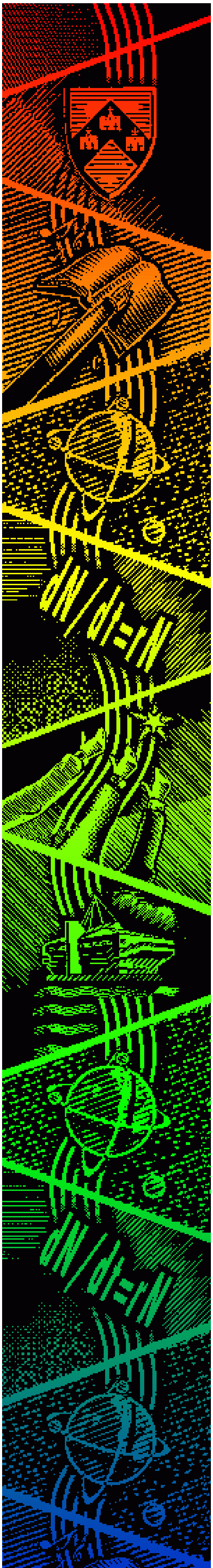

THE UNIVERSITY of York

Discussion Papers in Economics

No. 2007/19

The Asymmetric Outcome of Sticky Price Models

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June 2007

ABSTRACT. Empirical evidence shows demand shocks tend to have an asymmetric effect on output: it falls by a larger amount with a contraction than it rises with an expansion. We argue that introducing nominal rigidities in a framework where agents maximise their welfare can yield such an asymmetric outcome. We show that this is the case in the *Sticky Prices* framework, where each period an exogenously set fraction of firms fails to adjust prices. While the solution method commonly adopted by this literature, the log-linearization, delivers a perfectly symmetric response, methods that respect the original structure of the model yield an asymmetric one. We show that when products are good substitutes to each other and labour supply is inelastic, the model implies that the response of output is larger with monetary contractions than with expansions, even when the shock is small. We identify the origin of the asymmetry in that when not all firms adjust prices, some goods are cheaper than others and so more heavily consumed. With a positive shock, these goods are produced by the firms that fail to adjust, so that real income is not very much affected. But with a negative shock, they are produced by firms that adjust prices, causing a large swing in real income.

JEL CLASSIFICATION: E31, E32

KEYWORDS: Staggered price setting, Asymmetric response to shocks, Monetary business cycles.

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[†] I am grateful to Wouter Den Haan, Jean Imbs, Marco Ottaviani, Alexis Anagnostopoulos and Christos Genakos for very helpful comments and suggestions.

1. Introduction

An important stream of what has been called the New Keynesian synthesis has explained the link between monetary policy, inflation and the business cycle by incorporating Keynesian elements (imperfect competition and nominal rigidities) into a framework with optimizing agents. In particular, building on the work of Taylor (1980) and Calvo (1983), models of staggered price adjustment[1] (or *Sticky Prices* Models) have addressed the issue by introducing these elements into the framework traditionally associated with Real Business Cycle models. The result has been a class of microfounded macroeconomic models in which sluggish price adjustment, by causing non-neutral short-run effects of aggregate demand movements, provides an alternative to fluctuations in technology, intertemporal substitution and capital accumulation as a cause for business cycle fluctuations.

However, there is an important feature of monetary business cycles that these models do not explain. Several empirical studies have reported evidence of asymmetric effects of aggregate demand shocks on output. Macklem et al. (1996), Ravn and Sola (2004) and Devereux and Siu (2005) find negative shocks have larger real effects than positive ones. Cover (1992) finds positive shocks have a weak contractionary effect and negative shocks a large one.

Our objective is to show that the *Sticky Prices* framework can generate asymmetric responses to shocks that are broadly consistent with the empirical results described above. We argue that the introduction of nominal rigidities, by causing a substantial reallocation of demand across goods, is at the origin of the asymmetric outcome of the model. When not all firms adjust their prices, some goods are cheaper than the others and consumers direct their consumption towards the cheapest. When the optimal price increases, the most consumed goods are those produced by the firms that fail to adjust their prices, so that real income is not very much affected. But when the optimal price falls, the cheaper goods are those whose prices have been adjusted, causing a large swing in real income. We show that when labour supply is inelastic and products are good substitutes to each other[2], this reallocation of demand causes an asymmetric response of the optimal price, yielding a larger response of output with negative shocks. The reallocation of demand that follows a change in prices is to be found in an extended class of *Sticky Prices* models. But perhaps surprisingly, the asymmetry generated is largely unreported.

This is so because *Sticky Prices* models generally rely on a first-order Taylor expansion based solution method (or log-linearisation). We show that log-linearising around the steady state alters the basic properties of the aggregator of consumption through which the mechanism that drives the asymmetry is channelled. Using solution methods that respect the original (non-linear) structure of the equilibrium conditions (hence do not rely on linear approximations) such as the projection methods develop by Judd (1992) or the parameterised expectations approach of Den Haan and Marcet (1990), the true outcome of the model is revealed. We report important qualitative and quantitative differences between the log-linearised and the non-linearised solutions.

Economists often argue that nominal rigidities are asymmetric: prices are more

[1] e.g. Rotemberg and Woodford, 1997; Chari, Kehoe and McGrattan, 2000; Goodfriend and King, 1997; Galí, 2002; or Christiano, Eichenbaum and Evans (2005) to cite a few.

[2] These parameter values are widely accepted by microeconomic empirical studies.

flexible when going up than when going down. Often these asymmetries are simply assumed, or rely on an irrational aversion to nominal wage or price decreases. However, agents in the models we present are fully rational[3]. Several papers in the New Keynesian literature have tried to offer a theoretical account of the asymmetric response to shocks that is based on optimising behaviour. Tsiddon (1993) and Ball and Mankiw (1994) present models in which positive trend inflation causes asymmetric price responses. But the models we present do not assume trend inflation. Devereux and Siu (2005), in the context of a state-dependent pricing model find a distinct asymmetry in the response of prices. But our results do not rely on any form of strategic linkage between firms.

We first present the model, and analyse the implications of the failure of some firms to adjust their prices to a one-off aggregate demand shock in the static counterpart of a standard *Sticky Prices* model. The static analysis lays the intuition surrounding the mechanisms that drive the asymmetric outcomes of the model. We then explain why the log-linearisation fails to reveal them. We study next the outcome of a standard dynamic *Sticky Prices* model based on the widely used pricing mechanism introduced by Calvo (1983). To establish the connection with the static analysis, we first study a particular case where firms maximise myopic profits. We show that with a small 1% nominal shock, the first period response of output is 36% larger with the negative shock. We then introduce forward-looking behaviour, and show that because the response of the optimal price is more limited in this case, the asymmetry in the response of output has reduced: it is only 20% larger with the negative shock.

2. A Standard Sticky Prices Model

The economy is populated by a representative and infinitely lived household, and a large number of identical firms. The household consumes differentiated products, each of which is produced by a different firm. The representative household and the firms interact in a monopolistically competitive goods market and a perfectly competitive labour market. Because the goods market is monopolistically competitive, firms take as given downward sloping demand schedules for their products, and set the prices that maximise their profits. A fundamental friction is added to this economy: each period a fraction of firms fails to adjust their prices, keeping the one that was set in the previous period and thus failing to maximise profits. This pattern of price adjustment is known in the literature as the Calvo pricing mechanism (from Calvo (1982)). The basic experiment we perform is letting the amount of money change unexpectedly, and recording the response of the economy as it switches from the pre to the post-shock equilibrium.

2.1. Households. The household's maximisation program can be staged in two steps. First it maximises its overall utility, deciding how much to consume of an aggregate of consumption c_t that we define below, how much to work, l_t, and how much money to hold, M_t:

$$\max_{c_t,l_t,M_t} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma_c}}{1-\gamma_c} - a\frac{l_t^{1-\gamma_l}}{1-\gamma_l} \right\} \tag{1}$$

[3]Although some firms fail to maximise their profits, this is not generally deemed irrational behaviour.

subject to its budget constraint

$$c_t + \frac{M_t}{P_t} \leq w_t \cdot l_t + \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t} + f_t \tag{2}$$

and the cash-in-advance constraint

$$c_t \leq \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t} \tag{3}$$

where β is the discount factor, f_t the real dividends the consumer receives from the firms, w_t the real wage rate and P_t the aggregate price level, defined further on. $T_t = \tilde{M}_t - \tilde{M}_{t-1}$ are money transfers, taken as given by the household. Note that although $\tilde{M}_t$ equals M_t in equilibrium (because the agent is a representative one), the first one is taken as given by the household while the second is a choice variable. Further, the household also takes as given the wage rate, the dividends paid out by firms and the aggregate price level. We choose a separable functional form for the utility as is commonly adopted by the *Sticky Prices* literature (e.g. Galí (2002)). The rest of the model's structure draws on Cooley and Hansen (1989). The first-order conditions of the Lagrangian associated with the above program, with the constraints holding with equality, are given by the labour Euler equation,

$$a \cdot l_t^{-\gamma_l} = \kappa_t \cdot w_t \tag{4}$$

the money Euler equation

$$\frac{\kappa_t}{P_t} = \beta E_t \left[\frac{\kappa_{t+1} + \zeta_{t+1}}{P_{t+1}} \right] \tag{5}$$

and the consumption Euler equation,

$$c_t^{-\gamma_c} = \kappa_t + \zeta_t \tag{6}$$

where κ_t and ζ_t are the Lagrangian multipliers associated with the budget and cash in advance constraints respectively. The Frisch elasticity of labour supply is again given by $\xi = -1/\gamma_l$[4]. When $\gamma_l \to 0$, the utility function is linear in labour, and so the labour supply schedule is infinitely elastic. But as γ_l falls, labour supply becomes increasingly inelastic, so that large changes to the wage rate are needed for the household to change the amount of labour it is willing to provide.

In a second stage the household seeks to allocate optimally his aggregate level of consumption across a range of differentiated products taking as given the products' prices ($p_{i,t}$, $i \in [0,1]$) and its available level of expenditure, that we set exogenously. The household's preferences over the differentiated products it consumes are characterised by a constant elasticity of substitution function, commonly known as Dixit-Stiglitz aggregator ($c_{i,t}$ is the demand for each individual product, $i \in [0,1]$):

$$c_t = \left\{ \int_0^1 c_{i,t}^{\frac{\theta-1}{\theta}} di \right\}^{\frac{\theta}{\theta-1}} \tag{7}$$

[4] $\gamma_l \leq 0$ needs to be satisfied for the second order derivative of the objective to be negative and so our maximum to be an internal solution.

