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Vertical Production and Macroeconomic Persistence: The Case of an Emerging Market Economy

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Abstract

Empirical studies reveal persistence of macroeconomic variables to nominal shocks. However, theoretical models fail to match the data. This paper develops a Dynamic Stochastic General Equilibrium (DSGE) model with vertical input-output production, imperfect competition and staggered prices at each stage of production to reconcile theoretical models with empirical observations. We find that output response to stage-specific technological change and demand shock is more persistent the greater the number of production stages and the larger the share of intermediate goods in final good production. Depending on the source of technological change, we may either have contractionary or expansionary impact on macroeconomic variables.

JEL classification: E31; O33

Keywords: Vertical production chain; Staggered price contracts; Persistence; Technological change

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1 Introduction

New Neoclassical Synthesis models are unable to generate persistent responses of real economic variables to nominal shocks, as recently stressed by Chari, Kehoe and McGrattan (2000).[1] In calculating the "contract multiplier", which measures how much staggered price-setting increases the persistence of output relative to synchronised price-setting, they find that sticky prices in the presence of microeconomic foundations do not generate persistence. To solve the persistence problem, Dotsey and King (2006) propose various production-side features or "real flexibilities" in a general equilibrium framework. These features result in smaller variations in marginal cost, inducing smaller price adjustments to changes in aggregate demand. Hence, small price adjustments reinforce endogenous rigidity in nominal prices, and increase output persistence. This paper focuses on one of these "real flexibilities", namely the role of produced intermediate inputs in generating macroeconomic persistence to nominal shocks.

Dotsey and King's (2006) proposal introducing intermediate inputs captures firms' heterogeneity and stage-specific price dynamics as asserted by Bils and Klenow (2004). Among Bils and Klenow (2004) findings that goods with little value added display more frequent price change compared to goods with more value added. In plotting the rate of change in consumer price index (CPI) versus producer price index (PPI) for a selection of Emerging Market Economies (EMEs), Figure 1 shows that PPI displays more frequent change than CPI. This supports the role played by the intermediate sector in generating macroeconomic persistence is response to nominal shocks.

The model contributes to the literature by examining the transmission of nominal shocks, namely stage-specific productivity and demand shocks, in a Dynamic Stochastic General Equilibrium (DSGE) framework with vertical production structure, imperfect competition and staggered prices, and calibrating the model to EMEs.[2] We study the transmission mechanism of these nominal shocks and the multiplier effect of the intermediate good in driving price rigidity and macroeconomic persistence in these economies. In addition, we address the question whether a two-stage model can generate the macroeconomic persistence missing from a one-stage production model. We find that vertical input-output production structure interacts with staggered prices to deliver its

[1] Goodfriend and King (1997) define "New Neoclassical Synthesis" models as those that combine New Keynesian mechanisms of imperfect competition and staggered prices with real business cycle models' microeconomic foundations.

[2] In the paper the term "technological change" and productivity shocks are used interchangebly. Similarly, "preference" and demand shocks are used alternatively.

promise for generating macroeconomic persistence. Moreover, we conclude that depending on the source of technological change, there may either be contractionary or expansionary impact on macroeconomic variables.

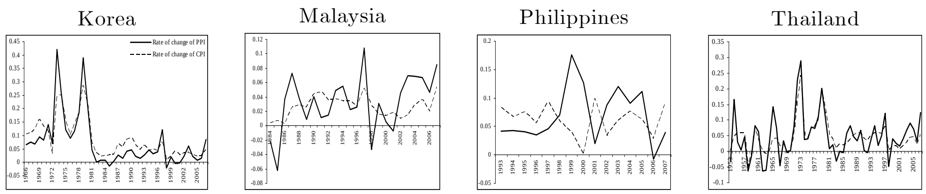

Fig. 1. *Rate of Change in CPI versus PPI for a Selection of EMEs*
Source: IMF, International Financial Statistics Yearbook

Incorporating empirically relevant production-side features in macroeconomic models, particularly the role of intermediate goods in production, goes back to an early study by Means (1936). Means (1936) provides evidence that the greater the interdependence of firms in an input-output chain, the higher the aggregate fluctuations in the economy, due to different industries having varying patterns of price and quantity changes. This finding triggered a wide range of literature to examine the interactions between input-output structure and aggregate fluctuations. Among these is Basu (1995), using a roundabout input-output production structure with imperfect competition studies the U.S. business cycle. The main factor driving Basu's results is intermediate goods acting as a multiplier for price stickiness, where price rigidity at an individual firm level leads to a large degree of economy-wide price stickiness.

Studies by Kydland and Prescott (1982) and Prescott (1986) stress the role of technology shocks as a major source of short-run fluctuations facing an economy. With the notable exception of Blanchard (1983), and Huang and Liu (2001, 2005), little theoretical work investigates technological change and demand shock transmission mechanisms embodied in a vertical input-output structure. Blanchard (1983) presents the production chain with staggered prices at various stages of production, and describes the price adjustment process by a 'snake effect' where staggered price decisions affect both the dynamics of the price level as well as the relative price structure of factor inputs along the production chain. Extending Blanchard's (1983) multiple stages of production and using intermediate inputs, Huang and Liu (2001, 2005) construct a general equilibrium model with price rigidity, imperfect competition among firms and optimising agents to study monetary policy shocks and optimal monetary policy, respectively. However, Blanchard (1983) features a non-optimising model while Huang and Liu (2001, 2005) focus solely on a monetary policy shock.

In constructing the model, we follow the lead of Blanchard (1983) and the recent extensions by Huang and Liu (2001, 2005) in presenting the production chain with staggered prices at various stages of production. However, our model differs from others by examining the transmission mechanism of stage-specific technology and demand shocks in the presence of vertical input-output production chain as well as the multiplier effect of intermediate good in driving price rigidity and macroeconomic persistence in EMEs. The paper conducts several robustness checks to key parameters, namely the share of intermediate goods in production and the degree of nominal rigidity, to study how sensitive the results are to various specifications of these parameters.

We present two alternative models to study the dynamic effects of stage-specific technology and demand shocks. Model I acts as a benchmark model with one-stage of production and price rigidity. In the benchmark model, the effect of the productivity shock does not last beyond the initial contract duration, allowing firms to quickly adjust their marginal cost and in turn prices, which results in insignificant persistence in aggregate output. Model II is characterised by a two-stage input-output production structure. In the case of multiple stages of production, finished-good firms face smaller changes in their marginal cost and thus have smaller incentives to change their prices than do intermediate-good firms. Therefore, the model is in accordance with Means (1936) main finding that prices movements are dampened through the production chain. Thus, the response of aggregate output in the two-stage production model dies out gradually, generating the macroeconomic persistence missing from the one-stage production model.

We calibrate Models I and II to a subset of EMEs, specifically South Korea, Malaysia, and Thailand. The simulation results are consistent with Blanchard (1983) findings that goods' prices at early stages of production are more volatile compared to goods further down the production chain, where the intermediate good price is almost twice as volatile as its finished good counterpart. Moreover, based on the variance decomposition from the two-stage production structure of Model II, the intermediate-stage technology shock accounts for the majority of macroeconomic fluctuations, contributing to 66 percent of aggregate output fluctuations. In contrast, finished-stage technology and demand shocks contribute to aggregate output fluctuations by 15 percent and 19 percent, respectively. In addition, the adverse response of hours worked to stage-specific technology shocks in Model II is broadly consistent with Galí (1999) and Basu *et al.* (2006) results.

The paper is organised as follows. Section 2 presents the vertical input-output production structure model with price rigidity and market clearing conditions. Section 3 examines the flexible-price and sticky-price equilibrium dynamics. Section 4 presents the benchmark parameterisation of the model. Section 5 analyses the simulation results and conducts robustness exercises. Section 6 concludes.

2 A Model with Vertical Production Chain

2.1 *Outline of the Model*

The economy is characterised by a two stages of production, where firms are linked in a vertical input-output production chain and each is subject to a stage-specific technology shock. At each stage of production there is a continuum of firms producing differentiated goods. There are two types of monopolistically competitive firms producing finished and intermediate goods. The production of the intermediate good requires homogeneous labour input provided by a representative household.

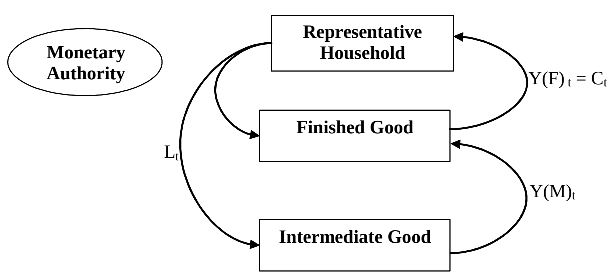

Fig. 2. *Flow Chart of the Economy with Vertical Production Chain*

The intermediate good is aggregated through a constant elasticity of substitution (CES) production function by perfectly competitive firms to yield a composite intermediate good. The production of the finished good requires a composite intermediate good and labour services. The finished good is aggregated through a CES production function by perfectly competitive firms to yield a composite finished good. The optimal price decision for all firms is modelled in the spirit of Calvo (1983). The economy is inhabited by infinitely lived households, whose preferences are defined over a composite finished good and leisure and subject to a preference shock. Figure 2 depicts a flow chart of the input-output production structure of goods, labor, and income in the modelled EME. The monetary authority sets the nominal interest rate based on a Taylor-type rule.

