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Noise and Bias in Eliciting Preferences

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ABSTRACT

In the context of eliciting preferences for decision making under risk, we ask the question: "which might be the 'best' method for eliciting such preferences?". It is well known that different methods differ in terms of the bias in the elicitation; it is rather less well-known that different methods differ in terms of their noisiness. The optimal trade-off depends upon the relative magnitudes of these two effects. We examine four different elicitation mechanisms (pairwise choice, willingness-to-pay, willingness-to-accept, and certainty equivalents) and estimate both effects. Our results suggest that economists might be better advised to use what appears to be a relatively inefficient elicitation technique (i.e. pairwise choice) in order to avoid the bias in better-known and more widely-used techniques.

Key words: pairwise choice, willingness-to-pay, willingness-to-accept, errors, noise, biases

JEL-classification: C91, C81

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1. INTRODUCTION

For all sorts of reasons economists frequently need to elicit people's preferences. There are different elicitation methods; sometimes economists use one method, sometimes another. This paper is concerned with trying to understand what might be the 'best' method.

In this paper we concentrate attention on a particular context – the elicitation of individuals' preferences when taking decisions under risk – though our concerns and results clearly have implications in other contexts. To keep our analysis simple and concentrate on the key issues, we presume that all the individuals whose preferences we are eliciting obey Expected Utility theory, and that we are concerned with the estimation of their (von Neumann-Morgenstern) utility functions using different kinds of elicitation mechanisms. The specific elicitation mechanisms that we are considering are the principal ones used by economists and are:

1. elicitation of preferences through pairwise choice preference questions;
2. elicitation of certainty equivalents through the statement of willingness-to-pay in an auction;
3. elicitation of certainty equivalents through the statement of willingness-to-accept in an auction;
4. elicitation of certainty equivalents using the Becker-Degroot-Marschak mechanism.

What we want to do is to compare the various elicitation methods - in terms of the noisiness of the subjects' responses and in terms of any apparent bias in the elicitation method. It is well known that subjects in experiments are noisy in their responses to pairwise choice questions (in that they give different answers when asked the same question on several occasions) and there is no reason to believe that they are not also noisy when it comes to stating their certainty equivalents. There may also be bias in their responses – not on the pairwise choice questions (as it is difficult to imagine that subjects have a bias for the question on the left or the question on the right) – but possibly on the certainty equivalents questions. Even if it has been explained carefully to subjects that their stated willingness-to-pay in a second-price auction ought to be equal to their certainty equivalent, it is apparent that there are subjects who deliberately and consistently under-bid.

Similarly, in attempts to elicit certainty equivalents through willingness-to-accept in second-price auctions, it would appear that many subjects over-ask. The Becker-Degroot-Marschak mechanism appears to be neutral in that there is no obvious bias in the procedure – but, nevertheless, it may be the case that subjects find the procedure too complicated and adopt some simple heuristic with an inbuilt bias. The purpose of this paper is to see if such biases exist and how noisy are the various elicitation methods. An interesting debate on such issues appeared in this journal in the 1980's with papers by Coursey *et al* (1987) and by Knetsch and Sinden (1984 and 1987). We return to this debate with new technologies for its resolution.

Our analysis is important because it would appear on *a priori* grounds that one of other of the certainty equivalents methods is potentially more informative than the pairwise choice method, since the latter only tells us which choice is preferred – but not by how much. However, if there is more noise in the certainty equivalents methods this might outweigh the inherent superiority of the method. Moreover, if there is bias in the certainty equivalents methods, it may be better to elicit utility functions through the unbiased preference method.

We have estimated the noise and bias in the various elicitation methods using experimental data. Section 2 describes the experimental design, while section 3 discusses the techniques used to estimate the noise and bias in the various methods. Section 4 presents some preliminary descriptive statistics and Section 5 presents a more formal analysis. Section 6 concludes.

2. EXPERIMENTAL DESIGN

The experiment was conducted at the Centre of Experimental Economics at the University of York with 24 participants. Each participant had to attend five separate sessions on five different days. After a subject had completed all five sessions, one question of one session was randomly selected and played out for real. The average payment to the subjects was £34.17 with £80 being the highest and £0 being the lowest payment.

In each of the five sessions subjects were presented the same 30 lottery pairs, 28 risky ones and two ambiguous ones (which are not analysed in this paper). All risky lotteries (e.g. see Figure 1) were composed of the four consequences £0, £10, £30, £40. The probabilities of these consequences are recorded in the Appendix Table for all 28 lottery pairs. In the experiment the lotteries were presented as segmented circles on the computer screen – as in Figure 1..

In the five sessions subjects had to perform altogether eight tasks six of which will be analysed in the present paper:

- three times report a preference in 28 pairwise choice questions (between two risky lotteries). We call this the CHOICE task;

- report a maximal buying price (bid) for each of the 56 lotteries. We call this the BID task;

- report a minimal selling price (ask) for each of the 56 lotteries. We call this the ASK task;

- report a certainty equivalent (CE) for each of the 56 lotteries. We call this the BDM task.

For all tasks we used incentive-compatible elicitation mechanisms. Bids and asks were elicited with second-price sealed-bid auctions while for the certainty equivalents we employed the Becker-DeGroot-Marschak mechanism. We now describe the various tasks with a little more detail.

In the CHOICE task, on the subject's computer screen both lotteries of the pair appeared as circles and subjects had to indicate whether they preferred the left lottery, or the right lottery, or neither. After pressing the corresponding key they had to confirm their choice by pressing the return key. If a pairwise choice question was selected as reward the subject could simply play out the preferred lottery. In case of stated indifference one of the lotteries was selected by the experimenter.

In the BID task the following question appeared under each lottery: "Submit your bid for this lottery in a second-price sealed-bid auction." That is, subjects were asked to assume they did not have the lottery and had to bid to get it. They had to type in their bid and confirm it by pressing

the return key. If a question of the bid treatment was selected for the reward, the subject who submitted the highest bid (among all subjects in the group with whom she completed the bid treatment) received a payment of £y where y is the highest amount in the corresponding lottery, and then the subject would pay the second highest bid and then play out the lottery (receiving whatever outcome resulted).

The ASK task was identical to the BID task except that for each lottery a different question was asked: "Submit your offer for this lottery in a second-price offer auction". That is, subjects were asked to assume that they owned the lottery and had to make an offer to dispose of it. If a question from the ask treatment was selected for the reward, and if the subject had submitted the lowest ask then the subject could play out the corresponding lottery. However, if he or she had not submitted the lowest offer (among all subjects in the group with whom she completed the ask task), he or she received the second lowest offer instead of playing out the lottery.

In the BDM task the following question appeared under each lottery: "State the amount of money such that you do not care whether you will receive this amount or the lottery". If a question of the BDM task was chosen as reward we employed the standard BDM mechanism: a number z was randomly drawn between zero and y where y is the highest possible prize in the given lottery. If z was greater or equal to the answer, the subject received £z, otherwise she or he could play out the given lottery.

