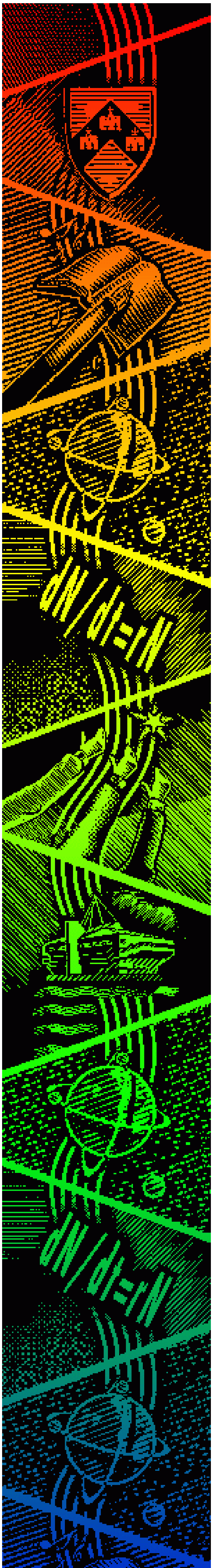


THE UNIVERSITY of York

Discussion Papers in Economics

No. 2007/05

Option Pricing under Stochastic Volatility and Trading Volume

By

Sadayuki Ono

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

Option Pricing under Stochastic Volatility and Trading Volume

Sadayuki Ono * †
University of York

March, 2007

* I would like to thank Massimo Guidolin, Wake Epps, Yiorgos Allayannis, Christopher Otrok, Gordon Gemmill, and the participants at the 32nd annual meeting of the European Finance Association for helpful comments and suggestions.

† Department of Economics and Related Studies, University of York, York, YO10 5DD, United Kingdom. Tel.: +44-(0)1904- 433-791. Fax: +44-(0)1904-433-759. E-mail address: so501@york.ac.uk.

Option Pricing under Stochastic Volatility and Trading Volume

Abstract

This paper presents a pricing formula for European options derived from a model in which changes in the underlying price and trading volumes are jointly determined by exogenous events. This specification makes increments to the volatility depend on the current level of volatility and news and thereby accounts for the observed persistence in volatility. Moreover, it makes volatility an observable variable. The model accounts well for time varying volatility smiles and term structures, and that out-of-sample price forecasts for a sample of call options are superior to the benchmark ad hoc procedure of plugging implicit volatilities into the Black-Scholes formula.

JEL Classification: G12; C52; C53

Keywords: option valuation; trading volume; the stochastic volatility and volume (SVV) model.

1 Introduction

The seminal works of Black and Scholes (1973) and Merton (1973) have contributed a major step to developing an option pricing model that has become known as the Black-Scholes (BS) model. However, one of the more firmly established facts in financial economics is that this BS option pricing formula cannot account for observed market option prices.[1]

A well known example is the "volatility smile," volatilities implied by the BS formula are not constant over moneyness (the ratio of the spot price and the strike price). Options which are deep in- and out-of-the-money show higher implied volatilities than volatilities at-the-money options. After the 1987 stock market crash the "smile" turned into an asymmetric shape, a "smirk:" out-of-the money puts and in-the-money calls exhibit higher implied volatilities than in-the-money puts and out-of-the money calls. There also seems to be a term structure pattern in volatility smiles: the smiles are strongest in short-term options and flatten out monotonically with increasing time-to-maturity.

These empirical biases are not surprising since the BS model is based on the strong assumption that the underlying asset price follows a one-dimensional diffusion process with a constant, instantaneous volatility parameter. Under this assumption returns on the underlying asset are normally distributed while asset returns empirically display strong volatility clustering, skewness and larger kurtosis in the conditional and unconditional return distributions than implied by normality.

A number of papers have investigated the implications for option prices of relaxing normal

[1] Examples of empirical research to test the BS model are Rubinstein (1994) for index options and MacBeth and Merville (1979) for equity options.

distribution assumption. Examples include the jump-diffusion BS model of Merton (1976) and the stochastic volatility models of Hull and White (1987) and Heston (1993). Duffie, Pan and Singleton (2000) recently developed a particularly versatile class of affine-diffusions with stochastic volatility which yield jumps in volatility.[2] These studies show that stochastic volatility improves the performance of option pricing models and stress the importance of using an appropriate measure of volatility when pricing options. However, there is a disadvantage to stochastic volatility: It involves latent factors, so that one faces the task of using observations on option prices to not only estimate parameters but also filter the unobservable factors. This technique becomes computationally demanding when the sample period grows since the number of latent volatility parameters to be estimated increases proportionally with the observation window.

To overcome the unobservability of volatility, some researchers have incorporated the generalized autoregressive conditional heteroskedastic (GARCH) asset return process of Bollerslev (1986) into pricing option models. The GARCH process is so desirable to fit empirical behavior of stock returns that it is capable of producing leptokurtosis and skewness in asset return distributions. For instance, Duan (1995) develops an option pricing model using local risk neutralization in which one-period ahead conditional volatility is invariant to a change in risk neutral measure when the variance of the underlying asset follows a GARCH process. Since options in this GARCH model are priced only by simulation, it can be computationally demanding in empirical applications. Heston and Nandi (2000) present a closed-form

[2] Other alternatives to the BS model are the stochastic volatility model with jumps of Bates (1996) and the stochastic volatility and interest rate models with jumps of Bakshi, Cao and Chen (1997) and Scott (1997).

solution of a nonlinear GARCH option pricing. By using S&P 500 index options, they show that out-of-sample fit of the NGARCH model is superior to that of the *ad hoc* BS model of Dumas, Fleming and Whaley (1998) that uses a flexible specification of volatility across strike prices and maturities. Heston and Nandi conclude that the improvement depends on the ability of the model to simultaneously capture the path dependence in volatility and the correlation between volatility and asset returns. As it is widely known, GARCH models are useful in modeling the time series behavior of conditional volatility in asset returns. However, they are a statistical model and might not explain the structural relationship between returns and volatility. It is thus important to consider what is a source of the GARCH effect.

This issue is discussed in Guidolin and Timmermann (2003) who point out that the stochastic properties of stock returns and volatility can be explained by learning effects concerning economic fundamentals. They show that stochastic volatility, which is endogenously determined by the path of investors' beliefs on parameters on dividends process, can account for the anomalies of implied volatility. While learning offers an excellent economic explanation of several empirical properties of option prices, the learning model needs to estimate investors' beliefs for empirical applications. However, data on economic fundamentals are released at a lower frequency than the daily basis at which asset prices are observed. In this sense beliefs are unobservable, being analogous to volatility in the stochastic volatility models.

Several empirical studies have identified a number of persistent patterns in trading volume and in the behavior of security prices.[3] One of the empirical regularities in the literature is

[3]Karpoff (1987) gives a comprehensive survey of the early literature.

a strong positive correlation between trading volume and return volatility as in the old Wall Street adage that "it takes volume to move prices."

In this paper, motivated by a strong connection between trading volume and the volatility of stock returns, I employ trading volume as an instrument for return volatility to derive an option pricing formula. This paper provides the following contributions. Firstly, I establish a stochastic volatility and volume (SVV) model in which a latent common factor for stock returns, volatility, and volume is represented as information arrivals and modeled as a discontinuous process. The SVV model is designed so that it captures several empirical regularities of stock returns, volatility, and volume. An option pricing formula for (European) vanilla options is then derived based on the assumption of no arbitrage. Option pricing depends crucially on the volatility of the underlying price process. On its turn, volatility is formulated as a stochastic process depending on trading volume. In this context, the trading volume can be a useful instrumental variable for estimating unobservable realizations of stochastic price volatility. I also examine whether the derived option formula can replicate the implied volatility patterns documented in the literature.

Secondly, I investigate the in-sample and out-of-sample performances of the SVV model in comparison to other structural option pricing models such as the Black-Scholes (BS) model, the jump-diffusion (JD) model of Merton (1976), the stochastic-volatility (SV) model of Heston (1993), and the stochastic volatility with jumps in returns (SVJ) model of Bates (1996). Besides these structural option pricing models, I select the *ad hoc* BS (ahBS) model of Dumas *et al.* (1998) that simply smooths the local volatility rate in the BS model across moneyness and time to maturity while it is not based on a theoretical framework. The *ad hoc*

BS model is selected because it is a primary model employed by financial practitioners and was also chosen as a benchmark model for model performance comparisons (S&P 500 index options) in several articles such as Dumas *et al.*, Guidolin and Timmermann, and Heston and Nandi. To estimate parameters on these models, I use data on individual equity options and an index option, on stock prices, an index, and trading volumes. By employing equity option data, this paper contributes to the literature since very few studies have empirically investigated the no-arbitrage valuation approaches to equity option prices.

I found two main implications of the SVV model. The first implication is that the SVV model is capable of reproducing several empirical regularities including various types of implied volatility patterns and significant conditional skewness and excess kurtosis of stock returns when it is calibrated to reasonable values of parameters. The second implication results from performing in-sample and out-of-sample tests to evaluate the model's fitting and one-period-ahead prediction employing data on eight equity options and one index option. I then find that the *ad hoc* BS model is more flexible in fitting data than any of the structural models. However, predictive performance, which evaluates economic significance in models, shows that the aggregate pricing errors from the SVV model are lower overall than those from the other option pricing models. According to this empirical result, I may conclude that the time-series information in trading volume provides a first-order importance in option valuation.

The literature about the relationship among trading volume, stock prices, and volatility is extensive both in the theoretical and empirical sense. Several theoretical models consider the contemporaneous relation of trading volume and price changes. Epps (1975) explores a model in which stock returns and trading volume are correlated so that volume on upticks of prices is greater than volume on downticks. He assumes that there are two groups of market participants, e.g., optimists and pessimists on a current opinion about an asset. This heterogeneity among investors leads trading to occur and price to change since the valuation of assets differs among the investors after they receive the news. This conclusion is reinforced by Copeland (1976) who develops a model in which information arrives sequentially to investors. As a statistical explanation of the positive correlation between trading volume and absolute price changes, Clark (1973), Epps and Epps (1976) and Tauchen and Pitts (1983) propose the "mixture of distributions hypothesis (MDH)" that posits a joint dependence of returns and volume on an underlying latent event or information flow variable. The existence of a common stochastic factor induces positive correlation between volume and the magnitude of the corresponding price changes. Harris (1987) presents empirical evidence that strong autocorrelations of trading volume and squared returns are propagated from a common factor. Andersen (1996) modifies the MDH based on the microstructure framework in which informed and uninformed investors exist who differently react to arriving information.

Another strand of the literature discusses the dynamic effects of trading volume on future volatility.[4] Shalen (1993) examines a noisy rational expectations model predicting a positive correlation between trading volume and future absolute price changes due to the dispersion of the future price expectations. Gallant, Rossi and Tauchen (1992) find that large price movements are followed by high volume by applying a semi-nonparametric estimation of the

[4] Campbell, Grossman and Wang (1993) present a model in which trading volume is primarily determined by liquidity needs. Their model implies that large trading volume tends to induce negative return autocorrelations. Alternatively, in the asymmetric information model of Wang (1994), the correlation turns out to be positive when trading is motivated by speculation.

joint process of price changes and volume. Lee and Rui (2002) examine the dynamic causal relation among stock market returns, trading volume and volatility. They find that there is a positive feedback effect between volume and volatility while volume does not help predict the level of returns. This finding suggests that information in returns is contained in trading volume indirectly through its predictability of return volatility. If this is the case, trading volume may be used as a proxy for information flow in the stochastic process generating volatility.

While there are many studies showing a strong relation among trading volume, stock prices, and volatility, the implication for derivative pricing has not been pursued yet. Most option pricing models in the literature are based on some distinguishing specification of the price process, possibly joint with an assumption on the volatility process, but volume has been disregarded altogether. The only exception is Howison and Lamper (2001), in which an option pricing model is developed under the assumption that stock return volatility is driven by the rate of information arrival. They estimate the rate of information flow from the number of transactions so that the volatility is observable. It is proven, by calibration, that the model implies a volatility smile.

My model differs from the model of Howison and Lamper (2001). Firstly, the common driving factor in my model is a Poisson jump process representing information release while Howison and Lamper model the rate of the information arrival as a mean-reverting process. According to Das and Sundaram (1999), a stochastic volatility model with a mean-reverting volatility process is not able to generate high enough levels of skewness and kurtosis at short maturities so that the shape of implied volatility becomes too shallow to match the one typically observed. Secondly, in my model, there are common jumps among prices, volatility, and trading volume whereas there are no common jumps in returns and volatility in Howison and Lamper (2001). Thirdly, Howison and Lamper (2001) do not deliver extensive empirical exercises, such as the estimation and evaluations of their model.

The remainder of the paper is organized as follows. Section 2 presents the data set and empirical regularities of implied volatilities, returns, and trading volume of eight individual stocks and one index. Section 3 introduces a stochastic volatility-volume model of price changes, volatility and trading volume. In Section 4 I derive the no arbitrage pricing formula of a European vanilla option. To study the model's ability to fit empirical regularities in option markets, the volatility patterns implied by the option formula are analyzed. Additionally, I derive closed-forms for conditional skewness and excess kurtosis of stock returns and show the model's capability of generating a considerable amount of the skewness and the kurtosis. Section 5 estimates option pricing models and evaluates their valuation performances. The last section concludes the paper.

2 Empirical Regularities of Call Options, Stock Returns, Return Volatility, and Trading Volume

This section presents data descriptions and provides several empirical regularities of the BS implied volatility, stock returns, return volatility, trading volume, and their relationships. In the following section, a stochastic process of the underlying asset and its trading volume will be constructed so that it qualitatively fits the empirical facts of underlying assets presented in this section.

2.1 Data

My data set includes daily observations on: (1) the most recent trading prices for individual stocks and the most recent level of an index before the markets close; (2) trading volume for individual stocks and all stocks in the New York Stock Exchange (NYSE); (3) the bid-ask midpoint prices for the equity and index call options; (4) the bid-ask midpoint prices for Treasury bills; and (5) prices for futures contracts of the index. Data for the trading prices and the trading volume for stocks and data for bid and ask prices of options are obtained from the homepage of the Chicago Board Options Exchange (CBOE). Because trading volume of the S&P index is not available, I employ the trading volume of all stocks listed in NYSE as the proxy. Data source for the trading volume of all stocks in NYSE is the homepage of NYSE. Data on T-bill prices are obtained from the Wall Street Journal. Futures prices are collected from the homepage of the Chicago Mercantile Exchange (CME).

I selected eight technology stocks (ticker) and one index: Advanced Micro Devices Inc. (AMD), AOL Time Warner Inc. (AOL), and International Business Machines Corp. (IBM), which is traded on the NYSE, and Oracle Corp. (ORCL), Cisco Systems Inc. (CSCO), Microsoft Corp. (MSFT), Intel Corp. (INTC), and Dell Computer Corp. (DELL), which are traded on the NASDAQ, and S&P 500 INDEX (SPX).[5] The sample period for the nine underlying assets is September 3rd, 2002 - August 29th, 2003, and the total number of observation dates is 251. These equity options are American style so that early exercise may be optimal. Since there is no early exercise advantage for call options unless dividends are paid out, the prices of European call options and American call options is in this case

[5]In September 2003, the AOL Time Warner corporate title changed to Time Warner Inc. The ticker also changed to TWX.

identical. Since a European option formula is only derived in this paper, I do not use data on put options for empirical experiments. Among the eight firms, AMD, AOL, ORCL, CSCO, and DELL did not pay dividends during the sample period.[6] Hence, pricing formulas for European options are applied to valuing these firms' call option prices. On the other hand, IBM and INTC periodically pays dividends, and MSFT paid its first dividend, eight cents per share on January 27th, 2003, which was in the sample period. To employ a European option formula for pricing IBM, MSFT, and INTC call options, call option contracts whose expiration date is beyond the dividend payment date were excluded.

In the empirical experiments, I use daily observations for which quotes of stocks and options at 4:00 pm (EST) and cumulated volumes from 9:00 am to 4:00 pm are recorded each day. Although the quotes of options are not the daily closing ones, these are used to be consistent with the daily closing quotes of stocks and level of the index.[7] I apply Bakshi *et al.* (1997) and Heston and Nandi (2000) by using quote midpoints for options rather than transaction prices to avoid measurement errors from bid-ask bounce.