2.2 *Households*

The economy is composed of a continuum of infinitely-lived individuals, whose total is normalised to unity. Households are assumed to have identical preferences over the consumption of goods and supply of labor. The paper adopts the simplifying assumption that money plays the role of a unit of account, in terms of which prices of goods and labour services are quoted. Hence, money does not appear in either the consumer's utility function or the budget constraint.[3]

The objective of the representative household is to maximize the discounted sum of the expected utility obtained from consuming goods C_t and supplying labour services L_t. Utility is additively separable in consumption and leisure and subject to a preference shock. The preference shock shifts the marginal utility of goods and marginal disutility of labour. The representative consumer's instantaneous utility function is given by:

$$U(C,L) \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\xi_t C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

where E is an expectation operator, C_t is real consumption, and L_t denotes labour hours supplied. The preference shock is denoted by ξ_t and follows a first-order autoregressive process $\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t^\xi$, where ε_t^ξ is serially uncorrelated independent and identically distributed process with mean-zero, standard error σ_ξ, and $\rho_\xi < 1$. Household related structural parameters are: $0 < \beta < 1$ is the subjective discount factor, σ the coefficient of relative risk aversion in consumption, $\varphi \geqslant 1$ is the intertemporal elasticity of marginal disutility with respect to labor supply, and χ is coefficient on labor supply in the utility function.

The composite consumption good C_t consists of differentiated finished goods $j \in (0,1)$ denotes the good variety. The aggregate consumption for the finished good is represented by a CES function defined by:

$$C_t \equiv \left[\int_0^1 y_{j,t}^{f} \,{}^{\frac{\theta^f-1}{\theta^f}} dj \right]^{\frac{\theta^f}{\theta^f-1}} \tag{2}$$

where $\theta^f > 1$ denotes the elasticity of substitution between differentiated finished goods in the composite consumption good.

The household's objective is to maximise its utility subject to a series of budget constraints for period $t = 0, 1, 2, ..., \infty$, given by:

$$\int_0^1 P_{j,t}^f C_{j,t} + E_t Q_{t,t+1} B_{t+1} \leq W_t L_t + \Pi_t^f + B_t - T_t \tag{3}$$

[3]This modelling strategy has been adopted by recent work, such as Galí and Monacelli (2005), and McCallum and Nelson (2000), among others.

The budget constraint implies that the representative household consumes differentiated finished goods j at the price $P^f_{j,t}$ and holds one-period nominally riskless discount bonds B_t. To isolate the role of vertical production chain in the transmission of productivity and demand shocks and for simplicity, the model assumes a complete financial market. As such, a unique stochastic discount factor is guaranteed to exist, which is denoted by $Q_{t,t+1}$ for a one-period riskless bond purchased in period t to mature in period $t+1$. Household consumption and bond holding decisions are financed by it's total wage income given by W_tL_t, net profits from firms in each period Π^f_t after deducting lump sum taxes T_t, and bonds acquired at period t to mature at period $t+1$, which represents the nominal value of financial wealth that the household takes into period t.

Optimisation implies that the representative household exhausts its intertemporal budget constraint. Thus, the representative household is subject to the following transversality condition or no-Ponzi game, a necessary condition for optimality as is eliminates the possibility of households financing consumption indefinitely by borrowing:

$$\lim_{s\to\infty} E_t\left[\Pi_{s=0}^{t+s-1}\left(\frac{1}{1+r_s}\right)B_{t+s}\right]=0 \tag{4}$$

where R_t is defined as the gross nominal risk-free rate, hence, the following relation must hold $R_tE_tQ_{t,t+1}=1$, with $R_t=1+r_t$.

The optimal allocation of any given expenditure in the composite consumption good C_t yields the price index for the composite good or consumer price index (CPI) and equivalent to the aggregate finished good price, and the demand for the differentiated finished good, respectively:

$$P^f_t=\left[\int_0^1 P^f_{j,t}{}^{1-\theta^f}dj\right]^{\frac{1}{1-\theta^f}} \tag{5}$$

$$Y^f_{j,t}=\left(\frac{P^f_{j,t}}{P^f_t}\right)^{-\theta^f}C_t \tag{6}$$

where the differentiated finished good $Y^f_{j,t}$ output is equivalent to $C_{j,t}$ being the demand for these differentiated products. Equation (6) implies that the more expensive is good j relative to other finished goods the lower is the relative demand for good j.

Finally, the household is to choose a strategy $\{C_t,l_t,B_t\}_{t=0}^{t=\infty}$ to maximize its expected lifetime utility defined by equation (1) subject to an intertemporal budget constraint in equation (3) and transversality condition (4). Solving the household utility maximisation problem yields the following optimality conditions:

$$\chi \frac{L_t^{\varphi}}{\xi_t C_t^{-\sigma}} = \frac{W_t}{P_t^f} \tag{7}$$

$$\xi_t C_t^{-\sigma} = \beta E_t \left[\xi_{t+1} C_{t+1}^{-\sigma} R_t \frac{P_t^f}{P_{t+1}^f} \right] \tag{8}$$

Equation (7) represents the labour supply decision by the household, and equation (8) is the stochastic Euler equation for the purchase of domestic currency bonds.

2.3 *Vertical Production Chain*

We follow Huang and Liu (2005) in characterising the production chain as composed of two sectors, each consisting of a continuum of firms producing differentiated goods. While Huang and Liu (2005) study monetary policy, we introduce and study the transmission mechanism of stage-specific technology and demand shocks in a vertical input-output production chain. Finished goods are consumed by households, while intermediate goods are used as factor inputs in the production of the finished good. All firms are price takers in the input market. Wages are determined in a competitive labour market. Firms in both stages engage in monopolistic competition and their optimal price decisions are set à la Calvo (1983).

2.3.1 *Composite Finished Good*

The aggregate finished good Y_t^f is a composite of differentiated finished goods $Y_{j,t}^f$ indexed by $j \in (0,1)$, where $Y_{j,t}^f$ denotes the j^{th} finished good. Thus, the goods are combined into an output index making use of the following CES technology:

$$Y_t^f = \left[\int_0^1 Y_{j,t}^f{}^{\frac{\theta^f - 1}{\theta^f}} dj \right]^{\frac{\theta^f}{\theta^f - 1}} \tag{9}$$

where $\theta^f > 1$ is the elasticity of substitution between differentiated finished goods, and the term $\left[\theta^f / \left(\theta^f - 1\right)\right]$ represents the price markup rate.

Firms producing the composite finished good through cost minimisation problem obtain the demand function for each of the differentiated good $Y_{j,t}^f$, which is equivalent to household demand for the finished good as derived in equation (6). Equation (6) is rewritten as:

$$Y_{j,t}^f = \left(\frac{P_{j,t}^f}{P_t^f} \right)^{-\theta^f} Y_t^f \tag{10}$$

where the corresponding price of the aggregate finished good P_t^f or the minimum cost per unit of output given individual good's price $P_{j,t}^f$, which is equivalent to household CPI derived in equation (5), is given by:

$$P_t^f = \left[\int_0^1 P_{j,t}^f \quad {}^{1-\theta^f} dj \right]^{\frac{1}{1-\theta^f}}$$

2.3.2 *Differentiated Finished Goods' Firms and Optimal Price Setting*

Each finished good producing firm produces a differentiated good indexed by $j \in (0, 1)$. A typical firm j uses a Cobb-Douglas production function to combine homogenous labour services $l_{j,t}^f$ and an intermediate good $Y_{j,t}^m$ using a constant returns to scale (CRS) technology characterised by diminishing marginal product and constant elasticity of substitution, given by:

$$Y_{j,t}^f = Y_{j,t}^m \quad {}^{\phi} \left[A_t^f l_{j,t}^f \right]^{1-\phi} \tag{11}$$

where ϕ is the share of intermediate inputs in total factor inputs of finished good production, (log) productivity $\log A_t^f = a_t^f$ is a stage-specific labour augmenting technology shock identical for all finished good producers. The technology shock follows a first-order autoregressive process, where $a_t^f = \rho_f a_{t-1}^f + \varepsilon_t^f$. In specifying the technology shock, we assume no growth trend in productivity. Thus, the technology factor follows a log-stationary process, where ε_t^f is a white noise process, independent of all other shocks with variance σ_f^2, and a persistence coefficient $\rho_f < 1$.

Firms are price takers in the input market and monopolistic competitors in the finished good market. Each firm meets a downward sloping demand curve given by equation (6). Without loss of generality and assuming symmetry among firms, the cost minimisation problem of the producer yields the demand for labour services and intermediate factor inputs given respectively by:

$$l_{j,t}^{f} = (1-\phi)\,\frac{V_t^f}{W_t} Y_{j,t}^{f(d)} \tag{12}$$

$$Y_{j,t}^{m} = \phi \frac{V_t^f}{P_t^m} \left(\frac{P_{i,t}^m}{P_t^m}\right)^{-\theta^m} Y_{j,t}^{f(d)} \tag{13}$$

$$V_t^f = \Phi P_t^m{}^{\phi} \left(\frac{W_t}{A_t^f}\right)^{1-\phi} \tag{14}$$

where V_t^f defined by equation (14) is the nominal marginal cost per unit of finished good production, θ^m is the elasticity of substitution between differentiated intermediate goods indexed by i, and P_t^m is the price of the intermediate good. An important mechanism at work is the productivity multiplier associated with the intermediate good, represented by $\Phi = \left[1/\left(\phi^{\phi}(1-\phi)^{1-\phi}\right)\right]$. The multiplier depends on the share of the intermediate factor input in producing the finished good. Based on our parameterisation for EMEs and due to the importance of intermediate good in EMEs production, the productivity multiplier is calculated to be 2. Thus, linkages between the two stages of production through the share of intermediate goods creates a productivity multiplier over the whole production chain.

