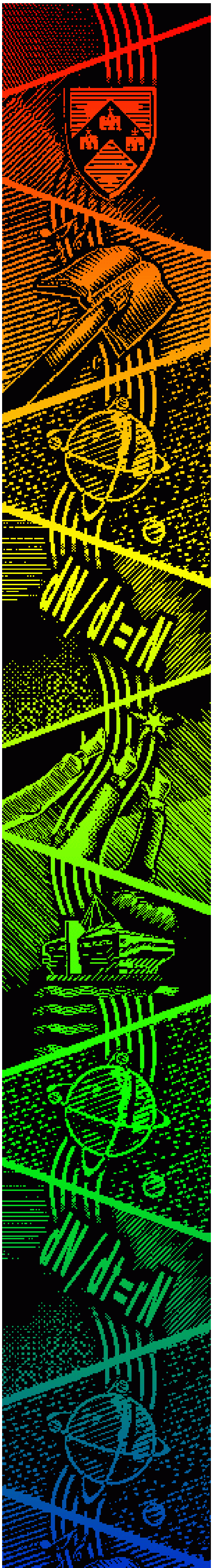

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Paying for performance with altruistic or motivated providers

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Paying for performance with altruistic or motivated providers

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Abstract

We present a model of optimal contracting between a purchaser (a principal) and a provider (an agent). We assume that: a) providers differ in efficiency and there are two types of provider; b) efficiency is private information (adverse selection); c) providers are partially altruistic or intrinsically motivated; d) they have limited liability. Four types of separating equilibrium can emerge, depending on the degree of altruism, characterised as *very low*, *low*, *high* and *very high*. i) For *very low* altruism the quantity of the efficient and inefficient types is distorted upwards and downwards respectively; the efficient type makes a positive profit. ii) For *low* altruism the quantity of the efficient and inefficient types is also distorted respectively upwards and downwards, but profits are zero for both types. iii) For *high* altruism the first best is attained: no distortions on quantities and zero profits. iv) For *very high* altruism the quantity of the inefficient type is distorted upwards, and the quantity of the efficient type is distorted either upwards or downwards. The inefficient type might have a positive profit. The quantity of the efficient type is higher than that of the inefficient type in all four possible equilibria. The transfer for the efficient type can be higher or lower than the inefficient one, unless altruism tends to zero in which case the transfer for the efficient type is higher. The utility of the efficient type is higher than that of the inefficient one when altruism is *very low*, *low* or *high*, though not necessarily when altruism is *very high*.

Keywords: altruism, motivated agents, performance. JEL: D82, I11, I18, L51.

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1 Introduction

Policymakers aim to design incentive schemes that encourage better performance in the public sector. In the health care sector, this is often referred to as *Paying for Performance*. For example, the Medicare Programme in the U.S., which provides public health care insurance to the elderly population, provides higher transfers to hospitals that perform well according to measurable quality indicators, such as rates of cervical cancer screening and hemoglobin testing for diabetic patients (Rosenthal, Frank, Li and Epstein, 2005). Similarly, in the United Kingdom general practitioners who perform well on certain quality indicators, such as the measurement of blood pressure and cholesterol in patients with ischemic heart disease, can receive substantial financial rewards (about 20% of a general-practitioner's budget; see Doran et al., 2006).

A major issue in rewarding performance is that providers differ in efficiency; some providers are more efficient than others, and efficiency is private information, i.e. there is adverse selection. Models of adverse selection have been extensively developed in the literature (starting from the seminal papers of Baron and Myerson, 1982; Laffont and Tirole, 1993; for a review see Laffont and Martimort, 2003). A key difference between the public sector (including health care) and other (perhaps private) sectors is that providers may be altruistic or intrinsically motivated. To some extent, purchasers and providers share the same objective function.

The present study analyses the optimal incentive scheme designed by a purchaser (a principal) in the presence of adverse selection and altruistic providers (or motivated agents). The main assumptions are: a) providers differ in efficiency and there are two types of provider; b) providers are partially altruistic: providers and purchasers partly share the same objective function; c) providers have limited liability: the financial surplus cannot be negative. Assumptions b) and c) are crucial.

We show that four types of separating equilibrium can emerge, depending on the degree of provider's altruism, which we refer to for convenience as *very low*, *low*, *high* and *very high*. i) For *very low* levels of altruism the quantity of the efficient and inefficient types is distorted upwards and downwards respectively; the efficient type makes a positive profit whereas the inefficient makes zero profit. ii) For *low* levels of altruism the quantity of the efficient and inefficient types is also distorted respectively upwards and downwards, but profits are zero for both types. iii) For *high* levels of altruism the first best is attained: there are no distortions on quantities, and profits are zero. iv) For *very high* levels of altruism the quantity of the inefficient type is distorted upwards, and the quantity of the efficient type is distorted either upwards or downwards. The inefficient provider may make a positive profit.

The quantity of the efficient provider is higher than the inefficient one in all four possible equilibria. A pooling equilibrium never arises. The transfer for the efficient provider can be higher or lower than for the inefficient one, unless altruism tends to zero, in which case the transfer for the efficient is higher. The utility of the efficient provider is higher than that of the inefficient one when altruism is *very low*, *low* or *high*.

Some of these results are surprising. 1) In contrast to the standard principal-agent model (in which only the quantity of the inefficient type is distorted), for *low* or *very low* levels of altruism both quantities are distorted (including the quantity of the efficient type). 2) The first best can be obtained for some values of altruism, but not necessarily for the highest values of altruism (in our classification this happens for *high* altruism but not for *very high* altruism). The purchaser may then be better off by having providers characterised by *high* altruism rather than *very high* altruism. 3) In many of these regimes, profits are zero (limited liability constraints are binding), but this does not necessarily imply that first best allocations can be obtained: profits are zero but quantities are optimally distorted. 4) Despite asymmetric information, the transfer for the efficient provider may be lower than the transfer for the inefficient one.

There is an analogy between this study and the principal-agent literature with state dependent utility functions (see Jullien, 2000, and Laffont and Martimort, 2003, ch.3). As in that literature, a range of equilibria arise, although the assumptions and the results differ. For instance, both quantities can be simultaneously distorted in the present model.

This study contributes to the literature on designing optimal incentive schemes in the public sector with motivated agents (see Besley and Ghatak, 2005, 2006; Dixit, 2005; Murdock, 2002). In these studies it is assumed that the agent shares to some extent the same objective function of the principal. This is similar to our assumption of altruism. None of these studies deals with adverse selection.[1]

The assumption of altruism or motivation seems reasonable within the public sector. In the healthcare sector doctors may care about the health of the patients. In the education sector teachers may care about the education of pupils. In ministries bureaucrats may subscribe to the mission set by the government (bureaucrats may be driven by 'Public Service Motivation', see Francois, 2000).

This study also contributes to the literature on provider incentives in health care. This literature often supposes that providers are at least partially altruistic, but does so mainly within a moral-hazard set-up (Ellis and McGuire, 1986; Eggleston, 2005; Chalkley and Malcomson, 1998a).[2] Some recent studies have introduced adverse selection, but they maintain the assumption that providers are surplus/profit maximisers (see De Fraja, 2000; Beitia, 2003; Chalkley and Malcomson, 2002).

This study introduces a model which combines adverse selection and altruistic providers. The literature sharing these assumptions is small. Choné and Ma (2007), Ma (2007) and Jack (2005) study the optimal incentive scheme when providers differ in altruism and the degree of altruism is private information. In contrast the present work considers the optimal incentive scheme when efficiency is private information and the degree of altruism is uniform across providers.

Choné and Ma (2007) and Ma (2007) assume limited liability, i.e. providers cannot make losses. The present work shares the assumption of limited liability. Jack (2005) shows that a separating equilibrium always arises: providers with high altruism hold informational rents. By adding limited-liability constraints, Choné and Ma (2007) show that pooling arises at least for providers with higher altruism. In equilibrium, the provider's utility is always positive, but providers with lower altruism make higher profits. In the model presented below, despite the limited-liability assumption, a pooling equilibrium never arises.

The paper is organised as follows. Section 2 introduces the main assumptions of the

[1] See also Benabou and Tirole (2003) for a model of intrinsic motivation.

[2] There is also an extensive literature which assumes that providers are surplus/profit maximisers (see Ma, 1994; Chalkley and Malcomson, 1998b; Ellis, 1998).

model. Sections 3 to 6 derive the equilibrium for different degrees of provider's altruism. Section 7 provides an example. Section 8 presents conclusions.

2 The model

2.1 The provider (the agent)

Define q as the quantity of output produced by the provider. This may be interpreted, for example, as number of discharges, as in De Fraja (2000); as a quality indicator as in Eggleston (2005); or as the amount of care received by the patients, as in Ma (2007) and Choné and Ma (2007); as a performance indicator for a general practitioner. Providers differ in efficiency θ due, for example, to differences in ability of the doctors. Efficiency is private information of the provider, and can take two possible values $\left\{\underline{\theta};\overline{\theta}\right\}$, with $\underline{\theta}<\overline{\theta}$. $\overline{\theta}$ denotes the inefficient provider and $\underline{\theta}$ the efficient one. The probabilities of types $\overline{\theta}$ and $\underline{\theta}$ are common knowledge and are respectively equal to λ and $(1-\lambda)$ (see Laffont and Tirole, 1993; Baron and Myerson, 1982; De Fraja, 2000; Beitia, 2003).

The cost function of a provider of type θ who provides quantity q is $C(\theta,q)$. We assume that $C_q>0$, $C_{qq}\geq 0$, $C_\theta>0$ and $C_{q\theta}>0$: the cost and the marginal cost is lower for the more efficient provider. Define $t(\theta)$ as the transfer to provider θ, $\pi(\theta)=t(\theta)-C(\theta,q)$ as the financial surplus, and $W(q)$ as the benefit for the patients (or welfare to consumers) upon receiving the quantity q. We assume that the purchaser is not able to audit the cost of the provider, so that the regulatory policy cannot be based on costs (as in Baron and Myerson, 1982). The contract offered by the purchaser specifies the quantity q that the provider is required to produce and the total transfer t paid by the purchaser.

The provider is assumed to be partially altruistic. Define α as a positive parameter denoting the degree of altruism (with $0\leq\alpha\leq 1$). As in Ellis and McGuire (1986) and Chalkley and Malcomson (1998b), the utility of the provider is given by the sum of the financial surplus and the altruistic component: $U(\theta)=\pi(\theta)+\alpha W(q)$. Alternatively, α can be interpreted as the degree of *intrinsic motivation* of the providers or the agents, as in Dixit (2005) and Besley and Ghatak (2005).

We assume limited liability, which implies that the financial surplus or profit cannot be negative: $\pi(\theta) \geq 0$ (see Ma, 2007; Choné and Ma, 2007; Besley and Ghatak, 2005 and 2006).

2.2 The purchaser (the principal)

The benefit to patients (consumers) receiving care from provider θ is $W(q(\theta))$, with $W_q > 0$ and $W_{qq} \leq 0$. To simplify notation, we denote $\underline{q} = q(\underline{\theta})$ and $\overline{q} = q(\overline{\theta})$ as the quantity produced by the efficient and the inefficient provider respectively, and similarly for transfers: $\underline{t} = t(\underline{\theta})$, $\overline{t} = t(\overline{\theta})$, profits: $\underline{\pi} = \pi(\underline{\theta})$, $\overline{\pi} = \pi(\overline{\theta})$ and utilities: $\underline{U} = U(\underline{\theta})$, $\overline{U} = U(\overline{\theta})$. The purchaser knows the distribution of efficiency θ, and maximises the sum of the benefits of the patients net of the transfer to the providers:[3]

$$\max_{\overline{q},\underline{q}} \ \lambda\left[W(\overline{q}) - \overline{t}\right] + (1-\lambda)\left[W(\underline{q}) - \underline{t}\right] \tag{1}$$

subject to the Incentive-Compatibility Constraints (IC), which suggest that each provider is better off by announcing its own type rather than the other type (below $U(\underline{\theta}, \overline{\theta})$ and $U(\overline{\theta}, \underline{\theta})$ denote the utility of the efficient (inefficient) provider when mimicking the inefficient (efficient) one):

$$IC : \underline{U} \geq U(\underline{\theta}, \overline{\theta}) \text{ and } \overline{U} \geq U(\overline{\theta}, \underline{\theta}), \tag{2}$$

the Participation Constraints (PC), which suggest that each provider has non-negative utility:

$$PC : \underline{U} \geq 0 \text{ and } \overline{U} \geq 0, \tag{3}$$

and the Limited-Liability Constraints (LC), which suggest that each provider has non-negative profit:

$$LC : \underline{\pi} \geq 0 \text{ and } \overline{\pi} \geq 0. \tag{4}$$

Lemma 1 *When the Limited-Liability Constraint is satisfied for type θ, the Participation Constraint is also satisfied for type θ.*

[3] A *zero* weight is implicitly attached to the utility function of the provider. The analysis would still hold if the provider utility were also included in the purchaser's objective function provided that the weight attached to the utility of the provider is less than one (as in Baron and Myerson, 1982). Alternatively, if the weight attached to the utility of the provider is one, the analysis still holds if the opportunity cost of public funds is positive (as in Laffont and Tirole, 1993).