The following exclusionary criteria are employed on the original data. Firstly, options with less than six trading days to expiration are omitted to mitigate expiration-related biases. In addition, I exclude option contracts whose time to maturity is more than 180 trading days because the long-term equity options are very thinly traded. Secondly, quoted prices lower than $3/8 were eliminated due to potential price discreteness-related biases. Thirdly, quotes violating a number of arbitrage restrictions were dropped.[8]

[6] AMD, ORCL, CSCO and DELL actually never paid out any dividends in their history and AOL has not paid dividends since June 1998.

[7] Option trading in CBOE ends at 4:15 pm.

[8] The no-arbitrage conditions are the upper and lower bounds for option prices. To prevent arbitrage, call

2.2 Call options

Table 1 provides sample properties of call options of the eight high-tech companies and the S&P 500 index. In the table, as with terminology in Bakshi *et al.* (1997), I classify call option data according to either moneyness or term to maturity. A call option is assigned to in-the-money (ITM), at-the-money (ATM), or out-of-the-money (OTM) if the ratio of the spot stock price to the strike price is greater than or equal to 1.03, greater than 0.97 and less than 1.03, or less than or equal to 0.97. A call option is also referred to short-term or medium-term if the term to maturity is less than 60 business days or from 60 to 180 trading days. The table displays the total number of observations as well as the sample averages of the mid-point of the bid and ask option quotes, of the effective bid-ask spread defined as ask price minus the bid-ask midpoint, of daily trading volume. As can be seen in the table, the nine call options show the following common patterns. In all of the cases, ITM options preserve more observations than ATM or OTM options after the exclusion criteria are applied. The liquidities of OTM and ATM options appears higher than the liquidity of ITM options, showing that the average trading volumes of OTM and ATM calls are larger and the average depths of OTM and ATM calls are smaller. In this context, short-term call options of any category of moneyness are more liquid than medium-term counterparts.

prices C_t and put prices P_t must always satisfy $\max(S_t - B_{t,\tau}K, 0) \leq C_t \leq S_t$ and $\max(B_{t,\tau}K - S_t, 0) \leq P_t \leq B_{t,\tau}K$, where S_t, $B_{t,\tau}$, and K are the price for the underlying asset, the price for a τ-period maturity bond, and a strike price, respectively. See Epps (2000, p. 167-171).

2.3 Implied volatility

I present empirical facts of implied volatilities of the nine call options. Firstly, two panels in Figure 1 show the typical cross-sectional character of implied volatilities for short- and medium-term call options. As shown in the two figures, the volatility curves of CSCO and S&P 500, which are observed on March 27th, 2003, tend to show a strong smirk pattern (higher volatility for in-the-money option than at- or out-of-the money options). Secondly, this skewed shape disappears as the time-to-maturity increases. The equity and index option data reveals various term structure shapes (increasing or decreasing) for the implied volatility curves. The difference between short-term and medium-term options is negatively large when options are in-the-money. The difference appears small in out-of - and at-the money options. Thirdly, the data show that implied volatilities substantially change over time. Figure 2 provides significant time variations in daily implied volatility of S&P 500 call options averaged across contracts. The volatility shows a decreasing trend in the sample period. Table 2 reports summary statistics of daily implied volatilities. Implied volatilities tend to decrease with the size of a company, and that they are highly persistent.[9] The table also shows that the sample standard deviations of the daily volatilities are in the order of percentage points while they would have been zero if the volatilities had been constant. In sum, the BS assumption of a constant spot volatility might be inconsistent.

[9] Several studies have found that a AR(1) specification captures time-series properties of implied volatility of many kinds of options. See, for instance, Sheikh (1993) for stock options, Poterba and Summers (1986) for index options, and Taylor and Xu (1994) for currency options.

2.4 Stock returns, return volatility, and trading volume

Table 3 presents descriptive statistics for continuously compounded daily returns for the eight technology stocks and an index. Because US markets entered a recovery phase in the sample period after the burst of the information technology bubble, all of the stocks and the index, on average, reveal positive returns. In this period, stock returns were very volatile, but return volatilities of large firms and the index tended to be smaller. For each of the eight stocks and the index, the first-order autocorrelations are not significant, indicating that stock markets are efficient. As can been seen, there is evidence of non-normality in the daily returns. For instance, these series display either positive or negative skewness and the kurtosises of most of the series are somewhat larger than three. From the statistical point of view, six returns show non-normality. For example, the Jarque-Bera test rejects the hypothesis that the six stock return series have normal distributions at least one percent significance.[10]

One of the empirical regularities in the literature is a strong positive correlation between trading volume and changes in returns. The two panels in Figure 3 display the stock price of CSCO and the S&P 500 index movements and their daily trading volume and notice that high trading volume simultaneously leads or follows large price movements. The third column of table 4, in fact, presents high values of the contemporaneous correlation between absolute changes in prices and volume. In case of seven technology stocks, the correlation coefficients

[10]The Jarque-Bera test is an asymptotic test of the null of normality with a joint statistic using skewness and kurtosis coefficients. The test statistic represents $JB = \frac{N}{6}\left[S^2 + \frac{(K-3)^2}{4}\right] \sim \chi^2_2$ Chi-square with two degrees of freedom, where N is the number of observations, S is a sample skewness and K is a sample kurtosis.

are significantly positive at one percent level. In contrast, there is no significant correlation between returns and trading volume, according to the fourth column of the table. Figure 4 shows cross-correlations patterns for trading volume and a measure of return volatility, such as the absolute value of returns. As revealed in the figure, the cross-correlation is strongly positive at the zero lag. Correlations quickly decay, but they remain slightly positive for several lags. These empirical facts suggest that there might be a notable dynamic relationship between price movements and trading volume.

2.5 The GARCH and trading volume

I also examine a linkage between return volatility estimated by a GARCH and trading volume. Table 5 reports estimation results of GARCH (1,1) models when volume is included as an explanatory variable in the conditional variance equation. Trading volume is significant for all stocks and the index. In addition, the persistence of volatility, as measured by the sum of the GARCH coefficients, decreases for the all returns except for returns on S&P 500 when volume is added in the model.[11] Figure 5 and the second column of table 4 indicate that the first-order autocorrelation functions of trading volume slowly decay. This slow decaying is also one of the GARCH effects. These empirical regularities imply that a common factor might move return volatility and trading volume.

[11] Lamoureux and Lastrapes (1990) find that ARCH effects tend to disappear when volume is included as an explanatory variable in the conditional variance equation.

3 The Model

In this section I will present the stochastic volatility and volume (SVV) model in which price changes, trading volume, and volatility follow a trivariate latent process under the physical probability measure. The next section will derive a European option pricing formula under this SVV model and the assumption of no arbitrage.

Let $(\Omega,\mathcal{F},\mathbb{P})$ be a complete probability space, where Ω denotes the set of all outcomes, $\mathcal{F}$ denotes the σ-field of subsets of Ω and $\mathbb{P}$ denotes a probability measure on $\mathcal{F}$. Let also $\{\mathcal{F}_t\}_{t\geq 0}$ be a filtration, a non-decreasing sequence of sub-σ-field ($\mathcal{F}_s \subseteq \mathcal{F}_t \subseteq \mathcal{F}$, $s \leq t$). $\mathcal{F}_t$ can be interpreted as the information set at time t available to agents. Epps (2000, p. 48 and p. 83) provides a more detailed treatment.

Under the physical probability measure, the price of an underlying non-dividend-paying risky security, S_t, follows a diffusion-jump process of the form:

$$\frac{dS_t}{S_{t-}} = (\alpha_t - \theta m)dt + \sigma_t dW_t + X dN_t, \tag{1}$$

where S_{t-} denotes limits from the left. $\{W_t : t \geq 0\}$ is a standard Wiener process, $\{N_t : t \geq 0\}$ is a stochastic Poisson process with an jump-intensity parameter, θ such that $\Pr(dN_t = 1) = \theta dt$ and $\Pr(dN_t = 0) = 1 - \theta dt$. dW_t and dN_t are assumed to be independent. The Wiener process and the Poisson process are defined on $(\Omega,\mathcal{F},\mathbb{P})$ and are adapted to $\{\mathcal{F}_t\}_{t\geq 0}$.

X is the random size of a jump in price and is assumed to be the sum of two random variables so that $\ln(1+X) = Y + ZH$, where Y is a normal random variable with parameters (μ, ϕ^2), Z is a random variable that can take either ρ (> 0) or $-\rho$ with probability 0.5, and H is an exponential random variable with mean ψ. m represents the mean of the jump size in price, X, and it depends on parameters in the two jump variables such that $m \equiv E(X) = \frac{\exp(\mu+\frac{\phi^2}{2})}{(1-\rho\psi)(1+\rho\psi)} - 1$. The condition $\rho\psi < 1$ satisfies the existence of the mean. α_t denotes the instantaneous expected return per unit of time. It is convenient to write the price in terms of its logarithm, $s_t = \ln(S_t)$. From the extended Itô's formula, the price process (1) can be written as

$$ds_t = (\alpha_t - \theta m - \frac{\sigma_t^2}{2})dt + \sigma_t dW_t + (Y + ZH)dN_t. \tag{2}$$

ds_t is interpreted as the continuously compounded return.

Next, the stochastic process of the cumulated trading volume, V_t, is assumed to be

$$dV_t = \gamma\sigma_t^2 dt + HdN_t, \tag{3}$$

where γ is a positive constant. γ is restricted to be positive because the increment in cumulated volume is nonnegative at any time.

Finally, the instantaneous variance of returns conditional on the jump not occurring is related to the trading volume so that

$$\sigma_t^2 = \beta \int_0^t e^{-\lambda(t-s)} dV_s, \tag{4}$$

where β, λ are positive constants. From this volatility specification, an increment in trading volume as a proxy of information flow thus leads to an increase in volatility. Being represented as the integral of current and past increments of volume, the volatility could be highly persistent. The advantage of this specification is that the local volatility is characterized as a path-dependent model while Dumas *et al.* (1998) suggest the limitation of path-independent models. I will derive the process of volatility from the volume process (3) and the variance equation (4). Applying the Leibniz 's rule to (4) yields

$$d\sigma_t^2 = -\lambda\sigma_t^2 dt + \beta dV_t. \tag{5}$$

Now, substituting (3) into (5) gives

$$d\sigma_t^2 = -(\lambda - \beta\gamma)\sigma_t^2 dt + \beta H dN_t. \tag{6}$$

Notice that the rate of mean reversion, $\lambda - \beta\gamma$ is restricted to be positive since the unconditional mean of σ_t^2 is a finite positive value, $\frac{\theta\beta\psi}{\lambda-\beta\gamma}$.

This specification of the trivariate process (2), (3) and (4) (or (6)) nests the Black and Scholes (1973) and the Merton jump model (1976). The restriction that delivers BS is that volatility is constant, or $\beta = \lambda = 0$, and there is no jump in returns, $\theta = 0$. The restriction that delivers the Merton jump model is that volatility is constant, or $\beta = \lambda = 0$. The SVV model can produce common jumps in returns and volatility. This feature is similar with the model of Duffie *et al.* (2000). However, because of stochastic property of Z , news can affect either positively or negatively to the valuation of the underlying asset in my model. In contrast, the model of Duffie *et al.* (2000) cannot capture this feature.

Here, I examine the qualitative characters of the latent factor process. Firstly, the volatility of returns arises from the continuous term and the jump term. For example, the instantaneous conditional return volatility is given by

$$\frac{dVar\,(ds_t|\mathcal{F}_t)}{dt} = \sigma_t^2 + \theta(1-\theta)(\phi^2 + \rho^2\psi^2). \tag{7}$$

Secondly, from the return process (2), the size of jumps in returns is composed of two parts. One is independent of jumps in trading volume (the jump size is Y) while the other is related to jumps in volume (the jump size is ZH). This jump size structure thus allows jumps to occur in returns but not in trading volume. Thirdly, returns in this process are not autocorrelated, which is found in the data, because the (continuous and discontinuous) risk factors are not autocorrelated. Fourthly, in contrast to zero return autocorelations, volume and volatility are positively autocorrelated because of their drift terms. The positive autocorrelations are consistent with the empirical findings or the GARCH effect. Finally, in the volume process (3) a trade happens due to the jump component which is generally shared in the return process (2). As a result, the absolute value of return changes and the trading volume can be positively correlated, which is consistent with empirical findings.

The economic intuition supporting these characters of the model can be described in the following manner. Suppose that there are two kinds of market participants, say informed traders and noise traders. The noise traders might be uninformed or liquidity traders. The second and third term of the right hand side of equation (2) represent the (continuous) effect from the noise traders and the (discontinuous) effect from the informed traders, respectively. Suppose that some random piece of information hits the market, *i.e.* a jump occurs. Then, trade might occur because the informed traders have different opinions ("bullish" or "bearish") on the information. This effect on trading volume is captured by the second term of the right hand side of equation (3). At the same time, the return might be affected by the jump term. Since the jump size of the trading volume is the sum of the squared jump size of returns and a positive random variable, the price will change when trade happens. This feature is consistent with the empirical finding of positive contemporaneous correlation of absolute price changes and volume. At this point, the information is gradually noticed by the noise traders who will join the trading. Trading by the noise traders affects volatility via equation (4) and also affects trading volume via the trend term in (3). This volume formation is in agreement with Andersen (1996), in which trades occur by noise traders and informed traders. Since the information gradually spreads, the volatility is positively autocorrelated (ARCH effect) with lagged volume in (4), and the volume is also positively autocorrelated because of the effect of the positively autocorrelated volatility, the trend term in (3). These positive autocorelations are solid empirical regularities.

For empirical purposes, I discretize the volatility function (4) as

$$\sigma_t^2 = \beta \sum_{s=\Delta}^{t} \left(\frac{1}{1+\lambda}\right)^{t-s} (V_s - V_{s-\Delta}), \text{ for } t \geq 1,\ V_0 = 0. \tag{8}$$

where Δ is a positive integer which represents a unit of time. Notice that the discrete-time volatility function (8) converges to its continuous-time counterpart (4) as $\Delta \to 0$. Equation (8) can be rewritten as

$$\sigma_t^2 = \beta(V_t - V_{t-\Delta}) + \left(\frac{1}{1+\lambda}\right)\sigma_{t-\Delta}^2, \text{ for } t \geq 1,\ \sigma_0^2 = 0. \tag{9}$$

This volatility equation is similar to the GARCH(1,1) except that the ARCH term is replaced by an increment in volume and there is no constant. As seen with GARCH models, this variance equation characterizes shocks to volatility as decaying over time. Furthermore, the impact of volume on volatility is symmetric, which is similar to linear GARCH models. For instance, the size of volume shocks proportionally affects volatility so that volatility dampens at the same rate independently of the size of volume shocks. In contrast with any GARCH models, the shock arises from a contemporaneous change in trading volume rather than from the lagged return shock used in GARCH models. In this sense, I compute volatility from current and past trading volumes while GARCH computes it from the history of asset prices.

4 Option Pricing

4.1 Derivation

In Section 3, the joint process of the three variables was expressed under the physical probability measure. At this point, I transform the stochastic process to find a risk-neutral probability measure. To this purpose, I follow Merton (1976) who assumes that jump risks are diversifiable. Under this assumption and in the absence of arbitrage opportunities, the market becomes complete so that there exists a unique risk-neutral probability measure, as pointed out by Harrison and Kreps (1979). The risk neutral process, then, can be derived by replacing the drift term α_t in the return process under the physical measure by the risk-free rate because the expected return on any asset, under the risk-neutral probability, must be equal to the risk-free rate to avoid arbitrage opportunities. The risk-neutral process of returns then follows

$$ds_t = (r_t - \theta m - \frac{\sigma_t^2}{2})dt + \sigma_t d\widehat{W}_t + (Y + \rho H)d\widehat{N}_t, \tag{10}$$

where r_t is the instantaneous spot interest rate, assumed to be deterministic.[12] $\{\widehat{W}_t : t \geq 0\}$ and $\{\widehat{N}_t : t \geq 0\}$ are a Wiener process and a Poisson process under the risk-neutral probability, respectively. The risk-neutral process of volume and volatility is the same as the volume-volatility process under the physical probability equations (3) and (6), except that the common jump process is a Poisson process under the risk-neutral probability.

