Imperfect quality information in a quality-competitive hospital market

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Abstract

We examine the implications of policies to improve information about the qualities of profit seeking duopoly hospitals which face the same regulated price and compete on quality. We show that if the hospital costs of quality are similar then better information increases the quality of both hospitals. However if the costs are sufficiently different improved information will reduce the quality of both hospitals.

Keywords: Uncertain quality. Information. Competition. Hospitals.

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1 Introduction

There is an increasing trend to in countries with public health care systems to increase the choice available to patients (Department of Health, 2005; Vrangbaek and Ostergren, 2006). In the UK National Health Service (NHS), where general practitioners act as gatekeepers for elective (non-emergency hospital care), the Department of Health has required that patients must be offered a choice of hospitals when they are referred by their general practitioner. As an integral part of its policy the Department of Health has introduced measures to increase the information about hospitals available to patients and GPs. Practices have been provided with software to provide information to patients on local hospitals. The Healthcare Commission, which regulates NHS hospitals actively publicises its website which has comparative information on the quality of hospitals. The website has information on rates of postoperative mortality, hospital acquired infections, and readmission rates. The Netherlands also has a “Kiesbeter” (“Choose better”) website with similar information.

In these public systems hospitals are paid on a per case basis with centrally regulated prices. The intention is that since hospitals cannot compete via prices they will focus on quality improvement as a means of increasing market share. A major justification of policy initiatives to improve the information about quality of hospitals is that better information will increase the incentives for hospitals to raise quality.

We examine the argument that better information about hospital quality will increase quality levels. We use a duopoly model in which two public funded profit-seeking hospitals face the same fixed price per case treated and compete for patients via the quality of services they offer. Patients receive an imperfect signal about the quality of services at each hospital which they use to inform choice of hospital. Hospitals differ in their costs of producing quality. In equilibrium the effect of increasing information on hospitals’ quality levels depends on the difference between their quality cost parameters. When quality costs are similar improved information increases quality at both hospitals. However, if quality cost functions differ sufficiently improved information will reduce quality at both hospitals. We also show that whether improved information makes patients better off depends crucially on whether one takes an ex ante or ex post view of patient utility.

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1 Healthcare Commission website address
2 Netherlands website address

1.1 Related literature

There is an extensive literature discussing consumers imperfect information on price (Salop, 1976) and price and quality (Schwartz and Wilde, 1985; Chan and Leland, 1982) in monopolistic competition. This literature characterises imperfect information by the search costs of consumers in finding a firms’ price/quality. Instead, we have costless but imperfect information with imprecise information signals.

A model more closely related to ours, including imperfect information signals, is Dranove and Satterthwaite (1992). Their model includes vertical and horizontal differentiation and consumers search sequentially through monopolistically competitive firms. Firms compete on price as well as quality. In contrast, we analyse quality-only competition in a duopoly, and allow for differing quality production technology across hospitals. In the Dranove and Satterthwaite model holding the level of information about price constant, improved quality information always increases equilibrium quality. In our model this is so only if hospitals have similar quality-producing technologies.

We use a random utility model for consumer choice which has some similarities with the product differentiation literature (Perloff and Salop, 1985; Wolinsky, 1986; Anderson et al, 1995). In this literature the error term in the consumer choice model is attributed to consumers taste differences. In contrast, we assume the error term in our model represents consumer imperfect information.

There is a US empirical literature on the effect the effects of publicly reported hospital, health plan and physician quality information ('report cards') (Beaulieu, 2002; Gaynor and Vogt, 2003; Dranove et al, 2003; Cutler et al, 2004; Zhe, 2006). Part of this literature investigates whether information increases patient outcomes by selection of more healthy patients, by matching severely ill patients with high quality providers (Dranove et al, 2003) or by improved quality through increased competition (Cutler et al, 2004).

We do not consider patient severity and selection. We concentrate on the question of whether increased information increases competition between hospitals, and hence hospital quality, and hence patient utility.

Montefiori (2005) considers consumer imperfect information about hospital quality, assuming a normal distribution of consumer uncertainty about quality and using a Taylor approximation in the analysis. The Taylor approximation constrains the model to consider ‘bounded uncertainty’ about hospital quality, where perceived hospital quality is ‘very close’ to actual quality. The accuracy of quality information signals differs between the two firms, rather than the quality production technologies. Equilibrium hospital
quality is only affected by the difference between the information about the hospitals quality. Better information about the quality at one hospital reduces quality. This is because patients are risk-averse with respect to quality and so will be willing to accept lower mean quality if it is less uncertain. By contrast, in our model such risk-aversion can play no role as there is same the degree of uncertainty about quality at both hospitals. Information affects the demand response to quality changes and we find that increasing information increases equilibrium quality if the two hospitals have similar quality technologies.

2 The model

2.1 Information

There are two hospitals \(H, L\) with quality levels \(q_H, q_L \geq 0\). All patients consume one unit of hospital care, so that the total demand for the two hospitals is constant. Hospital quality only influences the choice of hospital.

Patients obtain imperfect information about hospital quality from primary care physicians’ recommendations, their own past experiences, past experiences of friends and families, and from publicly provided websites. A patient observes a quality signal \(\hat{q}_j\) for hospital \(j\)

\[
\hat{q}_j = q_j + \epsilon_j, \quad \epsilon_j \sim U \left( -\frac{1}{2v}, \frac{1}{2v} \right)
\]

(1)

The errors \(\epsilon_j\) have uniform distributions and zero means. The errors in a patient’s signals about the two hospitals are independent, as are the errors in different patients’ signals about a hospital. \(v > 0\) measures the precision of the signal\(^3\) and increases in \(v\) improve the accuracy of patient observations.

A patient has no prior information about hospital quality and so her expectation of hospital quality after receiving information on quality is\(^4\)

\[
E[q_H|\hat{q}_H] = \hat{q}_H
\]

\(^3\)The variance of the error distribution is \(\frac{1}{4v^2}\).

\(^4\)We could assume that there is a minimum level of quality \(q^o > 1/v\) with the hospitals incurring costs to increase quality above the minimum. This would avoid the case in which some patients’ expectations of quality at a hospital are negative when \(q_j < 1/v\). However this would clutter the notation and make no difference to our results concerning the effect of improved information (larger \(v\)) on hospital choice of quality or on welfare.
2.2 Demand

Patient utility is strictly increasing in hospital quality which is the only characteristic of hospitals that affects utility.\(^5\) Thus the patient will choose hospital \(H\) rather than \(L\) iff \(\hat{q}_H \geq \hat{q}_L\). The mass of patients is 1 so that the demand for hospital \(H\) is the probability that a patient observes that quality in \(H\) is at least as high as in \(L\)

\[
D^H(q_H, q_L) = \Pr[\hat{q}_H \geq \hat{q}_L = \Pr[\varepsilon_L - \varepsilon_H \leq q_H - q_L)] \tag{3}
\]

Since the difference between two uniformly distributed variables is has a triangular distribution, demand for hospital \(H\) is the distribution function of a triangle distribution (see the Appendix for a derivation).