The Participation Constraint for provider θ is $U(\theta) = \pi(\theta) + \alpha W(q) \geq 0$. Therefore, even if profits are zero, the utility of the provider is non-negative. Lemma 1 suggests that we can ignore the participation constraints and focus only on the remaining four. For brevity we use the terms IC-eff, IC-ineff, LC-eff and LC-ineff to refer to the Incentive-Compatibility Constraint and the Limited-Liability Constraint respectively for the efficient and the inefficient provider.

Lemma 2 *The Incentive-Compatibility Constraints can be written as:*

$$\underline{\pi} \geq \overline{\pi} + C(\overline{\theta}, \overline{q}) - C(\underline{\theta}, \overline{q}) - \alpha \left(W(\underline{q}) - W(\overline{q}) \right) \quad \textit{(IC-eff)} \tag{5}$$

$$\overline{\pi} \geq \underline{\pi} + C(\underline{\theta}, \underline{q}) - C(\overline{\theta}, \underline{q}) + \alpha \left(W(\underline{q}) - W(\overline{q}) \right) \quad \textit{(IC-ineff)} \tag{6}$$

All proofs of Lemmas, Propositions and Corollaries are given in Appendix 1. The standard principal-agent model with adverse selection (see for example Laffont and Martimort, 2003) can be obtained as a special case by setting $\alpha = 0$.

Suppose that the quantity of the efficient type is higher than the quantity of the inefficient type (we show below that this is always the case in equilibrium). Higher altruism weakens the Incentive-Compatibility constraint of the efficient provider (IC-eff): by mimicking the inefficient provider the efficient provider has an extra loss of utility equal to $\alpha \left(W(\underline{q}) - W(\overline{q}) \right)$ which arises from the altruistic component. In contrast, higher altruism strengthens the Incentive-Compatibility constraint of the inefficient provider (IC-ineff): by mimicking the efficient provider the inefficient provider has an extra gain of utility equal to $\alpha \left(W(\underline{q}) - W(\overline{q}) \right)$ which arises from the altruistic component.

The next four sections derive the optimal contracts for different degrees of altruism. Four different types of separating equilibrium can arise. The four levels of altruism will be called *very low* (α^{VL}), *low* (α^{L}), *high* (α^{H}) and *very high* (α^{VH}). Define the thresholds $\widetilde{\alpha}$, $\widehat{\alpha}$ and $\overline{\alpha}$ such that: $0 \leq \alpha^{VL} \leq \widetilde{\alpha} \leq \alpha^{L} \leq \widehat{\alpha} \leq \alpha^{H} \leq \overline{\alpha} \leq \alpha^{VH} \leq 1$. We begin by analysing the equilibrium for *high* altruism ($\widehat{\alpha} \leq \alpha^{H} \leq \overline{\alpha}$), since in this interval the *First Best* can be attained. By *First Best* is meant that profits are zero and quantities are not distorted: the incentive-compatibility constraints are not binding and the quantities are the same as with symmetric information.

3 High altruism (or First Best): $\widehat{\alpha} \leq \alpha^H \leq \overline{\alpha}$

We maximise the purchaser function (Eq.1) subject to the Limited-Liability Constraints $\underline{\pi} \geq 0$ and $\overline{\pi} \geq 0$. We ignore the Incentive-Compatibility constraints, and then verify ex-post the values of α for which both constraints are satisfied. Since the Limited-Liability Constraints are binding, i.e. $\underline{\pi} = \overline{\pi} = 0$, then $\overline{t} = C(\overline{\theta}, \overline{q})$ and $\underline{t} = C(\underline{\theta}, \underline{q})$. The maximisation problem is:

$$\max_{\overline{q}, \underline{q}} \ \lambda \left[W(\overline{q}) - C(\overline{\theta}, \overline{q})\right] + (1-\lambda)\left[W(\underline{q}) - C(\underline{\theta}, \underline{q})\right] \tag{7}$$

whose FOCs are:

$$W_{\overline{q}}(\overline{q}^H) = C_{\overline{q}}(\overline{\theta}, \overline{q}^H) \tag{8}$$

$$W_{\underline{q}}(\underline{q}^H) = C_{\underline{q}}(\underline{\theta}, \underline{q}^H) \tag{9}$$

The optimal quantity for each provider is such that the marginal benefit is equal to the marginal cost. The SOC is $W_{qq} - C_{qq} < 0$. Note that $\underline{q}^H$ and $\overline{q}^H$ do not depend on the degree of altruism, and that $\underline{q}^H > \overline{q}^H$ because the efficient provider has a smaller marginal cost. The following corollary establishes the values of α for which the Incentive-Compatibility Constraints are satisfied.

Proposition 3 *The first best can be implemented when the level of altruism is such that*

$$0 < \frac{C(\overline{\theta}, \overline{q}^H) - C(\underline{\theta}, \overline{q}^H)}{W(\underline{q}^H) - W(\overline{q}^H)} \equiv \widehat{\alpha} \leq \alpha^H \leq \overline{\alpha} \equiv \frac{C(\overline{\theta}, \underline{q}^H) - C(\underline{\theta}, \underline{q}^H)}{W(\underline{q}^H) - W(\overline{q}^H)} \gtreqless 1.$$

Proposition 3 suggests that there exists a non-empty range of altruism values, $\widehat{\alpha} \leq \alpha^H \leq \overline{\alpha}$ (with $\widehat{\alpha} < \overline{\alpha}$), such that the Incentive-Compatibility Constraints are not binding. Intuitively, the efficient provider has no incentive to mimic the inefficient one, because its contracted quantity is higher, which gives extra utility. The inefficient provider has no incentive to mimic the efficient one because it is too costly. The equilibrium under asymmetry of information is identical to that which would arise with symmetry of information. Within this range of altruism, the quantities and profits are the same as those obtained when the purchaser knows the level of efficiency of the providers.[4] The following corollary summarises and compares quantities, transfers, profits and utilities between providers.

Corollary 1 $\underline{q}^H > \overline{q}^H$, $\frac{\partial \underline{q}^H}{\partial \alpha} = \frac{\partial \overline{q}^H}{\partial \alpha} = 0$, $\underline{\pi}^H = \overline{\pi}^H = 0$, $\underline{t}^H \lesseqgtr \overline{t}^H$, $\frac{\partial \underline{t}^H}{\partial \alpha} = \frac{\partial \overline{t}^H}{\partial \alpha} = 0$, $\underline{U}^H > \overline{U}^H$, $\frac{\partial \underline{U}^H}{\partial \alpha} > 0$, $\frac{\partial \overline{U}^H}{\partial \alpha} > 0$.

The quantity of the efficient provider is higher than that of the inefficient one as a result of the lower marginal cost. Quantities do not vary with altruism. Profits are zero. The transfer to the efficient provider may be higher or lower than to the inefficient one: inefficiency implies higher costs, but the efficient provider produces more output. The transfer does not vary with altruism, since quantities do not vary with altruism. The efficient provider has a higher utility than the inefficient provider because it produces more output, which increases utility. The utility of each provider increases with altruism.

4 Very low altruism: $0 \leq \alpha^{VL} \leq \widetilde{\alpha}$

In this section we determine the equilibrium when altruism is positive but *very low*. The LC-ineff and IC-eff are binding, so that $\overline{\pi} = 0$ and $\underline{\pi} = C(\overline{\theta}, \overline{q}) - C(\underline{\theta}, \overline{q}) - \alpha \left(W(\underline{q}) - W(\overline{q}) \right) > 0$. At this stage we ignore LC-eff and IC-ineff and verify ex-post whether they are satisfied. The solution when LC-eff is binding is presented in section 5. The problem then becomes:

$$\max_{\overline{q}, \underline{q}} \lambda \left[W(\overline{q}) - C(\overline{\theta}, \overline{q}) \right] + (1 - \lambda) \begin{bmatrix} W(\underline{q}) - C(\underline{\theta}, \underline{q}) \\ - \left(C(\overline{\theta}, \overline{q}) - C(\underline{\theta}, \overline{q}) - \alpha \left(W(\underline{q}) - W(\overline{q}) \right) \right) \end{bmatrix} \tag{10}$$

Upon rearranging the FOCs, we obtain:

$$(1 + \alpha) W_q(\underline{q}^{VL}) = C_q(\underline{\theta}, \underline{q}^{VL}) \tag{11}$$

$$W_q(\overline{q}^{VL}) = C_q(\overline{\theta}, \overline{q}^{VL}) + \frac{1 - \lambda}{\lambda} \left[\alpha W_q(\overline{q}^{VL}) + C_q(\overline{\theta}, \overline{q}^{VL}) - C_q(\underline{\theta}, \overline{q}^{VL}) \right] \tag{12}$$

[4] This result has some similarity to Lewis and Sappington (1988), who analyse optimal regulation under asymmetric information on demand: if the marginal cost of the provider is decreasing then asymmetric information does not reduce welfare, and the solutions under symmetric and asymmetric information coincide.

The optimal quantity is such that the marginal benefit is equal to the marginal cost. It is optimal to distort both quantities to reduce the profit of the efficient type: the quantity of the efficient provider is distorted *upwards,* whereas the quantity of the inefficient provider is distorted *downwards.* Since the marginal benefit for the efficient provider is higher and the marginal cost is lower relative to the inefficient provider, it follows that $\underline{q}^{VL} > \overline{q}^{VL}$, i.e. the optimal quantity for the efficient provider is higher.

In equilibrium, the profit of the efficient provider is positive, while the profit of the inefficient provider is zero ($\underline{\pi}^{VL} > 0$ and $\overline{\pi}^{VL} = 0$). In contrast to the standard model of regulation (Laffont, Tirole, 1993; Baron and Myerson, 1982), it is optimal to distort not only the quantity of the inefficient provider but also the quantity of the efficient one. By increasing the quantity of the efficient provider, it becomes less appealing for the efficient provider to mimic the inefficient provider due to the higher altruistic component, which reduces the informational rent (see Eq.5).

When altruism tends to zero ($\alpha \to 0$) the traditional principal-agent model with self-interested agents applies, so that a separating equilibrium exists but the quantity produced by the inefficiency provider is distorted downwards (see Laffont and Martimort, 2003).

So far we have neglected the Incentive-Compatibility constraint for the inefficient provider (IC-ineff). At the optimal solution outlined above this constraint is never binding.[5] The following corollary summarises the results, and also establishes how quantities and the profit of the efficient provider vary with altruism.

Corollary 2 *$\underline{q}^{VL} > \overline{q}^{VL}$, $\frac{\partial \underline{q}^{VL}}{\partial \alpha} > 0$, $\frac{\partial \overline{q}^{VL}}{\partial \alpha} < 0$, $\underline{\pi}^{VL} > 0$, $\overline{\pi}^{VL} = 0$, $\frac{\partial \underline{\pi}^{VL}}{\partial \alpha} < 0$, $\underline{t}^{VL} \gtrless \overline{t}^{VL}$ ($\underline{t}^{VL} > \overline{t}^{VL}$ if $\alpha \to 0$), $\frac{\partial \underline{t}^{VL}}{\partial \alpha} \gtrless 0$, $\frac{\partial \overline{t}^{VL}}{\partial \alpha} < 0$, $\underline{U}^{VL} > \overline{U}^{VL}$, $\frac{\partial \underline{U}^{VL}}{\partial \alpha} \gtrless 0$, $\frac{\partial \overline{U}^{VL}}{\partial \alpha} \lesseqgtr 0$.*

The marginal benefit of the quantity produced by the efficient provider and the marginal cost of the quantity of the inefficient provider increase with altruism. Consequently, the quantity of the efficient provider increases with altruism, and the quantity of the inefficient provider decreases with altruism. The profit of the efficient provider is positive and decreases with altruism, because higher altruism relaxes the Incentive-Compatibility

[5] Since $\overline{\pi} = 0$, and substituting the IC-eff into IC-ineff, the IC-ineff can be rewritten as: $0 > C(\overline{\theta}, \overline{q}) - C(\underline{\theta}, \overline{q}) - \alpha\left(W(\underline{q}) - W(\overline{q})\right) + C(\underline{\theta}, \underline{q}) - C(\overline{\theta}, \underline{q}) + \alpha\left(W(\underline{q}) - W(\overline{q})\right)$, which simplifies to $0 > C(\overline{\theta}, \overline{q}^{VL}) - C(\underline{\theta}, \overline{q}^{VL}) - \left(C(\overline{\theta}, \underline{q}^{VL}) - C(\underline{\theta}, \underline{q}^{VL})\right) < 0$ (recall $\underline{q}^{VL} > \overline{q}^{VL}$).

constraint of the efficient provider. The profit of the inefficient provider is zero. The transfer to the efficient provider may be higher or lower than to the inefficient one. However, the transfer to the efficient provider is higher for sufficiently low altruism. The transfer to the inefficient provider reduces with altruism, due to the lower contracted quantity. The transfer for the efficient provider may either increase or reduce with altruism: higher altruism increases output but implies a lower rent.

