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A Computational Study On General Equilibrium Pricing of Derivative Securities^{*}

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Abstract

This paper analyses the accuracy of replicating portfolio methods in predicting asset prices. In a two-period, general equilibrium model with incomplete financial markets and heterogeneous agents, a computational study is conducted under various distributional assumptions. We focus on the price of a call option on an underlying risky asset. There is evidence that the value of the (approximate) replicating portfolio is a good approximation for the general equilibrium price for CRRA preferences, but not for CARA preferences. Furthermore, there is strong evidence that the introduction of the call option reduces market incompleteness and that the price of the underlying asset is unchanged. There is, however, inconclusive evidence on whether the availability of the option increases agents' welfare.

Keywords: Asset pricing, General equilibrium, Incomplete markets. *JEL codes*: D52, G12.

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1 Introduction

Ever since the seminal contributions of Black and Scholes (1973), Merton (1973), and Harrison and Kreps (1979), there has evolved a whole industry concerned with asset pricing based on martingale methods. These methods are based on the two fundamental theorems of finance, the first of which states that, in absence of arbitrage opportunities, there exists a probability measure under which each asset's price is the expectation of the future dividends. The second fundamental theorem then states that this measure is unique in incomplete markets. In a two-period economy with finitely many future states and one consumption good, the equivalent martingale measure can be interpreted as a vector of state prices, representing the price of consumption in each uncertain state in future.

In essence, financial markets are complete if the economy is isomorphic to an economy with a complete set of contingent contracts (a so-called Arrow-Debreu economy, see Debreu (1959, Chapter 7)). Completeness allows the market to determine asset prices in such a way that all agents' present value vectors are equalised.¹ The present value vector of an agent lists her individual valuation of each future state (in terms of the consumption good) given all asset prices.

From a practical point of view the two fundamental theorems are extremely useful. If markets are complete then, in order to value a (new) financial asset, no knowledge whatsoever about preferences or initial endowments is needed. In complete markets any new asset is redundant and its no-arbitrage value can be computed as a linear combination of the prices of the existing assets. The weights are determined by the orthogonal projection of the dividend stream of the new asset on the market span and are called the replicating portfolio. We call the resulting price the *replicating portfolio value*, RPV. In other words, in complete markets, asset pricing can be achieved by using observable data on prices, without appealing to portfolio theory and utility maximisation.

In incomplete markets, however, the present value vectors are, generically, not equal in equilibrium. Only their orthogonal projections on the market span are. In other words, there are several valid vectors of state prices in equilibrium. This happens because the dimension of the subspace orthogonal to the market space is larger than one. This, in turn, implies that new assets are, generically, not redundant. Therefore, no replicating portfolio exists. The best one can hope for is to find an approximate replicating portfolio via an orthogonal projection on the market space and, therefore, an approximate value for the new asset. This method has been

¹This is nothing more than the famous "price equals marginal rate of substitution" rule taught in undergraduate microeconomics. Here the substitution is between uncertain future states.

advocated by, for example Föllmer and Sondermann (1986).

There are mainly two reasons why the RPV might lead to structural errors in predicting a new asset's value. One relating to the use of the wrong replicating portfolio due to market incompleteness, and one relating to the use of the wrong prices for the existing assets due to changes in the market span. As to the former point, market incompleteness leads to non-existence of a replicating portfolio. Whether or not markets are incomplete is open for debate. However, even if markets are, in fact, complete, asset valuation typically only uses a subset of all traded assets. That is, asset pricing then takes place *as if* markets are incomplete.² This implies that, generically, for practical purposes the dividend stream of a new asset does not lie in the market span and only a partial spanning asset, or approximate replicating portfolio, can be obtained.

Secondly, the introduction of a new asset might change the prices of all other traded assets in equilibrium as well, due to the resulting change in the market span. This then leads to an error due to the fact that the "wrong" asset prices are used to determine the RPV.³ There is empirical evidence that the introduction of new batches of options has substantially changed asset prices between 1973 and 1986 (see Conrad (1989) and Detemple and Jorion (1990)). Some theoretical papers have been devoted to this topic. Weil (1992) and Elul (1997) show that the introduction of a new asset permits agents to better share risk. This weakens the need for precautionary savings and, hence, leads to a higher interest rate. This, in turn, reduces the prices of all assets in the economy. Oh (1996) shows that in economies with mean-variance preference, or CARA preferences and normally distributed asset payoffs, the price of any risky asset relative to the riskless bond is unaffected by changes in the market span. In a recent paper, Calvet et al. (2004) show that in a CARA-normal economy with limited participation relative prices are in fact influenced by financial innovation.

This paper conducts a computational study to assess these issues. A two-period economy with three heterogeneous agents is studied. On the financial market two assets, a riskless bond and a risky asset, are initially traded. Under several distributional assumptions we compare the performance of RPV as a predictor for the general equilibrium price of a call option on the risky asset for different strike prices. Agents' utility functions are either of the CARA or CRRA type. We also study whether the prices of the existing assets change substantially after the introduction of the call option.

First of all, we find that the introduction of the call option significantly increases

²See Ross (2005) for an elaboration of this point.

³In fact, this is reminiscent of the "Lucas-critique", see Lucas (1976).

the degree of market completeness, which is measured by the fraction of the variance of total initial endowments that can be traded on the market. Our findings also indicate that, if agents have CRRA preferences, the RPV is a good predictor for the general equilibrium value (GEV) of the option. If, however, agents have CARA preferences, then the RPV is significantly different form the GEV. Furthermore, the price of the underlying asset does not significantly change after the introduction of the option. We also find inconclusive evidence on the effect of the introduction of the option on agents' welfare, as measured by the equivalent variation. In most cases we find that the equivalent variation does not change significantly. This seems to imply that, in equilibrium, the price of the option is such that the increase in utility resulting from better consumption smoothing over risky states disappears. It is surprising to see that even richer agents can not benefit from the introduction of the option. Since the option is in zero net-supply, we do see that richer consumers sell the option to poorer agents and, hence, provide insurance. Since the richer agents are risk-averse themselves, however, they need a higher price to take on the additional risk. On balance these effects seem to cancel out in most cases. Finally, we find that the prices of existing asset explain the variance in the GEV of the option substantially better for CARA than for CRRA preferences.