As is standard in New-Keynesian literature and given the assumption of monopolistic competition and staggered prices, each firm producing finished goods set their price in a staggered fashion in the spirit of Calvo (1983). In this framework, $(1-\alpha_f)$ fraction of firms adjust their prices optimally, where α_f is the probability that firm j does not change its price in period t. Hence, firms resetting the price choose the new price $P_t^{f(*)}$ to maximise their expected present value of future stream of real profit given by equation (15), subject to production technology (11), and meeting the demand for the good (6), as follows:

$$\Pi_{t+s}^{f} = E_t \sum_{s=0}^{\infty} \alpha_f^s \beta^s Q_{t,t+s} \left[P_t^{f(*)} Y_{j,t+s}^{f}(1+\tau_f) - V_{t+s}^{f} Y_{j,t+s}^{f}\right] \tag{15}$$

A typical finished good producing firm discounts its future stream of profit at the rate $\beta^s Q_{t,t+1}$, and takes as given the paths of its marginal cost V_t^f, the total demand for finished good Y_t^f, the aggregate price index of the good P_t^f, and the finished sector's subsidy τ_f. Thus, finished good producing firms' optimal price decision is given by:

$$P_t^{f(*)} = \frac{\mu^f}{(1+\tau_f)} \sum_{s=0}^{\infty} \frac{E_t\left[(\alpha_f\beta)^s Q_{t,t+s} v_{t+s}^f Y_{t+s}^f \frac{1}{P_{t+s}^f}\right]}{E_t\left[(\alpha_f\beta)^s Q_{t,t+s} Y_{t+s}^f \frac{1}{P_{t+s}^f}\right]} \tag{16}$$

Equation (16) states that firms adjusting their prices will choose one that is equal to the desired markup $\mu^f = \left[\theta^f / \left(\theta^f - 1\right)\right]$ over a weighted average of the future real marginal cost v_{t+s}^f. Thus, $P_t^{f(*)}$ is the average price of final good producing firms allowed to reset their prices in period t. At symmetric equilibrium[4], all firms adjusting their prices at period t choose the same price, otherwise firms choose last period's price. Thus, the average price of finished goods is given by:

$$P_t^f = \left[\alpha_f \left(P_{t-1}^f\right)^{1-\theta^f} + (1-\alpha_f)\left(P_t^{f(*)}\right)^{1-\theta^f}\right]^{\frac{1}{1-\theta^f}} \tag{17}$$

2.3.3 *Composite Intermediate Good*

Similar to the aggregate finished good Y_t^f production technology, the aggregate intermediate good Y_t^m is a composite of differentiated intermediate goods $Y_{i,t}^m$ indexed by $i \in [0,1]$, where $Y_{i,t}^m$ denotes the i^{th} intermediate good. Thus, differentiated intermediate goods are combined into an output index making use of the following CES technology:

$$Y_t^m = \left[\int_0^1 Y_{i,t}^m{}^{\frac{\theta^m-1}{\theta^m}} di\right]^{\frac{\theta^m}{\theta^m-1}} \tag{18}$$

where $\theta^m > 1$ is the elasticity of substitution between differentiated intermediate goods, and $[\theta^m / (\theta^m - 1)]$ is the firm's price markup rate.

From the cost minimisation problem, we solve for the differentiated good $Y_{i,t}^m$ demand by aggregate intermediate good producers, given by:

$$Y_{i,t}^m = \left(\frac{P_{i,t}^m}{P_t^m}\right)^{-\theta^m} Y_t^m \tag{19}$$

where $P_{i,t}^m$ is the price of the i^{th} good, and P_t^m is the corresponding price index for the aggregate intermediate good given by:

$$P_t^m = \left[\int_0^1 P_{i,t}^m{}^{1-\theta^m} di\right]^{\frac{1}{1-\theta^m}}$$

[4]Note that the j subscript is dropped due to assuming symmetry among firms.

2.3.4 *Differentiated Intermediate Goods' Firms and Optimal Price Setting*

Each firm produces a differentiated good $Y_{i,t}^m$ indexed by $i \in (0,1)$. A typical firm i uses a linear production function with homogenous labour services $l_{i,t}^m$ to produce the intermediate good, given by:

$$Y_{i,t}^m = A_t^m l_{i,t}^m \tag{20}$$

where (log) productivity $\log A_t^m = a_t^m$ is a stage-specific labour augmenting technology shock identical for all intermediate good producers. The technology shock follows a first-order autoregressive process, where $a_t^m = \rho_m a_{t-1}^m + \varepsilon_t^m$ and persistence term $\rho_m < 1$. We assume similar specifications for the technology shock as the finished good, where ε_t^m is a white noise process, independent of all other shocks and with variance σ_m^2.

Firms are price takers in the input market and monopolistic competitors in the intermediate good market. Each firm meets a downward sloping demand curve as in equation (13). Without loss of generality and by assuming symmetry among firms, the cost minimization problem for the intermediate good producing firm yields the demand labour:

$$l_{i,t}^m = \frac{1}{A_t^m} Y_{j,t}^{m(d)} \tag{21}$$

$$V_t^m = \frac{W_t}{A_t^m} \tag{22}$$

where V_t^m is the intermediate good producing firm's nominal marginal cost.

Firms producing the differentiated intermediate good solve a similar profit maximisation problem to the one final good producing firms solved earlier in section (1.3.2). Each firm in the intermediate stage of production sets price in a staggered fashion in the spirit of Calvo (1983). In this framework, $(1-\alpha_m)$ fraction of firms adjust their prices optimally, where α_m is the probability that firm i does not change its price in period t. Hence, firms resetting the price choose the new price $P_t^{m(*)}$ to maximize their expected present value of future stream of real profit (23) subject to production technology (20), and meeting the demand for the good (13), as follows:

$$\Pi_{t+s}^m = E_t \sum_{s=0}^{\infty} \alpha_m^s \beta^s Q_{t,t+s} \left[P_t^{m(*)} Y_{i,t+s}^m (1+\tau_m) - V_{t+s}^m Y_{i,t+s}^m \right] \tag{23}$$

Discounting the future stream of profit at the rate $\beta^s Q_{t,t+1}$, while taking as given the paths of marginal cost V_t^m, total demand Y_t^m, aggregate price index P_t^m, and the subsidy to intermediate good τ_m, the intermediate good producing firm optimal price decision is given by:

$$P_t^{m(*)} = \frac{\mu^m}{(1+\tau_m)} \sum_{s=0}^{\infty} \frac{E_t\left[(\alpha_m\beta)^s Q_{t,t+s} v_{t+s}^m Y_{t+s}^m \frac{1}{P_{t+s}^m}\right]}{E_t\left[(\alpha_m\beta)^s Q_{t,t+s} Y_{t+s}^m \frac{1}{P_{t+s}^m}\right]} \tag{24}$$

where $\mu^m = \dfrac{\theta^m}{\theta^m - 1}$

Equation (24) states that firms adjusting their prices will choose a price that is equal to the desired markup μ^m over a weighted average of the future real marginal cost v_{t+s}^m. Thus, $P_t^{m(*)}$ is the average price of firms at the intermediate stage allowed to reset their prices in period t. At symmetric equilibrium[5], all intermediate good producing firms adjusting their prices at period t choose the same price, otherwise firms choose last period's price. Thus, the average price of finished goods is:

$$P_t^m = \left[\alpha_m \left(P_{t-1}^m\right)^{1-\theta^m} + (1-\alpha_m)\left(P_t^{m(*)}\right)^{1-\theta^m}\right]^{\frac{1}{1-\theta m}} \tag{25}$$

2.4 *The Monetary Authority*

The instrument of the monetary policy authority is the short-term nominal interest rate r_t.[6] Taylor (1993) presents a nominal interest rate rule which is a linear function of the gap between the inflation rate and its target and the gap between real output and trend output. We adopt a variant of the Taylor-rule where the monetary authority sets the interest rate in response to the deviations of last period's interest rate from its steady state rate, CPI inflation π_t^f from its steady state level $\bar{\pi}^f$ and final-stage output Y_t^f from its flexible-price equilibrium level $Y_t^{f(*)}$. In addition, following the lead of Rotemberg and Woodford (1999) and Clarida, Galí and Gertler (2000), the central bank can smooth the nominal interest rate. Hence the interest rate rule includes lagged nominal interest rate. Due to the focus of this paper on the transmission of technology and demand shocks in a model with a vertical input-output production, and in order to isolate the model from other shocks, a monetary policy shock in the policy rule is ignored. Thus, the monetary policy rule is given by:

[5] Note that the i subscript is dropped due to assuming symmetry among firms.

[6] As the monetary policy rule is specified as an interest rate rate rule, we can abtract from money. This provides another reason for treating money as a unit of account than explicitly representing money in the utility function or the budget constraint.

$$r_t = (r_{t-1})^{\gamma_r} \left(\frac{\pi_t^f}{\bar{\pi}^f}\right)^{\gamma_\pi} \left(\frac{Y_t^f}{Y_t^{f(*)}}\right)^{\gamma_{y^f}} \tag{26}$$

where the parameter γ_r allows the monetary authority to engage in interest rate smoothing. The parameter γ_π allows the monetary authority to control CPI around a target rate being its steady state inflation $\bar{\pi}^f$, the higher this parameter the more strict the monetary authority is on deviation of CPI inflation from its target. The parameter γ_{y^f} is a measure of the weight the monetary authority places on the output gap, defined as the deviation of aggregate output Y_t^f from its flexible equilibrium level $Y_t^{f(*)}$.

2.5 *Market Clearing Conditions*

The bond market clearing condition implies that $B_t = 0$ for all t. The only role for fiscal policy that we allow is providing a production subsidy to firms that eliminates the distortions that arise even under flexible prices due to monopoly pricing by firms. Therefore, the government budget constraint states that production subsidies to both sectors are financed by lump sum taxes collected from the household, such that $T_t = \tau_f P_t^f Y_t^f + \tau_m P_t^m Y_t^m$. In the closed-economy model presented above, and in the absence of capital and government spending, the market clearing condition for the final good corresponds to real GDP equating to aggregate consumption. Thus, the market clearing condition for the composite finished good is satisfied by $Y_t^f = C_t$. In addition, the labour market clearing condition in the finished and intermediate good sectors is represented by $L_t = l_t^f + l_t^m$, where $l_t^f = \int_0^1 l_{j,t}^f dj$ and $l_t^m = \int_0^1 l_{i,t}^m di$.

