THE UNIVERSITY of York

Discussion Papers in Economics

No. 2006/27

Consumption and Habit Formation when Time Horizon is Finite

by

Viola Angelini

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

Consumption and habit formation when time horizon is finite*

Viola Angelini†

University of York and University of Padua

November 9, 2006

Abstract

This paper provides a closed-form solution under labour uncertainty for optimal consumption and the value function in a finite horizon life-cycle model with habit persistence.

Keywords: habit formation, life-cycle consumption, precautionary saving.

JEL classification: D91, C61.

*This paper is part of my PhD thesis. I wish to thank my supervisors Peter Simmons and Guglielmo Weber for their useful comments and suggestions. I am also grateful to Efrem Castelnuovo and Giorgia Marini for helpful discussions.

†Corresponding address: Dipartimento di Scienze Economiche "Marco Fanno", University of Padua, 35123 Padova, Italy. E-mail: viola.angelini@unipd.it. Telephone: +390498273848. Fax: +390498274211.

1 Introduction

The presence of internal habit formation is an intuitively appealing hypothesis to explain a number of consumption puzzles, such as excess smoothness and excess sensitivity, thus reconciling the life-cycle theory with the empirical findings. The idea behind it is that individuals derive utility not only from the level of current consumption, but also from the comparison of this level with a reference stock determined by their past consumption. As Deaton (1992) points out, simple model of habits formation "are worth exploring because of their insights into consequences of non-additive preferences". Using a Constant-Absolute-Risk-Aversion (CARA) utility function, Alessie and Lusardi (1997) find an exact solution for consumption under habit persistent behaviour. Guariglia and Rossi (2002) extend their model by assuming a hybrid utility function that is intertemporally isoelastic but still exponential in the risk component. Importantly, the results of Alessie and Lusardi (1997) and Guariglia and Rossi (2002) rely on the assumption that individuals optimise over an infinite horizon. Unfortunately, infinite horizon models cannot be used to assess the relative importance of habits on precautionary saving over the life cycle and do not allow direct estimate or testing on micro-data.

This paper contributes to the existing literature on habit persistence by providing an exact closed form solution for consumption and the value function in a finite horizon life-cycle model. The main advantage of obtaining an analytical solution is that the consumption function derived embodies more information on the habit persistence model. Furthermore, Alessie and Teppa (2002) argue that employing the Euler equation approach the habit persistence parameter might be underestimated because generally in surveys consumption is measured with error.

The paper is organised as follows. Section 2 outlines the model, section 3 presents the closed form solutions for the value function and optimal consumption when time horizon is finite and section 4 concludes.

2 The model

I assume that preferences are characterised by a CARA utility function and are non-separable over time in that current felicity depends also on last period consumption. The consumer enters each period t with non-human wealth $(1+r)A_{t-1}$, where r is the interest rate, and she receives labour income y_t. Then, the individual chooses the optimal level of consumption c_t, which in turns determines the stock of financial assets A_t to carry forward into next period. The length of life (T) is known in advance and the individual has to die without debt ($A_T = 0$). The optimisation problem can be stated as follows:

$$\max_{c_{t+i}} E_t \sum_{i=0}^{T-t} \phi^i \left[-\frac{1}{\theta} \exp(-\theta(c_{t+i} - \gamma c_{t+i-1})) \right]$$

subject to:

$$\begin{aligned} c_{t+i} &= y_{t+i} + (1+r)A_{t+i-1} - A_{t+i} \quad \text{for } 0 \le i \le T-t \\ c_T &= y_T + (1+r)A_{T-1} \end{aligned}$$

where E_t is the expectation operator conditional on the information available at t, $\phi = (1+r)^{-1}$ is the intertemporal discount factor (regulated by the real interest rate r), and θ indicates the coefficient of absolute risk aversion. The parameter γ represents the importance of the habit: if $\gamma = 0$, preferences are intertemporally separable, if $\gamma > 0$ there is habit formation (the higher γ, the stronger the habit) and $\gamma < 0$ indicates durability. As in Caballero (1991), I also assume that at t both A_{t-1} and c_{t-1} are known and the only source of uncertainty is labour income y, which follows a driftless random-walk process $y_t = y_{t-1} + w_t$ where the w_t are normally distributed, zero-mean i.i.d. shocks whose variance is σ^2.[1]

3 Optimal consumption and the value function

Following the methodology used by Berloffa and Simmons (2002) and Angelini and Simmons (2005), I derive the exact solution for the value function and optimal consumption in every period.

