to deviate to a higher price as $p_i = v - t$ given the price of the other merging and the non-merging firms is unchanged. This is equivalent to impose the following:

$$\frac{\partial \pi_i}{\partial p_i} |_{p_i=v-t, p^*_m,\, p^*_{nm}} = (p_i - c)\frac{\partial D_i}{\partial p_i} + D_i + (k-1)(p^*_m - c)\frac{\partial D_m}{\partial p_i} |_{p_i=v-t, p^*_m,\, p^*_{nm}} \leq 0$$

Solving for v drives to the conclusion that it holds if $v \geq v_{IDm}$ where:

$$v_{IDm} = c + \frac{t(4nN - 3n - 2N - kn - k^2 - 2k + 1)}{(2n + k - 2)(n - k)}$$

Moreover, in order for an equilibrium to exist for the merged firms, following Chen-Riordan, it has to be imposed that:

$$\pi^*_i \geq \pi^D_i$$

where π^D_i is:

$$\pi^D_i = \frac{2(v - t - c)D_i(p^*_m + t, p^*_{nm})}{N}$$

Solving for v, it is found that this condition holds for $v \leq \bar{v}_m(N, n, k, t, c)$. It follows that the equilibrium for merging firms exist for:

$$v_{IDm} \leq v \leq \bar{v}_m$$

Turn now to non-merging firms. If one of the non-merged firms, say j, tries to deviate it will raise price to $p_j = v - t$

$$\frac{\partial \pi_j}{\partial p_j} \mid_{p_m^*, p_{nm}^*, p_j = v-t} = (p_j - c)\frac{\partial D_j}{\partial p_j} + D_j \mid_{p_m^*, p_{nm}^*, p_j = v-t} \leq 0$$

Solving for v drives to the conclusion that it holds if $v \geq v_{IDnm}$ where:

$$v_{IDnm} = c + \frac{t(4Nn - 2kN - 4n - k^2 + 3k)}{(2n + k - 2)(n - k)}$$

Moreover, in order for an equilibrium to exist for the merged firms, following Chen-Riordan, it has to be imposed that:

$$\pi_j^* \geq \pi_j^D$$

where π_j^D is:

$$\pi_j^D = \frac{2(v - t - c)D_j(p_{nm}^* + t, p_m^*)}{N}$$

Solving for v, it is found that this condition holds for $v \leq \bar{v}_{nm}(N, n, k, t, c)$. It follows that the equilibrium for non-merging firms exist for:

$$v_{IDnm} \leq v \leq \bar{v}_{nm}$$

From the benchmark case, it is possible to recall that equilibrium in Region I was defined for:

$$c + \frac{2t(N-1)}{n-1} < v \leq \bar{v}(N, n)$$

Bringing together all the information, it is verified that an equilibrium which is incentive compatible before and after merger and for both merged and non-merged firms exist for the following subset of values of v:

$$v_{1D} = \max\{c + \frac{2t(N-1)}{n-1}, v_{IDm}, v_{IDnm}\} \leq v \leq \min\{\bar{v}_{bm}, \bar{v}_m, \bar{v}_{nm}\} = v_{1U}$$

Region 2

In this case the parameters are such that: $\frac{v-p^*_{bm}}{t} = 1$, $\frac{v-p^*_m}{t} = 1$ and $\frac{v-p^*_{nm}}{t} = 1$. Focus first on a given merging firm, say i. It must be ruled out that she has an incentive to raise her price to $p_i > v - t$ or decrease it to $p_i < v - t$. Consider a price increase, in that case the demand faced by the firm is as if she was in Region III, given the other firms stick to their equilibrium prices. This is not profitable if:

$$\frac{\partial \pi_i}{\partial p_i} |_{p_i > v-t, p^*_m,\, p^*_{nm}} = (p_i - c)\frac{\partial D_i}{\partial p_i} + D_i + (k-1)(p^*_m - c)\frac{\partial D_m}{\partial p_i} |_{p_i > v-t, p^*_m,\, p^*_{nm}} \leq 0$$