The paper is organised as follows. In Section 2, the two-period general equilibrium model with incomplete financial markets (GEI) is introduced. In Section 3 the replicating portfolio approach is described in detail and in Section 4 the computational study is presented. Section 5 discusses the simulation results, while Section 6 concludes.

2 The GEI Finance Economy

The General Equilibrium model with Incomplete markets (GEI) explicitly includes incomplete financial markets in a general equilibrium framework. In this paper the simplest version is used. It consists of two time periods, t = 0, 1, where t = 0 denotes the present and t = 1 denotes the future. At t = 0 the state of nature is known to be s = 0. Uncertainty over possible states of nature at t = 1 is modelled by a probability space S = (S, S, P), where S is assumed to be a finite set indexed (with slight abuse of notation) by $s = 1, \ldots, S$. In the economy there are $H \in \mathbb{N}$ agents, or investors or households, indexed by $h = 1, \ldots, H$. There is one consumption good, which can be interpreted as income. A consumption plan for agent $h \in \{1, \ldots, H\}$ is a vector $x^h \in \mathbb{R}^{S+1}_+$, where x^h_s gives the consumption level in state $s \in \{0, 1, \ldots, S\}$.⁴

⁴In general a vector $x \in \mathbb{R}^{S+1}$ is denoted $x = (x_0, x_1) \in \mathbb{R} \times \mathbb{R}^S$ to separate x_0 in period t = 0and $x_1 = (x_1, \ldots, x_S)$ in period t = 1.

Each agent h = 1, ..., H, is characterised by a vector of initial endowments, $\omega^h \in \mathbb{R}^{S+1}_+$, and a utility function $u^h : \mathbb{R}^{S+1}_+ \to \mathbb{R}$. We denote aggregate initial endowments by $\omega = \sum_{h=1}^{H} \omega^h$. Regarding the initial endowments and utility functions the following assumptions are made.

Assumption 1. The vector of aggregate initial endowments is strictly positive, i.e. $\omega \in \mathbb{R}^{S+1}_{++}$.

Assumption 2. For each agent h = 1, ..., H, the utility function, u^h , is continuous, strictly monotone and strictly quasi-concave on \mathbb{R}^{S+1}_+ .

Assumption 1 ensures that in each period and in each state of nature there is at least one agent who has a positive amount of the consumption good. Assumption 2 ensures that each agent's demand is a continuous function.

It is assumed that the market for the consumption good is a spot market. The agents can smoothen consumption over time and uncertain states by trading on the financial market, where $J \in \mathbb{N}$ financial contracts are traded, indexed by $j = 1, \ldots, J$. The future payoffs (dividends) of these assets are collected in a matrix

$$A = [d_s^j]_{s=1,\dots,S}^{j=1,\dots,J} \in \mathbb{R}^{S \times J},$$

where d_s^j is the payoff of one unit of asset j in state s. The following assumption is made with respect to the matrix A.

Assumption 3. There are no redundant assets, i.e. rank(A) = J.

Assumption 3 can be made without loss of generality; if there are redundant assets then a no-arbitrage argument guarantees that its price is uniquely determined by the other assets. Let the market subspace be denoted by $\langle A \rangle = Span(A)$. That is, the market subspace consists of those income streams that can be generated by trading on the financial markets. If S = J, the market subspace consists of all possible income streams, i.e. markets are complete. If J < S, there is idiosyncratic risk and markets are incomplete.

A *GEI economy* is defined as a tuple $\mathcal{E} = ((u^h, \omega^h)_{h=1,\dots,H}, A)$. Given a GEI economy \mathcal{E} , agent h can trade assets by buying a portfolio $\theta^h \in \mathbb{R}^J$ given the (row)vector of prices $q = (q_0, q_1) \in \mathbb{R}^{J+1}$, where q_0 is the price for consumption in period t = 0 and $q_1 = (q_1, \dots, q_J)$ is the vector of security prices with q_j the price of security $j, j = 1, \dots, J$.⁵ Given a vector of prices $q = (q_0, q_1) \in \mathbb{R}^{J+1}$, the budget set for agent $h = 1, \dots, H$ is given by

$$B^{h}(q) = \left\{ x \in \mathbb{R}^{S+1}_{+} \middle| \exists_{\theta \in \mathbb{R}^{J}} : q_{0}(x_{0} - \omega_{0}^{h}) \leq -q_{1}\theta, x_{1} - \omega_{1}^{h} = A\theta \right\}.$$
(1)

 $^{{}^{5}}$ We follow the convention of denoting prices in row vectors and quantities in column vectors.

Given the asset payoff matrix A we will restrict attention to asset prices that generate no arbitrage opportunities, i.e. asset prices q such that there is no portfolio generating a non-negative income stream. Such asset prices exclude the possibility of "free lunches". The link between utility maximisation follows from the following theorem (see, for example, Magill and Quinzii (1996)).

Theorem 1 (Fundamental Theorem of Asset Pricing). Let \mathcal{E} be a finance economy satisfying Assumption 2. Then the following conditions are equivalent:

- 1. $q \in \mathbb{R}^{J+1}$ permits no arbitrage opportunities,
- 2. $\forall_{h=1,\dots,H}$: $\arg\max\{u^h(x^h)|x^h\in B^h(q)\}\neq \emptyset$,
- $\exists. \ \exists_{\pi \in \mathbb{R}^S_{++}} : q_1 = \pi A,$
- 4. $B^h(q)$ is compact for all $h = 1, \ldots, H$.

The vector $\pi \in \mathbb{R}^{S}_{++}$ can be interpreted as a vector of state prices. Condition 3 therefore states that any no-arbitrage price for security j equals the present value of security j given some vector of positive state prices π . As a consequence of this theorem, in the remainder we restrict ourselves to the set of no-arbitrage prices

$$Q = \{ q \in \mathbb{R}^{J+1} | q_0 > 0, \exists_{\pi \in \mathbb{R}^S_{++}} : q_1 = \pi A \}.$$
(2)

An important consequence of Theorem 1 is that in complete markets state prices are uniquely determined up to normalisation.⁶ If one normalises state prices on the unit simplex, π can be interpreted as a probability measure. Since no-arbitrage prices are simply the expected value of asset payoffs under π , this probability measure is usually referred to as the *martingale measure*. Furthermore, note that Theorem 1 does not require equilibrium considerations at all. So, under complete markets only Assumption 2 is needed for asset pricing. If markets are incomplete, however, π is not uniquely determined. This is exactly the reason why asset pricing in incomplete markets is conceptually much more difficult.