The properties of \(D^H\) (and analogously for hospital \(L\)) are shown in Table 1.

<table>
<thead>
<tr>
<th>(q_H \in [0, q_L - \frac{1}{v}])</th>
<th>([q_L - \frac{1}{v}, q_L])</th>
<th>([q_L, q_L + \frac{1}{v}])</th>
<th>([q_L + \frac{1}{v}, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D^H)</td>
<td>(\frac{v^2}{2} (q_H - q_L + \frac{1}{v})^2)</td>
<td>(1 - \frac{v^2}{2} (q_H - q_L + \frac{1}{v})^2)</td>
<td>1</td>
</tr>
<tr>
<td>(D^H_H)</td>
<td>0</td>
<td>(v + v^2 (q_H - q_L))</td>
<td>0</td>
</tr>
<tr>
<td>(D^H_HH)</td>
<td>0</td>
<td>(v^2)</td>
<td>0</td>
</tr>
<tr>
<td>(D^H_HL)</td>
<td>0</td>
<td>(-v^2 (q_H - q_L + \frac{1}{v}))</td>
<td>(D^H_H &gt; 0)</td>
</tr>
<tr>
<td>(D^H_L)</td>
<td>0</td>
<td>(-v^2 (q_H - q_L + \frac{1}{v}))</td>
<td>(-v^2 (q_H - q_L + \frac{1}{v}))</td>
</tr>
<tr>
<td>(D^H_LL)</td>
<td>0</td>
<td>(-v^2)</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: \(D^H, D^H_H, D^H_H, D^H_L\) are continuous for \(q_H \geq 0\)

Demand is non-decreasing in own quality and non-increasing in the quality of the other hospital. The demand function is convex in own quality when \(H\) has lower quality than \(L\) and concave when it has higher quality. Figure 1 illustrates. The demand function would have a similar shape for other symmetric error distributions, such as the normal and logistic distributions. The triangle distribution is more tractable than these alternatives. Note from the last row of Table 1 that more precise information reduces demand for the lower quality hospital and increases it for the higher quality hospital.

2.3 Hospitals

The hospital cost functions are

\[
c_j(q_j, D^j) = cD^j + \frac{1}{2}\delta_j q_j^2; \quad j = H, L \tag{4}
\]

\(^5\)In general, the hospital market is horizontally as well as vertically differentiated. We extend the model to consider horizontal and vertical differentiation in section 4.
We assume that quality is a public good for the patients of a hospital, as in Gravelle and Masiero (2000) and Brekke et al (2006). The hospital incurs the same cost to achieve a given level of quality irrespective of the number of patients treated. Examples include investment in staff training and information systems.

The hospitals may have different costs of quality. Without loss of generality we assume that $\delta_L \geq \delta_H > 0$. (The subscripts are mnemonics for the quality of the hospitals, not their quality costs: hospital $H$ will turn out to be the higher quality hospital in equilibrium.)

Profits for hospital $j$ are

$$\Pi^j(q_H, q_L) = (p - c)D^j - \frac{1}{2}\delta_j q_j^2, \quad j = H, L$$

(5)

The regulated price $p$ is the same for both hospitals. We assume that hospital managers choose quality to maximise profits. If the hospitals are in public ownership this may because future pay or professional reputation is linked to profit. We discuss the implications of public and private ownership in section 3.

2.4 Nash equilibrium in qualities

The first order conditions for profit maximisation for hospital $H$ are

$$\Pi^H_{q_H} = (p - c)D^H_{q_H} - \delta_H q_H \leq 0, \quad q_H \geq 0, \quad \Pi^H_{q_H}q_H = 0$$

(6)

and analogously for hospital $L$. The first term $(p - c)D^H_{q_H}$ is the marginal net revenue from increasing quality: the increase in demand multiplied by the net profit per unit sold.

Although the quality cost function is convex, the profit function is not concave in own quality because of the non-concavity of the demand function $D^H$ in own quality. Thus (6) is a necessary but not sufficient condition for profit maximisation. $\Pi^H_{q_H} = 0$ may be satisfied at a local minimum and even when it is satisfied at a local maximum, the hospital may be making a loss and would do better with zero quality.

Consider Figure 2 which shows the effect on the profit maximising $q_H$ of increases in $q_L$. Each triangular curve show the marginal net revenue $(p - c)D^H_{q_H}$ from $q_H$ for a given level of $q_L$. From Table 1, for given $q_L$, $(p - c)D^H_{q_H}$ is increasing in $q_H$ and has slope $(p - c)D^H_{q_H} = (p - c)\nu^2$ when $q_H < q_L$ and is decreasing in $q_H$ and has slope $(p - c)D^H_{q_H} = -(p - c)\nu^2$ when $q_H > q_L$. The marginal net revenue triangles are further to the right for higher levels of $q_L$. Thus in the Figure $0 = q^0_L < q^1_L < ... < q^g_L$. 

5
The global profit maximising quality which satisfies (6) depends on the quality cost parameter $\delta^H$. We can distinguish 2 cases.

**Case (a).** Hospital $H$ has high marginal cost of quality in that

$$\delta^H \geq (p - c)v^2 \equiv \delta^o$$

(7)

(i) When $\delta^H > \delta^o$ hospital $H$ marginal cost curve will be steeper than the upward sloping portion of its marginal net revenue curve. Any $q_H^*$ satisfying $\Pi^H_{HH} = 0$ also satisfies the second order condition. Thus if $q_H^* > q_L$ (as when $q_L = q_L^1$ or $q_L = q_L^2$) we have

$$\Pi^H_{HH} = (p - c)D^H_{HH} - \delta^H = (p - c)v^2 - \delta^H < (p - c)v^2 - \delta^o = 0$$

(8)

and if $q_H^* < q_L$ (as when $q_L = q_L^3$) we have

$$\Pi^H_{HH} = (p - c)D^H_{HH} - \delta^H = (p - c)(-v^2) - \delta_H < 0$$

(9)

Thus any $q_H^*$ satisfying $\Pi^H_{HH} = 0$ is a local maximum.

At $q_H = q_L$, $\Pi^H_{HH}$ is discontinuous but it is obvious from Figure 2 that $q_H^* = q_L = q_L^2$ is a local profit maximiser since the marginal cost curve $\delta^H q_H$ cuts the net marginal revenue curve from below.

It is also apparent from Figure 2 that profit is positive at any $q_H^*$ satisfying the first order condition, since the area under the marginal cost curve is always less than the area under the net marginal revenue curve. Hence such $q_H^*$ are also global optima. Finally, we see that if $q_L \geq 1/v$ then the optimal $q_H^* = 0$.

(ii) When $\delta^H = \delta^o$ the second order condition is satisfied at $q_H^*$ satisfying $\Pi^H_{HH} = 0$ for $q_H^* > q_L$. When $q_L = 1/v$ the marginal cost curve coincides with the net marginal curve and profit is zero for all $q_H \in [0, q_L]$. We assume that hospital managers are lexicographically altruistic (if profit is unaffected they prefer to care for more patients rather than less) so that the hospital sets $q_H^* = q_L = 1/v$. When $q_L > 1/v$, profit is maximised at zero quality.