[12] The assumption of deterministic interest rates is realistic for short and medium term options because the rates do not significantly fluctuate. For instance, the annual standard deviation of the continuously compounded real return on a three-month T-bill is less than two percent. In contrast, the annual standard deviation of the continuously compounded real return on a US stock market index exceeds 15 percent. See Campbell (1999).

Let $P(s_t, \sigma_t^2, \tau)$ be the time t price of a European put option written on the stock with strike price K and time-to-maturity τ. This price also depends on the current price and volatility of the underlying asset. The absence of arbitrage implies that the discounted price is a martingale under the risk-neutral probability, $\widehat{\mathbb{P}}$, such that the conditional expectation of put option returns is given by $\widehat{E}_t \frac{dP}{P} = r_t$. Applying the extended Itô's formula to the underlying price processes, the option price must satisfy the partial differential equation:

$$
\begin{aligned}
0 &= -r_t P + (r - \theta m - \frac{\sigma_t^2}{2})P_s - (\lambda - \beta\gamma)\sigma_t^2 P_{\sigma^2} - P_\tau + \frac{\sigma_t^2}{2} P_{ss} \\
&\quad + \theta[\widehat{E}_t P(s_t + (Y + \rho H), \sigma_t^2 + \beta H, \tau) - P(s_t, \sigma_t^2, \tau)] \qquad (11)
\end{aligned}
$$

with terminal condition, $P(S_T, \sigma_T^2, 0) \equiv \overline{P}(S_T) = (K - S_T)^+$. In Appendix B, it is shown that the solution to the partial differential equation (11) is given by:

$$
P(S_t, \sigma_t^2, \tau, K) = B(t,T) K \widehat{F}(K; S_t, \sigma_t^2, \tau) - S_t \widehat{G}(K; S_t, \sigma_t^2, \tau), \qquad (12)
$$

where $B(t,T)$ represents the time t price of a discount bond with maturity τ so that $B(t,T) = e^{-r(T-t)}$, $\widehat{F}(K; S_t, \sigma_t^2, \tau) = \Pr(S_T < K | S_t, \sigma_t^2, \tau)$ is the risk-neutral probability that the option expires in the money conditional on the current stock price and volatility and time to maturity, and $\widehat{G}(K; S_t, \sigma_t^2, \tau) = \int_0^K \frac{s}{\widehat{E} S_T} d\widehat{F}(s; S_t, \sigma_t^2, \tau)$ is an alternative conditional probability that the option expires in the money. Notice that the local volatility, σ_t^2 is calculated by equation (9) given parameter values. The explicit form of the characteristic functions are presented in Appendix B.[13] The first and second terms on the right hand side of equation (12) can be interpreted as the present value of the strike price payment and the

[13]The PDE (12) involves two Fourier inversions which are computationally demanding. To reduce (12) to one Fourier inversion, Epps (2003) considers a derivative whose value is S_T^ς at expiration for any real number ς. The value of this derivative follows (12). Solving the PDE with the terminal condition, the value of a European put option is given by

present value of the underlying stock upon optimal exercise. Once the price of put options is obtained, the corresponding call option price with the same strike price and term-to-maturity is easily pinned down by the call-put parity condition,

$$C(S_t, \sigma_t^2, \tau, K) = P(S_t, \sigma_t^2, \tau, K) + S_t - B(t,T)K. \tag{13}$$

4.2 Option price properties

I will now investigate some qualitative implications of the stochastic volatility and volume (SVV) model for option prices. To this purpose, I will replicate the various surfaces of the BS implied volatility curves extracted from option prices generated by the SVV model. I will also analyze the sensitivity of these patterns to parameters entering the option pricing formula.

For implementing these numerical experiments, I choose benchmark parameter values of the SVV model by calibrating on the price for an underlying asset and trading volume data and using plausible values found in Bakshi *et al.* (1997). I need to identify eight structural parameters in the SVV model and three inputs for the underlying asset and the risk-free rate. For this purpose, I use information on S&P 500 index. Firstly, the values of β and λ follow directly from the estimates of a regression model for implied volatility and trading volume, $\beta = 1.26 \times 10^{-15}$ and λ $(= \frac{1}{\alpha^{251}} - 1) = 1.05 \times 10^5$.[14] Secondly, the parameter γ $(= 8.27 \times 10^{19})$

$$P(S_t, \sigma_t^2, \tau, K) = B(t,T)\frac{K}{2}\left[1 - \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{X^{-i\varsigma}}{i\varsigma + \varsigma^2}\Psi_{\widehat{F}}(\varsigma; s_t, \sigma_t^2, \tau)d\varsigma\right],$$

where $\Psi_{\widehat{F}}$ is derived in Appendix B. We will employ this formula to compute put option prices for empirical exercises.

[14] I estimate a regression to fit this relationship $IV_t = c + \alpha \cdot IV_{t-1} + \beta \cdot V_t + \varepsilon_t$, where V_t denotes an increment in trading volume of NYSE all stocks, and IV_t represents BS implied volatility of S&P 500 call options averaged over maturities (5 days $<$ maturity $<$ 180 days) and moneyness (0.9 $<$ moneyness $<$1.1).

is chosen so that the rate of mean-reversion of volatility κ ($= \lambda - \gamma\beta$) is 1.0 because Bakshi *et al.* (1997) found that plausible values of mean-reversion are positive but in the vicinity of one. Again, considering parameter estimates of several option pricing models in Bakshi *et al.* (1997), I set θ (jump intensity) equal to 0.6, μ (the mean of jump size in returns) to -0.1, ϕ (the standard deviation of jump size in returns) to 0.1. Thirdly, the parameter representing the mean jump size in trading volume ψ ($= 6.49 \times 10^{13}$) is set so that the long-term volatility $\sqrt{\frac{\theta\beta\psi}{\lambda-\beta\gamma}}$ equals the sample standard deviation of S&P 500 index returns from September 3rd, 2002 until August 29th, 2003 ($= 0.22$). Fourthly, ρ (the correlation between jump size in returns and in volatility) is set 3.36×10^{-29}, for the correlation between returns and trading volume of S&P 500 index and the model-implied one (the covariance $= 2\rho\theta\psi^2$) to be the same. Fifthly, the current values of the stock price S_t ($= 877.68$) to match the level of S&P 500 index on September 3rd, 2002. The current instantaneous volatility σ_t ($= 0.32$) is computed as the sample standard deviation of 39 ten-minute returns on September 3rd, 2002 transforming in a yearly basis multiplied by the root of 251×39. Finally, the risk-free rate r ($= 0.016$) is set to be the sample mean over time to maturity 2 to 177 days on September 3rd, 2002.

Figure 6 provides a view of the implied volatility surface implicit in SVV call option prices for a benchmark choice of parameter values. This figure shows that the SVV model is capable of replicating a variety of aspects of the implied volatility surface that have been documented in option markets. Firstly, the smirk shapes of implied volatility appear for maturities: the volatility is higher for in-the-money than for out-of-the money contracts and at-the-money contracts for all maturities. Secondly, smirks flatten with time-to-maturity.

Finally, the term structure of implied volatility for at-the-money is upward sloping while it is downward sloping for out-of-the-money and in-the-money options.

Next, I will explore other implications of the SVV model for the volatility smile. Figure 7 summarizes the effects of changes in the parameter values on the implied volatility surface at a fixed maturity (= 76 trading days). In this figure, I can observe three distinct groups of parameters. The first consists of parameters that can generate the asymmetric features of implied volatility smiles. The second group sharply increases the volatility for in-the-money and out-of-the-money options so that the smile is exacerbated with the level of volatility. The last group also strengthens the volatility smile, but this smile intensity occurs as the level of volatility decreases.

In the first group, μ characterizes the mean jump size in returns independently of volume and ρ characterizes the mean jump sizes in returns related to volume. In the first panel of Figure 7, μ is set to produce a mean total jump size in returns of -0.18, -0.09 and 0.23 on an annual basis, resulting in a significant asymmetry in the volatility smile. From Panel B, changes in ρ have similar effects because a change in the value of ρ leads to changes in the average return and ρ also affects the correlation between returns and volume (or volatility). The incorporation of non-zero mean jump size and correlation between returns and volatility affects the skewness of the conditional return distribution that explains the asymmetric distortions of the volatility smiles.

The second group of parameters that strengthen the volatility smile contains ϕ (the volatility of return jump size), ψ (the mean jump size of volume) and θ (the jump intensity). As revealed in Panels C-E, the degree of the smile curve increases as the level of the volatility increases. Since these parameters are positively related to the volatility of volatility, excess kurtosis increases with the value of ϕ, ψ and θ. The increase in excess kurtosis intensifies the "depth" of the volatility smile. Contrary to the parameters in the first group, the second group parameters do not affect the sign of skewness, thus leading to no change in asymmetric volatility pattern.

The third and final group of parameters consists of parameters in the volatility function, β, λ and γ. Panels F-H show that changes in these parameters tend to intensify the implied volatilities. Unlike the case of the second group parameters, the volatility curvature is strengthened as the volatility level decreases. This strength in smile results from the fact that these parameters affect the long-term volatility of returns , thus having the effect of shifting the level of implied volatilities.

Next, I will turn to analyze the term structure pattern of implied volatility. Figure 8 displays the implied volatility term structure for at-the-money options. The SVV model generates upward and downward sloping term structures. As it has been presented, upward and downward sloping term structures are observed in reality. The term structure depends on the long-term volatility, which is a function of parameters, β, λ and γ. When long-term volatility is relatively low ($= 0.22$) to the current volatility ($= 0.26$), volatility decreases with time to maturity. When long-term volatility is relatively high ($= 0.50$) to the current volatility, term structure is upward sloping. These term structure patterns can be interpreted as such that when current volatility is above long-term volatility, return volatility is expected to decline over time and thus implied volatility falls with maturity. In contrast, Das and Sundaram (1999) show that the term structure of at-the-money implied volatilities produced by Merton's (1976) jump-diffusion model is always a monotone increasing function of maturity, which is inconsistent with the data.

4.3 Conditional moments of stock returns

It has been shown that the SVV model is able to replicate several cross-sectional (in terms of moneyness and time-to-maturity) properties of implied volatility. Das and Sundaram (1999) show that skewness and excess kurtosis of asset returns (under a risk-neutral probability) are important factors in creating various shapes of implied volatility. I will thus examine whether the configuration of the risk-neutral stochastic process in equity returns assumed by the SVV model significantly produces conditional 3rd and 4th moments. I, moreover, compare these moments with those generated by the jump-diffusion model of Merton (1976).

The closed-form expression of skewness and excess kurtosis of stock returns are derived in Appendix A. It is easily observed that there is no skewness and excess kurtosis if no jumps occur, or the jump intensity parameter $\theta = 0$. Under parameter values already used as the benchmark in the previous subsection, the SVV model generates skewness of -4.32 for a one-month maturity, of -1.74 for a three-month maturity and of -0.97 for a six-month maturity; similarly, excess kurtosis is 30.37 for a one-month maturity, 7.81 for a three-month maturity, and 3.50 for a six-month maturity. In contrast, the jump-diffusion model merely generates -0.22, -0.13 and -0.09 for skewness of a one-month, a three-month and a six-month maturity, respectively; 0.56, 0.19 and 0.09 excess kurtosis for a one-month, a three-month and a six-month maturity, respectively, for the same parameter selection. The SVV model thus produces roughly ten to twenty times larger skewness in the absolute term and forty to fifty times larger kurtosis than the jump-diffusion model does. These results show that adding jumps in volatility generates significant skewness and excess kurtosis, thus producing the rich features of implied volatilities.

5 Empirical Exercises

In this section I will describe the empirical methods in order to estimate option pricing models and to evaluate their economic importance.

5.1 Estimation procedure

The option prices derived in the previous section are a deterministic function of stock and strike prices, time-to-maturity, the risk-free rate, trading volume, and parameters that according to the model should be known. This means that, apart from observation error, the theory leaves no room for any deviation of observed prices from prices implied by the model. I use call option prices to estimate the model since the model offers the same pricing formula of European and American calls unless the underlying asset pays dividends as mentioned in Section 2. Denoting by $\widetilde{C_j}(S_t, \tau_j, K_j)$ the observed price of the jth call option at time t, I assume that this price is related to the price implied by the model, $C_j(S_t, \sigma_t^2, \tau_j, K_j)$ according to

$$\widetilde{C_j}(S_t, \tau_j, K_j) = C_j(S_t, \sigma_t^2, \tau_j, K_j) + \widetilde{e}_{jt}, \tag{14}$$

where the $\widetilde{e}_{jt}$ are independently and identically distributed with mean zero and constant variance. Following Bakshi *et al.* (1997) and Dumas *et al.* (1998), I estimate the eight parameters $\Phi \equiv \{\beta, \lambda, \gamma, \theta, \mu, \phi, \psi, \rho\}$ from each cross-section of option prices, underlying prices and trading volumes so as to minimize the sum of squared differences between observed prices and model implied prices:

$$\underset{\{\beta,\lambda,\gamma,\theta,\mu,\phi,\psi,\rho\}}{Min}\sum_{t=1}^{T}\sum_{j=1}^{N_t}(\widetilde{C}(S_t,\tau_j,K_j)-C(S_t,\sigma_t^2,\tau_j,K_j))^2, \tag{15}$$

where T and N_t denote the number of observation times in the sample and the number of options at one point in time t, respectively.[15,16] Local volatility is determined by current and past trading volumes, or current trading volume and the one-lag local volatility in $\sigma_t^2=\beta(V_t-V_{t-1})+\left(\frac{1}{1+\lambda}\right)\sigma_{t-1}^2$. In this context, the volatility is observable up to the structural parameters, and this fact makes the estimation problem easier than in stochastic volatility models. As in Heston and Nandi (2000), the starting volatility, σ_0, is held invariant at the sample standard deviation which is computed from the whole sample of daily continuously compounded returns appropriately annualized.[17]

To evaluate the pricing performance of the SVV model, I compare its ability to fit cross-sectional option prices both in- and out-of sample relative to several alternative option pricing models in the literature. A natural alternative is of course the BS model. However, the choice seems to be inappropriate because the BS has only one unknown parameter. I thus select as an alternative model for comparison the *ad hoc* BS model of Dumas *et al.* (1998) and Christoffersen and Jacobs (2004), where the volatility of the original BS formula is modeled as:

$$\sigma=\max(0.01,a_0+a_1K+a_2K^2+a_3\tau+a_4\tau^2+a_5K\tau). \tag{16}$$

[15] As mentioned in Section 3, we impose restrictions on the parameter space such that $\beta\geq0$, $\lambda\geq0$, $\gamma\geq0$, $\theta\geq0$, $\phi\geq0$, $\varphi\geq0$, $\lambda-\beta\gamma>0$.

[16] I estimate the model at one point in time t when the number of options is more than or equal to eight that is the number of parameters of the SVV model.

[17] The annualized volatility is given by the standard deviation of daily log returns multiplied by $\sqrt{252}$, respectively. As a matter of facts, the year 2002 had 252 trading days.

While the *ad hoc* BS model is not theoretically consistent, it serves as a more solid benchmark than the original BS model because it is popular with practitioners and its pricing performance has been used as a benchmark in many papers (e.g., Dumas *et al.* (1998), Heston and Nandi (2000), Guidolin and Timmermann (2003)).

As another competing model, I choose Merton's (1976) jump-diffusion (JD) model. The reason for its selection is that the jump-diffusion model is nested in the SVV model and thus I may investigate additional effects of (1) jumps in volatility and (2) trading volume as instruments of volatility in option pricing.

Finally, the stochastic volatility (SV) model of Heston (1993) and the stochastic volatility / random jumps (SVJ) model of Bates (1996) are also compared in pricing performances since they are widely investigated in the option valuation literature. Under a risk-neutral probability, a stochastic structure of continuously compounded returns of the underlying non-dividend-paying stock for the SVJ model is given by

$$ds_t = (r_t - \theta m - \frac{\sigma_t^2}{2})dt + \sigma_t d\widehat{W}_t + X d\widehat{N}_t, \tag{17}$$

$$d\sigma_t^2 = (\kappa - \gamma\sigma_t^2)dt + \xi\sigma_t d\widehat{W}_{\sigma t}. \tag{18}$$

where $\{\widehat{W}_t : t \geq 0\}$ and $\{\widehat{W}_{\sigma t} : t \geq 0\}$ are Wiener processes with $Cov\left(d\widehat{W}_t, d\widehat{W}_{\sigma t}\right) = \rho dt$, $\{N_t : t \geq 0\}$ is a stochastic Poisson process with a jump-intensity parameter, θ, X is the random size of a jump in price, and $\ln(1 + X)$ is assumed to be a normal random variable with parameters $(\ln(1 + \mu) - \frac{\phi^2}{2}, \phi^2)$. Notice that the SVJ model nests the JD and the SV models. For example, the SVJ model alters the JD model when $\kappa = \gamma = \xi = 0$ and also converts the SV model when $\mu = \phi = 0$.