Under Assumption 2, Theorem 1 shows that the demand function $x^h(q)$, maximising investor h's utility function $u^h(x)$ on $B^h(q)$, is well-defined for all $h = 1, \ldots, H$, and all $q \in Q$. It can easily be shown that $x^h(q)$ and the security demand function, $\theta^h(q)$, determined by $A\theta^h(q) = x_1^h(q) - \omega_1^h$, are continuous on Q.

Define the excess demand function $f:Q\to \mathbb{R}^{J+1}$ by

$$f(q) = (f_0(q), f_1(q)) = \left(\sum_{h=1}^H (x_0^h(q) - \omega_0^h), \sum_{h=1}^H \theta^h(q) - \Theta\right),$$

⁶The space orthogonal to $\langle A \rangle$ is one-dimensional.

where $\Theta \in \mathbb{R}^J_+$ denotes each asset's net-supply. A financial market equilibrium (FME) for a GEI economy \mathcal{E} is a tuple $((\bar{x}^h, \bar{\theta}^h)_{h=1,\dots,H}, \bar{q})$ with $\bar{q} \in Q$ such that:

- 1. $\bar{x}^h \in \arg \max\{u^h(x^h) | x^h \in B^h(\bar{q})\}$ for all $h = 1, \dots, H$;
- 2. $A\bar{\theta}^h = \bar{x}^h_1 \omega^h_1$ for all h = 1, ..., H;

3.
$$\sum_{h=1}^{H} \bar{\theta}^h = 0.$$

It is easy to show that $q \in Q$ is an FME if and only if f(q) = 0. The following result is proved in e.g. Hens (1991) and Talman and Thijssen (2006).

Theorem 2. Let \mathcal{E} be a GEI economy satisfying Assumptions 1–3. Then there exists $q \in Q$ such that f(q) = 0.

3 The Value of a New Financial Asset

Let S = (S, S, P) be a (discrete) probability space and let $\mathcal{E} = (u, \omega, A)$ be a twoperiod GEI economy, with J assets. Suppose that a new asset is introduced with future dividends $d^{J+1} \in \mathbb{R}^S$. This new asset can be a new financial product which is actually going to be traded on the financial market, like a derivative security. It could also represent the risky payoffs of a real investment project of a firm. In this case the asset will not actually be traded, but the firm might wish to value the project as the shareholders would if the project were traded.⁷

In complete markets the new asset is redundant and its payoff can, hence, be written as a linear combination, θ , of the columns in A. The vector θ is called the *replicating portfolio* and the value of the redundant asset should equal θq , where q is the vector of prices of the J assets in A.

Generically, however, if financial markets are incomplete, then no unique replicating portfolio exists. Following Föllmer and Sondermann (1986) one could use the orthogonal projection of d^{J+1} on $\langle A \rangle$ instead. Let $\theta_A(d^{J+1})$ denote the (unique) replicating portfolio of $proj_{\langle A \rangle}(d^{J+1})$, where $proj_{\langle A \rangle}(x)$ denotes the projection in $|| \cdot ||_2$ of $x \in \mathbb{R}^S$ onto $\langle A \rangle$. The value of this approximate replicating portfolio, denoted by $RPV(d^{J+1})$, is then

$$RPV(d^{J+1}) = q\theta_A(d^{J+1}).$$
(3)

From a practical point of view the RPV approach is appealing, since $\theta_A(d^{J+1})$ is essentially obtained by performing a linear regression of d^{J+1} on the existing

⁷See Dixit and Pindyck (1994) for an introduction to the valuation of real investment projects using financial option pricing techniques.

assets in A. For example, the matrix A could consist of past observations on the dividends of the J stocks. If the new asset is, for example, a derivative security, which is written on one of the assets in A, then d^{J+1} can be computed for all observations, and $\theta_A(d^{J+1})$ can be obtained. In complete markets, the derivative security is redundant and, hence, the aforementioned regression should have $R^2 = 1$. So, $R^2 < 1$ is an indication of market incompleteness.

The replicating portfolio approach computes the expectation of the price of the new asset, conditional on the existing market span, using risk-neutral probabilities that are derived from market prices. A clear advantage of this procedure is that one only uses observable dividends and prices and no unobservables like preferences and endowments. However, if markets are incomplete, the replicating portfolio approach might lead to structural errors if it holds that $d^{J+1} \notin \langle A \rangle$. The reasons are two-fold. Firstly, agents in the economy will, typically, not agree on the valuation of the residual of d^{J+1} . This is the orthogonal projection of d^{J+1} on the orthogonal complement of $\langle A \rangle$, $\langle A \rangle^{\perp}$.⁸ Secondly, RPV assumes that in the economy with the new asset the prices of the existing assets do not change. There is, however, no guarantee that this will happen.

In order to account for market incompleteness and changes in the market span, $\langle A \rangle$, let $\tilde{A} = \begin{bmatrix} A & d^{J+1} \end{bmatrix} \in \mathbb{R}^{S \times (J+1)}$ be the asset payoff matrix *after* the derivative security has been introduced in the market. Note that, if initial endowments include asset holdings and the new security is introduced in non-zero net-supply, then initial endowments change as well to, say, $\tilde{\omega}$. That is, after a security with payoffs d^{J+1} has been introduced, the new GEI economy is $\tilde{\mathcal{E}} = (u, \tilde{\omega}, \tilde{A})$. Let $\tilde{q} \in \mathbb{R}^{J+1}$ be a vector of equilibrium prices in this economy. Then the general equilibrium value of the new asset, denoted by $GEV(d^{J+1})$, is

$$GEV(d^{J+1}) = \tilde{q}_{J+1}.$$
(4)

The main difference with the approximate replicating portfolio value is that the RPV values the investment project in the economy \mathcal{E} , whereas the GEV values the investment in the economy $\tilde{\mathcal{E}}$. Note that, if the new asset is perfectly correlated with a linear combination of the J original assets, then $GEV(d^{J+1}) = RPV(d^{J+1})$.

