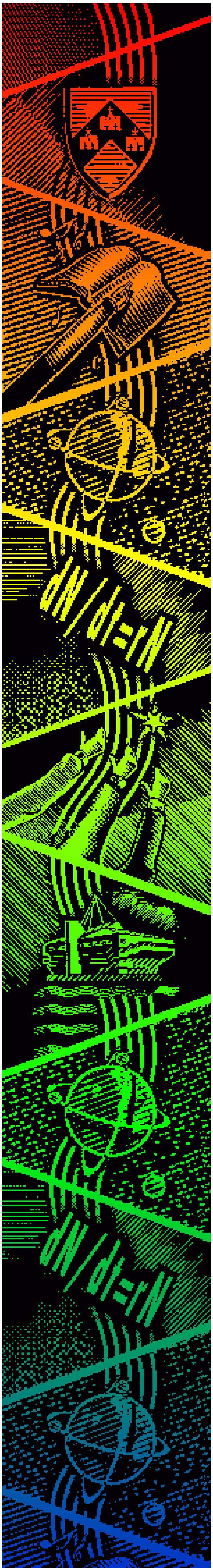

THE UNIVERSITY of York

Discussion Papers in Economics

No. 2008/05

Competition and quality in health care markets: a differential-game approach

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Competition and quality in health care markets: a differential-game approach

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Abstract

We investigate the effect of competition on quality in health care markets with regulated prices taking a differential game approach, in which quality is a stock variable. Using a Hotelling framework, we derive the open-loop solution (health care providers commit to an optimal investment plan at the initial period) and the feedback closed-loop solution (providers move investments in response to the dynamics of the states). Under the closed-loop solution competition is more intense in the sense that providers observe quality in each period and base their investment on this information. If the marginal provision cost is *constant*, the open-loop and closed-loop solutions coincide, and the results are similar to the ones obtained by static models. If the marginal provision cost is *increasing*, investment and quality are lower in the closed-loop solution (when competition is more intense). In this case, static models tend to exaggerate the positive effect of competition on quality.

Keywords: Health care markets; Competition; Quality.

JEL: H42; I11; I18; L13.

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1 Introduction

Quality is a major concern in health care. In recent years, health care markets in several countries have been subject to reforms introducing competition between health care providers, at least along some dimensions. In particular, the combination of prospective payment systems and free patient choice aims at giving hospitals incentives to attract patients (and thus payments) by improving their quality.[1]

The existing theoretical literature on quality competition in health care markets with fixed prices is almost unanimous in reporting a positive relationship between competition – measured either as a switch from monopoly to (imperfect) competition or as an increase in the degree of competition intensity – and quality (see, e.g., Calem and Rizzo, 1995; Gravelle, 1999; Lyon, 1999; Gravelle and Masiero, 2000; Beitia, 2003; Nuscheler, 2003; Brekke, Nuscheler and Straume, 2007; Karlsson, 2007).[2] This result is also in line with more general theoretical studies of quality competition in regulated markets (see, e.g., Ma and Burgess, 1993; Wolinsky, 1997; Brekke, Nuscheler and Straume, 2006; Matsumura and Matsushima, 2007).

The empirical evidence seems, however, to be more ambiguous. When empirically analysing the effect of competition on quality in health care markets with fixed prices, Kessler and McClellan (2000) and Tay (2003) find a positive effect, Gowrisankaran and Town (2003) find a negative effect, Shen (2003) finds mixed effects, while Shortell and Hughes (1988) and Mukamel, Zwanziger and Tomaszewski (2001) find no effects.[3]

In the present paper we revisit the modelling of quality competition in health care markets in order to shed some more light on the apparent divergence between theory and evidence. In most of the theoretical literature an important assumption is that quality can be adjusted instantaneously and permanently (at some costs). This is obviously a simplifying assumption

[1] Examples include the UK, where hospitals are paid a tariff for every patients treated (Payment by Results) and patients have been given a free choice of hospital. Similar reforms have been introduced in Norway, Denmark, Italy and several other European countries. Our study is also relevant for the US Medicare system, where hospitals are paid a fixed price per treatment within a specific diagnosis related group (DRG), a system that has been adopted by many European countries.

[2] One exception is Brekke, Siciliani and Straume (2008) who show that the positive relationship between competitition and quality might be reversed if health care providers have altruistic preferences.

[3] See Gaynor (2006) for a survey of theoretical and empirical literature on the relationship between hospital competition and quality.

that may be restrictive. A provider who wants to increase quality will have to invest in it. For example, a hospital will have to train existing doctors, hire more qualified doctors, buy new high-tech equipment and so on. Thus, there might be potentially important implications of the dynamic nature of quality investments that are ignored in a static analysis.

We extend the above mentioned studies by modelling quality as a stock and by adopting a dynamic approach. We develop a model of competition within a Hotelling framework, with two horizontally differentiated health care providers that potentially differ also with respect to the quality of the good they provide. We assume that quality is a stock that can be increased only if the investment in quality is higher than its deterioration. Modelling quality as a stock introduces a dynamic element into the analysis, which turns the problem into a capital accumulation game (Dockner, Jørgensen, Van Long and Sorger, 2000).

We use a differential-game approach to derive the equilibrium quality, under two different behaviour rules followed by providers: in the first setting, providers decide their optimal dynamic plans at the initial period and then stick to it forever (open-loop solution); in the second setting, providers do not commit to an optimal path and set their controls at any time in response to the current value of states (feedback closed-loop solution). The first solution concept might be appropriate when providers can commit to investment plans and stick to it for long periods of time. The second one is appropriate when providers do not or cannot commit to investment plans, but rather observe quality in each period and base their investments on this information. Hence, under the closed-loop solution competition is more intense.

We find that if the marginal cost of provision is *constant*, the open-loop and closed-loop solutions coincide: investment and quality are identical under the two solution concepts. This result holds for a general specification of the cost function (cost of quality is either zero, linear or quadratic in quality; investment and quality are either substitutes or complements in costs). We find, as is intuitive, that if the price is above marginal cost, a higher regulated price or lower travel costs (i.e., more competition) increase investment, and thus quality, in steady state. Cost complementarity (substitutability) between investment and quality will increase (decrease) the steady state levels of both. A higher time preference discount rate reduces quality and investment under weak regularity conditions, while a higher depreciation rate reduces quality but has an indeterminate effect on investment.

Similar results are obtained under the open-loop solution when we assume that the marginal cost of provision is *increasing*. However, in contrast to the case with constant marginal cost, investment and quality are lower under the closed-loop solution than under the open-loop solution. Therefore, our model predicts that when (i) quality investment is costly, (ii) the marginal cost is increasing and (iii) providers do not commit to investment plans, then the beneficial effects from competition in terms of higher quality are lower than expected from existing theoretical literature.

The intuition for this result is related to the fact that, when the marginal cost of provision is increasing, quality investments are strategic complements. In a static setting, this yields strong incentives for quality competition. However, in a dynamic setting, if the players use closed-loop investment rules, lower quality investment by one provider will induce a future reduction in quality investment by the other provider. Thus, due to these strategic dynamics, the players are able to obtain a more collusive outcome in steady state, compared with the open-loop solution or with the solution of a corresponding one-shot game. From the viewpoint of each player, the instantaneous demand loss due to lower quality investments is weighed against the future gains due to the strategic response – lower investment – by the competitor. As long as the players value future profits, the latter consideration reduces the incentive for quality investments, implying a lower steady state level of quality in the closed-loop solution. Thus, the dynamic strategic effect identified in the present paper may potentially help explaining the ambiguous empirical evidence on competition and quality, which in some cases finds no significant association between the two variables (Shen, 2003; Shortell and Hughes, 1988; Mukamel, Zwanziger and Tomaszewski, 2001).

The crucial assumption for the above described dynamics is that the marginal cost of provision is increasing. If, on the other hand, marginal production costs are constant, there is no longer any strategic interaction between the players in terms of quality investments, since a fixed price and constant marginal costs imply that marginal revenue is also constant. This explains why the open- and closed-loop solutions coincide in this particular case. Even if the players are able to update their investment plans according to the evolution of quality states, the absence of strategic interaction implies that the each player's optimal investment rule does not depend on the choices made by the other player.

Although quality competition between health care providers is what we have foremost in mind, it should be noted that our model might also be interpreted in the context of pharmaceutical markets. In most countries prices of prescription drugs are regulated and pharmaceutical companies use a considerable amount of resources on marketing in order to capture market shares. Although quality competition is not directly relevant in such markets, drug marketing and quality investments share the same essential feature, namely to increase consumers' willingness-to-pay for the good.[4] Thus, it is possible to reinterpret the quality variable in our model as a marketing variable.

It should also be noted that our results are in contrast with the results obtained within capital accumulation models with a Cournot framework (Dockner, 1992; Dockner, Jørgensen, Van Long and Sorger, 2000). In these models, the providers compete *a la Cournot* but face capacity constraints that can be relaxed by capital accumulation through investment. It turns out that the investment under the closed-loop solution is in this case *higher* than under the open-loop one: the more intense competition under the closed-loop solution generates an additional incentive to invest. In contrast, within the regulated markets described in our model, the investment in quality under the closed-loop solution is *lower* than under the open-loop one.[5]

The rest of the paper is organised as follows. The main assumptions of the model are presented in Section 2. Section 3 derives and characterises the equilibrium quality under the open-loop solution, while Section 4 derives and characterises the feedback closed-loop solution. Section 5 concludes the paper.