which implies:

$$v \geq v_{IIDm} = c + t\frac{4N - 2n - k - 1}{2N - n - k}$$

Consider instead a price decrease, in that case the demand faced by firm i is as if they were in Region I, given the other firms stick to their equilibrium prices. This is not profitable if:

$$\frac{\partial \pi_i}{\partial p_i} |_{p_i < v-t, p^*_m,\, p^*_{nm}} = (p_i - c)\frac{\partial D_i}{\partial p_i} + D_i + (k-1)(p^*_m - c)\frac{\partial D_m}{\partial p_i} |_{p_i < v-t, p^*_m,\, p^*_{nm}} \leq 0$$

implying:

$$v \leq v_{IIUm} = c + t\frac{2N - k - 1}{n - k}$$

Turn now to non-merging firms. An analogous reasoning allows to rule out possible deviations. Suppose in particular that firm j raises her price to $p_j > v - t$. In that case the demand faced by the firm is as if she was in Region III. In order for this not to be profitable it must be:

$$\frac{\partial \pi_j}{\partial p_j} |_{p^*_m, p^*_{nm}, p_j > v-t} = (p_j - c)\frac{\partial D_j}{\partial p_j} + D_j |_{p^*_m, p^*_{nm}, p_j > v-t} \leq 0$$

which in turn implies:

$$v \geq v_{IIDnm} = c + 2t$$

If firm j decreases her price, instead, to $p_j < v - t$. In that case the demand faced by the firm is as if she was in Region I. In order for this not to be profitable it must be:

$$\frac{\partial \pi_j}{\partial p_j} |_{p^*_m, p^*_{nm}, p_j < v-t} = (p_j - c)\frac{\partial D_j}{\partial p_j} + D_j |_{p^*_m, p^*_{nm}, p_j < v-t} \leq 0$$

implying:

$$v \leq v_{IIUnm} = c + 2t\frac{N-1}{n-1}$$

From the benchmark case, it is possible to recall that equilibrium in Region II was defined for:

$$c + 2t < v \leq c + 2t\frac{N-1}{n-1}$$

Bringing together all the information, it is verified that an equilibrium which is incentive compatible before and after merger and for both merged and non-merged firms exist for the following subset of values of v:

$$v_{2D} = \max\{c+2t, v_{IIDm}, v_{IIDnm}\} \leq v \leq \min\{c+2t\frac{N-1}{n-1}, v_{IIUm}, v_{IIUnm}\} = v_{2U}$$

Region 3

In this case the parameters are such that: $\frac{1}{2} < \frac{v-p^*_{bm}}{t} < 1$, $\frac{1}{2} < \frac{v-p^*_m}{t} < 1$ and $\frac{1}{2} < \frac{v-p^*_{nm}}{t} < 1$. It has to be checked that at the proposed equilibrium prices there are no incentives to deviate. In this case there are two possible deviations possibilities for the merged firms and for the non-merged: they can potentially deviate either to $p = v - t$ or to $p = v - \frac{t}{2}$.

Consider first one of the merging firms, say i. Suppose she raises the price to $p_i = v - \frac{t}{2}$. For the deviation not to be profitable the following must be imposed:

$$\frac{\partial \pi_i}{\partial p_i} |_{p_i = v - \frac{t}{2}, p^*_m, \, p^*_{nm}} = (p_i - c)\frac{\partial D_i}{\partial p_i} + D_i + (k-1)(p^*_m - c)\frac{\partial D_m}{\partial p_i} |_{p_i = v - \frac{t}{2}, \, p^*_m, p^*_{nm}} \leq 0$$

Solving for v drives to the conclusion that it holds if $v \geq v_{IIIDm}$ where:

$$v_{IIIDm} = c + \frac{t}{2}\frac{4N(4N-3n-k-3) + n(2n+3k) + 2(k+1)}{2N(4N-4n-2k-1) + n(2n+3k) - k^2 + 2k}$$