Proposition 1 *At t the value function takes the form:*

$$V_t = -\frac{\beta_t}{\theta} \exp(-\theta(\delta_t A_{t-1} + \zeta_t y_t - \gamma \eta_t c_{t-1})) \quad (1)$$

[1]This assumption is made for the sake of simplicity and could easily be relaxed to allow for a more general form of the income process.

where β_t, δ_t, ζ_t and η_t are defined recursively by

$$
\begin{aligned}
\eta_t &= \frac{\delta_{t+1} + \gamma\eta_{t+1}}{1 + \delta_{t+1} + \gamma\eta_{t+1}} \\
\beta_t &= \frac{1}{\eta_t}\left[\phi\beta_{t+1}\frac{\eta_t}{1-\eta_t}E_t \exp(-\theta\zeta_{t+1}w_{t+1})\right]^{1-\eta_t} \\
\delta_t &= \frac{\delta_{t+1}(1+r)}{1 + \delta_{t+1} + \gamma\eta_{t+1}} = \delta_{t+1}(1+r)(1-\eta_t) \\
\zeta_t &= \frac{\delta_{t+1} + \zeta_{t+1}}{1 + \delta_{t+1} + \gamma\eta_{t+1}} = (\delta_{t+1} + \zeta_{t+1})(1-\eta_t)
\end{aligned}
$$

Proof. See the appendix A1. ■

The recurrence relations in proposition 1 show that the habit formation parameter γ has an effect on all the terms in the value function. Figure 1 shows the four coefficients as functions of γ and time for $r = 0.025$ and $w_t \sim N(0, 0.1^2)$.

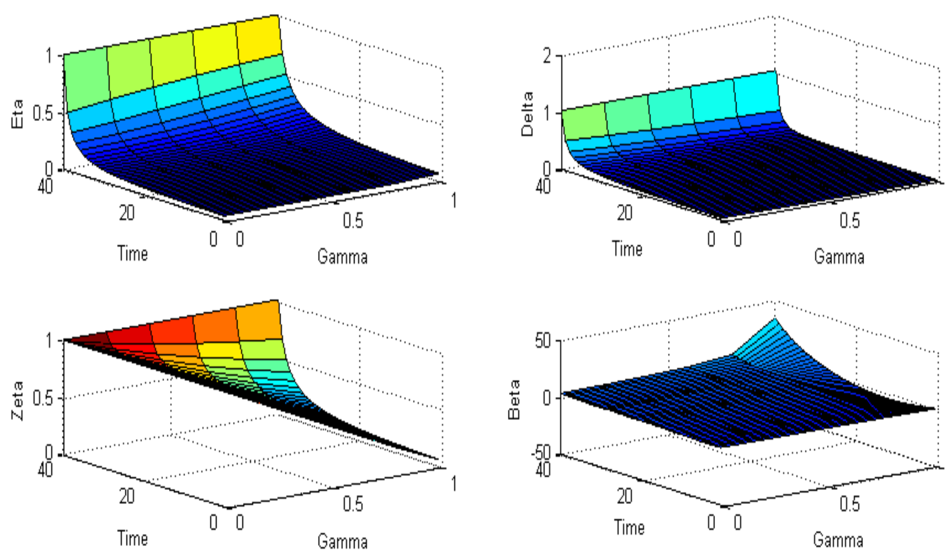

Figure 1. η_t, δ_t, ζ_t, β_t as functions of γ and time

In the last period

$$\begin{aligned} V_T &= -\frac{1}{\theta}\exp(-\theta[(1+r)A_{T-1}+y_T-\gamma c_{T-1}]) \\ &= -\frac{\beta_T}{\theta}\exp(-\theta(\delta_T A_{T-1}+\zeta_T y_T-\gamma\eta_T c_{T-1})) \end{aligned}$$

with $\delta_T = 1 + r$ and $\beta_T = \zeta_T = \eta_T = 1$.