[4] In markets for prescription drugs, marketing activities are also directed towards prescribing physicians in an attempt to increase their "willingness-to-prescribe" (see, e.g., Brekke and Kuhn, 2006; Königbauer, 2007).

[5] In a related paper (Brekke, Cellini, Siciliani and Straume, 2008), we assume that providers can adjust quality instantaneously but that the demand is sluggish. Sluggish demand implies that if a provider increases quality, it will take some time before the potential demand increase is fully realised.

2 Model

Our basic framework is the widely used Hotelling model (Hotelling, 1929) for quality competition with regulated prices.[6] Consider a market for medical treatment with two hospitals located at either end of the unit line $S = [0, 1]$. On this line segment there is a uniform distribution of patients, with total mass normalised to 1. We assume unit demand, where each patient needs – and demands – one hospital treatment in order to be cured. Assuming full market coverage, the decision patients make is to choose which provider to demand treatment from. The utility of a patient who is located at $x \in S$ and seeking treatment at hospital i, located at z_i, is given by[7]

$$U(x, z_i) = v + kq_i - \tau |x - z_i|, \tag{1}$$

where v is the gross valuation of medical treatment, $q_i \geq \underline{q}$ is the quality at hospital i, k is a parameter measuring the (marginal) utility of quality, and τ is a travelling cost parameter.[8] The lower bound $\underline{q}$ on hospital quality represents the minimum treatment quality hospitals are allowed to offer, implying that $q < \underline{q}$ can be interpreted as malpractice. For simplicity, we set $\underline{q} = 0$. Moreover, we normalise the marginal utility of quality to one, i.e., $k = 1$, without loss of generality. This implies that τ can be interpreted as the marginal disutility of travelling *relative* to quality. Thus, a low (high) τ means that quality is of relatively more (less) importance to the patient than travelling distance.

Since the distance between hospitals is equal to one, the patient who is indifferent between seeking treatment at hospital i and hospital j is located at x_i^D, given by

$$v - \tau x_i^D + q_i = v - \tau \left(1 - x_i^D\right) + q_j, \tag{2}$$

[6] The Hotelling model in a differential game framework is used, *inter alia*, by Laussel, de Montmarin and Van Long (2004) and by Piga (1998). Differently from our present paper, however, the former focuses on network effects and competition is on prices (rather than quality), while the latter studies the role of advertising and price competition.

[7] There is strong empirical evidence showing that distance and quality are main predictors of patients' choice of hospital, see, e.g., Kessler and McClellan (2000) and Tay (2003).

[8] We refer to distance in physical terms. However, one could also interpret the horizontal dimension in a disease space, where the location of a patient is associated with the disease she suffers from, and the two hospitals are differentiated with respect to the disease they are best able to cure, reflecting hospital specialisation or "service mix" (see, e.g., Calem and Rizzo, 1995, and Brekke, Nuscheler and Straume, 2007).

yielding

$$x_i^D = \frac{1}{2} + \frac{q_i - q_j}{2\tau}, \tag{3}$$

which is also the demand for medical treatment at hospital i, given the assumptions of (i) uniform patient distribution (with mass 1), (ii) exogenous locations of providers, and (iii) full market coverage.

The provider with a higher quality gets a market share which is more than half. Notice how lower travelling costs make it less costly for patients to switch between hospitals, increasing the demand responsiveness to changes in quality.

Most of the literature assumes that quality can be increased instantaneously and permanently (at some costs). This is, obviously, a simplifying assumption. Quality is a stock which may be increased if the appropriate investment in quality is chosen. In the hospital context, there are many examples of quality-enhancing investments. For instance, the purchase of MR-machines and CT-scanners improve diagnosing, which in turn will increase the treatment quality. Hospitals invest also in human capital in order to improve quality: they spend money on training of their medical staff, they hire highly skilled physicians (specialists) and nurses, etc. Finally, hospitals invest in facilities to improve, say, the quality of theaters, rooms, catering, etc.[9]

Define $I(t)$ as the investment in quality at time t, and δ as the depreciation rate of the quality stock. Analytically, the law of motion of quality is given by:

$$\frac{dq_i(t)}{dt} \equiv \dot{q}_i(t) = I_i(t) - \delta q_i(t), \tag{4}$$

where δ is the depreciation rate. Such an accumulation law, *à la* Solow, is widely used in industrial organization theoretical models to describe capacity accumulation (see Dockner, Jørgensen, Van Long and Sorger, 2000, or Cellini and Lambertini, 2003, for literature reviews).

We assume that hospitals maximise profits.[10] The instantaneous objective function of hos-

[9] Some investments might not just influence quality but also capacity. For instance, a new MR-machine might improve treatment quality, but also the number of patients that can be treated. Here, we restrict attention to pure quality investments. Capacity accumulation has been analysed in detail in several other papers (Dockner, Jørgensen, Van Long and Sorger, 2000).

[10] At first glance, this assumption may seem inappropriate for public and non-profit hospitals due to their constraints on profit distribution. However, hospitals may add to their reserves the financial surplus obtained.

pital i is assumed to be given by

$$\pi_i(t) = T + px_i^D(q_i(t), q_j(t)) - C\left(I_i(t), x_i^D(q_i(t), q_j(t)), q_i(t)\right) - F, \tag{5}$$

where p is a regulated price per treatment (patient) and T is a potential lump-sum transfer received from a third-party payer (insurer).[11]

On the cost-side, each hospital i faces a fixed cost F and variable cost $C(\cdot)$ that depends on the quality investment I_i and supply of x_i^D treatments at quality q_i. We assume that $C(\cdot)$ is increasing in the investment, $C_{I_i} > 0$, as investment in quality is obviously costly, at an increasing rate, $C_{I_iI_i} > 0$. We also assume that $C(\cdot)$ is weakly increasing in quality, $C_{q_i} \geqslant 0$.

The cost function is also increasing in the number of patients treated, $C_{x_i} > 0$. Below we will distinguish the cases when the cost of treatment is linear ($C_{x_ix_i} = 0$) and when it is convex ($C_{x_ix_i} > 0$).[12] We will show that this assumption has important implications for the results. We also make the simplifying assumption that the cost function is separable in the quantity x_i, implying that $C_{x_iq_i} = C_{x_iI_i} = 0$.[13,14]

In contrast, we assume that $C_{I_iq_i} \gtreqless 0$. If $C_{I_iq_i} < 0$, then quality and investment are complements and a higher quality reduces the marginal cost of investment. If $C_{I_iq_i} > 0$, then quality and investment are substitutes and a higher quality increases the marginal cost of investment.

Defining ρ as the (constant) preference discount rate, the hospital objective function over

Alternatively, managers may spend the surplus to pursue other objectives: they might increase the physician staff, the range of services, or even increase managerial perks (see e.g., Dranove and White, 1994, De Fraja, 2000, Chalkley and Malcomson, 1998a,b). Importantly, the empirical evidence shows little support for different behaviour of for-profit and not-for-profit hospitals (Sloan, 2000), suggesting that profit maximisation is a reasonable assumption.

[11] Alternatively, we could have assumed that the payments are collected directly from the consumers. It is, however, straightforward to show that this will not change our results.

[12] A convex variable cost function is normally supported by the evidence suggesting that economies of scale are quite rapidly exhausted in the hospital sector (see, e.g., Ferguson, Sheldon and Posnett, 1999, and Folland, Goodman and Stano, 2004, for literature surveys).

[13] The assumption of cost separability between quality and quantity is widely used in the related literature (see, e.g., Economides, 1989, 1993; Calem and Rizzo, 1995; Lyon, 1999; Gravelle and Masiero, 2000; Barros and Martinez-Giralt, 2002).

[14] The assumption of separability between investment and quantity is done to focus on quality dynamics rather than capacity dynamics. As mentioned above, we restrict attention to investments that affect quality only.

the infinite time horizon is

$$\int_{0}^{+\infty} \pi\left(t\right) e^{-\rho t} dt. \tag{6}$$

In reality, providers may not have an infinite-time horizon, but may have reasonably long finite horizons. If the optimal path does not differ significantly from the solution with a very large but finite horizon, the convenience of working with an infinite-horizon model may be worth the loss of realism (see Léonard and van Long, 1992, p. 285). Also, when doctors or managers retire, they may well be replaced by other doctors and managers with similar utility functions, thus generating an infinite-time horizon.

In this type of dynamic models with strategic interactions – i.e., differential games – there are two main solution concepts: a) *open-loop solution*, where each hospital knows the initial state of the system and then nothing else, i.e., each hospital knows the initial quality of the other provider, but not in the following periods; b) *closed-loop solution*, where each hospital knows the initial state of the system, but also later knows the state variable values, i.e., each hospital knows the quality of the other provider, not only in the initial state, but also in all of the subsequent periods. Within the closed-loop solutions, further distinctions can be made: if one assumes that players take into account only the initial state and the current state, the "memoryless" closed-loop solution is obtained; if players take into account the whole history of states, the "perfect state" closed-loop is obtained; finally, if players in each instant take into account the current value of states (i.e., the whole past history is summarized by the current value of states), the feedback rule is obtained. Typically, the feedback closed-loop solution is obtained based on the Bellman equation.

In order to establish which one is the most appropriate solution concept, it is essential to evaluate the relevant information set used by players when they take their decisions. In the cases in which the collection of information over time is difficult, it is reasonable to model the choice according to the open-loop rule; on the opposite, when players can observe the current state of the world and they behave accordingly, the closed rules are more appropriate. Clearly, closed-loop solutions are more appealing, but solving for closed-loop is more difficult. However, in some cases – and health care markets can be a good example – players might have to commit to investment plans and stick to them for long periods of time. In this case, the open-loop solution might be the relevant one. Nevertheless, there is a wide range of problems where the two solutions coincide.[15] Below, we compare the closed-loop and open-loop solutions. Section 3 provides the open-loop solution, while Section 4 provides the closed-loop one, under the specific rule of feedback behaviour.