On the other hand, if the firm decreases her price to $p_m = v - t$ then it has to be imposed that:

$$\frac{\partial \pi_i}{\partial p} |_{p_i = v-t, \, p^*_m, p^*_{nm}} = (p_1 - c)\frac{\partial D_1}{\partial p_1} + D_1 + (p_2 - c)\frac{\partial D_2}{\partial p_2} + D_2 \, |_{p_i = v-t, p^*_m, \, p^*_{nm}} \geq 0$$

Solving for v drives to the conclusion that it holds if $v \leq v_{IIIUm}$ where:

$$v_{IIIUm} = c + t\frac{4N(4N-4n-k-2)+n(4n+3k+3)-k^2+2k+1}{2N(4N-4n-2k-1)+n(2n+3k)-k^2+2k}$$

It follows that the equilibrium for merging firms exist for:

$$v_{IIIDm} \leq v \leq v_{IIIUm}$$

Turn now to non-merging firms. If one of the non-merged firms, say j, tries to deviate it will cut her price to $p_j = v - t$ then for the deviation not to be profitable it must be:

$$\frac{\partial \pi_j}{\partial p_j} \mid_{p_m^*, p_{nm}^*, p_j = v-t} = (p_j - c)\frac{\partial D_j}{\partial p_j} + D_j \mid_{p_m^*, p_{nm}^*, p_j = v-t} \geq 0$$

Solving for v drives to the conclusion that it holds if $v \leq v_{IIIUnm}$ where:

$$v_{IIIUnm} = c + \frac{t}{2}\frac{4N(4N-3n-k-3)+n(2n+k+6)-k^2+4k}{2N(4N-4n-k-2)+n(2n+k+2)-k^2+2k}$$

On the other hand, if j raises her price to $p_j = v - \frac{t}{2}$ then the following has to be imposed:

$$\frac{\partial \pi_j}{\partial p_j} \mid_{p_m^*, p_{nm}^*, p_j = v-\frac{t}{2}} = (p_j - c)\frac{\partial D_j}{\partial p_j} + D_j \mid_{p_m^*, p_{nm}^*, p_j = v-\frac{t}{2}} \leq 0$$

Solving for v drives to the conclusion that it holds if $v \geq v_{IIIDnm}$ where:

$$v_{IIIDnm} = c + t\frac{4N(4N-4n-k-2)+2n(2n+k+2)-k^2+3k}{2N(4N-4n-k-2)+n(2n+k+2)-k^2+2k}$$

It follows that the equilibrium for non-merging firms exist for:

$$v_{IIIDnm} \leq v \leq v_{IIIUnm}$$

From the benchmark case, it is possible to recall that equilibrium in Region I was defined for:

$$c + \frac{t}{2}\frac{4N-n-3}{2N-n-1} < v \leq c + 2t$$

Bringing together all the information, it is verified that an equilibrium which is incentive compatible before and after merger and for both merged and non-merged firms exist for the following subset of values of v:

$$v_{3D} = \max\{c+\frac{t}{2}\frac{4N-n-3}{2N-n-1}, v_{IIIDm}, v_{IIIDnm}\} \leq v \leq \min\{c+2t, v_{IIIUm}, v_{IIIUnm}\} = v_{3U}$$

Region 4

In this case the parameters are such that: $\frac{v-p^*_{bm}}{t} = \frac{1}{2}$, $\frac{v-p^*_{m}}{t} = \frac{1}{2}$ and $\frac{v-p^*_{nm}}{t} = \frac{1}{2}$. Focus first on a given merging firm, say i. It must be ruled out that she has an incentive to raise her price to $p_i > v - \frac{t}{2}$or decrease it to $p_i < v - \frac{t}{2}$. Consider a price increase, in that case the demand faced by the firm is as if she was a local monopolist, given the other firms stick to their equilibrium prices. This is not profitable if:

