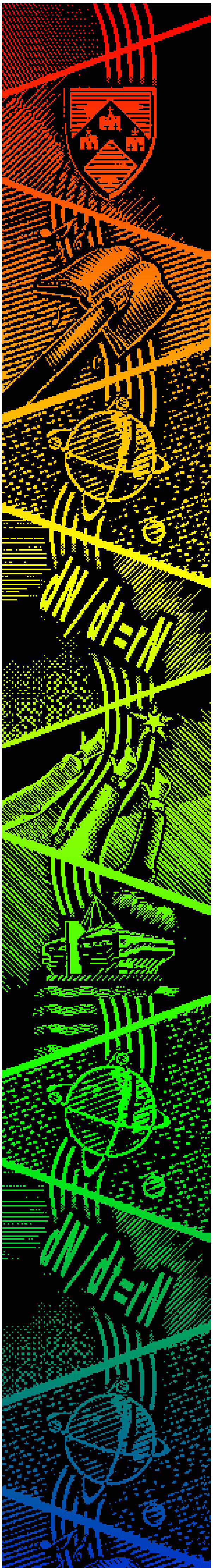

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Valuation of the Firm’s Liabilities when Equity Holders are also Creditors

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VALUATION OF THE FIRM'S LIABILITIES WHEN EQUITY HOLDERS ARE ALSO CREDITORS

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Abstract

This paper presents a tractable structural model whereby controlling equity holders are also among the creditors of the firm. As the firm approaches distress, equity holders can depauperate the firm and expropriate other creditors by repaying their credit before bankruptcy. The bankruptcy court's right to revoke such repayment protects arm's length creditors, reduces the cost of borrowing and induces equity holders to antici-

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pate repayment of their credit. Equity holders decide repayment neither too early nor too late, so as to reduce the risk of repayment revocation by the bankruptcy court. Similar conclusions apply to the preferential repayment of bank loans personally guaranteed by equity holders. The analysis also suggests that callable bearer bonds may be more valuable to equity holders than to other creditors.

Key words: equity holders's credit, debt repayment, assets liquidation, revocatoria, debt valuation, default, structural model.

JEL classification: G13;G33.

1 Introduction

Controlling equity holders of closely held private companies often lend to the very firm they own. For example Gelter (2005) studies this phenomenon in the US and in various European jurisdictions. Alternatively equity holders often offer personal guarantees to banks that concede loans to the firm they control. The accounting profession in Europe and US is familiar with these phenomena, which are often motivated by the quest for tax savings. In fact loans and loan guarantees given by equity holders increase leverage and in this way they are often instrumental to reducing the total tax burden on the firm and on the equity holders themselves. For example, Miller (1977) discussed the potential tax savings obtainable through equity holders' loans in the US setting. Cheris (1970) analysed the legal qualification and tax implications of equity holders' loans and the related *thin-capitalisation* rules. Indeed the US and many European countries envisage *thin-capitalisation* rules in order to counter the abuse of leverage for the purpose of tax avoidance, since leverage allows the firm to deduct interest charges from its taxable profit. Tax avoiding high leverage is typically achieved precisely through loans given by equity holders or through bank loans guaranteed by equity holders, since third party lenders would often not be willing to lend to extremely levered firms.

On the other hand this paper argues that, let alone tax considerations, equity holders' loans and guarantees can harm arm's-length creditors. As the firm approaches financial distress, it may repay equity holders' loan at face value, possibly by selling part of its assets, thus effectively making equity holders' loan senior to the claim of the other creditors. Hereafter we refer to this action as "repayment". The sale of assets and repayment would depauperate the firm and make default more likely. Equity holders would receive the face value of their loan, which, in the proximity of distress, would typically exceed the fair value of that same loan in the absence of repayment. Of course repayment would be detrimental to arm's length creditors, since it would reduce the amount of assets to satisfy their claims. Hence arm's length creditors are exposed to "repayment risk".

"Repayment risk" is a risk well know to bankruptcy courts and legislators. In order to protect creditors' interests, the bankruptcy codes of the US and most European countries envisage specific rules to avert extreme cases of debtor's misconduct of the type just described. For simplicity we mainly consider the rules of the US bankruptcy code (USC) and Uniform Fraudulent Transfer Act as points of reference. For example, sections 547-548 of USC address "preferences" and fraudulent transfers, which encompass "repayment" as above defined. Preferences regard the preferential treatment of some creditors and consist in the repayment or concession of privileges to some creditors to the detriment of other creditors. Fraudulent transfers consist of a variety of fraudulent ways in which the debtor can hide assets from the grab of creditors when bankruptcy is imminent. In order to void preferences and fraudulent transfers, the U.S. code gives bankruptcy trustees "avoiding powers". The exercise of such avoiding powers is intended to recuperate the assets that the debtor has "conveyed" outside the reach of creditors. In particular section 547 of USC gives the trustee the right to avoid repayments of loans to controlling equity holders, who are qualified as insiders, that took place within one year of bankruptcy filing. Gelter (2005) reports similar rules in Germany, Austria and Italy, whereby equity holders's loans are treated as equity by the bankruptcy court, at least when the loan was given in the relative proximity of distress.

Also repayments of bank loans personally guaranteed by controlling equity holders can be voided, which is of course a concern to the banking sector. For a U.S. example, Cullina (1991) and Brandt (1989) support such judicial decisions. Cullina explains how such repayments should be classified as cases of fraudulent conveyance. The rationale is that the equity holders' personal guarantee makes the personal assets of equity holders available to creditors in addition to the firm' assets. Thus repaying the guaranteed bank loan eliminates the equity holders' personal obligation to honour the guarantee they have conceded to the bank, with the indirect effect of decreasing the total pool of assets available for creditors. This judicial orientation entails that the analysis of equity holders' loans can be extended to bank loans personally guaranteed by equity holders, as is shown later on in the paper.

This paper analyses a tractable structural model of credit risk that hinges on "repayment risk" and that reveals a number of points. First, a potent deterrent against "repayment risk" is the bankruptcy court's or the bankruptcy trustee's right to avoid and revoke repayments to equity holders that took place prior to bankruptcy filing. The possible exercise of avoiding powers protects arm's length creditors, reduces the cost of borrowing and induces equity holders to anticipate repayment, so as to distance the time of repayment from the time of bankruptcy filing. In fact, as a necessary condition for the exercise of the bankruptcy trustee's avoiding powers, bankruptcy codes normally require that repayment precede bankruptcy filing by not more than one or two years.

Second, in the absence of avoiding powers the earlier the repayment is, the less detrimental it is to the firm's arm's length creditors. Instead it is in the interest of equity holders to delay repayment. But in the presence of avoiding powers equity holders decide repayment neither too early nor too late, in order to reduce excessive exposure to the risk of avoiding powers. The tax shield associated with the debt held by equity holders does not materially affect the repayment decision.

Third, the analysis of repayment risk is applicable not only to loans which equity holders concede to the firm, but also to callable bonds and callable convertible bonds held by equity holders, to callable preferred stock held by equity holders and even to injections of funds from equity holders into the firm that neither command any interest nor oblige the firm to repayment. "Repayment risk" is especially high when the firm has issued bearer callable bonds, which equity holders may anonymously buy at above market prices, because the bonds would be more valuable to themselves than to anybody else. Finally the analysis is applicable also to bank loans personally guaranteed by equity holders. The repayment policy of such loans is similar to that of equity holders' loans, since even such repayments could be voided by bankruptcy trustees.