To illustrate how the RPV can provide a poor proxy for the GEV, consider an economy with two agents, three states of nature and one asset. The asset is a contingent contract on the second state, i.e. $A = [e_2]$, where e_i is the *i*-th unit vector. So, $e_2 \in \mathbb{R}^3$ is a basis for $\langle A \rangle$. Initial endowments are taken to be $\omega^1 =$ (2,1,1,1) and $\omega^2 = (1,2,2,2)$. Suppose that both agents have identical, timeseparable von Neumann-Morgenstern utility functions with constant relative risk

 $^{^8\}mathrm{See},$ for example, Magill and Quinzii (1996) for an extensive discussion.

aversion (CRRA),

$$u^{h}(x_{0}, x_{1}) = \log(x_{0}) + 0.95 \sum_{s=1}^{3} \frac{1}{3} \log(x_{s}).$$

After normalising the price of date 0 consumption to 1, the equilibrium price for the asset in this economy is q = 0.3246. Suppose now that a contingent contract for the third state is introduced, i.e. $\tilde{A} = \begin{bmatrix} e_2 & e_3 \end{bmatrix}$. In this case $\{e_2, e_3\}$ is a basis for $\langle \tilde{A} \rangle$. Since the dividend stream of the new asset is orthogonal to $\langle A \rangle$, it immediately follows that $\theta_{\langle A \rangle}(e_3) = 0$, implying that $RPV(e_3) = 0$. Given the same endowments, the equilibrium prices of the two assets are $\tilde{q} = (0.3221, 0.3221)$. This example makes clear that RPV can be a bad approximation of the true value of the asset. It also shows that the prices of existing assets can change when a new asset is introduced.

This example is, of course, an extreme case. Most financial innovations will not be orthogonal to the existing market span. It does show, however, that in general it will be difficult to say how accurate the RPV predicts the value that the market will assign to a financial innovation. In the next section we will, therefore, conduct some simulation experiments to asses the accuracy of RPV for the introduction of a particular financial innovation, namely a call option on an existing risky asset.

4 A Simulation Analysis

In this section, the two valuation methods presented in Section 3 are numerically assessed, under several distributional assumptions, for the introduction of a call option on the *J*-th asset. We are interested in a number of empirical questions, the first of which is the accuracy of $RPV(d^{J+1})$ in predicting $GEV(d^{J+1})$. We also want to assess whether the equilibrium price of the underlying asset changes significantly. This would have important consequences in itself for the practise of derivative pricing. If, namely, the price of the underlying asset changes significantly with the introduction of an option, then the standard assumption of an exogenous and unchanging underlying asset price process (like a geometric Brownian motion) might be questioned.

Apart from the market value of a financial innovation, however, an important question is how much better off the agents in the economy are after the introduction of the new asset. In standard microeconomic models one often uses the equivalent or compensating variation to measure effects on agents' wealth of price and/or income changes. Since the equivalent variation takes current prices – asset prices before the introduction of the new asset – as the starting point this seems the better approach in the current setting. The equivalent variation of the new asset, $EV(d^{J+1})$, is

defined as

$$EV(d^{J+1}) = \sum_{h=1}^{H} e^{h}(q, u^{h}) - q\theta_{1}^{h},$$

where for all h, $e^h(q, u^h)$ solves

$$e^{h}(q,\bar{u}^{h}) = \min_{\{\theta \in \mathbb{R}^{J+1} | \omega^{h} + W\theta \ge 0\}} \{q_{\mathbf{1}}\theta | u^{h}(\omega_{0}^{h} - \frac{q_{\mathbf{1}}}{q_{0}}\theta, \omega_{\mathbf{1}}^{h} + \tilde{A}\theta) \ge \bar{u}^{h}\},$$

where $W = \begin{bmatrix} -q \\ \tilde{A} \end{bmatrix}$, q is an equilibrium price vector in the economy \mathcal{E} , and \bar{u}^h is the utility of agent h at prices q. That is, $EV(d^{J+1})$, measures the total amount agents would want to pay for the new asset under *current* equilibrium prices and is, as such, a monetary measure for the welfare change resulting from the introduction of the new asset.

Markets are incomplete when J < S. In itself, however, this does not give an indication of the degree of market incompleteness. The possibilities of agents to trade their initial endowments, for example, give a better indication. Indeed, if an agent's endowments lie in the market space, $\omega^h \in \langle A \rangle$, then for this agent, market are *effectively* complete, because her endowments are uniquely priced. Following Calvet et al. (2004), we define the degree of market completeness of the economy \mathcal{E} as the fraction of variability in total endowments that is traded on the market, i.e.

$$\alpha(\mathcal{E}) = \frac{Var(proj_{\langle A \rangle}(\omega^h))}{Var(\omega)}.$$

This leads to the following quantities of interest. Let q and \tilde{q} be the equilibrium price vectors in the economies \mathcal{E} and $\tilde{\mathcal{E}}$, respectively. For all simulations, we will test the following hypotheses,

$$H_0: \quad \alpha(\mathcal{E}) = \alpha(\tilde{\mathcal{E}}),\tag{5}$$

$$H_0: EV^h(d^{J+1}) = 0, \text{ all } h = 1, \dots, H,$$
 (6)

$$H_0: \quad q_j = \tilde{q}_j, \text{ all } j = 1, \dots, J, \tag{7}$$

$$H_0: \quad GEV(d^{J+1}) = RPV(d^{J+1}), \tag{8}$$

The first hypothesis tests whether market incompleteness changes significantly by the introduction of the call option on the risky asset. The second set of hypotheses test whether the agents in the economy are actually significantly better of with the new asset. The third hypothesis tests whether the equilibrium price of the underlying asset changes significantly. Finally, the last hypothesis tests whether the general equilibrium value and the (approximate) replicating portfolio value are significantly different. For each of these tests we compute the asymptotically normal z-statistic.

Furthermore, we investigate how good the asset prices (in \mathcal{E}) predict the equilibrium option price (in $\tilde{\mathcal{E}}$). This is achieved by estimating the regression equation

$$\tilde{q}_{J+1} = \beta_0 + \sum_{j=1}^J \beta_j q_j + \varepsilon, \qquad (9)$$

using ordinary least squares. The R^2 of this regression indicates how much of the variance in $GEV(d^{J+1})$ is explained by changes in the prices of the existing assets.

4.1 General Simulation Set-Up

It is assumed that there are J = 2 assets in the economy. The first asset is a riskless bond with payoff stream $d^1 = 1_S$, which is in zero supply. The second asset is a risky asset with

$$d^2 = \beta f + (1 - \beta)\varepsilon,$$

where $f \in \mathbb{R}^S$ is an economy-wide factor, $\varepsilon \in \mathbb{R}^S$ is an asset-specific shock, and $\beta \in [0,1]$ is the asset's exposure to the economy-wide factor. Throughout, β is drawn randomly from the interval [0.25,0.75]. The risky asset is assumed to be in unit supply.