The utility of the efficient provider is always higher than for the inefficient one: the efficient provider has higher output and holds the informational rent. The utility of the efficient provider may either increase or reduce with altruism: higher altruism increases utility directly, implies a higher quantity, but reduces the informational rent. For the inefficient provider the effect on utility is also indeterminate: higher altruism increases utility directly, but implies a lower quantity.

Since the profit of the efficient provider reduces with altruism, there is a level $\alpha = \widetilde{\alpha}$ such that the Limited-Liability Constraint of the efficient provider (LC-eff) is binding, and the equilibrium $\left\{\underline{q}^{VL}, \overline{q}^{VL}\right\}$ does not hold anymore. The following proposition determines how $\widetilde{\alpha}$ compares with the lower bound $\widehat{\alpha}$ obtained under the First-Best solution with *high* altruism.

Proposition 4 *There exists an $\alpha = \widetilde{\alpha} < \widehat{\alpha}$ where the LC-eff is binding.*

This proposition suggests that the equilibrium outlined in this section is valid only for $0 \leq \alpha^{VL} \leq \widetilde{\alpha} < \widehat{\alpha}$, when altruism is *very low.* Section 3 has analysed the equilibrium for $\widehat{\alpha} \leq \alpha^{H} \leq \overline{\alpha}$, when altruism is *high.* The next section analyses the intermediate case, when altruism is *low* and $\widetilde{\alpha} \leq \alpha^{L} < \widehat{\alpha}$.

5 Low altruism: $\widetilde{\alpha} \leq \alpha^{L} \leq \widehat{\alpha}$

In this section we derive the solution when the Limited-Liability Constraint of the efficient type (LC-eff) is binding. As in section 4, LC-ineff and IC-eff are also binding (whereas IC-ineff is not). From LC-ineff we have $\overline{\pi} = 0$, which implies $\overline{t} = C(\overline{\theta}, \overline{q})$. Since $\underline{t} = C(\underline{\theta}, \underline{q}) + \underline{\pi}$

by definition, the problem can be formulated as:

$$\max_{\overline{q},\underline{q},\underline{\pi}} \lambda \left[W(\overline{q}) - C(\overline{\theta},\overline{q})\right] + (1-\lambda)\left[W(\underline{q}) - C(\underline{\theta},\underline{q}) - \underline{\pi}\right] \tag{13}$$

subject to

$$\underline{\pi} \geq C(\overline{\theta},\overline{q}) - C(\underline{\theta},\overline{q}) - \alpha\left(W(\underline{q}) - W(\overline{q})\right) \quad \text{(IC-eff)} \tag{14}$$

$$\underline{\pi} \geq 0 \quad \text{(LC-eff)} \tag{15}$$

In this case the degree of altruism is high enough for the informational rent of the efficient provider to turn negative, if the Limited-Liability constraint were not binding. Therefore, both the IC-eff and LC-eff are binding with strict equality. Define $\mu_1 \geq 0$ and $\mu_2 \geq 0$ as the Lagrangean multipliers associated respectively with IC-eff and LC-eff.[6] The FOCs are:

$$(1-\lambda)\left[W_q(\underline{q}) - C_q(\underline{\theta},\underline{q})\right] + \mu_1 \alpha W_q(\underline{q}) = 0 \tag{16}$$

$$\lambda\left[W_q(\overline{q}) - C_q(\overline{\theta},\overline{q})\right] - \mu_1\left[C_q(\overline{\theta},\overline{q}) - C_q(\underline{\theta},\overline{q}) + \alpha W_q(\overline{q})\right] = 0 \tag{17}$$

$$-(1-\lambda) + \mu_1 + \mu_2 = 0 \tag{18}$$

From the last equation we obtain $\mu_1 = (1-\lambda) - \mu_2 > 0$. Upon substituting into (16-17) and rearranging, we obtain:

$$W_q(\underline{q}^L) + \frac{(1-\lambda)-\mu_2}{1-\lambda}\alpha W_q(\underline{q}^L) = C_q(\underline{\theta},\underline{q}^L) \tag{19}$$

$$W_q(\overline{q}^L) = C_q(\overline{\theta},\overline{q}^L) + \frac{(1-\lambda)-\mu_2}{\lambda}\left[C_q(\overline{\theta},\overline{q}^L) - C_q(\underline{\theta},\overline{q}^L) + \alpha W_q(\overline{q}^L)\right] \tag{20}$$

The optimal quantity is such that the marginal benefit is equal to the marginal cost. As in the previous section, the optimal quantity of the efficient type is distorted upwards, while the optimal quantity of the inefficient type is distorted downwards. Since the marginal benefit is higher for the efficient provider and the marginal cost is lower compared to the

[6] The Lagrangean is: $L = \lambda\left[W(\overline{q}) - C(\overline{\theta},\overline{q})\right] + (1-\lambda)\left[W(\underline{q}) - C(\underline{\theta},\underline{q}) - \underline{\pi}\right]$
$+\mu_1\left\{\underline{\pi} - \left[C(\overline{\theta},\overline{q}) - C(\underline{\theta},\overline{q}) - \alpha\left(W(\underline{q}) - W(\overline{q})\right)\right]\right\} + \mu_2\underline{\pi}$.

inefficient provider, it follows that $\underline{q}^L > \overline{q}^L$, i.e. the optimal quantity for the efficient provider is higher than the quantity of the inefficient one. In contrast to the previous section the profit for *both* providers is zero ($\overline{\pi}^L = \underline{\pi}^L = 0$).

For $0 \leq \alpha \leq \widetilde{\alpha}$ the LC-eff is not binding and $\mu_2 = 0$, so that we recover the solution with *very low* altruism. For $\widehat{\alpha} \leq \alpha \leq \overline{\alpha}$, the IC-eff is not binding and $\mu_1 = (1-\lambda) - \mu_2 = 0$, so that we recover the solution with *high* altruism.

So far we have neglected the IC-ineff. At the optimal solution outlined above, this constraint is always satisfied.[7] The following corollary summarises the results and establishes how quantities, transfers and utilities vary with α.

Corollary 3 $\underline{q}^L > \overline{q}^L$, $\frac{\partial \overline{q}^L}{\partial \alpha} > 0$, $\frac{\partial \underline{q}^L}{\partial \alpha} \lesseqgtr 0$ *(with* $\frac{\partial \underline{q}^L}{\partial \alpha} < 0$ *for* $\mu_1 \to 0$*)*, $\frac{\partial \mu_1}{\partial \alpha} < 0$ *and* $\frac{\partial \mu_2}{\partial \alpha} > 0$, $\underline{\pi}^L = \overline{\pi}^L = 0$, $\underline{t}^L \gtrless \overline{t}^L$, $\frac{\partial \overline{t}^L}{\partial \alpha} > 0$, $\frac{\partial \underline{t}^L}{\partial \alpha} \lesseqgtr 0$, $\underline{U}^L > \overline{U}^L$, $\frac{\partial \overline{U}^L}{\partial \alpha} > 0$, $\frac{\partial \underline{U}^L}{\partial \alpha} \gtrless 0$.

The quantity of the efficient provider is higher than for the inefficient one. The quantity of the inefficient provider increases with altruism. The quantity of the efficient provider can increase or reduce with altruism. However, for sufficiently high altruism (as α approaches $\widehat{\alpha}$) the quantity of the efficient provider reduces with altruism. These results are in clear contrast to those obtained under *very low* altruism, whereby the quantity of the inefficient provider decreases with altruism and the quantity of the efficient provider increases with altruism ($\frac{\partial \overline{q}^{VL}}{\partial \alpha} < 0$, $\frac{\partial \underline{q}^{VL}}{\partial \alpha} > 0$). In Eq.(20), although higher altruism α implies a higher marginal cost of the output for the inefficient provider, it also causes the Incentive-Compatibility Constraint for the efficient provider (IC-eff) to be less binding as reflected by the smaller value of $\mu_1 = (1-\lambda) - \mu_2$, reducing the marginal cost: this second effect always dominates, and the quantity of the inefficient provider always increases with altruism.

Profits are zero. The transfer for the efficient provider may be higher or lower than for the inefficient one: the efficient provider has lower per-unit cost, but it is contracted a higher output. The transfer for the inefficient provider increases with altruism due to the higher quantity. The transfer for the efficient provider reduces (increases) with altruism

[7] The IC-ineff is: $0 \geq -\left(C(\overline{\theta}, \underline{q}^L) - C(\underline{\theta}, \underline{q}^L)\right) + \alpha\left(W(\underline{q}^L) - W(\overline{q}^L)\right)$. From IC-eff we know that $\alpha\left(W(\underline{q}^L) - W(\overline{q}^L)\right) = C(\overline{\theta}, \overline{q}^L) - C(\underline{\theta}, \overline{q}^L)$, so that, after substitution, IC-ineff becomes: $0 \geq -\left(C(\overline{\theta}, \underline{q}^L) - C(\underline{\theta}, \underline{q}^L)\right) + C(\overline{\theta}, \overline{q}^L) - C(\underline{\theta}, \overline{q}^L) < 0$ (which is always satisfied since $\underline{q}^L > \overline{q}^L$).

when the contracted quantity reduces (increases). The utility of the efficient provider is always higher than for the inefficient provider, since the contracted quantity is higher. The utility of the inefficient provider increases with altruism: higher altruism increases utility directly, but also implies a higher quantity. The utility of the efficient provider may either increase or decrease with altruism: higher altruism increases utility directly, but output may increase or decrease with altruism.

It is interesting to compare quantities under the three scenarios considered so far. Comparison of Eqs. (8-9), (11-12) and (19-20) suggests that the quantity for the inefficient provider is higher under *high* altruism than under *low* or *very low* altruism, i.e. $\overline{q}^H \geq \left\{\overline{q}^L; \overline{q}^{VL}\right\}$. The quantity for the efficient provider is lower under *high* altruism than under *low* or *very low* altruism, i.e. $\underline{q}^H \leq \left\{\underline{q}^L; \underline{q}^{VL}\right\}$. By comparing Eqs. (19-20) with (11-12), we see that even if $\mu_2 > 0$, this does not imply that $\underline{q}^L < \underline{q}^{VL}$ and $\overline{q}^L > \overline{q}^{VL}$ because the altruism α is higher under *low altruism* than under *very low* altruism, so that $\underline{q}^L \gtrless \underline{q}^{VL}$ and $\overline{q}^L \gtrless \overline{q}^{VL}$.