The second maximisation problem the household faces is given by:

$$\max_{c_{i,t}} \left\{ \int_0^1 c_{i,t}^{\frac{\theta-1}{\theta}} di \right\}^{\frac{\theta}{\theta-1}} \quad s.t. \quad \int_0^1 p_{i,t} \cdot c_{i,t} \cdot di \leq E_t \tag{8}$$

where E_t is nominal aggregate demand and it is exogenous to the model. We define a price level P_t as the ratio of nominal to real aggregate demand, $P_t = E_t/c_t$. The parameter θ is the constant elasticity of substitution between any two goods ($\theta > 1$) and goods are perfect substitutes to each other when $\theta \to \infty$. Substituting (7) in this expression we obtain

$$c_{i,t} = [\lambda_t \cdot p_{i,t}]^{-\theta} \cdot c_t \tag{9}$$

where λ is the Lagrange multiplier of the maximisation problem. Substituting this expression in (7) we obtain

$$\lambda_t = \left\{ \int_0^1 p_{i,t}^{1-\theta} di \right\}^{\frac{1}{\theta-1}}$$

which is an expression for the multiplier as a function of the prices. Substituting the expression of the multiplier in (9) we obtain the demand for good c_i

$$c_{i,t} = \left\{ \frac{p_{i,t}}{\left\{ \int_0^1 p_{i,t}^{1-\theta} di \right\}^{\frac{1}{1-\theta}}} \right\}^{-\theta} \cdot c_t \tag{10}$$

The demand for good i depends negatively on the good's price (this is called the own-price effect), positively on all the other goods prices (the cross-price effect) and positively on real aggregate demand (or aggregate consumption, c). It is now possible to construct a relation between the aggregate price level P_t, and the products' prices ($p_{i,t}$, $i \in [0,1]$). Combining the definition of the price level ($P_t = E_t/c_t$) with (10) and the budget constraint in (8) gives

$$P_t = \left\{ \int_0^1 p_{i,t}^{1-\theta} di \right\}^{\frac{1}{1-\theta}} \tag{11}$$

So that we finally obtain the demand for good i in a more succinct form:

$$c_{i,t} = \left[\frac{p_{i,t}}{P_t} \right]^{-\theta} \cdot c_t \quad i \in [0,1] \tag{12}$$

These downward sloping demand schedules are taken as given by the firms when choosing their prices. We turn now to examine these.

2.2. Firms. The firms' production function is linear in labour ($y_{i,t} = l_{i,t}$, where $y_{i,t}$ is production and $l_{i,t}$ is labour employed), and so the firms' marginal cost is simply given by the real wage rate. After substituting the demand functions (12) the firms take as given, we can write the expression for the real profits made by each firm i, producer of good $c_{i,t}$ and setting $p_{i,t}$, in any given period τ as:

$$f_{i,\tau}(p_{i,t}) = \left[\frac{p_{i,t}}{P_\tau} - w_\tau\right] \cdot \left[\frac{p_{i,t}}{P_\tau}\right]^{-\theta} \cdot c_\tau$$

Each period, some of the firms fail to adjust their prices to the one that maximises their profits. The decision to adjust is taken when the firms receive a price changing signal, which comes every period with probability $1-\eta$. Those firms that do not receive the signal keep their posted prices. This pricing mechanism is due to Calvo (1983) and it greatly simplifies the setting and solution of the model. First, because the pricing signal is a random variable with a memoryless distribution, the probability to adjust in any given period does not depend on the firms' past decisions. Moreover, because there are a large number of firms, the law of large numbers implies that the fraction of firms that each period keep their prices fixed is equal to the probability of not receiving the signal (η that is). Last, the probability of adjustment can be related to the length of time firms keep their prices fixed on average, given by $(1-\eta)\sum_{k=0}^{\infty} k\eta^{k-1} = 1/(1-\eta)$.

Firms want the price they set today $p_{i,t}^{opt}$, to maximise not only current profits but also those future expected profits that depend on it. When choosing the optimal price in period t firms know they might not do so in the following periods. They therefore set the price that solves the following program:

$$\max_{p_{i,t}} E_t \left\{ \sum_{\tau=t}^{\infty} (\beta\eta)^{\tau-t} f_{i,\tau}(p_{i,t}) \right\}$$

where β is the discount factor. The solution to this problem is given by the optimal nominal price:

$$p_t^{opt} = (1+\mu)\frac{\sum_{\tau=t}^{\infty}(\beta\eta)^{\tau-t}E_t\left[P_\tau^\theta w_\tau c_\tau\right]}{\sum_{\tau=t}^{\infty}(\beta\eta)^{\tau-t}E_t\left[P_\tau^{\theta-1} c_\tau\right]} \tag{13}$$

where $\mu = 1/(\theta-1)$ is the markup over the marginal cost when firms maximise myopic profits. Note that because the left hand side is equal across firms, we have dropped the subscript i: all *adjusters* set the same price. When firms consider the path of future variables in the price they set, the current inflation rate is affected by expectations of future variables. An important case we examine in the following sections is given when the discount factor is set to 0, and so the optimal price is given by

$$p_t^{opt} = (1+\mu) P_t w_t \tag{14}$$

2.3. Characterisation of the Equilibrium. After the exogenous money stock $\tilde{M}_t$ changes unexpectedly, the representative household decides how much to consume of each one of the differentiated goods taking as given the prices, and how much labour to supply taking as given the real wage rate. Conditional on receiving a price changing signal, firms decide upon what price to set and how much labour to demand, taking as given the demand they face and the level of aggregate variables. After household and firms trade goods and labour, aggregate variables settle to their new equilibrium values. In equilibrium the amount of money available to the consumer is equal to the amount held the previous period plus the transfers T_t, and so we have that $M_t = \tilde{M}_t$. The exogenously set money stock $\tilde{M}_t$ evolves according to

$$\tilde{M}_t = g_t \cdot \tilde{M}_{t-1} \tag{15}$$

where g_t is a stochastic process whose logarithm follows a $NID(0, \sigma^2)$ distribution. It can be shown[5] that choosing $\gamma_c = 1$ (utility function logarithmic in consumption) as shall be the case hereafter, both constraints hold with equality under mild restrictions on the process of g_t and the discount factor β. The cash in advance constraint holds with equality iff $\beta E_t \left[g_{t+1}^{-1}\right] < 1$. In this case $E_t \left[g_{t+1}^{-1}\right] = \exp(\sigma^2/2)$ and $0 < \beta < 1$, so the first condition is satisfied when σ is small enough. When the cash-in-advance constraint holds with equality, we have that

$$c_t = \tilde{M}_t / P_t$$

Notice that this expression is identical to the relation between real aggregate demand and the price level we define above ($c_t = E_t/P_t$), if we substitute available expenditure by the level of the money stock and ignore the time subscripts. Therefore, a change in the money stock in this model is equivalent to a change in nominal aggregate demand. Combining this expression with the law governing the evolution of the money stock (15) allows us to write a relation between output, inflation, and the monetary shock:

$$c_t = g_t \cdot \frac{\tilde{M}_{t-1}}{P_{t-1}} \cdot (1 + \pi_t)^{-1} \tag{16}$$

where π_t is inflation, defined as the aggregate price level's growth rate. Thus, the cash in advance constraint describes an inverse relation between output and inflation, conditional on the shock and the value of the previous period's real money balances. Setting $\gamma_c = 1$ in the utility function and combining (4), (5) and (6) and defining $\chi = a / \left(\beta \cdot E_t \left[g_{t+1}^{-1}\right]\right)$ allows us to write the equilibrium labour supply schedule as

$$w_t = \chi \cdot l_t^{-\gamma_l} \cdot c_t^{\gamma_c} \tag{17}$$

Next, since labour is a homogenous good, it is naturally aggregated linearly, and its market clearing condition is given by

$$l_t = \int_0^1 l_{i,t} di$$

Equilibrium in the goods market is reached when the quantity supplied of each good covers its demand, $y_{i,t} = c_{i,t}$. We have that integrating the left hand side of this condition over the continuum of goods and using the firms' production function and the labour market equilibrium condition gives

$$\int_0^1 y_{i,t} di = \int_0^1 l_{i,t} di = l_t$$

[5] The proof is available on request from the author.

and aggregating the right hand side and using the consumer's demand functions (12) gives

$$\int_0^1 y_{i,t}^d di = \int_0^1 \left[\frac{p_{i,t}}{P_t}\right]^{-\theta} \cdot c_t \cdot di = R_t^{\theta} \cdot c_t \quad \text{where } R_t = \frac{P_t}{P_t^*}$$

where P_t^* is an alternative price aggregator given by

$$P_t^* = \left\{\int_0^1 p_{i,t}^{-\theta} di\right\}^{-\frac{1}{\theta}}$$

The aggregate labour demand schedule can thus be written

$$l_t = R_t^{\theta} \cdot c_t \tag{18}$$

The Calvo pricing mechanism allows the possibility of summarizing the whole history of prices set in the past in a single variable, the past aggregate price level P_{t-1}. In period t, a constant proportion η of firms sets the price they posted the previous period and the rest sets the nominal optimal price p_t^{opt}. In period $t-1$, η set the price they posted the previous period and the rest set p_{t-1}^{opt}, and so on. In period t we have that: $(1-\eta)$ set p_t^{opt}, $\eta(1-\eta)$ set p_{t-1}^{opt}, $\eta^2(1-\eta)$ set p_{t-2}^{opt}, and so on. We can use this decomposition to rearrange the expression of the two price aggregators as

$$P_t = \left\{\eta \cdot (P_{t-1})^{1-\theta} + (1-\eta) \cdot \left(p_t^{opt}\right)^{1-\theta}\right\}^{\frac{1}{1-\theta}} \tag{19}$$

$$P_t^* = \left\{\eta \cdot \left(P_{t-1}^*\right)^{-\theta} + (1-\eta) \cdot \left(p_t^{opt}\right)^{-\theta}\right\}^{-\frac{1}{\theta}} \tag{20}$$

We can now define an *equilibrium* in this economy as a function of the shock to nominal aggregate demand and the exogenously set fraction η of *adjusters*:

Definition 1. *an equilibrium in this economy is a collection of allocations for the representative household c_t, l_t, $c_{i,t}$ for $i \in [0,1]$; allocations for firms $l_{i,t}$ for $i \in [0,1]$; together with prices w_t, P_t, p_t^{opt} that satisfy the following conditions: (i) taking prices as given, consumer allocations solve the household's problem, satisfying (4), (5) and (6), (ii) taking all prices but his own as given adjusters satisfy (13) or (14), while non-adjusters keep their posted prices; the factor markets clear implying that (16), (17), (18), (19) and (20) ought to be satisfied.*

3. Consumer Choice Analysis

We explain now why the model we have just presented can yield an asymmetric response of real aggregate demand (c_t, equal to output in this model). Because we work with a general equilibrium framework, it is difficult to identify the causal links between the different variables. Changes in prices affect real variables when some firms fail to adjust prices and changes in real variables affect prices through their influence on profits. In section 3.1 we break this circle by assuming a perfectly symmetric change in the optimal price: it falls by as much with a contraction in nominal demand as it rises with an expansion. In this partial equilibrium scenario we show how the response of real variables is larger with a contraction in nominal aggregate demand than with an expansion. We also show how the reallocation of consumption from more expensive to cheaper goods is at the origin of this asymmetric pattern. In section 3.2 we show that under certain circumstances, the asymmetric pattern described by real variables is also followed by the optimal price. As a result, we find that real aggregate demand falls by more than it rises with a shock, while the opposite is the case for aggregate labour, the price level and the real wage rate Throughout the following two sections we consider only the first period after the shock, and we let firms maximise myopic profits. We then drop all time subscripts and let the optimal price be given by (14) rather than (13). The model we examine is then in all respects static. We consider the dynamic implications of our analysis in section 5.

