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The Company You Keep: Qualitative Uncertainty in Providing Club Goods

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Clubs are typically experience goods. Potential members cannot ascertain precisely beforehand their quality (dependent endogenously on the club's facility investment and number of users, itself dependent on its pricing policy). Members with unsatisfactory initial experiences discontinue visits. We show that a monopoly profit maximiser never offers a free trial period for such goods but, for a quality function homogeneous of any feasible degree, a welfare maximiser always does. When the quality function is homogeneous of degree zero, the monopolist provides a socially excessive level of quality to repeat buyers. In other possible regimes, the monopolist permits too little club usage.

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1. INTRODUCTION

Club goods - e.g., transport, health, education and leisure facilities - are important and pervasive. This article studies optimum provision and pricing rules for a

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club good. The quality of the club good is increasing in the supplier's investment in the club facility and decreasing in the usage of it. We compare the provision and pricing by a monopolist and a welfare maximiser to show that the monopolist is likely to over-provide quality and allow too little use of the club relative to the welfare optimum.

The particular feature of club goods that we emphasize is the qualitative uncertainty consumers face as club goods are essentially experience goods. Ex ante, a potential club user is uncertain how agreeable she will find membership. E.g., in a leisure club, the water in the swimming pool might be too tepid or too enervating, the food be more than she can stomach, maybe cigarette smoke gets in her eye or passive smoking just gets up her nose. Again, a consumer evaluating a private school for her child could have objective information on staff-pupil ratios, its position in examination league tables and the number of sport teams it fields, yet not know if her child will thrive in the school's particular disciplinary ethos. Such customers typically have to try the good before they really knows what they buy. Yet, this qualitative uncertainty is largely ignored in the club literature[3].

The one club paper that treats qualitative uncertainty that we know, by Todd Sandler, Frederic Sterbenz and John Tschirhart (1985), studies consumers who are certain about their own membership but uncertain about the congestion they will experience on any given visit. It focuses on the relationship between risk aversion and capacity provision and not, as we do, on the endogenous determination of club membership through the provider's pricing and facility investment strategy. Also, it explores neither the market provision of the club good nor members' self-selection.

As our club good is an experience good that generates the frequency of future visits to the club by its potential members, this paper is related to the literature on experience goods and repeat buying. Jacques Crémer (1984), Julia Liebeskind and Richard Rumelt (1989), Thomas Hoerger (1993), Daniel Krähmer (2002), J. Miguel Villas-Boas (2004) and Dirk Bergemann and Juuso Välimäki (2005) ana-

[3]This literature has dealt mainly with other important issues, such as multijurisdictionality in large economies with many competing clubs, congestion externalities or tiered pricing (see, e.g., Myrna Wooders (1978, 1999), Richard Cornes and Todd Sandler (1996), Suzanne Scotchmer (1985) and Amihai Glazer, Esko Niskanen and Scotchmer (1997)).

lyze how qualitative uncertainty associated with experience goods affects buyers' learning and intertemporal pricing by an imperfectly competitive firm. But, none compares the behaviour of a monopoly supplier of the experience good with that of a benchmark supplier, such as a welfare maximizer[4] . This comparison is important for the fact that the monopolist's and welfarist's regimes differ delimits the possible configurations of their choice variables.

Crémer and Bergemann and Välimäki, like us, look specifically at the behavior of a monopolist. Crémer shows that a monopolist will not offer first time buyers an 'introductory' price but will charge a lower price to repeat buyers. Bergemann and Välimäki show that a monopolist supplying an experience good actually faces two types of markets: a mass market (where buyers are willing to buy at the full information monopoly price) and a niche market (with uninformed buyers who are not) where pricing strategies differ. In the mass market, prices decline over time whereas, in the latter, higher prices follow lower ones.

We show that, consistent with Crémer and with Bergemann and Välimäki's mass market result, the monopoly club provider will not make an "introductory offer" that allows consumers to "try before they buy," but the welfare maximizer might (*Proposition 1 and Observation 2*). More specifically, we consider the class of club quality functions that are homogeneous in the facility investment and usage of the club. We show that (*Proposition 3*), in this class, the welfare maximiser will offer a free trial period for all degrees of homogeneity that lead to feasible outcomes (which necessitates homogeneity greater than or equal to minus unity). This is a very strong result. Also, we show that, under plausible assumptions, if the degree of homogeneity exceeds minus unity, the monopolist always invests in a greater level of facility provision per use of the club than does the welfarist. In the much discussed case of a quality function homogeneous of degree zero, this translates to the monopolist over-investing in the quality provided to repeat buyers compared to the welfare maximizer (*Proposition 4*).

[4]In fact, none of these authors analyse explicitly the case of club goods with specific features such as those mentioned in the paragraph above (although Cremer (1984) briefly mentions a club as an example of an experience good).

The key to these results is a very simple observation: the monopolist wants to make profits, the welfarist to produce utility and ex post equality. The welfarist sets a low (zero) trial price to, in essence, compensate those having such a poor trial that they wish to leave the club. It might rather do that than set a higher (positive) trial price that allows either greater break-even investment in the club facility (which benefits everyone, but those who remain members more), or reduce the price paid by those who remain members, which benefits only the latter. Conversely, for any given investment in the facility, the monopolist just wants to maximise revenues. By charging all those who try the good initially, it can reduce the price for those who wish to remain consumers subsequently. It thereby perhaps induces some of those with relatively bad trial experiences that it would otherwise lose to remain.

The other papers mentioned above, by Liebeskind and Rumelt (1989), Hoerger (1993), Krähmer (2002) and Villas-Boas (2004), have a different angle from ours. Liebeskind and Rumelt and Hoerger study the effects of product quality uncertainty in the presence of adverse selection on the producers' side, while Krähmer and Villas-Boas analyze how consumers' learning in the presence of quality uncertainty impacts on the pricing strategies of oligopolists.

In section 2, we present the basis of our two-period model and analyze first-time visitors' period 2 club membership decision. We show how membership is determined endogenously, depending on the provider's price and quality strategy. We also do comparative static analysis of the sensitivity of club membership to prices and quality. In subsection 2.2, we study the monopolist's pricing and investment decisions and, in 2.3, those for the social welfare maximizer. We compare their equilibrium pricing and investment decisions in 2.4. Section 3 presents our conclusions. The Appendix contains proofs and derives one of the key equations that drive some of our main findings.

2. THE MODEL

We consider a two-period model of club membership in an economy with a single private good and a single club good ("a club" for short) with a sole supplier. The private good is essential, but not the club. There are n consumers, n being very large, who are identical ex ante and uncertain about the club's quality. People must join and experience the club to learn their evaluation of its quality, which becomes their private information. So, ex-ante homogeneous consumers become heterogenous in their valuation ex-post once they join. To find his evaluation (which occurs perfectly rather than gradually), someone must visit the club a fixed number of times (normalised at unity) in period 1, irrespective of the supplier. Given his experience, he decides whether to remain a period 2 member or to quit, and how many visits to make if he stays. Thus, part of our focus is on exit decisions.

We assume that a typical member has a strictly concave time-separable utility function with per period utility given by $U((x_i, v_i, c(\varepsilon, y, V_i))$, where x_i is his period i's consumption of the private good, $i = 1, 2$, v_i is the number of visits he makes in period i, y is the quantity of the club good (equivalently, its facility size) *that, once provided, does not depreciate in value*, V_i is the total number of visits made by all members in period i, ε is a random-valued parameter capturing the 'qualitative uncertainty' and $c(\varepsilon, y, V)$ is the *quality* or *congestion* function.

An example can clarify the three influences on c. If, say, the club is centred on a swimming pool, everyone prefers a 50m pool to a 25m one, though it costs more (a larger y). This is like vertical differentiation. But, depending on their realised ε, some swimmers might find a given pool temperature too high, some too low and others just right. This is like horizontal differentiation. Lastly, all might agree that, from their standpoint, fewer swimmers (a smaller V) are better than more - again like vertical differentiation.

Assumption A1 specifies the utility function, A2 the distribution of ε and A3 says that club quality increases in the facility size but decreases in crowding:

A1. The function $U((x_i, v_i, c(\varepsilon, y, V_i))$ is quasi-linear of the form $U(.) = u(x_i) + \varepsilon v_i C(y, V_i)$, with $u(x_i)$ being strictly concave.