The economy consists of H = 3 agents with time-separable von Neumann–Morgenstern utility functions

$$u^{h}(x_{0}, x_{1}) = v^{h}(x_{0}) + \delta^{h} \sum_{s=1}^{S} p_{s}^{h} v^{h}(x_{s}),$$

where δ^h is the discount rate of agent h and p_s^h is the probability which agent h assigns to state s. Throughout, we take $\delta^h = 0.95$ and, unless otherwise stated, $p_s^h = 1/S$, all $h = 1, \ldots, H$ and $s = 1, \ldots, S$. The function $v(\cdot)$ is either of the CARA or the CRRA type, i.e.

$$v^{h}(x) = 1 - e^{-\gamma^{h}x}, \quad \text{or} \quad v^{h}(x) = \frac{x^{1-\gamma^{h}} - 1}{1 - \gamma^{h}},$$

where the rate of risk aversion, γ^h , is randomly drawn from $\{1, \ldots, 6\}$, all $h = 1, \ldots, H$.

Initial endowments consist of labour income (wages) and dividend income. For agent h, labour income is equal to

$$L^h = \zeta^h f + (1 - \zeta^h) l^h,$$

where $l^h \in \mathbb{R}^S$ is an agent specific wage shock and ζ^h is the exposure of agent *h*'s wages to the economy-wide factor. Throughout we take $\zeta^h = 0.5$, all $h = 1, \ldots, H$. The initial portfolios, θ_0^h , of risky asset holdings are $\theta_0^1 = 0$, $\theta_0^2 = 1/3$, and $\theta_3^2 = 2/3$. Endowments are assumed to consist for 2/3 of wages and for 1/3 of asset income, i.e.

$$\omega_1^h = \frac{2}{3}L^h + \frac{1}{3}d^2\theta_0^h.$$

Date 0 initial endowments are given by $\omega_0 = \begin{bmatrix} 2/3 & 1 & 4/3 \end{bmatrix}$.

We consider the valuation of a call option on the risky asset, i.e.

$$d^3 = \max\{0, d^2 - d_K^2 \mathbf{1}_S\},\$$

where the strike price, d_K^2 , is the K-th percentile of d^2 . Finally, every run consists of 100 simulations. All equilibria are computed using a differentiably implementable homotopy algorithm, constructed by Herings and Kubler (2002). ⁹

4.2 Tri-Nomial Shocks

As a first model, we study economies where the factor and the asset and wage specific shocks form a tri-nomial tree. That is, we take

$$f \in \{0.89, 1.02, 1.15\}$$

$$\varepsilon \in \{0.89, 1.02, 1.15\}$$

$$l^h \in \{0.92, 1.02, 1.12\} \text{ all } h = 1, \dots, H.$$

This implies that $\mathbb{E}(d^2) = \mathbb{E}(L^h) = 1.02$, which represents an average 2% increase in wages and a dividend rate of 2%. Also, $\sigma(L^h) = 0.125$, where $\sigma(\cdot)$ is the standard deviation. The variance of d^2 depends on the, randomly generated, factor loading β . For all simulations we take K = 0.5.

The results of the tests (5)-(8) as well as the point estimates, *t*-values and R^2 of the regression (9) are reported in Table 1 of Appendix B. Both for CARA and CRRA utility, the coefficients in the regression (9) of the prices of the riskless and the risky assets are negative and positive, respectively, and are significant at the 99% level. The R^2 is substantially higher, though, for the CARA regression.

Furthermore, in both cases the introduction of the option does not change the degree of market completeness (5) significantly. However, $GEV(d^3)$ is significantly different from $RPV(d^3)$ in both cases (at the 1% and 10% level for CARA and CRRA, respectively). The equivalent variation is only significant (at the 1% level) for agent 3 (the richest agent) in the CRRA simulations. This could indicate that

⁹see Appendix A for a description of the algorithm.

the additional utility that agents derive from better opportunities for risk-sharing due to the presence of the option are largely offset by the price they pay for this asset. Only agent 3 benefits significantly, possibly because this agents finds herself to be the provider of consumption smoothing possibilities over risky states to the other, less well-off, agents.

Finally, in either case, the price of the underlying asset does not significantly change after the introduction of the option. Only for the CRRA case does the price of the riskless asset change significantly (at the 10% level).

4.3 Uniform Shocks

In the second model, we study economies where the factor and the asset and wage specific shocks are uniformly distributed. That is, we take

$$f \sim U(0.8, 1.24)$$

 $\varepsilon \sim U(0.8, 1.24)$
 $l^h \sim U(0.85, 1.2)$ all $h = 1, \dots, H$.

Again, this implies that $\mathbb{E}(d^2) = \mathbb{E}(L^h) = 1.02$ and that the variance of labour income is lower than the variance of asset income. In the finite setting that is considered in this paper, these distributions are achieved by simulating 500 observations on f, ε , and l^h , according to the distributions specified above.¹⁰ For these, and the following, simulations the option's strike price K is drawn randomly from the interval [0.25,0.75].

The results of the tests (5)-(8) as well as the point estimates, *t*-values and R^2 of the regression (9) are reported in Table 2 of Appendix B. Both for CARA and CRRA utility, the coefficients in the regression (9) of the prices of the riskless and the risky assets are negative and positive, respectively, and are significant at the 99% level. The R^2 is substantially higher, again, for the CARA regression.

In both cases, the introduction of the option changes the degree of market completeness (5) significantly at the 1% level. However, $GEV(d^3)$ is significantly different from $RPV(d^3)$ (at the 1% level) only for the CARA specification. The equivalent variation is now only significant (at the 1% level) for agents 1 and 2 in the CARA specification, but not for agent 3 at all.

Finally, for either utility function, the price of the underlying asset does not significantly change after the introduction of the option. Only for the CRRA case does the price of the riskless asset change significantly (at the 5% level).

¹⁰That is, S = 500.