For each model, I investigate two notions of pricing performance: in- and out-of-sample. In the case of the in-sample exercise, I rely on call option prices, stock prices, and trading volumes as inputs to estimate the structural parameters in each model. The estimated parameters are then used to derive model-based option prices corresponding to the sample observations. For out-of-sample exercises, the current prices of call option contracts are computed by using the current stock price and interest rate and the estimates of the time invariant parameters (including the local volatility) obtained from the previous period. Then, to evaluate each model's out-of-sample pricing performance, several measures of accuracy are computed.

The root mean squared valuation error (RMSVE) is the square root of the sample (time) average of the squared mean (cross-sectional) differences of market prices from model's theoretical values defined as

(19) $$\frac{1}{T}\sum_{t=1}^{T}\sqrt{\sum_{j=1}^{N_t}\frac{\left(\widetilde{C}(S_t,\tau_j,K_j)-C(S_t,\sigma_{t-1}^2,\tau_j,K_j)\right)^2}{N_t}}.$$

The other error matrices have functional form

(20) $$\frac{1}{T}\sum_{t=1}^{T}\left(\frac{1}{N_t}\sum_{j=1}^{N_t}F(\widetilde{C},C)\right).$$

Each metric is determined by the following function $F(\widetilde{C},C)$. The mean absolute valuation error (MAVE) is the sample average of the absolute differences between market prices and the theoretical values, defined as

(21) $$F(\widetilde{C},C)=\left|\widetilde{C}(S_t,\tau_j,K_j)-C(S_t,\sigma_{t-1}^2,\tau_j,K_j)\right|.$$

The mean percentage pricing errors (MPPE) implies

$$F(\widetilde{C}, C) = \frac{\widetilde{C}(S_t, \tau_j, K_j) - C(S_t, \sigma_{t-1}^2, \tau_j, K_j)}{\widetilde{C}(S_t, \tau_j, K_j)}. \tag{22}$$

These measurements have been used to assess the quality of the fitted models in major empirical works, including Bakshi *et al.* (1997), Dumas *et al.* (1998), Guidolin and Timmermann (2003), and Heston and Nandi (2000). MPPE is represented as the sum of pricing errors in terms of percentage points and can take either a negative value or a positive one. As a result, rather than measuring pricing performance for comparison, MPPE checks whether or not a model is likely to overvalue option prices. On the other hand, the other two measures are in terms of dollar values so that they are easily interpreted, but they tend to embed over-weights on high price options.

5.2 In-sample parameter estimates

Employing the equity and index (call) option data illustrated in Section 2, I estimate the six option pricing models from the non-least-square estimation, which minimizes the in-sample sum of squared errors between model option values and market option prices. In addition to the exclusion criteria described in Section 2, for the estimations I select option contracts whose time to expiration is less than 100 trading days and spot-strike price ratio is between 0.9 and 1.1. Table 6 provides the square roots of the mean squared valuation errors for the nine options. Several observations are highlighted. Firstly, as seen in the table, the *ad hoc* BS model reveals the most flexibility. For example, its valuation performances in terms of the RMSVE are improved over the BS model by at least 65 percent for the all options (the highest: 86 percent for MSFT, the lowest: 66 percent for S&P 500). Also, the in-sample average of valuation errors of the *ad hoc* BS model are uniformly smaller than other structural models. This result is somewhat unexpected because the number of parameters estimated is smaller for the *ad hoc* BS model than for the SVJ and the SVV models. For in-sample estimation, the key element to reduce valuation errors is a cross-sectional model fit across option contracts, not the model's capture of time-variation in option prices. As a result, the smoothing volatility function of the ad hoc BS, which is a quadratic form of moneyness and time to maturity, is able to match cross-sectional volatility patterns of options.[18]

Secondly, the average pricing error of the SVV model is significantly smaller than that of the jump-diffusion model, except for AMD. For instance, the RMSVE are reduced by at least 20 percent. (the highest: 61 percent for AOL, the lowest: 21 percent for INTC). This empirical evidence indicates that taking stochastic jumps in volatility into account is of first-order importance in improving on the jump-diffusion model. Additionally, the SVV model does not under-perform the four structural option pricing models in terms of in-sample model fittings. Consequently, modeling local volatility as discontinuous jumps could be a better alternative to continuous local volatility models.

I will now turn to analyzing the stability of the SVV model's parameters. Whether or not estimated parameters are steady over time is an important element to judge model specifications because model parameters are assumed to be constant. Summary statistics of parameter estimates obtained for the SVV model are reported in Table 7. As can been seen in the table, the parameters estimated each day largely fluctuate for the nine options.

[18]According to Christoffersen and Jacobb (2002), Dumas *et al.* (1998) and Guidolin and Timmermann (2003), who use S&P 500 index options for empirical exercises, the *ad hoc* BS model outperforms structural models such as a stochastic volatility model, deterministic volatility models, a GARCH model and a Baysian belief model for in-sample pricing performances.

For instance, their sample standard deviation divided by the sample mean, for most cases, very large. There are also outliers of several parameter estimates that considerably affect the sample means so that the means significantly deviate from the mediums. Specifically, the large fluctuations in most of the parameters for AMD and S&P 500 are observed. This evidence shows that the in-sample estimates for the SVV model seem to be unstable. This instability of parameters and the relatively small in-sample pricing errors for the SVV model indicate the potential over-fitting of the model. In the next subsection, I will investigate the economic importance of the SVV, focusing on the model's valuation prediction errors.

5.3 Out-of-sample predictions

I have demonstrated that the in-sample model fittings are overall in the following order, the BS, the JD, the SV, the SVJ, the SVV and the *ad hoc* BS models. This inference may result, however, not from economic components, but from merely over-fitting of the data because the in-sample pricing performances tend to improve as the number of parameters increases. One method to evaluate the economic importance of option pricing models is to explore each model's out-of-sample cross-sectional price prediction.

Table 8 provides the average one-step-ahead prediction errors in terms of the three measurements. There seems to be a series of regularities in the out-of-sample prediction errors. Firstly, the prediction performance of the *ad hoc* BS model is faded compared with its in-sample fittings. For example, the out-of-sample RMSVEs increase by 713 percent (IBM) to 277 percent (S&P500). This significant deterioration suggests that the *ad hoc* BS model achieves the in-sample flexibility simply by over-fitting the data, which is not surprising because the model is not based on any economic considerations.[19]

Secondly, the aggregate prediction error measures, in general, rank the SVV as the best model. The SVV model outperforms the *ad hoc* BS model in the RMSVE and AMVE and all options except for the AMVE of S&P 500, more importantly the SVV model is the best in out-of-sample prediction performances except for AMD and S&P 500. For instance, the SVV model improves the RMSVE of the *ad hoc* BS model by 3 percent (S&P 500) to 67 percent (IBM). These results suggest that information on trading volume of underlying assets possibly captures time-variations in return volatility for one-period-ahead predictions and that incorporating trading volume into option pricing models thus leads to first-order improvement in price predictions. A reason why the SVV fails to beat the other models in forecasting performances for AMD and S&P 500 options might be due to large instability in parameters seen in Table 7. In addition, trading volume of all NYSE stocks might not be a good proxy for trading volume of stocks in S&P 500. Finally, according to MPPE, all models except for the jump-diffusion model tend to underprice the call options.

6 Conclusion

Based on extant empirical evidence that trading volume provides information on changes in stock returns, I have derived an option pricing formula established on a joint, trivariate process for stock returns, trading volumes, and volatility. This model offers a novel approach for option valuation that formulates the volatility and volume of the underlying asset to be stochastic. Firstly, a key element of the stochastic volatility and volume (SVV) model is

[19]Using S&P 500 index options, Heston and Nandi (2000) and Guidolin and Timmermann (2003) find the similar result by the *ad hoc* BS model.

that a latent factor modeled as a discontinuous process generates a simultaneous jump in returns, volatility, and volume. Additionally, the SVV model incorporates trading volume as an instrument of unobservable volatility so that the volatility is computed by using current and past volumes. Finally, the local volatility parameter is predictable in this model setting. I analyze the model's implications for the implied volatility surface. Under plausible values of the parameters, I show that the SVV model provides a good description of cross-sectional properties and a term structure of the implied volatility.

I also have conducted empirical evaluations of the model by investigating the in-sample fitting and out-of-sample option price prediction from data on prices of the eight equity and one index options and have compared it with other option pricing models including the BS, the *ad hoc* BS, the jump-diffusion and stochastic volatility models. In terms of in-sample pricing errors, the SVV model is superior to other structural models, while the estimated parameters are considerably unstable in the sample. Despite this instability of the parameter estimates, the SVV model is, in most of the cases, the best in one-period ahead predictions of option prices. These empirical findings suggest that incorporating stochastic jumps in volatility and trading volume is important to option valuation.

There is a course for future research projects applying the SVV model. I may estimate the parameters in the model using time-series data on returns and volume, putting aside option data. The SVV model has the advantage that the closed forms of conditional characteristic function enable this estimation to be tractable, using estimation methods developed by Singleton (2001). I can then compare the estimated parameters on the physical probability measure with those on the risk-neutral probability measure estimated in this paper.

Appendix

A Derivation of Conditional Skewness and Kurtosis

Let $\Psi(\varsigma; s_t, \sigma_t^2, \tau)$ be the characteristic function of a return at time $t+\tau$ conditional on the current log price and current instantaneous volatility of an underlying asset. Notice that ς is a real number. Under the process of returns (2) and the volatility process (6), the Kolmogorov backward equation of the conditional characteristic function is given by

$$0 = -\Psi_\tau + (\alpha_t - \theta m - \frac{\sigma_t^2}{2})\Psi_s - (\lambda - \beta\gamma)\sigma_t^2\Psi_{\sigma^2} + \frac{\sigma_t^2}{2}\Psi \tag{A1}$$

$$+\theta[\widehat{E}_t\Psi(\varsigma; s_t + (Y+ZH), \sigma_t^2 + \beta H, \tau) - \Psi(\varsigma; s_t, \sigma_t^2, \tau)],$$

$$\text{with initial condition } \ \Psi(\varsigma; s_t, \sigma_t^2, 0) = e^{is_t\varsigma}. \tag{A2}$$

Next, I guess and verify that the solution is of the form

$$\Psi(\varsigma; s_t, \sigma_t^2, \tau) = e^{g(\tau;\varsigma) + h(\tau;\varsigma)\sigma_t^2 + i\varsigma s_t}. \tag{A3}$$

Substituting equation (A3) into (A1) and (A2) yields the system of ordinary differential equations

$$\begin{cases} g_\tau(\tau;\varsigma) = (\alpha_t - \theta m)i\varsigma + \theta E[e^{h(\tau;\varsigma)\beta H + i\varsigma(Y+ZH)} - 1] \\ h_\tau(\tau;\varsigma) = \frac{i\varsigma}{2}(-1 + i\varsigma) - (\lambda - \beta\gamma)h(\tau;\varsigma) \end{cases} \tag{A4}$$

with $g(0;\varsigma) = h(0;\varsigma) = 0$. The solutions are

$$g(\tau;\varsigma) = (\alpha_t - \theta m)i\varsigma\tau + \theta E[e^{i\varsigma(Y+ZH)}\int_0^\tau e^{h(s;\varsigma)\beta H} ds - \tau], \tag{A5}$$

$$h(\tau;\varsigma) = \frac{i\varsigma(1 - i\varsigma)\{e^{-\kappa\tau} - 1\}}{2\kappa}. \tag{A6}$$

Hence I have inserted back the proposed solution into the PDE (A1) and confirmed that the solution satisfies the PDE.

A standard argument (see, Epps (2000, p. 62 and p. 64)) establishes now that the mean, variance, skewness, and excess kurtosis of returns are

(A7) $$\text{Mean } = \mu_1, \text{ Variance } = \mu_2, \text{ Skewness } = \frac{\mu_3}{\mu_2^{\frac{3}{2}}}, \text{ Excess Kurtosis } = \frac{\mu 4}{\mu_2^2},$$

where $\mu_k = \left.\frac{d^k \ln \Psi}{d\varsigma^k}\right|_{\varsigma=0}$. Since the characteristic function has an exponential-affine form, the expressions for the moments in Proposition are given by the derivatives of $g(\tau;\varsigma)$ and $h(\tau;\varsigma)$ with respect to ς. Notice that computing the integral in equation (A5) becomes less cumbersome after taking derivatives of $g(\tau;\varsigma)$.

The closed-form expression of skewness and excess kurtosis of stock returns conditional on the current price and current volatility level in the diffusion term are given by, respectively,

(A8) $$\frac{\theta\{8\left(EX^3\right)\tau+\left[24(EXH)-12(EX^2H)\right]\delta A_1+6\left[(EXH^2)+16(EH^2)\right]\delta^2 A_2-(EH^3)\delta^3 A_3\}}{8\left[A_0\sigma_t^2+\theta\left\{(EX^2)\tau+[(EH)-(EXH)]\delta A_1+\frac{1}{2}(EH^2)\delta^2 A_2\right\}\right]^{\frac{3}{2}}}$$

(A9) $$\frac{\theta\{\left(EX^4\right)\tau+\left[128(EX^2H)-8(EX^3H)\right]\delta A_1+48\left[EH^2(1+2X+0.5X^2)\right]\delta^2 A_2-8\left[EH^3(3-X)\right]\delta^3 A_3+(EH^4)\delta^4 A_4\}}{16\left[A_0\sigma_t^2+\theta\left\{(EX^2)\tau+[(EH)-(EXH)]\delta A_1+\frac{1}{2}(EH^2)\delta^2 A_2\right\}\right]^{\frac{3}{2}}}$$

where

$$\begin{aligned}
\kappa &= \lambda-\beta\gamma,\ \delta=\frac{\beta}{\kappa},\ A_0=\frac{1-e^{-\kappa\tau}}{\kappa},\ A_1=\tau-\frac{1-e^{-\kappa\tau}}{\kappa},\ A_2=\tau-\frac{3-4e^{-\kappa\tau}+e^{-2\kappa\tau}}{2\kappa}, \\
A_3 &= \tau-\frac{11-18e^{-\kappa\tau}+9e^{-2\kappa\tau}-2e^{-3\kappa\tau}}{6\kappa}, \\
A_4 &= \tau-\frac{25-48e^{-\kappa\tau}+36e^{-2\kappa\tau}-16e^{-3\kappa\tau}+3e^{-4\kappa\tau}}{12\kappa}.
\end{aligned}$$

$X=Y+\rho H$ and H represent the sizes of jumps in returns and in volume, respectively.