1.1 Other related literature

This paper is motivated by the fact that the bankruptcy law literature has long recognised and analysed preferences, fraudulent conveyance and avoiding powers, but the finance literature has kept almost mute on the topic with rare exceptions. Hence the contribution of this paper to the finance literature consists in assessing how bankruptcy code provisions concerning preferences and fraudulent conveyance affect the prices of liabilities as well as capital structure decisions.

As for the legal literature, Rose-Ackerman (1985, p. 951) long predicted that, in the absence of appropriate code provisions, "the volume of loans would be inefficiently low and interest rates inefficiently high to take account of this possibility of hiding assets from creditors." The analysis of this paper to some extent supports this conclusion.

As for the economics literature, Gelter (2005) studied the social welfare effects of the subordination of controlling equity holders' loans in bankruptcy, a provision envisaged in both continental Europe and the US, which goes under the name of "equitable subordination". Gelter concentrates on loans given by equity holders to the firm when the firm is already in the relative proximity of financial distress. He stresses that such loans can represent rescue attempts and that equity subordination can discourage not only inefficient rescue attempts, but also efficient ones.

Heaton (2000) showed how fraudulent conveyance laws improve firms' debt capacity and cost of borrowing. This papers differs from Heaton's since it concentrates on loans and guarantees given by controlling equity holders and since the analysis hinges on arbitrage pricing of the firm's liabilities. Also Bond and Krishnamurthy (2004) stress the importance of bankruptcy rules addressing fraudulent conveyance for the proper regulation and functioning of credit markets.

This article is also close in spirit to Morellec's (2001). Morellec assumes that, as the firm's prospects worsen, the firm's assets are progressively liquidated and the proceeds are distributed to equity holders. Such liquidations progressively erode the collateral value of the firm to the detriment of creditors. Also in this paper the firm's assets are liquidated, although not progressively as in Morellec, but the proceeds are specifically used to repay debt held by equity holders, which is again detrimental to other creditors. Notice that if, as in Morellec, equity holders did not hold any debt, it may be difficult or impossible for the firm to distribute the assets sales proceeds to equity holders: there may not be enough reserves of retained earnings to distribute, especially when the firm approaches distress, or debt covenants may limit the distribution of dividends, and reduction of share capital is often seen with suspicion by the courts especially when the firm approaches distress. Instead repayment of debt held by equity holders does not pose any of these limitations and seems an effective way for equity holders to distribute some of the firm's wealth to themselves even if the firm approaches default. In the absence of repayment the market value of the equity holders' loan would typically be below face value. But repayment is effected at face value, which would amount to a gift from the firm to equity holders in so far as face value exceeds market value in the absence of repayment. On the other hand debt repayment may be revoked by the bankruptcy court, as discussed later. These features distinguish the present contribution from Morellec's.

Most model assumptions in this article follow Goldstein-Ju-Leland (2001): in particular the building block of the analysis is the *before-tax* value of the firm, so that the tax advantage of the equity holders' loan is properly modelled.

The paper is organised as follows. The next two sections illustrate the model with and without repayment. Then another section explores the model predictions and yet another section generalises the analysis. Finally the analysis is extended to the repayment of guaranteed bank loans and then the conclusions are drawn.

2 The model without repayment

This section introduces the model, which follows the tradition of contingent claims analysis and structural models. Assume that, under universal risk neutrality, the value of the firm's assets V follows the process

$$dV = V(r - b)\,dt + V\sigma dz \tag{1}$$

where r is the default-free interest rate, which is is assumed constant, b is the rate at which the firm generates EBIT (earnings before interest and taxes), σ is the volatility parameter and dz is the differential of a Wiener process. Following Goldstein-Ju-Leland (2001), V is the *before-tax* value of the firm, to be divided among debt holders, equity holders and the Government tax claim. Let t_c denote the corporate tax rate. Assume that all net earnings produced by the firm are immediately distributed to equity holders and that dividends enjoy no tax credit, rather equity holders need to pay a dividend withholding tax at the rate t_d. This assumption acknowledges the trend of European countries away from dividend tax credits and towards taxation of dividends. Moreover it is not inconsistent to assume that all net earnings are distributed, even if distributions entail a sort of double taxation. Of course this is true as long as such double taxation is not extreme and the reason is that equity value, as is typical of this type of models, rises if net earnings are distributed as soon as possible. It follows that the *after-tax* value of the un-levered firm is $V(1 - t_c)(1 - t_d)$.

The balance sheet equation of the levered firm is

$$V = E(V) + T(V) + D(V) + D_s(V) + B(V) \tag{2}$$

where $E(V)$ is the value of equity, $T(V)$ is the value of total future taxes to be paid by the firm and by equity holders, $D(V)$ is the value of debt not held by equity holders, $D_s(V)$ is the value of debt held by equity holders and $B(V)$ is the value of future possible bankruptcy costs. $D(V)$ continuously pays coupons at the yearly rate c and has face value F. $D(V)$ can be thought of as a bank loan of indefinite or very long maturity, or a bank loan that is periodically "rolled-over". $D_s(V)$ continuously pays coupons at the yearly rate c_s and has face value F_s. Following Kim-Ramaswamy-Sundaresan (1993), Fan-Sundaresan (2000) and others, default and bankruptcy are triggered by a "liquidity" condition: default takes place as soon as V drops to the barrier level $V_{dd} = \frac{c+c_s}{b}$. Bankruptcy follows default. Bankruptcy costs are a proportion a of V_{dd}, so the firm can be sold for an *after-tax* rescue value of $V_{dd}(1-a)(1-t_c)(1-t_d)$. Notice that those who buy the firm would just be willing to pay for the *after-tax* value of the firm. According to a widespread judicial orientation, reported for example in Gelter (2005), in bankruptcy $D(V)$ is regarded as senior to $D_s(V)$, despite the lack of explicit subordination covenants. Gelter reports that courts, both in US and Europe, typically tend to treat equity holders' loans as equity and thus as subordinated to those of arm's length creditors. Hence the recovery values of the debt claims are respectively

$$D\left(V_{dd}\right)=\min\left(V_{dd}\left(1-a\right)\left(1-t_{c}\right)\left(1-t_{d}\right),F\right) \tag{3}$$

$$D_{s}\left(V_{dd}\right)=\min\left(\max(V_{dd}\left(1-a\right)\left(1-t_{c}\right)\left(1-t_{d}\right)-F,0),F_{s}\right). \tag{4}$$

The firm continuously produces EBIT at rate $Vbdt$, and such earnings are divided as follows:

- cdt is the interest gross of withholding tax paid on $D\left(V\right)$, i.e. paid to creditors other than equity holders;

- $c_{s}\left(1-t_{i}\right)dt$ is the interest on $D_{s}\left(V\right)$ paid to equity holders net of withholding tax, whose rate is t_{i};

- $c_{s}t_{i}dt$ is the withholding tax on interest paid to equity holders;

- $\left(Vb-c-c_{s}\right)\left(1-t_{c}\right)\left(1-t_{d}\right)dt$ are the dividends paid out to equity holders net of corporate tax and the dividend withholding tax;

- $\left(Vb-c-c_{s}\right)t_{c}dt$ is the corporate tax;

- $\left(Vb-c-c_{s}\right)\left(1-t_{c}\right)t_{d}dt$ is the dividend withholding tax.