3 Open-loop solution

Provider i's maximisation problem is given by

$$\text{Maximise} \int_0^{+\infty} \pi_i(t)\, e^{-\rho t} dt, \tag{7}$$

$$\begin{aligned} \text{subject to} \quad \dot{q}_i(t) &= I_i(t) - \delta q_i(t), && (8)\\ \dot{q}_j(t) &= I_j(t) - \delta q_j(t), && (9)\\ q_i(0) &= q_{i0} > 0, && (10)\\ q_j(0) &= q_{j0} > 0. && (11) \end{aligned}$$

Let $\mu_i(t)$ and $\mu_j(t)$ be the current value co-state variables associated with the two state equations. The current-value Hamiltonian is:[16]

$$H_i = T + px_i^D(q_i, q_j) - C(I_i, x_i^D(q_i, q_j), q_i) - F + \mu_i(I_i - \delta q_i) + \mu_j(I_j - \delta q_j), \tag{12}$$

[15] Games where this coincidence arises are presented in Clemhout and Wan (1974); Reinganum (1982); Mehlmann and Willing (1983); Dockner, Feichtinger and Jørgensen (1985). See also Mehlmann (1988), Fershtman, Kamien and Muller (1992), Dockner, Jørgensen, Van Long and Sorger (2000, ch. 7) for review.

[16] The indication of time (t) is omitted, to ease notation.

The solution is given by (a) $\partial H_i/\partial I_i = 0$, (b) $\dot{\mu_i} = \rho\mu_i - \partial H_i/\partial q_i$, (c) $\dot{q_i} = \partial H_i/\partial \mu_i$, (d) $\dot{\mu_j} = \rho\mu_j - \partial H_i/\partial q_j$, or more extensively:

$$\begin{aligned} \mu_i &= C_{I_i}, && (13) \\ \dot{\mu_i} &= -\frac{1}{2\tau}\left(p - C_{x_i}\right) + C_{q_i} + \mu_i\left(\delta + \rho\right), && (14) \\ \dot{q_i} &= I_i - \delta q_i, && (15) \\ \dot{\mu_j} &= \frac{p}{2\tau} + \mu_j\left(\delta + \rho\right), && (16) \end{aligned}$$

to be considered along with the transversality condition $\lim_{t\to+\infty} \mu_i(t)q_i(t) = 0$. The second order conditions are satisfied if the Hamiltonian is concave in the control and state variables (Léonard and Van Long, 1992).[17]

Totally differentiating equation (13) with respect to time we obtain $\dot{\mu_i} = C_{I_iI_i}\dot{I_i} + C_{I_iq_i}\dot{q_i}$ or, after substitution, $\dot{\mu_i} = C_{I_iI_i}\dot{I_i} + C_{I_iq_i}\left(I_i - \delta q_i\right)$. Substituting into equation (14), and using $\mu_i = C_{I_i}$, under symmetry we obtain:

$$\dot{I_i} = \frac{-\frac{1}{2\tau}\left(p - C_{x_i}\right) + C_{q_i} + \left(\delta+\rho\right)C_{I_i} - C_{I_iq_i}\left(I_i - \delta q_i\right)}{C_{I_iI_i}}, \tag{17}$$

which, together with $\dot{q_i} = I_i - \delta q_i$, describe the dynamics of the equilibrium.

Totally differentiating the locus of investment, $\dot{I_i} = 0$, and rearranging yields

$$\frac{\partial I_i}{\partial q_i}\Big|_{\dot{I_i}=0} = -\frac{\left(\frac{1}{2\tau}\right)^2\left(C_{x_ix_i}\right) + C_{q_iq_i} + (2\delta+\rho)C_{I_iq_i} - C_{I_iq_iq_i}\left(I_i - \delta q_i\right)}{\left(\delta+\rho\right)C_{I_iI_i} - C_{I_iI_iq_i}\left(I_i - \delta q_i\right)}. \tag{18}$$

Sufficient but not necessary conditions for the locus of investment, $\dot{I_i} = 0$, to be negatively sloped are $C_{I_iq_i} > 0$ and $C_{I_iq_iq_i} = C_{I_iI_iq_i} = 0$. The second locus is $\dot{q_i} = 0$, or $I_i = \delta q_i$, with $\frac{\partial I_i}{\partial q_i} > 0$.

[17] This is the case since (a) $H_{I_iI_i} = -C_{I_iI_i} < 0$; (b) $H_{q_iq_i} = -C_{q_iq_i} - [1/\left(2\tau\right)^2]C_{x_ix_i} < 0$; (c) $H_{I_iI_i}H_{q_iq_i} > \left(H_{I_iq_i}\right)^2$ or $C_{I_iI_i}\left\{C_{q_iq_i} + [1/\left(2\tau\right)^2]C_{x_ix_i}\right\} > C^2_{I_iq_i}$.

The dynamics of investment and quality can be represented in matrix form as follows:

$$\begin{bmatrix} \dot{I}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} (\delta+\rho) - \frac{C_{I_i I_i q_i}}{C_{I_i I_i}}(I_i - \delta q_i) & \frac{\begin{bmatrix} \left(\frac{1}{2\tau}\right)^2 (C_{x_i x_i}) + (2\delta+\rho) C_{I_i q_i} \\ + C_{q_i q_i} - C_{I_i q_i q_i}(I_i - \delta q_i) \end{bmatrix}}{C_{I_i I_i}} \\ 1 & -\delta \end{bmatrix} \begin{bmatrix} I(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} -(p-c)/(2\gamma\tau) \\ 0 \end{bmatrix}, \tag{19}$$

where the 2-by-2 matrix is the Jacobian J of the dynamic system. As for the dynamic properties of the system, suppose that this is evaluated around the steady state (or alternatively that third-order derivatives of the cost function are set to zero: $C_{I_i I_i q_i} = C_{I_i q_i q_i} = 0$). Then, it is immediate to check that the Jacobian matrix J in (19) is such that $tr(J) = \rho > 0$, and $\det(J) = -\delta(\delta+\rho) - \frac{\left(\frac{1}{2\tau}\right)^2 (C_{x_i x_i}) + C_{q_i q_i} + (2\delta+\rho) C_{I_i q_i}}{C_{I_i I_i}} < 0$, implying that the equilibrium is stable in the saddle sense. The solution is described in Figure 1.

[Figure 1 about here]

Let q^s be the steady state level of quality and suppose we start off equilibrium at level $q_0 < q^s$. The solution is then characterised by a period of increasing quality and decreasing investment in quality. Suppose instead that $q_0 > q^s$. In this case, we should observe a period of decreasing quality and increasing investment. Notice how off equilibrium investment and quality move over time in opposite directions, while in the steady state investment is proportional to quality ($I_i = \delta q_i$).

In Figure 1 we have assumed the investment locus, $\dot{I}_i = 0$, to be negatively sloped , implying that the equilibrium is a saddle point. The equilibrium can still be a saddle point also when $\dot{I}_i = 0$ is positively sloped, as long as it is not too positively sloped. This case arises, for example, when the marginal cost of quality and the marginal cost of provision are constant but quality and investment are complements, which implies $C_{I_i q_i} < 0$. The equilibrium is still a saddle point if $\det(J) = -\delta(\delta+\rho) - \frac{(2\delta+\rho)C_{I_i q_i}}{C_{I_i I_i}} < 0.$[18]

Figure 2 shows some comparative dynamics. Suppose the system is in an initial steady state. Figure 2 shows the effect of an unexpected increase in price or reduction in travel costs (more competition) on investment and quality. The locus $\dot{I}_i = 0$ shifts upwards. The shock generates a positive jump in investment (overshooting), and it is followed by a period of decreasing investment and increasing quality.

[Figure 2 about here]

In the following, we will consider two special cases: constant and increasing marginal cost of treatment. Later on, the specification of the treatment cost function will be shown to have crucial implications with respect to the comparison of the open-loop and closed-loop solutions.

3.1 Constant marginal treatment cost

Suppose that marginal cost of treatment is constant, while it is quadratic in investment and quality:

$$C(I_i, x_i^D, q_i) = cx_i^D + \frac{\gamma}{2}I_i^2 + \frac{\beta}{2}q_i^2 + \varphi q_i I_i. \tag{20}$$

With this specification, the solution of the Hamiltonian system leads to

$$\dot{I}_i = -\frac{1}{2\tau\gamma}(p-c) + (\delta+\rho)I_i + \frac{1}{\gamma}\left[\beta + (2\delta+\rho)\varphi\right]q_i. \tag{21}$$

In the steady state we have $\dot{q}_i = 0$, implying $I_i = \delta q_i$, which, substituted into $\dot{I}_i = 0$, yields

$$I_{OL}^s = \frac{(p-c)\delta}{2\tau\left[\beta + \gamma\delta(\delta+\rho) + \varphi(2\delta+\rho)\right]} \tag{22}$$

and

$$q_{OL}^s = \frac{I_{OL}^s}{\delta}, \tag{23}$$

[18] If the determinant is positive, then we have an unstable node.

where superscript s and subscript OL indicate the steady state levels in the open-loop solution, respectively.