$$\frac{\partial \pi_i}{\partial p_i} |_{p_i > v-\frac{t}{2}, p^*_m,\, p^*_{nm}} = (p_i - c)\frac{\partial D_i}{\partial p_i} + D_i + (k-1)(p^*_m - c)\frac{\partial D_m}{\partial p_i} |_{p_i > v-\frac{t}{2}, p^*_m,\, p^*_{nm}} \leq 0$$

which implies:

$$v \geq v_{IIDm} = c + t$$

Consider instead a price decrease, in that case the demand faced by firm i is as if they were in Region III, given the other firms stick to their equilibrium prices. This is not profitable if:

$$\frac{\partial \pi_i}{\partial p_i} |_{p_i > v-\frac{t}{2}, p^*_m,\, p^*_{nm}} = (p_i - c)\frac{\partial D_i}{\partial p_i} + D_i + (k-1)(p^*_m - c)\frac{\partial D_m}{\partial p_i} |_{p_i < v-\frac{t}{2}, p^*_m,\, p^*_{nm}} \leq 0$$

implying:

$$v \leq v_{IVUm} = c + \frac{t}{2}\frac{4N-n-k-2}{2N-n-1}$$

Turn now to non-merging firms. An analogous reasoning allows to rule out possible deviations. Suppose in particular that firm j raises her price to $p_j > v - \frac{t}{2}$. In that case the demand faced by the firm is the one of a local monopolist. In order for this not to be profitable it must be:

$$\frac{\partial \pi_j}{\partial p_j} |_{p^*_m, p^*_{nm}, p_j > v-\frac{t}{2}} = (p_j - c)\frac{\partial D_j}{\partial p_j} + D_j |_{p^*_m, p^*_{nm}, p_j > v-\frac{t}{2}} \leq 0$$

which in turn implies:

$$v \geq v_{IVDnm} = c + t$$

If firm j decreases her price, instead, to $p_j < v - \frac{t}{2}$. In that case the demand faced by the firm is as if she was in Region III. In order for this not to be profitable it must be:

$$\frac{\partial \pi_j}{\partial p_j} \mid_{p^*_m, p^*_{nm}, p_j < v - \frac{t}{2}} = (p_j - c)\frac{\partial D_j}{\partial p_j} + D_j \mid_{p^*_m, p^*_{nm}, p_j < v - \frac{t}{2}} \leq 0$$

implying:

$$v \leq v_{IVUnm} = c + \frac{t}{2}\frac{4N - n - 3}{2N - n - 1}$$

From the benchmark case, it is possible to recall that equilibrium in Region II was defined for:

$$c + t < v \leq c + \frac{t}{2}\frac{4N - n - 3}{2N - n - 1}$$

Bringing together all the information, it is verified that an equilibrium which is incentive compatible before and after merger and for both merged and non-merged firms exist for the following subset of values of v:

$$v_{4D} = \max\{c+t, v_{IVDm}, v_{IVDnm}\} \leq v \leq \min\{c+\frac{t}{2}\frac{4N - n - 3}{2N - n - 1}, v_{IVUm}, v_{IVUnm}\} = v_{4U}$$

C Appendix: Proofs

Proof of Proposition 1

It has to be established that: $\pi^*_{nm} > \frac{\pi^*_m}{k} > \pi^*_{bm}$. Start with: $\pi^*_{nm} > \frac{\pi^*_m}{k}$. The profit differential:

$$\begin{aligned} \pi^*_{nm} - \frac{\pi^*_m}{k} &= \frac{t(2n-k)(n-1)(2N-n-1)[4Nn + (n+1)(k-2n)]}{N(N-1)[k^2 + k(n-2) - 2n(n-1)]^2} \\ &- \frac{t[2N(2n-1) - 2n^2 - n + 1]^2}{N(N-1)(2n+k-2)[k^2 + k(n-2) - 2n(n-1)]} \end{aligned}$$

can be rearranged to give:

$$\pi^*_{nm} - \frac{\pi^*_m}{k} = t\frac{(k-1)[n(k+1) - k](2N - n - 1)^2}{(2n + k - 2)^2(N-1)N(n-k)^2}$$

which is undoubtedly positive under the assumptions adopted. This implies that $\pi^*_{nm} > \frac{\pi^*_m}{k}$ for all the feasible combinations parameters.