Figure 2 shows that as $q_L$ increases from zero the optimal $q_H^*$ is initially increasing as $H$ moves up its marginal cost curve from $a_0$, to $a_1$ and then $a_2$. At $a_2$ we have $q_H^* = q_L^2$ and $\delta_H q_H^* = (p - c)D^H_{H} = (p - c)v$ so that $q_H^*$ is increasing with $q_L$ up to $q_L = (p - c)v/\delta_H$. Further increases in $q_L$ move $H$ back down its marginal cost curve from $a_2$, to $a_3$ and when $q_L \geq q_L^4 = 1/v$ the optimal $q_H^*$ is zero.
Thus if \( \delta_H \geq \delta^o \) holds the reaction function for \( H \) is

\[
\begin{align*}
  r^H_a(q_L, \delta_H) &= \frac{(p-c)(v+v^2q_L)}{\delta_H + v^2(p-c)}, & q_L \in [0, (p-c)v/\delta_H] \\
  &= \frac{(p-c)(v-v^2q_L)}{\delta_H - v^2(p-c)}, & q_L \in [(p-c)v/\delta_H, 1/v] \\
  &= 0, & q_L \geq 1/v
\end{align*}
\]

The reaction function is illustrated in Figure 2. Its intercept \( r^H_a(0, \delta_H) \) on the \( q_H \) axis is at or below \( \frac{1}{2v} \) since \( \delta_H \geq \delta^o = (p-c)v^2 \) and its intercept on the \( q_L \) axis is at \( \frac{1}{v} \). When \( \delta_H = \delta^o \) the reaction function \( r^H_a(q_L, \delta^o) \) starts at \( q_H = \frac{1}{2v} \) and increases with \( q_L \) to \( \frac{1}{v} \) at the 45° line and jumps downward to zero for \( q_L > \frac{1}{v} \).

**Case (b).** Now suppose that \( \delta_H < \delta^o \). From Figure 2 we see that when \( q_L < 1/v \) the marginal cost curve \( \delta^b_Hq_H \) with \( \delta^b_H > \delta^o \) cuts the marginal net profit curve once from below and so any \( q^*_H \) satisfying the first order condition is both a local and global profit maximiser. When \( q_L > 1/v \) (as when \( q_L = q^*_L, q_L = q^*_L^6 \) or \( q_L = q^*_L^7 \)) the marginal cost curve cuts the net marginal revenue curve twice. The second order condition is satisfied only on the downward sloping part of the net marginal revenue curve where \( q^*_H > q_L \).

For \( q_L \) close to \( 1/v \), the \( q^*_H \) satisfying the first and second order conditions is also a global maximiser since the area under the marginal cost curve is less than the area under the marginal net profit curve. But, as \( q_L \) increases the area under the marginal cost curve up to \( q^*_H \) increases whereas the area under the triangular net at the optimal point decrease. Hence when \( \delta_H < \delta^o \) there exists a \( q^*_L \) such that the profit at the solution satisfying the first and second order conditions is negative for \( q_L > q^*_L \). Hence the optimal \( q^*_H = 0 \) when \( q_L > q^*_L \). In Figure 2 \( q^*_L = q^*_L^6 \). As \( q_L \) increases hospital \( H \) moves up its marginal cost curve \( \delta^b_Hq_H \) from \( b_0 \) (when \( q_L = 0 \)) to \( b_1, ..., b_5 \) until \( b_6 \) where \( q^*_L^5 = q^*_L \). When \( q_L = q^*_L^7 > q^*_L, b_7 \) satisfies the necessary and sufficient conditions for a local maximum but profit is negative and the global maximum is at zero quality.

Thus in case (b) where \( \delta_H < \delta^o \), the reaction function for \( H \) is

\[
\begin{align*}
  r^H_b(q_L, \delta_H) &= \frac{(p-c)(v+v^2q_L)}{\delta_H + v^2(p-c)}, & q_L \in [0, q^*_L] \\
  &= 0, & q_L \geq q^*_L
\end{align*}
\]

In Figure 3 the reaction function has an intercept \( r^H_b(0, \delta_H) \) between \( \frac{1}{2v} \) and \( \frac{1}{v} \) on the \( q_H \) axis and is upward sloping until \( q_L = q^*_L \) where it jumps downward to zero.
Hospital $L$ has the analogous reaction functions and Figure 4 plots the case (a) and (b) reaction functions for both hospitals. If both hospitals have a low cost parameter ($\delta_H, \delta_L < \delta^o$) then the reaction functions are $r^H_b(q_L, \delta_H)$, $r^L_b(q_H, \delta_L)$ and there is no Nash equilibrium in pure strategies because the reaction functions do not intersect.

Since without loss of generality we have assumed $\delta_H \leq \delta_L$, there are three types of Nash equilibria. First, when the cost parameters are $\delta_H = \delta_L = \delta^o$, there is a Nash equilibrium (not shown) on the 45° line at $q_H = q_L = \frac{1}{v}$ if both hospitals prefer to produce a positive quality rather than no quality when both qualities yield the same profit. However this equilibrium is not robust to small downward perturbations in one of the cost parameters.

Second, when $\delta_H \leq \delta^o < \delta_L$, there is an equilibrium at $NE_{ba}$ above the 45° line where $r^H_b(q_L, \delta_H) = r^L_b(q_H, \delta_L)$. Third, when $\delta^o < \delta_H \leq \delta_L$, the equilibrium is at $NE_{oa}$ on or above the 45° where $r^H_a(q_L, \delta_H) = r^L_a(q_H, \delta_L)$.

Sufficient conditions for the stability of the equilibrium are (Dixit, 1986)

$$0 > \Pi^j_{jj}, \quad j = H, L \quad (12)$$
$$0 < \Pi^H_{HL} - \Pi^H_{HH} \Pi^L_{HH} \quad (13)$$

where

$$\Pi^j_{jj} = (p - c)D^j_{jj} - \delta_j, \quad j = H, L \quad (14)$$
$$\Pi^H_{HL} = (p - c)D^H_{HL}, \quad \Pi^L_{HH} = (p - c)D^L_{HL} \quad (15)$$

The second order conditions $\Pi^j_{jj} < 0$ for profit maximisation are satisfied on the reaction functions. Referring to Table 1, (13) is

$$[-(p - c)D^L_{LL} - \delta_H][D^L_{LL} - \delta_L] + (p - c)^2(D^L_{LL})^2 = \delta_H \delta_L + (p - c)D^L_{LL}(\delta_L - \delta_H) > 0$$

since $\delta_L \geq \delta_H > 0$ and so the second and third types of Nash equilibria are stable.