Worth emphasising is that $T\left(V\right)$ is the present value of the total tax payments due by the firm and by the equity holders, i.e. the present value of the cash flow stream equal to

$$\left[\left(Vb-c-c_{s}\right)\left(t_{c}+\left(1-t_{c}\right)t_{d}\right)+c_{s}t_{i}\right]dt \tag{5}$$

over any infinitesimal period dt until default takes place. In this setting the absence of arbitrage opportunities allows us to derive the formulae for $E\left(V\right)$,

$D(V)$, $D_s(V)$, $T(V)$ and $B(V)$ that are reported in Appendix A.1. In the next section the current setting is enriched with the main new feature of this paper: before default the firm repays the debt held by equity holders by selling part of its assets. We refer to this event as "repayment".

3 The model with repayment

We now consider the same model as before, but for the right of equity holders to deliberate repayment of $D_s(V)$, which is the debt held by themselves. Repayment is assumed to be optimally decided by equity holders as soon as V drops to the level V^*, where we impose the condition that $V^* > V_{dd}$ in order for repayment to precede default. This condition is needed as it becomes binding when liquidation costs are negligible and repayment cannot be revoked by the court, although in other cases imposing that $V^* > V_{dd}$ is redundant because of the way equity holders will optimally set V^*. We will determine V^* later. Upon repayment $D_s(V)$ is reimbursed at face value F_s. In order to secure the liquidity to repay F_s, the firm needs to liquidate a part of its assets, whose *before-tax* and *before-liquidation* value we denote as x. Thus liquidation of assets causes V^* to drop to $V^* - x$. Liquidation of assets entails liquidation costs proportional to x, which we denote with kx, where $0 \leq k \leq 1$ is a constant. Moreover those who buy the part of the assets that are liquidated, would only be willing to pay the *after-tax* value of such assets net of liquidation costs, which amounts to $x(1-k)(1-t_c)(1-t_d)$. Then x, which is the *before-tax* and *before-liquidation* amount of assets to sell in order to enable the firm to repay F_s, must be such that

$$F_s = x\,(1-k)\,(1-t_c)\,(1-t_d) \tag{6}$$

giving $x = \frac{F_s}{(1-k)(1-t_c)(1-t_d)}$. In the setting of this section, before repayment the firm's balance sheet equation is

$$V = E\,(V) + T\,(V) + D\,(V) + D_s\,(V) + B\,(V) + L\,(V) \tag{7}$$

where $L\,(V)$ is the present value of liquidation costs kx induced by repayment. After repayment the balance sheet equation becomes

$$V = E'\,(V) + T^{'}\,(V) + D'\,(V,\tau) - R\,(V,\tau) + B'\,(V)\,. \tag{8}$$

where the superscripts denote claim values after repayment. $R\,(V,\tau)$ is the value of the bankruptcy court's right to revoke the repayment of F_s before default and bankruptcy. τ denotes the time period left until the last day on which declaration of bankruptcy would enable the court to revoke repayment of F_s. The term $-R\,(V,\tau)$ in the above balance sheet equation signifies a potential liability for equity holders, who have received repayment of their credit, but may have to return such payment to the bankruptcy trustee due to the trustee's avoiding powers. The trustee would then use the sum thus recuperated to pay arm's length creditors. Correspondingly the value of the claim of arm's length creditors $D'(V,\tau)$ increases by $R(V,\tau)$ after repayment. The possible exercise of avoiding powers will be critical in the analysis to follow.

Thus repayment of F_s can be revoked through the trustee's avoiding powers, even if such repayment took place before the bankruptcy date. The bankruptcy courts of the US and European countries typically envisage similar avoiding powers aimed at protecting the interest of the mass of creditors against detrimental acts of the debtor previous to bankruptcy or against acts in violation of the equal treatment of all creditors. Pursuant to the exercise of avoiding powers equity holders would give back F_s to the firm and concur with the other creditors for the satisfaction of their claim in bankruptcy. Hereafter we denote with τ_o the value of τ at the time of repayment. Typically $0 \leq \tau_o \leq 2$ meaning that avoiding powers typically apply to repayments that precede the bankruptcy declaration date by more than two years.

We notice that, to rule out arbitrage opportunities, we need to impose the following conditions at the moment of repayment

$$V^* = E(V^*) + T(V^*) + D(V^*) + B(V^*) + L(V^*) \tag{9}$$

$$E(V^*) = E'(V^* - x) \tag{10}$$

$$T(V^*) = T'(V^* - x) + x(1-k)(t_c + (1-t_c)t_d) \tag{11}$$

$$D(V^*) = D'(V^* - x, \tau_o) \tag{12}$$

$$D_s(V^*) = F_s - R(V^* - x, \tau_o) \tag{13}$$

$$B(V^*) = B'(V^* - x) \tag{14}$$

$$L(V^*) = kx \tag{15}$$

Conditions 10 and 12 state that the values of equity and of the arm's length credit before repayment must be the same as those after repayment at the very time of repayment. Condition 11 states a similar condition for the tax claim, while accounting for the fact that repayment entails a reduction in the debt induced tax shield and hence an increase in the value of the tax claim equal to $x(1-k)(t_c + (1-t_c)t_d)$. Condition 13 states that at V^* the value of debt held by equity holders is equal to its face value, since such debt is reimbursed, minus the value of the court's right to revoke such reimbursement should the firm file for bankruptcy no later than τ_o years since the date of reimbursement. Condition 14 states that at V^* the value of potential bankruptcy costs before repayment approaches the value of potential bankruptcy costs after repayment. Repayment causes the value of the firm's assets to drop from V^* to $V^* - x$ and hence it affects the probability of default and the value of potential bankruptcy costs. Equation states that at V^* the firm incurs costs equal to kx due to the partial liquidation of the firm's assets. These conditions imply that

$$V^*-x = E'\left(V^*-x\right)+T^{'}\left(V^*-x\right)+D'\left(V^*-x,\tau_o\right)-R\left(V^*-x,\tau_o\right)+B'\left(V^*-x\right). \tag{16}$$

When repayment is possible, the above conditions and the absence of arbitrage opportunities also imply that $E\left(V\right)$, $T\left(V\right)$, $D\left(V\right)$, $D_s\left(V\right)$, $B\left(V\right)$ and $L\left(V\right)$ satisfy the following ODE's and conditions. $D_s\left(V\right)$ satisfies

$$\frac{\partial^2 D_s}{\partial V^2}\frac{1}{2}\sigma^2V^2+\frac{\partial D_s}{\partial V}V\left(r-b\right)-rD_s+c_s\left(1-t_i\right)=0 \tag{17}$$
$$D_s\left(V\to\infty\right)\to\frac{c_s\left(1-t_i\right)}{r},\ D_s\left(V^*\right)=F_s-R\left(V^*-x,\tau_o\right)$$

The solution to the above ODE is

$$D_s\left(V\right)=\frac{c_s\left(1-t_i\right)}{r}+\left(-\frac{c_s\left(1-t_i\right)}{r}+F_s-R\left(V^*-x,\tau_o\right)\right)\left(\frac{V}{V^*}\right)^q \tag{18}$$
$$q=\frac{-\left(r-b-\frac{1}{2}\sigma^2\right)-\sqrt{\left(r-b-\frac{1}{2}\sigma^2\right)^2+2r\sigma^2}}{\sigma^2}. \tag{19}$$