A2. The parameter ε is distributed over the interval $[\underline{\varepsilon}\ ,\ \overline{\varepsilon}]$ with density function $f(\varepsilon)$ and CDF $F(\varepsilon)$. The supplier knows f and F, but not any individual's realisation of ε. Other things equal, $c(\varepsilon, y, V_i)$ increases in ε.

A3. $C_y(y, V) > 0; C_V(y, V) < 0$.

The club good is supplied by a profit-maximizing monopolist that acts as a Stackelberg leader in choosing the level of provision y and prices p_i for the periods $i = 1$, 2 at the start of period 1. We consider only linear pricing[5]. Also, we assume that the unit cost of providing the club good is constant at unity.

Let $\overline{V}$ denote the aggregate number of visits made in period 1 - i.e., $\bar{V} = n(= V_1)$. With M_i, $i = 1, 2$, being the period i income of consumers, budget constraints of a member in periods 1 and 2 are then, respectively:

$$M_1 - p_1 = x_1$$

and

$$M_2 - p_2 v_2 = x_2$$

The sequence of events is:

- *Period 1.* The leader sets y, $p_1$and p_2. People then decide to join (or not) the club and make a visit. After experiencing it, they become heterogeneously (and privately) informed about its quality, based on which they decide whether to stay in the club or to exit.

- *Period 2.* If a customer remains with the club, he then decides how many visits to make in period 2, given his private valuation of it.

As each consumer's realisation of ε is private information, the firm has no more information about period 2's demand at the start of period 2 than it did at the start of period 1. So it cannot, on that score, gain from setting p_2 at the start of period 2 rather than period 1.

[5] As first period visits are fixed, a consumer effectively has to pay a lump sum to join the club and try the club good. So, pricing has the flavour of intertemporal two-part pricing.

2.1. The members' problem in period 2.

2.1.1. The exit decision and club membership

For convenience, we denote v_2 by v and V_2 by V from now on. Suppose each member treats V (which is determined endogenously later) as parametric and chooses v to maximize period 2 utility subject to the budget constraint[6]. For a given p_2 and y, we assume that both a supplier and consumers can infer the V that will occur in an equilibrium. Additionally, given the large number of consumers, V is taken to equal its expected value (or decision makers take it as so when making their decisions). A typical member then solves the following in period 2:

$$\max_{v} \; u(M_2 - p_2 v) + \varepsilon v C(y, V)$$

The first order condition (FOC) yields:

$$-p_2 u_{x_2}(M_2 - p_2 v) + \varepsilon C(y, V) \leqq 0 \; for \; v \geqq 0 \tag{1}$$

with $-p_2 u_{x_2}(M_2) + \varepsilon C(y, V) \leqq 0$ if $v = 0$. Now, with quality taken as parametric, $-p_2 u_{x_2}(M_2) + \varepsilon C(y, V)$ is increasing in ε. Then, given a plausible assumption on C[7] and a sufficiently wide support for $f(\varepsilon)$, there exists an ε^* such that

$$-p_2 u_{x_2}(M_2) + \varepsilon C(y, V) \gtreqless 0 \;\; \text{according as } \varepsilon \gtreqless \varepsilon^*.$$

Call $\varepsilon^* \in [\underline{\varepsilon}, \overline{\varepsilon}]$ the *marginal quality valuation* - i.e., ε^* solves

$$-p_2 u_{x_2}(M_2) + \varepsilon^* C(y, V) = 0. \tag{2}$$

So, ε^* just leaves the consumer indifferent between choosing some club consumption and not. Clearly, ε^* is a function of p_2 and y (as well as other parameter values, e.g., M_2). Note that the number of visits at the marginal quality valuation is zero:

[6] Formally, this requires n to represent a continuum. None of the ensuing results change if we explicitly treat the continuum case, but the mathematical notation is much complicated.

[7] We assume $C(0, V) > 0$, $\overline{\varepsilon}$ is sufficiently large so that $\overline{\varepsilon} > p_2 u_x(M_2)/C(y, V)$ holds for all possible values of p_2 and C.and $\underline{\varepsilon}$ is sufficiently small (e.g., zero). Then ε^* is strictly between its lower and upper bounds.

$v(\varepsilon^*) = 0$. The following Lemma, proven in the Appendix, shows how period 2 club membership gets determined depending upon the realization of ε.

LEMMA 1. *(Single Crossing) Members with $\varepsilon \geqq \varepsilon^*$ remain in the club, those with $\varepsilon < \varepsilon^*$ exit.*

A member who stays in the club has visits $v = v(\varepsilon, p_2, y, V)$ solving

$$-p_2 u_{x_2}(M_2 - p_2 v(\varepsilon, p_2, y, V)) + \varepsilon C(y, V) = 0 \tag{3}$$

Thus, ex ante (when seen from period 1), for a given p_2 and y, the expected number of visits by a member is given by

$$\int_{\varepsilon^*}^{\overline{\varepsilon}} v(\varepsilon, p_2, y, V) dF(\varepsilon). \tag{4}$$

Denoting $v(\varepsilon, p_2, y, V)$ by only $v(\varepsilon)$ from now on, unless otherwise necessary, the expected number of visits in *aggregate* is therefore given by

$$V = n \int_{\varepsilon^*}^{\overline{\varepsilon}} v(\varepsilon) dF(\varepsilon) \tag{5}$$

The only technical task remaining in this subsection is to prove the existence and uniqueness of an equilibrium in expected visits for a given p_2 and y. The following Lemma is proven in the Appendix:

LEMMA 2. *For a given y and p_2, a unique equilibrium in expected period 2 visits exists.*

2.1.2. Some comparative statics

The following comparative statics for consumers' responses to magnitudes that they take as parametric are used to solve the leader's problem (see the Appendix):

LEMMA 3. *(i) $\partial V/\partial y > 0$; (ii) $\partial V/\partial p_2 < 0$; (iii) $\partial \varepsilon^*/\partial y < 0$; (iv) $\partial v(\varepsilon)/\partial \varepsilon > 0$; (v) $sign(\partial \varepsilon^*/\partial p_2) = sign(C + VC_V)$.*

Thus: (i) aggregate (and individual) visits increase with the level of facility provision; (ii) an increase in second period price reduces aggregate (and individual) visits; (iii) more people stay with the club if the level of provision increases; (iv) period 2 demand for the club good increases with the favourableness of period 1's experience; (v) a change in the second period price has an ambiguous effect on how many use the club. This is the result of independent interest. We show below that p_2 can be set at a level where a further increase would produce either a rise or no change in club membership, depending on the club provider's objective.

2.2. The monopolist's problem

The monopolist acts as a Stackelberg leader, choosing $(y,\, p_1,\, p_2)$ to maximize its profit knowing that members behave as described above. She maximizes subject to the constraint that agents join the club in the first period. We confine attention to a pure strategy equilibrium[8]. The maximization problem is:

$$\max_{p_1, p_2, y} n\left\{p_1 + \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} p_2 v(\varepsilon) dF(\varepsilon)\right\} - y$$

subject to the participation constraint (PC):

$$\begin{aligned} u(M_1 - p_1) + C(y, \overline{V})E(\varepsilon) + \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \left[u(M_2 - p_2 v(\varepsilon)) + \varepsilon v(\varepsilon) C(y, V)\right] dF(\varepsilon) \geqq \\ u(M_1) + \delta u(M_2)(1 - F(\varepsilon^*)) \end{aligned} \tag{6}$$

Here, $E(\varepsilon) = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon f(\varepsilon) d\varepsilon$ and δ is the discount factor. Letting superscript "m" show magnitudes for the monopolist, $\mathcal{L}^m$ the Lagrangian and λ^m the multiplier,

$$\begin{aligned} \mathcal{L}^m \;=\; & n\{p_1 + \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} p_2 v(\varepsilon) dF(\varepsilon)\} - y + \lambda^m [u(M_1 - p_1) + C(y, \overline{V})E(\varepsilon) \\ & + \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \left[u(M_2 - p_2 v(\varepsilon)) + \varepsilon v(\varepsilon) C(y, V)\right] dF(\varepsilon) - u(M_1) - \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} u(M_2) dF(\varepsilon^*) \end{aligned}$$

[8] Our model is a full commitment one wherein the monopolist commits deterministically to its pricing and quality strategies in period 1. We thereby rule out any "ratcheting effects" à la Laffont-Tirole - which normally give rise to mixed strategy equilibria in non-commitment games.