The reaction function for hospital $H$ has the same form above the 45° whether the Nash equilibrium is $NE_{ba}$ or $NE_{oa}$. Hence solving the reaction functions we have

**Proposition 1** There is a unique stable Nash equilibrium if and only if $\delta_L > \delta^o \equiv (p - c)v^2$ (equivalently if and only if $v < v^o \equiv \sqrt[2]{\delta_L/(p - c)}$). The Nash Equilibrium qualities are

$$q^*_H = \frac{v(p - c)\delta_L}{\delta_H \delta_L + v^2(p - c)(\delta_L - \delta_H)} \quad (16)$$
$$q^*_L = \frac{v(p - c)\delta_L}{\delta_H \delta_L + v^2(p - c)(\delta_L - \delta_H)} \quad (17)$$
and $0 < q_L^* \leq q_H^* < \frac{1}{v}$

In the symmetric case where $\delta_H = \delta_L = \delta$

$$q_H^* = q_L^* = q^* = \frac{(p - c)v}{\delta}$$  \hspace{1cm} (18)

Notice that the relationship between the Nash equilibrium qualities must satisfy the ratio condition

$$\frac{q_H^*}{q_L^*} = \frac{\delta_L}{\delta_H}$$  \hspace{1cm} (19)

so that ratio of the quality production parameters defines the relative difference between $q_H^*$ and $q_L^*$.

### 2.5 Comparative Statics

Differentiation of (16) and (17) yields the comparative static properties of the Nash Equilibrium. We summarise the effects of changes in prices and cost parameters in

**Proposition 2**

(i) The quality of both hospitals is increasing in the price and decreasing in the unit cost of production $(\partial q_j^*/\partial (p - c) > 0, j = H, L)$ and decreasing in each hospital’s own quality cost parameter $(\partial q_j^*/\partial \delta_j < 0, j = H, L)$.

(ii) When the quality cost parameters differ $(\delta_L > \delta_H)$, the quality of hospital $H$ is decreasing in the quality cost parameter of hospital $L$ $(\partial q_H^*/\partial \delta_L < 0)$ and hospital $L$ quality is increasing in the quality cost parameter of hospital $H$ $(\partial q_L^*/\partial \delta_H > 0)$.

The propositions can also be demonstrated diagrammatically by making use of the fact that the equilibrium is defined equivalently by either of the reaction functions ((16) or (17)) and the ratio condition (19). Thus in Figure 5 the initial equilibrium is at $\text{NE}_0$ where the two reaction functions and the ratio condition locus intersect. The intersection of the reaction function $r_a^L$ with the $45^\circ$ line is at $q = v(p - c)/\delta_L$. The ratio condition locus depends only on the quality cost parameters. Thus an increase in $(p - c)$ will pivot $r_a^L$ upwards (not shown) and shift the equilibrium up the ratio condition locus, increasing both $q_H^*$ and $q_L^*$.

Consider the slightly less intuitive result that changes in the other hospital’s quality parameter have the opposite effects on $q_H^*$ and $q_L^*$. Suppose that $\delta_L$ increases, thereby increasing $(\delta_L/\delta_H)'$ to $(\delta_L/\delta_H)''$ and steepening the
ratio condition locus. The increase in $\delta_L$ has no effect on $r^H_a$ so that equilibrium shifts from NE$_0$ to NE$_2$. Hence increasing $\delta_L$ reduces both $q^*_H$ and $q^*_L$. Suppose instead that the initial ratio condition locus is $(\delta_L/\delta_H)'$ and the equilibrium is at NE$_1$. Now let $\delta_H$ increase, shifting the ratio condition locus to $(\delta_L/\delta_H)$. $\delta_H$ has no effect on $r^L_a$ and so the equilibrium shifts to NE$_0$ from NE$_1$. The increase in $\delta_L$ reduces $q^*_L$ but increases $q^*_H$.

Our main interest is in the effect of better information (higher $v$). The diagrammatic analysis is more complicated in this case because increases in $v$ shift the intercepts of both reaction curves on the vertical axis and the 45° line. For example, the intercept of the $r^L_x$ reaction curve shifts down and the intercept on the 45° line shifts up, so that the equilibrium could be shifted up or down the ratio condition locus. Thus quality could be increased or reduced by better information. Since the ratio condition locus is unaffected by $v$ we do know however that the hospitals qualities will move in the same direction.

We establish

**Proposition 3** (i) *Improvement in information increase (reduce) the quality of both hospitals iff $\delta_H > (\leq) \hat{\delta} \equiv \delta^o \delta_L/(\delta^o + \delta_L)$ where $\delta^o = v^2(p - c)$.*

(ii) *Improvement in information increases the quality of both hospitals if the quality cost parameter of the lower cost hospital is large enough: $\delta_H > \delta^o \Rightarrow \partial q^*_j/\partial v > 0$, $j = H, L$.*

(iii) *Improvement in information increases the quality of both hospitals if the relative difference in the quality cost parameters is small enough: $\delta_H > \frac{1}{2} \delta_L \Rightarrow \partial q^*_j/\partial v > 0$, $j = H, L$.*

(iv) *If $\delta_L > \delta_H$ quality at both hospitals is maximised with respect to information at $v = \hat{v} \equiv \{\delta_H\delta_L/[(\delta_L - \delta_H)(p - c)]\}^{\frac{1}{2}}$ and is increasing in $v$ for $v \in (0, \hat{v})$ and decreasing in $v$ for $v \in (\hat{v}, \delta^o)$ where $\delta^o \equiv [\delta_L/(p - c)]^{\frac{1}{2}}$.*

(v) *If $\delta_L = \delta_H$ quality at both hospitals is always increased by better information for $v \in (0, \delta^o)$.*

**Proof.** Part (i) follows from the differentiation of (16) and (17) and the definition (7) of $\delta^0$. For part (ii) note that $\hat{\delta}$ is strictly increasing in $\delta_L$ and $\lim_{\delta_L \to \delta^0} \hat{\delta} = \delta^o$ so that $\delta_H \geq \delta^o$ implies $\delta_H \geq \hat{\delta}$.

For part (iii) refer to Figure 6 and note that $\lim_{\delta_L \to \delta^0} \hat{\delta} = \frac{1}{2} \delta^o$. The derivative of $\hat{\delta}$ with respect to $\delta_L$ is $[\delta^o/(\delta^o + \delta_L)]^2$ which is decreasing in $\delta_L$ and $\lim_{\delta_L \to \delta^0} d\hat{\delta}/d\delta_L = \frac{1}{4}$. Thus $\hat{\delta}$ always lies below the ray from the origin through $(\delta^o, \frac{1}{2} \delta^o)$ with slope $\frac{1}{2}$ or equivalently $\frac{1}{2} \delta_L > \hat{\delta}$.

For part (iv) substitute $v^2(p - c)$ for $\delta^o$ in the condition $\delta_H \geq \hat{\delta} \equiv \delta^o \delta_L/(\delta^o + \delta_L)$ for $q^*_j$ to increasing or decreasing in $v$. Part (v) follows from substituting $\delta_H$ for $\delta_L$ in the definition of $\hat{\delta}$. ■
Figure 6 illustrates the relationship between the quality cost parameters, the existence of Nash Equilibrium and the effect of better information on quality. By the assumption that $\delta_L \geq \delta_H$ and Proposition 1, there is a Nash Equilibrium if and only if the quality cost parameters are in the region below the 45° line and to the right of $\delta'$. Better information increases quality in the region above the $\delta$ locus.