The results are reasonable. If the price is above the marginal treatment cost, then lower travel costs (τ) or a higher price (p) increase quality and investment. Similarly, a higher marginal cost of quality (β) or of investment (γ) reduce quality and investment. If quality and investment are complements ($\varphi > 0$), then quality and investment are higher in the steady state; if they are substitutes, quality and investment are lower in the steady state. A higher time preference discount rate (ρ) reduces quality and investment whenever $\gamma\delta + \varphi > 0$.[19] Notice also how a higher depreciation rate of quality (δ) is associated with lower steady state quality, while the effect on investment is indeterminate.[20]

3.2 Increasing marginal treatment cost

Suppose that marginal cost of treatment is increasing, while it is still quadratic in investment and quality:

$$C(I_i, x_i^D, q_i) = \frac{c}{2}\left(x_i^D\right)^2 + \frac{\gamma}{2}I_i^2 + \frac{\beta}{2}q_i^2 + \varphi q_i I_i. \tag{24}$$

With this particular specification, the solution of the Hamiltonian system yields

$$\dot{I}_i = -\frac{1}{2\tau\gamma}\left(p - c\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau}\right)\right) + \frac{\beta}{\gamma}q_i + (\delta+\rho)\, I_i + \frac{\varphi}{\gamma}q_i\,(2\delta+\rho)\,. \tag{25}$$

The symmetric steady state equilibrium, $\dot{I}_i = 0$ and $\dot{q}_i = 0$, together with $I_i = \delta q_i$, yield

$$I_{OL}^s = \frac{\left(p - \frac{c}{2}\right)\delta}{2\tau\left[\beta + (\delta+\rho)\,\gamma\delta + \varphi\,(2\delta+\rho)\right]} \tag{26}$$

[19] From (22) we obtain

$$\frac{\partial I_{OL}^s}{\partial \rho} = -\frac{(p-c)\,\delta\,(\gamma\delta+\varphi)}{2\tau\left[\beta + \gamma\delta\,(\delta+\rho) + \varphi\,(2\delta+\rho)\right]^2}.$$

[20] From (22) we obtain

$$\frac{\partial I_{OL}^s}{\partial \delta} = \frac{(p-c)\left(\rho\varphi + \beta - \gamma\delta^2\right)}{2\tau\left[\beta + \varphi\,(2\delta+\rho) + \delta\gamma\,(\delta+\rho)\right]^2}.$$

The sign of this expression is determined by the sign of $\left(\rho\varphi + \beta - \gamma\delta^2\right)$, which is indeterminate.

and

$$q_{OL}^s = \frac{I_{OL}^s}{\delta}, \tag{27}$$

which is similar to the solution with constant marginal cost, and requires no further comments.

4 Closed-loop solution

The open-loop rule is realistic in cases where the players compute their investment plans at the beginning, and then stick to them forever. In other cases, however, players take their decisions while observing the evolution of states, and the relevant rule is then of the closed-loop type. In this section we derive the so-called feedback rule, where the controls set by the players depend on the current level of states. Open-loop solutions are simpler to compute, but they are only weakly time consistent. On the opposite, closed-loop solutions – which are more involved to compute – are strongly time consistent. There is a wide body of literature on the cases of coincidence between the time path of controls and states under different solution concepts. In the lucky case in which they coincide, the (more easily computable) open-loop solutions are strongly time consistent (see Mehlman, 1988).

4.1 Constant marginal treatment cost

Applying the cost specification in (20), the optimal rule for player i in the closed-loop stable solution is given by[21]

$$I_i(t) = \phi_i^{CL}(q_i(t), q_j(t)) = \frac{1}{\gamma}[\alpha_1 + (\alpha_3 - \varphi)\, q_i(t)], \tag{28}$$

where

$$\alpha_1 = \frac{p - c}{\tau\left[\rho + \sqrt{4\frac{\beta+\rho\varphi+2\delta\varphi}{\gamma} + (\rho+2\delta)^2}\right]} > 0 \tag{29}$$

[21] See Appendix A for the details of this solution. As we show in the appendix, there are three solutions, of which two are unstable. We focus on the stable solution (Dockner, Jørgensen, Van Long and Sorger, 2000, p. 251-2).

and

$$\alpha_3 - \varphi = \gamma \left(\frac{\rho}{2} + \delta \right) - \sqrt{\gamma \left(\beta + \rho \varphi + 2\delta \varphi \right) + \gamma^2 \left(\frac{\rho}{2} + \delta \right)^2} < 0. \tag{30}$$

Using that fact that $I_i = \delta q_i$ in steady state, it is shown in Appendix A that the only stable steady-state equilibrium in the closed-loop solution coincides with the previously derived open-loop steady-state equilibrium, given by (22)-(23), i.e., $I_{CL}^s = I_{OL}^s = \delta q_{OL}^s = \delta q_{CL}^s$. This constitutes the first main result of the paper:

Proposition 1 *If the marginal treatment cost is constant, the open-loop and closed-loop solutions coincide in steady state.*

The coincidence between the two solutions clearly holds also for the special cases when $\varphi = 0$ (or $\varphi = \beta = 0$). We can also show that the same result holds when the cost of quality is linear rather than convex (i.e., $C(I_i, x_i^D, q_i) = cx_i^D + \frac{\gamma}{2} I_i^2 + \beta q_i + \varphi q_i I_i$; the proof is omitted).

The important contributing factor to this coincidence result is that there is a constant marginal (instantaneous) revenue gain of quality investments, implying that the optimal dynamic investment rule for each player, as given by (28), is independent of the quality level provided by the other player. In other words, the optimal investment path for hospital i does not depend on the quality stock of hospital j, implying an absence of strategic interaction in this particular respect. Given that the cost function is separable in quantity, this feature – constant marginal revenue – is always present when the treatment price is fixed and the marginal treatment cost is constant. Consequently, our analysis suggests that, if the marginal cost is constant, the solution within a dynamic approach (regardless of the open- or closed-loop solution) is qualitatively similar to the ones that would be obtained within a static approach. We can thus conclude that the static analysis is reasonably robust in this particular case.

4.2 Increasing marginal treatment cost

Suppose instead that the marginal treatment cost is increasing, with the cost function given by (24). To keep the analysis simple, we assume that quality does not affect the marginal cost of investment, i.e., $\varphi = 0$. Moreover, we set the marginal cost of investment γ equal to one ($\gamma = 1$).

This normalisation is without loss of generality. What matters is the marginal cost of quality β relative to the marginal cost of investment γ. Clearly, as we have shown above, this does not affect the coincidence result in Proposition 1. With this specification, the optimal investment rule for player i is given by[22]

$$I_i(t) = \phi_i^{CL}(q_i(t), q_j(t)) = \alpha_1 + \alpha_3 q_i(t) + \alpha_5 q_j(t), \tag{31}$$

where

$$\alpha_1 = \frac{2p-c}{4\tau}\frac{1}{\delta+\rho-\alpha_3} > 0, \tag{32}$$

$$\alpha_3 = \left(\frac{\rho}{2}+\delta\right) + \frac{18\tau^2}{c}\alpha_5^3 - \frac{\tau^2\left(5c+16\beta\tau^2+16\tau^2\left(\frac{\rho}{2}+\delta\right)^2\right)}{2c}\alpha_5 < 0, \tag{33}$$

and

$$\alpha_5 = \frac{1}{6}\sqrt{\frac{3c}{\tau^2}+8\left(\beta+\left(\frac{\rho}{2}+\delta\right)^2\right)-4\sqrt{\left(\beta+\left(\frac{\rho}{2}+\delta\right)^2\right)\left[\frac{3c}{\tau^2}+4\left(\beta+\left(\frac{\rho}{2}+\delta\right)^2\right)\right]}} > 0. \tag{34}$$

As to the steady state of the dynamic system, note that applying (31)-(34) to the steady state condition $I_i = \delta q_i$, the only stable steady-state investment and quality in the closed-loop solution are then given by

$$I_{CL}^s = \frac{2p-c}{4\tau}\frac{\delta}{(\delta+\rho-\alpha_3)(\delta-\alpha_3-\alpha_5)} > 0 \tag{35}$$

and

$$q_{CL}^s = \frac{I_{CL}^s}{\delta} > 0, \tag{36}$$

Thus, the investment and quality under closed-loop solution is positive and generally different from the quality under open-loop solution. The following proposition compares the two solutions.

Proposition 2 *If the marginal treatment cost is increasing, the steady state levels of investment and quality are lower under the closed-loop solution compared to the investment and quality under the open-loop solution, i.e.* $I_{OL}^s > I_{CL}^s$ *and* $q_{OL}^s > q_{CL}^s$.

[22] The derivation of the closed-loop solution with increasing marginal costs is presented in Appendix B. There are four possible solutions of which three are unstable. We focus on the stable solution.

See Appendix B for proof. When marginal treatment costs are non-constant, marginal revenue is no longer independent of quality levels, since quality changes will affect demand and, in turn, the marginal treatment cost facing each hospital. This introduces a strategic interaction in the sense that the optimal investment rule for player i depends, at each point in time, on the quality stock of player j, as we can see from (31). This dynamic interaction is reflected in the parameter α_5. More specifically, with increasing marginal costs, quality investments are strategic complements ($\alpha_5 > 0$). A quality increase by hospital i will shift demand away from hospital j, implying that the marginal cost of hospital j decreases. Since the price is constant, this increases the profit margin of hospital j, making quality investments more profitable on the margin for this hospital. Conversely, a quality reduction by hospital i will be strategically followed by a quality reduction by hospital j.

This strategic interaction has important implications for the players' dynamic competition incentives. From the perspective of the profit-maximising providers, the business-stealing effect of quality investments constitutes a form of "destructive competition". This is particularly pronounced in the case of inelastic total demand, where quality investments have a purely business-stealing effect.