4.4 Log-Normal Shocks

In the third model, we study economies where the factor and the asset and wage specific shocks are log-normally distributed. We simulate, as before, 500 draws for f, ε , and l^h , where

$$f \sim LN(1.02, 0.01)$$
 (10)

$$\varepsilon \sim LN(1.02, 0.01) \tag{11}$$

$$l^h \sim LN(1.02, 0.0025)$$
 all $h = 1, \dots, H.$ (12)

Again, we assume that all random variables grow at an average rate of 2%, with a smaller variance for labour income than dividends.

All empirical results are reported in Table 3 of Appendix B. Again, the coefficients in the regression (9) of the prices of the riskless and the risky assets are negative and positive, respectively, and are significant at the 99% level. The R^2 is substantially higher, again, for the CARA regression.

In both cases, the introduction of the option changes the degree of market completeness (5) significantly at the 1% level. However, $GEV(d^3)$ is significantly different from $RPV(d^3)$ (at the 1% level) only for the CARA specification. In this model, the equivalent variation is now significant at the 1% level for agent 1 in both specifications, and at the 5% level for agent 3 only for the CRRA specification. This time, the equivalent variation for agent 1 is insignificant for both specifications. Finally, for either utility function, the prices of both the riskless asset and the underlying asset do not change significantly after the introduction of the option.

4.5 Log-Normal Shocks with Downward Jumps

One implication of assuming preferences exhibiting risk aversion is that agents not only care about mean and variance, but also about higher moments. In order to ascertain the influence of such higher moments, we study simulations where the dividends and wages are skewed. This is achieved in the following way. After drawing 500 observations for f, ε , and l^h according to (10)–(12), respectively, we randomly draw 25 observations of f, which we replace by the smallest draw of f. This then represents a log-normal distribution with downward jumps.

All empirical results are reported in Table 4 of Appendix B. Again, the coefficients in the regression (9) of the prices of the riskless and the risky assets are negative and positive, respectively, and are significant at the 99% level. The R^2 is substantially higher, again, for the CARA regression.

The degree of market completeness again changes significantly at the 1% level. Also, most equivalent variations are, again, insignificant. The general equilibrium value, $GEV(d^3)$, is significantly different from $RPV(d^3)$ at the 1% level only for the CARA specification, and the change in the price of the underlying asset is insignificant. At the 10% level, the price change for the riskless asset in the CARA specification is significant.

4.6 Heterogeneous Beliefs

Finally, we look at the influence of heterogeneous beliefs. So far, it has been assumed that all agents believe that all states $s = 1, \ldots, S$, are equally likely. In this simulation run we allow beliefs to be heterogeneous. Wages and dividends are constructed in the same way as in the previous model. In addition, three different types of beliefs are assumed: uniform, pessimistic, and optimistic. An agent with uniform beliefs attaches equal probabilities to all states of nature. An agent with pessimistic beliefs assigns higher probabilities to states of nature with a low outcome. This is achieved in the following way. The 33% of states with the lowest 33% of values for the factor f are assigned a probability twice as high as the 33% of states with the highest 33% of values for f. An agent with optimistic beliefs constructs beliefs just the other way around. In every simulation round and for every agent, the belief type is drawn randomly.

All empirical results are reported in Table 5 of Appendix B. Again, the coefficients in the regression (9) of the prices of the riskless and the risky assets are negative and positive, respectively, and are significant at the 99% level. The R^2 is substantially higher, again, for the CARA regression.

The degree of market completeness again changes significantly at the 1% level. For the CRRA specification, all agents equivalent variations are significantly different from zero (at the 5% or 1% level). The general equilibrium value, $GEV(d^3)$, is significantly different from $RPV(d^3)$ at the 1% level only for the CARA specification. The changes in the prices of both the riskless asset and the underlying asset are insignificant.

5 Discussion

Based on the simulations, several observations can be made on the effects on the model economy of the introduction of new financial assets. We can also say something on the robustness of the (approximate) replicating portfolio method as a proxy for the general equilibrium value of a new asset in economies with heterogeneous agents and incomplete financial markets. A first observation is that, apart from the tri-nomial model, the introduction of a call option on the risky asset significantly changes (at the 1% level) the degree of market completeness as defined in (5). Since all z_{α} are positive, this implies that in an economy with risk-averse agents, the new asset should have positive value. If, however, the welfare change of the agents is assessed using the equivalent variation, no clear picture for this value emerges. Only in some models, for some preference specifications and for some agents the equivalent variation is significantly different from zero. This implies that the price for additional consumption smoothing over risky states that the new asset provides is such that no agent actually makes a profit on it; not even the richest agent. This might happen because all agents are risk averse and, therefore, have a positive demand for additional consumption smoothing. Since the option is in zero supply, however, the richer agent will provide this product to the poorer ones, but needs a higher price to compensate for the loss of her own consumption smoothing resulting from selling the option (i.e. from taking on more risk).¹¹

Secondly, the price of the existing risky asset does not significantly change due to the introduction of the new asset, even though the new asset is not perpendicular to the existing risky asset. In other words, the demand for the risk sharing opportunities provided by the risky asset remains unchanged. The price of the riskless asset also remains unchanged in virtually all models. From this it can be concluded that in economies with incomplete markets and heterogeneous risk averse agents, the demand for a new asset is solely determined by the *additional* market span it provides.

Thirdly, for all distributions, the CARA specification leads to significantly different general equilibrium values compared to the RPV (at the 1% level). For the CRRA specification, however, the difference is not significant (at the 1% level) for any distribution. This seems to indicate that RPV is a better predictor for CRRA than for CARA preferences. This result might be explained by the following fact. It is well-known (cf. Magill and Quinzii (1996, Section 16)) that if agents have *identical* CRRA preferences, then the date 1 consumption of each agent is proportional to total endowments. Since initial endowments do not change after the introduction of the option in our simulated economies (the option is in zero supply), linearity of the consumption pattern with respect to initial endowments in \mathcal{E} will not change in $\tilde{\mathcal{E}}$. This linearity might imply that the orthogonal projection of the dividend stream of the new asset on the existing market stream leads to a good proxy of the new asset's price. Apparently, heterogeneity of the rate of relative risk aversion does not

¹¹A crucial assumption for this result might be that options are in zero supply. If options are in unit supply (managerial stock options for example) then they change initial endowments and might give their owners a possibility to insure non-owners against risk without a loss of utility to themselves.

change this result too much.