3.1. Origin of the Asymmetry. We examine here how the consumer's optimal allocation of consumption across the goods it wishes to consume determine asymmetric patterns for the responses of real aggregate demand and aggregate labour to nominal changes. These two magnitudes are aggregates of the same household's demands for the different goods. However, these demands are aggregated differently, and as we show in section 3.2, the wedge driven between the two aggregates is crucial to understanding the model's outcome. We assume first a perfectly symmetric change in the optimal price and study the forces that this change generates. At the end of the section we show how a very particular choice of parameter values justifies our assumption in general equilibrium.

a. Optimal allocation of Consumption. A typical household in this economy, faced with a distribution of prices and a given level of available expenditure, allocates optimally his consumption across the goods it wishes to consume. After the shock to nominal aggregate demand, a fraction η of firms adjust prices and so receive the same demand for their products ($c^{adj} = (p^{opt}/P)^{-\theta}c$), while the rest keep their posted prices ($c^{kee} = (\bar{p}/P)^{-\theta}c$). We can then write the consumption aggregator (7) as

$$c = \left\{\eta\left(c^{adj}\right)^{\frac{\theta-1}{\theta}} + (1-\eta)\left(c^{kee}\right)^{\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}}$$

This is both a measure of utility derived from consumption and the level of real aggregate demand. We can consider the problem in terms of a traditional consumer choice analysis. We start by rearranging the previous expression as

$$c^{kee} = \left[\frac{c^{\frac{\theta-1}{\theta}}}{\eta} - \frac{1-\eta}{\eta}\left(c^{adj}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

that gives all the combinations c^{adj} and c^{kee} yielding a c level of real aggregate demand (the indifference curve). Performing the same operation with the budget constraint of the household's second stage maximisation program (8) gives

$$c^{kee} = \frac{E_k}{\eta \cdot \bar{p}} - \frac{1-\eta}{\eta}\frac{p^{opt}}{\bar{p}} \cdot c^{adj}$$

Here E_k is available expenditure to a typical household k, that we hold fix in our analysis. We consider now the effects of a change in the optimal price (the price of

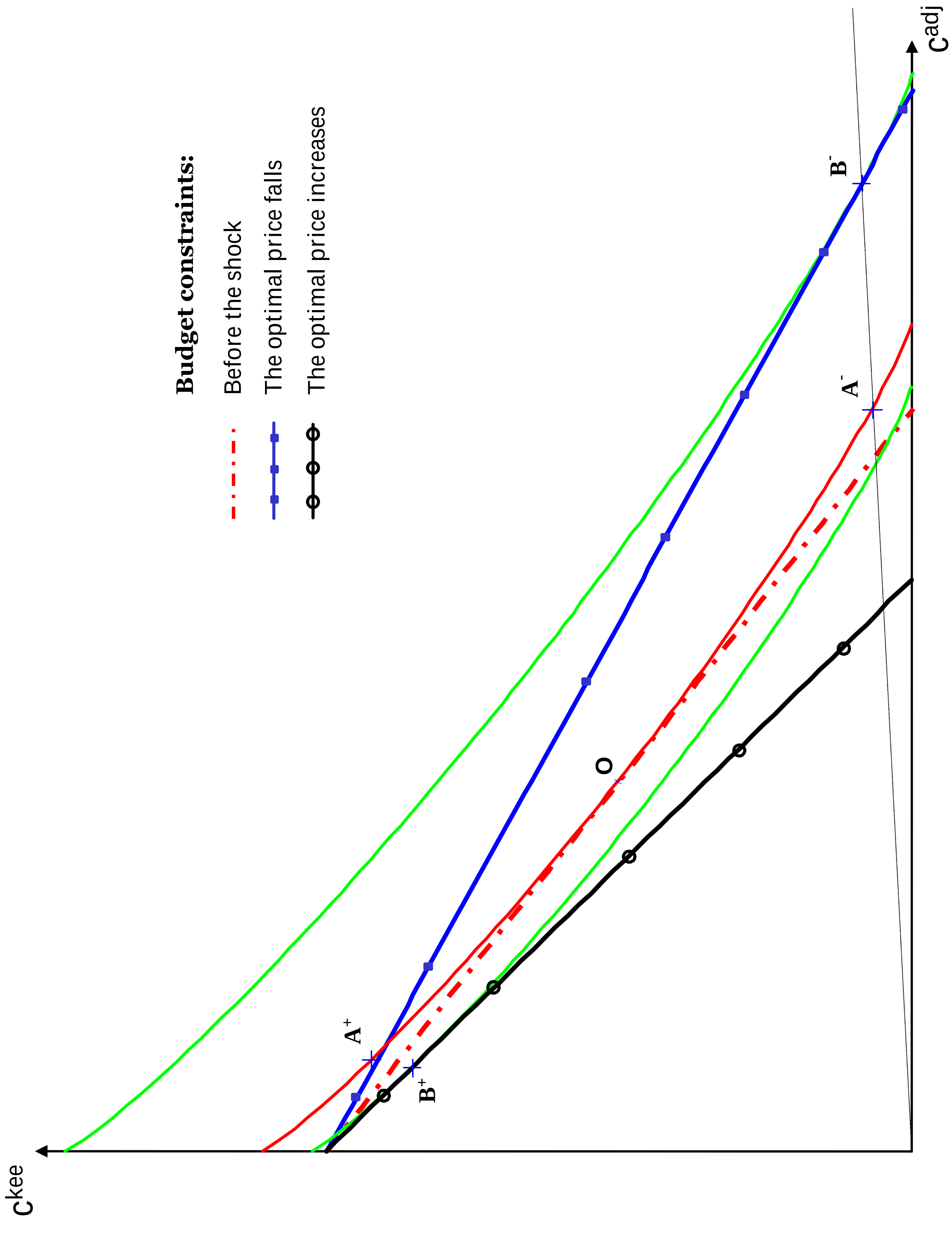

Figure 1: Hicks decomposition of the effect of changes in the optimal price on the optimal allocations of consumption (A^+, B^+ for a rise and A^-, B^- for a fall).

adjusters' goods) on the household's choices. We have represented in figure 1 the changes in the optimal allocation of consumption for a fall and a rise (of equal size) of the optimal price while keeping E_k fixed.

The budget constraint pivots from its original position (through O) upwards with a fall in p^{opt} and downwards with an increase. Starting from the allocations at O, where all prices are the same an increase in the optimal price leads to B^+ while a decrease leads to B^-. We have also represented in the figure the Hicksian decomposition into income and substitution effects of a change in prices. The change from O to A^+ and A^- is caused by the substitution effect, while the change from A^+ to B^+ and from A^- to B^- is caused by the income effect of the price change[6].

As can be appreciated from the figure, the change in real aggregate demand (the shift in the indifference curve) from A^+ to B^+ is substantially smaller than the change from A^- to B^-. The difference is wholly attributable to the income effect of the price change, since the substitution effect by definition causes no change in real aggregate demand and so allocations are on the original indifference curve. When prices increase the goods whose price has changed are relatively poorly consumed, since they are more expensive. When prices decrease, this has a large effect because the goods whose price has changed are consumed to a large extent, since they are now the cheapest. The income effect on real aggregate demand of a decrease in the optimal price is larger than the income effect of an increase.

b. General Equilibrium Response. Let us now consider the general equilibrium effects of a change in nominal aggregate demand E. We start with the determinants of the optimal price in (14). Because firms are monopolistic competitors, they set this price at a constant markup over the nominal marginal cost ($P \cdot w$). In this section, we set $\gamma_c = 1$ so that utility is logarithmic in consumption, and $\gamma_l = 0$ so that labour supply is infinitely inelastic. In this case the the equilibrium labour supply schedule (17) is given by $w = a \cdot c$ and so the real wage rate is independent of aggregate labour in equilibrium. Substituting the relation between real and nominal aggregate demand ($c = E/P$) we obtain $P \cdot w = a \cdot E$. Thus, with a shock to nominal aggregate demand, the optimal price changes by the same amount as the shock. A symmetric change in E then causes a symmetric change in p^{opt}, justifying the perfectly symmetric optimal price response we have assumed so far in our analysis.

We explained above why a symmetric change in the optimal price has an asymmetric effect on real aggregate demand, holding the household's available expenditure fixed. Let us now consider the effect of a change in available expenditure -equal to nominal aggregate demand- on real aggregate demand, taking as given the change in the optimal price. When E increases, the household's budget constraint (8) shifts up, causing an increase in the amount consumed of all goods and so in real aggregate demand[7], proportionally to the increase in E. When E falls, the opposite happens, and the amount consumed of all goods and real aggregate demand fall proportionally. Therefore, a symmetric change in E causes a symmetric change in real aggregate demand.

[6] The line through the origin, A^+ and B^+, and the line through the origin and A^- and B^- are the income expansions paths, since the indifference curves are convex to the origin.

[7] That again is a characteristic of the Dixit-Stiglitz aggregator.

Let us finally combine the effect of a change in available expenditure with the effects of a change in the optimal price. Both changes have opposing effects on real aggregate demand: the change in the optimal price limits the effects of the change in available expenditure. But because the change in the optimal price has asymmetric effects on real aggregate demand, we have that an increase in E is only limited to a small extent by an increase in the optimal price, while a decrease in E is limited to a large extent by a decrease in the optimal price. As a result, an expansion in nominal aggregate demand has larger effects on real aggregate demand than a contraction.

We have represented the responses of optimal price and real aggregate demand to a range of nominal demand shocks in figure 2. The solid lines correspond to the response to a positive shock while the dashed ones give the absolute value of the response to a negative shock. While the response of the optimal price is perfectly symmetric (dashed line coincides with the solid one), the response of real aggregate demand is asymmetric, and this asymmetry grows with the size of the shock.