3. ESTIMATION METHOD

We use the different kinds of data to estimate the subjects' (Neumann-Morgenstern) utility functions. In this section we discuss the main conceptual issues of our estimation method; details are presented in a technical appendix.

The estimation of the parameters of the utility function from *pairwise choice* data follows the procedure adopted in Hey and Orme (1994). Let us denote the two lotteries in the pairwise choice by L and R, and the Expected Utility of them by *EUL* and *EUR* respectively. Then, if there is

no noise or error in the subject's responses, he or she will report a preference for *L(R)*, if and only if *EUL > (<) EUR*. This is equivalent to saying that *L (R)* is reported as preferred if and only if *CEL > (<) CER,* where *CEL (CER)* denotes the Certainty Equivalent of *L (R).* However, as we know from the existing literature, subjects' responses are typically affected by noise. We assume that this noise affects the Certainty Equivalents. Let us denote the error in measuring the *difference* between the Certainty Equivalents by *ε*. With this error the subject will report a preference for *L(R)*, if and only if *CEL - CER + ε > (<) 0*, that is, if and only if *ε > (<) CER - CEL.* We can now write the probability that the subject reports a preference for *L (R)* as Prob*{ε > (<) CER - CEL}*.

To proceed to the estimation of the parameters using maximum likelihood methods, we need to specify the distribution of the measurement error. We assume this to be normally distributed with mean *0* and variance *s²*. The magnitude of *s* measures the noisiness of the subject's responses: if *s = 0* then the subject makes no mistakes – as *s* increases, the noise gets larger and larger. In the limit, when *s* is infinite, there is no information content in the subject's responses. There is a slight complication when the subject reports indifference (as was allowed in the experiment[4]). Following Hey and Orme (1994) we assume that those subjects expressing indifference do so when *- τ < CEL - CER + ε < τ* where *τ* is some *threshold.* We estimate *τ* along with the other parameters.

For the certainty equivalent methods, we follow the same route. If the subject is asked to provide his or her certainty equivalent for some gamble *G*, we assume that the subject calculates the Expected Utility of the gamble, *EUG*, according to his or her utility function, and then calculates the certainty equivalent *V* - that is, certain amount of money that yields the same utility. We can now write $V = u^{-1}(EUG)$. Incorporating the error and modelling it as above, then we have that $V = u^{-1}(EUG) + ε$, and hence that the probability density of *V* being reported as the certainty equivalent of the gamble is given by $f[V - u^{-1}(EUG)]$, where *f(.)* is the probability density function of *ε*. If we now make the same assumption about the distribution of the measurement error *ε* - namely that it is *N(0,s²)* – we can now proceed to the estimation of the parameters of the utility function. As will be

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[4] It is not clear why a subject should report indifference, and the modelling we have done is only one of several ways of proceed.

seen, we allow for a different variance s^2 for each of the elicitation mechanisms. We assume that subjects have a Constant Relative Risk Aversion (CRRA) utility function[5] and we adopt the following specific form, which embodies the normalisation that $u(0) = 0$ and $u(40) = 1$:

$$u(x) = (x/40)^r$$

We need to estimate only the parameter r (the relative risk aversion coefficient) as it fully describes the utility function of the individual. As noted above, we assume that the standard deviation of the noise - that is, the magnitude of s - is different for the different elicitation methods (i.e. choice and price), and we estimate them individually. We also test to see if there is any bias in the certainty equivalent elicitation methods - in the following way. We assume that there is a *true* valuation V and a *reported* valuation v which are related by

$$V = a + bv$$

Here the parameters a and b determine the *bias* in the reporting of the certainty equivalents. If $a=0$ and $b=1$ there is no bias. We assume that there is no bias in the pairwise choice elicitation method. In this regard, some further explanation is perhaps needed. With the certainty equivalent methods, particularly with the willingness-to-pay and the willingness-to-accept questions, there are well known biases: when asked how much they are willing to pay, it is well-known that subjects underbid; when they are asked how much they are willing to accept, they over-ask. This is partly because subjects do not appear to fully understand the question and perceive of it as some kind of strategic game. In contrast, a pairwise choice question is not open to such a misinterpretation, particularly in the context of the usual incentive mechanism; in other words, if the subject knows that his or her stated choice on any pairwise choice question is to be played out (and him or her is to paid according to the outcome), what (conscious or unconscious) reason is there for not replying according to his or her true preferences?

4. RESULTS

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[5] We have investigated other specifications – most notably that of CARA. CRRA fits significantly better. Details are available on request.

We now report the results obtained from the experiment. We consider the results obtained from the certainty equivalent data (BDM, BID, and ASK) combined with the pairwise choice data (PC). We estimate individual preferences functions subject by subject as subjects are clearly different. That is, we estimate the parameter *r* subject by each subject. We also estimate the standard deviation *s* for each of the estimation methods and also the parameters *a* and *b* which determine the bias in the reporting of the certainty equivalents[6].

In Table 1 we report the estimations obtained[7]. As a broad conclusion it seems to be the case that the ASK method and the BDM method are equally prone to error (*s-bdm* is not statistically different from *s-ask*). Both elicitation methods are, however, less prone to error than BID method. In fact, at a 10% significance level we reject the hypothesis that that *s-bdm = s-bid* and that *s-ask = s-bid* in favour of the alternative that *s-bdm < s-bid* and *s-ask < s-bid*. Columns 4 and 5 of Table 1 report the bias estimates for the BDM method; columns 7 and 8 report those for the BID method; and columns 10 and 11 report those for the ASK method. As noted earlier, lack of bias would have implies a value of *a* equal to 0 and a value of *b* equal to 1. Since this is clearly not always the case here, we can conclude that there is a degree of bias in the stated certainty equivalents.

Clearly the bias varies from subject to subject, as well as from elicitation method to elicitation method. Figure 2 gives an overall view, plotting the stated valuation as a function of the true valuation for each subject. In these figures there is also plotted the 45-degree line in which stated valuations are equal to the true valuations. It will be seen that for the BDM method the lines are clustered close to the 45-degree line, though there are some exceptions. In contrast, the BID lines are generally *below* the 45-degree line, indicating that subjects generally underbid in willingness-to-pay questions. The lines for the ASK method are somewhat dispersed though

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[6] It may be of interest to report the results of tests of whether the estimated functions differ from data source to data source. Using standard likelihood ratio tests we cannot reject the following null hypotheses: that the utility function estimated from each of BDM, BID and ASK is the same; that the utility function estimated from the choice data and from the certainty equivalents is the same; that the utility function estimated from the choice data and from the BDM, BID and ASK is the same.