Finally, consider the regression (9). For all models the following results hold. Firstly, the coefficient of the price of the riskless asset is negative and significant at the 1% level. Secondly, the coefficient of the price of the underlying risky asset is positive and significant at the 1% level. In other words, the higher the interest rate¹² or the price of the underlying asset, the higher the price of the option. Also, the R^2 of the simulations for the CARA specification is consistently and substantially higher than for the CRRA specification. So, for CARA preferences, the variability in the prices of the riskless and underlying risky assets explain more of the variability in the option's price than for CRRA preferences. This seems puzzling since existing asset prices predict the option's price better via RPV for CRRA preferences than for CARA preferences.

6 Conclusions

Over the past decades, the literature on asset pricing has developed separately from the literature on optimal portfolio choice. For a large part this is due to the two fundamental theorems of asset pricing. The first fundamental theorem tells us that, in the absence of arbitrage opportunities, asset prices are a linear combination of the assets' dividends. The coefficients are called state prices. The second theorem, in addition, establishes that if markets are complete then these state prices are unique. This has led to an entire industry which computes the value of new assets based on the assumptions of no-arbitrage and market completeness. The big advantage of these assumptions is that asset prices can be computed without any knowledge of the agents' preferences and endowments. Since each new asset is redundant in complete markets, there exists a unique replicating portfolio, the value of which should be the no-arbitrage price of the new asset.

If, however, markets are incomplete, then the vector of state prices is not uniquely determined and there is no replicating portfolio. As a result, there is no clear-cut way in which to obtain *the* arbitrage-free price of a new – and as yet non-traded – asset. Instead, there is a plethora of prices, each sustained by a state price vector which does not admit arbitrage opportunities. If one still assumes that in such circumstances the orthogonal projection of the dividends of the new asset on the market span is the replicating portfolio, then one risks making a systematic error in determining the asset's value.

A simulation study shows that under several distributional assumptions this is

¹²The interest rate, r, can be computed from the price of the riskless asset, q_1 , in the following way, $q_1 = \frac{1}{1+r}$.

indeed the case if agents have CARA preferences. For CRRA preferences, however, the approximation via the (approximate) replicating portfolio is statistically insignificantly different from the general equilibrium value.

Several additional points can be raised, however. In this paper – and in most of the literature – it is assumed that option payoffs are a function of variables whose distributions are exogenously determined. In the Black-Scholes world, for example, the underlying asset's price follows a geometric Brownian motion. In this paper, the option's payoffs is determined by the underlying asset's dividends, which are endogenously determined. In reality, however, option payoffs are endogenously determined by the underlying asset's equilibrium price. This could provide a channel through which asset prices might change after the introduction of options, which is a well-recorded empirical phenomenon (see Conrad (1989) and Detemple and Jorion (1990)). This is a topic for future research.

One can take this point even further. Since most empirical research in asset pricing uses cross-sectional analyses based on the iid assumption,¹³ such studies might make a systematic error either if changes in the composition of listed firms occur (due to bankruptcies or mergers and acquisitions), or if listed firms engage in investment projects during the sample period. In such cases, the market span is likely to change and, hence, so does the underlying probability space.

Appendix

A Homotopy Methods in GEI Analysis

The equilibria in this paper are computed by using a homotopy method developed in Herings and Kubler (2002). This is a differentiable homotopy obtained by replacing excess demand functions by the first order conditions of utility maximisation, an approach proposed by Garcia and Zangwill (1981). The advantage of this approach is that the number of agents, H, and the number of assets, J, determine the dimensionality of the homotopy instead of the number of states, S, which is typically very large. Furthermore, there is no need to explicitly compute agents' demand function and the set of no-arbitrage prices Q, both typically non-trivial. Instead one merely needs the Jacobians of the utility functions.

The Herings-Kubler (HK) homotopy is designed for two-period GEI economies with no consumption at t = 0. A standard way of transforming any GEI economy \mathcal{E} to an economy with no consumption at time is presented in Hens (1991) and consists

 $^{^{13}}$ See, for example, Campbell et al. (1997) for an overview.

of replacing the asset payoff matrix A by the matrix

$$\bar{A} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & A_1^1 & \dots & A_1^J \\ \vdots & \vdots & \ddots & \dots \\ 0 & A_S^1 & \dots & A_S^J \end{bmatrix}$$

That is, t = 0 consumption is translated to t = 1. by introducing an artificial state s = 0 and an artificial asset j = 0. The analysis that follows is concerned with the economy $\bar{\mathcal{E}} = (u, \omega, \bar{A})$.

The following additional assumption is made with respect to investors' preferences.

Assumption A.1. For all h = 1, ..., H, the utility function u^h is three times continuously differentiable such that for all $x \in \mathbb{R}^S_{++}$ it holds that

- 1. $\partial u^h(c) \in \mathbb{R}^S_{++};$
- 2. $\forall_{y\neq0:\partial u^h(c)y=0}: y \top \partial^2 u^h(c)y < 0;$
- 3. $\{\tilde{c} \in \mathbb{R}^{S}_{++} | u^{h}(\tilde{c}) \ge u^{h}(c)\}$ is closed in \mathbb{R}^{S} .

Let \hat{f} and \hat{q} denote the excess demand function and an asset price system, respectively, with the first entry removed. The algorithm starts from an initial price system q^0 in Q, with the price of t = 0 consumption normalised to 1. Equilibria of $\bar{\mathcal{E}}$ are computed by the homotopy $\mathcal{H}: [0,1] \times Q \to \mathbb{R}^J$

$$\mathcal{H}(t,q) = t\hat{f}(q) + (1-t)(\hat{q}^0 - \hat{q}).$$
(A.1)

Herings and Kubler (2002) prove the following theorem.

Theorem A.1. Let $\Omega \subset \mathbb{R}^{HS}_{++}$ be an open set with full Lebesgue measure. For all initial endowments $\omega \in \Omega$ it holds that

- 1. $\mathcal{H}^{-1}(\{0\})$ is a compact C^2 one-dimensional manifold with boundary $\mathcal{H}^{-1}(\{0\}) \cap (\{0,1\} \times Q);$
- 2. there is an odd number of solutions in $\mathcal{H}^{-1}(\{0\}) \cap (\{1\} \times Q);$
- 3. there is one solution in $\mathcal{H}^{-1}(\{0\}) \cap (\{0\} \times Q);$
- 4. there is no sequence $(t^n, q^n)_{n \in \mathbb{N}}$ in $\mathcal{H}^{-1}(\{0\})$ with limit $(t, q) \in [0, 1] \times \partial Q$ or such that $||(t^n, q^n)||_2 \to \infty$.