B Derivation and Solution of the Partial Differential Equation

To derive a solution to the partial differential equation (PDE) of a European put option price, I follow the characteristic function based method in Heston (1993), Bates(1996) and Epps (2000). Firstly, I guess a solution of the PDE (11) as

$$\text{(B1)} \qquad P^E(s_t,\sigma_t^2,\tau) = B(t,T)K\int_0^K d\widehat{F}_t(S;s_t,\sigma_t^2,\tau) - S_t\int_0^K d\widehat{G_t}(S;s_t,\sigma_t^2,\tau)$$

$$= B(t,T)K\widehat{F}(K;s_t,\sigma_t^2,\tau) - S_t\widehat{G}(K),$$

where $\widehat{F}(K) \equiv \widehat{F}_t(K;s_t,\sigma_t^2,\tau)$ and $\widehat{G}(K) \equiv \widehat{G_t}(K;s_t,\sigma_t^2,\tau)$. Substituting the conjectured solution (B1) into equation (11) gives the PDEs that are satisfied by $\widehat{F}$ and $\widehat{G}$

$$\text{(B2)} \qquad 0 = -\widehat{F}_\tau + (r_t - \theta m - \frac{\sigma_t^2}{2})\widehat{F}_s - (\lambda-\beta\gamma)\sigma_t^2\widehat{F}_{\sigma^2} + \frac{\sigma_t^2}{2}\widehat{F}_{ss}$$

$$+\theta[\widehat{E}_t\widehat{F}(s_t+(Y+\rho H),\sigma_t^2+\beta H,\tau) - \widehat{F}(s_t,\sigma_t^2,\tau)],$$

$$\text{(B3)} \qquad 0 = -\widehat{G}_\tau + (r_t - \theta m + \frac{\sigma_t^2}{2})\widehat{G}_s - (\lambda-\beta\gamma)\sigma_t^2\widehat{G}_{\sigma^2} + \frac{\sigma_t^2}{2}\widehat{G}_{ss}$$

$$+\theta[\widehat{E}_t(1+X)\widehat{G}(s_t+(Y+\rho H),\sigma_t^2+\beta H,\tau) - \widehat{G}(s_t,\sigma_t^2,\tau)],$$

with boundary conditions

$$\text{(B4)} \qquad \widehat{F}_t(K;s_T,\sigma_T^2,0) = \widehat{G_t}(K;s_T,\sigma_T^2,0) = 1_{[0,K)}e^{s_T}.$$

From the definition, the conditional characteristic functions for $\widehat{F}$ and $\widehat{G}$ are determined as

$$\text{(B5)} \qquad \Psi_{\widehat{J}}(\varsigma;s_t,\sigma_t^2,\tau) = \int_{-\infty}^{\infty} e^{i\varsigma s}d\widehat{J}(s;s_t,\sigma_t^2,\tau),$$

where $\widehat{J} \in \{\widehat{F}, \widehat{G}\}$. The corresponding characteristic functions also satisfy the similar PDEs (B2) and (B3):

$$\nabla_{\widehat{J}} \Psi_{\widehat{J}}(\varsigma; s_t, \sigma_t^2, \tau) = 0, \tag{B6}$$

where $\nabla_{\widehat{J}} = -\frac{\partial}{\partial \tau} + (r_t - \theta m + a_{\widehat{J}} \frac{\sigma_t^2}{2}) \frac{\partial}{\partial s} - (\lambda - \beta\gamma)\sigma_t^2 \frac{\partial}{\partial \sigma^2} + \frac{\sigma_t^2}{2} \frac{\partial^2}{\partial s^2} + \theta[\widehat{E}_t b_{\widehat{J}} \widehat{J}(s_t + (Y + \rho H), \sigma_t^2 + \beta H, \tau) - \widehat{J}(s_t, \sigma_t^2, \tau)]$, and $a_{\widehat{J}} = -1$ for $\widehat{J} = \widehat{F}$, $+1$ for $\widehat{J} = \widehat{G}$, $b_{\widehat{J}} = +1$ for $\widehat{J} = \widehat{F}$, $1 + X$ for $\widehat{J} = \widehat{G}$, the initial conditions are $\Psi_{\widehat{J}}(\varsigma; s_T, \sigma_T^2, 0) = e^{i\varsigma s_T}$.

In a similar manner as in Appendix A, to solve for the characteristic function explicitly, I consider the functional form

$$\Psi_{\widehat{J}}(\varsigma; s_t, \sigma_t^2, \tau) = \exp[g_{\widehat{J}}(\tau; \varsigma) + h_{\widehat{J}}(\tau; \varsigma)\sigma_t^2 + i\varsigma s_t], \tag{B7}$$

Now consider of the case of $\widehat{J} = \widehat{F}$. Since the PDE is satisfied for any s_t and σ_t^2, inserting the functional form equation (B7) into the PDE (B6) yields

$$g'_{\widehat{F}}(\tau; \varsigma) = (r_t - \theta m)i\varsigma + \theta \widehat{E}[\exp\{(h_{\widehat{F}}(\tau; \varsigma)\beta H + i\varsigma(Y + \rho H)\} - 1], \tag{B8}$$

$$h'_{\widehat{F}}(\tau; \varsigma) = \frac{i\varsigma}{2}(-1 + i\varsigma) - \kappa h_{\widehat{F}}(\tau; \varsigma), \tag{B9}$$

where $\kappa = \lambda - \beta\gamma$. The initial condition (B4) implies that

$$g_{\widehat{F}}(0; \varsigma) = h_{\widehat{F}}(0; \varsigma) = 0. \tag{B10}$$

Solving the resulting system of ordinary differential equations (B8) and (B9) with its initial condition (B10) produces

$$g_{\widehat{F}}(\tau; \varsigma) = (r - \theta m)i\varsigma\tau + \Upsilon_{\widehat{F}}(h_{\widehat{F}}(\tau; \varsigma)), \tag{B11}$$

(B12) $$h_{\widehat{F}}(\tau;\varsigma) = \frac{i\varsigma(1-i\varsigma)\left(e^{-\kappa\tau}-1\right)}{2\kappa},$$

where

$$\Upsilon_{\widehat{F}}(h_{\widehat{F}}(\tau;\varsigma)) = \theta\left\{\tau\left(\frac{e^{i\varsigma\mu-\frac{\varsigma^2\phi^2}{2}}}{A(\varsigma)}-1\right)+\ln\left|\frac{A(\varsigma)+B(\varsigma)e^{-\kappa\tau}}{A(\varsigma)+B(\varsigma)}\right|\left(\frac{e^{i\varsigma\mu-\frac{\varsigma^2\phi^2}{2}}}{\kappa A(\varsigma)}\right)\right\},.$$

$$A(\varsigma) = 1-i\rho\psi\varsigma+\frac{i\beta\psi\varsigma(1-i\varsigma)}{2\kappa},\ B(\varsigma) = -\frac{i\beta\psi\varsigma(1-i\varsigma)}{2\kappa},\ h_{\widehat{F}}(\tau;\varsigma) = \frac{i\varsigma(1-i\varsigma)\left(e^{-\kappa\tau}-1\right)}{2\kappa}.$$

In a similar way, I can derive

(B13) $$g_{\widehat{G}}(\tau;\varsigma) = (r-\theta m)i\varsigma\tau+\Upsilon_{\widehat{G}}(h_{\widehat{G}}(\tau;\varsigma)),$$

(B14) $$h_{\widehat{G}}(\tau;\varsigma) = \frac{i\varsigma(1+i\varsigma)\{1-e^{-\kappa\tau}\}}{2\kappa},$$

where

$$\Upsilon_{\widehat{G}}(h_{\widehat{G}}(\tau;\varsigma)) = \theta\left\{\tau\left(\left[\frac{e^{(i\varsigma+1)\mu+\frac{(i\varsigma+1)^2\phi^2}{2}}}{A(\varsigma)}\right]-m-1\right)\right.$$
$$\left.+\ln\left|\frac{A(\varsigma)+B(\varsigma)e^{-\kappa\tau}}{A(\varsigma)+B(\varsigma)}\right|\left(\frac{e^{(i\varsigma+1)\mu+\frac{(i\varsigma+1)^2\phi^2}{2}}}{\kappa A(\varsigma)}\right)\right\},.$$

$$A(\varsigma) = 1-i\rho\psi(\varsigma-i)-\frac{i\beta\psi\varsigma(1+i\varsigma)}{2\kappa},\ B(\varsigma) = -\frac{i\beta\psi\varsigma(1+i\varsigma)}{2\kappa},\ h_{\widehat{G}}(\tau;\varsigma) = \frac{i\varsigma(1+i\varsigma)\{1-e^{-\kappa\tau}\}}{2\kappa}.$$

Hence I have inserted back the "guess" into the PDEs and confirmed that the "guess" satisfies the PDE. As a result, I can confirm that the "guess" is the conditional characteristic functions for $\widehat{F}$ and $\widehat{G}$. Given the conditional characteristic functions, $\widehat{F}$ and $\widehat{G}$ are recovered by using the following inversion formula:

(B15) $$\widehat{F}(K;s_t,\sigma_t^2,\tau) = \frac{1}{2}-\lim_{c\to\infty}\int_{-c}^{c}\frac{\exp(-i\varsigma\ln(K))}{2\pi i\varsigma}\Psi_{\widehat{F}}(\varsigma;s_t,\sigma_t^2,\tau)d\varsigma,$$

(B16) $$\widehat{G}(K;s_t,\sigma_t^2,\tau) = \frac{1}{2}-\lim_{c\to\infty}\int_{-c}^{c}\frac{\exp(-i\varsigma\ln(K))}{2\pi i\varsigma}\Psi_{\widehat{G}}(\varsigma;s_t,\sigma_t^2,\tau)d\varsigma.$$

References

[1] Andersen, T. G. "Return Volatility and Trading Volume: An Information Flow Interpretation of Stochastic Volatility." *Journal of Finance*, 51 (1996), 169-204.

[2] Bakshi, G., Cao, C. and Z. Chen. "Empirical Performance of Alternative Option Pricing Models," *Journal of Finance*, 52 (1997), 2003-2049.

[3] Bates, D. "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutschemark Options," *Review of Financial Studies*, 9 (1996), 69-108.

[4] Black, F., and M. Scholes. "The Pricing Options and Corporate Liabilities," *Journal of Political Economy*, 81 (1973), 637-659.

[5] Bollerslev, T. "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics*, 31 (1986), 307-327.

[6] Campbell, J.Y. "Asset Prices, Consumption, and the Business Cycle," *Handbook of Macroeconomics,* Vol.1, Taylor, John B., and Michael Woodford, eds., Amsterdam, North Holland Press, Chapter 19 (1999), 1231-1303.

[7] Campbell, J.Y., Grossman, S.J., and Wang, J. "Trading Volume and Serial Correlation in Stock Returns," *Quarterly Journal of Economics*, 108 (1993), 905-939.

[8] Christoffersen, P., and K. Jacobs. "The Importance of the Loss Function in Option Valuation," *Journal of Financial Economics*, 72 (2004), 291-318.

[9] Clark, P.K. "A Subordinated Stochastic Process with Finite Variance for Speculative Prices," *Econometrica*, 41 (1973), 135-156.

[10] Copeland, T.E. "A Model of Asset Trading under Assumption of Sequential Information Arrival," *Journal of Finance*, 31 (1976), 1149-1168.

[11] Das, S.R., and R.K. Sundaram. "Of Smiles and Smirks: A Term Structure Perspective," *Journal of Financial and Quantitative Analysis*, 34 (1999), 211-239.

[12] Duan, J. "The GARCH Option Pricing Model," *Mathematical Finance*, 5 (1995), 13-32.

[13] Duffie, D., Pan, J., and K. Singleton. "Transform Analysis and Asset Pricing for Affine Jump-Diffusions," *Econometrica*, 68 (2000), 1343-1375.

[14] Dumas, B., J. Fleming and B. Whaley. "Implied Volatility Functions: Empirical Tests," *Journal of Finance*, 53 (1998), 2059-2106.

[15] Epps, T.W. "Security Price Changes and Transaction Volumes: Theory and Evidence," *American Economic Review*, 65 (1975), 586-97.

[16] Epps, T.W. *Pricing Derivative Securities*, World Scientific Press (2000).

[17] Epps, T.W. "Option Pricing under Stochastic Volatility with Jumps," working paper, University of Virginia (2003).

[18] Epps, T.W., and M.L. Epps. "The Stochastic Dependence of Security Price Changes and Transaction Volumes: Implications for the Mixture-of-Distributions Hypothesis," *Econometrica*, 44 (1976), 305-321.

[19] Gallant, A.R., Rossi, P.E., and G. Tauchen "Stock Prices and Volume," *Review of Financial Studies*, 5 (1992), 199-242.

[20] Guidolin, M., and A. Timmermann. "Option Prices and Implied Volatility Dynamics under Bayesian Learning," *Journal of Economic Dynamics and Control*, 27 (2003), 717-769.

[21] Harris, L. "Transaction Data Tests of the Mixture of Distributions Hypothesis," *Journal of Financial and Quantitative Analysis*, 22 (1987), 127-141.

[22] Harrison, M., and D. Kreps. "Martingales and Arbitrage in Multi-Period Securities Markets," *Journal of Economic Theory*, 20 (1979), 381-408.

[23] Heston, S. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options," *Review of Financial Studies*, 6 (1993), 327-343.

[24] Heston, S., and S. Nandi. "A Closed Form GARCH Option Pricing Model," *Review of Financial Studies*, 13 (2000), 585-625.

[25] Howison, S., and D. Lamper. "Trading Volume in Models of Financial Derivatives," *Applied Mathematical Finance*, 8 (2001), 119-135.

[26] Hull, J.C., and A. White. "The Pricing of Options on Assets with Stochastic Volatilities," *Journal of Finance*, 42 (1987), 281-300.

[27] Karpoff, J.M. "The Relation Between Price Changes and Trading Volume: A Survey," *Journal of Financial and Quantitative Analysis*, 22 (1987), 109-126.

[28] Kendall, Murice .G., and Jean D. Gibbons. *Rank Correlation Methods*, Hafner Publishing Co., Inc., New York (1990).

[29] Lamoureux, C.G., and W.D. Lastrapes. "Heteroskedasticity in Stock Return Data: Volume versus GARCH Effects," *Journal of Finance*, 45 (1990), 221-229.

[30] Lee, B.S., and O.M. Rui. "The Dynamic Relationship Between Stock Returns and Trading Volume: Domestic and Cross-Country Evidence," *Journal of Banking and Finance*, 26 (2002), 51-78.

[31] MacBeth, J.D., and L.J. Merville. "An Empirical Examination of the Black-Scholes Call Option Pricing Model," *Journal of Finance*, 34 (1979), 1173-1186.

[32] Merton, R.C. "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science*, 4 (1973), 141-183.

[33] Merton, R.C. "Option Pricing When Underlying Stock Returns are Discontinuous," *Journal of Financial Economics*, 3 (1976), 125-144.

[34] Poterba, J., and L. Summers. "The Persistence of Volatility and Stock Market Fluctuations," *American Economic Review*, 76 (1986), 1142-1151.

[35] Rubinstein, M. "Implied Biomial Tree," *Journal of Finance*, 40 (1994), 455-480.

[36] Scott, L. "Pricing Stock Options in a Jump-Diffusion Model with Stochastic Volatility and Interest Rates: Applications of Fourier Inversion Methods," *Mathematical Finance*, 7 (1997), 413-426.

[37] Shalen, C.T. "Volume, Volatility, and the Dispersion of Beliefs," *Review of Financial Studies*, 6 (1993), 405-434.

[38] Sheikh, A.M. "The Behavior of Volatility Expectations and their Effects on Expected Returns," *Journal of Business*, 66 (1993), 93-116.

[39] Singleton, K. "Estimation of Affine Asset Pricing Models Using the Empirical Characteristic Function," *Journal of Econometrics*, 102 (2001), 111-141.

[40] Tauchen, G., and M. Pitts. "The Price Variability-Volume Relationship on Speculative Markets," *Econometrica*, 51 (1983), 485-505.

[41] Taylor, S.J., and X. Xu. "The Term Structure of Volatility Implied by Foreign Exchange Options," *Journal of Financial and Quantitative Analysis*, 29 (1994), 57-74.

[42] Wang, J. "A Model of Competitive Stock Trading Volume," *Journal of Political Economy*, 102 (1994), 127-168.

Table 1: Descriptions for Call Options Data

This table reports the total number of observations and sample averages for daily quoted bid-ask midpoint prices, effective bid-ask spreads computed as the ask price minus the bid-ask mid point, and daily trading volume. The averages are computed over the whole sample period, that is September 3rd, 2002 - August 29th, 2003. Short-Term and Medium-Term stand for short-term (more than 5 and less than 61 trading days) contracts and medium-term (more than 60 days and less than 181 trading days) contracts, respectively. OTM denotes out-of-the-money options satisfying that the spot-strike price ratio (= Spot/Strike) is less than or equal to 0.97, ATM represents at-the-money options satisfying that the spot-strike price ratio is more than 0.97 and less than 1.03, ITM is in-the-money option satisfying that the spot-strike ration is more than or equal to 1.03.