After simplification, the FOC's for this maximization problem are[9]:

$$\frac{\partial \mathcal{L}^m}{\partial p_1^m} = n - \lambda^m u_{x_1} \leqq 0 \quad for \quad p_1^m \geqq 0 \tag{7}$$

$$\begin{aligned}\frac{\partial \mathcal{L}^m}{\partial p_2^m} &= n\delta[\int_{\varepsilon^*}^{\overline{\varepsilon}} \left\{ v(\varepsilon) + p_2^m \frac{\partial v(\varepsilon)}{\partial p_2} \right\} dF(\varepsilon)] + \\ & \lambda^m \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \{\varepsilon v(\varepsilon) C_V \frac{\partial V}{\partial p_2} - v(\varepsilon) u_{x_2}\} dF(\varepsilon) \geqq 0 \text{ for } p_2^m \geqq 0\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{\partial \mathcal{L}^m}{\partial y^m} &= n[C_y(y, \overline{V})E(\varepsilon) + \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\{C_y + C_V \frac{\partial V}{\partial y}\} dF(\varepsilon)] + \\ & n\delta u_{x_1} \int_{\varepsilon^*}^{\overline{\varepsilon}} p_2 \frac{\partial v(\varepsilon)}{\partial y} dF(\varepsilon) \leqq u_{x_1} \quad for \quad y^m \geqq 0.\end{aligned} \tag{9}$$

From equation (7), $\lambda^m \geqq n/u_{x_1} > 0$. So, PC binds - thus (as expected), consumers get no rent in equilibrium, whether $p_1^m > 0$ or not. Second, $p_2^m > 0$. Otherwise, the demand for period 2 club visits would be infinite and this maximization would have no solution. But can p_1^m be zero? I.e., could the monopolist make an "introductory offer" (interpreted as "a free trial period") on the club good?[10] Proposition 1 (proven in the Appendix) shows the answer is no, for reasons stressed in the Introduction: the monopolist wants to extract revenue from period 1 users so it can make repeat buying more attractive in period 2.

PROPOSITION 1. *The monopolist does not make an introductory offer on the club good - i.e.,* $p_1^m > 0$.

Now, $p_2^m > 0$ implies (8) holds as an equality. Substituting $\lambda^m = n/u_{x_1}$, using (23) in the Appendix for $\partial V/\partial p_2$ and simplifying, we get (see the Appendix)

$$\left[u_{x_1} \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2 u_{xx}} dF(\varepsilon) - \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v(\varepsilon) dF(\varepsilon) \right] \left[\frac{V}{C} C_V + 1 \right] = 0 \tag{10}$$

Define the visit elasticity of quality by $\eta_v = \frac{V}{C}\frac{\partial C}{\partial V}$ (< 0, since $C_V < 0$). As

[9] The monopolist's strategy space is closed and bounded and its objective and constraint functions are continuous, so equilibria will exist and be characterised by these FOC's. It is also easy to see that these FOCs will identify an equilibrium in pure strategies. Consumers cannot deviate from it and improve welfare by not joining the club: their utility outside the club is just the reservation expected utility they get from membership. Given consumers do not deviate, the monopolist maximises profit if satisfying these FOC's.

[10] Note that $p_1^m = 0$ is conceivable, given $v_1 = 1$ is fixed.

$[u_{x_1} \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2 u_{xx}} dF(\varepsilon) - \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v(\varepsilon) dF(\varepsilon)] < 0$, we note the following about the monopolist's period 2 pricing rule:

Observation 1. *The monopolist sets* $p_2^m (> 0)$ *such that* $\mid \eta_v \mid = 1$.

$\left| \frac{V}{C} \frac{\partial C}{\partial V} \right| = 1$ is analogous to conditions found elsewhere -e.g., in the efficiency wage hypothesis. It is a marginal revenue $= 0$ condition. Having chosen y and p_1, the monopolist picks a p_2 that maximizes VC, the quality-adjusted aggregate expected visits, thereby maximizing consumers' willingness to pay for the club good.

Lastly, we cannot rule out at this stage the possibility that y^m can be zero.

2.3. A benchmark: social welfare maximisation (under an identical informational constraint).

As a benchmark, consider the club good being provided by a benevolent social welfare maximizer. Like the monopolist, she also knows members' behaviour, as described in sections 2.1.1 and 2.1.2, but cannot observe agents' ex-post valuation of the good. (So, she cannot engage in discriminatory pricing ex post.) She uses this information while solving the following social welfare maximization problem:

$$\begin{aligned} \max_{p_1, p_2, y} \quad & n[u(M_1 - p_1) + C(y, \overline{V}) E(\varepsilon) + \\ & \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \{u(M_2 - p_2 v(\varepsilon)) + \varepsilon v(\varepsilon) C(y, V)\} dF(\varepsilon) + \delta \int_{\underline{\varepsilon}}^{\varepsilon^*} u(M_2) dF(\varepsilon)] \end{aligned}$$

subject to

$$np_1 + n\delta \int_{\varepsilon^*}^{\overline{\varepsilon}} p_2 v(\varepsilon) dF(\varepsilon) \geqslant y \tag{11}$$

Here (11) is the constraint that the expected revenue raised from the club good must cover its provision cost. We can now reasonably ignore the participation constraint (6) on the following ground. If it binds with a profit maximizer making positive profits, it will certainly be slack with a welfarist that just breaks even and leaves some surplus with consumers. The optimal values of the choice variables ε^*, p_2, etc., here will therefore generally differ from the corresponding values in the monopolist's problem. Let superscript "s" denote magnitudes in the welfarist's regime.

With $\mathcal{L}^s$ the Lagrangian for the welfarist's optimization, the FOC are[11]:

$$\frac{\partial \mathcal{L}^s}{\partial p_1^s} = -\, u_{x_1}^s + \lambda^s \leqq 0 \ \text{ for } p_1^s \geqq 0 \tag{12}$$

$$\begin{aligned} \frac{\partial \mathcal{L}^s}{\partial p_2^s} &= n\delta[\int_{\varepsilon^*}^{\overline{\varepsilon}} \{-v(\varepsilon)u_{x_2} + \varepsilon v(\varepsilon) C_V \frac{\partial V}{\partial p_2^s}\} dF(\varepsilon) + \\ &\lambda^s \int_{\varepsilon^*}^{\overline{\varepsilon}} \{v(\varepsilon) + p_2^s \frac{\partial v(\varepsilon)}{\partial p_2^s}\} dF(\varepsilon)] \leqq 0 \text{ for } p_2^s \geqq 0 \end{aligned} \tag{13}$$

$$\begin{aligned} \frac{\partial \mathcal{L}^s}{\partial y^s} &= n[C_y E(\varepsilon) + \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon) \{C_y + C_V \frac{\partial V}{\partial y^s}\} dF(\varepsilon)] + \\ &\lambda^s n\delta \int_{\varepsilon^*}^{\overline{\varepsilon}} p_2^s \frac{\partial v(\varepsilon)}{\partial y^s} dF(\varepsilon) - \lambda^s \leqq 0 \text{ for } y^s \geqq 0 \end{aligned} \tag{14}$$

By the same argument as with monopoly, $p_2^s > 0$ in equilibrium. So, (13) holds with equality. Second (after substituting for $\partial V/\partial y$), as

$$\{C_y + C_V \frac{\partial V}{\partial y}\} = \frac{C_y}{1 + C_V n \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_{xx}} dF(\varepsilon)} > 0 \tag{15}$$

(14) indicates $\lambda^s > 0$ - i.e., the revenue constraint binds[12]. This, with $p_2^s > 0$, implies $y^s > 0$ at the welfarist's optimum. So, (14) holds with strict equality. Thus, members receive some rents at the welfarist's optimum (as opposed to in monopoly). But, as shown formally below with homogeneous $C\,(.)$ (in Proposition 3), even with $\lambda^s > 0$, (12) can have a corner solution. So we have

Observation 2. *The welfarist could make an introductory offer on the club good (i.e., $p_1^s = 0$).*

[11] As with monopoly (see note 9), the welfarist's strategy space is compact and its objective and constraint functions are continuous. So, equilibria exist and satisfy these FOC. Now, consumers get positive expected surplus in the welfarist equilibrium. They cannot deviate and improve their welfare by not joining the club. This just yields their reservation expected utility. Given consumers do not deviate, the welfarist cannot do better than satisfy these FOC.