The size of the asymmetry depends crucially on the value of the elasticity of substitution. The better substitutes products are, the larger the extent to which the household substitutes consumption from the most expensive to the cheaper goods after the optimal price changes. The more intense the reallocation of demand, the weaker the income effect of the optimal price change is with an expansion and the stronger with a contraction. Thus, the larger the elasticity of substitution is, the larger the size of the asymmetry. To see that, consider the extreme case where goods are perfect substitutes to each other and so the elasticity of substitution tends to infinity. With a contraction in nominal aggregate demand and so a fall in the optimal price, the household stops consuming altogether the most expensive goods (produced by *non-adjusters*), since an equivalent alternative exists at cheaper prices. With an expansion, it is the goods produced by *adjusters* that are consumed no longer. As a result, a contraction leaves *real aggregate demand unaffected*. But with an expansion, *real aggregate demand changes by the full amount of the shock*.

As can be appreciated from figure 2, the size of the asymmetry in the response of aggregate labour is larger than that of real aggregate demand. We proceed now to analyse the difference between the two responses, a difference that is fundamental to understanding the general equilibrium response of the optimal price that we analyse in the following section. Both labour and output are aggregates of the same individual demands and the linear aggregator of labour is a special case (when $\theta \to \infty$) of the Dixit-Stiglitz aggregator of consumption. The equilibrium condition (18) in the static context we are examining is given by $l = R^{\theta} \cdot c$. The following proposition establishes a key property of the relation between the two aggregates.

Proposition 2. *The ratio of the two price aggregators is larger than one whenever some firms set a different price than the others: $R > 1$ if $p^{opt} \neq \bar{p}$.*

Proof. See appendix. ■

Thus, we have that $l > c$ and so the response of labour exceeds output's with a positive shock falls short of it with a negative one ($\hat{l}^{+} > \hat{c}^{+}$ and $\hat{l}^{-} > \hat{c}^{-}$). As a result the asymmetry of in the response of labour is larger than in the response of real aggregate demand.

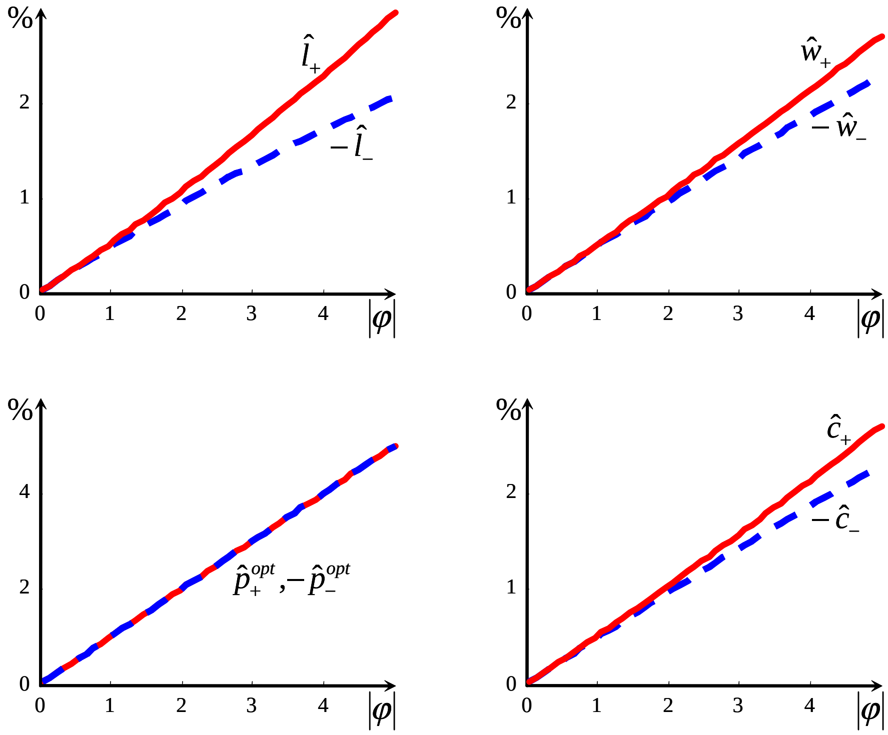

Figure 2: General equilibrium responses to a range of nominal shocks when labour supply is infinitely elastic, $\gamma_l = 0$ ($\gamma_c = 1$, $\eta = 0.5$, $\theta = 7.7$).

3.2. The Asymmetry in the response of the optimal price. The analysis in the previous section first assumed a symmetric response of the optimal price, and then presented general equilibrium results for a particular set of parameter values ($\gamma_c = 1$, $\gamma_l = 0$) for which the response of the optimal price was actually symmetric. We concluded in this setting that the response of real aggregate demand is larger with positive shocks than with negative. We also found that the response of labour systematically exceeds output's with a positive shock and falls short of it with a negative one.

We now let labour supply become more inelastic ($|\gamma_l|$ to increase) and show how our results can change dramatically. We have represented in figure 3 the response of real aggregate demand for two alternative values of γ_l. When $\gamma_l = -1$ the asymmetric pattern is as described in the previous section: real aggregate demand response is larger with positive shocks. But when $\gamma_l = -3$, the asymmetric pattern has reversed: the response is now larger with negative shocks. Moreover, the response to both positive and negative shocks is considerably smaller. We show now how when labour supply is sufficiently inelastic, the asymmetric patterns described in the previous section are channelled into an asymmetric response of the optimal price, and how this causes real aggregate demand's asymmetric pattern to reverse.

Let us consider again the determinants of the optimal price (14) in equilibrium. It is set at a constant markup over the marginal cost, the wage rate in our model. In equilibrium, the real wage rate is governed by the consumer's first order condition (17)

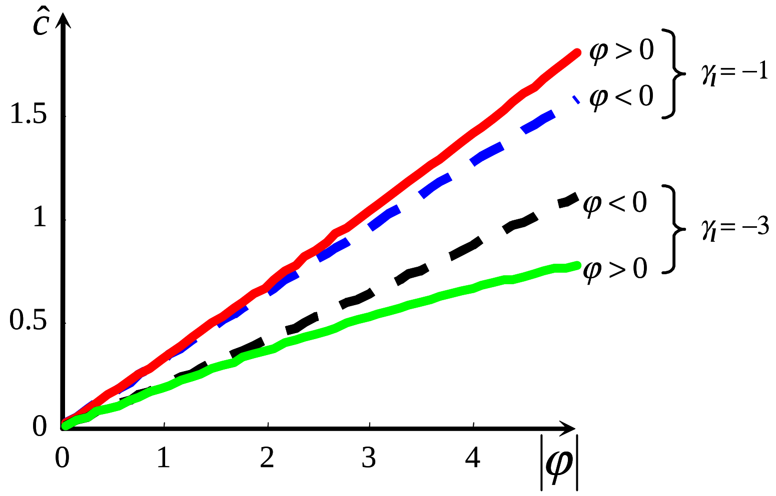

Figure 3: Response of real aggregate demand to a range of shocks for alternative values of the elasticity of labour supply $\xi = -1/\gamma_l$ ($\gamma_c = 1$, $\eta = 0.5$, $\theta = 7.7$).

that we rewrite here for convenience

$$w = a \cdot l^{-\gamma_l} \cdot c^{\gamma_c} \tag{21}$$

The response of the real wage rate to a change in nominal demand depends on the responses of aggregate labour and real demand and their respective weights ($-\gamma_l$ and γ_c). If we retain $\gamma_c = 1$ so that utility is logarithmic in consumption, and we let labour supply become more inelastic ($|\gamma_l|$ to increase) the response of the optimal becomes asymmetric.

We have represented in figure 4 the response of the optimal price for two alternative values of the elasticity of labour supply. Setting $\gamma_l = 0$ as in the previous section, labour supply is infinitely elastic and the response of the optimal price is perfectly symmetric: solid lines are identical in absolute value for positive and negative shocks. But the income effect of these changes in the optimal price (solid lines) is asymmetric. The income effect of the fall in the optimal price is larger in absolute value than the income effect of an identical rise. As we have explained, the better substitutes products are (larger θ), the larger the asymmetry in the income effect. As households substitute consumption to a larger extent to the cheaper goods, real aggregate demand is affected to a greater extent by a fall in the optimal price and to a lesser extent by an increase.

Let us consider now these same general equilibrium responses when labour supply is inelastic. As before, we choose $\gamma_l = -6.7$ so that the elasticity of labour supply is given by $\xi = 0.15$. In this case, aggregate labour carries a larger weight than real aggregate demand in determining the equilibrium real wage rate in (21). We have represented the response of the optimal price to $\pm 5\%$ shocks to nominal aggregate demand in figure 4 (dashed lines). The response of the optimal price is now clearly asymmetric: in absolute value, it is larger with a positive shock than with a negative one. Moreover, the asymmetry increases the better substitutes products are (larger θ). This very same pattern, as discussed in section 3.1 is followed by the response of aggregate labour. Precisely because aggregate labour carries a large weight in (21) the

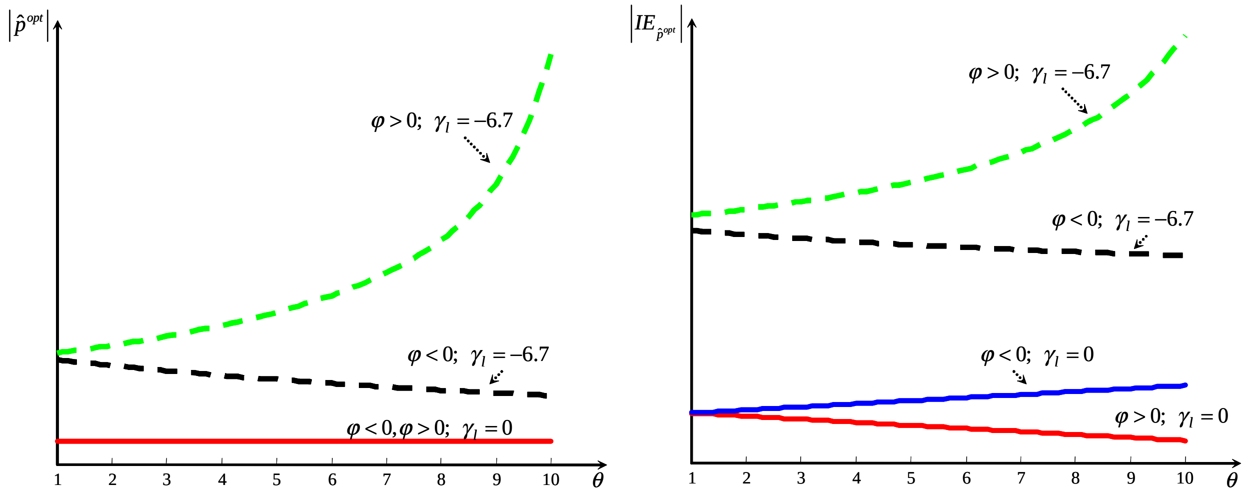

Figure 4: General equilibrium response of the optimal price and its income effect to $\varphi = \pm 5\%$ shocks for $\gamma_l = -6.7$ and $\gamma_l = 0$, as a function of θ ($\gamma_c = 1$, $\eta = 0.5$).

real wage rate describes this asymmetric pattern that carries through to the optimal price. Thus, large values of $|\gamma_l|$ and θ are essential for the response to a positive shock to be substantially larger than the response to a negative shock. When θ is large, the response of aggregate labour is highly asymmetric, and when $|\gamma_l|$ is large this response is translated to a larger extent to the response of the optimal price.