	OTM	ATM	ITM	OTM	ATM	ITM
	Short-Term			Medium-Term		
AMD						
Midpoint of Ask and Bid ($)	0.94	0.72	2.64	0.90	1.49	3.35
Effective B-A spread	0.07	0.07	0.12	0.07	0.10	0.13
Daily trading volume	217	170	40	103	168	27
Number of observations	95	98	1076	673	89	1027
AOL						
Midpoint of Ask and Bid ($)	0.63	0.89	5.38	0.93	1.76	6.02
Effective B-A spread	0.06	0.07	0.11	0.07	0.09	0.14
Daily trading volume	895	877	128	194	240	34
Number of observations	134	180	2060	754	172	1997
ORCL						
Midpoint of Ask and Bid ($)	0.59	0.78	4.26	0.86	1.48	4.82
Effective B-A spread	0.04	0.04	0.09	0.06	0.06	0.10
Daily trading volume	862	877	148	180	153	24
Number of observations	143	135	1483	776	183	1891
IBM						
Midpoint of Ask and Bid ($)	1.36	3.23	20.52	2.46	7.11	22.59
Effective B-A spread	0.08	0.11	0.27	0.11	0.16	0.30
Daily trading volume	416	645	53	69	87	12
Number of observations	767	519	3377	2170	419	3024

Table 1: Descriptions for Call Options Data (continued)

	OTM	ATM	ITM	OTM	ATM	ITM
		Short-Term			Medium-Term	
CSCO						
Midpoint of Ask and Bid ($)	0.63	0.90	5.99	0.96	1.84	6.38
Effective B-A spread	0.04	0.04	0.10	0.05	0.07	0.11
Daily trading volume	1748	2188	205	479	520	42
Number of observations	171	130	1927	796	112	1861
DELL						
Midpoint of Ask and Bid ($)	0.86	1.29	7.27	1.36	2.89	9.14
Effective B-A spread	0.06	0.07	0.12	0.08	0.11	0.13
Daily trading volume	430	595	117	130	87	14
Number of observations	383	320	2468	1525	342	2908
INTC						
Midpoint of Ask and Bid ($)	0.72	1.09	6.02	1.13	2.32	7.11
Effective B-A spread	0.05	0.05	0.09	0.06	0.07	0.10
Daily trading volume	821	1149	156	192	196	22
Number of observations	277	202	1958	1092	182	1975
MSFT						
Midpoint of Ask and Bid ($)	1.07	1.89	10.77	1.78	4.24	11.54
Effective B-A spread	0.06	0.07	0.11	0.08	0.11	0.14
Daily trading volume	1293	1319	117	357	176	24
Number of observations	427	390	3543	1395	316	3074
S&P 500						
Midpoint of Ask and Bid ($)	10.38	25.17	148.95	15.85	53.38	193.98
Effective B-A spread	0.40	0.79	1.11	0.60	1.01	1.12
Daily trading volume	787	1360	106	181	522	26
Number of observations	4540	3692	9518	5576	1335	5709

Table 2: Summary Statistics of the BS Implied Volatility

BS implied volatility is calculated from filtered call options data. Observations of implied volatility are restricted on 0.9 < Spot/Strike < 1.1. Mean, StDev, and A(1) refer to the sample mean, the sample standard deviation, and the sample first-order autocorrelation coefficient of implied volatility. Obs. indicates the number of date contains at least one observation. The sample period is September 3rd, 2002 - August 29th, 2003. The total number of observation dates is 251.

	Mean	StDev	A(1)	Obs.
AMD	0.750	0.149	0.963	161
AOL	0.492	0.119	0.958	242
ORCL	0.518	0.115	0.958	231
IBM	0.344	0.077	0.973	251
CSCO	0.489	0.120	0.969	245
DELL	0.384	0.076	0.964	251
INTC	0.487	0.099	0.959	251
MSFT	0.383	0.069	0.955	251
S&P500	0.249	0.052	0.949	251

Table 3: Empirical Properties of Daily Returns on Technology Stocks and an Index

Mean, StDev, A(1), Skewness, and Kurtosis are sample statistics such as the mean, the standard deviation, the first-order autocorrelation, the skewness, the kurtosis of daily returns, respectively. JBstat refers to a test statistic that evaluates the hypothesis that returns have a normal distribution with unspecified mean and variance against the alternative that they do not have a normal distribution. The test scores in bold numbers indicate a 1 percent significance of the test. The sample period is September 3rd, 2002 - August 29th, 2003. The returns are not adjusted for dividend payments. The transaction prices are used as the prices for all stocks.

	Mean	StDev	A(1)	Skewness	Kurtosis	JBstat
AMD	0.0010	0.054	-0.02	-1.25	16.5	**1937.2**
AOL	0.0011	0.030	0.01	-0.99	7.4	**240.8**
ORCL	0.0012	0.031	-0.14	0.10	3.1	0.5
IBM	0.0003	0.022	-0.07	0.74	7.1	**191.9**
CSCO	0.0013	0.029	-0.05	-0.27	4.0	**12.8**
DELL	0.0008	0.020	-0.04	0.49	4.8	**41.5**
INTC	0.0022	0.032	-0.18	-0.75	8.3	**312.0**
MSFT	0.0003	0.021	-0.14	0.16	3.6	4.8
S&P500	0.0004	0.014	-0.15	0.15	3.4	2.6

Table 4: Autocorrelation of Trading Volume and Correlation between Trading Volume and Returns

A(1) is the first-order autocorrelation of trading volume. C(r,tv) and C(|r|,tv) indicate the contemporaneous correlation between returns and volume and between the absolute value of returns and volume, respectively. The corresponding Spearman's rank correlation coefficients, which are used for nonparametric tests of association of two random variables, are shown in parentheses on the fifth and sixth columns. The test scores in bold numbers indicate a 1 %-level rejection of the test of the null hypothesis of no correlation between returns and volume or of no positive correlation between the absolute value of returns and volume. See Appendix Table 11 in Kendall and Gibbons (1990) for the test statistics. The sample covers September 3rd, 2002 - August 29th, 2003. The returns are not adjusted for dividend payments. The transaction prices are used as the prices for all stocks.

	A(1)	C(|r|,tv)	C(r,tv)
AMD	0.59	0.48 (**0.37**)	0.01 (0.09)
AOL	0.37	0.46 (**0.29**)	-0.12 (0.16)
ORCL	0.37	0.45 (**0.41**)	-0.04 (0.03)
IBM	0.54	0.64 (**0.36**)	0.05 (0.05)
CSCO	0.66	0.56 (**0.49**)	-0.00 (0.02)
DELL	0.52	0.52 (**0.39**)	0.10 (0.09)
INTC	0.41	0.50 (**0.33**)	0.08 (0.17)
MSFT	0.64	0.15 (0.07)	0.13 (0.14)
S&P500	0.48	0.28 (0.18)	0.17 (0.10)

Table 5: Maximum Likelihood Estimation of GARCH (1,1) with and without Trading Volume

This table reports maximum likelihood estimates of GARCH (1,1) with ($\gamma \neq 0$) and without trading volume ($\gamma = 0$). The GARCH (1,1) model with volume is:

$$r(t) = c + (h(t)^{0.5})\varepsilon(t),$$

$$h(t) = \omega + \alpha\varepsilon(t-1)+\beta h(t-1)+\gamma V(t),$$

where r(t) is the continuously compounded daily return, V(t) is daily cumulated trading volume, c, ω, α, β, γ are constants, and ε(t) ~ i.i.d.(0,1). This specification is the same estimated by Lamoureux and Lastrapes (1990). The inferences use standard errors robust to the distributional assumptions on ε(t). α+β measures the degree of mean reversion of the volatility process. Bold numbers indicate a 1 %-level rejection of the test of the null hypothesis of negative or zero coefficients. The sample period is September 3rd, 2002 - August 29th, 2003.

	α	β	γ (×10^{-9})	α + β	Log-liklihood
AMD (γ = 0)	0.026	**0.961**		0.987	401.3
	0.236	0.102	**0.278**	0.338	444.3
AOL (γ = 0)	-0.019	**1.014**		0.995	549.8
	-0.011	-0.160	**0.061**	-0.171	565.6
ORCL (γ = 0)	-0.005	**0.997**		0.992	540.8
	0.150	0.113	**0.027**	0.263	545.5
IBM (γ = 0)	0.085	**0.893**		0.978	634.4
	0.106	**0.621**	**0.024**	0.727	651.0
CSCO (γ = 0)	-0.014	**1.013**		0.999	558.9
	0.059	**0.411**	**0.009**	0.471	571.2
DELL (γ = 0)	-0.012	**1.015**		1.003	628.1
	0.047	-0.132	**0.023**	-0.085	648.2
INTC (γ = 0)	-0.014	**1.009**		0.994	532.1
	-0.020	-0.249	**0.024**	-0.270	542.0
MSFT (γ = 0)	-0.019	**0.999**		0.981	624.1
	0.136	-0.037	**0.010**	0.099	620.8
S&P500 (γ = 0)	0.026	**0.965**		0.991	731.6
	0.032	**0.962**	**0.0000054**	0.994	731.7

Table 6: Model's Valuation Errors from Non-Linear Least Square Estimates Using Data on Daily Call Option Prices

This table reports the aggregate in-sample valuation errors of option pricing models using eight equity call options and one index call option. The reported valuation errors are the root mean squared error (RMSVE). BS, ahBS, JD, SV, SVJ and SVV are represented as the BS, the ad hoc BS, the Merton jump model, the stochastic volatility model, the stochastic volatility-jump model, and the stochastic volatility-volume model. Average Price, Contracts, and Dates denote the average of call option prices in dollar, the total number of call option contracts, and the total dates, used for estimation, respectively. The sample period is September 3rd, 2002 - August 29th, 2003, 251 days. Bold numbers indicate the smallest for each option.

	BS	ahBS	JD	SV	SVJ	SVV	Average Price ($)	Contracts\Dates
AMD	0.095	**0.019**	0.060	0.076	0.057	0.055	1.98	407\40
AOL	0.099	**0.018**	0.051	0.052	0.034	0.020	3.03	1687\150
ORCL	0.062	**0.014**	0.027	0.037	0.02	0.017	2.38	1219\126
IBM	0.335	**0.057**	0.134	0.092	0.089	0.070	13.92	5991\251
CSCO	0.081	**0.019**	0.034	0.042	0.032	0.021	3.34	1727\173
DELL	0.156	**0.031**	0.058	0.059	0.050	0.044	5.38	4351\247
INTC	0.093	**0.018**	0.039	0.058	0.033	0.031	3.73	2675\239
MSFT	0.197	**0.028**	0.067	0.059	0.051	0.048	8.35	4713\215
S&P500	2.020	**0.679**	1.196	0.956	0.739	0.717	90.27	20699\251

Table 7: Summary Statistics for Parameters Estimates of the SVV Model

This table provides sample means, medians, the fifth and the ninety-fifth percentiles and sample standard deviations of the SVV model's parameters estimated daily in the sample periods using the non-linear least squares.

	β ($\times10^{-9}$)	λ	γ ($\times10^{9}$)	μ	ϕ	θ	ψ ($\times10^{9}$)	ρ ($\times10^{-9}$)
AMD								
Mean	546.03	258.05	0.00	-2.40	0.31	1.20	0.52	3.97
StDev	3367.30	1625.70	0.01	6.93	1.20	1.68	1.10	16.57
5th percentile	0.30	0.00	0.00	-18.42	0.00	0.00	0.00	0.00
Median	1.26	0.01	0.00	-0.39	0.00	0.38	0.00	0.00
95th percentile	59.21	7.18	0.01	-0.04	0.79	3.32	2.73	18.22
AOL								
Mean	5.19	1.84	0.06	-1.95	0.13	0.82	89.43	7.47
StDev	6.67	3.49	0.57	5.48	0.68	0.98	830.00	80.29
5th percentile	0.20	0.02	0.00	-18.42	0.00	0.11	0.00	0.00
Median	1.88	0.43	0.00	-0.54	0.00	0.52	0.00	0.00
95th percentile	17.20	6.76	0.19	-0.10	0.44	2.96	2.64	2.10
ORCL								
Mean	4.07	3.38	0.22	-1.12	0.05	1.53	796.19	0.04
StDev	5.24	7.94	1.89	3.63	0.24	1.77	6335.90	0.15
5th percentile	0.14	0.04	0.00	-1.93	0.00	0.10	0.00	0.00
Median	2.89	1.84	0.00	-0.44	0.00	0.98	0.04	0.00
95th percentile	10.08	7.88	0.32	-0.11	0.24	4.63	1.32	0.19
IBM								
Mean	7.30	3.18	0.04	-0.29	0.03	2.22	0.26	0.34
StDev	7.43	5.02	0.12	1.08	0.07	2.79	3.00	4.55
5th percentile	0.44	0.12	0.00	-0.46	0.00	0.42	0.00	0.00
Median	4.91	1.36	0.00	-0.23	0.00	1.30	0.03	0.00
95th percentile	20.53	10.21	0.27	-0.05	0.20	5.99	0.32	0.02

Table 7: Summary Statistics for Parameters Estimates of the SVV Model (continued)

	β (×10^{-9})	λ	γ (×10^{9})	μ	φ	θ	ψ (×10^{9})	ρ (×10^{-9})
CSCO								
Mean	2.83	6.03	0.06	-1.04	0.05	1.15	0.28	1.04
StDev	2.99	13.42	0.21	3.17	0.11	1.28	0.85	9.26
5th percentile	0.23	0.25	0.00	-1.76	0.00	0.12	0.00	0.00
Median	2.17	2.46	0.00	-0.39	0.00	0.78	0.06	0.00
95th percentile	6.85	18.06	0.71	-0.12	0.32	3.98	0.72	1.05
DELL								
Mean	2.94	2.83	0.06	-0.70	0.04	1.46	0.16	0.08
StDev	2.42	4.48	0.15	2.89	0.16	1.12	0.25	1.02
5th percentile	0.30	0.15	0.00	-0.66	0.00	0.23	0.00	0.00
Median	2.18	1.35	0.00	-0.24	0.00	1.20	0.09	0.00
95th percentile	7.50	7.90	0.39	-0.09	0.22	3.71	0.60	0.02
INTC								
Mean	1.96	1.67	0.08	-0.72	0.04	0.71	0.18	0.09
StDev	1.32	1.50	0.19	1.59	0.15	0.59	0.48	0.26
5th percentile	0.29	0.25	0.00	-1.34	0.00	0.05	0.00	0.00
Median	1.71	1.14	0.00	-0.47	0.00	0.56	0.01	0.00
95th percentile	4.20	4.17	0.49	-0.14	0.20	1.62	0.95	0.55
MSFT								
Mean	1.42	1.93	0.14	-0.66	0.05	1.29	2.37	0.02
StDev	1.26	2.40	0.33	2.36	0.14	1.48	29.14	0.08
5th percentile	0.10	0.06	0.00	-1.07	0.00	0.07	0.00	0.00
Median	1.10	0.98	0.00	-0.28	0.00	0.84	0.11	0.00
95th percentile	4.07	6.19	0.96	-0.09	0.27	3.66	1.31	0.11
S&P 500								
Mean	0.03	8.63	6.80	-1.76	0.12	1.28	4571.90	0.13
StDev	0.03	29.67	47.08	4.50	0.62	2.05	6128300	1.41
5th percentile	0.00	0.00	0.00	-15.43	0.00	0.04	0.00	0.00
Median	0.03	3.04	0.00	-0.26	0.00	0.69	0.00	0.00
95th percentile	0.06	23.85	23.94	-0.03	0.14	4.34	50.68	0.03

Table 8: Out-of-Sample Prediction Errors

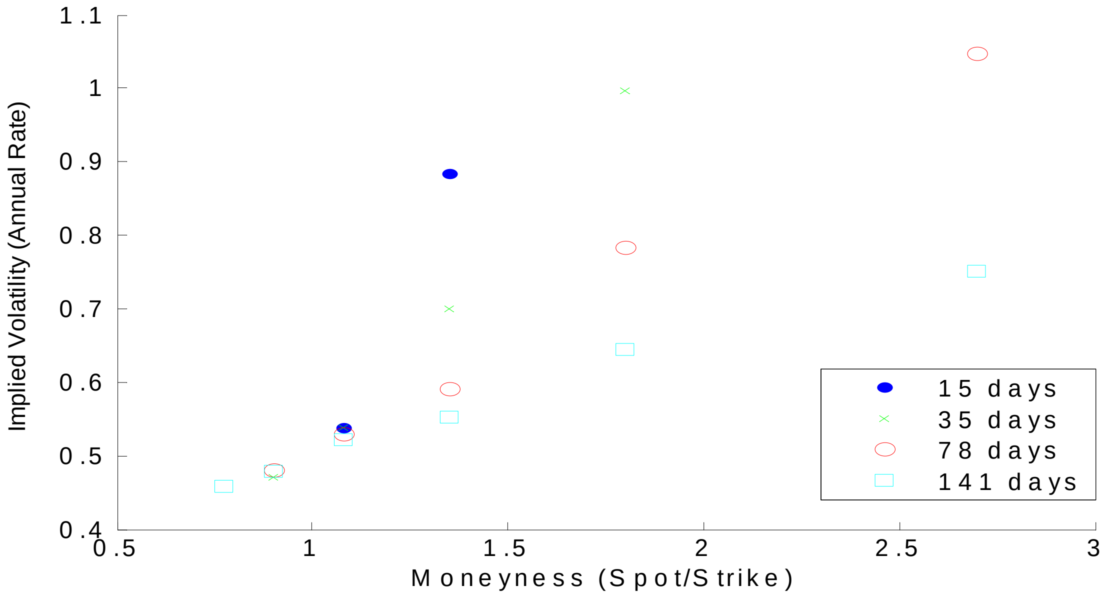
This table presents the aggregate out-of-sample prediction errors for the six models and eight equity and one index call options. Four measures are reported. RMSVE, AMVE, MPPE represent the root mean squared valuation error (in $), the absolute mean valuation error (in $), the mean percentage pricing error (in %), respectively. Bold numbers indicate the smallest for each option's RMSVE and AMVE.