After repayment $R\left(V,\tau\right)$ represents a potential liability for equity holders and a potential asset for the other creditors. In fact $R\left(V,\tau\right)$ is the present value of a claim that pays off if only if V drops to the post-repayment default barrier level V_d within τ_o years of the repayment date. In such event, the bankruptcy code retrospectively assimilated repayment to either a preference of as fraudulent conveyance and requires that equity holders need to pay back to arm's length creditors an amount equal to the minimum between the repayment amount F_s and the loss suffered by arm's length creditors $\max\left(F - V_d\left(1-a\right)\left(1-t_c\right)\left(1-t_d\right), 0\right)$. In other words, it is as if, in case of default soon after repayment, equity holders had to pay arm's length creditors the amount

$$R\left(V_d\right) = \min\left(\max\left(F - V_d\left(1-a\right)\left(1-t_c\right)\left(1-t_d\right), 0\right), F_s\right). \tag{20}$$

Thus, after repayment, the claim $R\left(V,\tau\right)$ satisfies, over the region $[V_d, \infty[\times [0, \tau_o]$, the following equation

$$\begin{aligned}
&\frac{\partial R}{\partial \tau} = \frac{\partial^2 R}{\partial V^2}\frac{1}{2}\sigma^2 V^2 + \frac{\partial R}{\partial V}V\left(r-b\right) - rR \qquad (21)\\
&R\left(V \to \infty\right) \to 0\\
&R\left(V_d\right) = \min\left(\max\left(F - V_d\left(1-a\right)\left(1-t_c\right)\left(1-t_d\right), 0\right), F_s\right)\\
&R\left(\tau = 0\right) = 0.
\end{aligned}$$

Notice that after repayment the default barrier is $V_d = \frac{c}{b}$, unlike $V_{dd} = \frac{c+c_s}{b}$ in the absence of repayment as in the previous section. $R\left(V_d\right)$ is the payoff to the claim $R\left(V,\tau\right)$ due to the exercise of the bankruptcy trustee's avoiding powers. It is the amount that, after repayment, equity holders need to pay back to other creditors in case of bankruptcy.

The expression for $R(V_d)$ does not consider that, if avoiding powers are exercised, equity holders have the right to concur with other creditors to satisfy their claim, since such claim is subordinated to others creditors and virtually worthless. The solution to the above PDE is

$$\begin{aligned} R(V,\tau) &= R(V_d)\left(\frac{V}{V_d}\right)^q - R(V_d)\left(\frac{1}{V_d}\right)^q e^{(-r+n)\tau}\cdot \\ &\cdot\left[V^q N\left(d_1(V,\tau)\right) - \left(\frac{V}{V_d}\right)^{-2\frac{n}{q\cdot\sigma^2}}(V_d)^q N\left(d_2(V,\tau)\right)\right] \end{aligned} \tag{22}$$

where

$$N(y) = \int_{-\infty}^{y}\frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}du,\ n = (r-b)q + \frac{1}{2}q(q-1)\sigma^2$$

$$d_1(V,\tau) = \frac{q\ln\left(\frac{V}{V_d}\right) + \left(n+\frac{1}{2}\sigma^2q^2\right)\tau}{q\cdot\sigma\sqrt{\tau}},\ d_2(V,\tau) = \frac{q\ln\left(\frac{V_d}{V}\right) + \left(n+\frac{1}{2}\sigma^2q^2\right)\tau}{q\cdot\sigma\sqrt{\tau}}.$$

Appendix A.2 shows that the solutions for $D(V)$, $T(V)$, $B(V)$ and $L(V)$ are

$$D(V) = \frac{c}{r} + \left(-\frac{c}{r} + D'(V^*-x,\tau_o)\right)\left(\frac{V}{V^*}\right)^q \tag{23}$$

$$D'(V,\tau) = \frac{c}{r} + \left(-\frac{c}{r} + \min\left(V_d(1-a)(1-t_c)(1-t_d), F\right)\right)\left(\frac{V}{V_d}\right)^q + R(V,\tau)$$

$$\begin{aligned}
T(V) &= \left(V - \frac{c+c_s}{r}\right)(t_c + (1-t_c)t_d) + \frac{c_s t_i}{r} + \qquad (24)\\
&+ \left(\begin{array}{c} -\left(V^* - \frac{c+c_s}{r}\right)(t_c + (1-t_c)t_d) - \frac{c_s t_i}{r} \\ +T'(V^* - x) + x(1-k)(t_c + (1-t_c)t_d) \end{array}\right)\left(\frac{V}{V^*}\right)^q \\
T'(V) &= \left(V - \frac{c}{r}\right)(t_c + (1-t_c)t_d) + \\
&+ \left(\begin{array}{c} -\left(V_d - \frac{c}{r}\right)(t_c + (1-t_c)t_d) \\ +V_d(1-a)((1-t_c)t_d + t_c) \end{array}\right)\left(\frac{V}{V_d}\right)^q .
\end{aligned}$$

$$\begin{aligned}
B(V) &= B'(V^* - x)\left(\frac{V}{V^*}\right)^q \qquad (25)\\
B'(V) &= aV_d\left(\frac{V}{V_d}\right)^q .
\end{aligned}$$

and

$$L(V) = xk\left(\frac{V}{V^*}\right)^q . \qquad (26)$$

We are left with the problem of how to determine V^*, which represents the optimal repayment policy.

3.1 The optimal policy

We assume that equity holders choose V^* so as to maximise the total value of their claims $E(V) + D_s(V)$, even though scrupulous equity holders may not follow this policy since it is detrimental to other creditors. Then equity holders will also ensure that repayment does not precipitate immediate default, which would force them to immediately give F_s back to the firm. This imposes the constraint $V^* - x > V_d$.

Equity holders cannot deliberate repayment if default has already taken place. This imposes the further constraint $V^* > V_{dd}$, where V_{dd} is the default barrier in case of no repayment. Finally it can be shown that equity holders will decide repayment only if V^* is at or below the current level V, i.e. only if $V^* \leq V$, else they would have already decided to repay their loan or would not have lent in the first place. These considerations suggest that the optimal repayment policy is represented by V^* such that

$$\begin{aligned} &\max_{V^*} \left[E\left(V\right) + D_s\left(V\right) \right] \\ &st : V^* - x > V_d, V^* > V_{dd}, V^* \leq V. \end{aligned} \tag{27}$$

Thus we can only find V^* numerically though a non-linear optimisation algorithm. Next we explore the model predictions when V^* is determined as just described.

4 Comparative statics and model predictions

Comparative statics reveal a number of points. To focus the arguments, we assume a base case scenario with average realistic parameters as illustrated in Table 1. Input parameter values (in Italic) are similar to those typically recurring in papers on structural models of credit risk.

[Table 1 about here]

4.1 The case of no repayment

The first column of Table 1 illustrates the case whereby equity holders are not repaid their credit. Assuming that $V = 100$, equity value is $E(V) = 31.5$, the tax claim is worth $T(V) = 29.5$ and the debt claim held by equity holders is worth $D_s(V) = 13.1$, which is far less than its face value of $F_s = 25$, since $D_s(V)$ recovers nothing in case of default, i.e. since $D_s(V_{dd}) = 0$.