6 Very high altruism: $\alpha^{VH} \geq \overline{\alpha}$

With *very high* altruism, the level of altruism is so high that it is the inefficient provider who has an incentive to mimic the efficient provider (see Eq.6). In this section the optimal solution is derived when the Incentive-Compatibility Constraint of the inefficient provider (IC-ineff) and the Limited-Liability Constraint of the efficient provider (LC-eff) are binding. We ignore the Incentive-Compatibility Constraint of the efficient provider (IC-eff), and verify ex-post that it is satisfied at the optimal solution. The Limited-Liability Constraint of the inefficient provider (LC-ineff) may or may not bind. From LC-eff we set $\underline{\pi} = 0$, which implies $\underline{t} = C(\underline{\theta}, \overline{q})$. By definition $\overline{t} = C(\overline{\theta}, \underline{q}) + \overline{\pi}$, so that we can write the problem as:

$$\max_{\overline{q}, \underline{q}, \overline{\pi}} \lambda \left[W(\overline{q}) - C(\overline{\theta}, \overline{q}) - \overline{\pi}\right] + (1-\lambda)\left[W(\underline{q}) - C(\underline{\theta}, \underline{q})\right] \tag{21}$$

subject to:

$$\overline{\pi} \geq C(\underline{\theta},\underline{q}) - C(\overline{\theta},\underline{q}) + \alpha\left(W(\underline{q}) - W(\overline{q})\right) \qquad \text{(IC-ineff)} \tag{22}$$

$$\overline{\pi} \geq 0 \qquad \text{(LC-ineff)} \tag{23}$$

From IC-ineff, the first term on the RHS is negative. Consequently a necessary but not sufficient condition for this constraint to be satisfied is that $\alpha\left(W(\underline{q}) - W(\overline{q})\right) > 0$, and therefore $\underline{q} > \overline{q}$: at the optimal solution the quantity of the efficient provider is always higher than the quantity of the inefficient provider. Define $\mu_1 \geq 0$ and $\mu_2 \geq 0$ as the Lagrangean multipliers associated respectively with IC-ineff and LC-ineff.[8] The FOCs are:

$$\lambda\left[W_q(\overline{q}) - C_q(\overline{\theta},\overline{q})\right] + \mu_1 \alpha W_q(\overline{q}) = 0 \tag{24}$$

$$(1-\lambda)\left[W_q(\underline{q}) - C_q(\underline{\theta},\underline{q})\right] + \mu_1\left\{C_q(\overline{\theta},\underline{q}) - C_q(\underline{\theta},\underline{q}) - \alpha W_q(\underline{q})\right\} = 0 \tag{25}$$

$$-\lambda + \mu_1 + \mu_2 = 0 \tag{26}$$

The above conditions can be rewritten as:

$$W_q(\overline{q}^{VH})\left(1 + \frac{\mu_1 \alpha}{\lambda}\right) = C_q(\overline{\theta},\overline{q}^{VH}) \tag{27}$$

$$W_q(\underline{q}^{VH}) + \frac{\mu_1}{1-\lambda}\left\{C_q(\overline{\theta},\underline{q}^{VH}) - C_q(\underline{\theta},\underline{q}^{VH}) - \alpha W_q(\underline{q}^{VH})\right\} = C_q(\underline{\theta},\underline{q}^{VH}) \tag{28}$$

$$\mu_1 = \lambda - \mu_2 > 0 \tag{29}$$

If LC-ineff is not binding, then $\mu_2 = 0$ and $\mu_1 = \lambda$. If LC-ineff is binding, then $\mu_2 > 0$ and $\mu_1 = \lambda - \mu_2 > 0$. In general, LC-ineff may or may not bind (see also Proposition 5 below). The quantity of the inefficient type is distorted upwards, but the quantity of the efficient type can be distorted either upwards or downwards (downwards for sufficiently high altruism). Increasing the quantity of the inefficient provider makes it less attractive the inefficient provider to mimic the efficient provider and reduces the informational

[8] The Lagrangean is: $L = \lambda\left[W(\overline{q}) - C(\overline{\theta},\overline{q}) - \overline{\pi}\right] + (1-\lambda)\left[W(\underline{q}) - C(\underline{\theta},\underline{q})\right]$
$+\mu_1\left\{\overline{\pi} - \left[C(\underline{\theta},\underline{q}) - C(\overline{\theta},\underline{q}) + \alpha\left(W(\underline{q}) - W(\overline{q})\right)\right]\right\} + \mu_2\overline{\pi}$.

rent that arises from the altruistic component (see Eq.22). So far we have neglected the Incentive-Compatibility Constraint of the efficient provider (IC-eff). At the optimal solution outlined above, this constraint is never binding.[9] Finally, if IC-ineff is not binding for $\alpha \leq \overline{\alpha}$ then $\mu_1 = 0$, and we recover the solution with *high* altruism.

Corollaries 4 and 5 summarise the results and establish how quantities, transfers and utilities vary with α when the Limited–Liability Constraint for the inefficient provider (LC-ineff) is binding and is not binding.

Corollary 4 *If LC-ineff is binding, then* $\underline{q}^{VH} > \overline{q}^{VH}$, $\frac{\partial \overline{q}^{VH}}{\partial \alpha} \gtrless 0$, $\frac{\partial \underline{q}^{VH}}{\partial \alpha} \gtrless 0$, $\underline{\pi}^{VH} = \overline{\pi}^{VH} = 0$, $\underline{t}^{VH} \gtrless \overline{t}^{VH}$, $\frac{\partial \overline{t}^{VH}}{\partial \alpha} \gtrless 0$, $\frac{\partial \underline{t}^{VH}}{\partial \alpha} \gtrless 0$, $\underline{U}^{VH} > \overline{U}^{VH}$, $\frac{\partial \overline{U}^{VH}}{\partial \alpha} \gtrless 0$, $\frac{\partial \underline{U}^{VH}}{\partial \alpha} \gtrless 0$.

If LC-ineff is binding, the quantity of the efficient provider is higher than that of the inefficient one. However, the effect of altruism on quantities is indeterminate. Profits are zero for both types. The transfer to the efficient provider can be higher or lower than to the inefficient one: the efficient provider has lower per-unit cost, but is contracted at a higher quantity. The transfer for each provider increases (decreases) with altruism when the contracted quantity increases (decreases) with altruism. The utility of the efficient provider is higher than the inefficient one because of the higher quantity. The utility for each provider may increase or decrease with altruism: altruism increases utility directly, but the contracted quantity may increase or decrease.

Corollary 5 *If LC-ineff is not binding, then* $\underline{q}^{VH} > \overline{q}^{VH}$, $\frac{\partial \overline{q}^{VH}}{\partial \alpha} > 0$, $\frac{\partial \underline{q}^{VH}}{\partial \alpha} < 0$, $\underline{\pi}^{VH} = 0$, $\overline{\pi}^{VH} > 0$, $\frac{\partial \overline{\pi}^{VH}}{\partial \alpha} \gtrless 0$, $\underline{t}^{VH} \gtrless \overline{t}^{VH}$, $\frac{\partial \overline{t}^{VH}}{\partial \alpha} \gtrless 0$, $\frac{\partial \underline{t}^{VH}}{\partial \alpha} < 0$, $\underline{U}^{VH} \gtrless \overline{U}^{VH}$, $\frac{\partial \overline{U}^{VH}}{\partial \alpha} \gtrless 0$, $\frac{\partial \underline{U}^{VH}}{\partial \alpha} \gtrless 0$.

If LC-ineff is not binding, the quantity of the efficient provider is higher than that of the inefficient one. The quantity of the inefficient provider increases with altruism, and the quantity of the efficient provider decreases with altruism. The efficient provider has zero profit. The inefficient provider has positive profit, which might increase or decrease

[9] We need to check whether or not the IC-eff is binding. Recall that IC-eff is: $\underline{\pi} \geq \overline{\pi} + C(\overline{\theta}, \overline{q}) - C(\underline{\theta}, \overline{q}) - \alpha \left(W(\underline{q}) - W(\overline{q})\right)$. If $\overline{\pi} > 0$ then $\overline{\pi} = C(\underline{\theta}, \underline{q}) - C(\overline{\theta}, \underline{q}) + \alpha \left(W(\underline{q}) - W(\overline{q})\right)$, and if $\overline{\pi} = 0$ then $\alpha \left(W(\underline{q}) - W(\overline{q})\right) = C(\overline{\theta}, \underline{q}) - C(\underline{\theta}, \underline{q})$. In any case, after substitution, IC-eff simplifies to: $0 \geq C(\overline{\theta}, \overline{q}) - C(\underline{\theta}, \overline{q}) - \left(C(\overline{\theta}, \underline{q}) - C(\underline{\theta}, \underline{q})\right)$, which is satisfied if $\underline{q}^{VH} > \overline{q}^{VH}$ (this is always the case when IC-ineff in (22) is binding).

with altruism. The transfer to the efficient provider can be higher or lower than to the inefficient one: the efficient provider has lower per-unit cost but is contracted at a higher quantity; the inefficient provider has a higher per-unit cost, has a rent but is contracted at a lower quantity. The transfer to the efficient provider decreases with altruism due to the lower quantity. The transfer to the inefficient provider may increase or reduce with altruism: higher altruism implies higher quantity, but profits may increase or reduce. The utility of the efficient provider can be higher or lower than that of the inefficient one: the inefficient provider has lower quantity, but a positive rent. The utility of the efficient provider may increase or decrease with altruism: altruism increases utility directly but the contracted quantity is lower. Similarly, for the inefficient provider: altruism increases utility directly, quantity is higher but profits may be lower.

The following proposition refines the equilibrium with *very high* altruism.

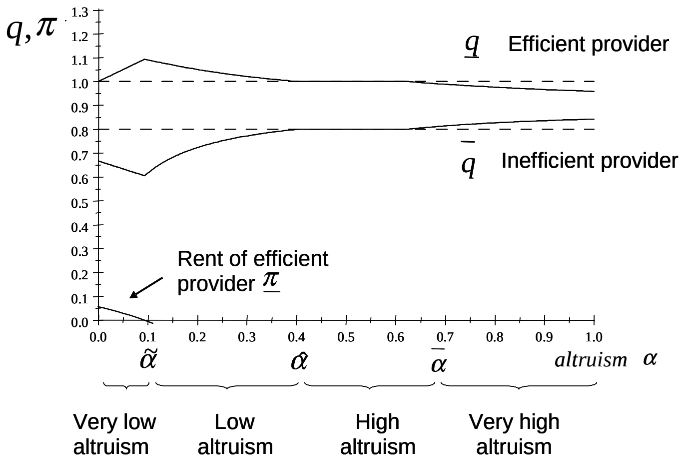
Proposition 5 *At $\alpha = \overline{\alpha}$ the LC-ineff is binding and $\overline{\pi}^{VH} = \underline{\pi}^{VH} = 0$.*

This proposition establishes that, at the lower bound of the equilibrium with *very high* altruism (when $\alpha = \overline{\alpha}$), the Limited-Liability Constraint of the inefficient provider is binding, so that the solution outlined in Corollary 4 holds. However, it is difficult to establish when LC-ineff is binding for higher values of altruism when $\alpha > \overline{\alpha}$. This is because the profit function of the inefficient provider when the LC-ineff is not binding, as outlined in Corollary 5, is not a monotonic function of altruism.

7 A numerical example

Suppose that the marginal benefit is linear and the cost function is quadratic, so that $W = aq$ and $C = \frac{\theta}{2}q^2$, where a is a positive parameter. Figure 1 provides the solution for $\lambda = 0.5$, $\overline{\theta} = 1.25$, $\underline{\theta} = 1$ and $a = 1$, so that $\overline{\theta} - \underline{\theta} = 0.25$.

Figure 1. Optimal level of output for efficient and inefficient provider



We have four possible equilibria (all proofs are in Appendix 2).

1) If altruism is *high* then:

$$
\begin{aligned}
\underline{q}^H &= \frac{a}{\underline{\theta}} = 1,\ \overline{q}^H = \frac{a}{\overline{\theta}} = 0.8 \\
\overline{\pi}^H &= \underline{\pi}^H = 0 \\
0.4 &= \frac{\underline{\theta}}{2\overline{\theta}} = \widehat{\alpha} \leq \alpha^H \leq \overline{\alpha} = \frac{\overline{\theta}}{2\underline{\theta}} = 0.625
\end{aligned}
$$

2) If altruism is *very low* then

$$
\begin{aligned}
\underline{q}^{VL} &= \frac{(1+\alpha)a}{\underline{\theta}} = 1 + \alpha \\
\overline{q}^{VL} &= \frac{a\,(1-\alpha)}{2\overline{\theta} - \underline{\theta}} = \frac{2(1-\alpha)}{3} \\
\overline{\pi}^{VL} &= 0 \\
\underline{\pi}^{VL} &= \left(\overline{\theta} - \underline{\theta}\right)\overline{q}^2/2 - \alpha a\left(\underline{q} - \overline{q}\right) = 0.056 - 0.44\alpha - 1.6\alpha^2 \\
0 &\leq \alpha^{VL} \leq \widetilde{\alpha} = \frac{5\underline{\theta}\overline{\theta} + 2\left(2\overline{\theta} - \underline{\theta}\right)\sqrt{\overline{\theta}\left(\overline{\theta} - \underline{\theta}\right)} - 4\overline{\theta}^2 - \underline{\theta}^2}{8\overline{\theta}^2 + \underline{\theta}^2 - 5\underline{\theta}\overline{\theta}} = 0.0934
\end{aligned}
$$

3) If altruism is *low* then

$$
\overline{q}^L = \left(A - \frac{8\alpha^2 - 2.7\alpha}{A} - 4\alpha\right)
$$

where $A = \sqrt[3]{16\alpha^2 + 80\alpha^3 + \sqrt{6912\alpha^6 + 2048\alpha^5 + 426.7\alpha^4 - 19\alpha^3}}$,

$$\begin{aligned}
\underline{q}^L &= \overline{q}^L + \frac{\left(\overline{\theta} - \underline{\theta}\right)}{\alpha a} \frac{\left(\overline{q}^L\right)^2}{2} = \overline{q}^L + \frac{0.25}{\alpha} \frac{\left(\overline{q}^L\right)^2}{2} \\
\overline{\pi}^L &= \underline{\pi}^L = 0
\end{aligned}$$

4) If altruism is *very high*, then

$$\underline{q}^{VH} = 4\alpha - \frac{3.2\alpha^2 + 2.1\alpha}{B} + B$$

where $B = \sqrt[3]{12.8\alpha^2 - 51.2\alpha^3 + \sqrt{2654.2\alpha^6 - 1245.2\alpha^5 + 207.5\alpha^4 + 9.7\alpha^3}}$

$$\begin{aligned}
\overline{q}^{VH} &= \underline{q}^{VH} - \left(\overline{\theta} - \underline{\theta}\right) \frac{\left(\underline{q}^{VH}\right)^2}{2a\alpha} \\
\overline{\pi}^{VH} &= \underline{\pi}^{VH} = 0
\end{aligned}$$

For zero altruism ($\alpha = 0$), the quantity of the efficient provider is not distorted, but the quantity for the inefficient is distorted downwards. The profit for the efficient provider is positive, whereas it is zero for the inefficient one. With *very low* altruism the quantity of the efficient provider is also distorted, but upwards, while the quantity for the inefficient provider is distorted even more downwards. For $\alpha > 0.09$ under *low* altruism, the profit of the efficient provider is zero, but both quantities are still distorted, upwards and downwards for the efficient and inefficient provider respectively. However, the higher the altruism, the more quantities converge towards the first best values, which can be implemented for $0.4 \leq \alpha \leq 0.625$. For *very high* altruism ($0.625 \leq \alpha \leq 1$) the quantity of the efficient type is distorted downwards, and that for the inefficient type is distorted upwards. The higher the level of altruism, the lower is the difference in the quantities provided by the two types. Profits are zero.