From the first order conditions of the maximisation problem it is possible to derive the optimal consumption function.

Proposition 2 *The consumption function is given by:*

$$\begin{aligned} c_t &= -\frac{1-\eta_t}{\theta}\ln\left[\phi\beta_{t+1}\frac{\eta_t}{1-\eta_t}E_t\exp(-\theta\zeta_{t+1}w_{t+1})\right] \\ &\quad +\delta_t A_{t-1}+\zeta_t y_t+(1-\eta_t)\gamma c_{t-1} \end{aligned} \tag{2}$$

Proof. See the appendix A2. ■

The closed form solution for consumption is additive and depends on four components: a precautionary premium, financial wealth, current non-capital income and past consumption. When $\gamma = r = 0$, equation (2) simplifies to the solution of Caballero (1991). Note that if $w \sim N(0,\sigma^2)$, then $E_t\exp(-\theta\zeta_{t+1}w_{t+1}) = \exp\left(\frac{\theta^2\zeta_{t+1}^2\sigma^2}{2}\right)$ and the term that measure the effect of uncertainty on consumption becomes:

$$-(1-\eta_t)\frac{\theta\zeta_{t+1}^2\sigma^2}{2}$$

Since $(1-\eta_t)$ and ζ_t are decreasing functions of γ, habit formation has a negative effect on precautionary savings. The stronger the habit, the less important the effect of labor income risk *ceteris paribus.* The reason behind it is that an individual with habit forming preferences consumes less out of labour income and total assets (δ_t and ζ_t are decreasing in γ) and, therefore, has more (non-precautionary) savings. Hence, the presence of habits affects optimal consumption not only directly via c_{t-1} but also indirectly, making the precautionary component smaller in absolute value and reducing the effect of labour income and financial wealth. Figure 2 shows the life-cycle consumption profiles for different levels of γ. On the one hand, habits provide a further motivation to consume: the stronger the habit, the less the utility derived from a given consumption level and the larger must be purchases to generate the same benefit. This explains why in the second half of the life-cycle consumption increases with the strength of the habit. On the other, anticipating this, a rational individual with non-separable preferences will tend to choose a lower consumption level early on in the life cycle to achieve higher consumption growth.

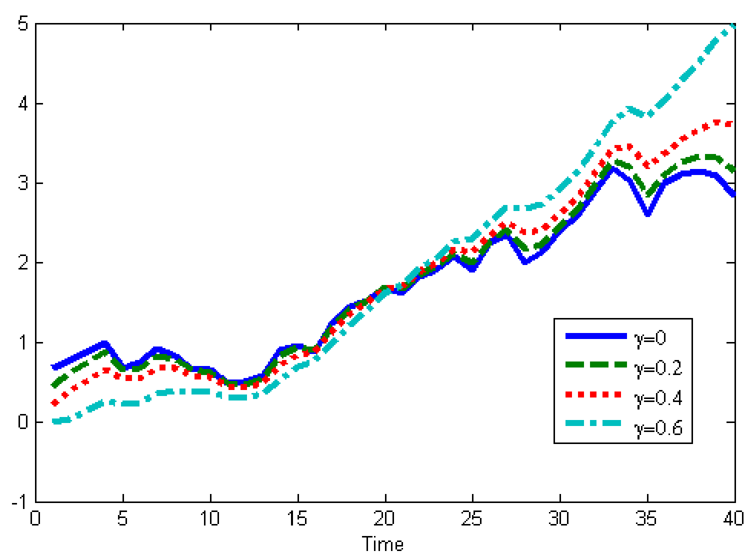

Figure 2. Life-cycle consumption profiles for different levels of γ.