An unreported simulation shows that, if $c_s = 0$ and $F_s = 0$ and all else is still as assumed in the first column, then equity value becomes $E(V) = 43.5 < (31.5 + 13.1)$ and the tax claim rises to $T(V) = 32.5$. This means that equity holders are better off lending to their firm even if they do not get repaid either before or after the firm's default. The reason is that lending enables the firm to save $c_s t_c$ of corporate tax and equity holders to save $c_s(1 - t_c)t_d$ of dividend tax even though they have to pay tax on coupons equal to $c_s t_i$. This entails a tax saving as long as $c_s(t_c + (1 - t_c)t_d - t_i) > 0$, as it had long been recognised by Miller (1977). Such tax savings are a real possibility in some jurisdictions, even though *thin-capitalisation* and similar tax rules often tend to limit the amount c_s that the firm can deduce for corporate tax purposes. Hence, especially when banks refuse cheap loans because of the firm's already high leverage, equity holders' lending may be motivated by the quest for tax savings. This can be the case irrespective of the repayment decision that we consider next.

4.2 The case of repayment

The second column of Table 1 shows the value of the firm's liabilities if, all else as in the first column, equity holders are repaid their loan at face value as soon as the firm's value drops to $V^* = 68.9$. *Ceteris paribus* repayment clearly makes equity holders better off: $D_s(V)$ rises from 13.1 to 25.2 and $E(V)$ decreases from 31.5 to 22.1. At the same time $D(V)$ decreases from 22.6 to 17 and $T(V)$ rises from 29.5 to 33.8. This means that, if equity holders are expected to deliberate repayment of their own loan, the credit spread on $D(V)$ rises from 1.53% (without repayment) to 3.34% (with repayment) while the credit spread on $D_s(V)$ drops from 5.53% (without repayment) to 0.97% (with repayment). Thus repayment at V^* transfers value from $D(V)$ and $E(V)$ to $D_s(V)$ and $T(V)$. $D_s(V)$ rises because, whereas in the absence of repayment $D_s(V)$ is subordinated to $D(V)$ and recovers nothing in bankruptcy, repayment at V^* effectively makes $D_s(V)$ senior to $D(V)$ and practically default-free, even though the possible exercise of the trustee's avoiding powers mitigates this conclusion. Equity holders's loan is repaid at face value, which *ceteris paribus* is usually much higher than the loan market value under the assumption of no repayment. Repayment would then entail a gift from the firm to equity holders. Unscrupulous equity holders would thus have the option to expropriate other creditors by making a gift to themselves: such option is optimally exercised at $V^* = 68.9$.

This example highlights the general result that repayment harms arm's length creditors and increases the cost of borrowing. This conclusion is some-how mitigated by the fact that repayment would also decrease the default barrier from $V_{dd} = \frac{c+c_s}{b}$ to $V_d = \frac{c}{b}$. This affect somewhat alleviates default risk.

The prospect of repayment also raises the "expected tax bill" $T(V)$, since repayment reduces leverage and the tax shield associated with leverage. When assets worth x are sold at V^* for a net realised price of $x(1-k)(1-t_c)(1-t_d)$, the sold assets are debt-free and for simplicity we do not consider the possibility that the asset buyer may increase its leverage to reduce its own tax burden. Then at V^* the Government is thought to receive $x(1-k)(t_c+(1-t_c)t_d)$. The reason why we abstract from the buyer's option to increase his leverage and tax shield is that the buyer should only be willing to pay for the *after-tax* value of assets. Thus our estimates of all claim values, apart from $T(V)$, would not be affected if the buyer increased his leverage and tax shield after purchasing the assets.

4.2.1 Optimal repayment

We note that, in order for repayment at V^* not to trigger immediate default, which would again make $D_s(V)$ subordinated to $D(V)$, we need to impose that $V^* - x > V_d$, which in our case means $68.9 - 42.6 > 15.6$. Simulations show that generally such constraint is met if $\tau_o > 0$ and that $\lim_{\tau_o \to 0}(V^* - x) \to V_d$. Thus, if the trustee's avoiding powers cannot be exercised, for example because the bankruptcy code does not envisage them, equity holders will tend to repay their loan as late as possible, i.e. just before V drops to the level $x + V_d$. Instead, if the trustee's avoiding powers can be exercised, equity holders will decide to repay earlier. This point is now further explained.

In the absence of avoiding powers equity holders tend to repay as late as possible. In fact we can view equity holders' decision as one of minimising the value of the arm's length credit $D(V)$, because, as V^* decreases, $E(V)$ rises and $D(V)$ drops: what equity holders may gain through later repayment is associated with a loss to arm's length creditors and vice versa. The lower V^* is, the lower is $D(V)$, in the absence of avoiding powers, hence equity holders will tend to repay their credit as late as possible, i.e. when $V^* = x + V_d$. This point is confirmed by inspection of equation 23. In fact, if $\tau = 0$, then $R(V, \tau) = 0$ and $D(V)$ becomes

$$D(V) = \frac{c}{r} + \left(-\frac{c}{r} + \min\left(V_d(1-a)(1-t_c)(1-t_d), F\right)\right)\left(\frac{V^* - x}{V_d}\right)^q \left(\frac{V}{V^*}\right)^q. \tag{28}$$

Now, whenever $\left(-\frac{c}{r} + \min\left(V_d(1-a)(1-t_c)(1-t_d), F\right)\right) < 0$, the value of V^* that minimises $D(V)$ also maximises $\left(\frac{V^*-x}{V_d}\frac{V}{V^*}\right)^q$. Then, given that $q < 0$ and given the constraint $V^* \geq x + V_d$, $V^* = x + V_d$ indeed guarantees that $\left(\frac{V^*-x}{V_d}\frac{V}{V^*}\right)^q$ be maximised subject to the mentioned constraint. The economic intuition behind this result is that repayment entails a transfer of value from arm's length creditors to equity holders, and such transfer of value is greater if repayment is carried out when $D(V)$ is more sensitive to a change in the level of assets V. And of course as V decreases $\frac{\partial D(V)}{\partial V}$ rises, so that the later repayment is set the greater the mentioned value transfer is. Notice also that, in order for V^* not to affect $D(V)$, we would need to impose that $x = fV^*$, where f is an arbitrary constant. We conclude that in the absence of avoiding powers unscrupulous equity holders would be tempted to increase equity value by delaying repayment of their own loan as much as possible.

In the presence of avoiding powers equity holders will not delay repayment as much and V^* becomes an interior point. The reason is that the exercise of avoiding powers becomes more unlikely as V^* rises, which means that $R(V,\tau)$ decreases in V^*, and this entails that $D(V)$ may decrease as well as increase in V^* as shown in Figure 1 for the base case scenario in the presence of avoiding powers. Similarly in the presence of avoiding powers $D_s(V)$ may increase as well as decrease in V^* as shown in Figure 2 again for the base case scenario. Clearly there is a value of V^* than maximises $D_s(V)$ and one that minimises $D(V)$ and the two a quite close.

[Figures 1 and 2 about here]

Figures 1 and 2 show that $D(V)$ is much more sensitive to V^* than $D_s(V)$ is. This entails that, since the optimal value of V^* involves a trade-off between minimising $D(V)$ and maximising $D_s(V)$, equity holders chose optimal repayment by primarily trying to minimise the value of the arm's length credit rather than by maximising the value of their loan. Put it another way the optimal value of V^* involves a trade-off for equity holders between maximising $D_s(V)$ and maximising $E(V)$, and equity holders will decide repayment mainly to maximise $E(V)$, which is much more sensitive to V^* than $D_s(V)$ is.