	BS	ahBS	JD	SV	SVJ	SVV
AMD						
RMSVE	0.059	0.065	0.057	0.058	**0.055**	0.057
AMVE	0.050	0.049	**0.044**	0.048	0.045	**0.044**
MPPE	-0.007	0.010	-0.012	-0.010	-0.013	-0.015
AOL						
RMSVE	0.103	0.099	0.068	0.066	0.055	**0.045**
AMVE	0.086	0.071	0.056	0.055	0.044	**0.037**
MPPE	-0.015	-0.020	0.012	-0.009	-0.002	-0.003
ORCL						
RMSVE	0.067	0.084	0.040	0.049	0.038	**0.037**
AMVE	0.057	0.064	0.033	0.041	0.032	**0.031**
MPPE	-0.008	-0.026	0.003	-0.003	0.001	0.000
IBM						
RMSVE	0.348	0.406	0.170	0.151	0.147	**0.134**
AMVE	0.288	0.279	0.131	0.122	0.118	**0.107**
MPPE	-0.058	-0.043	0.002	-0.012	-0.012	-0.005
CSCO						
RMSVE	0.090	0.052	0.055	0.061	0.057	**0.048**
AMVE	0.074	0.039	0.042	0.049	0.046	**0.038**
MPPE	-0.011	-0.003	0.004	-0.005	-0.005	-0.001
DELL						
RMSVE	0.165	0.142	0.086	0.090	0.087	**0.081**
AMVE	0.133	0.098	0.064	0.070	0.066	**0.061**
MPPE	-0.033	-0.019	-0.004	-0.008	-0.007	-0.006
INTC						
RMSVE	0.099	0.101	0.059	0.075	0.059	**0.054**
AMVE	0.083	0.076	0.048	0.063	0.049	**0.046**
MPPE	-0.018	-0.020	0.002	-0.013	-0.008	-0.005
MSFT						
RMSVE	0.206	0.134	0.096	0.098	0.092	**0.087**
AMVE	0.168	0.100	0.078	0.080	0.075	**0.071**
MPPE	-0.036	-0.014	-0.003	-0.006	-0.005	-0.006
S&P 500						
RMSVE	2.360	1.883	1.862	1.726	**1.662**	1.816
AMVE	1.768	1.377	1.385	1.303	**1.271**	1.428
MPPE	-0.048	-0.014	-0.006	-0.026	-0.019	-0.018

Figure 1: BS Implied Volatility

The two panels show BS implied volatility of CSCO and S&P 500 index call options over moneyness observed on March 27th, 2003. The legends indicate time to expiration of option contracts in terms of trading days.

Panel A: CSCO



Panel B: SPX

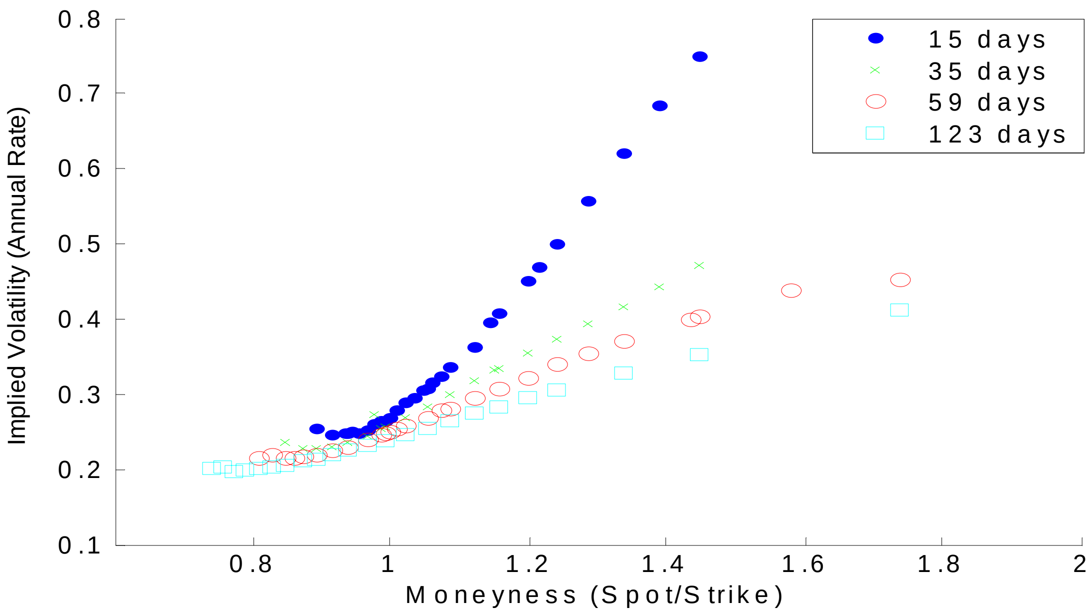


Figure 2: Dynamics of Implied Volatility of S&P 500 Index Call Option

The figure illustrates time-variations in BS implied volatility of S&P 500 index call options. Daily implied volatility is computed as the average over moneyness (0.9 < Spot/Strike <1.1) and time to expiration (more than 5 trading days and less than 181 trading days). The sample period is September 3rd, 2002 - August 29th, 2003. The total number of observation dates is 251.

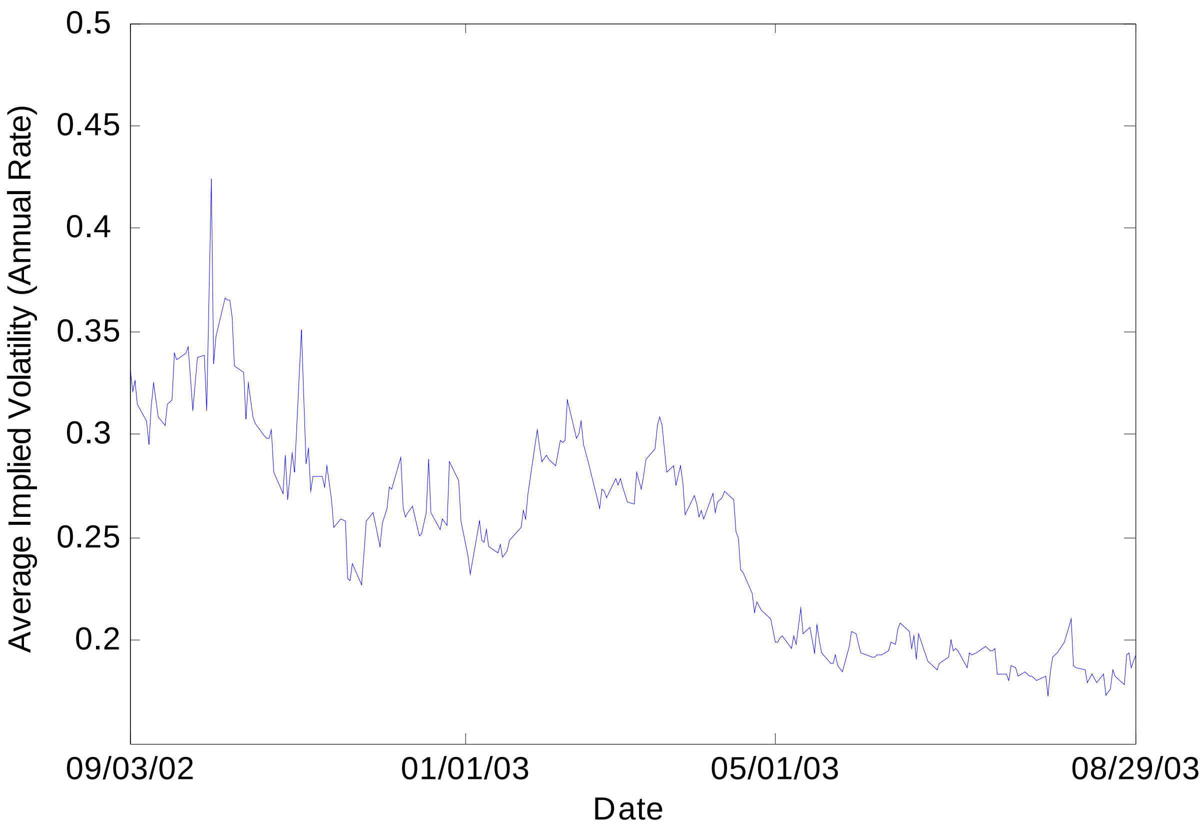
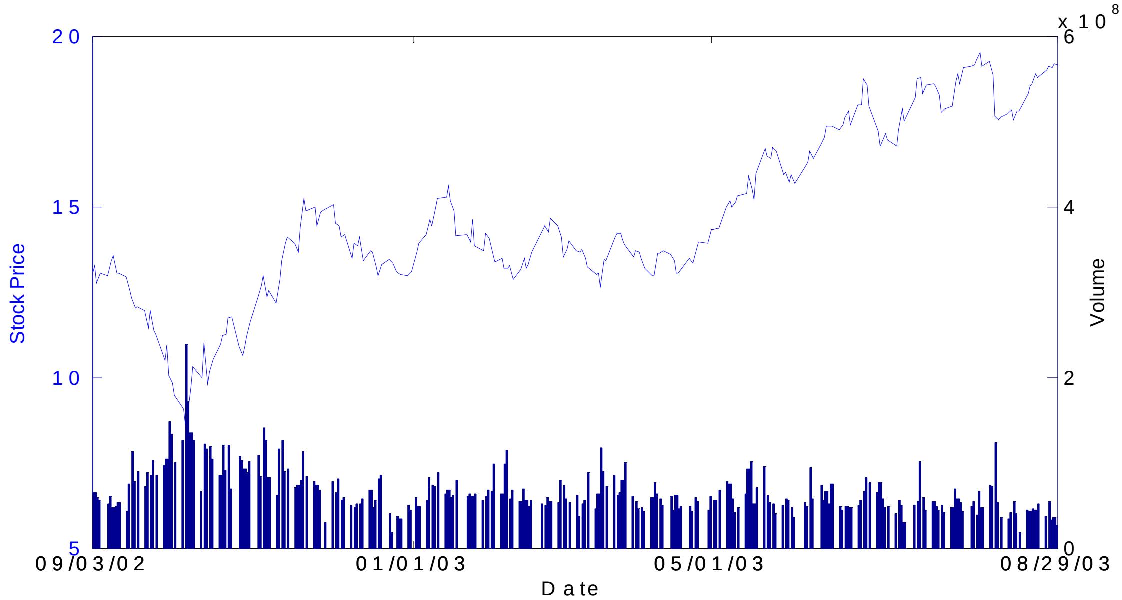


Figure 3: Stock Price or Index and Trading Volume (CSCO, SPX)

Panel A: CSCO



Panel B: SPX

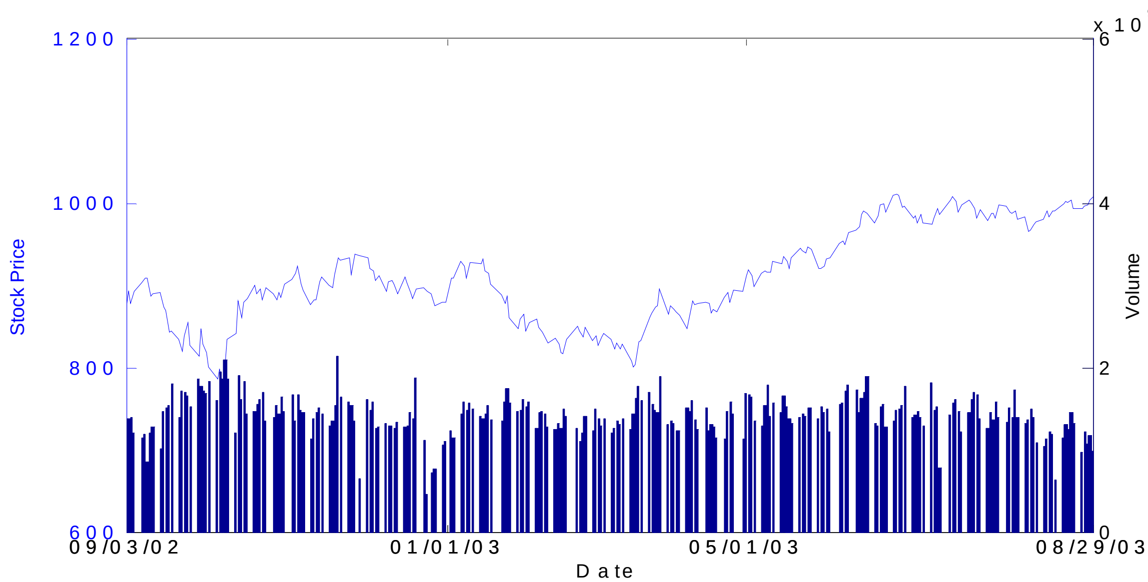


Figure 4: Correlations between Trading Volume (t + i) and Absolute Value of Returns (t)