Compared with the outcome of a static game, the open-loop solution in a dynamic setting does not (qualitatively) produce a less competitive outcome, since it has the essential characteristics of a one-shot game, where the players make once-and-for-all commitments to their investment plans at the outset of the game. This is also true for the closed-loop solution in the case of constant marginal costs, due to the aforementioned absence of strategic interaction.

However, the presence of increasing marginal costs introduces a dynamic strategic interaction in terms of quality investments, as explained above. When the players revise their investment plans according to the evolution of quality states, a decrease in quality investment by hospital i will invoke an investment-reducing response by hospital j. From the viewpoint of hospital i, the instantaneous loss in market share by reducing the supply of quality is weighed against the future gain of a quality reduction – a strategic response – by hospital j. As long as the players value future profits, the dynamic strategic interaction will drive the supply of quality in the market to a lower level in steady state, muting the effect of competition in the market.

To summarise the above analysis, our general message is that, in the presence of convex production costs, when dynamic strategic interaction is taken into account, the closed-loop solution indicates that the benefits of competition with respect to quality provision are overestimated in the existing theoretical literature, which is based on static competition models.[23]

5 Conclusions

We have investigated the effect of competition on quality in health care markets within a Hotelling framework. Differently from the existing literature we assume that quality cannot adjust instantaneously, but rather is a stock variable which increases when investment in quality is higher than the depreciation of quality. We investigate the optimal open-loop solution (when providers commit to an optimal plan of investment at the initial period) and the equilibrium closed-loop solution (under the feedback rule, where the providers move their investment in response to the dynamics of the states).

We find that if the marginal cost of provision is *constant*, the open-loop and closed-loop solution coincide: investment and quality are identical under the two solution concepts. This result suggests that previous predictions obtained from static models are robust to a dynamic specification. For example, if the price is above the marginal cost, a higher regulated price or lower travel costs (i.e., more competition) increase quality and investment. Moreover, we show (differently from static analyses) that a higher time preference discount rate reduces quality and investment under weak regularity conditions. Also, a higher depreciation rate reduces quality but has an indeterminate effect on investment.

However, if we assume that the marginal cost is *increasing*, then the open-loop and closed-loop solution do not coincide, which implies that the main results from closed-loop solutions

[23] We restricted the analysis to the case of constant or increasing marginal cost. Suppose that the marginal cost is instead decreasing: $C(I_i, x_i^D, q_i) = ax_i^D - \frac{b}{2}\left(x_i^D\right)^2 + \frac{\gamma}{2}I_i^2 + \frac{\beta}{2}q_i^2 + \varphi q_i I_i$, where a and b are positive parameters with $\frac{\partial C}{\partial x_i^D} = a - bx_i^D > 0$ and $\frac{\partial^2 C}{\partial^2 x_i^D} = -b < 0$. Also assume that $-\beta + \frac{b}{(2\tau)^2} < 0$ to make sure the problem is well behaved and the Second Order Conditions are satisfied. The profit function is: $\pi_i = (p-a)\,x_i^D + \frac{b}{2}\left(x_i^D\right)^2 - \frac{\gamma}{2}I_i^2 - \frac{\beta}{2}q_i^2 - \varphi q_i I_i - F$, which is analogous to the problem already analysed, the only difference being that the price is now replaced with $(p-a)$ and $c = -b$. It is straightforward to show that if the marginal cost is decreasing, then the quality under the open-loop solution is higher than the quality under the closed-loop solution.

departs from the predictions of static models. Investment and quality are lower under the closed-loop solution than under the open-loop solution. Therefore, our model predicts that the beneficial effects from competition in terms of higher quality are lower than expected from existing theoretical literature.

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Appendix A. Closed-loop solution with constant marginal cost

We assume that the marginal cost of treatment is constant, while it is quadratic in investment and quality so that $C(I_i, x_i^D, q_i) = cx_i^D + \frac{\gamma}{2}I_i^2 + \frac{\beta}{2}q_i^2 + \varphi q_i I_i$. The purchaser instantaneous objective function is

$$T + (p-c)\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau}\right) - \frac{\gamma}{2}I_i^2 - \frac{\beta}{2}q_i^2 - \varphi q_i I_i, \tag{A1}$$

which – faced with the linear dynamic constraint – gives rise to a linear-quadratic problem. Hence, define the value function as

$$V^i(q_i, q_j) = \alpha_0 + \alpha_1 q_i + \alpha_2 q_j + (\alpha_3/2)q_i^2 + (\alpha_4/2)q_j^2 + \alpha_5 q_i q_j. \tag{A2}$$

Define $I_i = \phi_i(q_i, q_j)$ and $I_j = \phi_j(q_i, q_j)$. The value function has to satisfy the Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho V^i(q_i, q_j) = \max \left\{ \begin{array}{c} T + (p-c)\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau}\right) - \frac{\gamma}{2}I_i^2 - \frac{\beta}{2}q_i^2 - \varphi q_i I_i \\ + V_{q_i}^i(q_i, q_j)\left(I_i - \delta q_i\right) + V_{q_j}^i\left(\phi_j(q_i, q_j) - \delta q_j\right) \end{array} \right\}. \tag{A3}$$

Maximisation of the right-hand-side yields $V_{q_i}^i = \gamma I_i + \varphi q_i$, which after substitution gives

$$I_i = \phi_i(q_i, q_j) = \frac{\alpha_1 + (\alpha_3 - \varphi)\, q_i + \alpha_5 q_j}{\gamma}. \tag{A4}$$

Similarly, we obtain

$$I_j = \phi_j(q_i, q_j) = \frac{\alpha_1 + (\alpha_3 - \varphi)\, q_j + \alpha_5 q_i}{\gamma}. \tag{A5}$$

Substituting $I_i = \phi_i(q_i, q_j)$, $I_j = \phi_j(q_i, q_j)$, $V_{q_i}^i(q_i, q_j) = \alpha_1 + \alpha_3 q_i + \alpha_5 q_j$, $V_{q_j}^j(q_i, q_j) = \alpha_1 + \alpha_3 q_j + \alpha_5 q_i$, $V_{q_j}^i = \alpha_2 + \alpha_4 q_j + \alpha_5 q_i$ into the (HJB) equation, we obtain

$$\rho V^i(q_i, q_j) = \left\{ \begin{array}{c} \left(T + \frac{p-c}{2}\right) + \frac{p-c}{2\tau}q_i - \frac{p-c}{2\tau}q_j - \frac{\gamma}{2}\left(\frac{\alpha_1 + (\alpha_3 - \varphi)q_i + \alpha_5 q_j}{\gamma}\right)^2 \\ -\frac{\beta}{2}q_i^2 - \varphi q_i\left(\frac{\alpha_1 + (\alpha_3 - \varphi)q_i + \alpha_5 q_j}{\gamma}\right) \\ + (\alpha_1 + \alpha_3 q_i + \alpha_5 q_j)\left(\frac{\alpha_1 + (\alpha_3 - \varphi)q_i + \alpha_5 q_j}{\gamma} - \delta q_i\right) \\ + (\alpha_2 + \alpha_4 q_j + \alpha_5 q_i)\left(\frac{\alpha_1 + (\alpha_3 - \varphi)q_j + \alpha_5 q_i}{\gamma} - \delta q_j\right) \end{array} \right\}, \tag{A6}$$

which can be rewritten as

$$\begin{aligned}
&\left(\rho\alpha_0 - T - \frac{p-c}{2} - \frac{1}{2\gamma}\alpha_1^2 - \frac{1}{\gamma}\alpha_1\alpha_2\right) \\
&+q_i\left(\alpha_1(\rho+\delta) - \frac{p-c}{2\tau} - \frac{\alpha_1\alpha_3}{\gamma} - \frac{\alpha_2\alpha_5}{\gamma} - \frac{\alpha_1\alpha_5}{\gamma} + \frac{\alpha_1\varphi}{\gamma}\right) \\
&+q_j\left(\alpha_2(\rho+\delta) + \frac{p-c}{2\tau} - \frac{\alpha_2\alpha_3}{\gamma} - \frac{\alpha_1\alpha_4}{\gamma} - \frac{\alpha_1\alpha_5}{\gamma} + \frac{\alpha_2\varphi}{\gamma}\right) \\
&+q_i^2\left(\alpha_3\left(\frac{\rho}{2}+\delta\right) - \frac{\alpha_3^2}{2\gamma} - \frac{\alpha_5^2}{\gamma} + \frac{\beta}{2} + \frac{\varphi\alpha_3}{\gamma} - \frac{\varphi^2}{2\gamma}\right) \\
&+q_j^2\left(\alpha_4\left(\frac{\rho}{2}+\delta\right) - \frac{\alpha_3\alpha_4}{\gamma} - \frac{\alpha_5^2}{2\gamma} + \frac{\varphi\alpha_4}{\gamma}\right) \\
&+q_iq_j\left(\alpha_5\left((\rho+2\delta) - \frac{2\alpha_3}{\gamma} - \frac{\alpha_4}{\gamma} + \frac{2\varphi}{\gamma}\right)\right) = 0.
\end{aligned} \tag{A7}$$