We may expect a second effect determining V^*: earlier repayment reduces the tax shield for the firm and equity holders when

$$t_c + (1 - t_c)\, t_d - t_i > 0.$$

In particular, as the interest withholding tax rate t_i decreases, the tax shield due to the equity holders' loan rises and V^* should decrease, since repayment entails foregoing a more valuable tax shield. But simulations reveal that V^* hardly rises as t_i decreases. Thus tax rules do not seem effective to induce earlier repayment. This highlights that the tax shield is of second order importance for the optimal repayment policy, even though the equity holders' loan may have been originally motivated by tax-avoidance.

V^* is hardly sensitive to c_s. *Ceteris paribus* V^* moves from 69.93 in the base case, whereby $c_s = 5\%$ and $r = 4\%$, to 70.38 when $c_s = 0$. In other words V^* hardly moves even if equity holders cut coupons to zero. This may seem puzzling, but the main reason is that, even if $c_s = 0$, equity holders do not lose that much since what they lose in terms of foregone interest they receive back in terms of correspondingly higher dividends (net of taxes). Thus lending at no interest will hardly make equity holders more impatient to be repaid. Hence controlling equity holders may want to lend to their own firms even without requiring any interest. Similar considerations are valid even when c_s is much higher than the risk free interest rate r. For example Table 2 shows the same scenario as in Table 1 but for the fact that $c_s = 0.15$. In particular V^* only changes form 69.93 in the base case to 67.42. This highlights how V^* is hardly sensitive to c_s.

[Table 2 about here]

V^* is quite insensitive also to the bankruptcy cost parameter a. V^* tends to rise in assets volatility σ, since higher volatility makes default and exercise of avoiding powers after repayment more likely.

As expected $D(V)$ increases in τ_o through a rise in $R(V^* - x, \tau_o)$, since the exercise of avoiding powers becomes more likely as the law extends the "suspect period before bankruptcy filing". This entails that the cost of borrowing from arm's length lenders is reduced as the avoid powers of bankruptcy trustees are extended. Of course as τ_o rises so does V^*, since equity holders anticipate repayment.

V^* generally rises in the rate b at which the firm generates earnings. The default barrier V_d rises in b, thus making the exercise of avoiding powers more likely. This prospects again "pushes" V^* upward.

As expected V^* rises with the size of the assets liquidation costs as captured by k. The higher the k, the more costly it is to liquidate the assets, i.e. the less "liquid" the assets are. We can think of k as either direct or indirect liquidation costs. When $k = 0$ the firm is probably just using cash or cash equivalents to pay F_s, so that it need not sell any of its "assets-in-place". V^* is sensitive to k. For example $V^* = 68.83$ in the base case, whereby $k = 0$, but V^* rises to 73.4 if $k = 0.1$.

As for the third column of Table 1, which shows *ceteris paribus* the values of the various claims after repayment, it is worth highlighting that the total value of the claims held by equity holders after repayment $E'(V) - R(V, \tau)$ may well be negative. This is clearly due to the possibility that the bankruptcy trustee exercise avoiding powers.

Overall simulations reveal that in most cases V^* can be expected to be between 120% and 200% of the face value $F + F_s$ of total outstanding debt. Thus repayments of equity holders' loans not only can have material valuation consequences, but are also quite likely events. *Inter alia* these considerations seem important since equity holders' lending is widespread among private small and medium sized firms.

We can conclude that unscrupulous equity holders, when deciding repayment, do not so much face a trade-off between maximising the value of the tax shield and minimising the risk of avoiding powers. Rather their trade-off is one between maximising the expropriation of arm's length creditors, which would typically entail delaying repayment, and minimising the risk of avoiding powers. As a consequence, the legislator should be aware that, in order to attenuate the incentive for equity holders to play the "repayment-game", it should increase the "suspect period" rather that reduce the magnitude of the tax shield associated with equity holders' lending.

4.3 Remedies to attenuate "repayment risk"

Now we consider possible remedies to attenuate the "repayment risk" faced by debt holders. The most effective remedy against repayment risk seems to be a covenant that prohibits the sale of assets. Similarly Morellec (2001) suggests that a pledge on the firm's assets may protect debt holders against the risk of depauperation of the firm's assets. But a pledge can only be obtained on fixed assets or some current assets other than cash or cash equivalents, whose level typically fluctuates enormously over time. Thus a pledge on fixed or on some current assets would not be very effective if the firm had plenty of cash with which to repay the debt due to equity holders. Moreover, whereas it may be relatively easy to require and monitor a debt covenant that limits the distribution of dividends, we can hardly think of a covenant that prevents the firm from repaying its debt by using cash.

Assets sales restrictions do not solve the problem of creditors' expropriation when they induce the firm to refinance equity holders' debt repayment with newly issued debt, let alone that it may be difficult for the firm to borrow when assets are worth V^* since financial distress would be "near". Even in this case, since repayment of $D_s(V)$ is at or close to par and since the fair value of $D_s(V)$ would be much less in the absence of repayment, other creditors would again be expropriated.

An alternative remedy to the "repayment-game" is for arm's length creditors to impose covenants whereby the firm cannot repay equity holders' credit before repaying the arm's length creditors'. But such covenant may be very restrictive for the firm, although the above analysis suggests that ruling out repayment could materially reduce the cost of borrowing.

Having shown how "repayment risk" should be a real concern to arm's length debt holders, in the following we discuss generalisations of the above analysis.

5 Generalisations of the analysis

We now discuss how the preceding results can be generalised to different default conditions, different forms of equity holders' credit and how they impact optimal capital structure.

5.1 Renegotiation, endogenous default and avoiding powers

Past literature (e.g. Anderson and Sundaresan (1996) or Mella-Barral and Perraudin (1997)) has proposed structural models whereby equity holders and debt holders renegotiate as the firm approaches financial distress. Renegotiation is intended to avert costly bankruptcy. Also in the above model we can envisage that, after repaying the debt due to themselves, equity holders renegotiate the firm's remaining debt obligations with creditors as the firm approaches distress. But such renegotiation, which is subsequent to repayment, should consider the possibility that the trustee exercise avoiding powers, which would strengthen debt holders' bargaining power and increase their payoff in bankruptcy. Indeed debt holders may want to accelerate bankruptcy if the deadline for exercise of avoiding powers approaches.

On the other hand if, after repayment, equity holders could inject cash into the firm, they would tend to do so at least to temporarily stave-off default until when the trustee's right to exercise avoiding powers expires. Equity holders may consider equity injections irrespective of any renegotiation with creditors.

These equity injections have not been explicitly modelled in the above model, since the default barrier would then become time dependent and scupper the tractability of the model. Moreover equity holders may not have deep pockets or may not even be cohesively willing to inject funds into the firm in the proximity of distress. Why then should they want repayment of their credit, which is the opposite of injecting funds, in the first place? Finally at the time of repayment the value $R(V^*, \tau_o)$ of the claim associated with avoiding powers, which represents a potential liability for equity holders, seems a good proxy of the present value of the cash injections that equity holders can be expected to make to keep the firm solvent. Thus its seems of second order importance to explicitly model cash injections after repayment.

5.2 Equity holders' credit in the form of bonds

Equity holders' credit may simply be a loan, which the firm can decide to repay at any time. But equity holders' credit can also be in the form of callable bonds, which again the firm can repay before maturity. In considering the case of bonds we extend the previous analysis to a setting that is more typical for firms whose stock trades in the stock market. Even controlling equity holders or board directors of such firms can play the "repayment-game" we discussed before.