3 Equilibrium dynamics

The equilibrium consists of 14 endogenous variables C_t, L_t, B_{t+1} for the representative household; $Y_{j,t}^f, Y_{j,t}^m, l_{j,t}^f$ and $P_{j,t}^f$ for finished good producers $j \in (0,1)$; $Y_{i,t}^m, l_{i,t}^m$, and $P_{i,t}^m$ for intermediate good producers $i \in (0,1)$; together with prices P_t^f, P_t^m, r_t and the nominal wage W_t, which satisfy the following conditions: (i) the household's allocations solve its utility maximisation problem when taking prices and wages as given; (ii) finished good producer's allocations solve its profit maximisation problem, when taking all input prices except its own as given; (iii) intermediate good producer's allocations solve its profit maximisation problem, when taking all input prices except its own as given; (iv) markets for bonds, labour, and each good along the production chain clears; (v) monetary authority short-term interest rate rule is described by (26).

3.1 *Steady State*

At symmetric equilibrium, the sector's price index coincides with the optimal pricing rule for each individual firm in the sector. Given no trend growth in productivity, then at steady state $A^f = A^m = 1$, and the demand shock $\xi = 1$. Hence, the optimal pricing decision for the finished and intermediate good producing firms in (16) and (24) at steady state reduces to:[7]

$$P^f = \frac{\mu^f}{(1+\tau_f)} \upsilon^f, \qquad P^m = \frac{\mu^m}{(1+\tau_m)} \upsilon^m \tag{27}$$

To obtain the real wage at steady state, we use the definition of marginal cost for both the finished and intermediate goods in (14) and (22), in addition to the aggregate labour supply by the household in (7), to get:

$$\frac{W}{P^f} = \Phi^{-1} \left(\frac{1+\tau_f}{\mu^f} \right) \left(\frac{1+\tau_m}{\mu^m} \right)^{\phi} \tag{28}$$

where the productivity multiplier Φ associated with the intermediate good plays a central role in the determination of steady state level of real variables, such as real wage, sectoral employment, aggregate and sectoral output. To solve for aggregate employment at steady state, first we need to define finished and intermediate sector demand for labour in equations (12) and (21), which are represented at steady state by:

$$l^f = (1-\phi)\,\Phi \left(\frac{1+\tau_m}{\mu^m} \right)^{-\phi} C, \quad l^m = \phi\Phi \left(\frac{1+\tau_m}{\mu^m} \right)^{1-\phi} C \tag{29}$$

Using the labour and goods market clearing conditions, in addition to the above sectoral labour employment yields in aggregate labour employment, given by:

$$L = l^f + l^m = \eta C \tag{30}$$

where $\eta = \Phi \left[(1+\tau_m)/\mu^m \right]^{-\phi} \{1 + \phi \left[(1+\tau_m-\mu^m)/\mu^m \right] \}$. Using the the steady state equations for the real wage and aggregate employment, we solve for the steady state relation between consumption and employment, which is given by:

$$\frac{WL}{P^f C} = \left(\frac{1+\tau_m}{\mu^m} \right) \left[1 + \phi \left(\frac{1+\tau_m-\mu^m}{\mu^m} \right) \right] \tag{31}$$

[7] Variables at steady state are distinguished from others by dropping the time subscript.

3.2 *The Flexible-Price or Efficient Equilibrium*

Under flexible-prices all firms adjust prices optimally each period, therefore individual firm's indices j and i are dropped.[8] To solve for flexible-price equilibrium conditions, a similar procedure to the one used to obtain the steady state equilibrium is employed. Starting with the optimal pricing decision for the finished and intermediate good producing firms. We make use of the assumption of isoelastic demand curve implying that firms will choose a markup given by $\mu^f = \left[\theta^f / \left(\theta^f - 1\right)\right]$ and $\mu^m = \left[\theta^m / \left(\theta^m - 1\right)\right]$ for the finished and intermediate goods firms, respectively. Hence, the markup is common across firms in each sector and constant over time. Using the definitions for sectoral markup, the optimal pricing rule for finished and intermediate good sectors' reduces to:

$$P_t^{f(*)} = \frac{\mu^f}{(1+\tau_f)} v_t^f, \quad P_t^{m(*)} = \frac{\mu^m}{(1+\tau_m)} v_t^m \tag{32}$$

where the real marginal cost for the final v_t^f and intermediate v_t^m goods is equal to the constant markup. Making use of the definition of the flexible price decisions above in combination with the household labour supply in (7), and solving for the real wage rate under flexible-price equilibrium yields the following definition for the real wage:

$$\left(\frac{W_t}{P_t^f}\right)^* = \frac{1}{\Phi} \frac{(1+\tau_f)}{\mu^f} \left(\frac{1+\tau_m}{\mu^m}\right)^\phi A_t^{f\ (1-\phi)} A_t^{m\ \phi} \tag{33}$$

To obtain aggregate employment under flexible-price equilibrium, first we solve for sectoral labour demand:

$$l_t^{f(*)} = (1-\phi)\,\Phi \left(\frac{\mu^m}{1+\tau_m}\right)^\phi \left(\frac{1}{A_t^m}\right)^\phi \left(\frac{1}{A_t^f}\right)^{1-\phi} C_t^{(*)} \tag{34}$$

$$l_t^{m(*)} = \phi\Phi \left(\frac{\mu^m}{1+\tau_m}\right)^{\phi-1} \left(\frac{1}{A_t^m}\right)^\phi \left(\frac{1}{A_t^f}\right)^{1-\phi} C_t^{(*)} \tag{35}$$

where $l_t^{f(*)}, l_t^{m(*)}$, and $C_t^{(*)}$ represent the final and intermediate sector demand for labour and aggregate consumption under flexible-price equilibrium, respectively. Hence, through the production chain the demand for labour at each stage of production is driven by the multiplier effect associated with the intermediate good, the stage-specific productivity shocks for each sector and aggregate consumption. Using the above two

[8] Variables with a star superscript denote flexible-price equilibrium variables.

equations for the sectoral labour demand and the labour market clearing condition, we derive aggregate employment under flexible-price equilibrium:

$$L_t^{(*)} = l_t^{f(*)} + l_t^{m(*)} = \eta A_t^{m\ -\phi} A_t^{f\ -(1-\phi)} C_t^{(*)} \tag{36}$$

where $\eta = \Phi \left[(1+\tau_m)/\mu^m \right]^{-\phi} \{1 + \phi \left[(1+\tau_m - \mu^m)/\mu^m \right] \}$. The above expression denoting aggregate employment under flexible-price equilibrium shows the role of the intermediate good multiplier Φ in amplifying productivity shocks originating from each stage of production. To clarify, a 1% increase in the finished good productivity shock A_t^f in the presence of the intermediate good multiplier cushions total employment from the full effect of the positive technological change, by resulting in less than 1% decrease in total employment, especially that in EMEs the intermediate good multiplier is large and equivalent to 2.

Moreover, we derive an expression for the relative price of intermediate-to-finished good denoted by $Q_t^{(*)} = P_t^{m(*)}/P_t^{f(*)}$. This expression plays a key role in driving the marginal cost for each of the sectors and in turn affects price inflation in both the finished and intermediate sectors. Combining the optimal pricing rules and the marginal cost equations in both sectors yields the relative price of intermediate-to-final good $Q_t^{(*)}$, given by:

$$Q_t^{(*)} = \frac{1}{\Phi} \frac{(1+\tau_f)}{\mu^f} \left(\frac{\mu^m}{1+\tau_m} \right)^{1-\phi} \left(\frac{A_t^f}{A_t^m} \right)^{1-\phi} \tag{37}$$

From the above equation, we see that the relative price of intermediate-to-finished good is a function of the inverse of the intermediate good multiplier, the stage-specific markup and stage-specific productivity shocks weighed by the share of intermediate factor input in final good production.

3.3 *The Sticky-Price Equilibrium*

At symmetric equilibrium, under the assumption of sticky prices, firms adjusting their prices at period t choose the same price at the probability $(1-\alpha_f)$ and $(1-\alpha_m)$ for finished and intermediate good sectors respectively.[9] While firms not adjusting their price choose last period's price, therefore we drop the i and j subscript from the equilibrium conditions. While variables that take the difference between the log-deviation of variables under sticky-price equilibrium and their flexible-price counterparts are denoted by a tilda. Based on this, the output gap is expressed

[9]Variables with a hat denote the log-deviation of variables under sticky-price equilibrium from their steady state counterparts.

by $\widetilde{c}_t = \ln\left(\frac{C_t}{C}\right) - c_t^*$, the relative price gap of intermediate-to-finished goods is expressed by $\widetilde{q}_t = \ln\left(\frac{Q_t}{Q}\right) - q_t^*$, and the employment gap is expressed by $\widetilde{l}_t = \ln\left(\frac{L_t}{L}\right) - l_t^*$.