For the equality to hold, the terms in brackets in the above equation have to be equal to zero. Notice that the last three terms do not depend on α_0, α_1 and α_2 but only on α_3, α_4 and α_5. We therefore focus on the system of three equations in three unknowns α_3, α_4 and α_5:

$$\begin{aligned}
\alpha_3\left(\frac{\rho}{2}+\delta\right) - \frac{\alpha_3^2}{2\gamma} - \frac{\alpha_5^2}{\gamma} + \frac{\beta}{2} + \frac{\varphi\alpha_3}{\gamma} - \frac{\varphi^2}{2\gamma} &= 0 \\
\alpha_4\left(\frac{\rho}{2}+\delta\right) - \frac{\alpha_3\alpha_4}{\gamma} - \frac{\alpha_5^2}{2\gamma} + \frac{\varphi\alpha_4}{\gamma} &= 0 \\
\alpha_5\left((\rho+2\delta) - \frac{2\alpha_3}{\gamma} - \frac{\alpha_4}{\gamma} + \frac{2\varphi}{\gamma}\right) &= 0
\end{aligned}$$

Define

$$A := \sqrt{\gamma(\beta+\rho\varphi+2\delta\varphi) + \gamma^2\left(\frac{\rho}{2}+\delta\right)^2}. \tag{A8}$$

There are six possible solutions:

$$
\begin{array}{llll}
1) & \alpha_3 = \varphi + \gamma\left(\frac{\rho}{2} + \delta\right) - A, & \alpha_4 = 0, & \alpha_5 = 0; \\
2) & \alpha_3 = \varphi + \gamma\left(\frac{\rho}{2} + \delta\right) - \frac{1}{3}A, & \alpha_4 = \frac{2}{3}A, & \alpha_5 = -\frac{2}{3}A; \\
3) & \alpha_3 = \varphi + \gamma\left(\frac{\rho}{2} + \delta\right) - \frac{1}{3}A, & \alpha_4 = \frac{2}{3}A, & \alpha_5 = \frac{2}{3}A; \\
4) & \alpha_3 = \varphi + \gamma\left(\frac{\rho}{2} + \delta\right) + A, & \alpha_4 = 0, & \alpha_5 = 0; \\
5) & \alpha_3 = \varphi + \gamma\left(\frac{\rho}{2} + \delta\right) + \frac{1}{3}A, & \alpha_4 = -\frac{2}{3}A, & \alpha_5 = \frac{2}{3}A; \\
6) & \alpha_3 = \varphi + \gamma\left(\frac{\rho}{2} + \delta\right) + \frac{1}{3}A, & \alpha_4 = -\frac{2}{3}A, & \alpha_5 = -\frac{2}{3}A.
\end{array}
\tag{A9}
$$

We disregard solutions 4, 5 and 6 because $\alpha_3 > 0$, which implies that the maximisation problem is convex rather than concave (those solutions provide minimum, not maximum). α_3 is negative for solution 1 whenever $\varphi \leq 0$ or $\varphi > 0$ but sufficiently small (i.e. $\varphi^2 < \beta\gamma$). α_3 is also negative for solutions 2 and 3 for sufficiently high β. Moreover, global asymptotic stability requires (Dockner, Jørgensen, Van Long and Sorge, 2000, p.252): a) $\alpha_3 + \alpha_5 < 0$, which is satisfied under solution 1 and 2 but not for solution 3 as $\alpha_3 + \alpha_5 = \varphi + \gamma\left(\frac{\rho}{2} + \delta\right) + \frac{1}{3}A > 0$; b) $\alpha_3 - \alpha_5 < 0$, which is satisfied by solution 1 but not 2 as $\alpha_3 - \alpha_5 = \varphi + \gamma\left(\frac{\rho}{2} + \delta\right) + \frac{1}{3}A > 0$. We therefore disregard solutions 2 and 3 as asymptotically unstable and focus on the only stable solution 1.

Substituting solution 1 in the second equation of (A7) gives

$$
\alpha_1 = \frac{p - c}{\tau} \frac{1}{\rho + \sqrt{4\frac{\beta + \rho\varphi + 2\delta\varphi}{\gamma} + (\rho + 2\delta)^2}}. \tag{A10}
$$

Recall that $I_i = \frac{\alpha_1 + (\alpha_3 - \varphi)q_i + \alpha_5 q_j}{\gamma} = \frac{\alpha_1 + (\alpha_3 - \varphi)q_i}{\gamma}$. Since in the steady state $I_i = \delta q_i$, we obtain that

$$
I_{CL}^s = \frac{\alpha_1}{\gamma} \frac{1}{(1 - \frac{(\alpha_3 - \varphi)}{\delta\gamma})}, \tag{A11}
$$

which after substituting for α_1 and α_3 gives

$$
I_{CL}^s = \frac{(p - c)\,\delta}{2\tau\left[\beta + \gamma\delta\,(\delta + \rho) + \varphi\,(2\delta + \rho)\right]} \tag{A12}
$$

which coincides with the steady state open-loop solution, as given by (22) in Section 3.

Appendix B. Closed-loop solution with increasing marginal cost

We normalise $\varphi = 0$ and $\gamma = 1$. The purchaser's instantaneous objective function is:

$$T + p\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau}\right) - \frac{c}{2}\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau}\right)^2 - \frac{1}{2}I_i^2 - \frac{\beta}{2}q_i^2. \tag{B1}$$

Define the value function as

$$V^i(q_i, q_j) = \alpha_0 + \alpha_1 q_i + \alpha_2 q_j + (\alpha_3/2)q_i^2 + (\alpha_4/2)q_j^2 + \alpha_5 q_i q_j. \tag{B2}$$

The value function has to satisfy the HJB equation:

$$\rho V^i(q_i, q_j) = \max \left\{ \begin{array}{c} \left(T + \frac{p}{2} - \frac{c}{8}\right) + \left(\frac{2p-c}{4\tau}\right) q_i - \left(\frac{2p-c}{4\tau}\right) q_j \\ -\left(\frac{c}{8\tau^2} + \frac{\beta}{2}\right) q_i^2 - \frac{c}{8\tau^2} q_j^2 + \frac{c}{4\tau^2} q_i q_j - \frac{1}{2} I_i^2 \\ +V^i_{q_i}(q_i, q_j)\left(I_i - \delta q_i\right) + V^i_{q_j}\left(\phi_j(q_i, q_j) - \delta q_j\right) \end{array} \right\}. \tag{B3}$$

Maximisation of the RHS yields

$$\begin{aligned} I_i &= \phi_i(q_i, q_j) = V^i_{q_i}(q_i, q_j) = \alpha_1 + \alpha_3 q_i + \alpha_5 q_j, && \text{(B4)} \\ I_j &= \phi_j(q_i, q_j) = V^j_{q_j}(q_i, q_j) = \alpha_1 + \alpha_3 q_j + \alpha_5 q_i, && \text{(B5)} \\ V^i_{q_j} &= \alpha_2 + \alpha_4 q_j + \alpha_5 q_i. && \text{(B6)} \end{aligned}$$

After substitution, we obtain

$$\rho V^i(q_i, q_j) = \left\{ \begin{array}{c} \left(T + \frac{p}{2} - \frac{c}{8}\right) + \left(\frac{2p-c}{4\tau}\right) q_i - \left(\frac{2p-c}{4\tau}\right) q_j \\ -\left(\frac{c}{8\tau^2} + \frac{\beta}{2}\right) q_i^2 - \frac{c}{8\tau^2} q_j^2 + \frac{c}{4\tau^2} q_i q_j - \frac{1}{2} I_i^2 \\ +V^i_{q_i}\left(V^i_{q_i} - \delta q_i\right) \\ +V^i_{q_j}\left(\phi_j(q_i, q_j) - \delta q_j\right) \end{array} \right\}, \tag{B7}$$

which provides

$$\rho V^i(q_i,q_j) = \left\{ \begin{array}{c} \left(T+\frac{p}{2}-\frac{c}{8}\right)+\left(\frac{2p-c}{4\tau}\right)q_i-\left(\frac{2p-c}{4\tau}\right)q_j \\ -\left(\frac{c}{8\tau^2}+\frac{\beta}{2}\right)q_i^2-\frac{c}{8\tau^2}q_j^2+\frac{c}{4\tau^2}q_iq_j \\ -\frac{1}{2}\left(V^i_{q_i}\right)^2+\left(V^i_{q_i}\right)^2-\delta q_iV^i_{q_i} \\ +V^j_{q_j}V^i_{q_j}-\delta q_jV^i_{q_j} \end{array} \right\} \tag{B8}$$

or

$$\rho V^i(q_i,q_j) = \left\{ \begin{array}{c} \left(T+\frac{p}{2}-\frac{c}{8}\right)+\left(\frac{2p-c}{4\tau}\right)q_i-\left(\frac{2p-c}{4\tau}\right)q_j \\ -\left(\frac{c}{8\tau^2}+\frac{\beta}{2}\right)q_i^2-\frac{c}{8\tau^2}q_j^2+\frac{c}{4\tau^2}q_iq_j \\ +\frac{1}{2}\left(V^i_{q_i}\right)^2-\delta q_iV^i_{q_i}+V^j_{q_j}V^i_{q_j}-\delta q_jV^i_{q_j} \end{array} \right\}. \tag{B9}$$

Recall:

$$\begin{aligned} V^i_{q_i}(q_i,q_j) &= \alpha_1+\alpha_3q_i+\alpha_5q_j, && \text{(B10)} \\ V^j_{q_j}(q_i,q_j) &= \alpha_1+\alpha_3q_j+\alpha_5q_i, && \text{(B11)} \\ V^i_{q_j} &= \alpha_2+\alpha_4q_j+\alpha_5q_i. && \text{(B12)} \end{aligned}$$