As before, the income effect of this response is asymmetric, but the pattern has now reversed. The income effect of the change in the optimal price (dashed lines) is larger with a positive shock than with a negative one, and the asymmetry grows with the elasticity of substitution. The forces driving the result we described in the previous section are still at play, but the asymmetry in the optimal price that these same forces generate moves against them. Because of the asymmetry in the income effect of the change in the optimal price, real aggregate falls further with a negative shock than it rises with a positive one. Moreover, because the response of the optimal price is very large when labour supply is inelastic, so is its income effect and real aggregate demand suffers a more limited response to both positive and negative shocks.

We represent in figure 5 the response of aggregate labour, real wage rate, optimal price and real aggregate demand to a range of nominal shocks (0 to 5%). The dashed lines give (minus) the response to a negative shock and the solid lines the response to a positive one. As before, the distance between the two curves gives us the size of the asymmetry and the position its direction. As in Ball and Romer (1990), we have chosen $\theta = 7.7$ and $\gamma_l = -6.7$. This value of the elasticity of substitution is commonly adopted by the New-Keynesian literature, although sometimes larger values are chosen (see Goodfriend, M. and King, R. (1997)), and it implies a markup over the marginal cost before the shock of 15%. The value we choose for γ_l implies a Frisch elasticity of labour supply of $\xi = 0.15$, which is in line with several microeconomic studies that have estimated it[8]. However, both the real business cycle and new Keynesian literatures

[8] MaCurdy (1981), Holtz-Eakin et al. (1988), Ziliak and Kniesner (1999), using variation in hours and wages data estimate a small elasticity, ranging from 0 to 0.5.

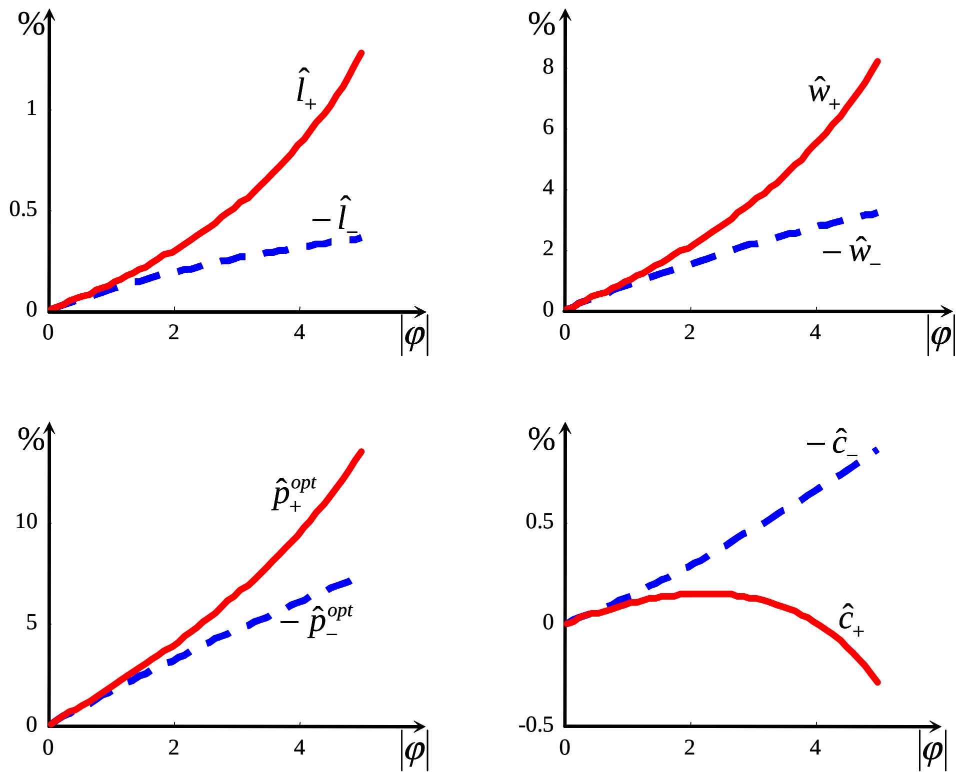

Figure 5: General equilibrium responses to a range of shocks when labour supply is inelastic, $\gamma_l = -6.7$ ($\gamma_c = 1$, $\eta = 0.5$, $\theta = 7.7$).

typically use larger values for this parameter: around $\xi = 1$, so that $\gamma_l = -1$. We keep these parameter values in the remainder of the paper.

The response of aggregate labour is larger with an expansion in nominal aggregate demand than with a contraction, and the size of the asymmetry grows with the absolute value of the shock. The real wage rate follows this same pattern (because labour supply is considerably inelastic) and so does then the optimal price. As we have explained, the income effect of the change in the optimal price is therefore also stronger with positive than with negative shocks. As a result, real aggregate demand displays a larger response with negative shocks than with positive ones. It is also important to note that in the event of a large positive shock, real aggregate demand suffers a negative response. This is so because the optimal price has experienced a large response, following that of the real wage rate. We can trace back this result to our choice of parameter values: labour supply being considerably inelastic, small changes in labour are associated with large movements in the real wage rate. Thus, the qualitative outcome of the model of the model differs not only with the sign of the shock, but also with its size. With small ($\pm 1\%$) shocks, real aggregate demand suffers a small asymmetric response ($-0.13\%, 0.10\%$). With large ($\pm 5\%$) shocks, the asymmetry has grown disproportionately ($-0.86\%, -0.29\%$) and the response to a positive shock has become negative.

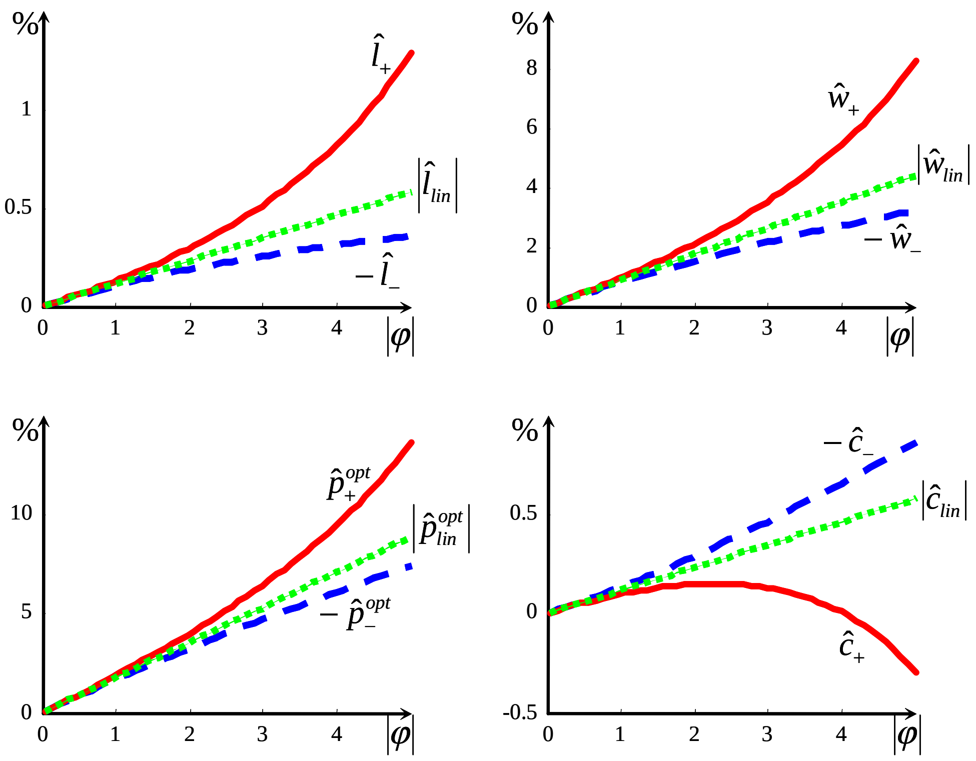

Figure 6: Linear and non-linear responses to shocks ($\gamma_l = -6.7$, $\gamma_c = 1$, $\eta = 0.5$).

4. Shortcomings of the Linearisation

The *Sticky prices* literature does not generally provide a solution to the original structure of the model. It offers instead the solution to an approximated version of the model, obtained by taking a first-order Taylor expansion of the equilibrium conditions. This procedure is called log-linearisation and it delivers by construction a perfectly symmetric response to shocks. We examine in this section the quantitative and qualitative differences between the true (non-linear) outcome of the model and the approximated (log-linear) one. We have represented in figure 6 the actual and log-linearised[9] responses to symmetric shocks to nominal demand. The dotted lines give the perfectly symmetric log-linearised responses: the response to a positive shock matches the absolute value to the negative one, so both lines coincide. Because the actual response is asymmetric, the log-linearised one lies between the true positive and negative ones. The gap between them is apparent for sufficiently large shocks (larger than 1%). We proceed now to explain why such quantitative and qualitative differences occur.

As we exposed in section 2.1, the composition of the aggregate price level is derived from combining demand functions with the Dixit-Stiglitz aggregator of consumption. The household's preferences over goods can then be derived from the aggregate price level's structure. The log-linearisation procedure fails to reveal the true implications of the model we have introduced because it distorts the true outcome of the aggregate price level. This is because the log-linearisation effectively eliminates the influence

[9] The log-linearised solution is given in the appendix.

the elasticity of substitution has on the weight of the different prices in the overall composition of the aggregate price level. The aggregate price level is a constant-elasticity of substitution function (given by 19) that we rewrite here for convenience

$$P = \left\{\eta \cdot (\bar{p})^{1-\theta} + (1-\eta) \cdot \left(p^{opt}\right)^{1-\theta}\right\}^{\frac{1}{1-\theta}}$$

The importance of the different prices in the overall value of the aggregate price level is governed by the parameters η (fraction of firms that fail to adjust) and θ (elasticity of substitution). Taking a first-order Taylor expansion around the steady state allows us to write the expression for the approximate response of the aggregate price level to the monetary shock:

$$\hat{P} \simeq (1-\eta) \cdot \hat{p}^{opt} \tag{22}$$

Notice that the parameter θ, the elasticity of substitution, has disappeared from the expression relating the prices set by the different firms and the aggregate price level ($\bar{p}$ is a constant and so disappears from the expression). The weight the response of the different prices carry in the response of the aggregate price level is then solely governed by η, and so the log-linearisation eliminates the channel through which the model delivers an asymmetric outcome. To illustrate how the elasticity of substitution affects the gap between the true value of the aggregate price level and its Taylor approximation, we graph both in figure 7, fixing the response of the *adjusters'* price to $\hat{p}^{opt} = +5\%$ and $\hat{p}^{opt} = -5\%$.