Turn now to $\frac{\pi^*_m}{k} > \pi^*_{bm}$:

$$\frac{\pi^*_m}{k} - \pi^*_{bm} = \frac{t[2N(2n-1) - 2n^2 - n + 1]^2}{N(N-1)(2n+k-2)[k^2 + k(n-2) - 2n(n-1)]} - \frac{t(2N-n-1)^2}{(n-1)N(N-1)}$$

can be re-expressed as:

$$\frac{\pi^*_m}{k} - \pi^*_{bm} = t\frac{(k-1)[n(3k-1) + k^2 - 3k + 1](2N-n-1)^2}{(N-1)N(2n+k-2)^2(n-k)(n-1)}$$

which is surely positive under the assumptions made. This establishes that $\frac{\pi^*_m}{k} > \pi^*_{bm}$ for all the feasible combinations parameters.

Given that $\pi^*_{nm} > \frac{\pi^*_m}{k}$ then *a fortiori* $\pi^*_{nm} > \pi^*_{bm}$. *Q.E.D.*

Proof of Proposition 3

It has to be established that: $\pi^*_{nm} > \frac{\pi^*_m}{k} > \pi^*_{bm}$. Start with: $\pi^*_{nm} > \frac{\pi^*_m}{k}$. The profit differential:

$$\begin{aligned}\pi^*_{nm} - \frac{\pi^*_m}{k} &= \frac{(2N-n-1)(4N-2n-k)^2[2(N-n)(v-c) + t(n-1)]^2}{tN(N-1)[4N(4N-5n+k+1) + 6n^2 + n(3k+2) - k^2 + 2k]^2} \\ &\quad - \frac{(2N-n-k)(4N-2n-1)^2[2(N-n)(v-c) + t(n-1)]^2}{tN(N-1)[4N(4N-5n+k+1) + 6n^2 + n(3k+2) - k^2 + 2k]^2}\end{aligned}$$

can be rearranged to give:

$$\pi^*_{nm} - \frac{\pi^*_m}{k} = \frac{(k-1)[(2N-n)(k-1) - k][2(N-n)(v-c) + t(n-1)]^2}{tN(N-1)[4N(4N-5n-k-1) + 6n^2 + 2n + 3kn - k^2 + 2k]^2}$$

which is undoubtedly positive under the assumptions adopted. This implies that $\pi^*_{nm} > \frac{\pi^*_m}{k}$ for all the feasible combinations parameters.

Turn now to $\frac{\pi^*_m}{k} > \pi^*_{bm}$:

$$\begin{aligned}\frac{\pi^*_m}{k} - \pi^*_{bm} &= \frac{(2N-n-k)(4N-2n-1)^2[2(N-n)(v-c) + t(n-1)]^2}{tN(N-1)[4N(4N-5n+k+1) + 6n^2 + n(3k+2) - k^2 + 2k]^2} \\ &\quad - \frac{(2N-n-1)[2(v-c)(N-n) + t(n-1)]^2}{tN(N-1)(4N-3n-1)^2}\end{aligned}$$

can be re-expressed as:

$$\frac{\pi^*_m}{k} - \pi^*_{bm} = \frac{A(N,n,k)[2(N-n)(v-c)+t(n-1)]^2(k-1)}{tN(N-1)(4N-3n-1)^2[4N(4N-5n+k+1)+6n^2+n(3k+2)-k^2+2k]^2}$$

where $A(N, n, k)$ is a long and uninteresting positive expression under the assumptions made. The previous expression is then positive itself. This establishes that $\frac{\pi^*_m}{k} > \pi^*_{bm}$ for all the feasible combinations parameters.

Given that $\pi^*_{nm} > \frac{\pi^*_m}{k}$ then *a fortiori* $\pi^*_{nm} > \pi^*_{bm}$. *Q.E.D.*