8 Concluding remarks

This study adds to the literature on optimal incentive schemes with altruistic or motivated agents. We have derived the optimal contract between a purchaser (a principal) and a provider (an agent) in the presence of adverse selection on efficiency, altruistic agents and limited liability. Four types of separating equilibriums emerge, depending on the size of the altruism. For *very low* altruism the quantity of the efficient and inefficient types are respectively distorted upwards and downwards; the efficient type makes a positive profit. For *low* altruism the quantity of the efficient and inefficient types are also distorted upwards and downwards respectively, but profits are zero for both types. For *high* altruism the first best is attained: quantities are not distorted and profits are zero. The equilibrium under symmetric and asymmetric information is the same. For *very high* altruism the quantity of the inefficient type is distorted upwards, and the quantity of the efficient type is distorted either upwards or downwards. The inefficient provider may make a positive profit.

One implication of this study is that, if providers are partially altruistic, the incentive scheme does not break down: a pooling equilibrium, where both types of provider receive the same transfer in exchange of the same quantity, never arises. The efficient provider always produces a higher quantity in equilibrium than the inefficient provider. However, despite the presence of asymmetric information on efficiency, the efficient provider does not necessarily receive a higher transfer. For example, for *high* altruism the efficient provider receives a higher transfer only if the differences in the contracted quantities (between the efficient and the inefficient provider) is sufficiently high, and if the difference in efficiency is sufficiently low.

A final implication is that the purchaser is not necessarily better off when the degree of altruism of the providers is higher. For example, under *very high* altruism (as opposed to *high* altruism), the inefficient provider has an incentive to mimic the efficient provider, being attracted by the higher contracted quantity of the efficient provider. In this case, the purchaser needs to distort quantities and eventually leave a rent to the inefficient provider. As a result, the purchaser is overall worse off when providers have *very high* altruism rather than *high* altruism.

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Appendix 1

Proof of Lemma 2. IC-eff is $\underline{U} \geq U\left(\underline{\theta}, \overline{\theta}\right)$ or, more extensively, $\underline{U} \geq \overline{t} - C(\underline{\theta}, \overline{q}) + \alpha W(\overline{q}) = \overline{t} - C(\overline{\theta}, \overline{q}) + \alpha W(\overline{q}) + C(\overline{\theta}, \overline{q}) - C(\underline{\theta}, \overline{q})$, which can be rewritten as $\underline{\pi} + \alpha W(\underline{q}) \geq \overline{\pi} + \alpha W(\overline{q}) + C(\overline{\theta}, \overline{q}) - C(\underline{\theta}, \overline{q})$. IC-ineff is $\overline{U} \geq U\left(\overline{\theta}, \underline{\theta}\right)$ or $\overline{U} \geq \underline{t} - C(\overline{\theta}, \underline{q}) + \alpha W(\underline{q}) = \underline{t} - C(\underline{\theta}, \underline{q}) + \alpha W(\underline{q}) + C(\underline{\theta}, \underline{q}) - C(\overline{\theta}, \underline{q})$, which can in turn be rewritten as $\overline{\pi} + \alpha W(\overline{q}) > \underline{\pi} + \alpha W(\underline{q}) + C(\underline{\theta}, \underline{q}) - C(\overline{\theta}, \underline{q})$. ■

Proof of Proposition 3. We need to determine the values of α for which the Incentive-Compatibility Constraints are satisfied. The IC-eff is: $\underline{\pi} \geq \overline{\pi} + C(\overline{\theta}, \overline{q}^H) - C(\underline{\theta}, \overline{q}^H) - \alpha\left(W(\underline{q}^H) - W(\overline{q}^H)\right)$ or, after substitution, $0 \geq 0 + C(\overline{\theta}, \overline{q}^H) - C(\underline{\theta}, \overline{q}^H) - \alpha\left(W(\underline{q}^H) - W(\overline{q}^H)\right)$, which implies $\alpha^H \geq \widehat{\alpha} = \frac{C(\overline{\theta}, \overline{q}^H) - C(\underline{\theta}, \overline{q}^H)}{W(\underline{q}^H) - W(\overline{q}^H)} > 0$. The IC-ineff is: $\overline{\pi} \geq \underline{\pi} + C(\underline{\theta}, \underline{q}^H) - C(\overline{\theta}, \underline{q}^H) + \alpha W(\underline{q}^H) - \alpha W(\overline{q}^H)$ or, after substitution, $0 \geq C(\underline{\theta}, \underline{q}^H) - C(\overline{\theta}, \underline{q}^H) + \alpha\left(W(\underline{q}^H) - W(\overline{q}^H)\right)$ which implies $\alpha^H \leq \overline{\alpha} = \frac{C(\overline{\theta}, \underline{q}^H) - C(\underline{\theta}, \underline{q}^H)}{W(\underline{q}^H) - W(\overline{q}^H)}$. Notice that $\widehat{\alpha} < \overline{\alpha}$, since $\frac{C(\overline{\theta}, \overline{q}^H) - C(\underline{\theta}, \overline{q}^H)}{W(\underline{q}^H) - W(\overline{q}^H)} < \frac{C(\overline{\theta}, \underline{q}^H) - C(\underline{\theta}, \underline{q}^H)}{W(\underline{q}^H) - W(\overline{q}^H)}$ or $C(\overline{\theta}, \overline{q}^H) - C(\underline{\theta}, \overline{q}^H) < C(\overline{\theta}, \underline{q}^H) - C(\underline{\theta}, \underline{q}^H)$. ■

Proof of Corollary 1. $\overline{t}^H = C(\overline{\theta}, \overline{q}^H)$ and $\underline{t}^H = C(\underline{\theta}, \underline{q}^H)$ where $\overline{\theta} > \underline{\theta}$ and $\underline{q}^H > \overline{q}^H$; $\underline{U}^H = \alpha W(\underline{q}^H) > \overline{U}^H = \alpha W(\overline{q}^H)$ since $\underline{q}^H > \overline{q}^H$; $\partial \underline{U}^H / \partial\alpha = W(\underline{q}^H)$ and $\partial \overline{U}^H / \partial\alpha = W(\overline{q}^H)$. ■

Proof of Corollary 2. Upon differentiating Eq.s (11) and (12), we obtain

$$\frac{\partial \underline{q}^{VL}}{\partial \alpha} = \frac{W_q(\underline{q}^{VL})}{-[(1+\alpha)W_{qq}(\underline{q}^{VL}) - C_{qq}(\underline{\theta}, \underline{q}^{VL})]} > 0 \tag{30}$$

$$\frac{\partial \overline{q}^{VL}}{\partial \alpha} = \frac{-\frac{1-\lambda}{\lambda} W_q(\overline{q}^{VL})}{-\left\{W_{qq}(\overline{q}^{VL})(1 - \alpha\frac{1-\lambda}{\lambda}) - C_{qq}(\overline{\theta}, \overline{q}^{VL}) - \frac{1-\lambda}{\lambda}\left[C_{qq}(\overline{\theta}, \overline{q}^{VL}) - C_{qq}(\underline{\theta}, \overline{q}^{VL})\right]\right\}} < 0 \tag{31}$$

Also, $\underline{\pi}^{VL} = C(\overline{\theta}, \overline{q}^{VL}) - C(\underline{\theta}, \overline{q}^{VL}) - \alpha\left(W(\underline{q}^{VL}) - W(\overline{q}^{VL})\right)$ so that

$$\begin{aligned} \frac{\partial \underline{\pi}^{VL}}{\partial \alpha} &= -\left(W(\underline{q}^{VL}) - W(\overline{q}^{VL})\right) - \alpha W_q(\underline{q}^{VL}) \frac{\partial \underline{q}^{VL}}{\partial \alpha} \\ &\quad + \left[C_q(\overline{\theta}, \overline{q}^{VL}) - C_q(\underline{\theta}, \overline{q}^{VL}) + \alpha W_q(\overline{q}^{VL})\right] \frac{\partial \overline{q}^{VL}}{\partial \alpha} \end{aligned} \tag{32}$$

so that $\frac{\partial \underline{\pi}^{VL}}{\partial \alpha} < 0$. The transfers are

$$\begin{aligned}
\underline{t}^{VL} &= C(\underline{\theta}, \underline{q}^{VL}) + \underline{\pi}^{VL} && (33)\\
&= C(\underline{\theta}, \underline{q}^{VL}) + C(\overline{\theta}, \overline{q}^{VL}) - C(\underline{\theta}, \overline{q}^{VL}) - \alpha \left(W(\underline{q}^{VL}) - W(\overline{q}^{VL}) \right) \\
\overline{t}^{VL} &= C(\overline{\theta}, \overline{q}^{VL}) && (34)
\end{aligned}$$

Now, $\underline{t}^{VL} > \overline{t}^{VL}$ if $C(\underline{\theta}, \underline{q}^{VL}) - C(\underline{\theta}, \overline{q}^{VL}) > \alpha \left(W(\underline{q}^{VL}) - W(\overline{q}^{VL}) \right)$ or if $\underline{\pi}^{VL} > C(\overline{\theta}, \overline{q}^{VL}) - C(\underline{\theta}, \underline{q}^{VL})$. If $\alpha = 0$, this condition is always satisfied. By differentiating w.r.t. α, we obtain

$$\begin{aligned}
\frac{\partial \underline{t}^{VL}}{\partial \alpha} &= C_q(\underline{\theta}, \underline{q}^{VL}) \frac{\partial \underline{q}^{VL}}{\partial \alpha} + \frac{\partial \underline{\pi}^{VL}}{\partial \alpha} && (35)\\
&= -\left(\alpha W_q(\underline{q}^{VL}) - C_q(\underline{\theta}, \underline{q}^{VL}) \right) \frac{\partial \underline{q}^{VL}}{\partial \alpha} - \left(W(\underline{q}^{VL}) - W(\overline{q}^{VL}) \right) \\
&\quad + \left[C_q(\overline{\theta}, \overline{q}^{VL}) - C_q(\underline{\theta}, \overline{q}^{VL}) + \alpha W_q(\overline{q}^{VL}) \right] \frac{\partial \overline{q}^{VL}}{\partial \alpha}
\end{aligned}$$

The first term is positive (from the FOC on $\underline{q}^{VL}$), while the second and the third terms are negative. Overall, the effect is indeterminate in sign.