[12] To show that (14) indicates $\lambda^s > 0$, suppose not, thus $\lambda^s = 0$ by Kuhn-Tucker theory. Then (14) collapses to $n[\overline{v} C_y E(\varepsilon) + \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon) \{C_y + C_V \frac{\partial V}{\partial y^s}\} dF(\varepsilon)] \leqq 0$. This cannot be, given $C_y + C_V \frac{\partial V}{\partial y^s} > 0$ and $C_y > 0$.

Observation 2 helps us to prove the following in the Appendix:

PROPOSITION 2. *If the welfarist makes an introductory offer, then she also sets* $p_2^s(>0)$ *so that* $|\ \eta_v\ |> 1$ *holds. Further, there is 'overprovision' of the good in the Samuelson rule sense that willingness to pay for the marginal investment in the club facility is less than its cost.*

It is also worth noting that (13) holding with equality can be rearranged to

$$n\delta[-\{\frac{V}{C}C_V+1\}\int_{\varepsilon^*}^{\overline{\varepsilon}}\varepsilon v(\varepsilon)dF(\varepsilon)+\lambda^s\int_{\varepsilon^*}^{\overline{\varepsilon}}\{v(\varepsilon)+p_2^s\frac{\partial v(\varepsilon)}{\partial p_2^s}\}dF(\varepsilon)]=0 \qquad (16)$$

So, as $VC_V+C<0$ at the welfarist's optimum when it makes an introductory offer, it follows that $\int_{\varepsilon^*}^{\overline{\varepsilon}}\{v(\varepsilon)+p_2^s\frac{\partial v(\varepsilon)}{\partial p_2^s}\}dF(\varepsilon)<0$. I.e., other things equal, the welfarist could increase its expected revenue by lowering p_2. The rationale is simple: if $VC_V+C<0 \Longleftrightarrow |\ \eta_v\ |>1$, quality is very sensitive to visits at the welfarist's optimum and it will wish to discourage visits, other things equal. It can do so by raising p_2^s to above the profit-maximizing level, given its choice of y^s and p_1^s.

2.4. Monopolist's versus welfarist's equilibrium

At the monopoly equilibrium $p_1^m>0$ and $p_2^m>0$, although $y^m=0$ is possible; at the social optimum $p_2^s>0$ and $y^s>0$, while $p_1^s=0$ is possible. Also, all the choice variables cannot *simultaneously* be positive for both the monopolist and the welfarist.[13] Thus, there are only three possible ways in which the monopolist's equilibrium can differ from the social optimum:

1.*Regime (a).* $p_1^m>0$, $p_2^m>0, y^m>0$; $p_1^s=0$, $p_2^s>0, y^s>0$;

2. *Regime (b).* $p_1^m>0$, $p_2^m>0, y^m=0$; $p_1^s=0, p_2^s>0, y^s>0$; and

3. *Regime (c).* $p_1^m>0$, $p_2^m>0, y^m=0$; $p_1^s>0$, $p_2^s>0, y^s>0$.

2.5. The characterisation of different regimes

This section explores which one(s) of the above regimes is (are) likely to occur and their characteristics. We first study cases when the quality function, $C(y,V)$,

[13]Since that makes their first order conditions exactly identical, which cannot be possible given that the monopolist maximises profit while the welfarist breaks even!

is homogeneous[14]. Our general result (proven in the Appendix) is the following:

PROPOSITION 3. *Suppose that the quality function $C(y, V)$ is homogeneous of degree k. Then: (i) regime (c) cannot occur for any k; (ii) regime (b) occurs if and only if $k = -1$; (iii) only regime (a) can occur for all k satisfying $k + 1 > 0$.*

Although $C(y, V)$ might not be homogeneous, homogeneity is a convenient simplification for visualising the consequences of different extents of qualitative returns to scale. An implication of Proposition 3 is that, with sufficiently large qualitative scale diseconomies, the monopolist will find it suboptimal to invest in the club facility. E.g., if $k = -1$, doubling y and V keeps the facility provision per use of the club constant but halves the quality as perceived by its customers, simply because crowding per se causes them such detriment. The monopolist would then find it more profitable to not spend on the facility and keep visits low if it wishes to maintain quality. But, an even more striking and important implication of Proposition 3 is: the welfarist will *always* offer a free trial period for all degrees of homogeneity of $C(.)$ that lead to a feasible solution (which the Appendix shows requires $k+1 \geq 0$). This behaviour contrasts starkly with the monopolist's, which (from Proposition 1) *never* offers a free trial period whether or not $C(.)$ is homogeneous.

In the context of Proposition 2, Proposition 3 means that the welfarist oversupplies the club facility in the Samuelson rule sense for all feasible k. Conversely, in regime (a), as $y^m > 0$, (9) means that the monopolist's provision satisfies Samuelson's rule. If $k = -1$, regime (b) holds. Then, (9) indicates that, generically, the monopoly overprovides under this rule, although $y^m = 0$. This seems paradoxical. But, it just implies that any facility provision by the monopolist would be socially excessive, given the configuration of its other choice variables. It also highlights the well-known fact that Samuelson's rule need not have a straightforward implication in terms of levels of provision of a shared good.

In the arbitrary k-degree homogeneous case, the quality function satisfies $C(y.V) = V^k c(y/V)$ for some function $c(.)$. It is easy to show that the monopolist then always

[14] Robert Barro and Paul Romer (1987), Clive Fraser (2000) and Serge-Christophe Kolm (1974), among others, study some of the implications of homogeneous club quality or congestion functions.

wishes to offer a higher level of facility provision per visit than does the welfarist if the facility provision elasticity of quality, $(y/V)\, c^{/}\,(y/V)\,/c\,(y/V)$, is monotonic in the facility provision per visit, $z \equiv y/V$. First, we show in the next Lemma (proven in the Appendix) that $zc^{/}\,(z)\,/c\,(z)$ is decreasing in z at both the monopolist's and welfarist's equilibrium. Hence, if it is monotonic, it must be decreasing everywhere.

LEMMA 4. *If there are diminishing returns to an investment in the facility provision (i.e., $c^{//} < 0$) and the facility provision elasticity of quality, $(y/V)\, c^{/}\,(y/V)\,/$ $c\,(y/V)$, is monotonic in $z \equiv y/V$, then it is decreasing everywhere.*

If the conditions of Lemma 4 are satisfied, the monopolist will always invest in a greater level of facility provision per visit than the welfarist: as their equilibria satisfy $z^s c^{/}(z^s)/c(z^s) > k + 1 = z^m c^{/}(z^m)/c(z^m)$, we must have $z^m > z^s$.

When $C\,(.)$ is homogeneous of degree zero ("h.o.d.0") is much discussed. Then quality just depends on the facility investment per use of the club. E.g., patients at a health clinic might find the quality of care depends on the average time doctors spend with each patient and average drug and equipment spending per treatment, or swimmers might think the quality of a swimming pool is determined by the average area each swimmer has to herself, and the construction cost per square metre of pool is constant[15]. In the h.o.d.0. case, $C(y, V) = c(y/V)$, for some function $c\,(.)$,with $c^{/}\,(y/V) > 0$. The following proposition is an immediate implication of Lemma 4 and the fact that $z^m > z^s$ if the conditions of this Lemma are satisfied.

PROPOSITION 4. *If $C\,(.)$ is h.o.d.0. and the elasticity of quality w.r.t. facility provision is monotonic, then $C\,(y^m, V^m) = c\,(y^m/V^m) > c\,(y^s/V^s) = C\,(y^s, V^s)$: the monopolist invests in socially excessive quality provision for period 2.*

The rationale for this result is that the monopolist both wishes to extract rent from those who try in period 1 but not buy in period 2 (hence it sets $p_1^m > 0$) and to provide an incentive for many period 1 tryers to remain period 2 buyers. It can do this by ensuring a high period 2 quality, which relaxes the participation constraint. The welfarist, conversely, is concerned about equity as well as efficiency.

[15] Fraser (2000) and Kolm (1974) study implications of an h.o.d.0. $C\,(y, V)$ for club theory.