First, substituting for the relative price of intermediate-to-finished goods and the household labour supply in the real marginal cost definition, and calculating the log-deviation of the real marginal cost of the final and intermediate sectors from their steady state, we are able to re-write the real marginal cost as a function of the output, price, and employment gaps, as follows:

$$\widetilde{v}_t^f = \phi\widetilde{q}_t + (1-\phi)\,\sigma\widetilde{c}_t + \varphi\widetilde{l}_t \qquad \widetilde{v}_t^m = \sigma\widetilde{c}_t - \widetilde{q}_t + \varphi\widetilde{l}_t \tag{38}$$

Second, the optimal pricing decision rules (16) and (24) are log-linearised around zero steady state rate of inflation and by making use of the log-linearised relations between the price indices and pricing decisions in both sectors we obtain the finished and intermediate sectors price inflation, respectively:

$$\widehat{\pi}_t^f = \beta E_t\left\{\widehat{\pi}_{t+1}^f\right\} + \kappa_f\left(\phi\widetilde{q}_t + (1-\phi)\,\sigma\widetilde{c}_t + \varphi\widetilde{l}_t\right) \tag{39}$$

$$\widehat{\pi}_t^m = \beta E_t\left\{\widehat{\pi}_{t+1}^m\right\} + \kappa_m\left(\sigma\widetilde{c}_t - \widetilde{q}_t + \varphi\widetilde{l}_t\right) \tag{40}$$

$$\text{where } \kappa_f = \left\{\left[(1-\alpha_f)\,(1-\alpha_f\beta)\right]/\alpha_f\right\}$$
$$\kappa_m = \left\{\left[(1-\alpha_m)\,(1-\alpha_m\beta)\right]/\alpha_m\right\}$$

To obtain the Euler equation in terms of the gaps, we log-linearise the intertemporal Euler equation (8) around its steady state and subtracting its flexible-price counterpart yields:

$$\widetilde{c}_t = E_t\left\{\widetilde{c}_{t+1}\right\} - \frac{1}{\sigma}\left(\widehat{r}_t - E_t\left\{\widehat{\pi}_{t+1}^f\right\} - \widehat{rr}_t^*\right) \tag{41}$$

where $\widehat{r}_t$ and $\widehat{\pi}_t^f$ are the log-deviations of the nominal interest rate and the CPI inflation rate from steady state and $\widehat{rr}_t^*$ is the real interest rate under flexible-price equilibrium.

Finally, the log-linear version of the difference between the relative price gap under sticky-price and flexible price equilibrium is given by:

$$\widetilde{q}_t = \widetilde{q}_{t-1} + \widehat{\pi}_t^m - \widehat{\pi}_t^f - \Delta q_t^* \tag{42}$$

where $\Delta q_t^* = q_t^* - q_{t-1}^*$ and as we log-linearise the flexible-price equilibrium relative price in (37) and substitute it in (42) provides:

$$\widetilde{q}_t = \widetilde{q}_{t-1} + \widehat{\pi}_t^m - \widehat{\pi}_t^f - (1-\phi)\left[\Delta\widehat{a}_t^f - \Delta\widehat{a}_t^m\right] \tag{43}$$

From the above equation, the relative price gap $\widetilde{q}_t$ will *not* respond to sectoral productivity shocks, when the two sectors have identical shocks, implying that $\Delta\widehat{a}^f_t = \Delta\widehat{a}^m_t$, or if intermediate good is the only input producing the final good, i.e. $\phi = 1$. In this case, the relative price gap of the intermediate-to-finished good is driven by last period's relative price gap, and finished and intermediate good price inflation. Table 1 summarises the equilibrium dynamics under sticky prices for the two-stage vertical production model.

Table 1

Equilibium Conditions for Two-Stage Vertical Production Model

$$\hat{y}^f_t = \widehat{c}_t$$

$$\hat{c}_t = E_t\left\{\hat{c}_{t+1}\right\} - \frac{1}{\sigma}\left(\hat{r}_t - E_t\left\{\hat{\pi}^f_{t+1}\right\} + E_t\left\{\widehat{\xi}_{t+1} - \widehat{\xi}_t\right\}\right)$$

$$\widehat{l}^f_t = \left[\phi\widetilde{q}_t + (1-\phi)\,\sigma\widehat{x}_t + \varphi\widetilde{l}_t\right] - \widehat{w}_t + \widehat{p}^f_t + \hat{y}^f_t$$

$$\widehat{y}^m_t = \left[\phi\widetilde{q}_t + (1-\phi)\,\sigma\widehat{x}_t + \varphi\widetilde{l}_t\right] - \widetilde{q}_t + \hat{y}^f_t$$

$$\widehat{w}_t - \widehat{p}^f_t = \sigma\hat{c}_t + \varphi\widehat{l}_t - \widehat{\xi}_t$$

$$\widehat{y}^f_t = \phi\widehat{y}^m_t + (1-\phi)\,\widehat{l}^f_t + (1-\phi)\,\widehat{a}^f_t$$

$$\widehat{y}^m_t = \widehat{l}^m_t + \widehat{a}^m_t$$

$$\widehat{p}^f_t = \alpha^f\widehat{p}^f_{t-1} + \left(1-\alpha^f\right)\left(1-\alpha^f\beta\right)\left[\phi\widetilde{q}_{t+1} + (1-\phi)\,\sigma\widehat{x}_{t+1} + \varphi\widetilde{l}_{t+1}\right]$$

$$\widehat{\pi}^f_t = \beta E_t\left\{\hat{\pi}^f_{t+1}\right\} + \kappa_f\left[\phi\widetilde{q}_t + (1-\phi)\,\sigma\widehat{x}_t + \varphi\widetilde{l}_t\right]$$

where $\kappa_f = \frac{\left(1-\alpha_f\right)\left(1-\alpha_f\beta\right)}{\alpha_f}$

$$\widehat{p}^m_t = \alpha^m\widehat{p}^m_{t-1} + (1-\alpha^m)\,(1-\alpha^m\beta)\left[\sigma\widehat{x}_{t+1} - \widetilde{q}_{t+1} + \varphi\widetilde{l}_{t+1}\right]$$

$$\widehat{\pi}^m_t = \beta E_t\left\{\hat{\pi}^m_{t+1}\right\} + \kappa_m\left[\sigma\widehat{x}_t - \widetilde{q}_t + \varphi\widetilde{l}_t\right]$$

where $\kappa_m = \frac{(1-\alpha_m)(1-\alpha_m\beta)}{\alpha_m}$

$$\hat{r}_t = \gamma_r\hat{r}_{t-1} + \gamma_{\pi^f}\hat{\pi}^f_t + \gamma_r\widehat{x}_t$$

$$\widehat{rr}^*_t = \Psi_m E_t\left\{\Delta\widehat{a}^m_{t+1}\right\} - \Psi_f E_t\left\{\Delta\widehat{a}^f_{t+1}\right\} + \Psi_\xi E_t\left\{\Delta\widehat{\xi}_{t+1}\right\}$$

where $\Psi_m \equiv \sigma\left[\frac{\phi(1+\varphi)}{\varphi+\sigma}\right]$, $\Psi_f \equiv \sigma\left[\frac{\phi(1-\phi)(1+\varphi)}{\varphi+\sigma}\right]$, $\Psi_\xi \equiv \sigma\left[\frac{1}{\varphi+\sigma}\right]$

$$\widehat{x}_t = \hat{y}^f_t - \hat{y}^{f\ *}_t$$

$$\hat{y}^{f\ *}_t = \tau_m\widehat{a}^m_t - \tau_f\widehat{a}^f_t + \tau_\xi\widehat{\xi}_t$$

where $\tau_m \equiv \frac{\phi(1+\varphi)}{\varphi+\sigma}, \tau_f \equiv \frac{\phi(1-\phi)(1+\varphi)}{\varphi+\sigma}, \tau_\xi \equiv \frac{1}{\varphi+\sigma}$

$$\widetilde{q}_t = \widehat{q}_t - \widehat{q}^*_t$$

$$\widetilde{q}_t = \widetilde{q}_{t-1} + \widehat{\pi}^m_t - \widehat{\pi}^f_t - (1-\phi)\left[\Delta\widehat{a}^f_t - \Delta\widehat{a}^m_t\right]$$

$$\widehat{q}^*_t = (1-\phi)\,\widehat{a}^f_t - (1-\phi)\,\widehat{a}^m_t$$

$$\widehat{l}_t = \widehat{l}^f_t + \widehat{l}^m_t$$

$$\widetilde{l}_t = \widehat{l}_t - \widehat{l}^*_t$$

$$\widehat{l}^*_t = \widehat{c}^*_t - \phi\widehat{a}^m_t - (1-\phi)\,\widehat{a}^f_t$$

4 Parameterisation

In this section we report the benchmark parameter values used in solving the model. The model is calibrated using emerging market economies data, and in particular South Korea, Malaysia and Thailand. Benchmark parameters are summarised in Table 2.

Table 2

Calibration of the Model for Emerging Market Economies

Description	Parameter	Value
Household's subjective discount factor	β	0.985
Coefficient of relative risk aversion	σ	2
Intertemporal elasticity of marginal disutility of labor	φ	0.98
Coefficient on labor in the utility function	χ	1
Share of intermediate good in final good	ϕ	0.5
Probability final good price does not change	α^f	0.63347
Probability intermediate good price does not change	α^m	0.81115
Elasticity of substitution for each sector's differentiated goods	$\theta^f = \theta^m$	11
Weight in Taylor rule on interest rate smoothing	γ_r	0.9
Weight in Taylor rule on CPI inflation target	γ_{π^f}	1.5
Weight in Taylor rule on output gap	γ_{y^f}	0.5

4.1 *Preferences*

This section specifies the parameters that govern household consumption, labour supply, and asset holding decisions. Some standard parameter values govern household preferences, such as the intertemporal elasticity of substitution in consumption assumed to be 0.5, which inverse implies a coefficient of relative risk aversion σ of 2 as reported by Backus *et al.* (1994). Devereux *et al.* (2006) use a quarterly discount factor β of 0.985, which implies a steady state annual real interest rate of 6% in the case of EMEs. Following Christiano *et al.* (1997), we set the elasticity of labour supply parameter φ to unity. The parameter χ in the utility function measures the weight on leisure in the utility function is set to unity.