Figure 7 shows the effect of better information on quality for two cases. In the first, an example of the quality cost configuration above the $\delta$ locus in Figure 6, an increase in $v$ increases the qualities of both hospitals. The second, where the relative difference $(\delta_L / \delta_H)$ is much greater, is an example of a cost configuration below the $\delta$ locus in Figure 6.

To get some intuition for the conclusion that in some circumstances better information leads to a reduction in quality note that the equilibrium can be defined by the interior proﬁt maximisation condition $\Pi_H = 0$ (6) for hospital $H$ and the fact that the equilibrium qualities are proportional $q_H \delta_H - q_L \delta_L = 0$ (19). In the case $q_H > q_L$ totally differentiating with respect to $v$ gives (see Table 1)

$$\frac{\partial q_H}{\partial v} = \frac{\delta_L \Pi_H^H}{- [\Pi_H^H \delta_L + \delta_H \Pi_H^L]} = \frac{\delta_L (p - c) D_H^H}{(p - c)v^2(\delta_L - \delta_H)}$$

(20)

Thus $\text{sgn } \frac{\partial q_H}{\partial v}$ (and $\text{sgn } \frac{\partial q_L}{\partial v}$ since $\delta_L^* = q_H^* \delta_H / \delta_L$) is

$$\text{sgn } \frac{\partial q_H}{\partial v} = \text{sgn } \Pi_H^H = \text{sgn } D_H^H \text{sgn } [1 - 2v(q_H - q_L)]$$

(21)

Equilibrium quality decreases if and only if better information reduces the marginal revenue from increasing quality.

Patients choose the hospital with the highest perceived quality. A patient’s choice is not affected by the magnitude of her perceived diﬀerence in quality, only by its sign. Hence the demand for hospital $H$ is the distribution function $D_H = F(q_H - q_L; v) = F(\Delta q; v)$. An increase in $q_H$ increases demand for hospital $H$ at the rate $f(q_H - q_L; v)$. This density of the diﬀerence in patient quality perceptions is unimodal with mode at $\Delta q = 0$. An increase in information $v$ is equivalent to a mean preserving contraction in the distribution, shifting probability mass from the tails to the centre, increasing $f$ near to $\Delta q = 0$ and decreasing $f$ when $\Delta q$ is suﬃciently large.. Hence when hospital $H$ quality is similar to that of hospital $L$, an improvement in information shifts up $f(\Delta q; v)$ and increases its marginal revenue from quality.

Figure 8 illustrates. As the figure shows an increase in $v$ steepens the marginal revenue curve for the firm and also shifts its intercepts on the horizontal axis. Thus when $q_H$ is close to $q_L$ the marginal revenue curve
is shifted up and optimal quality for hospital \( H \) increases. When \( q_H \) and \( q_L \) are sufficiently different the marginal revenue curve is shifted down and the optimal quality for hospital \( H \) is reduced.

In the next section we make assumptions about the welfare function in order to analyse the welfare implications of improvements in information. However, even in the absence of a welfare function, Proposition 3 is policy relevant in showing that improved information may not increase quality if hospitals have sufficiently different quality cost parameters.

Policy makers looking to encourage quality competition, may improve the information consumers have about hospital quality. However, the model suggests they should also ensure that hospitals have relatively equal access to capital investment and labour markets for management and doctors, represented in the model by \( \delta_H \) and \( \delta_L \) to enable them to compete for patients on quality. Where hospitals have very unequal resources for improving quality, our model suggests that increasing information levels can reduce equilibrium quality of both high and low quality hospitals.

3 Welfare

3.1 Average patient utility

Although patients care only about the quality of the hospital they choose: \( u = \max\{q_H + \varepsilon_H, q_L + \varepsilon_L\} \), the welfare implications of improved information depend crucially on whether one takes an ex ante or ex post view of welfare.

Ex ante patient expected utility is

\[
U^A = \int \int_{\varepsilon_H + q_H - q_L} (q_H + \varepsilon_H) f(\varepsilon_H; v) f(\varepsilon_H; v) d\varepsilon_L d\varepsilon_H + \int \int_{\varepsilon_H + q_H - q_L} (q_L + \varepsilon_L) f(\varepsilon_L; v) f(\varepsilon_H; v) d\varepsilon_L d\varepsilon_H
\]

where the first part is expected utility from choosing hospital \( H \) (when \( q_H + \varepsilon_H - q_L \geq \varepsilon_L \)) and the second from choosing hospital \( L \) (when \( q_H + \varepsilon_H - q_L < \varepsilon_L \)). Since \( \varepsilon_j \) is uniformly distributed on \([\frac{1}{2v}, \frac{1}{2v}]\) ex ante expected patient utility can be written as

\[
U^A(q_H, q_L, v) = \frac{1}{2}(q_H + q_L) + \frac{1}{2}v(\Delta q)^2 - \frac{v^2}{6}(\Delta q)^3 + \frac{1}{6v}
\]

\(^6\)Patients either pay no price for care or they pay the same price whichever hospital is chosen. In the latter case the welfare function would contain a term equal to the total amount paid by patients multiplied by \((\sigma - \lambda)\). Since total demand is constant this term has no bearing on the welfare analysis of information policy.
where $\Delta q = q_H - q_L$.

Using the ex ante specification of expected patient utility in the welfare function requires that we respect patient imperfect observations about quality as well as their preferences. An alternative justification for the ex ante form is that $q_j + \varepsilon_j$ reflect variations in actual quality delivered to a patient at hospital $j$. This requires a rather strained interpretation of increases in $v$ as due to a policy which reduces the amount of quality variation at both hospitals. An example might be promulgation of best practice guidelines.

The effect of an increase in hospital $H$ quality at given $q_L, v$ is

$$U_{qH}^A = \frac{1}{2} + v\Delta q - \frac{v^2}{2} (\Delta q)^2 = D_H$$

and similarly for an increase in $q_L$. An increase in quality at a hospital causes some patients to change their choice of hospital. But these are the patients who are indifferent between the two hospitals given their observations of quality and so they do not gain or lose from the small change in quality. Thus the only effect of the quality increase is on the patients who choose that hospital.

The effect of better information (higher $v$) at given $q_H, q_L$ is

$$U_v^A = \left[ \frac{1}{2} - \frac{v}{3} \Delta q \right] (\Delta q)^2 - \frac{1}{6v^2}$$

The first term in (25) is positive since $\left[ \frac{1}{2} - \frac{v}{3} \Delta q \right] > \left[ \frac{1}{2} - v\Delta q + \frac{v^2}{2} (\Delta q)^2 \right] = D_L > 0$. This is intuitive: when quality differs better information improves patient choices and one would expect the increase in patient utility to be greater the larger is the difference in qualities.