It is interested in equalizing the actual utility of stayers and leavers as nearly as possible. It prefers, therefore, to not charge in period 1, though this means relatively less funds are available for facility provision to enhance period 2 quality.

By pricing in this way, the welfarist essentially operates a limited system of random redistributive taxation. Only consumers with sufficiently good period 1 experiences are taxed to pay for the club good. Their tax increases with the favourableness of their experience as their club demand increases in ε. Indeed, were transfers possible, the welfarist might wish to make ex post equalising transfers to those who choose to not use the club in period 2 due to their bad period 1 experiences. It is limited for, by assumption, transfers are impossible and setting $p_1^s = 0$ is the best it can do.

Surprisingly, this scenario is reminiscent of the literature on monopoly pricing under asymmetric information where high prices signal high-quality product quality (e.g., cf. Paul Milgrom and John Roberts 1986, Kyle Bagwell and Michael Riordan 1991, Kenneth Judd and Riordan 1994). To signal the product quality, the monopolist may charge a price well above the full information profit maximizing one. Ours is not a signalling model, yet it can have an observationally equivalent implication. When the quality of the club good is yet to be learnt by visitors, the monopolist credibly provides a higher quality club good than the welfarist would if it charges a higher first period price than does the latter (i.e., $p_1^m > p_1^s = 0$).

An Example. Suppose $C(y,V) = [(y/V)+\gamma]^{\vartheta}$, for some scalars $\gamma < 0$ and $\vartheta \in (0,1)$. Then, it is easy to show[16] that $y^m/V^m = \frac{\gamma}{\vartheta-1} > y^s/V^s$, hence $C(y^m,V^m) = [(y^m/V^m)+\gamma]^{\vartheta} > C(y^s,V^s) = [(y^s/V^s)+\gamma]^{\vartheta}$.

From Proposition 3, we know that homogeneity of $C(.)$ severely restricts the possibility of regimes (b) and (c). So, we will now suppose that $C(y,V)$ is not homogeneous and that these regimes are possible. What might the characteristics of these regimes be? We will make the following reasonable assumption:

[16] From (10), $-(y^m/V^m)[\gamma+(y^m/V^m)]^{\vartheta-1}\vartheta + [\gamma+(y^m/V^m)]^{\vartheta} = 0 \iff -(y^m/V^m)\vartheta + [\gamma+(y^m/V^m)] = 0 \iff (y^m/V^m)(1-\vartheta)+\gamma = 0 \iff y^m/V^m = \frac{\gamma}{\vartheta-1}$. Likewise, from Proposition 2, $-(y^s/V^s)[\gamma+(y^s/V^s)]^{\vartheta-1}\vartheta + [\gamma+(y^s/V^s)]^{\vartheta} < 0$, which simplifies to $y^s/V^s < \frac{\gamma}{\vartheta-1}$.

A4. $C_{VV} \leqq 0$ (increasing marginal disutility of congestion); $C_{Vy} \geqq 0$ (increased facility provision ameliorates the negative impact of increased club usage).[17]

In comparing monopoly and welfarist regimes now, the visit elasticity of quality plays the same pivotal role as in the homogeneous case (cf. the proofs of Propositions 3-4 and Lemma 4). In regime (b), the inequality $\mid \eta_v^s \mid > \mid \eta_v^m \mid = 1$ holds as $\mid \eta_v^m \mid = 1$ by Observation 1 and $\mid \eta_v^s \mid > 1$ by Proposition 2. Conversely, under regime (c), $\mid \eta_v^s \mid = 1$ (combining equations (12) and (13) when $p_1^s > 0$) - i.e., under regime (c), $\mid \eta_v^s \mid = \mid \eta_v^m \mid = 1$ holds. We can use this, together with the properties of η_v when C is non-homogeneous, to show that monopoly will plausibly result in less period 2 use of the club than is socially optimal in regimes (b) and (c).

To see how the elasticity $\eta_v \equiv VC_V(y, V) / C(y, V)$ behaves in response to changes in y and V, we can totally differentiate and rearrange to obtain

$$d\eta_v = C^{-2} \left[\left(CC_V + CVC_{VV} - VC_V^2 \right) dV + \left(CVC_{Vy} - VC_V C_y \right) dy \right] \tag{17}$$

Given our assumptions, $\left(CC_V + CVC_{VV} - VC_V^2 \right) < 0$ and $\left(CVC_{Vy} - VC_V C_y \right) > 0$. Thus, other things equal, an increase in V will decrease η_v (make it more negative), while increasing y will increase it. In both these regimes $y^m = 0$. So, to compare the monopolist and the welfarist's behavior in them, we can let $dy = y^s > 0 = y^m$. Then, to satisfy $V^m C_V(0, V^m) / C(0, V^m) = -1 \geq V^s C_V(y^s, V^s) / C(y^s, V^s)$ and (17), we must have $V^s > V^m$. This establishes the following:

Proposition 5. *Under regimes (b) and (c) and A4, the aggregate second period visits to the club under monopoly are less than the socially optimal level:* $V^m < V^s$.

Note that $C_{Vy} \geqq 0$ in A.4 can not hold and yet we get $V^s > V^m$ in regimes (b) and (c). E.g., if $C(y, V) = h(y) / g(V)$ for some positive increasing functions h and g, then $C_{Vy} = -h^{/}(y) g^{/}(V) / g(V)^2 < 0$. Yet, direct calculation shows that $CVC_{Vy} - VC_V C_y = 0$ in this case and, so, we must have $V^s > V^m$ as before.

We cannot compare period 2 quality levels in regimes (b) and (c) because,

[17] We also assume $C(0, V) > 0$. If not, regimes (b)-(c) could not occur as the monopolist would not get any period 2 customers and the participation constraint could not be met if $y^m = 0$.

although $y^s > 0 = y^m$, $V^s > V^m$ might still mean $C(0, V^m) > C(y^s, V^s)$ occurs. But, as total period 1 visits $(\overline{V})$ are the same under monopoly and welfarism, $C\left(y^s, \overline{V}\right) > C\left(0, \overline{V}\right)$ holds: the welfarist offers a higher period 1 quality than the monopolist in these regimes. This is consistent with the suggestion that, compared with the welfarist, the monopolist is more focused on treating retained customers well, even if at the expense of disappointed first period customers. These arguments suggest that the monopolist could offer a higher quality to repeat customers, yet a lower quality to first-time and once-only customers, than does the welfarist. So, unlike in a single-period model, we cannot say unambiguously that the monopolist will over- or under-supply quality.

3. CONCLUSIONS

We have examined the pricing and investment strategies of a club good provider when potential club users are uncertain about the quality of the shared facility. Essentially, club goods are experience goods: " you have to try before you know you want to buy." Yet this aspect of clubs has not been studied in the literature. Incorporating this feature is important as it can give rise to very contrasting strategies for a monopoly profit maximizer and a welfare maximizer, as we have shown.

In our model, potential members are unsure of the quality of a club's facilities beforehand. They must make a fixed number of visits in period 1 to ascertain their evaluation of the quality, which they then learn perfectly. Based on this learning experience, they then decide whether to continue their membership and the number of visits to make, or to leave the club for good. Pricing strategies announced in period 1 and the investment the provider undertakes to maintain the shared facilities are therefore crucial in determining the club's ultimate membership.

In this scenario, one might expect a provider to offer an introductory discount to consumers who have no prior knowledge of the quality of the good they are about to experience. But, we show that is not necessarily so - it depends upon who the provider is. If it is a social welfare maximizer, she might indeed give consumers an "introductory offer" of a free trial period in which to decide whether it is agreeable to them - and definitely does so if the quality function is homogeneous. She does this to reduce the disparity in welfare between those who try the product, find it unsatisfactory and therefore leave the club, and those who find it satisfactory and wish to continue as consumers. In the extreme, only the latter pay for providing the club facility. Conversely, if the provider is a monopolist, her focus is on extracting as much rent as possible from consumers. As a result, the monopolist never makes an introductory offer. Thus all consumers, whether stayers or leavers, contribute to any cost of facility provision and to profits. This enables the monopolist to increase (in some cases) the size of the club facility (thereby its quality), therefore increasing the incentive for consumers to remain with it.