The utilities are: $\underline{U}^{VL} = \alpha W(\underline{q}^{VL}) + \underline{\pi}^{VL} \geq 0$ and $\overline{U}^{VL} = \alpha W(\overline{q}^{VL})$ with

$$\begin{aligned}
\frac{\partial \underline{U}^{VL}}{\partial \alpha} &= W(\overline{q}^{VL}) + \left[C_q(\overline{\theta}, \overline{q}^{VL}) - C_q(\underline{\theta}, \overline{q}^{VL}) + \alpha W_q(\overline{q}^{VL}) \right] \frac{\partial \overline{q}^{VL}}{\partial \alpha} \gtrless 0 && (36)\\
\frac{\partial \overline{U}^{VL}}{\partial \alpha} &= W(\overline{q}^{VL}) + \alpha W_q(\overline{q}^{VL}) \frac{\partial \overline{q}^{VL}}{\partial \alpha} \gtrless 0. && (37)
\end{aligned}$$

In each of the equations the first term is positive and the second is negative. ■

Proof of Proposition 4. The profit of the efficient provider when $\alpha = \widehat{\alpha}$ is

$$\begin{aligned}
\underline{\pi}(\alpha &= \widehat{\alpha}, \underline{q}^{VL}, \overline{q}^{VL}) = C(\overline{\theta}, \overline{q}^{VL}) - C(\underline{\theta}, \overline{q}^{VL}) - \widehat{\alpha} \left(W(\underline{q}^{VL}) - W(\overline{q}^{VL}) \right) && (38)\\
&= C(\overline{\theta}, \overline{q}^{VL}) - C(\underline{\theta}, \overline{q}^{VL}) - \frac{C(\overline{\theta}, \overline{q}^{H}) - C(\underline{\theta}, \overline{q}^{H})}{W(\underline{q}^{H}) - W(\overline{q}^{H})} \left(W(\underline{q}^{VL}) - W(\overline{q}^{VL}) \right) \\
&= C(\overline{\theta}, \overline{q}^{H}) - C(\underline{\theta}, \overline{q}^{H}) \left(\frac{C(\overline{\theta}, \overline{q}^{VL}) - C(\underline{\theta}, \overline{q}^{VL})}{C(\overline{\theta}, \overline{q}^{H}) - C(\underline{\theta}, \overline{q}^{H})} - \frac{W(\underline{q}^{VL}) - W(\overline{q}^{VL})}{W(\underline{q}^{H}) - W(\overline{q}^{H})} \right).
\end{aligned}$$

Notice that: (a) since $\overline{q}^{VL} < \overline{q}^{H}$ then $\frac{C(\overline{\theta}, \overline{q}^{VL}) - C(\underline{\theta}, \overline{q}^{VL})}{C(\overline{\theta}, \overline{q}^{H}) - C(\underline{\theta}, \overline{q}^{H})} < 1$; (b) since $\overline{q}^{VL} < \overline{q}^{H}$ and since $\underline{q}^{VL} > \underline{q}^{H}$ then $\frac{W(\underline{q}^{VL}) - W(\overline{q}^{VL})}{W(\underline{q}^{H}) - W(\overline{q}^{H})} > 1$. It follows that $\underline{\pi}(\alpha = \widehat{\alpha}, \underline{q}^{VL}, \overline{q}^{VL}) < 0$. Since

$\underline{\pi}(\alpha = 0, \underline{q}^{VL}, \overline{q}^{VL}) > 0$, $\underline{\pi}(\alpha = \widehat{\alpha}, \underline{q}^{VL}, \overline{q}^{VL}) < 0$ and $\frac{\partial \underline{\pi}(\alpha, \underline{q}^{VL}, \overline{q}^{VL})}{\partial \alpha} < 0$, then $\exists\ \widetilde{\alpha}$, with $0 < \widetilde{\alpha} < \widehat{\alpha}$, such that $\underline{\pi}(\alpha = \widetilde{\alpha}, \underline{q}^{VL}, \overline{q}^{VL}) = 0$. What is the value of $\widetilde{\alpha}$? It is the value such that: $\underline{\pi}^{VL}(\widetilde{\alpha}) = C(\overline{\theta}, \overline{q}^{VL}(\widetilde{\alpha})) - C(\underline{\theta}, \overline{q}^{VL}(\widetilde{\alpha})) - \alpha \left(W(\underline{q}^{VL}(\widetilde{\alpha})) - W(\overline{q}(\widetilde{\alpha})) \right) = 0$. ■

Proof of Corollary 3. If $\underline{\pi}^L = 0$, then $\partial L / \partial \mu_1 = 0$, $\partial L / \partial \underline{q}^L = 0$ and $\partial L / \partial \overline{q}^L = 0$ are respectively (the subscript $(.)^L$ is omitted in the following)

$$\begin{aligned}
-\left[C(\overline{\theta}, \overline{q}) - C(\underline{\theta}, \overline{q}) - \alpha \left(W(\underline{q}) - W(\overline{q}) \right) \right] &= 0 \\
(1 - \lambda) \left[W_q(\underline{q}) - C_q(\underline{\theta}, \underline{q}) \right] + \mu_1 \alpha W_q(\underline{q}) &= 0 \\
\lambda \left[W_q(\overline{q}) - C_q(\overline{\theta}, \overline{q}) \right] - \mu_1 \left[C_q(\overline{\theta}, \overline{q}) - C_q(\underline{\theta}, \overline{q}) + \alpha W_q(\overline{q}) \right] &= 0
\end{aligned}$$

The determinant of the corresponding matrix is

$$\begin{aligned}
\Delta = & -\left(\alpha W_q(\underline{q}) \right)^2 \left\{ \lambda \left[W_{qq}(\overline{q}) - C_{qq}(\overline{\theta}, \overline{q}) \right] - \mu_1 \left[C_{qq}(\overline{\theta}, \overline{q}) - C_{qq}(\underline{\theta}, \overline{q}) + \alpha W_{qq}(\overline{q}) \right] \right\} \\
& - \left[C_q(\overline{\theta}, \overline{q}) - C_q(\underline{\theta}, \overline{q}) + \alpha W_q(\overline{q}) \right]^2 \left\{ (1 - \lambda) \left[W_{qq}(\underline{q}) - C_{qq}(\underline{\theta}, \underline{q}) \right] + \mu_1 \alpha W_{qq}(\underline{q}) \right\}
\end{aligned} \tag{39}$$

From Cramer's rule, we obtain

$$\begin{aligned}
\frac{\partial \underline{q}^L}{\partial \alpha} = & \ (1/\Delta) \left(W(\underline{q}) - W(\overline{q}) \right) \\
& \times \left\{ \alpha W_q(\underline{q}) \left[\lambda (W_{qq}(\overline{q}) - C_{qq}(\overline{\theta}, \overline{q})) - \mu_1 (C_{qq}(\overline{\theta}, \overline{q}) - C_{qq}(\underline{\theta}, \overline{q}) + \alpha W_{qq}(\overline{q})) \right] \right\} \\
& + (1/\Delta) \mu_1 W_q(\underline{q}) \left[C_q(\overline{\theta}, \overline{q}) - C_q(\underline{\theta}, \overline{q}) + \alpha W_q(\overline{q}) \right] \left[C_q(\overline{\theta}, \overline{q}) - C_q(\underline{\theta}, \overline{q}) \right] \gtreqless 0.
\end{aligned} \tag{40}$$

The first term is negative, and the second is positive. The effect is in general indeterminate in sign. However, for $\mu_1 \to 0$ then $\frac{\partial \underline{q}^L}{\partial \alpha} < 0$.

$$\begin{aligned}
\frac{\partial \overline{q}^L}{\partial \alpha} = & \ (1/\Delta) \alpha W_q(\underline{q}) \mu_1 W(\underline{q}) \left[C_q(\overline{\theta}, \overline{q}) - C_q(\underline{\theta}, \overline{q}) \right] \\
& - (1/\Delta) \left(W(\underline{q}) - W(\overline{q}) \right) \left\{ (1 - \lambda) \left[W_{qq}(\underline{q}) - C_{qq}(\underline{\theta}, \underline{q}) \right] + \mu_1 \alpha W_{qq}(\underline{q}) \right\} \\
& \times \left[C_q(\overline{\theta}, \overline{q}) - C_q(\underline{\theta}, \overline{q}) + \alpha W_q(\overline{q}) \right] > 0
\end{aligned} \tag{41}$$

which is positive.

$$\begin{aligned}\frac{\partial \mu_1}{\partial \alpha} &= -(1/\Delta)\left(W(\underline{q})-W(\overline{q})\right)\left\{(1-\lambda)\left[W_{qq}(\underline{q})-C_{qq}(\underline{\theta},\underline{q})\right]+\mu_1\alpha W_{qq}(\underline{q})\right\} \\ &\quad\times\left\{\lambda\left[W_{qq}(\overline{q})-C_{qq}(\overline{\theta},\overline{q})\right]-\mu_1\left[C_{qq}(\overline{\theta},\overline{q})-C_{qq}(\underline{\theta},\overline{q})+\alpha W_{qq}(\overline{q})\right]\right\} \\ &\quad+(1/\Delta)\alpha\mu_1 W_q^2(\underline{q})\left\{\lambda\left[W_{qq}(\overline{q})-C_{qq}(\overline{\theta},\overline{q})\right]-\mu_1\left[C_{qq}(\overline{\theta},\overline{q})-C_{qq}(\underline{\theta},\overline{q})+\alpha W_{qq}(\overline{q})\right]\right\} \\ &\quad+(1/\Delta)\mu_1 W_q(\overline{q})\left[C_q(\overline{\theta},\overline{q})-C_q(\underline{\theta},\overline{q})+\alpha W_q(\overline{q})\right] \\ &\quad\times\left\{(1-\lambda)\left[W_{qq}(\underline{q})-C_{qq}(\underline{\theta},\underline{q})\right]+\mu_1\alpha W_{qq}(\underline{q})\right\}<0\end{aligned} \tag{42}$$

which is negative. $\underline{t}^L = C(\underline{\theta},\underline{q}^L) \gtrless \overline{t}^L = C(\overline{\theta},\overline{q}^L)$ since $\underline{q}^L > \overline{q}^L$. $\partial t^L(\theta)/\partial\alpha = C_q(\theta,q^L(\theta))\partial q^L(\theta)/\partial\alpha$. $\underline{U}^L = \alpha W(\underline{q}^L) > \overline{U}^L = \alpha W(\overline{q}^L)$ since $\underline{q}^L > \overline{q}^L$. $\partial U^L(\theta)/\partial\alpha = \alpha W_q(q^L(\theta))\partial q^L(\theta)/\partial\alpha$. ■

Proof of Corollary 4. If $\overline{\pi}^{VH}=0$ then $\partial L/\partial\mu_1 = 0$, $\partial L/\partial\underline{q}^{VH}=0$ and $\partial L/\partial\overline{q}^{VH}=0$ are respectively (the subscript $(.)^{VH}$ is omitted in the following):

$$\begin{aligned}-\left[C(\underline{\theta},\underline{q})-C(\overline{\theta},\underline{q})+\alpha\left(W(\underline{q})-W(\overline{q})\right)\right] &= 0 \\ (1-\lambda)\left[W_q(\underline{q})-C_q(\underline{\theta},\underline{q})\right]+\mu_1\left[C_q(\overline{\theta},\underline{q})-C_q(\underline{\theta},\underline{q})-\alpha W_q(\underline{q})\right] &= 0 \\ \lambda\left[W_q(\overline{q})-C_q(\overline{\theta},\overline{q})\right]+\mu_1\alpha W_q(\overline{q}) &= 0\end{aligned}$$

The determinant of the corresponding matrix is:

$$\begin{aligned}\Delta &= -\left[C_q(\overline{\theta},\underline{q})-C_q(\underline{\theta},\underline{q})-\alpha W_q(\underline{q})\right]^2\left\{\lambda\left[W_{qq}(\overline{q})-C_{qq}(\overline{\theta},\overline{q})\right]+\mu_1\alpha W_{qq}(\overline{q})\right\} \\ &\quad-\left(\alpha W_q(\overline{q})\right)^2\left\{(1-\lambda)\left[W_{qq}(\underline{q})-C_{qq}(\underline{\theta},\underline{q})\right]+\mu_1\left[C_{qq}(\overline{\theta},\underline{q})-C_{qq}(\underline{\theta},\underline{q})-\alpha W_{qq}(\underline{q})\right]\right\}\end{aligned} \tag{43}$$

Using Cramer's rule we obtain:

$$\begin{aligned}\frac{\partial \underline{q}^{VH}}{\partial\alpha} &= (1/\Delta)\left(W(\underline{q})-W(\overline{q})\right) \\ &\quad\times\left\{\left[\lambda\left[W_{qq}(\overline{q})-C_{qq}(\overline{\theta},\overline{q})\right]+\mu_1\alpha W_{qq}(\overline{q})\right]\right\}\left[C_q(\underline{\theta},\underline{q})-C_q(\overline{\theta},\underline{q})+\alpha W_q(\underline{q})\right] \\ &\quad+(1/\Delta)\alpha W(\overline{q})\left\{\mu_1 W_q(\underline{q})\left[C_q(\underline{\theta},\underline{q})-C_q(\overline{\theta},\underline{q})+\alpha W_q(\underline{q})\right]-\alpha\mu_1 W_q(\underline{q})W_q(\overline{q})\right\}\end{aligned} \tag{44}$$

which is indeterminate in sign. If $C_q(\underline{\theta},\underline{q})-C_q(\overline{\theta},\underline{q})+\alpha W_q(\underline{q})>0$, then the first term is negative while the second is indeterminate in sign.