The case whereby equity holders' credit is in the form of bonds presents some important complications. In fact, whereas in the case of loans it is clear that equity holders are creditors toward the firm and arm's length creditors can value their claims accordingly, in the case of bonds this may not be clear, since it may not be possible to know who holds the bonds at any given time. This is particularly true when bond holders can retain anonymity as is the case of bearer bonds, which can change hands unnoticed. This lack of transparency and the possibility that equity holders may buy a substantial part of outstanding bonds from current bond holders pose significant pricing problems: the other bond holders would not know whether to price bonds considering or excluding repayment to equity holders. The pricing difference between the two cases could be substantial as shown above. Moreover notice that equity holders may well be willing to pay more for the bonds than the fair market price, since the bonds in their hands would be worth more than in anybody else's hands. This is because they can expropriate other bonds holders buy deciding early repayment of the bonds they have bought. In fact repayment of some bonds at V^* would amount to the firm exercising an out-of-the-money bond call option, i.e. to a gift to bond-holders. But if the called bonds are held by equity holders, equity holders are really making a gift to themselves. Indeed by calling the bonds at V^* they are making the maximum possible gift to themselves given the amount of bonds they have bought. Of course such is not the case if the bonds are held by others, in which case the bond would be called at a price above rather than below the current bond price. We have assumed one single repayment of equity holders' credit, but a succession of repayments are theoretically possible: we abstract from this case for simplicity.

The policy to call the bonds at V^* can help explain why callable straight bonds are called late (see e.g. Vu (1986), Longstaff and Tuckman (1994), King and Mauer (2000)), i.e. why they are called when their market prices are higher than their call prices. Of course such call policy presupposes that callable bonds are at least partly owned by equity holders and that they often tend to trade above their face value, especially when the risk of exercise of avoiding powers is limited. In fact King and Mauer (2000) find that none of the traditional explanations fits the average call policy they observe for callable straight bonds. Furthermore the same arguments are applicable to convertible bonds, even though Asquith (1995) showed that convertibles are not called as late as previously thought. As for convertible bonds owned by equity holders, conversion would clearly rule out the risk of depauperating the company through call and repayment, although conversion may take place at higher stock prices as the prospect of repayment at V^* increases the value of the "straight bond" component of the convertible.

If the bonds were neither callable nor puttable, but simply amortising, and if the amortisation were done by randomly selecting the bonds to be repaid at any given date, and if equity holders did not lend to the firm in any other way, then the problem of "expropriation through repayment" would disappear and the valuation of the firm's liabilities would be much simpler. Indeed the market may be willing to pay a premium for such "simplicity".

All these considerations suggest that, if a third party bond holder knew that he was selling callable bonds to a controlling equity-holder, he could ask a higher price than if he sold the bonds to anybody else. What has been said about bonds is also applicable to preferred stock. Moreover, what has been said under the assumption that $D_s(V)$ belongs to equity holders is also applicable if $D_s(V)$ belongs to a third party in secret agreement with equity holders. Equity holders and this third party could agree to share the gains due to repayment. This collusion would be particularly difficult to detect.

5.3 Distribution of capital reserves

The above analysis concerns no only loans and bonds held by equity holders, but also "capital reserves". Here by "capital reserves" we mean components of the book value of equity other than shareholders' capital and other than reserves accumulated through retained profit. Capital reserves are constituted by equity holders' injections of funds into the firm, without the duty for the firm to repay such funds. Of course no interest is paid to on capital reserves. But notice that the firm can, in absence of covenants that forbid it, repay capital reserves to equity holders. In fact what was said above for the case whereby $c_s = 0$ is applicable also to the distributions of such "capital reserves" formed with injections equity holders's funds. Thus the whole operation of injecting funds and repaying them can mimic an interest-free loan or interest-free callable bond and may explain why covenants limiting dividends limit also distributions of "capital reserves".

5.4 Dividends rather than debt repayment

It can be argued that equity holders could depauperate a distressed firm by simply distributing dividends rather than by having the firm repay the debt due to themselves. But in fact there may be important limitations to distributing dividends. As the firm approaches distress retained profits may have been eroded by losses. Debt covenants may restrict the amount of dividend distributions and dividends may be taxed when distributed. Distributions through reduction of share capital are subject to close scrutiny by the courts and typically require implicit consent by creditors. Instead the repayment of debt to equity holders offers a particularly quick and unencumbered way for equity holders to depauperate the firm. Finally, what seems particularly dangerous is that equity holders distribute dividends and finance such distribution by lending to the firm. This could transform equity reserves, which are junior to creditors, into debt that is effectively senior to other debt claims, in so far as it is repaid before bankruptcy.

5.5 Agency costs of debt

At this point it is worth mentioning that, by lending to their own firm, equity holders can generally align their interests more closely to those of arm's length creditors. The reason is that the payoffs to equity holders would become more similar to the payoffs of other creditors. In other words equity holders' loans can attenuate some agency costs like assets substitution or under-investment as studied by Jensen-Meckling (1976) and Myers (1977). But this alignment of interests and similarity of payoffs to those of arm's length creditors break down in case the loan is repaid before bankruptcy. In other words the equity holders' loan repayment considered in this paper entails that such loan no longer alleviates the mentioned agency costs associated with borrowing from arm's length creditors.

5.6 Optimal capital structure

Now we consider the effect of "repayment risk" on optimal capital structure. In fact in the previous analysis of "repayment risk" we have considered a realistic capital structure but not an optimal one. Optimal capital structure for given coupon rates $\frac{c}{F}$ and $\frac{c_s}{F_s}$ respectively for $D(V)$ and $D_s(V)$ is characterised as $[F^*, F_s^*]$ such that

$$[F^*, F_s^*] = \arg \max_{F, F_s, V^*} [D(V) + E(V) + D_s(V)] \tag{29}$$

$$st : V^* - x > V_d, V^* > V_{dd}, , V^* \leq V, F \geq 0, F_s \geq 0.$$

This means that, for a given F, equity holders will set F_s^* and V^* so as to maximise the value of their claims $E(V) + D_s(V)$ subject to the usual constraints listed above. By repeating this maximisation for all values of F we can find the optimal values F^*, F_s^* and V^*. Here we abstract from possible thin-capitalisation rules which may limit deduction of c_s for corporate tax purposes as F_s varies. Table 3 reports results using base case parameter values, with the only difference that F, F_s and V^* are now different form the base case.

[Table 3 about here]

As expected the optimal value $F^* + F_s^*$ implies a trade-off between minimising bankruptcy costs $B(V)$ and minimising the tax burden $T(V)$. As in the base case we assume that $k = 0$ so that $L(V) = 0$ for simplicity. The first column of Table 2 shows that, when $F = 1$, $V^* \simeq V_d + x$ since there is hardly any risk of avoiding powers. But when F grows the risk of avoiding powers increases so that $V^* > V_d + x$ because equity holders anticipate repayment.

The optimal capital structure is found for $20 \leq F^* \leq 25$: notice that in the table $E(V) + D(V) + D_s(V)$ is highest (63.45) for $F = 20$. Table 2 reveals that, as F rises, also the optimal values F_s^* and V^*, which maximise $E(V) + D_s(V)$, rise. V^* rises since higher F_s and F imply greater risk of avoiding powers: the avoiding powers payoff $R(V_d) = \min(\max(F - V_d(1-a)(1-t_c)(1-t_d), 0), F_s)$ increases in F_s and F. F_s^* rises with F because the more the debt the firm owes to arm's length creditors, the greater the opportunity for unscrupulous equity holders to expropriate arm's length creditors through repayment. This result highlights how optimal capital structure does not entail a direct trade-off between F_s^* and F. Of course this conclusion is mitigated by the risk of avoiding powers, which makes repayment to expropriate other creditors less effective.