The latter results about the monopoly provider are consistent with those from models of monopoly pricing with experience goods and repeat purchases (such as Crémer (1984) and Bergemann and Välimäki (2005)), and also with ones that establish a signalling role about the future quality of the product played by today's price in a dynamic setting (such as Bagwell and Riordan (1991), Milgrom and Roberts (1986) and Judd and Riordan (1994)). However, none of these other papers have considered explicitly, as we have done, the implications of the peculiar features of clubs - such as the congestion externality and the endogenous determination of quality arising both from the utilization choice of members and the entrepreneurial club provider's pricing strategy and level of facility provision.

It is worth stressing again, finally, that archetypal clubs like leisure facilities are not the only ones with characteristics that might be unverifiable prior to use. For example, the ethos of a school and its teachers' dedication can make a difference to its quality, whatever the resources spent on books and other equipment. Simultaneously, different consumers of the same services might take contrasting stances on the balance between concentration on the "3 Rs" and, say, pastoral care at a chosen establishment. In the same vein, many welfare states try to ensure equality of opportunity to ex ante identically treated individuals by providing a fixed amount of primary and secondary education free at the point of delivery. Only those consumers who reveal a preference or a particular aptitude for education have to pay for additional amounts in the tertiary system. The predictions of our welfarist analysis in the homogeneous case mimics this scenario. Our model therefore provides a rationalisation for why we observe partial tax financing of such goods and partial financing by user charges, a rationalisation different from that based on ex ante differences between consumers.

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APPENDIX

Proof of Lemma 1. The marginal quality valuation, ε^*, satisfies $-p_2u_{x_2}(M_2)+\varepsilon^*C(y,V)=0$ ((2) in the text). Given p_2 and C, $-p_2u_{x_2}(M_2)+\varepsilon C(y,V)$ is increasing in ε and equals zero at $\varepsilon=\varepsilon^*$. Hence, for $\varepsilon>\varepsilon^*$, $-p_2u_{x_2}(M_2-p_2v)+\varepsilon C(y,V)=$

0 can be satisfied for some $v > 0$. But this implies that members having $\varepsilon \geqq \varepsilon^*$ remain in the club and make positive visits (the marginal member "remains" in the club but makes zero visit). Obviously, for $\varepsilon < \varepsilon^*$, members make zero visits and exit the club as $-p_2 u_{x_2}(M_2) + \varepsilon C(y, V) < 0$.■

Proof of Lemma 2. For a given p_2, y and V, the club usage choice of someone with experience ε is a continuous and differentiable mapping $v(\varepsilon, p_2, y, V)$: $[0, M_2/p_2] \to [0, M_2/p_2]$ satisfying (3).The ex ante expected visits for this consumer satisfy (4) and those for all consumers must satisfy $V = n\int_{\varepsilon^*}^{\overline{\varepsilon}} v(\varepsilon, p_2, y, V)dF(\varepsilon)$ uniquely if a unique equilibrium exists. Define the aggregate expected visit mapping $V(p_2, y, V)$ by $V(p_2, y, V) = n\int_{\varepsilon^*}^{\overline{\varepsilon}} v(\varepsilon, p_2, y, V)dF(\varepsilon) : [0, nM_2/p_2] \to [0, nM_2/p_2]$. This mapping is also continuous and differentiable. By differentiating, $n\partial\left[\int_{\varepsilon^*}^{\overline{\varepsilon}} v(\varepsilon, p_2, y, V)dF(\varepsilon)\right]/\partial V = -n\left(C_V(y, V)/C(y, V)\right)\int_{\varepsilon^*}^{\overline{\varepsilon}} p_2\frac{u_{x2}}{u_{xx}}dF(\varepsilon) < 0$, using (3) and Leibnitz's rule. So, $V(p_2, y, V)$ is monotonically decreasing in V and takes it maximum value at $V(p_2, y, 0)$, where $nM_2/p_2 > V(p_2, y, 0) > 0$, with the first inequality following from the fact that the private good is essential. As $nM_2/p_2 > V(p_2, y, 0) > V(p_2, y, nM_2/p_2)$, the graph of $V(p_2, y, V)$ against V must cross the 45^0 line uniquely from above at a point where $V(p_2, y, V) = V$. Thus, a unique equilibrium in expected visits exists for a given p_2 and y. ■

Proof of Lemma 3. (i) Differentiation of (5) with respect to y, (using Leibnitz's rule) yields

$$\frac{\partial V}{\partial y} = n[\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\partial v(\varepsilon)}{\partial y}dF(\varepsilon) - v(\varepsilon^*)\frac{\partial \varepsilon^*}{\partial y}] = n\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\partial v(\varepsilon)}{\partial y}dF(\varepsilon) \tag{18}$$

since $v(\varepsilon^*) = 0$. Differentiating (3) with respect to v and y and integrating over ε, we obtain

$$\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\partial v(\varepsilon)}{\partial y}dF(\varepsilon) = -[C_y + C_V\frac{\partial V}{\partial y}]\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_{xx}}dF(\varepsilon) \tag{19}$$

where u_{xx} is the second derivative with respect to x_2. Hence

$$\frac{\partial V}{\partial y} = -n\left\{C_y\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_{xx}}dF(\varepsilon)\right\} / \left\{1 + nC_V\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_{xx}}dF(\varepsilon)\right\} > 0. \tag{20}$$

So, as the level of provision increases, both individual and aggregate visits increase.

(ii) Differentiating (5) with respect to p_2, we find

$$\frac{\partial V}{\partial p_2} = n[\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\partial v(\varepsilon)}{\partial p_2} dF(\varepsilon) - v(\varepsilon^*)\frac{\partial \varepsilon^*}{\partial p_2}] = n\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\partial v(\varepsilon)}{\partial p_2} dF(\varepsilon) \tag{21}$$

Differentiation of (3) with respect to v and p_2 yields

$$\frac{\partial v(\varepsilon)}{\partial p_2} = \frac{\left\{-\varepsilon C_V \frac{\partial V}{\partial p_2} - u_{xx} p_2 v(\varepsilon) + u_x\right\}}{p_2^2 u_{xx}} \tag{22}$$

Thus, integrating and rearranging to isolate $\partial V/\partial p_2$,

$$\frac{\partial V}{\partial p_2} = -n\int_{\varepsilon^*}^{\overline{\varepsilon}} \left[\frac{v(\varepsilon)}{p_2} - \frac{u_x}{p_2^2 u_{xx}}\right] dF(\varepsilon) / \left[1 + nC_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_{xx}} dF(\varepsilon)\right] < 0 \tag{23}$$

(iii) Differentiating (2) w.r.t. y, using (20) and rearranging yields

$$\frac{\partial \varepsilon^*}{\partial y} = -C_y \varepsilon^* / C\left[1 + nC_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_{xx}} dF(\varepsilon)\right] < 0. \tag{24}$$

(iv) By (3) in the text, $p_2^2 u_{xx} \partial v(\varepsilon)/\partial\varepsilon + C = 0$, so $\partial v(\varepsilon)/\partial\varepsilon = -C/p_2^2 u_{xx} > 0$.

(v) From the condition defining the marginal quality valuation, using (23),

$$\begin{aligned}
\frac{\partial \varepsilon^*}{\partial p_2} &= \left\{u_x(M_2) - \varepsilon^* C_V \frac{\partial V}{\partial p_2}\right\}\frac{1}{C} \\
&= \left\{C + C_V n\int_{\varepsilon^*}^{\overline{\varepsilon}} v(\varepsilon) dF(\varepsilon)\right\}\left[1 + nC_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_{xx}} dF(\varepsilon)\right]^{-1} \frac{\varepsilon^*}{p_2} \\
\therefore\ \ & sign\left(\frac{\partial \varepsilon^*}{\partial p_2}\right) = sign\left\{C + C_V V\right\} \lesseqqgtr 0
\end{aligned} \tag{25}$$

Proof of Proposition 1. As the participation constraint binds in equilibrium (whether or not $p_1^m > 0$), rewrite (6) as

$$\begin{aligned}
& \delta\int_{\varepsilon^*}^{\overline{\varepsilon}} \left[u(M_2 - p_2 v(\varepsilon)) + \varepsilon v(\varepsilon) C(y, V) - u(M_2)\right] dF(\varepsilon) \\
= \ & u(M_1) - U(M_1 - p_1) - C(y, \overline{V}) E(\varepsilon)
\end{aligned}$$