Less intuitive is the contribution of the second term in (25) which reduces the gain from better information and may make $U_v^A < 0$. Indeed when there is no difference in quality between the two hospitals better information reduces expected utility. The rationale is that utility is $\max\{q_H + \varepsilon_H, q_L + \varepsilon_L\}$. If $q_H = q_L = q$, expected utility is $q + E[\max\{\varepsilon_H, \varepsilon_L\}]$ and the expected value of the maximum of two independent draws from the uniform distribution on $[\frac{-1}{2v}, \frac{1}{2v}]$ is $\frac{1}{3v}$. This is smaller the higher is $v$: there is less chance that at least one of the observations will exceed any specified value.

A simpler specification of average patient utility is that we ignore patient errors in observing quality and evaluate the care they receive at its true quality: we take an ex post perspective. Ex post expected or average patient
utility is
\[
U^P(q_H, q_L, v) = D^H(q_H, q_L, v)q_H + D^L(q_H, q_L, v)q_L = \frac{1}{2}(q_H + q_L) + v(\Delta q)^2 - \frac{v^2}{2}(\Delta q)^3
\]  
(26)

Using the fact that \(D_j^k = -D_k^j > 0\) we see that ex post utility is increasing in \(q_H\)
\[
U^P_{q_H} = D^H + D^H_H q_H + D^H_L q_L = D^H + D^H_H(q_H - q_L) > 0
\]  
(27)

whereas increases in \(q_L\) may reduce ex post utility
\[
U^P_{q_L} = D^L + D^L_H q_H + D^L_L q_L = D^L - D^L_L(q_H - q_L)
\]  
(28)

When \(q_H > q_L\) increases in \(q_j\) increase the utility of those who chose hospital \(j\) and induce some patients to switch to hospital \(j\) from hospital \(k\). If the patients are switching to a higher quality hospital they gain ex post as well. But if the patients switch from the high quality to the low quality hospital then they are worse off ex post as result of the quality increase. With the ex ante expected utility criterion increases in quality always raises patient utility.

A second contrast with \(U^A\) is that \(U^P\) is never decreased when information improves:
\[
U^P_v = (\Delta q)^2 - v(\Delta q)^3 = (\Delta q)^2[1 - v\Delta q] \geq 0
\]  
(29)

As a result of the better information patients switch from the lower to the higher quality provider. This direct switching benefit from switching is decreasing in \(v\) as there are fewer patients at hospital \(L\) to switch to hospital \(H\) information improves.

We summarise the implications of the ex ante and ex post views in

**Proposition 4** Patients are always made better ex ante off by improvements in quality but may be made worse off by improvements in information. Patients may be made worse off ex post by improvements in the quality of the lower quality hospital but are never made worse off by improvements in information.

3.2 Welfare function

For the derivation and analysis of equilibrium hospital quality we assumed that hospital managers aimed to maximise hospital profit whether the hospitals were public or privately owned. But in order to undertake welfare
analysis it is necessary to specify hospital ownership ie who is the residual claimant for their profits and losses. There are two possibilities compatible with the analysis in the previous sections. Under private ownership the managers act as perfect agents in maximising profit for private owners. Under public ownership the managers care about profit for intrinsic reputational reasons and the taxpayers are the residual claimants.

Remembering that total demand is fixed and normalised to 1, we specify the welfare function as

\[ W^k(q_H, q_L, v) = \alpha (\Pi^H + \Pi^L) + \sigma U^k(q_H, q_L, v) - \lambda [p(D^H + D^L) + g(v)] \]

\[ = \alpha \left( p - c - \frac{1}{2} \delta_H q_H^2 - \frac{1}{2} \delta_L q_L^2 \right) + \sigma U^k(q_H, q_L, v) - \lambda (p + g(v)) \]

(30)

where \( U^k \) is either ex ante \((k = A)\) or ex post \((k = P)\) patient utility and \( g(v) \) is the cost of improving information. \( \alpha \) is the welfare weight on hospital profit, \( \sigma \) the welfare weight on expected patient utility and \( \lambda (> 1) \) either measures the shadow price of public funds or the welfare weight on taxpayers.\(^7\) To characterise public ownership of hospitals we can set \( \alpha = \lambda \) and for private ownership we would assume that \( a < \lambda \). Changes in \( v \) alter patients’ choices of hospitals not their total demand for care. The welfare effects of improved information arise from its effect on the total cost of producing quality and patient utility. Hospital revenues and government expenditure do not depend on the amount of information.

Policy makers potentially have two instruments: they can provide better information about hospital quality (increase \( v \)) and they may be able to set the price \( p \). Both \( v \) and \( p \) can be used to influence hospital quality, but information also has a direct effect on patient welfare at given quality levels because it improves their decisions about which hospital to choose. The policy rhetoric accompanying policies to improve information suggests that policy makers want to use information to increase competition and thereby drive up quality. This implies that policy makers do not regard price as means of affecting quality. However, for the sake of completeness we derive optimal conditions on information and price.

\(^7\)We do not enquire about the nature of the contract between owners and managers nor do we specify the relative welfare weights on private owners and managers. If there is a profit sharing contract then the welfare weight on firm profit depends on the profit share and the relative welfare weights on owners and managers but since owner and manager utility will be proportional to profit we set the resulting mix of welfare weights and profit shares to \( \alpha \). If there is a forcing contract so that managers get a fixed salary but nothing if they fail to maximise profit, we can drop their constant utility from the welfare function.
Recall from section 2.4 that if \( \delta_L < v^2(p-c) \) there is no Nash equilibrium. We therefore impose the constraint \( \delta_L \geq v^2(p-c) \) on the choice of policy instruments.\(^8\) Writing the policy Lagrangean as \( W^k + \gamma[\delta_L - v^2(p-c)] \), necessary conditions for optimal policy are

\[
\sum_j [\sigma U^k_{q_j} - \alpha \delta_j q^*_j]q^*_v + \sigma U^k_v - \lambda g'(v) - \gamma 2v(p-c) = 0 \tag{31}
\]

\[
\sum_j [\sigma U^k_{q_j} - \alpha \delta_j q^*_j]q^*_p - (\lambda - \alpha) - \gamma v^2 = 0 \tag{32}
\]

where \( q^*_j, q^*_p \) are the partial derivatives of the equilibrium quality \( q^*_j \) with respect to the information parameter and price.

If the hospitals have the same quality technologies so that \( \delta_H = \delta_L \), and \( g'(v) \) is such that the constraint does not bind at the optimum so that \( \gamma = 0 \), (31) simplifies to

\[
\frac{dL^k}{dv} = (\sigma - \alpha \delta q^*)q^*_v + \sigma U^k_v - g'(v) = 0 \tag{33}
\]

Since \( U^P_v = 0 > U^A_v \) and \( q^*_v > 0 \), we see that the optimal quality will be less than the “first best” \( q = \sigma/2\alpha \delta \). Optimal quality will be lower under public than under private ownership because the welfare weight on costs will be less. The optimal level of information will be lower when the ex ante view of patient welfare is taken. If hospitals have different quality technologies it is no longer possible to say whether information would be greater under the ex post or ex ante view of patient utility.