$$\frac{\partial\overline{q}^{VH}}{\partial\alpha} = (1/\Delta)\left[C_q(\underline{\theta},\underline{q}) - C_q(\overline{\theta},\underline{q}) + \alpha W_q(\underline{q})\right]^2 \mu_1 W_q(\overline{q}) \tag{45}$$
$$-(1/\Delta)\left\{\left[C_q(\underline{\theta},\underline{q}) - C_q(\overline{\theta},\underline{q}) + \alpha W_q(\underline{q})\right]\alpha\mu_1 W_q(\underline{q})W_q(\overline{q})\right\}$$
$$-(1/\Delta)\left(W(\underline{q}) - W(\overline{q})\right)\alpha W_q(\overline{q})\left\{\begin{array}{c}(1-\lambda)\left[W_{qq}(\underline{q}) - C_{qq}(\underline{\theta},\underline{q})\right]\\ +\mu_1\left[C_{qq}(\overline{\theta},\underline{q}) - C_{qq}(\underline{\theta},\underline{q}) - \alpha W_{qq}(\underline{q})\right]\end{array}\right\}$$

The first and third term are positive, while the second is indeterminate in sign.

$$\frac{\partial\mu_1}{\partial\alpha} = (1/\Delta)\left(W(\underline{q}) - W(\overline{q})\right)\left\{\begin{array}{c}(1-\lambda)\left[W_{qq}(\underline{q}) - C_{qq}(\underline{\theta},\underline{q})\right]\\ +\mu_1\left[C_{qq}(\overline{\theta},\underline{q}) - C_{qq}(\underline{\theta},\underline{q}) - \alpha W_{qq}(\underline{q})\right]\end{array}\right\}$$
$$\times\left\{\lambda\left[W_{qq}(\overline{q}) - C_{qq}(\overline{\theta},\overline{q})\right] + \mu_1\alpha W_{qq}(\overline{q})\right\}$$
$$+(1/\Delta)\left[C_q(\underline{\theta},\underline{q}) - C_q(\overline{\theta},\underline{q}) + \alpha W_q(\underline{q})\right]\mu_1 W_q(\underline{q})$$
$$\times\left\{\lambda\left[W_{qq}(\overline{q}) - C_{qq}(\overline{\theta},\overline{q})\right] + \mu_1\alpha W_{qq}(\overline{q})\right\}$$
$$+(1/\Delta)\alpha\mu_1 W_q^2(\overline{q})\left\{(1-\lambda)\left[W_{qq}(\underline{q}) - C_{qq}(\underline{\theta},\underline{q})\right] + \mu_1\left[C_{qq}(\overline{\theta},\underline{q}) - C_{qq}(\underline{\theta},\underline{q}) - \alpha W_{qq}(\underline{q})\right]\right\}$$

The first term is positive, the second is indeterminate in sign, and the third is negative. $\underline{t}^{VH} = C(\underline{\theta},\underline{q}^{VH}) \gtreqless \overline{t}^{VH} = C(\overline{\theta},\overline{q}^{VH})$ since $\underline{q}^{VH} > \overline{q}^{VH}$. $\partial t^{VH}(\theta)/\partial\alpha = C_q(\theta, q^{VH}(\theta))\partial q^{VH}(\theta)/\partial\alpha$. $\underline{U}^{VH} = \alpha W(\underline{q}^L) > \overline{U}^{VH} = \alpha W(\overline{q}^{VH})$ since $\underline{q}^{VH} > \overline{q}^{VH}$. $\partial U^{VH}(\theta)/\partial\alpha = \alpha W_q(q^{VH}(\theta))\partial q^{VH}(\theta)/\partial\alpha$.

■

Proof of Corollary 5. By setting $\mu_2 = 0$, $\mu_1 = \lambda$ and differentiating Eq.s (27) and (28) w.r.t. α, we obtain

$$\frac{\partial\overline{q}^{VH}}{\partial\alpha} = \frac{W_q(\overline{q}^{VH})}{-[(1+\alpha)W_{qq}(\overline{q}^{VH}) - C_{qq}(\overline{\theta},\overline{q}^{VH})]} > 0 \tag{46}$$

$$\frac{\partial\underline{q}^{VH}}{\partial\alpha} = \frac{-\frac{\lambda}{1-\lambda}W_q(\underline{q}^{VH})}{-\left\{W_{qq}(\underline{q}^{VH})(1-\alpha\frac{\lambda}{1-\lambda}) - C_{qq}(\underline{\theta},\underline{q}^{VH}) + \frac{\lambda}{1-\lambda}\left[C_{qq}(\overline{\theta},\underline{q}^{VH}) - C_{qq}(\underline{\theta},\underline{q}^{VH})\right]\right\}} < 0 \tag{47}$$

and

$$\frac{\partial^2\overline{q}^{VH}}{\partial\alpha^2} = \frac{W_q(\overline{q}^{VH})W_{qq}(\overline{q}^{VH})}{[-(1+\alpha)W_{qq}(\overline{q}^{VH}) + C_{qq}(\overline{\theta},\overline{q}^{VH})]^2} < 0 \tag{48}$$

$$\frac{\partial^2 \underline{q}^{VH}}{\partial \alpha^2} = \frac{\left(\frac{\lambda}{1-\lambda}\right)^2 W_q(\underline{q}^{VH}) W_{qq}(\underline{q}^{VH})}{\left\{-W_{qq}(\underline{q}^{VH})(1-\alpha\frac{\lambda}{1-\lambda}) + C_{qq}(\underline{\theta},\underline{q}^{VH}) - \frac{\lambda}{1-\lambda}\left[C_{qq}(\overline{\theta},\underline{q}^{VH}) - C_{qq}(\underline{\theta},\underline{q}^{VH})\right]\right\}^2} > 0 \tag{49}$$

The profit is: $\overline{\pi}^{VH} = -\left(C(\overline{\theta},\underline{q}^{VH}) - C(\underline{\theta},\underline{q}^{VH})\right) + \alpha\left(W(\underline{q}^{VH}) - W(\overline{q}^{VH})\right)$ with

$$\begin{aligned}\frac{\partial \overline{\pi}^{VH}}{\partial \alpha} &= \left(W(\underline{q}^{VH}) - W(\overline{q}^{VH})\right) - \left(C_q(\overline{\theta},\underline{q}^{VH}) - C_q(\underline{\theta},\underline{q}^{VH})\right)\frac{\partial \underline{q}^{VH}}{\partial \alpha} \\ &+\alpha\left(W_q(\underline{q}^{VH})\frac{\partial \underline{q}^{VH}}{\partial \alpha} - W_q(\overline{q}^{VH})\frac{\partial \overline{q}^{VH}}{\partial \alpha}\right).\end{aligned} \tag{50}$$

The first two terms are positive and the third is negative. Consequently, $\overline{\pi}^{VH}$ is not monotonic in α. Notice that $\overline{\pi}^{VH}$ is also not convex or concave, since

$$\begin{aligned}\frac{\partial^2 \overline{\pi}^{VH}}{\partial \alpha^2} &= 2\left(W_q(\underline{q}^{VH})\frac{\partial \underline{q}^{VH}}{\partial \alpha} - W_q(\overline{q}^{VH})\frac{\partial \overline{q}^{VH}}{\partial \alpha}\right) \\ &- \left(C_{qq}(\overline{\theta},\underline{q}^{VH}) - C_{qq}(\underline{\theta},\underline{q}^{VH})\right)\left(\frac{\partial \underline{q}^{VH}}{\partial \alpha}\right)^2 \\ &+\alpha\left(\begin{array}{c} W_{qq}(\underline{q}^{VH})\left(\frac{\partial \underline{q}^{VH}}{\partial \alpha}\right)^2 - W_{qq}(\overline{q}^{VH})\left(\frac{\partial \overline{q}^{VH}}{\partial \alpha}\right)^2 \\ +W_q(\underline{q}^{VH})\frac{\partial^2 \underline{q}^{VH}}{\partial \alpha^2} - W_q(\overline{q}^{VH})\frac{\partial^2 \overline{q}^{VH}}{\partial \alpha^2}\end{array}\right)\end{aligned} \tag{51}$$

The first two terms are always negative. The first part of the third term is indeterminate in sign, while the second part is positive. Therefore $\frac{\partial^2 \overline{\pi}^{VH}}{\partial \alpha^2} \gtreqless 0$.

$\underline{t}^{VH} = C(\underline{\theta},\underline{q}^{VH}) \gtreqless \overline{t}^{VL} = \overline{\pi}^{VH} + C(\overline{\theta},\overline{q}^{VH})$ since $\underline{q}^{VH} > \overline{q}^{VH}$ and $\overline{\pi}^{VH} > 0$. $\frac{\partial \underline{t}^{VH}}{\partial \alpha} = C_q(\underline{\theta},\underline{q}^{VH})\frac{\partial \underline{q}^{VH}}{\partial \alpha} < 0$. $\frac{\partial \overline{t}^{VH}}{\partial \alpha} = C_q(\overline{\theta},\overline{q}^{VH})\frac{\partial \overline{q}^{VH}}{\partial \alpha} + \frac{\partial \overline{\pi}^{VH}}{\partial \alpha} \gtreqless 0$: the first term is positive but the second is indeterminate. $\underline{U}^{VH} = \alpha W(\underline{q}^{VH}) \gtreqless \overline{U}^{VH} = \alpha W(\overline{q}^{VH}) + \overline{\pi}^{VH}$ since $\underline{q}^{VH} > \overline{q}^{VH}$ but $\overline{\pi}^{VH} > 0$. $\frac{\partial \underline{U}^{VH}}{\partial \alpha} = W(\underline{q}^{VH}) + \alpha\frac{\partial \underline{q}^{VH}}{\partial \alpha} \gtreqless 0$ since the first term is positive and the second is negative.

$$\begin{aligned}\frac{\partial \overline{U}^{VH}}{\partial \alpha} &= W(\overline{q}^{VH}) + \alpha\frac{\partial \overline{q}^{VH}}{\partial \alpha} + \frac{\partial \overline{\pi}^{VH}}{\partial \alpha} \\ &= W(\underline{q}^{VH}) - \left(C_q(\overline{\theta},\underline{q}^{VH}) - C_q(\underline{\theta},\underline{q}^{VH})\right)\frac{\partial \underline{q}^{VH}}{\partial \alpha} + \alpha W_q(\underline{q}^{VH})\frac{\partial \underline{q}^{VH}}{\partial \alpha} \gtreqless 0.\end{aligned} \tag{52}$$

The first two terms are positive and the third is negative. ■

Proof of Proposition 5. The proof is by contradiction. If LC-ineff is not binding, then $\mu_2 = 0$, $\mu_1 = \lambda$ and the FOCs are

$$(1+\alpha)W_q(\overline{q}^{VH}) = C_q(\overline{\theta}, \overline{q}^{VH}) \tag{53}$$

$$W_q(\underline{q}^{VH}) + \frac{\lambda}{1-\lambda}\left(C_q(\overline{\theta}, \underline{q}^{VH}) - C_q(\underline{\theta}, \underline{q}^{VH})\right) = C_q(\underline{\theta}, \underline{q}^{VH}) + \frac{\lambda}{1-\lambda}\alpha W_q(\underline{q}^{VH}) \tag{54}$$

so that the rent $\overline{\pi}^{VH}$ is minimised. Since $\overline{\pi}^{H}(\overline{\alpha}) = -\left(C(\overline{\theta}, \underline{q}^{H}) - C(\underline{\theta}, \underline{q}^{H})\right) + \overline{\alpha}\left(W(\underline{q}^{H}) - W(\overline{q}^{H})\right) = 0$, then $\overline{\pi}^{VH}(\overline{\alpha}) = -\left(C(\overline{\theta}, \underline{q}^{VH}) - C(\underline{\theta}, \underline{q}^{VH})\right) + \overline{\alpha}\left(W(\underline{q}^{VH}) - W(\overline{q}^{VH})\right) < 0$. Therefore LC-ineff is binding at $\alpha = \overline{\alpha}$. ■

Appendix 2

High altruism. Define $\Delta\theta = \overline{\theta} - \underline{\theta}$. Then,

$$\overline{q}^H = \frac{a}{\overline{\theta}}, \underline{q}^H = \frac{a}{\underline{\theta}} \tag{55}$$

$$\frac{\underline{\theta}}{2\overline{\theta}} = \frac{\Delta\theta\overline{q}^H}{a\left(\underline{q}^H - \overline{q}^H\right)} = \widehat{\alpha} \leq \alpha^H \leq \overline{\alpha} = \frac{\Delta\theta\underline{q}^H}{a\left(\underline{q}^H - \overline{q}^H\right)} = \frac{\overline{\theta}}{2\underline{\theta}}. \tag{56}$$