[7] We omit two of the 24 subjects (subjects 21 and 22) who answered all questions as if they were perfect expected-value maximisers. For them *r = 1*, all the *s* and *a* values are 0, and all the *b* values are 1.

generally they are clustered around the 45-degree line, indicating no tendency to over-ask in willingness-to-accept questions.

5. BIAS AND NOISE

In this section we compare the bias and noise across elicitation methods. Information on mean values is given in Table 2 where the bias is calculated at £0 and £40. Comparing the noise of the single methods, column 2 of Table 2 shows that PC is the least noisy on average and then BDM and then ASK and then BID. In order to take in to account both the noise of the statistics and its bias we calculate the Mean Square Error. From this analysis we have very clear conclusion: PC performs best, BDM performs better then ASK, which, in turn, performs better then BID. However, it should be noted that these results are based on averages: for individual subjects the pattern is different.

To put these results in perspective, we return to our original question: suppose we want to elicit the true preferences of an individual, what might be the best method? One way of answering this is the following. Suppose we want to know whether our individual prefers a lottery A to a lottery B or vice versa. We could simply ask the individual which he or she prefers, or we could ask the individual to value the two lotteries and then see for which lottery the valuation is the highest. The problem – as is clear from the above – is that there is noise (and bias) in the subject's responses. So let us ask the more appropriate question: suppose A is genuinely preferred to B by the subject, what is the probability that (either through pairwise choice or by valuations) the individual actually expresses the correct preference? This depends on how far apart A and B are in the subject's preferences. Accordingly, we consider three cases: where the difference in the true evaluations are £1, £2 and £3. Clearly as the difference increases the probability of expressing the true preference increases, but how it does so depends upon the noise and the bias. For each subject we calculate these probabilities (using the estimates of Table 1) and present the results in Table 3 and Figure 3. These show that *generally but not always* the PC method has the greatest probability

of eliciting the true preferences. This may be because the PC method is more easily understandable by subjects in experiments than the other methods.

6. CONCLUSIONS

In this study we have been concerned with the question: which is the 'best' elicitation method for eliciting preferences. We have analysed four standard elicitation methods, pairwise choice, maximal buying prices, minimal selling prices, and certainty equivalents. A particular feature of our analysis is the explicit distinction between noise and bias induced by the single methods. Our experimental data show that maximal buying prices induce the lowest noise but, at the same time, the largest bias. Altogether, we find evidence that pairwise choice may be regarded as the best method in general, though for certain subjects one of the other methods may be preferable. But if one does not know anything about an individual subject, it may be best to use the pairwise choice method.

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Table 1: Estimation using all the data

Subject	Relative Risk aversion coefficient	Goodness of fit	Bias		Standard deviation of the measurement	Bias		Standard deviation of the measurement	Bias		Standard deviation of the measurement	Indifference	Standard deviation of the measurement
			Intercept	Slope	Error	Intercept	Slope	Error	Intercept	Slope	error		Error
	r	Log-likelihood	a-bdm	b-bdm	s-bdm	a-bid	b-bid	s-bid	a-ask	b-ask	s-ask	τ	s-pc
1	0.625	-19.189	-5.466	1.171	2.691	4.718	1.022	4.261	7.942	0.893	6.955	0.383	1.435
2	0.644	-18.395	-1.619	0.953	2.784	15.171	0.789	7.555	-0.115	0.958	2.511	1.234	1.196
3	1.005	-7.680	0.204	1.000	0.512	0.201	1.000	1.346	0.488	0.990	0.295	0.234	2.355
4	0.687	-17.483	-6.548	1.123	2.648	12.591	1.034	6.258	-3.213	1.015	2.596	0.028	1.144
5	0.531	-19.386	-8.045	1.191	5.675	8.073	1.004	5.770	-1.868	1.061	3.033	0.123	2.695
6	0.218	-19.206	-0.280	0.931	5.078	11.078	0.814	6.368	0.776	1.070	4.014	0.000	1.520
7	0.997	-12.34	-0.572	0.998	1.216	1.053	0.947	1.102	0.955	0.964	1.371	0.313	2.041
8	0.465	-21.131	9.764	0.412	8.632	6.868	0.568	5.541	4.931	0.711	5.262	0.000	4.051
9	1.445	-17.740	11.614	0.536	6.838	1.660	0.994	1.634	4.692	0.917	2.378	1.356	14.858
10	1.555	-19.132	0.357	0.963	3.175	13.419	1.138	6.495	2.819	0.812	3.466	0.000	95957.85
11	0.394	-18.656	-6.888	1.219	3.826	5.457	1.116	3.982	0.999	1.309	4.824	1.348	1.771
12	0.944	-11.672	1.508	0.977	1.12	1.306	0.985	0.815	1.279	0.928	1.445	0.251	1.636
13	4.366	-19.138	9.105	0.752	3.684	27.067	0.233	7.245	23.48	0.235	6.144	0.000	1.802
14	0.531	-14.946	-0.573	1.018	2.565	3.903	0.900	2.171	-1.788	1.001	2.360	0.090	1.481
15	0.630	-20.790	-2.613	1.114	5.942	4.426	0.986	5.676	-13.676	1.801	6.787	0.073	1.313
16	0.554	-22.314	-1.789	1.304	6.891	11.927	1.348	7.526	8.086	0.582	8.755	0.850	1.13
17	2.722	-20.339	9.259	0.751	3.171	21.559	0.156	7.503	21.372	0.123	6.165	0.000	10.595
18	0.607	-19.356	-4.311	1.091	3.535	6.617	0.963	4.873	2.329	0.901	5.888	0.265	1.232
19	0.711	-20.877	-7.902	1.009	6.181	9.026	0.641	4.381	5.143	0.685	6.052	0.226	1.322
22	1.013	-4.723	0.063	1.000	0.337	0.096	0.995	0.740	0.291	0.987	0.267	1.676	18.193
23	1.153	-13.904	1.874	0.857	2.609	0.567	0.966	0.897	0.817	0.949	2.057	0.257	2.257
24	0.712	-19.642	-4.263	1.057	4.046	8.456	1.079	4.878	0.971	0.779	6.335	0.000	1.292

Table 2: Average noise, bias and mean square error

	s	bias(£0)	bias(£40)	mse(£0)	mse(£40)
BDM	3.780	-0.324	-1.366	14.392	16.152
BID	4.410	7.965	3.744	82.895	33.461
ASK	4.044	3.032	-1.202	25.546	17.796
PC	3.587	0.000	0.000	12.864	12.864

Table 3: Probabilities of eliciting the true preference using the different methods