4 Conclusion

In this paper I have extended the results of Alessie and Lusardi (1997) and Guariglia and Rossi (2002) and I have shown that, using a simple model of habit formation, it is possible to derive an explicit solution for the consumption function under labour income uncertainty even in the case in which the time horizon is finite. The main advantage of obtaining such a closed-form solution is that it is possible to gain several insights on the effect of habit persistence on the level of consumption and savings over the life-cycle.

References

[1] Alessie, R. and A. Lusardi, 1997. Consumption, saving and habit formation. Economics Letters 55, 103-108.

[2] Alessie, R. and F. Teppa, 2002. Saving and habit formation: evidence from Dutch panel data. CentER DP 2002-62.

[3] Angelini, V. and P. Simmons, 2005. Household debt, employment risk and consumption. Discussion Paper in Economics No. 2005/07, University of York.

[4] Berloffa, G. and P. Simmons, 2003. Unemployment risk, labour force participation and savings. Review of Economic Studies 70, 521-539.

[5] Caballero, R.J., 1991. Earnings uncertainty and aggregate wealth accumulation. American Economic Review 81, 859-871.

[6] Deaton, A., 1992. Understanding consumption (Oxford University Press: Oxford).

[7] Guariglia, A. and Rossi, M., 2002. Consumption, habit formation and precautionary saving: evidence from the British Household Panel Survey. Oxford Economic Papers 54, 1-19.

A Appendix

A.1 Proof of proposition 1

I guess that the value function takes the form:

$$V_t = -\frac{\beta_t}{\theta}\exp(-\theta(\delta_t A_{t-1} + \zeta_t y_t - \gamma\eta_t c_{t-1})) \tag{3}$$

where β_t, δ_t, ζ_t and η_t are unknown functions that need to be determined. Taking expectations gives:

$$\begin{aligned}
E_{t-1}V_t &= -\frac{\beta_t}{\theta}E_{t-1}\exp(-\theta(\delta_t A_{t-1} + \zeta_t(y_{t-1} + w_t) - \gamma\eta_t c_{t-1})) \\
&= -\frac{\beta_t}{\theta}E_{t-1}\exp(-\theta\zeta_t w_t)\exp(-\theta(\delta_t A_{t-1} + \zeta_t y_{t-1} - \gamma\eta_t c_{t-1}))
\end{aligned}$$

Therefore, the maximisation problem at t can be written as:

$$\begin{aligned}
V_t &= \max_{c_t}\{-\frac{1}{\theta}\exp(-\theta(c_t - \gamma c_{t-1})) \\
&\quad -\frac{\phi}{\theta}\beta_{t+1}E_t\exp(-\theta\zeta_{t+1}w_{t+1})\exp(-\theta(\delta_{t+1}A_t + \zeta_{t+1}y_t - \gamma\eta_{t+1}c_t))\} \\
&= \max_{c_t}\{-\frac{1}{\theta}\exp(-\theta(c_t - \gamma c_{t-1})) \\
&\quad -\frac{\phi}{\theta}\beta_{t+1}E_t\exp(-\theta\zeta_{t+1}w_{t+1}) \\
&\quad \exp(-\theta(\delta_{t+1}((1+r)A_{t-1} + y_t - c_t) + \zeta_{t+1}y_t - \gamma\eta_{t+1}c_t))\} \\
&= \max_{c_t}\{-\frac{1}{\theta}\exp(-\theta(c_t - \gamma c_{t-1})) \\
&\quad -\frac{\phi}{\theta}\beta_{t+1}E_t\exp(-\theta\zeta_{t+1}w_{t+1}) \\
&\quad \exp(-\theta\delta_{t+1}(1+r)A_{t-1})\exp(-\theta(\delta_{t+1} + \zeta_{t+1})y_t)\exp(\theta(\delta_{t+1} + \gamma\eta_{t+1})c_t))\}
\end{aligned}$$