The thick lines (P^+ and P^-) represent the true responses of the aggregate price level, while the dashed ones ($\hat{P}^+$ and $\hat{P}^-$) are the first-order Taylor expansions, given by (22). As the parameter θ grows, the gap between the outcome produced by a first-order Taylor expansion and the true value increases substantially. To understand why, take the limit of the aggregate price level as θ approaches the extreme values it can take ($\theta > 1$). When θ approaches one, the aggregate price level is given by:

$$\lim_{\theta \to 1} P = (\bar{p})^{\eta} \cdot \left(p^{opt}\right)^{1-\eta}$$

A first-order Taylor expansion in this case gives a full account of the aggregate price level's response (higher order expansions add no information). In this case then, the log-linearised solution portrays the model accurately. But when θ approaches infinity (goods become perfect substitutes), the aggregate price level is given by:

$$\lim_{\theta \to \infty} P = \min\left(\bar{p}, p^{opt}\right)$$

In this case the log-linearised version of the aggregate price level is most inaccurate. The true response of the aggregate price level to a monetary contraction is $\hat{P} = \hat{p}^{opt}$ and the true response to an expansion is $\hat{P} = 0$, while the log-linearised response is yielded by (22), an average of the two. The gap between the first-order Taylor expansion and the true aggregate price level is arbitrarily large in this case. Therefore, taking a first-order Taylor expansion of the constant elasticity of substitution function will only be a valid option when the goods consumed are highly differentiated (low θ). It is interesting to point out here that a second order Taylor expansions, represented

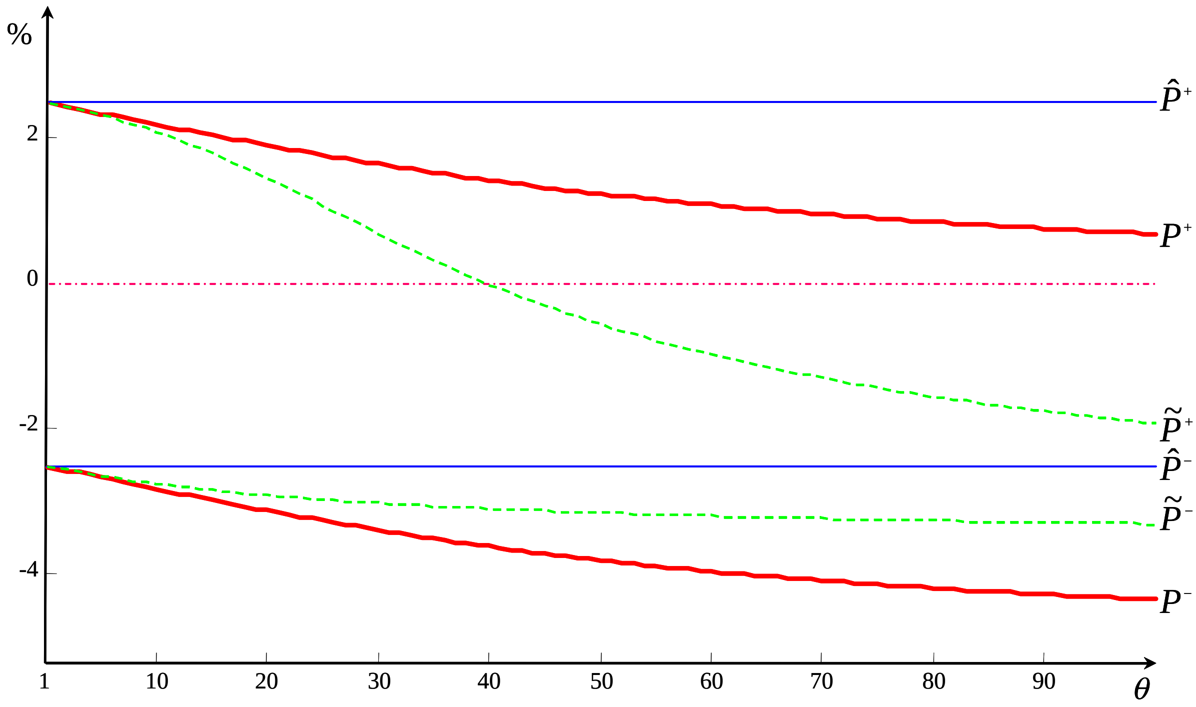

Figure 7: Response of the price level when $\hat{p}^{opt} = \pm 5\%$ as a function of θ. P is the true value of the price level, $\hat{P}$ is the first order approximation and $\tilde{P}$ is the second order approximation.

in figure 7 by $\tilde{P}^+$ and $\tilde{P}^-$, does not always improve the quality of the approximation. While it does for a fall in the optimal price, it does not for an increase.

The fact that a first-order Taylor expansion eliminates the elasticity of substitution from the composition of the aggregate price level has further consequences. Because linearising the two alternative price aggregators - P given by (19) and P^* given by (20)- yields identical expressions, the log-linearisation eliminates the ratio R from the relation between labour and consumption. This alters significantly the relation between labour and consumption and hence the determination of the equilibrium wage rate. Recall equation (18) related aggregate employment and aggregate output as:

$$l = R^\theta \cdot c$$

Linearising this expression gives:

$$\hat{l} \simeq \hat{c}$$

Because R is eliminated from the expression when taking a first-order Taylor expansion, the literature generally dismisses its importance. But the fact is that large values of the elasticity of substitution cause it to significantly affect the relation between labour and consumption. While in steady state the ratio is equal to one, we have shown in proposition 2 that in response to both positive and negative shocks, R becomes larger than 1 (so that $l > c$), implying that with a negative shock, the response of labour exceeds that of consumption in the event of a monetary expansion and falls short of it in a contraction. Note that as we have discuss in the previous section, this feature is instrumental in determining the direction of the asymmetric in the response of output.

5. Dynamic Analysis

We consider first the dynamic response under the assumption that we have use so far: those firms that adjust prices maximise myopic profits. This solution of this model does not involve numerical computations of any expectation terms. The solution of the model is given by a non-linear solver that combines the equilibrium conditions of the model. The accuracy level of the solution can be set arbitrarily high[10]. In the first period after the shock, the results of this model are identical to those we have presented so far. We first consider and how the asymmetry evolves over time and then reintroduce forward-looking behaviour in the analysis. In figure 8, the solid line corresponds to the response to the positive shock and the dashed one to the negative one. We consider first the results for the aggregate amount of labour, the real wage rate and the optimal price.

The response of the aggregate amount of labour in the first period is clearly asymmetric, with the response to a positive shock exceeding the negative one $(0.13\%, -0.10\%)$. The real wage rate again follows this asymmetric pattern $(0.96\%, -0.82\%)$, with its response being significantly larger than labour's because labour supply is inelastic (we have set $\gamma_l = -6.7$). In the second period after the shock, a second round of adjustment of prices causes both aggregates to quickly converge to their long-run steady state values. The response to both positive and negative shocks has reduced to $(0.02\%, -0.01\%)$ for aggregate labour and $(0.10\%, -0.10\%)$ for the real wage rate. The optimal price is set by *adjusters* at a constant markup over the wage rate. Thus, in the first period after the shock it follows the wage rate's asymmetric pattern $(1.87\%, -1.69\%)$. In the second period, following again the wage rate's, the response of the optimal price is considerably reduced $(1.10\%, -1.08\%)$. It is thus very close in this second period to its steady value $(+1\%, -1\%)$ and the asymmetry in its response has nearly vanished. Convergence towards the steady state is quick for the three magnitudes: beyond the second period they display close to negligible responses.

The asymmetry in the response of the optimal price (that *adjusters* set) generates a far larger income effect for the positive shock than for a negative one. The response of real aggregate demand departs in several respects from that of the other magnitudes we have described. In the first period after the shock, it is asymmetric $(0.10\%, -0.13\%)$, with the response to a negative shock 30% larger than to a positive one. Thus, the response of real aggregate demand to a negative shock exceeds that to a positive one, while the reverse is the case for the optimal price, aggregate labour and the real wage rate. In the second period after the shock, the response of real aggregate demand to the positive shock is close to negligible and it is very small for the negative one (-0.03%). As is well known (see Chari et al. (1999)), monetary shocks fail to produce persistent movements of output in this model. As a result the asymmetry in the response of output is also short-lived.

We can now introduce in our analysis the forward looking behaviour of firms, re-establishing (13), instead of (14) and so having firms maximising an infinite sum of expected future profits. Unlike the myopic firms model, we need now to compute

[10]The tolerance level we set in our myopic firms model is 10^{-14}.

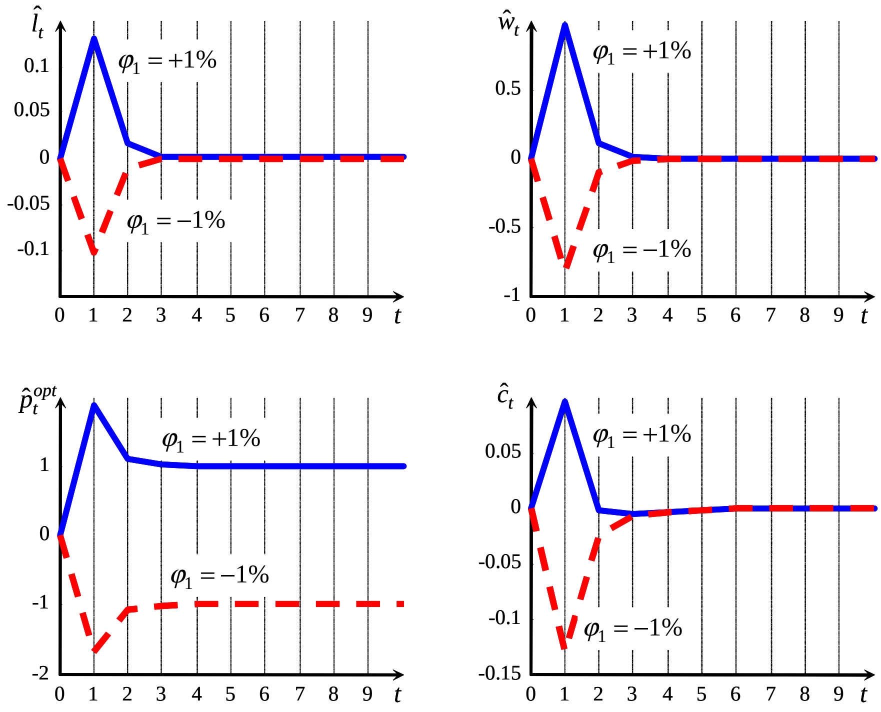

Figure 8: Impulse responses to small first period shocks ($\varphi_1 = \pm 1\%$) in the myopic firms model ($\gamma_l = -6.7$, $\gamma_c = 1$, $\eta = 0.5$).

numerically expectation terms. The results are obtained using two different non-linear numerical methods: the projection methods develop by Judd (1992) and the Parameterised Expectations Approach of Den Haan and Marcet (1990), delivering very close results[11]. Because firms are now forward looking the response of the optimal price has changed. We have drawn the response of the optimal price to $\pm 1\%$ shocks in the two models in the top panel of figure 9.