True difference = £1				True difference = £2				True difference = £3			
pc	*bdm*	*bid*	*ask*	pc	bdm	bid	ask	*pc*	*bdm*	*bid*	*ask*
.76	.59	.56	.55	.92	.67	.63	.59	.98	.75	.69	.63
.80	.61	.55	.62	.95	.7	.59	.72	.99	.79	.64	.81
.66	.92	.7	.99	.80	1.00	.85	1.00	.90	1.00	.94	1.00
.81	.59	.54	.61	.96	.68	.59	.70	1.00	.76	.63	.79
.64	.54	.55	.59	.77	.58	.60	.67	.87	.62	.64	.75
.74	.56	.55	.57	.91	.62	.61	.63	.98	.67	.66	.69
.69	.72	.75	.70	.84	.88	.91	.86	.93	.96	.98	.95
.6	.58	.59	.57	.69	.65	.67	.65	.77	.72	.75	.71
.53	.58	.67	.63	.55	.65	.81	.74	.58	.72	.90	.83
.5	.59	.54	.60	.50	.68	.58	.69	.50	.76	.61	.77
.71	.56	.56	.54	.87	.62	.62	.59	.95	.68	.68	.63
.73	.74	.81	.70	.89	.90	.96	.85	.97	.97	1.00	.94
.71	.60	.66	.69	.87	.70	.80	.84	.95	.78	.90	.93
.75	.61	.64	.62	.91	.71	.77	.73	.98	.79	.86	.82
.78	.54	.55	.52	.94	.58	.60	.55	.99	.63	.65	.57
.81	.53	.53	.56	.96	.56	.56	.61	1.00	.59	.58	.66
.54	.62	.73	.82	.57	.72	.89	.97	.61	.81	.97	1.00
.79	.57	.56	.55	.95	.64	.62	.61	.99	.71	.67	.66
.78	.55	.60	.57	.93	.59	.69	.63	.99	.63	.77	.7
.52	.98	.83	1.00	.54	1.00	.97	1.00	.57	1.00	1.00	1.00
.67	.62	.79	.64	.81	.74	.95	.77	.91	.83	.99	.86
.78	.57	.55	.56	.94	.63	.61	.61	.99	.69	.66	.67

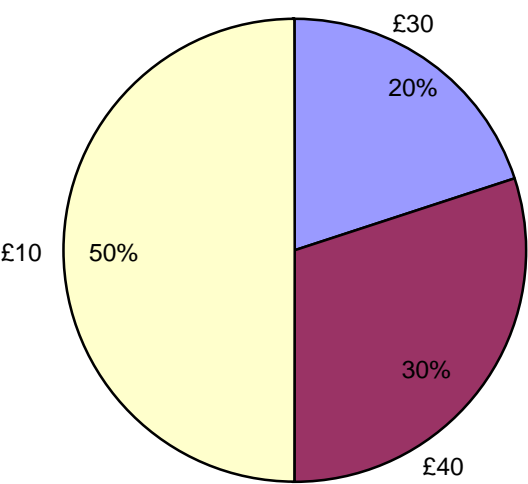**Figure 1: Presentation of the lotteries in the experiment**

Figure 2: The relationships between the stated valuation and the true valuation

BDM   BID   ASK

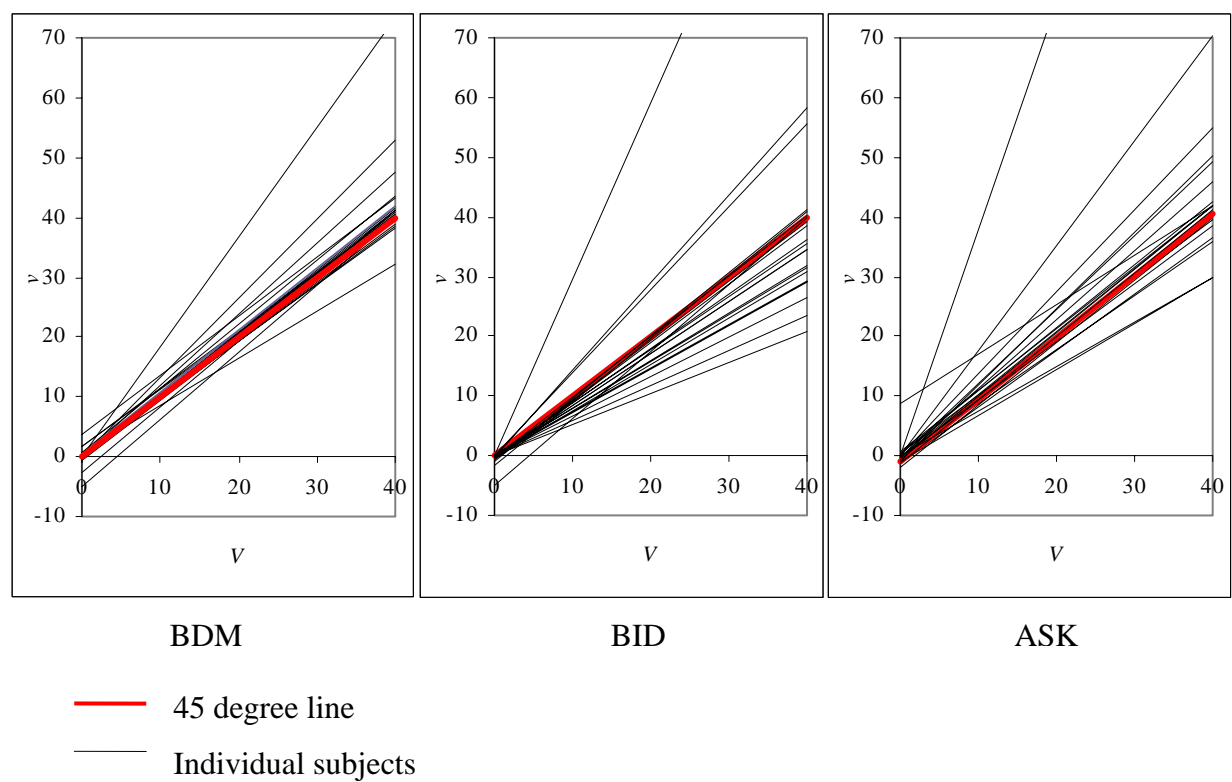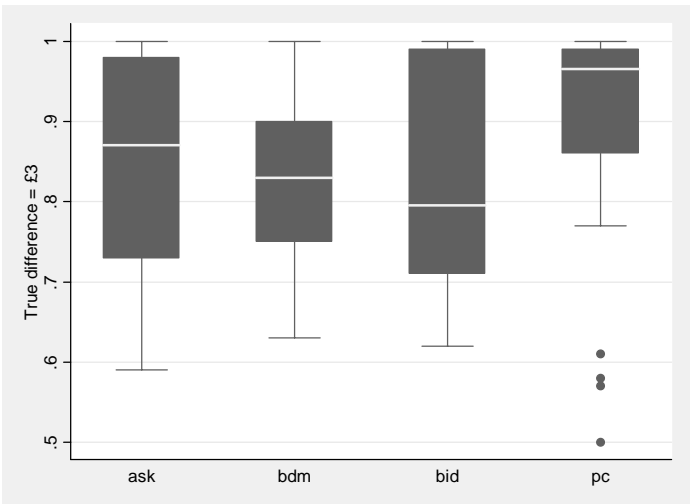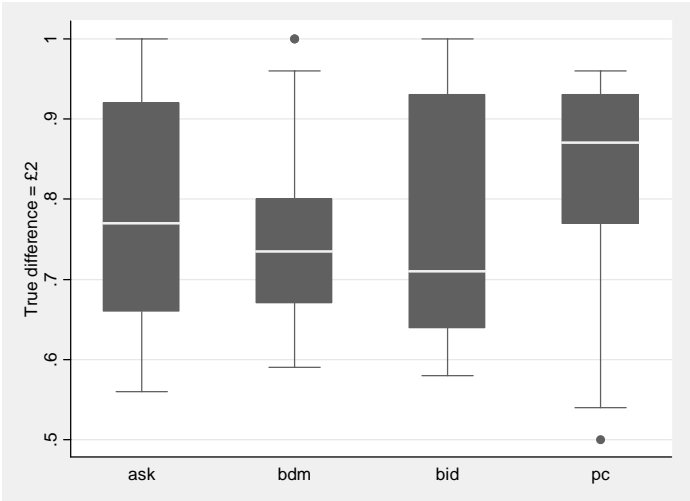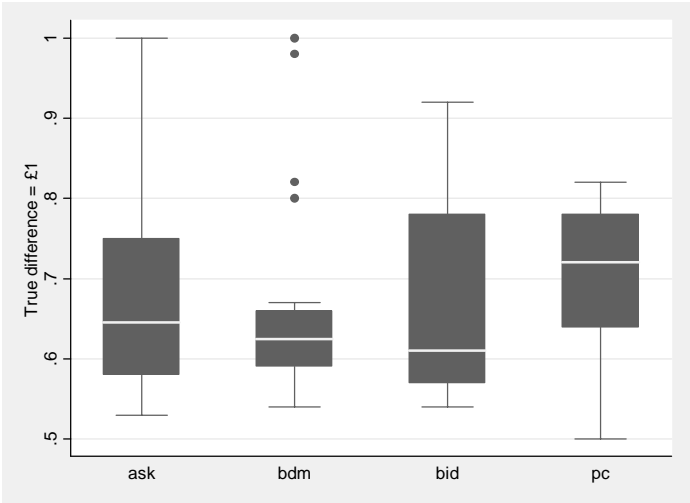——— 45 degree line