4.2 *Vertical Production Structure*

Production in the modelled economy is characterised by a vertical input-output production structure with finished and intermediate goods, the parameter $\phi = 0.5$ governs the share of intermediate good input in finished good production. This share is consistent with Basu's (1995) estimate as well as the average share of intermediate goods in production based on input-output tables calculated over 48 industries in various

EMEs, as reported by Yamano and Ahmad (2006). The degree of finished and intermediate good price rigidity is governed by the parameters α^f and α^m, respectively. Evidence by Bils and Klenow (2004) and Ortega and Rebei (2006) shows considerable degree of heterogeneity in price setting practices across different sectors. Due to limited sectoral data for EMEs, we follow estimates for sectoral price rigidity by Phaneuf and Rebei (2008) and Devereux *et al.* (2006). The parameter α^f and α^m representing the probabilities that finished and intermediate good price does not change are taken as 0.63347 and 0.81115, respectively. These probabilities imply that finished good prices are reset every 2.9 quarters on average, while intermediate good prices re-optimised every 4 quarters. In addition, the elasticity of substitution between varieties of finished and intermediate goods θ^f and θ^m is set to 11, as calibrated in Devereux *et al.* (2006). The choice of θ^f and θ^m implies a steady state mark-up in each of two sectors of 10%.

4.3 *Monetary Policy*

We adopt a variant of the Taylor-rule presented in equation (26) where the monetary authority sets the interest rate in response to the deviations of last period's interest rate from its steady state rate, CPI inflation π_t^f from its steady state level $\bar{\pi}^f$ and finished-stage output Y_t^f from its flexible-price equilibrium level $Y_t^{f(*)}$. Taylor (1993) presents a nominal interest rate rule which is a linear function of the gap between the inflation rate and its target and of the gap between real output and trend output, with the weights $\gamma_{_{\pi^f}} = 1.5$ and $\gamma_{_x} = 0.5$, respectively. Similar values are assigned to $\gamma_{_{\pi^f}}$ and $\gamma_{_x}$ in the case of EMEs by Cook and Devereux (2006). Fraga *et al.* (2003) study inflation targeting in EMEs and provide evidence that central banks in EMEs practice interest rate smoothing, and estimated the weight to be $\gamma_{_r} = 0.9$. We use parameters specified by both studies to put the weights on the various objectives in the monetary authority's interest rate rule.

5 The Dynamics of the Model

This section describes the impact of stage-specific productivity and demand shocks on sectoral output, consumption, employment and inflation. In addition, it undertakes robustness checks on the sensitivity of the model to different calibrations of key parameters driving the results. The linearised model is solved analytically as presented in section 2. First-order approximations are used to compute moments, variance decomposition and impulse response functions presented below via Dynare where the model is solved using the method of generalised Schur decomposition. [10] The impulse responses for Model II with a two-stage input-output production are plotted against Model I acting as benchmark model with one-stage of production to highlight the transmission mechanism of shocks in the absence of a vertical production chain, where one period in the model corresponds to one quarter.

5.1 *Sources of Short-Run Dynamics*

What are the most important shocks driving short-run fluctuations in the modelled EME? We address this question by examining the variance decomposition for selected macroeconomic variables following stage-specific technology and demand shocks.

Table 3

Variance Decomposition: One-Sector vs. Two-Sector Model (%)

Model I: One-Sector Model			Model II: Two-Sector Model			
Variable	ε_t^f	ε_t^ξ	Variable	ε_t^f	ε_t^m	ε_t^ξ
y_t	0.09	99.91	y_t^f	14.64	66.41	18.96
l_t	95.27	4.73	y_t^m	6.76	85.01	8.23
w_t	100	0	l_t	17.68	35.48	46.84
p_t	0.38	99.62	w_t	6.24	91.98	1.78
r_t	0.38	99.62	p_t^f	96.9	1.59	1.52
rr_t^*	0.38	99.62	p_t^m	26.33	72.66	1.01
y_t^*	0.38	99.62	r_t	79.39	12.42	8.2
x_t	0.38	99.62	rr_t^*	0.02	89.93	10.04
$\widetilde{l}_t$	0.38	99.62	$y_t^{f\ *}$	0	68.26	31.74
			x_t	91.06	2.54	6.4
			$\widetilde{l}_t$	85.33	8.1	6.57
			$\widetilde{q}_t$	32.89	64.9	2.21

Table 3 shows that the intermediate-stage productivity shock mainly dominates short-run fluctuations of macroeconomic variables in the two-sector model by contributing to the variance of finished output and consumption by 66%, real wages by 92%, relative price of intermediate-to-finished good by 65%, and total hours by 35%. In addition, the intermediate-stage shock explains 85% of the variance of intermediate-stage output, 73% of the variance of the intermediate-stage prices, and 54% of the variance of intermediate sector employment. However, demand shock explains 46.8% of the variance of total labour hours, and 56.8% and 26% of the variance of finished and intermediate sector labour, respectively.

[10] See Klein (2000) and Collard and Juillard (2003).

The variance decomposition for selected variables following stage-specific technology and demand shocks shows that the intermediate-stage productivity shock contributes to the majority of short-run dynamics in the two-stage production structure model. This highlights the key role the intermediate goods and its multiplier play in amplifying shocks all through the production chain and in turn driving short-run fluctuations in the modelled EME. While in Model I the demand shock mainly drives the short-run dynamics, except for total hours and wages which are driven mainly by finished-stage productivity shock.

5.2 *Stage-Specific Technology and Demand Shocks*

The model examines the dynamic effects of stage-specific technological change in the finished and intermediate good sectors, and a demand shock. This section of the paper compares the dynamic effects of these shocks between two models. Model I acts as a benchmark model with one-stage of production and price rigidity. In the benchmark model, the effect of the productivity shock does not last beyond the initial contract duration, allowing firms to quickly adjust their marginal cost and in turn prices, which results in insignificant persistence in aggregate output. While Model II is characterised by a two-stage production structure, where finished and intermediate sectors are linked in a vertical input-output production chain. This setup allows for a comparison between the two models' dynamics in response to shocks, with a focus on the role of a vertical production chain in generating macroeconomic persistence.

5.2.1 *Effect of a Positive Finished-Stage Technological Change*

The short-run dynamics of a 1% positive finished-stage productivity shock in both the benchmark model and Model II are summarised in Figure 3. The response of the benchmark model to a positive finished-stage technology shock complies with the findings of Galí (1999) and Basu, Fernald and Kimball (2006), where a favorable technology shock induces short-run decline in aggregate finished-good output and total employment and in turn a rise in CPI inflation. This is depicted by the circled line in panels A, C and H of Figure 3. From panel G, we see that it is the case where the monetary authority does not fully accommodate the shock, therefore total employment and real wages decline by 1%, while the nominal interest rate shows a minor increase of 0.003%. In this simple model with one-stage production, imperfect competition and sticky prices, even though all firms experience a decline in their marginal costs only a fraction of them adjust their prices downward in the short-run. Accordingly, CPI slightly increases partly offsetting the increase in productivity. In addition, the drop in wages, as household source of labour income, causes demand for finished goods to fall and in turn does not support the increase in productivity combined with the slight increase in CPI result in a decline in aggregate output. Thus, the benchmark model presents the case of a one-stage of production where a positive technological progress has persistently adverse effect on aggregate output and total employment in the short-run.

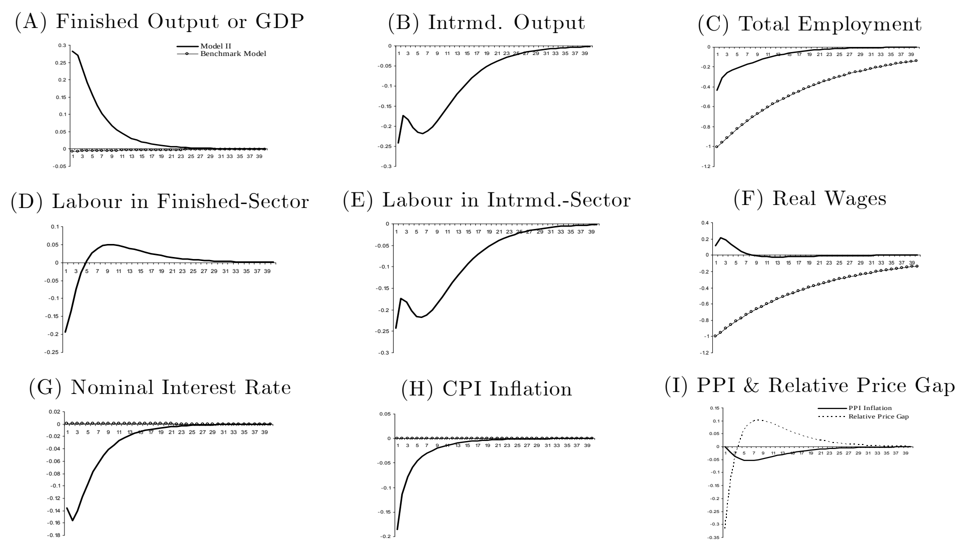

Fig. 3. *Impulse Response to 1% +ve Finished-Stage Technology Shock*

The response of a two-stage production model or Model II to a positive finished-stage productivity shock, depicted by a solid line in Figure 3, results in a drop in the price of the finished good and thus an increase in the relative price ratio of intermediate-to-finished good. This is reflected in panel H and I of Figure 3. The increase in the relative price ratio of intermediate-to-finished good leads finished good firms to change their factor input mix by reducing demand for both labour and intermediate good input, as shown in panels D and B. In turn, the intermediate-stage of production reduces its output and labour demand, putting pressure on both the real wage rate and its own price, as shown in panels B, E, F and I. The combined decrease in labour demand in both sectors results in a decline in total employment, reflected in panel C. However, total employment drop in Model II of 0.4% is significantly less than that experienced in the benchmark model of 1%. This prediction of the model conforms with the intermediate good multiplier effect through the production chain, which cushions aggregate employment from the total impact of a positive technological change.