Substituting, we obtain

$$\rho V^i(q_i,q_j) = \left\{ \begin{array}{c} \left(T+\frac{p}{2}-\frac{c}{8}\right)+\left(\frac{2p-c}{4\tau}\right)q_i-\left(\frac{2p-c}{4\tau}\right)q_j \\ -\left(\frac{c}{8\tau^2}+\frac{\beta}{2}\right)q_i^2-\frac{c}{8\tau^2}q_j^2+\frac{c}{4\tau^2}q_iq_j \\ +\frac{1}{2}\left(\alpha_1+\alpha_3q_i+\alpha_5q_j\right)^2-\delta q_i\left(\alpha_1+\alpha_3q_i+\alpha_5q_j\right) \\ +\left(\alpha_1+\alpha_3q_j+\alpha_5q_i\right)\left(\alpha_2+\alpha_4q_j+\alpha_5q_i\right)-\delta q_j\left(\alpha_2+\alpha_4q_j+\alpha_5q_i\right) \end{array} \right\}. \tag{B13}$$

For the equality to hold, the following terms in brackets have to be equal to zero:

$$\begin{aligned}
&\left(\rho\alpha_0 - T + \frac{c}{8} - \frac{p}{2} - \frac{\alpha_1^2}{2} - \alpha_1\alpha_2\right) \\
&+q_i\left(\alpha_1(\rho+\delta) - \frac{2p-c}{4\tau} - \alpha_1\alpha_3 - \alpha_2\alpha_5 - \alpha_1\alpha_5\right) \\
&+q_j\left(\alpha_2(\rho+\delta) + \frac{2p-c}{4\tau} - \alpha_2\alpha_3 - \alpha_1\alpha_4 - \alpha_1\alpha_5\right) \\
&+q_i^2\left(\alpha_3\left(\frac{\rho}{2}+\delta\right) + \frac{c}{8\tau^2} - \frac{\alpha_3^2}{2} - \alpha_5^2 + \frac{\beta}{2}\right) \\
&+q_j^2\left(\alpha_4\left(\frac{\rho}{2}+\delta\right) + \frac{c}{8\tau^2} - \alpha_3\alpha_4 - \frac{\alpha_5^2}{2}\right) \\
&+q_iq_j\left(\alpha_5(\rho+2\delta) - 2\alpha_3\alpha_5 - \alpha_4\alpha_5 - \frac{c}{4\tau^2}\right) = 0
\end{aligned} \tag{B14}$$

Again, notice that the last three terms do not depend on α_0, α_1 and α_2 but only on α_3, α_4 and α_5. We therefore focus on the system of three equations in three unknowns α_3, α_4 and α_5:

$$\begin{aligned}
\alpha_3\left(\frac{\rho}{2}+\delta\right) + \frac{c}{8\tau^2} - \frac{\alpha_3^2}{2} - \alpha_5^2 + \frac{\beta}{2} &= 0 \\
\alpha_4\left(\frac{\rho}{2}+\delta\right) + \frac{c}{8\tau^2} - \alpha_3\alpha_4 - \frac{\alpha_5^2}{2} &= 0 \\
\alpha_5(2\delta+\rho) - 2\alpha_3\alpha_5 - \alpha_4\alpha_5 - \frac{c}{4\tau^2} &= 0
\end{aligned} \tag{B15}$$

Define

$$\begin{aligned}
m &:= \frac{\rho}{2} + \delta \\
z &:= \frac{c}{\tau^2} \\
B &:= \sqrt{3z + 8\beta + 8m^2 - 4\sqrt{(\beta+m^2)(3z+4\beta+4m^2)}} \\
C &:= \sqrt{3z + 8\beta + 8m^2 + 4\sqrt{(\beta+m^2)(3z+4\beta+4m^2)}}
\end{aligned} \tag{B16}$$

where $B > 0$ as $\left(3z + 8\beta + 8m^2\right)^2 - 16\left(\beta + m^2\right)\left(3z + 4\beta + 4m^2\right) = 9z^2$.

Solving the system of three equations in (B15) for α_3, α_4 and α_5, we obtain six candidates for the solution:

$$
\begin{array}{llll}
1) & \alpha_3 = m + \frac{B^3}{12z} - \frac{\left(5z+16\beta+16m^2\right)B}{12z}, & \alpha_4 = -\frac{1}{6}B, & \alpha_5 = \frac{1}{6}B; \\
2) & \alpha_3 = m - \frac{B^3}{12z} + \frac{\left(5z+16\beta+16m^2\right)B}{12z}, & \alpha_4 = \frac{1}{6}B, & \alpha_5 = -\frac{1}{6}B; \\
3) & \alpha_3 = m + \frac{C^3}{12z} - \frac{\left(5z+16\beta+16m^2\right)C}{12z}, & \alpha_4 = -\frac{1}{6}C, & \alpha_5 = \frac{1}{6}C; \\
4) & \alpha_3 = m - \frac{C^3}{12z} + \frac{\left(5z+16\beta+16m^2\right)C}{12z}, & \alpha_4 = \frac{1}{6}C, & \alpha_5 = -\frac{1}{6}C; \\
5) & \alpha_3 = m - \frac{1}{6}\sqrt{z+4\beta+4m^2}, & \alpha_4 = \frac{1}{3}\frac{\sqrt{z+4\beta+4m^2}\left(-5z+16\beta+16m^2\right)}{4z+16\beta+16m^2}, & \alpha_5 = \frac{\sqrt{z+4\beta+4m^2}}{3}; \\
6) & \alpha_3 = m + \frac{1}{6}\sqrt{z+4\beta+4m^2}, & \alpha_4 = -\frac{1}{3}\frac{\sqrt{z+4\beta+4m^2}\left(-5z+16\beta+16m^2\right)}{4z+16\beta+16m^2}, & \alpha_5 = -\frac{\sqrt{z+4\beta+4m^2}}{3}.
\end{array}
\tag{B17}
$$

We disregard solution 2 and 6 because $\alpha_3 > 0$, which implies that the maximisation problem is convex rather than concave (those solutions provide a minimum, not a maximum). The proof that $\alpha_3 > 0$ for solution 6 is straightforward. For solution 2, after substituting for B, we obtain:

$$\alpha_3 = m + \frac{B}{12z}\left(4\sqrt{\left(\beta+m^2\right)\left(3z+4\beta+4m^2\right)} + 2z + 8\beta + 8m^2\right) > 0. \tag{B18}$$

Moreover, stability requires that: $\alpha_3 + \alpha_5 < 0$. This is clearly not satisfied for solution 5 as $\alpha_3 + \alpha_5 = m + \frac{1}{6}\sqrt{z+4\beta+4m^2} > 0$. It is also not satisfied for solution 3, since

$$\alpha_3 + \alpha_5 = m + \left(\sqrt{3z\left(\beta+m^2\right) + 4\left(\beta+m^2\right)^2} - 2\left(\beta+m^2\right)\right)\frac{C}{3z} > 0. \tag{B19}$$

Stability also requires that $\alpha_3 - \alpha_5 < 0$. This is not satisfied for solution 4 since

$$\alpha_3 - \alpha_5 = m + \frac{C}{3z}\left(z + 2\left(\beta+m^2\right) - \sqrt{\left(\beta+m^2\right)\left(3z+4\beta+4m^2\right)}\right) > 0 \tag{B20}$$

as $\left(z+2\left(\beta+m^2\right)\right)^2 - \left(\beta+m^2\right)\left(3z+4\beta+4m^2\right) = m^2z + z^2 + \beta z > 0$. We are therefore left with solution 1, for which all the regularity conditions are satisfied, as we show below.

First, we need $\alpha_3 < 0$ (concavity condition). After substitution of B we obtain for solution 1 that

$$\alpha_3 = m - \frac{\begin{array}{c}\left[\sqrt{3z+8\beta+8m^2 - 4\sqrt{\left(\beta+m^2\right)\left(3z+4\beta+4m^2\right)}}\right. \\ \left.\times\left(2\sqrt{\left(\beta+m^2\right)\left(3z+4\beta+4m^2\right)} + z + 4\beta + 4m^2\right)\right]\end{array}}{6z}. \tag{B21}$$

Therefore, α_3 is negative if

$$\sqrt{\begin{matrix} 3z+8\left(\beta+m^2\right) \\ -4\sqrt{\left(\beta+m^2\right)\left(3z+4\beta+4m^2\right)} \end{matrix}} \left(\begin{matrix} 2\sqrt{\left(\beta+m^2\right)\left(3z+4\beta+4m^2\right)} \\ +z+4\left(\beta+m^2\right) \end{matrix} \right) > 6zm. \tag{B22}$$

Taking the square on both sides and subtracting the left-hand side from the right-hand side expression, we have that α_3 is negative if

$$z^2\left(20\beta+3z+\sqrt{64\beta\left(3z+4\beta+4m^2\right)+64m^2(3z+4\beta)+\left(16m^2\right)^2}-16m^2\right)>0. \tag{B23}$$

which is clearly satisfied.

Second, we need the stability condition $\alpha_3+\alpha_5<0$ to be satisfied. After substitution of B we obtain for solution 1 that

$$\alpha_3+\alpha_5=m-\frac{\left(\begin{matrix} \sqrt{\left(\beta+m^2\right)\left(3z+4\beta+4m^2\right)} \\ +2\left(\beta+m^2\right) \end{matrix} \right) \sqrt{\begin{matrix} 3z+8\beta+8m^2 \\ -4\sqrt{\left(\beta+m^2\right)\left(3z+4\beta+4m^2\right)} \end{matrix}}}{3z}. \tag{B24}$$

Therefore, $\alpha_3+\alpha_5<0$ if

$$\left(\begin{matrix} \sqrt{\left(\beta+m^2\right)\left(3z+4\beta+4m^2\right)} \\ +2\left(\beta+m^2\right) \end{matrix} \right) \sqrt{\begin{matrix} 3z+8\beta+8m^2 \\ -4\sqrt{\left(\beta+m^2\right)\left(3z+4\beta+4m^2\right)} \end{matrix}} > 3zm. \tag{B25}$$

Taking the square of both sides and subtracting the left-hand side from the right-hand side expression, we have that the condition is satisfied if $9z^2\beta>0$, which is clearly the case. Therefore, for solution 1 we have $\alpha_3+\alpha_5<0$.