If $p_1^m = 0$, then the RHS will be strictly negative while the LHS will be strictly positive, given that for $\varepsilon > \varepsilon^*$ the person get more utility in the club than out. This would violate the participation constraint. Hence $p_1^m > 0$.■

Derivation of equation (10). From (8) in the text, substituting $\lambda = n/u_{x_1}$,

$$n\delta u_{x_1}\left[\int_{\varepsilon^*}^{\overline{\varepsilon}}\{v(\varepsilon)+p_2^m\frac{\partial v(\varepsilon)}{\partial p_2^m}\}dF(\varepsilon)\right]+n\delta\left[\int_{\varepsilon^*}^{\overline{\varepsilon}}\{-v(\varepsilon)u_{x_2}+\varepsilon v(\varepsilon)C_V\frac{\partial V}{\partial p_2^m}\}dF(\varepsilon)\right]=0$$

Rearranging and cancelling $n\delta > 0$, this becomes

$$\int_{\varepsilon^*}^{\overline{\varepsilon}}(u_{x_1}-u_{x_2})\,v\left(\varepsilon\right)dF(\varepsilon)+u_{x_1}p_2^m\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{\partial v(\varepsilon)}{\partial p_2^m}dF(\varepsilon)+C_V\frac{\partial V}{\partial p_2^m}\int_{\varepsilon^*}^{\overline{\varepsilon}}\varepsilon v\left(\varepsilon\right)dF(\varepsilon)=0.$$

As $\frac{\partial V}{\partial p_2} = \frac{-n\int_{\varepsilon^*}^{\overline{\varepsilon}}\left[\frac{v(\varepsilon)}{p_2}-\frac{u_x}{p_2^2u_{xx}}\right]dF(\varepsilon)}{\left[1+nC_V\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{\varepsilon}{p_2^2u_{xx}}dF(\varepsilon)\right]} \equiv -n\int_{\varepsilon^*}^{\overline{\varepsilon}}\left[\frac{v(\varepsilon)}{p_2}-\frac{u_x}{p_2^2u_{xx}}\right]dF(\varepsilon)/D$ from (23), the last equation can be written as

$$\begin{aligned}&\int_{\varepsilon^*}^{\overline{\varepsilon}}(u_{x_1}-u_{x_2})\,v\left(\varepsilon\right)dF(\varepsilon)-\\&\int_{\varepsilon^*}^{\overline{\varepsilon}}\left[\frac{v(\varepsilon)}{p_2}-\frac{u_x}{p_2^2u_{xx}}\right]dF(\varepsilon)\left\{u_{x_1}p_2^m+nC_V\int_{\varepsilon^*}^{\overline{\varepsilon}}\varepsilon v\left(\varepsilon\right)dF(\varepsilon)\right\}/D=0\end{aligned}$$

Or

$$\begin{aligned}&\int_{\varepsilon^*}^{\overline{\varepsilon}}(u_{x_1}-u_{x_2})\,v\left(\varepsilon\right)dF(\varepsilon)\left[1+nC_V\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{\varepsilon}{p_2^{m2}u_{xx}}dF(\varepsilon)\right]-\\&\int_{\varepsilon^*}^{\overline{\varepsilon}}\left[\frac{v(\varepsilon)}{p_2^m}-\frac{u_x}{p_2^{m2}u_{xx}}\right]dF(\varepsilon)\left\{u_{x_1}p_2^m+nC_V\int_{\varepsilon^*}^{\overline{\varepsilon}}\varepsilon v\left(\varepsilon\right)dF(\varepsilon)\right\}=0\end{aligned}$$

Now, from the first and second terms,

$$\begin{aligned}&\int_{\varepsilon^*}^{\overline{\varepsilon}}(u_{x_1}-u_{x_2})\,v\left(\varepsilon\right)dF(\varepsilon)-u_{x_1}p_2^m\int_{\varepsilon^*}^{\overline{\varepsilon}}\left[\frac{v(\varepsilon)}{p_2^m}-\frac{u_x}{p_2^{m2}u_{xx}}\right]dF(\varepsilon)\\=&\int_{\varepsilon^*}^{\overline{\varepsilon}}\left[u_{x_1}\frac{u_{x_2}}{p_2^mu_{xx}}-u_{x_2}v\left(\varepsilon\right)\right]dF(\varepsilon)\equiv(A).\end{aligned}$$

From the terms in C_V, we have

$$\begin{aligned}
& -\int_{\varepsilon^*}^{\overline{\varepsilon}} \left(\frac{\varepsilon}{p_2^{m2} u_{xx}}\right) dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v\left(\varepsilon\right) dF(\varepsilon) + \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2^m u_{xx}} dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v\left(\varepsilon\right) dF(\varepsilon) \\
= \;& -\int_{\varepsilon^*}^{\overline{\varepsilon}} \left(\frac{\varepsilon}{p_2^{m2} u_{xx}}\right) dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v\left(\varepsilon\right) dF(\varepsilon) + \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2^m u_{xx}} dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v\left(\varepsilon\right) \frac{p_2^m}{C} dF(\varepsilon)
\end{aligned}$$

= (using $\varepsilon = p_2^m u_{x_2}/C$ from the FOC for an agent with period 1 experience ε)

$$\begin{aligned}
& -\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{p_2^m}{C} \left(\frac{u_{x_2}}{p_2^{m2} u_{xx}}\right) dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v\left(\varepsilon\right) dF(\varepsilon) \\
& + \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2^m u_{xx}} dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v\left(\varepsilon\right) \frac{p_2^m}{C} dF(\varepsilon) = 0
\end{aligned}$$

The residual terms in C_V equal

$$nC_V \left[\int_{\varepsilon^*}^{\overline{\varepsilon}} \left(\frac{\varepsilon}{p_2^{m2} u_{xx}}\right) dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_1} v\left(\varepsilon\right) dF(\varepsilon) - \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{v(\varepsilon)}{p_2^m} dF\left(\varepsilon\right) \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v\left(\varepsilon\right) dF(\varepsilon)\right] \equiv (B).$$

Amalgamating (A) and (B),

$$\begin{aligned}
& u_{x_1} \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2^m u_{xx}} dF\left(\varepsilon\right) + u_{x_1} nC_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \left(\frac{\varepsilon}{p_2^{m2} u_{xx}}\right) dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} v\left(\varepsilon\right) dF(\varepsilon) \\
= \;& u_{x_1} \left[\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2^m u_{xx}} dF\left(\varepsilon\right) + C_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \left(\frac{\varepsilon}{p_2^{m2} u_{xx}}\right) dF(\varepsilon) V\right] \\
= \;& \left[u_{x_1} \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2^m u_{xx}} dF\left(\varepsilon\right)\right] \left[1 + \frac{C_V V}{C}\right] \quad \text{(using } \varepsilon = p_2^m u_{x_2}/C \text{ again)}
\end{aligned}$$

Also,

$$\begin{aligned}
& -\int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v\left(\varepsilon\right) dF\left(\varepsilon\right) - nC_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{v(\varepsilon)}{p_2^m} dF\left(\varepsilon\right) \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v\left(\varepsilon\right) dF(\varepsilon) \\
= \;& -\int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v\left(\varepsilon\right) dF\left(\varepsilon\right) \left[1 + \frac{C_V V}{C}\right] \text{ (again using } \varepsilon = p_2^m u_{x_2}/C\text{)}
\end{aligned}$$

So, resubstituting, the FOC (8) becomes

$$\left[u_{x_1} \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2^m u_{xx}} dF\left(\varepsilon\right) - \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v\left(\varepsilon\right) dF\left(\varepsilon\right)\right] \left[1 + \frac{C_V V}{C}\right] = 0$$

which is (10) in the text.