In comparing the monetary policy response in Model I versus Model II, we see that the effect of a positive finished-stage productivity shock on the former does not last beyond the initial contract duration, allowing firms to quickly adjust their marginal cost and in turn prices, which results in insignificant persistence in aggregate output, CPI inflation and in turn a minor increase of 0.003% in nominal interest rate. While in the presence of a vertical input-output production, Model II, the presence of nominal rigidity in both stages of production, and the role of the intermediate good multiplier in amplifying shocks all through the production chain generates macroeconomic persistence shown in the drop and persistent return to steady state in both CPI inflation by 0.18% and nominal interest rate by 0.14%, in panels G and H of Figure 3. Thus, the results comply with and give evidence to Dotsey and King (2006) proposition that vertical input-output production structure provides the "real flexibility" needed to resolve the persistence problem. This is shown from the highly persistent aggregate output, CPI, PPI, and nominal interest rate responses to the finished-stage productivity shock in Model II compared to the benchmark model.

5.2.2 *Effect of a Positive Intermediate-Stage Technological Change*

The short-run dynamics of a 1% positive intermediate-stage productivity shock in Model II are summarised in Figure 4. Following the shock, the price of the intermediate good and producer price index (PPI) drop, which results in a decrease in the relative price of the two goods by 0.2% compared to its flexible-price counterpart which drops to 0.5%, as shown in panel I. The drop in the intermediate good price results in an increase in its demand, which rises sharply and persistently producing a hump-shaped response to the technology shock, as reflected in panel B. This triggers a reduction in intermediate sector demand for labour, as seen in panel E.

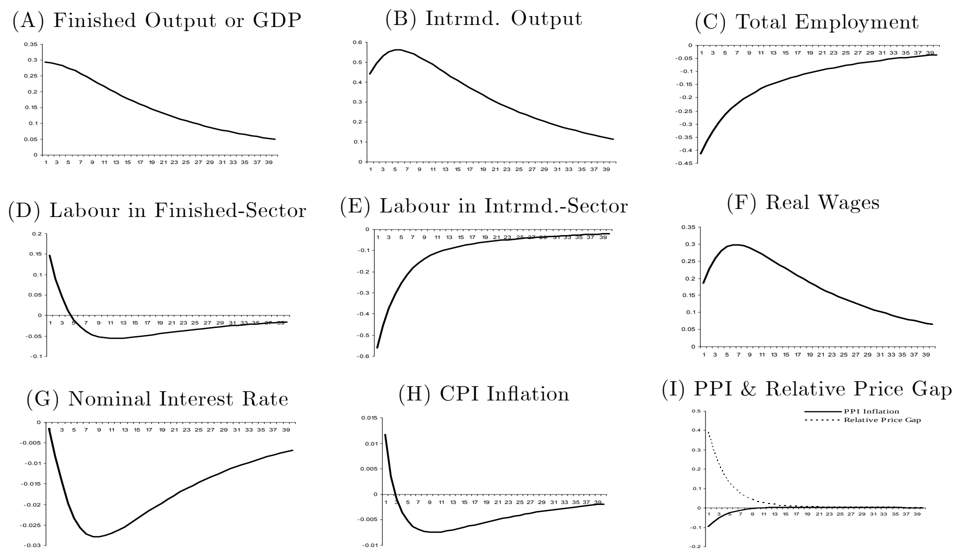

Fig. 4. *Impulse Response to 1% +ve Intrmd.-Stage Technology Shock*

In addition, the drop in intermediate-good price changes the factor input mix for the finished-good producing firm, resulting in higher demand for its factor inputs, namely labour and the intermediate input, as shown in panels D and B. Hence, finished good output increases causing both its price and CPI inflation to rise by 0.002% and 0.012%, respectively. The net effect of a drop in intermediate-good sector labour demand by 0.56% and an increase in finished-good sector labour demand by 0.15% is a net decrease in total employment by 0.45%. Thus, the overall effect of an intermediate-stage technology shock is a rise in both finished and intermediate output and a net effect of a drop in total employment as the drop in intermediate sector labour demand outweighs the rise in finished sector labour demand.

5.2.3 *The Role of the Intermediate-Good Sector Multiplier*

Model II with a vertical input-output production chain has different implications for the adjustment of aggregate output and total employment depending on the source of technological change it faces. Figure 5, panels A-C, presents a comparison in the reaction of real variables to finished-stage A_t^f (circled line) and intermediate-stage A_t^m (solid line) productivity shocks. Our main results, from panels A-C, suggest that a positive intermediate-stage productivity shock in comparison to its finished-stage counterpart has an expansionary effect on both aggregate finished and intermediate output, with an amplified and more persistent drop in total employment, and thus a less expansionary monetary policy compared to finished-stage productivity shock. This prediction of the model is mainly driven by the linkages between the two stages of production through the share of intermediate in finished good production, which creates a productivity multiplier amplifying the impact of the intermediate-stage shock over the whole production chain.

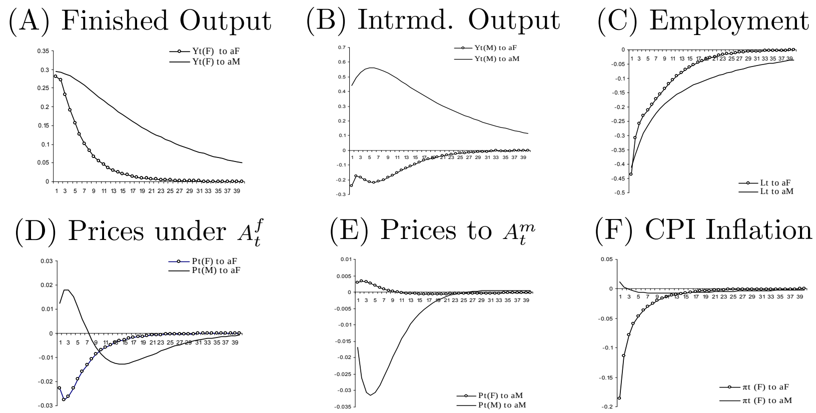

Fig. 5. *Source of Technological Change in a Vertical Production Chain*

Figure 5, panels D-F, presents the reaction of nominal variables to each of the two stage-specific technology shocks in the case of a two-sector model. The model captures clearly Means (1936) observation on the price dynamics across the production chain. As seen in panel D and E, the nominal price of the intermediate good p_t^m, being at an early stage of production, is significantly more volatile than the prices of goods further down the production chain, namely the finished good p_t^f. The model with a vertical two-stage production chain predicts that the variability in intermediate good price is 1.7 times more that of finished stage price, which is consistent with recent evidence by Huang and Liu (2001) and Phaneuf and Rebei (2008).

Thus, our evidence suggests that a positive technology shock may either have a contractionary or expansionary effect on output and employment depending on the source of technological change. In comparing the transmission mechanism of the two stage-specific technology shocks, the paper provides evidence for the dominating effect of the intermediate-stage technology shock on short-run fluctuations, and the role of the intermediate good multiplier in driving the results.

5.2.4 *Effect of a Demand Shock*

The short-run dynamics of a 1% positive demand shock in both the benchmark model and Model II are summarised in Figure 6. The response of the benchmark model to a positive demand shock, designated by a dotted line in Figure 6, induces short-run increase in aggregate finished output, total employment, real wages, as well as a rise in CPI inflation and nominal interest rate, as shown in panels A, C, F and G.

Thus, at the outset of a demand shock, the demand for finished good increases which induces a higher demand for its factor input, namely labour and therefore a rise in the real wage rate. This rise in demand increases the price of the finished good and CPI inflation. Thus, the benchmark model presents the case where preference shock has an expansionary and persistent effect on employment and output in the short-run.

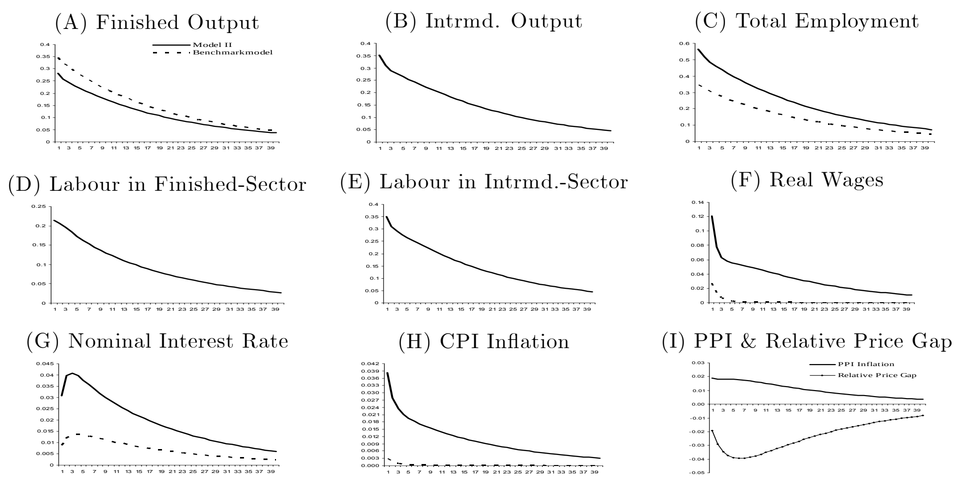

Fig. 6. *Impulse Response to 1% +ve Demand Shock*

A positive demand shock to Model II, from Figure 6 shown in solid line, causes firms to increase their production to meet the rising demand. As a result, finished good firms increase their demand for both labour and the intermediate inputs. In turn, the intermediate-stage of production increases its output and labour demand, putting pressure on both the real wage rate and its own price, thus increasing PPI inflation. The increase in factor inputs prices due to higher demand and the rise in finished good price result in increasing CPI inflation. The rise of finished and intermediate good prices result in decreasing the relative price gap of intermediate-to-finished good, shown in panel I. The combined increase in labour demand in both sectors results in an increase in total employment, as shown in panel C.