That is, generically, there exists a path from q^0 to an FME q; there is only one solution at q^0 ; there is an odd number of solutions; and the algorithm does not diverge or converge to the boundary.

The homotopy \mathcal{H} has the advantage that one does not have to compute the set Q explicitly. Unfortunately, however, it is usually non-trivial to compute the excess demand function f analytically. One can use (A.1), but at every step n, the function value $\hat{f}(q^n)$ has to be computed numerically, which is highly time consuming. Instead one can replace \mathcal{H} with the diffeomorphic implementable homotopy $\mathcal{H}^* : [0,1] \times Q \times \mathbb{R}^{H(J+1) \times \mathbb{R}^H} \to \mathbb{R}^{(H+1)(J+2)-2}$, defined by

$$\mathcal{H}^*(t,q,\theta,\lambda) = \begin{cases} t \sum_{h=1}^H \theta_j^h + (1-t)(q_j^0 - q_j), & j = 1,\dots, J\\ \left(\partial u^h(\omega^h + \bar{A}\theta^h)\bar{A}\right)^\top - \lambda^h q^\top, & h = 1,\dots, H\\ q\theta^h, & h = 1,\dots, H, \end{cases}$$
(A.2)

where λ is the vector of Lagrange multipliers obtained from utility maximisation. We have the following result.

Theorem A.2 (Herings and Kubler (2002)). $(\mathcal{H}^*)^{-1}(\{0\})$ is C^2 diffeomorphic to $\mathcal{H}^{-1}(\{0\})$.

This implies that, generically, the homotopy \mathcal{H}^* converges to an FME. The homotopy (A.2) is implemented in Matlab via a four-step Adams-Bashforth predictorcorrector method.

B Empirical Results of Simulations

In all tables below, z_{α} denotes the z-value of (5), z_{EV^h} denotes the z-value of (6), all $h = 1, \ldots, H$, z_{q_j} denotes the z-value of (7), all $j = 1, \ldots, J$, and z_{RPV} denotes the z-value of (8). Significance at the 10%, 5%, and 1% level is indicated by *, **, and * * *, respectively. Also reported are the point estimates, t-values, and R^2 of the regression (9).

	CARA	CRRA
$ z_{\alpha} $	0.2775	1.4840
$ z_{EV^1} $	0.0597	1.0672
$ z_{EV^2} $	-0.2999	1.0050
$ z_{EV^3} $	0.2537	3.1744^{***}
$ z_{q_1} $	1.3287	1.8498^{*}
$ z_{q_2} $	0.2718	0.5368
$ z_{RPV} $	3.7077^{***}	1.7521^{*}
β_0	0.0597^{***}	-0.0079
	(7.1162)	(-1.1781)
β_1	-0.2999^{***}	-0.2489^{***}
	(-31.4897)	(-14.2343)
β_2	0.2537^{***}	0.2854^{***}
	(16.6493)	(13.1762)
R^2	0.9290	0.6703

Table 1: Results from the simulations with tri-nomial shocks.

	CARA	CRRA
$ z_{\alpha} $	6.5163***	5.7539***
$ z_{EV^1} $	3.5240***	1.0924
$ z_{EV^2} $	4.0834***	1.1292
$ z_{EV^3} $	1.0051	0.9379
$ z_{q_1} $	1.5128	2.2863^{**}
$ z_{q_2} $	0.4182	0.0975
$ z_{RPV} $	2.9002***	0.8231
β_0	0.0804***	0.0557^{***}
	(4.3275)	(4.0394)
β_1	-0.3216^{***}	-0.1854^{***}
	(-18.2227)	(-14.2343)
β_2	0.2628***	$0.16\overline{16^{***}}$
	(10.0558)	(5.1843)
R^2	0.7713	0.4698

Table 2: Results from the simulations with uniform shocks.

	CARA	CRRA
$ z_{\alpha} $	5.3582***	5.7658***
$ z_{EV^1} $	0.0925	1.5532
$ z_{EV^2} $	6.2474^{***}	2.8028^{***}
$ z_{EV^3} $	1.1347	2.2927^{**}
$ z_{q_1} $	0.2992	1.3780
$ z_{q_2} $	1.0109	0.9918
$ z_{RPV} $	3.7591^{***}	0.1496
β_0	0.0572***	0.0053
	(4.1619)	(0.7220)
β_1	-0.2478^{***}	-0.1654^{***}
	(-13.3233)	(-6.3322)
β_2	0.2018***	0.1860***
	(6.9699)	(7.8222)
R^2	0.7278	0.4194

Table 3: Results from the simulations with log-normal shocks.

	CARA	CRRA
$ z_{\alpha} $	9.5549***	10.5868^{***}
$ z_{EV^1} $	3.0290***	1.6352
$ z_{EV^2} $	1.0052	1.0077
$ z_{EV^3} $	2.6206***	1.1950
$ z_{q_1} $	1.9190^{*}	1.0112
$ z_{q_2} $	0.1150	1.6057
$ z_{RPV} $	3.8904***	0.6204
β_0	0.0744^{***}	-0.0331^{***}
	(5.3914)	(-3.7909)
β_1	-0.1822^{***}	-0.1122^{***}
	(-11.9330)	(-4.7600)
β_2	0.1167^{***}	0.1780^{***}
	(3.9683)	(6.7325)
R^2	0.8201	0.3900

Table 4: Results from the simulations with log-normal shocks and downward jumps.

	CARA	CRRA
$ z_{lpha} $	11.6678***	11.5432***
$ z_{EV^1} $	1.6561^{*}	2.3374**
$ z_{EV^2} $	1.0051	2.7879^{***}
$ z_{EV^3} $	1.0050	2.6024^{***}
$ z_{q_1} $	0.6767	1.2342
$ z_{q_2} $	0.1511	1.2020
$ z_{RPV} $	2.8618***	0.1424
eta_0	0.0004	0.0356^{***}
	(0.0409)	(4.0974)
β_1	-0.2508^{***}	-0.1430^{***}
	(-10.0304)	(-3.5832)
β_2	0.2788***	0.1307^{***}
	(9.5215)	(3.7760)
R^2	0.5016	0.1259

Table 5: Results from the simulations with log-normal shocks, downward jumps, and heterogeneous beliefs.

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