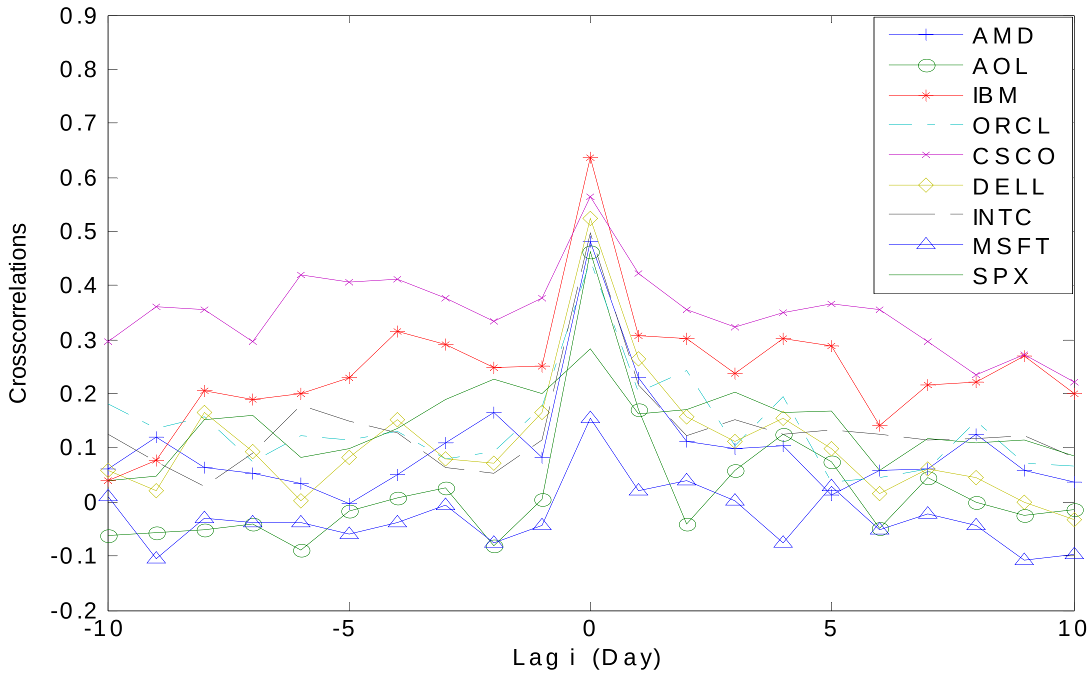


Figure 5: Autocorrelation Function of Trading Volume

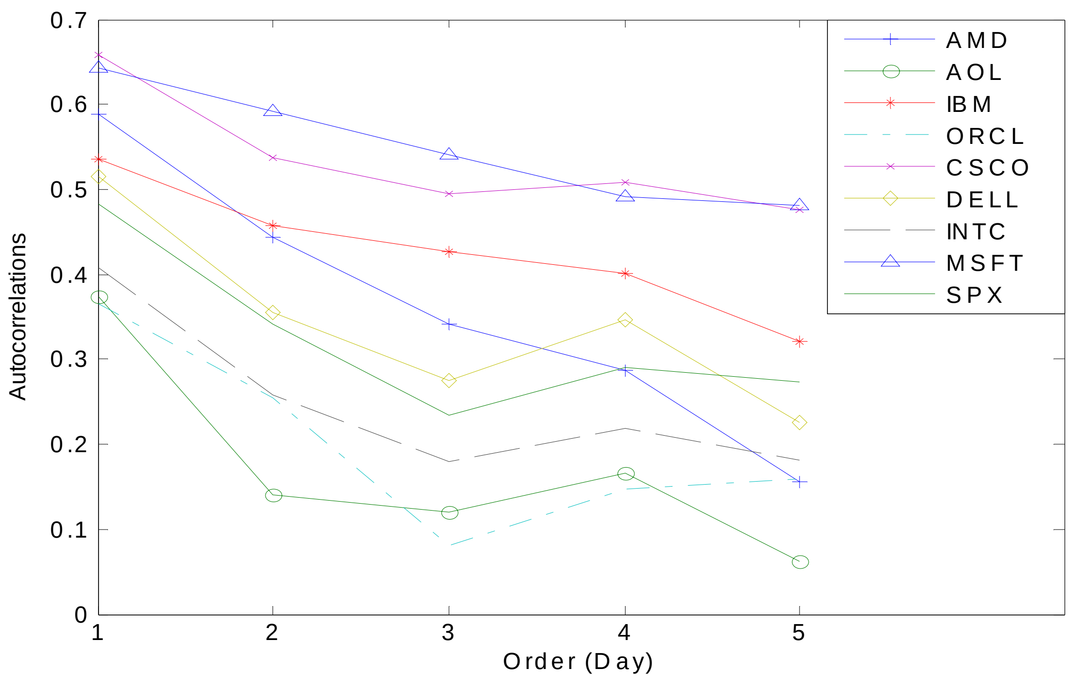


Figure 6: Call Implied Volatility Surface Generated by the Stochastic Volatility and Volume Model

A Benchmark Selection of Parameters: $\beta = 1.26\times10^{-15}$, $\lambda = 1.05\times10^{5}$, $\gamma = 8.27\times10^{19}$, $\theta = 0.60$, $\mu = -0.10$, $\phi = 0.10$, $\psi = 6.49\times10^{13}$, $\rho = 3.36\times10^{-29}$. The current price of the underlying stock, the current annual volatility of the return, and the annual risk-free rate are set to be 41.22 (the close level of S&P 500 index on September 3rd, 2002), 0.32 (the daily standard deviations on September 3rd, 2002 for returns on the S&P 500 index, and the daily standard deviations are computed as sample standard deviations of 10 minute returns on September 3rd, 2002 transforming in a yearly basis by multiplied by the root of 252×39), 0.016 (the average of the midpoint of ask and bid T-bill rates on September 3rd, 2002 over time to maturity 2 to 176 days), respectively.

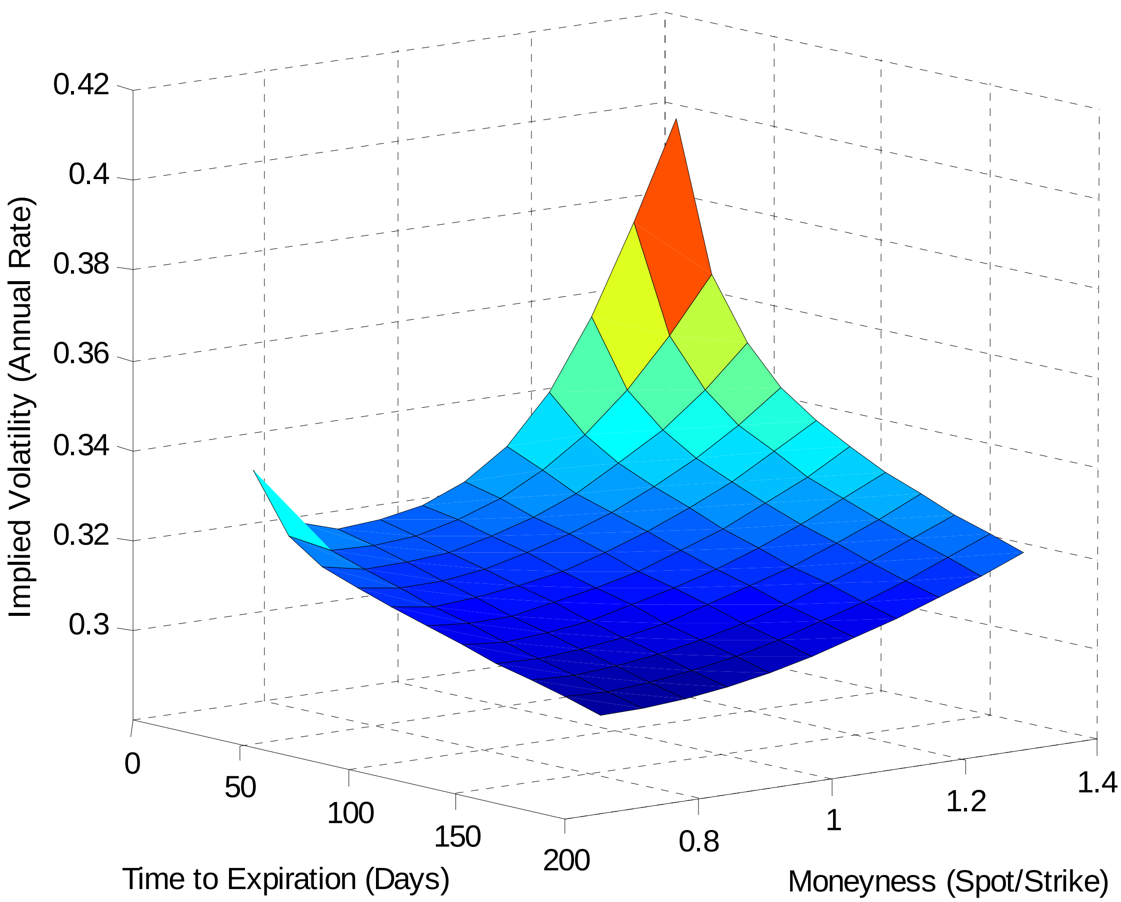
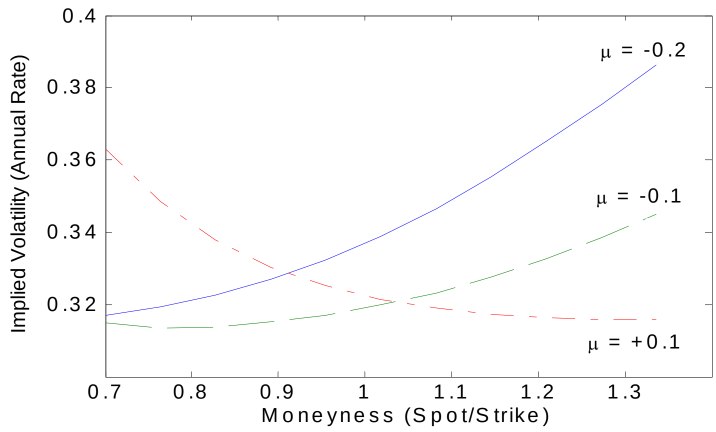


Figure 7: Effects of Parameters in the SVV Model on Implied Volatility

In each figure, the value of a parameter in the SVV model is changed. The other parameter values are the same with the benchmark selection. The current price of the underlying stock, the current volatility of the return, and the risk-free rate are kept the same values in Figure 6. The time to expiration is set as 76 trading days.

Panel A



Panel B

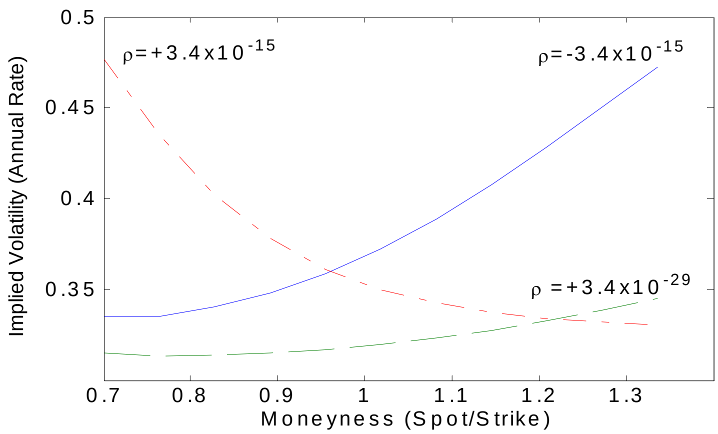
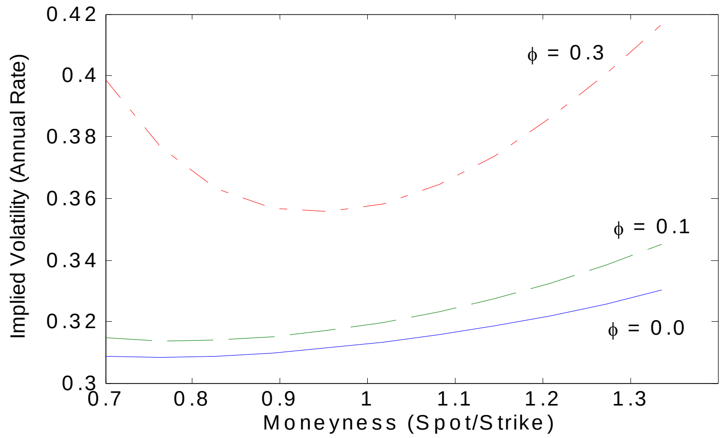
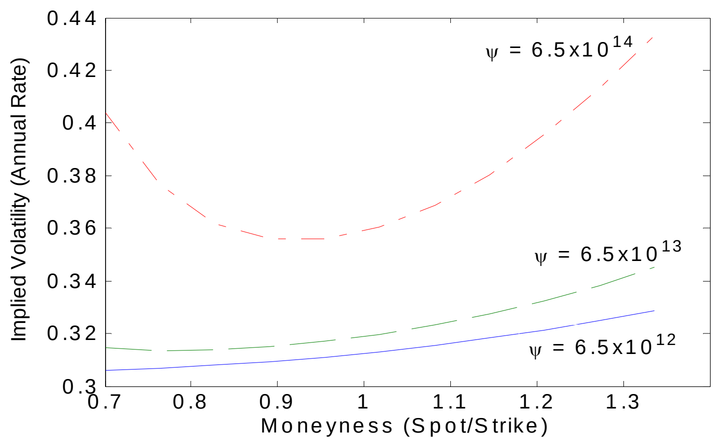


Figure 7: Effects of Parameters in the SVV Model on Implied Volatility (continued)

Panel C



Panel D



Panel E

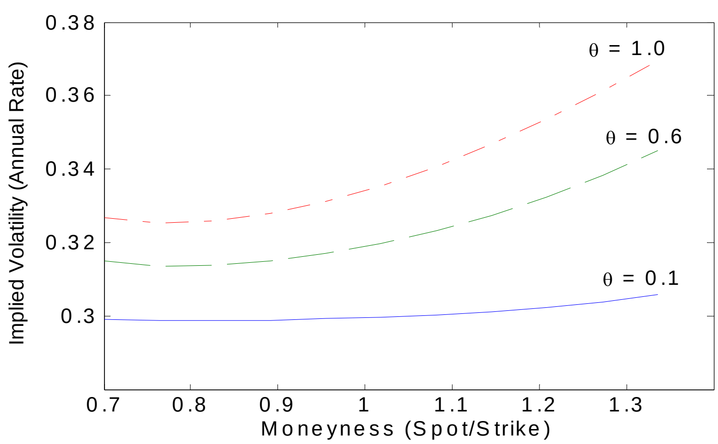
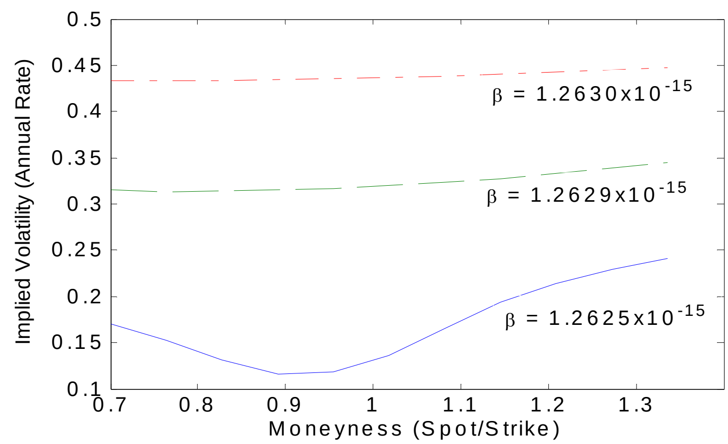
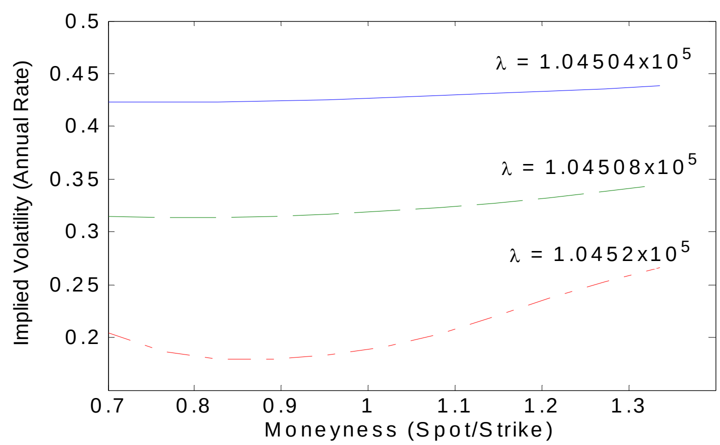


Figure 7: Effects of Parameters in the SVV Model on Implied Volatility (continued)

Panel F



Panel G



Panel H

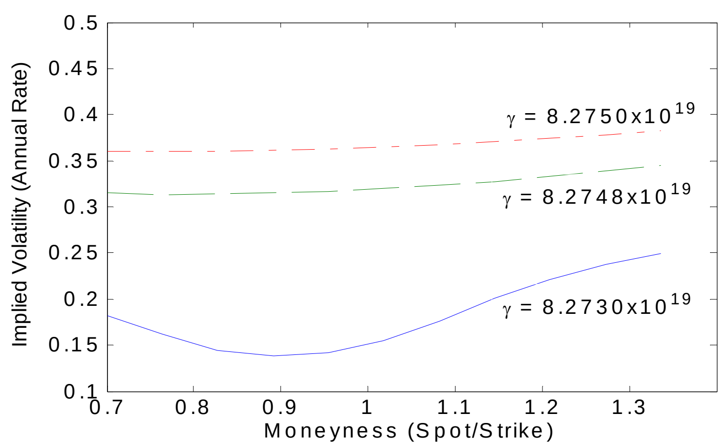


Figure 8: Term Structure of Implied Volatilities

The implied volatilities are those of at-the-money call options with the ratio of the spot price to strike rate of one. LVOL denotes the long-term volatility in an annual rate. The selection of parameters corresponds to the benchmark choice in Figure 6, except for λ. Two λs are set so that the long-term volatility is 0.22 (the benchmark selection) and 0.5.