——— Individual subjects

Figure 3: Box plot presenting the probabilities of observing the correct preferences

Appendix Table 1: The lotteries in the experiment

No.	£0	£10	£30	£40	No.	£0	£10	£30	£40	No.	£0	£10	£30	£40
1	.000	.000	1.000	.000	20	.000	.200	.700	.100	39	.000	.500	.000	.500
2	.750	.000	.250	.000	21	.000	.000	.500	.500	40	.500	.250	.000	.250
3	.300	.600	.100	.000	22	.500	.000	.500	.000	41	.200	.000	.400	.400
4	.000	.600	.100	.300	23	.250	.500	.250	.000	42	.100	.000	.200	.700
5	.000	1.000	.000	.000	24	.000	.500	.000	.500	43	.800	.000	.000	.200
6	.000	.500	.500	.000	25	.500	.250	.000	.250	44	.400	.000	.500	.100
7	.500	.500	.000	.000	26	.000	.250	.500	.250	45	.400	.000	.000	.600
8	.000	.000	.700	.300	27	.000	.000	.750	.250	46	.700	.000	.000	.300
9	.800	.000	.140	.060	28	.250	.250	.500	.000	47	.200	.000	.000	.800
10	.200	.000	.740	.060	29	.200	.000	.000	.800	48	.200	.000	.400	.400
11	.000	.200	.800	.000	30	.800	.000	.000	.200	49	.100	.000	.000	.900
12	.500	.100	.400	.000	31	.320	.600	.000	.080	50	.600	.000	.000	.400
13	.000	.200	.600	.200	32	.020	.600	.000	.380	51	.300	.500	.000	.200
14	.000	.100	.300	.600	33	.700	.000	.000	.300	52	.200	.200	.000	.600
15	.200	.800	.000	.000	34	.350	.000	.500	.150	53	.600	.100	.000	.300
16	.100	.400	.500	.000	35	.850	.000	.000	.150	54	.000	.350	.000	.650
17	.000	.400	.600	.000	36	.150	.000	.000	.850	55	.000	.100	.250	.650
18	.500	.200	.300	.000	37	.830	.000	.000	.170	56	.250	.350	.000	.400
19	.000	.200	.300	.500	38	.230	.000	.600	.170					

TECHNICAL APPENDIX – NOT FOR PUBLICATION

This Appendix describes the mathematics lying behind the estimation and the GAUSS programs used in the estimation. We assume throughout that the subjects make monetary evaluations of the various gambles with a normally distributed error.

In the experiment there were 4 possible outcomes £0, £10, £30 and £40. We denote the utilities of these by u_1, u_2, u_3 and u_4.

We assume a CRRA utility function:

$$u(x) = (x/40)^r \qquad (1)$$

where we have normalised the function so that $u_1 = 0$ and $u_4 = 1$. A risk-neutral person has $r=1$. The inverse of the utility function is

$$x = u^{-1}(u) = 40u^{1/r}$$

Estimation using the Certainty Equivalent data.

Denote by c_j the certainty equivalent reported by the subject on question number j $(j = 1,..,J)$. Let us drop the subscript j to save notational clutter. Suppose the probabilities on the question are p_1, p_2, p_3 and p_4. Then the Expected utility of the gamble (for given parameters) is

$$EUG = p_1u_1 + p_2u_2 + p_3u_3 + p_4u_4 = 0.25^r p_2 + 0.75^r p_3 + p_4 \quad (2)$$

Hence the true certainty equivalent, t, of the gamble G is given by

$$t = 40 (0.25^r p_2 + 0.75^r p_3 + p_4)^{1/r} \qquad (3)$$

The difference between the stated certainty equivalent and the true one

$$c - t = c - 40 (0.25^r p_2 + 0.75^r p_3 + p_4)^{1/r}$$

We assume that this difference $c - t$ is error, normally distributed with standard deviation s. The normal *pdf* of this is:

$$f(x) = \frac{1}{\sqrt{2\pi^2}} \exp\left[-\frac{x^2}{2s^2}\right] \qquad (4)$$

The log of the *pdf* is therefore

$$\ln f(x) = -x^2 / 2s^2 - 0.5\ln(2\pi s^2) \tag{5}$$

It follows that the log of the probability density of the difference is:

$$\ln f(c-t) = -(c-t)^2 / 2s^2 - \ln(s) - 0.5\ln(2\pi) \tag{6}$$

This is the contribution to the log-likelihood from the certainty equivalent data.