The thin lines correspond to the response of the optimal price in the model where firms maximise myopic profits (as in (14)) and the thick ones to the model where they maximise an infinite discounted sum of future profits, as described in section 2. In the first period after the shock, the response of the optimal price when firms are forward looking $(1.78\%, -1.62\%)$ is more limited to both positive and negative shocks than when they are myopic $(1.87\%, -1.69\%)$. Because they look ahead when adjusting, they know the effects of the nominal shock are short-lived, and real activity will progressively converge back to the steady state. As a result, the response of the optimal price, which is a function of the future path of real activity, is smoothed out relative to the myopic firms case.

The price level records a parallel response, although more limited since some firms fail to adjust prices. Hence, real aggregate demand records a larger response to both positive and negative shocks, as represented in figure 9. With a positive shock, because

[11]In both methods we fit functional forms to the expectation terms, as a function the vector of states given by g_t, $\tilde{M}_{t-1}/P_{t-1}$ and R_{t-1}. The results of accuracy tests are described in the appendix. We describe the solution method in a technical appendix, available from the author.

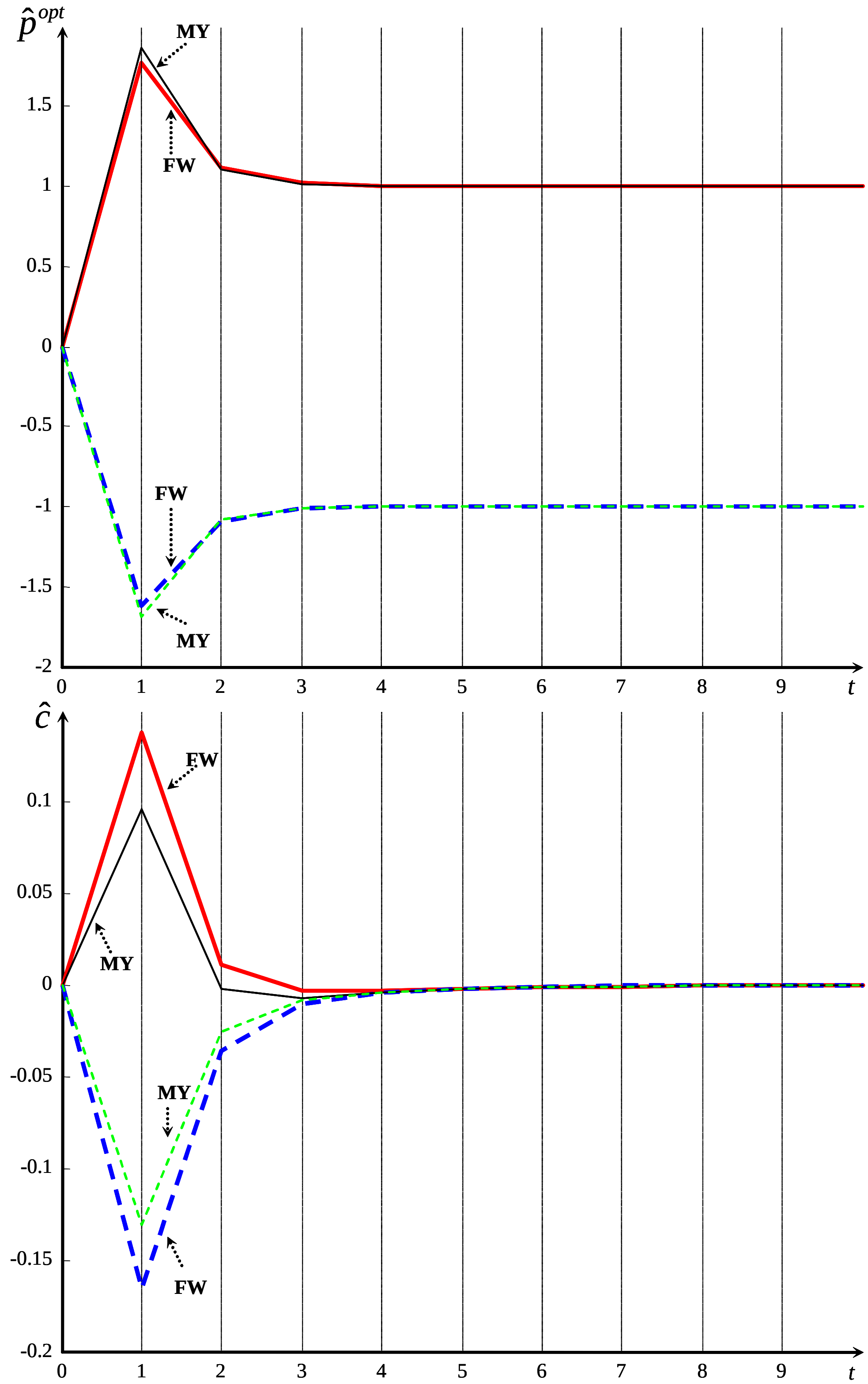

Figure 9: Comparison of myopic (thin lines) and forward-looking (thick ones) firms models with small ±1% shocks. Optimal price responses in the top panel and output responses in the bottom one.

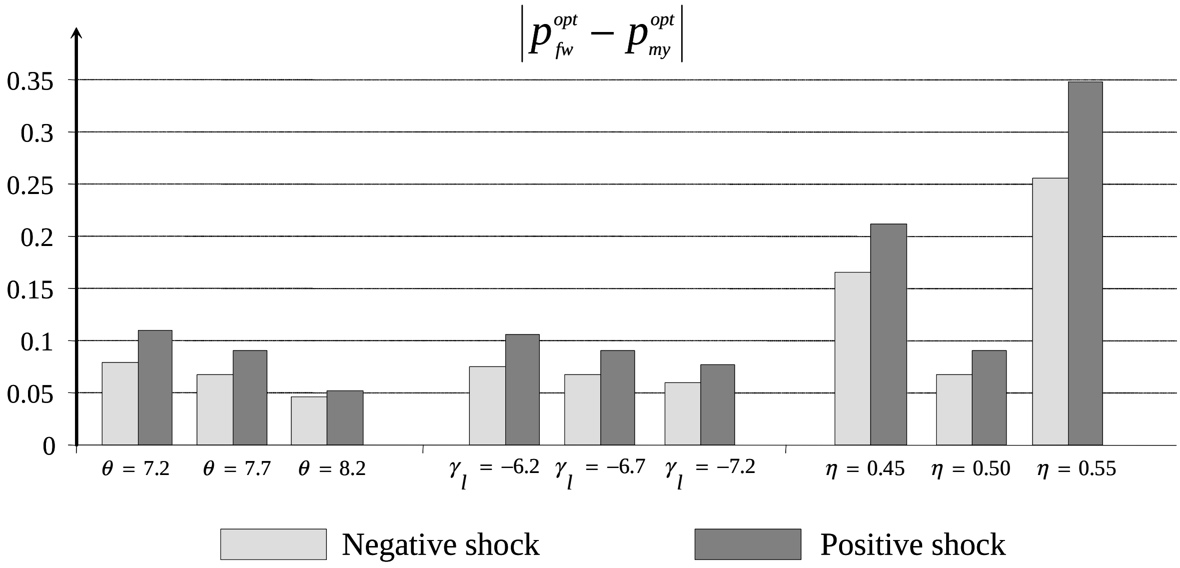

Figure 10: Comparison of the myopic and forward-looking firms models: first period response to $\pm 1\%$ nominal shocks. Unless otherwise specified: $\theta = 7.7$, $\gamma_l = -6.7$, $\eta = 0.5$ (middle bars in each grouping).

the response of the optimal price is more limited when firms are forward-looking, its income effect is weaker and real aggregate demand increases further than when firms are myopic. With a negative shock, the more limited response of the optimal price level yields a weaker negative response of the price level, and so real aggregate demand experiences a larger fall. Thus, forward looking behaviour magnifies the impact of the nominal shock on real aggregate demand.

Importantly, because the response of the optimal price is more limited when firms are forward-looking, the reallocation of demand becomes also less intense. Because the reallocation of demand is at the origin of the asymmetric outcome of the model, the introduction of forward-looking behaviour limits the size of the asymmetry. While the first period response of output with a negative -1% shock is 36% above the response to the positive shock[12] when firms maximise myopic profits, it is only 20% when firms are forward-looking. In the second and subsequent periods, the response of the optimal price is very close between the two models we are comparing (indistinguishable in the top panelof figure 9). However, a significant gap still remains between the responses of real aggregate demand in the two models. While the asymmetry remains, it becomes weaker as more firms adjust prices.

We find that the more limited response of the optimal price when firms are forward looking than when they are myopic holds in small brackets around our chosen parameter values[13]. We have represented in figure 10 the absolute value of the change in the first period's response of the optimal price, for the same small variations from our benchmark choice as before, from its forward looking value to its myopic one for

[12] Computed as $100 \cdot (-\hat{c}_1^- - \hat{c}_1^+)/\hat{c}_1^+$

[13] However, this is not universally true. One can find combinations of parameter values for which the response of the optimal price is actually larger when firms are forward-looking.