Now suppose that health care policy makers can also control the price paid to hospitals. Using (32) to solve for \( \sigma U^k_{q_L} - \delta_j q^*_L \) and substituting into (31) gives

\[
\frac{dL^k}{dv} = \left[ \sigma U^k_{q_L} - \alpha \delta_j q^*_L \right] \left[ q^*_v - q^*_p \right] + \left[ (\lambda - \alpha) + \gamma v^2 \right] \frac{q^*_v}{q^*_p} + \sigma U^k_v - \lambda g'(v) - \gamma 2v(p-c)
\]

\[
= \sigma U^k_v - \lambda g'(v) + \left[ (\lambda - \alpha) + \gamma v^2 \right] \frac{q^*_v}{q^*_p} - \gamma 2v(p-c)
\]

\[
= \sigma U^k_v - \lambda g'(v) - (\lambda - \alpha) \frac{\partial p}{\partial v} \bigg|_q + \frac{\gamma}{v^2} \left[ \frac{q^*_v}{q^*_p} - \frac{\delta^2}{\delta^2} \right] \tag{34}
\]

\(^8\)The comparative static properties were derived on the assumption that \( \delta_L > v^2(p-c) \) which was sufficient for a stable unique equilibrium. There is also an equilibrium with \( \delta_L = \delta_H = v^2(p-c) \) but it is not robust to small increases in \( v \) and \( p \). We include this equilibrium possibility in the welfare analysis to ensure that the feasible set is closed. By assuming a sufficiently high marginal cost on information we can restrict attention to equilibria where \( \delta_L > v^2(p-c) \).
If price is set optimally there are four effects of better information on welfare. First, better information may make patients better off. Second, better information may have a marginal cost. Third, both information and price affect quality. Under private ownership $\lambda > \alpha$ and a lower price improves welfare if quality fixed. Hence if better information increases quality it permits a reduction in the price and thereby increases welfare in the case of private hospitals. Finally, the last term is the gain in welfare from an increase in $v$ if $v$ has, relative to $p$, a bigger effect on quality than in tightening the constraint.

If hospitals are publicly owned and the constraint does not bind, then the only policy relevant effects of information is its direct effect on patient utility and its marginal cost.

4 Horizontal differentiation

We have so far assumed that hospitals differ only in vertical quality in order to focus on the effects of improved information on quality and to derive welfare propositions. However, this simplification carries the cost that we have had to ignore one obvious policy which achieves the first best: since quality has a fixed cost independent of the number of patients served, the first best can be achieved by closing the hospital with the highest cost of quality. The obvious reason why hospital closure may not be welfare improving is that the hospitals are also horizontally differentiated: perfectly informed patients care about which hospital they use even if the quality of care is the same in the two hospitals. We therefore now sketch a Hotelling type model (Brekke et al 2006) with horizontal differentiation in which better information can reduce equilibrium quality but closure of the hospital with the higher cost of quality is not first best.

We extend our simple model of vertical quality differences to the case where there are two types of patient. The first type have mass 1 and are uniformly distributed at $s \in [0, 1]$ along a road or product space with hospital $H$ located at $s = 0$ and $L$ located at $s = 1$. A patient located at $s$ has perceived utility $q_H + \varepsilon_H - ts$ from hospital $H$ and $q_L + \varepsilon_L - t (1 - s)$ from hospital $L$, where $t > 0$ is a travel cost parameter. We make the same assumptions about the error distributions as before (section 2.1). The second type of patient has a very high travel or mismatch parameter $\tau$ and mass $m_L$ of them are located at $s = 1$. We can always find a sufficiently high $\tau$ so that in equilibrium all of these patients will choose $L$ ($\tau > \Delta q + \frac{1}{2\tau}$) and the cost saving from shutting down $L$ ($q_L^2 \delta_L / 2$) is less than the loss in expected utility ($m_L (\tau - \Delta q)$) they would suffer from being forced to use hospital $H$.

Now consider the equilibrium, which is determined by competition for the
patients who are uniformly distributed along the road and have transport cost \( t < \tau \). Each of these patients always prefers treatment to no treatment and chooses hospital \( H \) if and only if \( q_H + \varepsilon_H - ts \geq q_L + \varepsilon_L - t(1-s) \), or \( \Delta q + t(1-2s) \geq \varepsilon_L - \varepsilon_H \).

We derive their demand functions by first calculating the demand by patients located at \( s \) and then integrating these location specific demands over \( s \in [0,1] \) to get demand for hospital \( H \) (and hence \( L \)) as a function of the hospital qualities, distance cost parameter \( t \), and the information precision parameter \( v \). There are three critical locations. All patients at \( s \) will choose \( H \) if \( q_H + \varepsilon_H - ts \geq q_L + \varepsilon_L - t(1-s) \) or if

\[
\begin{align*}
1 - \frac{v^2}{2} \left( -\Delta q - t + \frac{1}{v} + 2st \right)^2
= 1 - 2v^2t^2(-s_1 + s)^2
\end{align*}
\]

where \( s_i = \min\{\max\{0, s_i\}, 1\}, i = 1, 2, 3 \) and \( D^H_i(s, \Delta q) \) is the location specific demand. Total demand for \( H \) is

\[
D^H = \int_{s_1}^{s_2} D^{H1} ds + \int_{s_1}^{s_2} D^{H2} ds + \int_{s_2}^{s_3} D^{H3} ds + \int_{s_3}^{1} D^{H4} ds
\]

All patients of the first type are treated and so \( D^L = 1 - D^H + m_L \).

Depending on the parameters \( t, v \) and the quality difference \( \Delta q \), the market for the first type of patient can have six configurations: complete
monopoly for $H$ when $s_1 > 1$, local monopoly for $H$ when $0 < s_1 < 1 < s_3$, local monopolies for $H$ and $L$ when $0 < s_1 < s_3 < 1$, full competition when $s_1 < 0 < 1 < s_3$, local monopoly for $L$ when $s_1 < 0 < s_3 < 1$, and complete monopoly for $L$ when $s_3 < 0$.

Since the demand function depends on $\Delta q$, we have, as in the simpler model of earlier sections, $D^H_H = D^L_L$. Hence equilibria in which both hospitals produce positive quality must satisfy the same conditions as in the simpler model: interior profit maximisation $\Pi^H_H = 0$ (6) and proportional equilibrium qualities $q_H \delta_H - q_L \delta_L = 0$ (19).

The Nash Equilibria take different forms depending on the equilibrium market configuration. Establishing that an equilibrium with a particular market configuration is proof against deviations by either firm which alter $\Delta q$ and hence may shift the market to another configuration requires much tedious manipulation. We sketch two example equilibria.