Proof of Proposition 2. (i) First, we show that if $p_1^s = 0$ then $u_{x_1}^s > \lambda^s$. Suppose otherwise, so $p_1^s = 0$ yet $u_{x_1}^s = \lambda^s$. Suppose the welfarist were then to increase p_1^s to $p_1^s = \varepsilon > 0$, for some very small ε. To first-order, the loss of welfare in first period utility is exactly counter-balanced by the value of extra funds, λ^s. Thus the welfarist could equally well set $p_1^s = \varepsilon > 0$, contradicting the unique optimality of $p_1^s = 0$. Next, note from (13), since $u_{x_1}^s > \lambda^s > 0$,

$$n\delta[\int_{\varepsilon^*}^{\overline{\varepsilon}} \{-v(\varepsilon)u_{x_2} + \varepsilon v(\varepsilon)C_V \frac{\partial V}{\partial p_{2^s}}\}dF(\varepsilon) + u_{x_1}^s \int_{\varepsilon^*}^{\overline{\varepsilon}} \{v(\varepsilon) + p_2^s \frac{\partial v(\varepsilon)}{\partial p_2^s}\}dF(\varepsilon)] > 0$$

which, after simplification (similar to the derivation of (10) above), yields

$$\left[u_{x_1} \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2 u_{xx}} dF(\varepsilon) - \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v(\varepsilon) dF(\varepsilon)\right] \left[\frac{V}{C} C_V + 1\right] > 0$$

Since $\left[u_{x_1} \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2 u_{xx}} dF(\varepsilon) - \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v(\varepsilon) dF(\varepsilon)\right] < 0$, hence $\left[\frac{V}{C} C_V + 1\right] < 0 \Rightarrow \mid \eta_v \mid > 1$ as $C_V < 0$.

(ii) Using the fact that $u_{x_1}^s > \lambda^s$ for $p_1^s = 0$, equation (14) - which holds with equality as $y^s > 0$ - can be rewritten as

$$\begin{aligned} & n\left[C_y E(\varepsilon) + \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\{C_y + C_V \frac{\partial V}{\partial y^s}\}dF(\varepsilon)\right] + u_{x_1}^s n\delta \int_{\varepsilon^*}^{\overline{\varepsilon}} p_2^s \frac{\partial v(\varepsilon)}{\partial y^s} dF(\varepsilon) \\ < \;& u_{x_1}^s \end{aligned}$$

The left hand side is the (expected) marginal 'valuation' of increased facility provision. The first $n\,[.]$ term is the expected benefit from increased facility size (taking into account any direct and indirect impact on quality, the latter from any induced change in congestion), at unchanged total usage of the club. The second term, $u_{x_1}^s n\delta \int_{\varepsilon^*}^{\overline{\varepsilon}} p_2^s \frac{\partial v(\varepsilon)}{\partial y^s} dF(\varepsilon)$, is the valuation of the expenditure on extra visits induced by the increased facility provision. The right hand side is the utility value of the cost incurred to increase the facility size. There is overprovision of the club good in the Samuelson rule sense since the valuation of the good induced by an increase in facility size falls short of the cost of providing that increase in the facility.■

Proof of Proposition 3. Our proof strategy is to show, first, that if $C(y, V)$ is homogeneous, then the monopolist's behaviour under regimes (b)-(c) (i.e., $y^m = 0$) occurs *iff* $C(y, V)$ is homogeneous of degree -1 (abbreviated "h.o.d.-1"). We then show that $C(y, V)$ being h.o.d.-1 is inconsistent with the welfarist's behaviour under regime (c). So, if $C(y, V)$ is h.o.d.-1, then only regime (b) holds. For all other k, only regime (a) is possible. But, the monopolist's behaviour under regime (a) is only consistent with $C(y, V)$ being h.o.d.k , where $k + 1 > 0$.

Suppose that $C(y, V)$ is h.o.d.k. i.e.

$$C(ty, tV) = t^k C(y, V) \text{ for all } t > 0 \tag{26}$$

Then, by Euler's theorem,

$$yC_y + VC_v = kC(y, V) \tag{27}$$

At the monopoly equilibrium: $|\eta_v^m| = 1 \Rightarrow V^m C_v^m = -C(y^m, V^m)$. Substituting in (27) then yields:

$$y^m C_y^m = (k + 1)C(y^m, V^m) \tag{28}$$

Proof of part (i): *regime (c) cannot occur for any k.*

In regime (c), $p_1^m > 0$, $p_2^m > 0, y^m = 0$; $p_1^s > 0$, $p_2^s > 0, y^s > 0$.With $y^m = 0$ for the monopolist and $C(0, V) > 0$ (see footnote 4), (28) then implies the only possible value of k, for this regime to occur is $k = -1$.However, as $p_1^s > 0$,we must have $[\frac{V^s}{C^s} C_v^s + 1] = 0 \Rightarrow V^s C_v^s = -C^s$ for the welfarist which then yields, similar to the monopoly case, the following form of (27): $y^s C_y^s = (k + 1)C(y^s, V^s) \Rightarrow y^s = 0$ if $k = -1$ thereby contradicting the fact that $y^s > 0$ in this regime.

Proof of part (ii): *regime (b) occurs if and only if* $k = -1$.

In regime (b), $p_1^m > 0$, $p_2^m > 0, y^m = 0$; $p_1^s = 0$, $p_2^s > 0, y^s > 0$.The '$\Rightarrow$'part: If $k = -1$, then (28) implies $y^m C_y^m = 0$. As $C_y(0, V) > 0$ by (A3), we must have $y^m = 0$.This means, from the monopolist's point of view, both regimes (b) and

(c) are possible. However, as just shown above, with $k = -1$, for the welfarist, regime (c) is not possible. Therefore, the only candidate for a plausible regime, when $k = -1$ is regime (b). We need to verify that $y^s > 0$ is consistent with regime (b). We do that as follows: By part (i) of the proof of proposition 2, at the welfarist equilibrium in regime (b) we have

$$[\frac{V^s}{C^s} C_v^s + 1] < 0 \Rightarrow V^s C_v^s + C^s < 0 \tag{29}$$

Now, from (27), $y^s C_y^s + V^s C_v^s = kC(y^s, V^s)$. Rewrite this by adding $C(.)$ on both sides,

$$\begin{aligned} y^s C_y^s + V^s C_v^s + C(y^s, V^s) &= (k+1)C(y^s, V^s) \qquad (30) \\ i.e., V^s C_v^s + C(y^s, V^s) &= (k+1)C(y^s, V^s) - y^s C_y^s \qquad (31) \end{aligned}$$

Then, using (29),

$$\begin{aligned} (k+1)C(y^s, V^s) - y^s C_y^s &< 0 \qquad (32) \\ i.e., (k+1)C(y^s, V^s) &< y^s C_y^s \qquad (33) \end{aligned}$$

When $k = -1$, (33) $\Rightarrow$

$$y^s C_y^s > 0 \Rightarrow y^s > 0 \text{ as } C_y^s > 0 \tag{34}$$

Thus, if C is h.o.d.-1, then only regime (b) holds.

Proof of part (iii): *Only regime (a) can occur for all k satisfying $k + 1 > 0$.*

We know from parts (i)-(ii) that we can rule out regimes (b) and (c) *iff* $k+1 \neq 0$. So, if $k + 1 \neq 0$, only regime (a) can occur and we must have $p_1^s = 0$, $y^m > 0$, and $y^s > 0$. Now, for the monopolist, $y^m C_y^m = (k+1)C^m$ (equation (28)) implies $y^m > 0 \Leftrightarrow k+1 > 0$, by (A.3). ■

Proof of Lemma 4 By definition, if $C(.)$ is homogeneous of arbitrary degree k, then $C(y, V) = V^k c(y/V)$ for some function $c(.)$. As $V^m C_V^m + C^m = 0$ at the monopoly equilibrium and $C_V = kV^{k-1}c\,(y/V) - yV^{k-2}c\,(y/V)$ then, using $z^m \equiv y^m/V^m$, $z^m c^{/}(z^m)/c(z^m) = k+1$. Likewise, as $V^s C_V^s + C^s < 0$ at the welfarist equilibrium, we can show $z^s c^{/}(z^s)/c(z^s) > k+1$. Now, by differentiation, $d\left[zC^{/}(z)/C(z)\right]/dz = \left[C^{/}(z)\right]^{-2}\left[zC(z)\,C^{//}(z) + C^{/}(z)\left\{C(z) - zC^{/}(z)\right\}\right]$. As $C(z^m) - zC^{/}(z^m) = 0$, then $d\left[z^m C^{/}(z^m)/C(z^m)\right]/dz < 0$ must hold. Likewise, $C(z^s) - zC^{/}(z^s) < 0$, so $d\left[z^s C^{/}(z^s)/C(z^s)\right]/dz < 0$ also. Therefore, if $zC^{/}(z)/C(z)$ is monotonic, it must be decreasing everywhere. ■