Estimation using the Preference data.

Again we assume that subjects make normally distributed errors when evaluating lotteries. When comparing two lotteries they compare the estimated certainty equivalents. Suppose we have two gambles *L* and *R*. The Expected Utilities are *EUL* and *EUR*. Their monetary evaluations are *ML* = $u^{-1}(EUL)$ and are *MR* = $u^{-1}(EUR)$ The treatment is different according as to whether the subject reports indifference or not.

*1) The subject **never** reports indifference*. In this case, we have that *L* is reported as preferred to *R* if *ML* – *MR* + $\varepsilon \geq$ *0* and that *R* is reported as preferred if *ML* – *MR* + ε < *0*. Hence the probability that *L* is reported as preferred is *Prob($\varepsilon \geq$ MR-ML)* and the probability that *R* is reported as preferred is *Prob(ε< MR-ML)*. Hence the probability of *L (R)* is:

$$\text{Prob}(\varepsilon \geq MR\text{-}ML) \quad (\text{Prob}(\varepsilon < MR\text{-}ML)) \tag{9}$$

Now we need to find expressions for the probabilities. If we denote the normal *cdf* by *Ψ(x/s)* (this is the integral of (4)) we can then write that the probability of *L (R)* is

$$1 - \Psi((MR - ML)/s) \quad (\Psi((MR - ML)/s)) \tag{10}$$

Hence the *log-likelihood* is

$$\ln(1 - \Psi(MR - ML)) \quad (\ln(\Psi(MR - ML))) \tag{11}$$

*2) The subject **sometimes** reports indifference.* This is almost the same but we need some story about when the subject reports indifference. We say that *L* is reported as preferred when if *ML* –

$MR + \varepsilon \geq \tau$ that R is reported as preferred if $ML - MR + \varepsilon < -\tau$ and that indifference is reported when $-\tau \leq ML - MR + \varepsilon < \tau$ Hence the probability that L is reported as preferred is $Prob(\varepsilon \geq MR-ML + \tau)$ the probability that R is reported as preferred is $Prob(\varepsilon < MR-ML - \tau)$ and the probability that indifference is reported is $Prob(MR-ML - \tau \leq \varepsilon < MR-ML + \tau)$

Estimating bias in the certainty equivalents.

We simply assume that there is a *true* valuation V and a *reported* valuation v which are related by

$$V = a + b v$$

Here the parameters a and b determine the *bias* in the reporting of the certainty equivalents. If $a=0$ and $b=1$ there is no bias. In the text tables we report the estimated values of a and b for each of the certainty equivalent methods. We assume no bias in the pairwise choice elicitation method.

GAUSS PROGRAM – NOT FOR PUBLICATION

/*
This is CRRA.EST.

It uses GAUSS to fit preference functionals to combined
preference AND certainty equivalent data using the constant
relative risk aversion form for the utility function.
Thus, u(x) = (x/40)^r so u(0) = 0 and u(40) = 1.

This version combines the three preference data sets (from occasions A, B and C) with three
sources of certainty equivalent data: BDM, BID and ASK data.
The three occasions for the PC data are in the files p.a p.b and p.c.
For the CE methods we have that BDM is c.1, BID is c.2 and ASK is c.3.

This version uses the normal pdf for the error terms.

It is appropriate for subjects who sometimes declare indifference on the
preference questions. I use

indiff=-1 for subjects 20 and 21 who are always exactly risk-neutral (subjects 20 and 21).
indiff=0 for those subjects who never express indifference (excluding subjects 20 and 21).
indiff=1 for those subjects who sometimes express indifference.

I am going to estimate using various combinations of the data sets:

ds=1 just the BDM data
ds=2 just the BID data
ds=3 just the ASK data
ds=4 all the CE data together
ds=5 just the PC data
ds=6 all the CE data plus the PC data

THIS PROGRAM CORRECTS FOR THE BIAS
u(a+C*b)=Eu(G)+e.
a and b are the intercept and the slope of the function relating the expressed CE to the true value.
abdm, bbdm, abid, bbid, aask, bask are the intercept and the slope of the bias in the 3 data sets.

I assume no bias in the pairwise choice decisions.
*/

library maxlik;
maxset;

output file = d:/active/people/schmidt/two/crra/crra.out;
output off;

k = 24;   /* k is the number of subjects */
n = 56;   /* n is the number of certainty equivalent questions*/
m = 28;   /* m is the number of pairwise choice questions*/

on=ones(n,1);
om=ones(m,1);
let _max_MaxIters = 1000;
let vars = 1 2 3 4 5 6 7 8;
eps=0.000000001;   /* drop this later   */