Finally, stability also requires that $\alpha_3-\alpha_5<0$. This is clearly satisfied for solution 1 as $\alpha_3<0$ while $\alpha_5>0$. We conclude that solution 1 is stable and also satisfies the concavity condition.

We now calculate for solution 1 (i.e., the only stable solution) the values of α_1 and α_2. Notice that for solution 1 we have $\alpha_4=-\alpha_5$ which we can conveniently use to calculate α_1 and α_2.

The second and third equation of the system (B14) are

$$\alpha_1(\rho+\delta)-\frac{2p-c}{4\tau}-\alpha_1\alpha_3-\alpha_2\alpha_5-\alpha_1\alpha_5=0,$$
$$\alpha_2(\rho+\delta)+\frac{2p-c}{4\tau}-\alpha_2\alpha_3+\alpha_1\alpha_5-\alpha_1\alpha_5=0. \tag{B26}$$

Solving for α_1 and α_2, we have

$$\alpha_1 = \frac{2p-c}{4\tau}\frac{1}{\delta+\rho-\alpha_3}>0,$$
$$\alpha_2 = -\frac{2p-c}{4\tau}\frac{1}{\delta+\rho-\alpha_3}<0. \tag{B27}$$

The optimal investment rule is

$$I_i=\alpha_1+\alpha_3 q_i+\alpha_5 q_j. \tag{B28}$$

In the symmetric equilibrium and steady state, this implies $\delta q=\alpha_1+\alpha_3 q+\alpha_5 q$, which gives (recall that $\alpha_3+\alpha_5<0$):

$$q^s_{CL}=\frac{\alpha_1}{\delta-\alpha_3-\alpha_5}>0. \tag{B29}$$

Substituting for α_1, we have

$$q^s_{CL}=\frac{2p-c}{4\tau}\frac{1}{(\delta+\rho-\alpha_3)(\delta-\alpha_3-\alpha_5)}>0, \tag{B30}$$

where

$$\alpha_5=\frac{1}{6}\sqrt{\frac{3c}{\tau^2}+8\beta+8\left(\frac{\rho}{2}+\delta\right)^2-4\sqrt{\left(\beta+\left(\frac{\rho}{2}+\delta\right)^2\right)\left(\frac{3c}{\tau^2}+4\beta+4\left(\frac{\rho}{2}+\delta\right)^2\right)}}>0 \tag{B31}$$

and

$$\alpha_3=\left(\frac{\rho}{2}+\delta\right)+\frac{18\tau^2}{c}\alpha_5^3-\frac{\tau^2\left(5c+16\beta\tau^2+16\tau^2\left(\frac{\rho}{2}+\delta\right)^2\right)}{2c}\alpha_5<0. \tag{B32}$$

Finally, we compare the closed-loop solution with the open-loop solution. First, note that when

$\gamma = 1$ and $\varphi = 0$, the steady-state quality under the open-loop solution is equal to

$$q_{OL}^s = \frac{2p-c}{4\tau} \frac{1}{\beta + (\delta+\rho)\,\delta}. \tag{B33}$$

Therefore, comparing with (B30), we have that $q_{OL}^s > q_{CL}^s$ if (recall $m = \frac{\rho}{2} + \delta$)

$$\begin{aligned} & \frac{2}{3}\left(\beta + m^2\right) - \frac{\rho^2}{4} - \frac{\rho}{12}\sqrt{3z + 8\left(\beta + m^2\right) - 4\sqrt{\left(\beta + m^2\right)\left(3z + 4\beta + 4m^2\right)}} \\ & + \frac{1}{6}\sqrt{\left(\beta + m^2\right)\left(3z + 4\beta + 4m^2\right)} \\ > \ & \beta + (\delta + \rho)\,\delta, \end{aligned} \tag{B34}$$

which can be re-written as

$$\begin{aligned} & \frac{1}{6}\sqrt{\left(\beta + m^2\right)\left(3z + 4\beta + 4m^2\right)} - \frac{1}{3}\left(\delta^2 + \delta\rho + \frac{\rho^2}{4} + \beta\right) \\ > \ & \frac{\rho}{12}\sqrt{3z + 8\left(\beta + m^2\right) - 4\sqrt{\left(\beta + m^2\right)\left(3z + 4\beta + 4m^2\right)}} \end{aligned} \tag{B35}$$

Now, note that the LHS of inequality (B35) is positive as

$$\frac{\left(\beta + \left(\frac{\rho}{2} + \delta\right)^2\right)\left(3z + 4\beta + 4\left(\frac{\rho}{2} + \delta\right)^2\right)}{36} - \left(\frac{\delta^2}{3} + \frac{\rho\delta}{3} + \frac{\rho^2}{12} + \frac{\beta}{3}\right)^2 = \frac{z}{12}\left(\delta^2 + \delta\rho + \frac{\rho^2}{4} + \beta\right) > 0. \tag{B36}$$

Therefore, we can take the square of the expression on the LHS and the RHS of inequality (B35) and obtain

$$\begin{aligned} & \left(\frac{1}{6}\sqrt{\left(\beta + m^2\right)\left(3z + 4\beta + 4m^2\right)} - \left(\frac{1}{3}\delta^2 + \frac{1}{3}\delta\rho + \frac{1}{12}\rho^2 + \frac{1}{3}\beta\right)\right)^2 \\ & - \frac{\rho^2\left(3z + 8\left(\beta + \left(\frac{\rho}{2} + \delta\right)^2\right) - 4\sqrt{\left(\beta + m^2\right)\left(3z + 4\beta + 4m^2\right)}\right)}{144} > 0, \end{aligned} \tag{B37}$$

or, equivalently,

$$\frac{2}{9}\left(\beta+\delta^2\right)^2+\frac{z}{12}\left(\beta+\delta^2+\delta\rho\right)+\frac{\rho^2}{18}\left(\beta+\delta\rho+5\delta^2\right)+\frac{4}{9}\rho\delta\left(\beta+\delta^2\right)> \frac{\left(\delta\rho+\delta^2+\beta\right)}{9}\sqrt{4m^4+8m^2\beta+3zm^2+4\beta^2+3z\beta}. \tag{B38}$$

Taking the square of the expression on the LHS and RHS of inequality (B38) we obtain:

$$\begin{aligned} & \left(\frac{2}{9}\left(\beta+\delta^2\right)^2+\frac{z}{12}\left(\beta+\delta^2+\delta\rho\right)+\frac{\rho^2}{18}\left(\beta+\delta\rho+5\delta^2\right)+\frac{4}{9}\rho\delta\left(\beta+\delta^2\right)\right)^2 \\ & -\frac{\left(\delta\rho+\delta^2+\beta\right)^2}{81}\left(4\left(\frac{\rho}{2}+\delta\right)^4+8\left(\frac{\rho}{2}+\delta\right)^2\beta+3z\left(\frac{\rho}{2}+\delta\right)^2+4\beta^2+3z\beta\right) \\ = & \frac{z^2}{144}\left(\left(\beta+\delta^2\right)^2+\rho\left(2\beta\delta+2\delta^3+\delta^2\rho\right)\right)>0, \end{aligned} \tag{B39}$$

which is always satisfied. We conclude the the steady-state quality under the open-loop solution is always higher than the steady-state quality under the closed-loop solution.

Figure 1. Equilibrium is a saddle point

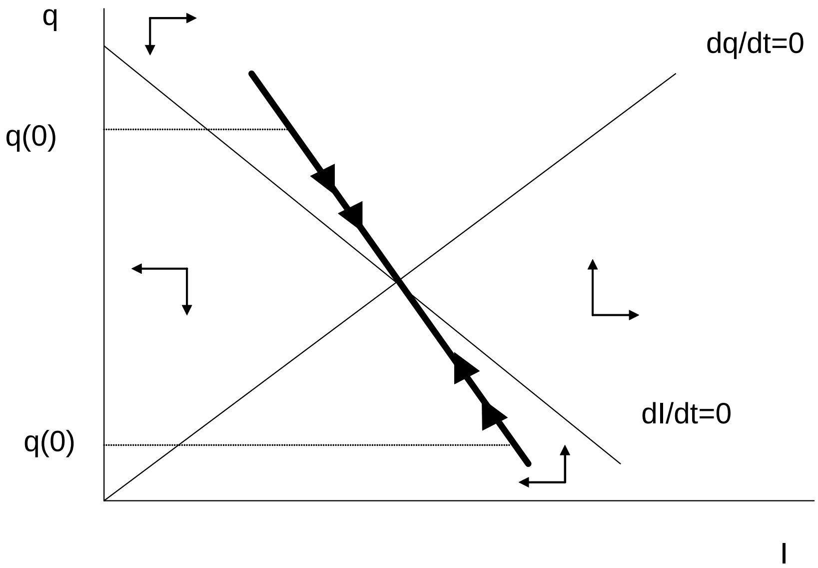

Figure 2. Increase in price or reduction in travel costs (more competition)