The first order condition with respect to c_t is:

$$\begin{aligned}
\frac{\partial}{\partial c_t} &= \exp(-\theta(c_t-\gamma c_{t-1})) \\
&\quad -\phi\beta_{t+1}\gamma(\delta_{t+1}+\gamma\eta_{t+1})E_t\exp(-\theta\zeta_{t+1}w_{t+1})\exp(-\theta\delta_{t+1}(1+r)A_{t-1}) \\
&\quad \exp(-\theta(\delta_{t+1}+\zeta_{t+1})y_t)\exp(\theta(\delta_{t+1}+\gamma\eta_{t+1})c_t))
\end{aligned}$$

Rearranging the terms:

$$\begin{aligned}
&\exp(-\theta(1+\delta_{t+1}+\gamma\eta_{t+1})c_t) \\
=\ &\phi\beta_{t+1}(\delta_{t+1}+\gamma\eta_{t+1})E_t\exp(-\theta\zeta_{t+1}w_{t+1})\exp(-\theta\delta_{t+1}(1+r)A_{t-1}) \\
&\exp(-\theta(\delta_{t+1}+\zeta_{t+1})y_t)\exp(-\theta\gamma c_{t-1})
\end{aligned} \tag{4}$$

By substituting back into the value function:

$$\begin{aligned}
V_t &= -\frac{1}{\theta}\exp(-\theta(c_t-\gamma c_{t-1})) - \frac{1}{\theta\gamma(\delta_{t+1}+\gamma\eta_{t+1})}\exp(-\theta(c_t-\gamma c_{t-1})) \\
&= -\frac{1}{\theta}\left[\frac{1+\delta_{t+1}+\gamma\eta_{t+1}}{\delta_{t+1}+\gamma\eta_{t+1}}\right]\exp(-\theta(c_t-\gamma c_{t-1})) \\
&= -\frac{1}{\theta}\left[\frac{1+\delta_{t+1}+\gamma\eta_{t+1}}{\delta_{t+1}+\gamma\eta_{t+1}}\right] \\
&\quad \left[\phi\beta_{t+1}(\delta_{t+1}+\gamma\eta_{t+1})E_t\exp(-\theta\zeta_{t+1}w_{t+1})\right]^{\frac{1}{(1+\delta_{t+1}+\gamma\eta_{t+1})}} \\
&\quad \exp\left(-\theta\frac{\delta_{t+1}(1+r)}{1+\delta_{t+1}+\gamma\eta_{t+1}}A_{t-1}\right)\exp\left(-\theta\frac{\delta_{t+1}+\zeta_{t+1}}{1+\delta_{t+1}+\gamma\eta_{t+1}}y_t\right) \\
&\quad \exp\left(\theta\frac{\delta_{t+1}+\gamma\eta_{t+1}}{1+\delta_{t+1}+\gamma\eta_{t+1}}\gamma c_{t-1}\right)
\end{aligned}$$

The comparison of this expression with equation (3) gives the recurrence relations for the unknown functions.

A.2 Proof of proposition 2

From equation (4), the consumption function is:

$$
\begin{aligned}
c_t \quad = \quad & -\frac{1}{\theta(1+\delta_{t+1}+\gamma\eta_{t+1})}\ln[\phi\beta_{t+1}(\delta_{t+1}+\gamma\eta_{t+1})E_t\exp(-\theta\zeta_{t+1}w_{t+1})] \\
& +\frac{\delta_{t+1}(1+r)}{1+\delta_{t+1}+\gamma\eta_{t+1}}A_{t-1}+\frac{\delta_{t+1}+\zeta_{t+1}}{1+\delta_{t+1}+\gamma\eta_{t+1}}y_t+\frac{1}{1+\delta_{t+1}+\gamma\eta_{t+1}}c_{t-1}
\end{aligned}
$$