sp = 5.0;
output on;
print "starting s = " sp;
print "";

```
output off;

load c1[k,n] = d:/active/people/schmidt/two/crra/c.1;    /* this is the matrix of BDM observations */
load c2[k,n] = d:/active/people/schmidt/two/crra/c.2;    /* this is the matrix of BID observations */
load c3[k,n] = d:/active/people/schmidt/two/crra/c.3;    /* this is the matrix of ASK observations */
load pa[k,m] = d:/active/people/schmidt/two/crra/p.a;    /* this is matrix of observations of preferences in occasion A*/
load pb[k,m] = d:/active/people/schmidt/two/crra/p.b;    /* this is matrix of observations of preferences in occasion B*/
load pc[k,m] = d:/active/people/schmidt/two/crra/p.c;    /* this is matrix of observations of preferences in occasion C*/


load p[n,4] = d:/active/people/schmidt/two/crra/p.inp;                /* this is matrix of probabilities in the lotteries in the
CE questions */
load q[m,8] = d:/active/people/schmidt/two/crra/pcprobs.inp;
                                                         /* this is matrix of probabilities in the PC questions */

ql=q[.,1]~q[.,2]~q[.,3]~q[.,4];                           /* this is the matrix of probabilities of the left gamble */
qr=q[.,5]~q[.,6]~q[.,7]~q[.,8];                           /* this is the matrix of probabilities of the right gamble */

/* the next few lines work out the maximum payoff in each of the CE lotteries */
mx = ones(n,1);
i = 1;
do while i <= n;
if p[i,4] > 0; mx[i] = 40;endif;
if mx[i] <40;
if p[i,3] > 0; mx[i] = 30; endif;
endif;
if mx[i] < 30;
if p[i,2] > 0; mx[i] = 10; endif;
endif;
if mx[i] < 10;
if p[i,1] > 0; mx[i] = 0; endif;
endif;
i = i + 1;
endo;
/*end of working out max payoff */


ds=6;
do while ds<=6;
screen off; output on;format /rd 7,3;
print "";
print "";
if ds==1; print "Estimation of CRRA Model with BDM data" ; endif;
if ds==2; print "Estimation of CRRA Model with BID data" ; endif;
if ds==3; print "Estimation of CRRA Model with ASK data" ; endif;
if ds==4; print "Estimation of CRRA Model with (all) CE data" ; endif;
if ds==5; print "Estimation of CRRA Model with PC data" ; endif;
if ds==6; print "Estimation of CRRA Model with CE and PC data" ; endif;

if ds==1;print "  subj exit c log-lik abdm   bbdm   sbdm      r";endif;
if ds==2;print "  subj exit c log-lik abid   bbid   sbid      r";endif;
if ds==3;print "  subj exit c log-lik aask   bask   sask      r";endif;
if ds==4;print "  subj exit c log-lik abdm   bbdm   sbdm   abid   bbid   sbid   aask   bask   sask      r ";endif;
if ds==5;print "  subj exit c log-lik spc      r    tau";endif;
if ds==6;print "  subj exit c log-lik abdm   bbdm   sbdm   abid   bbid   sbid   aask   bask   sask   spc      r
 tau";endif;
print "";
output off; screen on;

j = 1;
do while j<=24;
```

```
if j==20 or j==21;indiff=-1;endif;
if j==1 or j==2 or j==3 or j==4 or j==5 or j==7 or j==9 or j==11 or j==12 or j==14 or j==15 or j==16 or j==18 or
j==19 or j==22 or j==23;indiff=1;endif;
if j==6 or j==8 or j==10 or j==13 or j==17 or j==24;indiff=0;endif;


/*reading in the decisions of the subjects*/
/* first the pairwise choice on the 3 occasions */
wa = (pa[j,.])';
wb = (pb[j,.])';
wc = (pc[j,.])';
/* now the certainty equivalents of the three types*/
w1 = (c1[j,.])';
w2 = (c2[j,.])';
w3 = (c3[j,.])';



if indiff==0;

if ds==1;_max_active=1|1|1|0|0|0|0|0|0|0|1|0;endif;
if ds==2;_max_active=0|0|0|1|1|1|0|0|0|0|1|0;endif;
if ds==3;_max_active=0|0|0|0|0|0|1|1|1|0|1|0;endif;
if ds==4;_max_active=1|1|1|1|1|1|1|1|1|0|1|0;endif;
if ds==5;_max_active=0|0|0|0|0|0|0|0|0|1|1|0;endif;
if ds==6;_max_active=1|1|1|1|1|1|1|1|1|1|1|0;endif;
start= 0.0|1.0|sp|0.0|1.0|sp|0.0|1.0|sp|sp|1.0|0.0;
/*starting values always 0 for a, 1 for b, sp for s, 0.0 for r and 0.0 for tau */
{x,f,g,c,retcode} = maxlik(q,vars,&ll,start);
output on;screen off;
if ds==1;print j~retcode~f~x[1]~x[2]~scf(x[3])~abs(x[11]);endif;
if ds==2;print j~retcode~f~x[4]~x[5]~scf(x[6])~abs(x[11]);endif;
if ds==3;print j~retcode~f~x[7]~x[8]~scf(x[9])~abs(x[11]);endif;
if ds==4;print j~retcode~f~x[1]~x[2]~scf(x[3])~x[4]~x[5]~scf(x[6])~x[7]~x[8]~scf(x[9])~abs(x[11]);endif;
if ds==5;print j~retcode~f~scf(x[10])~abs(x[11]);endif;
if ds==6;print j~retcode~f~x[1]~x[2]~scf(x[3])~x[4]~x[5]~scf(x[6])~x[7]~x[8]~scf(x[9])~scf(x[10])~abs(x[11]);endif;
screen on;output off;
endif;



if indiff==1;
if ds==1;_max_active=1|1|1|0|0|0|0|0|0|0|1|1;endif;
if ds==2;_max_active=0|0|0|1|1|1|0|0|0|0|1|1;endif;
if ds==3;_max_active=0|0|0|0|0|0|1|1|1|0|1|1;endif;
if ds==4;_max_active=1|1|1|1|1|1|1|1|1|0|1|1;endif;
if ds==5;_max_active=0|0|0|0|0|0|0|0|0|1|1|1;endif;
if ds==6;_max_active=1|1|1|1|1|1|1|1|1|1|1|1;endif;
start= 0.0|1.0|sp|0.0|1.0|sp|0.0|1.0|sp|sp|1.0|0.0;
/*starting values always 0 for a, 1 for b, sp for s, 1.0 for r and 0.0 for tau */
{x,f,g,c,retcode} = maxlik(q,vars,&ll,start);
output on;screen off;
if ds==1;print j~retcode~f~x[1]~x[2]~scf(x[3])~abs(x[11]);endif;
if ds==2;print j~retcode~f~x[4]~x[5]~scf(x[6])~abs(x[11]);endif;
if ds==3;print j~retcode~f~x[7]~x[8]~scf(x[9])~abs(x[11]);endif;
if ds==4;print j~retcode~f~x[1]~x[2]~scf(x[3])~x[4]~x[5]~scf(x[6])~x[7]~x[8]~scf(x[9])~abs(x[11]);endif;
if ds==5;print j~retcode~f~scf(x[10])~abs(x[11])~1/(1+exp(-x[12]));endif;
if ds==6;print
j~retcode~f~x[1]~x[2]~scf(x[3])~x[4]~x[5]~scf(x[6])~x[7]~x[8]~scf(x[9])~scf(x[10])~abs(x[11])~tcf(x[12]);endif;
screen on;output off;
endif;
```

```
j = j+1;
endo;
ds=ds+1;
endo;


proc ll(x,y);