Very low altruism. From the two FOCs: a) $(1+\alpha)a = \underline{\theta}\underline{q}$ so that $\underline{q}^{VL} = \frac{(1+\alpha)a}{\underline{\theta}}$. b) $a = \overline{\theta}\overline{q} + \frac{1-\lambda}{\lambda}\left(\alpha a + \overline{\theta}\overline{q} - \underline{\theta}\overline{q}\right)$ so that $\overline{q}^{VL} = \frac{a\left(1-\alpha\frac{1-\lambda}{\lambda}\right)}{\overline{\theta}+\frac{1-\lambda}{\lambda}\left(\overline{\theta}-\underline{\theta}\right)}$; notice that $\overline{q}^{VL} = \frac{a(1-\alpha)}{2\overline{\theta}-\underline{\theta}}$ if $\lambda = 0.5$. c) We need to check that LC-eff is satisfied: $\underline{\pi}^{VL} = \left(\overline{\theta} - \underline{\theta}\right)\frac{(\overline{q}^{VL})^2}{2} - \alpha a\left(\underline{q}^{VL} - \overline{q}^{VL}\right) \geq 0$. Assume that $\lambda = 0.5$. Notice that $\underline{q}^{VL} - \overline{q}^{VL} = \frac{a(1+\alpha)}{\underline{\theta}} - \frac{a(1-\alpha)}{2\overline{\theta}-\underline{\theta}} = 2a\frac{\overline{\theta}(1+\alpha)-\underline{\theta}}{\underline{\theta}\left(2\overline{\theta}-\underline{\theta}\right)}$. Therefore, $\underline{\pi}^{VL} = \frac{\overline{\theta}-\underline{\theta}}{2}\left(\frac{a(1-\alpha)}{2\overline{\theta}-\underline{\theta}}\right)^2 - 2\alpha a^2\frac{\overline{\theta}(1+\alpha)-\underline{\theta}}{\underline{\theta}\left(2\overline{\theta}-\underline{\theta}\right)}) \geq 0$ which is non-negative when $-\frac{(\overline{\theta}-\underline{\theta})}{2}\frac{2(1-\alpha)}{2\overline{\theta}-\underline{\theta}} - 2\alpha\frac{\overline{\theta}(1+\alpha)-\underline{\theta}}{\underline{\theta}} \geq 0$ or $\alpha^2\left(5\underline{\theta}\overline{\theta} - \underline{\theta}^2 - 8\overline{\theta}^2\right) + \alpha\left(10\underline{\theta}\overline{\theta} - 2\underline{\theta}^2 - 8\overline{\theta}^2\right) + \underline{\theta}\overline{\theta} - \underline{\theta}^2 \geq 0$. The solution of the quadratic polynomial is $\alpha = -\frac{-5\underline{\theta}\overline{\theta}+4\overline{\theta}^2-2\underline{\theta}\sqrt{\overline{\theta}^2-\underline{\theta}\overline{\theta}}\pm4\overline{\theta}\sqrt{\overline{\theta}^2-\underline{\theta}\overline{\theta}+\underline{\theta}^2}}{-5\underline{\theta}\overline{\theta}+8\overline{\theta}^2+\underline{\theta}^2}$ so that $\underline{\pi}^{VL} \geq 0$ when $0 \leq \alpha^{VL} \leq \widetilde{\alpha} = \frac{5\underline{\theta}\overline{\theta}+2\left(2\overline{\theta}-\underline{\theta}\right)\sqrt{\overline{\theta}\Delta\theta}-4\overline{\theta}^2-\underline{\theta}^2}{8\overline{\theta}^2+\underline{\theta}^2-5\underline{\theta}\overline{\theta}}$.

Low altruism. If the level of altruism is low, then the problem is to $\max_{\overline{q},\underline{q}}\lambda\left(a\overline{q} - \frac{\overline{\theta}}{2}\overline{q}^2\right) + (1-\lambda)\left(a\underline{q} - \frac{\underline{\theta}}{2}\underline{q}^2\right)$ subject to $\underline{q} = \overline{q} + \frac{(\overline{\theta}-\underline{\theta})}{\alpha a}\frac{\overline{q}^2}{2}$. More extensively

$$\max_{\overline{q}} \lambda\left(a\overline{q} - \frac{\overline{\theta}}{2}\overline{q}^2\right) + (1-\lambda)\left[a\left(\overline{q} + \frac{\left(\overline{\theta}-\underline{\theta}\right)}{\alpha a}\frac{\overline{q}^2}{2}\right) - \frac{\underline{\theta}}{2}\left(\overline{q} + \frac{\left(\overline{\theta}-\underline{\theta}\right)}{\alpha a}\frac{\overline{q}^2}{2}\right)^2\right] \tag{57}$$

The FOC for $\underline{q}^L$ is:

$$\lambda\left(a - \overline{\theta}\overline{q}^L\right) + (1-\lambda)\left[a + \frac{\left(\overline{\theta}-\underline{\theta}\right)}{\alpha}\overline{q}^L - \underline{\theta}\left(\overline{q}^L + \frac{\left(\overline{\theta}-\underline{\theta}\right)}{\alpha a}\frac{\left(\overline{q}^L\right)^2}{2}\right)\left(1 + \frac{\left(\overline{\theta}-\underline{\theta}\right)}{\alpha a}\overline{q}^L\right)\right] = 0 \tag{58}$$

The above can be cumbersome, so we solve numerically. Let $\lambda = 0.5$, $\overline{\theta} = 1.25$, $\underline{\theta} = 1$ and $a = 1$, so that $\overline{\theta} - \underline{\theta} = 0.25$. The FOC is $-\frac{\underline{\theta}\left(\overline{\theta}-\underline{\theta}\right)^2}{2\alpha^2 a^2}q^3 - \frac{3\underline{\theta}}{2}\frac{\overline{\theta}-\underline{\theta}}{\alpha a}q^2 + \left(\frac{\overline{\theta}-\underline{\theta}}{\alpha} - \overline{\theta} - \underline{\theta}\right)q + 2a = 0$ or $-0.03125\frac{q^3}{\alpha^2} - 0.375\frac{q^2}{\alpha} + \left(0.25\frac{1}{\alpha} - 2.25\right)q + 2 = 0$, whose solution is:

$$\overline{q}^L = \left(A - \frac{8\alpha^2 - 2.67\alpha}{A} - 4\alpha\right) \tag{59}$$

where

$$A = \sqrt[3]{16\alpha^2 + 80\alpha^3 + \sqrt{6912\alpha^6 + 2048\alpha^5 + 426.67\alpha^4 - 18.963\alpha^3}} \tag{60}$$

$$\underline{q}^L = \overline{q}^L + \frac{\left(\overline{\theta} - \underline{\theta}\right)}{\alpha a} \frac{\left(\overline{q}^L\right)^2}{2}. \tag{61}$$

Very high altruism. We assume that $\overline{\alpha} = \frac{\overline{\theta}}{2\underline{\theta}} < 1$. From the FOCs we obtain:

a) $(1+\alpha)a = \overline{\theta}\overline{q}$ from which $\overline{q}^{VH} = \frac{(1+\alpha)a}{\overline{\theta}}$.

b) $a = \underline{\theta}\underline{q} + \frac{\lambda}{1-\lambda}\left[-\left(\overline{\theta} - \underline{\theta}\right)\underline{q} + \alpha a\right]$ from which $\underline{q}^{VH} = a\frac{1-\lambda(1+\alpha)}{\underline{\theta}-\lambda\overline{\theta}}$; if $\lambda = 0.5$ then $\underline{q}^{VH} = \frac{a(1-\alpha)}{2\underline{\theta}-\overline{\theta}}$. Notice that we require $2\underline{\theta} - \overline{\theta} > 0$ or $2\underline{\theta} > \overline{\theta}$ (or $\underline{\theta} > \left(\overline{\theta} - \underline{\theta}\right)$) so that $\underline{q}^{VH} > 0$.

c) We first show that the LC-ineff type is always binding with strict equality:

$$\begin{aligned}
\overline{\pi}^{VH} &= -\left(\frac{\overline{\theta}}{2}(\underline{q}^{VH})^2 - \frac{\underline{\theta}}{2}(\underline{q}^{VH})^2\right) + \alpha\left(a(\underline{q}^{VH}) - a(\overline{q}^{VH})\right) \\
&= -\frac{\overline{\theta} - \underline{\theta}}{2}\left(\frac{a(1-\alpha)}{2\underline{\theta} - \overline{\theta}}\right)^2 + \alpha\left(\frac{a^2(1-\alpha)}{2\underline{\theta} - \overline{\theta}} - \frac{a^2(1+\alpha)}{\overline{\theta}}\right) \\
&= a^2\left(-\frac{\overline{\theta} - \underline{\theta}}{2}\frac{(1-\alpha)^2}{\left(2\underline{\theta} - \overline{\theta}\right)^2} + \frac{2\alpha\left(\overline{\theta} - \underline{\theta} - \underline{\theta}\alpha\right)}{\left(2\underline{\theta} - \overline{\theta}\right)\overline{\theta}}\right) \\
&= \frac{a^2}{\left(2\underline{\theta} - \overline{\theta}\right)}\left(-\frac{\left(\overline{\theta} - \underline{\theta}\right)(1-\alpha)^2}{2\left(2\underline{\theta} - \overline{\theta}\right)} - \frac{2\alpha\left(\underline{\theta} + \underline{\theta}\alpha - \overline{\theta}\right)}{\overline{\theta}}\right)
\end{aligned} \tag{62}$$

so that $\overline{\pi}^{VH} < 0$ for any $\alpha > \frac{\overline{\theta}-\underline{\theta}}{\underline{\theta}}$. But since $\frac{\overline{\theta}-\underline{\theta}}{\underline{\theta}} < \frac{\overline{\theta}}{2\underline{\theta}} = \overline{\alpha}$ when $2\underline{\theta} - \overline{\theta} > 0$, then $\overline{\pi}^{VH} < 0$ for $\overline{\alpha} < \alpha \leq 1$. Also notice that if $\alpha = 1$, then $\overline{\pi}^{VH} = \frac{a^2}{(2\underline{\theta}-\overline{\theta})}\left(-\frac{2\left(2\underline{\theta}-\overline{\theta}\right)}{\overline{\theta}}\right) < 0$. In summary, the LC-ineff is always binding.

d) Since LC-ineff is binding, the problem becomes:

$$\max_{\overline{q},\underline{q}} \lambda\left(a\overline{q} - \frac{\overline{\theta}}{2}\overline{q}^2\right) + (1-\lambda)\left(a\underline{q} - \frac{\underline{\theta}}{2}\underline{q}^2\right) \tag{63}$$

subject to:

$$\overline{q} = \underline{q} - \left(\overline{\theta} - \underline{\theta}\right)\frac{\underline{q}^2}{2a\alpha}. \tag{64}$$

More extensively:

$$\max_{\underline{q}} \lambda\left[a\left(\underline{q} - \left(\overline{\theta} - \underline{\theta}\right)\frac{\underline{q}^2}{2a\alpha}\right) - \frac{\overline{\theta}}{2}\left(\underline{q} - \left(\overline{\theta} - \underline{\theta}\right)\frac{\underline{q}^2}{2a\alpha}\right)^2\right] + (1-\lambda)\left(a\underline{q} - \frac{\underline{\theta}}{2}\underline{q}^2\right) \tag{65}$$

The FOC is:

$$(1-\lambda)\left(a-\underline{\theta}\underline{q}\right)+\lambda\left[a\left(1-\left(\overline{\theta}-\underline{\theta}\right)\frac{\underline{q}}{a\alpha}\right)-\overline{\theta}\left(\underline{q}-\left(\overline{\theta}-\underline{\theta}\right)\frac{\underline{q}^2}{2a\alpha}\right)\left(1-\left(\overline{\theta}-\underline{\theta}\right)\frac{\underline{q}}{a\alpha}\right)\right]=0 \tag{66}$$

Substituting $\lambda = 0.5$, $\overline{\theta} = 1.25$, $\underline{\theta} = 1$ and $a = 1$, we have:

$$-0.39\frac{\underline{q}^3}{\alpha^2}+0.47\frac{\underline{q}^2}{\alpha}-\left(2.25+\frac{0.25}{\alpha}\right)\underline{q}+2=0 \tag{67}$$

with solution

$$\underline{q}^{VH}=3.99\alpha-\frac{3.22\alpha^2+2.13\alpha}{B}+B \tag{68}$$

where

$$B=\sqrt[3]{12.800\alpha^2-51.200\alpha^3+\sqrt{2654.2\alpha^6-1245.2\alpha^5+207.53\alpha^4+9.7087\alpha^3}} \tag{69}$$

$$\overline{q}^{VH}=\underline{q}^{VH}-\left(\overline{\theta}-\underline{\theta}\right)\frac{\left(\underline{q}^{VH}\right)^2}{2a\alpha}. \tag{70}$$