In comparing the results of the benchmark model to Model II, it is evident that a vertical two-stage production chain following a demand shock drives higher employment across the production chain and real wages at all stages. Again the role of the intermediate good and its multiplier in amplifying the demand shock through the production chain is clear in Model II.

5.3 *Sensitivity Analysis*

To examine the robustness of the model predictions, we study the sensitivity of the results to various specifications to key parameters. These key parameters are: the share of intermediate input in finished good production ϕ and the degree of price rigidity for finished and intermediate sectors, α^f and α^m, respectively.

5.3.1 *Share of Intermediates in Finished Good Production*

We study the sensitivity of the model prediction under finished and intermediate-stage technology shocks to various shares of the intermediate input, as shown in Figure 7. The benchmark calibration of the share of intermediate good input in aggregate production is 0.5 in the case of EMEs. We pick two arbitrarily smaller shares for the intermediate input of 0.05 and 0.2. Figure 7 reflects that the role intermediate inputs share plays in the short-run dynamics of the model depends on the source of technological change.

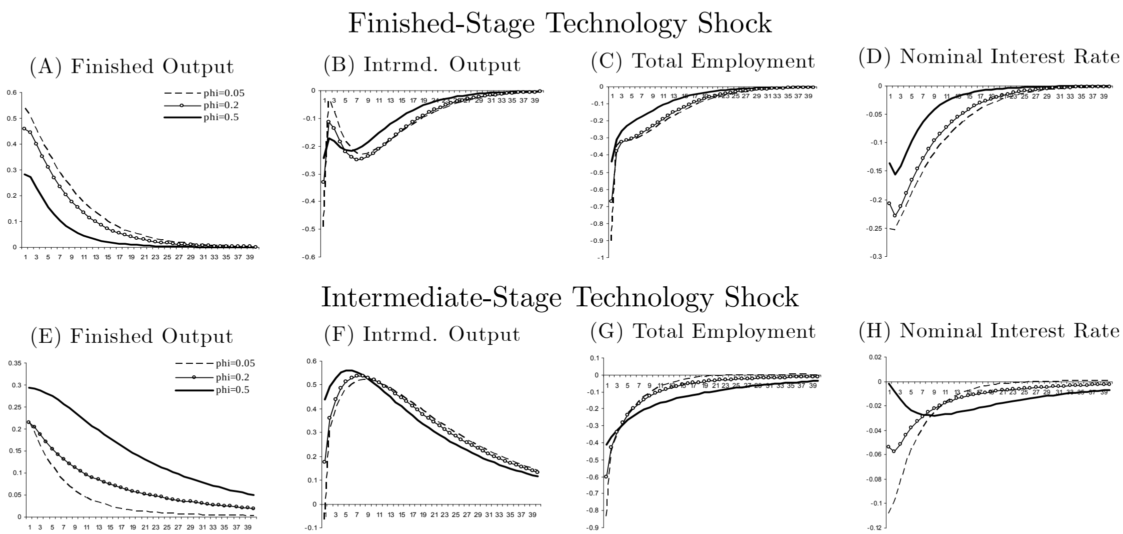

Fig. 7. *Share of Intrmd. Inputs and Stage-Specific Technology Shocks*

In response to a finished-stage technology shock, panels A-D of Figure 7, finished-good firms reduce their demand for both labour and intermediate factor inputs. From panel C, increasing the share of the intermediate input in finished good production from 0.05 to 0.5, insulates the economy from an increased drop in total employment due to the technology shock. From panel D, the nominal interest rate drops implying an expansionary monetary policy, however less expansionary compared to the alternative scenarios considered. Hence, the higher the share of the intermediate good, the lower the adverse effects of the finished-stage technology shock.

The response of the model to an intermediate-stage productivity shock and the effect of the intermediate good multiplier, in panels E-H of Figure 7, when increasing the share of the intermediate factor input generates an increase in both finished and intermediate output, while a lower drop in.total employment. From panel H, we see that the higher the share of the intermediate input the more stable monetary policy compared to lower values to the parameter ϕ. Thus, the higher the share ϕ in finished good production, the higher and more persistent the response of finished and intermediate output, and total employment.

5.3.2 *Degree of Price Rigidity*

Given the importance of the price rigidity parameter in driving persistence in the model, we compare the model predictions under the following alternative pricing assumptions: (i) perfectly flexible prices (dotted line), where firms of both sectors reset their prices at every period, i.e. $\alpha^f = \alpha^m = 0$, (ii) staggered prices at finished good sector only (circled line), α^f=0.63347, α^m=0, and (iii) staggered prices at both stages of production (solid line), i.e. α^f=0.63347, α^m=0.81115.

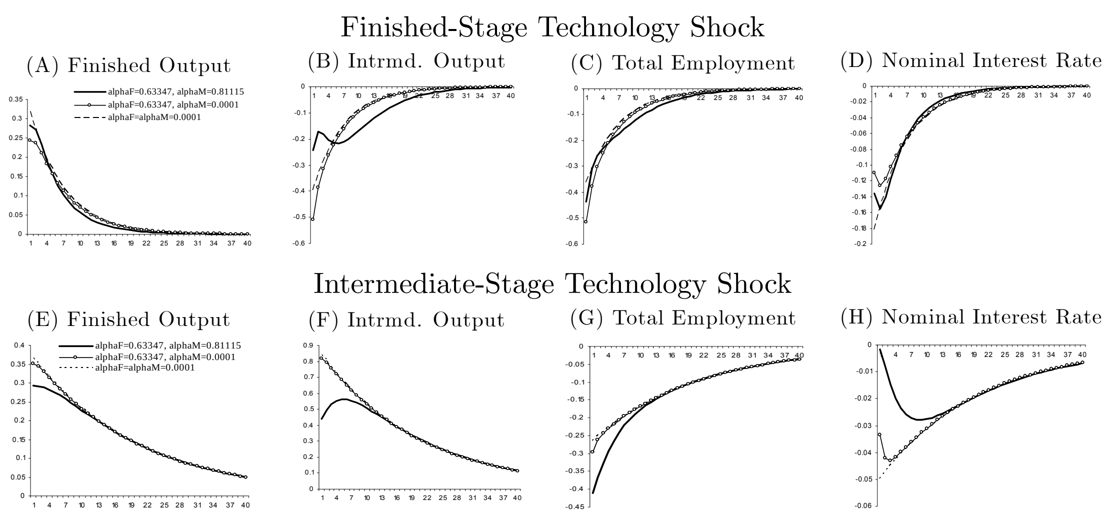

Fig. 8. *Nominal Price Rigidity and Stage-Specific Technology Shocks*

Figure 8, panels A-D, presents the response of the two-stage model to a finished-stage productivity shock in the case of perfectly flexible prices. Assuming that nominal prices are perfectly flexible at both stages of production has a minor impact on the results compared to other pricing assumptions. For example, finished output is higher by 0.05%, total employment drops to 0.38% instead of 0.5%, and monetary policy is slightly more expansionary compared to alternative pricing assumptions. Although nominal prices are revised each period under the assumption of perfectly flexible prices, employment continues to fall following a final-stage productivity shock, as shown in panel D.

In contrast, different specifications of the nominal price rigidity parameter plays a key role in driving the results of the model following an intermediate-stage productivity shock. The response of the model to an intermediate-stage productivity shock when assuming perfectly flexible prices is more favorable compared to alternative pricing assumptions. The absence of price rigidity at each stage of production results in higher sectoral output, lower drop in employment, and a more expansionary monetary policy when compared to different combinations of price rigidity at each stage of production. In addition, the higher the nominal price rigidity, the more stable monetary policy is compared more flexible price settings that allow for more fluctuations in prices and thus in interest rates.

6 Conclusion

The paper proposes a framework for Emerging Market Economies in which empirically relevant vertical production chain plays a key role in the transmission of stage-specific technological change and demand shocks. We show that a model with a vertical input-output production structure and staggered prices at each stage of production can generate significantly different responses to technological change depending on the source of the productivity shock. Thus, specifying the source of technological change can not be dismissed as a main factor driving short-run dynamics of macroeconomic variables and monetary policy design in EMEs. Therefore, we share Basu *et al.* (2006) recommendation that "to the extent that policy makers can better assess technological movements and respond decisively to them, monetary policy might be improved in the future".[11]

Our main results indicate that a positive intermediate-stage productivity shock in comparison to its finished-stage counterpart has an expansionary effect on both finished and intermediate output, with an amplified and more persistent drop in total employment, and thus a less expansionary monetary policy. Based on our findings a positive technology shock may either have a contractionary or expansionary effect on output and employment depending on the source of technological change. In studying the variance decomposition of nominal shocks presented in the model, we show the dominating effect of the intermediate-stage tech-

[11] Basu *et al.* (2006), p. 1444.

nology shock on short-run fluctuations of macroeconomic variables, and the role of the intermediate good multiplier in driving the results.

The model captures Blanchard's (1983) finding that nominal price of the intermediate good, being at an early stage of production, is significantly more volatile than the prices of good further down the production chain. Thus, the model conforms with evidence by Means (1936), Blanchard (1983) and Huang and Liu (2001) that output responses are more persistent, the greater number of production stages and the larger share of intermediate goods. We examine this result further as we compare a vertical two-stage production chain versus one-stage production structure, our results indicate that the former yields a significantly amplified and persistent responses of macroeconomic variables to positive finished and intermediate-stage productivity and demand shocks, due to the presence of an intermediate good sector and its multiplier effect. Thus, a two-stage model can generate the macroeconomic persistence missing from a one-stage production model.

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