/* THIS IS THE CRRA LIKELIHOOD FUNCTION*/

local ll,lla,lla1,lla2,lla3,llb,llb1,llb2,llb3,llc,llc1,llc2,llc3,f1,f2,f3,ll1,ll2,ll3;
local abdm,bbdm,sbdm,abid,bbid,sbid,aask,bask,sask,spc,tau,r;
local nw1,nw2,nw3;
local u1,u2,u3,u4,uv,eul,eur,eug,cel,cer,ced,ced1,ced2,ced3;
local x1a,x2a,x3a,x4a, u1a,u2a,u3a,u4a,ua,euga,cega;

x1a=ones(n,1);
x2a=ones(n,1);
x3a=ones(n,1);
x4a=ones(n,1);
euga=ones(n,1);
ced2=ones(n,1);
u1a=ones(n,1);
u2a=ones(n,1);
u3a=ones(n,1);
u4a=ones(n,1);
cega=ones(n,1);

abdm=x[1];
bbdm=x[2];
sbdm=scf(x[3]);
abid=x[4];
bbid=x[5];
sbid=scf(x[6]);
aask=x[7];
bask=x[8];
sask=scf(x[9]);

spc=scf(x[10]);

r=abs(x[11]);
if indiff==1;tau=tcf(x[12]);endif;
if indiff==0;tau=0.0;endif;


/* we start with the PC estimation */
u1=0;u2=0.25^r;u3=0.75^r;u4=1;
uv=u1|u2|u3|u4;
eul=ql*uv;eur=qr*uv;
cel=40*(eul^(1/r));cer=40*(eur^(1/r));          /*these are the CE of left and the CE of right     */
ced=cer-cel;                             /*this is the difference between the two CEs          */


/*print r;print cer~cel~wa~wb~wc;pause(5);*/


/*if ds==5;print ced~(ced-tau);pause(5);endif;*/

if indiff==0;
f1 = cdfnc(ced/spc);                     /*this is the prob that z is greater than ced     */
f3 = cdfn(ced/spc);                      /*this is the prob that z is smaller than ced     */
lla1= 0.5*(3*om-wa).*(2*om-wa).*ln(f1+eps*om);
lla3=0.5*(wa-2*om).*(wa-om).*ln(f3+eps*om);
```

```
lla=lla1+lla3;                          /* this is the log-likelihood for PC occasion A    */
llb1= 0.5*(3*om-wb).*(2*om-wb).*ln(f1+eps*om);
llb3=0.5*(wb-2*om).*(wb-om).*ln(f3+eps*om);
llb=llb1+llb3;                          /* this is the log-likelihood for PC occasion B    */
llc1= 0.5*(3*om-wc).*(2*om-wc).*ln(f1+eps*om);
llc3=0.5*(wc-2*om).*(wc-om).*ln(f3+eps*om);
llc=llc1+llc3;                          /* this is the log-likelihood for PC occasion C    */
endif;


if indiff==1;
f1 = cdfnc((ced+tau)/spc);                   /*this is the prob that z is greater than ced+tau    */
f3 = cdfn((ced-tau)/spc);                    /*this is the prob that z is smaller than ced-tau    */
f2=om-f1-f3;                         /*this is the prob that z is between ced-tau and ced+tau */
lla1= 0.5*(3*om-wa).*(2*om-wa).*ln(f1+eps*om);
lla2=(3*om-wa).*(wa-om).*ln(f2+eps*om);
lla3=0.5*(wa-2*om).*(wa-om).*ln(f3+eps*om);
lla=lla1+lla2+lla3;                 /* this is the log-likelihood for PC occasion A    */
llb1= 0.5*(3*om-wb).*(2*om-wb).*ln(f1+eps*om);
llb2=(3*om-wb).*(wb-om).*ln(f2+eps*om);
llb3=0.5*(wb-2*om).*(wb-om).*ln(f3+eps*om);
llb=llb1+llb2+llb3;                 /* this is the log-likelihood for PC occasion B    */
llc1= 0.5*(3*om-wc).*(2*om-wc).*ln(f1+eps*om);
llc2=(3*om-wc).*(wc-om).*ln(f2+eps*om);
llc3=0.5*(wc-2*om).*(wc-om).*ln(f3+eps*om);
llc=llc1+llc2+llc3;                 /* this is the log-likelihood for PC occasion C    */
endif;


nw1=abdm*on+bbdm*w1;
nw2=abid*on+bbid*w2;
nw3=aask*on+bask*w3;

eug=p*uv;                           /* this is the EU of the Gambles                   */
ced1=nw1-40*(eug^(1/r));                  /* this is the difference between the stated CE and
                               that implied by the utility function: BDM data    */
x1a=(mx-nw2)/40;
x2a=(mx-nw2+10)/40;
x3a=(mx-nw2+30)/40;
x4a=(mx-nw2+40)/40;

i=1;
do while i<=n;
if x1a[i]>0;u1a[i]=(x1a[i])^r;endif;
if x1a[i]==0;u1a[i]=0;endif;
if x1a[i]<0;u1a[i]=-(-x1a[i])^r;endif;
if x2a[i]>0;u2a[i]=(x2a[i])^r;endif;
if x2a[i]==0;u2a[i]=0;endif;
if x2a[i]<0;u2a[i]=-(-x2a[i])^r;endif;
if x3a[i]>0;u3a[i]=(x3a[i])^r;endif;
if x3a[i]==0;u3a[i]=0;endif;
if x3a[i]<0;u3a[i]=-(-x3a[i])^r;endif;
if x4a[i]>0;u4a[i]=(x4a[i])^r;endif;
if x4a[i]==0;u4a[i]=0;endif;
if x4a[i]<0;u4a[i]=-(-x4a[i])^r;endif;
i=i+1;
endo;

ua=u1a~u2a~u3a~u4a;


i=1;
```

```
do while i<=n;
euga[i]=p[i,.]*ua[i,.]';
i=i+1;
endo;


i=1;
do while i<=n;                                      /* this is the EU of the augmented gamble   */
if euga[i]>0;cega[i]=40*((euga[i])^(1/r));endif;
if euga[i]==0;cega[i]=0;endif;
if euga[i]<0;cega[i]=-40*((-euga[i])^(1/r));endif;                /* this is the difference between xmax and .... */
i=i+1;
endo;

ced2=mx-cega;


/*
if r<0.00000001;
print mx~w2~nw2~r*on~x1a~x2a~x3a~x4a~ua~euga~cega~ced2;pause(5);
endif;
*/

ced3=nw3-40*(eug^(1/r));                   /* this is the difference between the stated CE and
                                             that implied by the utility function: ASK data    */

/* below is some old code for the BID data*/
/* ced2=nw2-40*(eug^(1/r)); */
                    /* this is the difference between the stated CE and
                                             that implied by the utility function: BID data    */


/* coming up are some pdfs */
ll1=-ced1.*ced1/(2*sbdm^2)-ln(sbdm)-0.5*ln(6.283185308);          /*this is the log density of ced1*/
ll2=-ced2.*ced2/(2*sbid^2)-ln(sbid)-0.5*ln(6.283185308);         /*this is the log density of ced2*/
ll3=-ced3.*ced3/(2*sask^2)-ln(sask)-0.5*ln(6.283185308);          /*this is the log density of ced3*/


if ds==1;ll=on'*(ll1);endif;
if ds==2;ll=on'*(ll2);endif;
if ds==3;ll=on'*(ll3);endif;
if ds==4;ll=on'*(ll1+ll2+ll3);endif;
if ds==5;ll=om'*(lla+llb+llc);endif;
if ds==6;ll=om'*(lla+llb+llc)+on'*(ll1+ll2+ll3);endif;


retp(ll);
endp;

proc scf(s);
retp(abs(s));
/*retp(100/(1+exp(-s)));*/
endp;

proc tcf(t);
/*retp(abs(t));*/
retp(10/(1+exp(-t)));
endp;

closeall;
end.
```