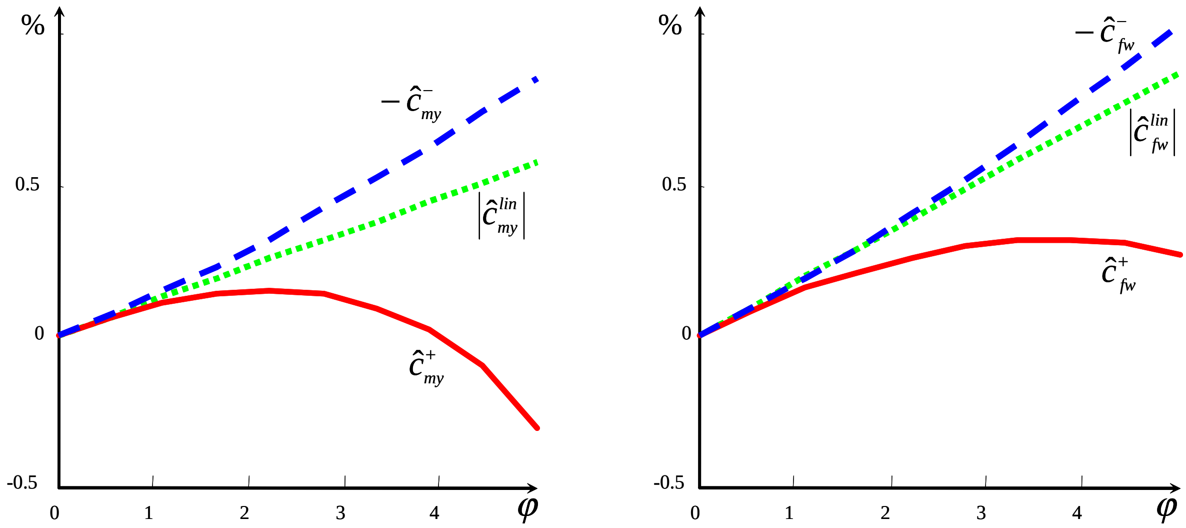

Figure 11: Response of real aggregate demand in the first period to a range of shocks. Myopic firms in the first panel, forward-looking in the second ($\gamma_l = -6.7$, $\gamma_c = 1$, $\eta = 0.5$).

both positive and negative shocks: $\left|\hat{p}^{opt}_{fr} - \hat{p}^{opt}_{my}\right|$. It is consistently negative for positive shocks and positive for negative ones: in both cases, the response is smaller when firms are forward looking than when they are myopic. However, the size of the change in the response of the optimal price between the two models does vary significantly with the value of the parameters.

Let us finally consider the response of real aggregate demand for different values of the nominal shock. The first panel in figure 11 corresponds to the first period response when firms are myopic, whereas the second corresponds to the forward-looking case. Because the response of the optimal price is more limited when firms are forward-looking, the response of real aggregate demand to both positive and negative shocks is larger. As a result, the introduction of forward-looking behaviour causes the response to large positive shocks to be positive, instead of negative as in the myopic firms case. Also, the size of the asymmetry has reduced considerably, and the accuracy of the log-linearisation has improved. A more limited response of the optimal price implies a weaker reallocation of demand towards the cheaper goods, explaining both results.

6. Conclusion

We have shown that the introduction of nominal rigidities in a framework where agents maximise their welfare leads to asymmetric effects of nominal aggregate demand shocks. When labour supply is sufficiently inelastic, real aggregate demand records a larger response with negative shocks. When labour supply is only mildly inelastic, the response of real aggregate displays the opposite pattern: it is larger with positive shocks. Finally, when the elasticity of substitution is small, positive shocks have similar effects to negative ones. We show that the size of the asymmetry is significantly reduced by the introduction of forward-looking behaviour. Finally, we illustrate the important quantitative and qualitative differences between our non-linear solution and the log-linearised one.

While studying the log-linearised version of the model has certain advantages, we believe it has important drawbacks. While the loss in accuracy of the method is small when the shock is, the loss in terms of our understanding of the forces driving the model's outcomes can actually be substantial. The new Keynesian Phillips curve is a convenient reduced form of the log-linearised supply side of the model, apt to be estimated or incorporated in a more complex framework. One of the claimed advantages of using this relation is that it to is derived from solid microfoundations. But the forces at work in the microfounded model are to a certain extent lost in its ultimate log-linearised reduced form, thus limiting our capacity to understand them.

7. APPENDIX

7.1. Proof to proposition 2:. *The ratio of the two price aggregators is larger than one whenever some firms set a different price than the others:* $R > 1$ *if* $p^{opt} \neq \bar{p}$.

Proof. We want to prove that the ratio of the two price alternative price aggregators satisfies $R > 1$ when $p^{opt} \neq \bar{p}$ for $\bar{p}, p^{opt} > 0$, $\theta > 1$ and $0 < \eta < 1$. The ratio of the two alternative price aggregators, $R = P/P^*$ can be written, using (**??**) and (**??**) as:

$$R = \frac{\left\{\eta \cdot (\bar{p})^{1-\theta} + (1-\eta) \cdot (p^{opt})^{1-\theta}\right\}^{\frac{1}{1-\theta}}}{\left\{\eta \cdot (\bar{p})^{-\theta} + (1-\eta) \cdot (p^{opt})^{-\theta}\right\}^{-\frac{1}{\theta}}}$$

Define first $d = p^{opt}/\bar{p} - 1 > 0$ and

$$S(d) = \left\{\eta + (1-\eta) \cdot (1+d)^{1-\theta}\right\}^{\frac{\theta}{\theta-1}} - \eta - (1-\eta) \cdot (1+d)^{-\theta}$$

Rearranging $R > 1$,

$$\begin{aligned}
\frac{\left\{\eta \cdot (\bar{p})^{1-\theta} + (1-\eta) \cdot (p^{opt})^{1-\theta}\right\}^{\frac{1}{1-\theta}}}{\left\{\eta \cdot (\bar{p})^{-\theta} + (1-\eta) \cdot (p^{opt})^{-\theta}\right\}^{-\frac{1}{\theta}}} &> 1 \Leftrightarrow \\
\left\{\eta \cdot (\bar{p})^{1-\theta} + (1-\eta) \cdot \left(p^{opt}\right)^{1-\theta}\right\}^{\frac{\theta}{\theta-1}} &< \eta \cdot (\bar{p})^{-\theta} + (1-\eta) \cdot \left(p^{opt}\right)^{-\theta} \Leftrightarrow \\
\left\{\eta + (1-\eta) \cdot \left(\frac{p^{opt}}{\bar{p}}\right)^{1-\theta}\right\}^{\frac{\theta}{\theta-1}} &< \eta + (1-\eta) \cdot \left(\frac{p^{opt}}{\bar{p}}\right)^{-\theta} \Leftrightarrow \\
S(d) &< 0
\end{aligned}$$

Therefore we need to prove that $S(d) < 0$. When $p^{opt} = \bar{p}$, $d = 0$ and $S(0) = 0$ $(R = 1)$. We then need to prove that $\partial S/\partial d > 0$ when $d < 0$ and $\partial S/\partial d < 0$ when $d > 0$. We have that

$$\frac{\partial S(d)}{\partial d} = -\theta\,(1-\eta)\left\{\eta + (1-\eta) \cdot (1+d)^{1-\theta}\right\}^{\frac{1}{\theta-1}} (1+d)^{-\theta} + \theta\,(1-\eta)\,(1+d)^{-\theta-1}$$

I need to prove that this derivative is larger than zero whenever d is:

$$\begin{aligned}
\frac{\partial S(d)}{\partial d} &= -\theta(1-\eta)\left\{\eta+(1-\eta)\cdot(1+d)^{1-\theta}\right\}^{\frac{1}{\theta-1}}(1+d)^{-\theta}+\theta(1-\eta)(1+d)^{-\theta-1}>0\\
0 &< -\theta(1-\eta)\left\{\eta+(1-\eta)\cdot(1+d)^{1-\theta}\right\}^{\frac{1}{\theta-1}}(1+d)^{-\theta}+\theta(1-\eta)(1+d)^{-\theta-1}\\
1 &> \left\{\eta+(1-\eta)\cdot(1+d)^{1-\theta}\right\}^{\frac{1}{\theta-1}}(1+d) \quad \text{since } \theta>0,\ \eta<1\\
1 &> (1+d)^{\theta-1} \quad \text{since } \theta>1,\ \eta<1
\end{aligned}$$

Thus, $\partial S/\partial d > 0$ when $d < 0$ and $\partial S/\partial d < 0$ when $d > 0$, and so $R > 1$ when $p^{opt} \neq \bar{p}$.
■

7.2. Accuracy tests. The degree of accuracy of the solution obtained varies with the parameter values chosen and with the size and sign of the shock. While it is at its highest with the parameter values that yield the analytical solution ($\gamma_c = 1$, $\gamma_l = 0$, $\theta = 2$), it falls as the homothopy reaches the more distant areas of the parameter space.. Following Judd (1998) we can give the level of accuracy in economic terms by calculating the one period optimisation error as a function of the nominal price set. We give the results of the accuracy test on the response of the optimal price to different shocks sizes (both positive and negative) for different values of the elasticity of labour supply in table 1. We base these on the forward-looking impulse response of the optimal price in section 5, for the first 8 periods after the shock. The degree of accuracy is lowest when labour supply is inelastic, due to the fact that the functional form we use to parameterise expectation has difficulties to absorb the non-linearity in the model's structure in this case, specially when the shock is large. In the worst case represented in table 1, firms make a 2.49% mistake when they set the optimal price based on the functional forms used to approximate the expectations. This error is small enough for our conclusions not to be affected by it.

	Period	1	2	3	4	5	6	7	8
γ_l=0,|φ|=1%	(+) shock	-0.0055%	-0.0047%	-0.0047%	-0.0047%	-0.0047%	-0.0047%	-0.0047%	-0.0047%
	(-) shock	-0.0042%	-0.0050%	-0.0049%	-0.0048%	-0.0048%	-0.0048%	-0.0048%	-0.0048%
γ_l=0,|φ|=5%	(+) shock	-0.0071%	-0.0066%	-0.0050%	-0.0046%	-0.0045%	-0.0045%	-0.0045%	-0.0045%
	(-) shock	-0.0009%	-0.0075%	-0.0057%	-0.0052%	-0.0051%	-0.0050%	-0.0050%	-0.0050%
γ_l=−6.7,|φ|=1%	(+) shock	0.3700%	0.0700%	0.0400%	0.0400%	0.0400%	0.0400%	0.0400%	0.0400%
	(-) shock	-0.1800%	0.0100%	0.0300%	0.0400%	0.0400%	0.0400%	0.0400%	0.0400%
γ_l=−6.7,|φ|=5%	(+) shock	2.4900%	0.3400%	0.0300%	0.0300%	0.0300%	0.0300%	0.0400%	0.0400%
	(-) shock	0.1600%	-0.1300%	0.0100%	0.0300%	0.0400%	0.0400%	0.0400%	0.0400%

Table 1: Accuracy in the computation of the optimal price

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