If $0 < s_1$ and $s_3 < 1$ then each hospital monopolises one section of the market for the first type of patient. The demand function (38) simplifies to

$$D^H = \hat{s} = s_2 = \frac{\Delta q + t}{2t}$$

(39)

Each hospital always gets half of the patients in the competitive segment $(s_1, s_3)$ since in this segment the difference in quality $\Delta q$ is on average offset by the distance cost and so patients’ choices are random. Increases in $v$ increase the monopoly segments, but do so at the same rate. Hence the market share of firms is unaffected by the uncertainty about quality. Since the information parameter $v$ has no effect on demand it has no effect on firm’s choice of quality.

**Proposition 5** If $vt > 1$ there exist equilibria in which improvements in information have no effect on equilibrium quality.

**Proof.** (Sketch.) Suppose that $\delta_H = \delta_L$, then the equilibrium will have $\Delta q = 0$. If $\Delta q = 0$ then $vt > 1$ implies both $0 < s_1$ and $s_3 < 1$ and so the demand for both hospitals is unaffected by $v$.

The rationale for the condition $vt > 1$ is that with large $v$ (little uncertainty about quality) or large $t$ (high distance costs) patients near a hospital will not choose the alternative hospital even if they believe the nearest hospital has lower quality. Any error in quality perception will be small and the perceived quality difference will be insufficient to outweigh the transport costs.

In the second example, the firms compete across the entire market for the first type of patient ($s_1 < 0$, $s_3 > 1$) and we assume that the quality
difference is large enough that $s_2 > 1$. The demand function for $H$ is then

$$D^H = \int_0^1 D^{H2} ds = \int_0^1 [1 - 2v^2 t^2 (-s_1 + s)^2] ds \quad (40)$$

Solving the conditions (6) and (19) for the interior solution Nash Equilibrium yields

$$q^*_H = \frac{v(p - c)\delta_L}{\delta_H \delta_L + v^2(p - c)(\delta_L - \delta_H)} \quad (41)$$

$$q^*_L = \frac{v(p - c)\delta_H}{\delta_H \delta_L + v^2(p - c)(\delta_L - \delta_H)} \quad (42)$$

which is the same as when there is no horizontal differentiation.

Differentiation of (41) or equivalently of $D^H$ with respect to $v$ shows that an improvement in information (increase in $v$) increases or decreases equilibrium quality as

$$v^2(p - c) \gtrless \frac{\delta_H \delta_L}{\delta_L - \delta_H} \quad (43)$$

We must also check that the conditions $s_1 < 0$, $s_2 > 1$ are satisfied so the demand function is (40). These conditions are satisfied if $t < \Delta q < \frac{1}{v} - t$ where, from (19), $\Delta q = q_H - q_H(\delta_L - \delta_H)/\delta_L$. Thus we have

**Proposition 6** An improvement in information will reduce vertical quality when there is horizontal differentiation if $\frac{\delta_H \delta_L}{\delta_L - \delta_H} \in \left(v^2(p - c) \left(\frac{vt}{1-vt}\right), v^2(p - c)\right)$.

## 5 Conclusions

Our results give some insights about how changes in information affect hospital quality competition. We model patient information as an imperfect signal about true hospital quality and focus on the precision of the signal as a policy instrument. One contribution we make is highlighting the influence of heterogeneous quality-production technologies. The model shows that increasing information will increase hospital quality only if the level of information is relatively low, and/or the hospitals have similar quality-producing technologies. Furthermore, the level of information that maximises quality is lower the higher is the gap between hospitals’ quality-producing technology.

Governments looking to encourage quality competition may hope to do so by improving the information consumers have about hospital quality. However, our model suggests that governments must also ensure that hospitals have relatively equal access to capital investment and labour markets for management and doctors, represented in the model by $\delta_H$ and $\delta_L$, if they want to improve quality by improving information.
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References


Appendix. Derivation of demand function

The line $ed_b$ in Figure A1 plots $H_L = k$ where $k < 0$. The distance $0ce = k$. The square is support of the joint distribution of error terms $\varepsilon_j \sim U\left(\frac{-1}{2v}, \frac{1}{2v}\right)$, $j = H, L$. The distances $0a = 0c = \frac{1}{2v}$ so that the area of support is $\frac{1}{v}$. The integral over the support equals 1, so that the joint density over the support is $\frac{1}{v}$.

With the mass of patients equal to 1, the demand for hospital $H$ is the probability that patient chooses $H$:

$$D^H(q_H, q_L; v) = \Pr(q_H + \varepsilon_H \geq q_L + \varepsilon_L) = \Pr(\varepsilon_H - \varepsilon_L \leq q_H - q_L) \quad (A1)$$

When $q_H - q_L = k$ the probability that patient chooses $H$ is the area $def$ divided by the area of the square. The area of $def$ is $\frac{df*fb}{2} = \frac{(df)^2}{2}$ by similar triangles. Further

$$df = cf - cd = cf - ce = \frac{1}{2v} - (0c - 0c) = \left(\frac{|k| - \frac{1}{2v}}{v}\right)$$

$$= \frac{1}{v} - |k| \quad (A2)$$

Hence the probability that the patient chooses $H$ when the quality gap $q_H - q_L = k$ is

$$D^H(q_H, q_L; v) = \Pr(\varepsilon_H - \varepsilon_L \leq q_H - q_L) = \frac{\left(\frac{1}{v} - |k|\right)^2}{2v^2} = \frac{v^2}{2} \left(\frac{|k| - \frac{1}{v}}{v}\right)^2 \quad (A3)$$

which is the expression in Table 1 for $D^H$ when $q_H \in \left[q_L - \frac{1}{v}, q_L\right]$. When $q_H - q_L = k > 0$ the probability that the patient chooses $H$ is found by subtracting the corresponding triangle (in the top left corner of the square support) from the square and dividing by $\frac{1}{v^2}$ to get the expression in Table 1 for $D^H$ when $q_H \in \left[q_L, q_L + \frac{1}{v}\right]$.
Figure 1. Demand for hospital $H$ is the distribution function $F(q_H - q_L)$ of the difference of two variates distributed uniformly on $[-1/2v, 1/2v]$.

Figure 2. Profit maximizing quality for hospital $H$. 
Figure 3. Hospital $H$ reaction functions at different values of $\delta_H$

Figure 4. Nash equilibria
Figure 5. Effects of changes in cost parameters on quality

Figure 6. Quality cost parameters and effect of information on quality
Figure 7. Equilibria defined by intersection of ratio condition loci ($\delta_L/\delta_H$) and hospital $L$ reaction functions $r^L_a$. Increase in information ($v$ to $v'$) increases quality when the quality cost differential is small ($r^L_a$ are solid lines) but reduces it when the cost differential is large ($\delta'_L$, $r'^L_a$ are dashed lines).

Figure 8. Better information (increase in $v$) may increase or reduce hospital $H$ marginal revenue and increase or reduce profit maximizing $q_H$, depends on marginal cost of quality.
Figure A1. Support of jointly uniform error distribution and probability of choice of hospital $H$ when $q_H - q_L = k$