6 Bank loans and personal guarantees

The above model can also be re-interpreted and extended to analyse and price the effect of guarantees conceded by controlling equity holders to banks that lend to the firm. For simplicity and as a reference point, we assume that equity holders are personally solvent even when the firm defaults. In other words the guarantee they give to the bank is itself default-free. Then we can re-interpret the above pricing model as follows. $D(V)$ is the value of debt held by a bank that has no guarantee. Then, assuming that $t_i = 0$, we can re-write

$$D_s(V) = D_{sb}(V) - G(V) \tag{30}$$

where

$$D_{sb}(V) = \frac{c_s}{r} + \left(-\frac{c_s}{r} + F_s\right)\left(\frac{V}{V^*}\right)^q \tag{31}$$

$$G(V) = R(V^* - x, \tau_o)\left(\frac{V}{V^*}\right)^q. \tag{32}$$

$D_{sb}(V)$ is the value of the guaranteed bank loan, which is going to be repaid at face value F_s when the value of the firm's assets equals V^*. In the absence of repayment before default, the loan would be repaid just at default when the value of assets equals V_{dd}. In the latter case the formula for $D_{sb}(V)$ would be the same as 31 but for the fact that V_{dd} would replace V^*. Notice that $D_{sb}(V)$ is essentially default-free in either case since either the full face value F_s is repaid at V^* or it is repaid at V_{dd} as the shareholders guarantee that the bank can always fully recover their credit, even in default.

$G(V)$ is the value of the claim associated with the loan guarantee offered by creditors to the bank. In fact repayment effectively eliminates the risk that the guarantee will have to be honoured, effectively liberating equity holders from their obligation toward the bank. In this way repayment reduced the total pool of assets available to satisfy creditors. Hence, in case bankruptcy follows repayment within one year, according to US courts the bankruptcy trustee could avoid repayment since repayment would then be qualified as fraudulent conveyance. For example Cullina (1991) explains this under a legal point of view. The trustee's right to avoid repayment entails a potential liability for equity holders and such potential liability is worth $G(V)$ before repayment and $R(V^* - x, \tau_o)$ at the very time of repayment.

Equation 30 suggests we can still use the model and results in the previous sections to analyse also the present setting. In particular the qualification of repayment as fraudulent conveyance protects the firm's other creditors and reduced the cost of borrowing from them. Equity holders will decide to repay the bank loan $D_{sb}(V)$ neither to early nor too late, again in order to reduce exposure to the risk of the bankruptcy trustee's avoid powers. The bank itself may be quite indifferent to repayment only if equity holders are certain to be personally solvent even if the firm they control enters bankruptcy. In drawing these conclusions we keep assuming that the bank loan $D_{sb}(V)$ would be subordinated to $D(V)$ in case of default. For example this can be the case if $D(V)$ is a secured loan due to a second bank that is not assisted by personal guarantees of the equity holders. Of course if the bank loan $D_s(V)$ was senior to $D(V)$ or secured on some of the firm's assets the incentive for equity holders to decide repayment before default would be much weaker, if any.

7 Conclusion

This paper has analysed the valuation of the firm's liabilities when equity holders are at the same time creditors. This is a frequent phenomenon especially in private companies and it has a material valuation effect on the firm's liabilities. By anticipating repayment of their own credit before the firm's default, equity holders can effectively make their credit senior to the claim of arm's-length creditors.

Possible revocation of repayment by the bankruptcy court protects arm's-length creditors, reduces the cost of borrowing and induces equity holders to repay their credit earlier. Early repayment is generally less detrimental to arm's-length creditors. Equity holders effectively have an option to "put" their loan bank to the firm, which they do not exercise too late in order to avoid excessive exposure to the risk of repayment revocation by the court. The tax shield due to equity holders' lending does not materially affect the repayment decision. Repayment is delayed as assets volatility increases and as the time period for the court's repayment revocation is lengthened.

The analysis is applicable not only to loans due by the firm to equity holders, but also to bank loans personally guaranteed by equity holders. Moreover the analysis is applicable to callable bonds and to callable preferred stock, whose value to equity holders may be higher than to anybody else, since equity holders can govern the call and repayment decision. Equity holders could buy the callable bonds at above market prices and then get the firm to call them. This could not be done with non-callable bonds.

The results of the paper underscore the importance of requiring that the firm's financial statements disclose the amount of debt due to equity holders. Finally company law and/or the bankruptcy code should envisage that the bankruptcy court be given the unconditional right to revoke at least debt repayments to equity holders occurred one or two years before bankruptcy filing. The longer such period, the stronger the protection of arm's length creditors against repayment and the less the cost of borrowing.

A Appendix

A.1 The model without repayment

In case of no repayment, standard no-arbitrage arguments entail the following equations and solutions. As for $D_s(V)$

$$\frac{\partial^2 D_s}{\partial V^2}\frac{1}{2}\sigma^2 V^2 + \frac{\partial D_s}{\partial V} V(r-b) - rD_s + c_s(1-t_i) = 0 \tag{33}$$
$$D_s(V \to \infty) \to \frac{c_s(1-t_i)}{r},\ D_s(V_{dd}) = \min\left(\max(V_{dd}(1-a)(1-t_c)(1-t_d) - F), F_s\right)$$

with solution

$$D_s(V) = \frac{c_s(1-t_i)}{r} + \left(-\frac{c_s(1-t_i)}{r} + \min\left(\max(V_{dd}(1-a)(1-t_c)(1-t_d) - F), F_s\right)\right)\left(\frac{V}{V_{dd}}\right)^q \tag{34}$$

where $q = \frac{-\left(r-b-\frac{1}{2}\sigma^2\right)-\sqrt{\left(r-b-\frac{1}{2}\sigma^2\right)^2+2r\sigma^2}}{\sigma^2}$ and $V_{dd} = \frac{c+c_s}{b}$. Notice that $D_s\left(V_{dd}\right)$ wold often equal 0. As for $D\left(V\right)$

$$\frac{\partial^2 D}{\partial V^2}\frac{1}{2}\sigma^2 V^2 + \frac{\partial D}{\partial V}V\left(r-b\right) - rD + c = 0 \tag{35}$$
$$D\left(V \to \infty\right) \to \frac{c}{r},\ D\left(V_{dd}\right) = \min\left(V_{dd}\left(1-a\right)\left(1-t_c\right)\left(1-t_d\right), F\right)$$

with solution $D\left(V\right) = \frac{c}{r} + \left(-\frac{c}{r} + \min\left(V_{dd}\left(1-a\right)\left(1-t_c\right)\left(1-t_d\right), F\right)\right)\left(\frac{V}{V_{dd}}\right)^q$.

As for $T\left(V\right)$

$$\frac{\partial^2 T}{\partial V^2}\frac{1}{2}\sigma^2 V^2 + \frac{\partial T}{\partial V}V\left(r-b\right) - rT + \left(Vb-c-c_s\right)\left(t_c + \left(1-t_c\right)t_d\right) + c_s t_i \tag{36}$$
$$T\left(V \to \infty\right) \to V\left(t_c + \left(1-t_c\right)t_d\right),\ T\left(V_{dd}\right) = V_{dd}\left(1-a\right)\left(\left(1-t_c\right)t_d + t_c\right)$$

with solution

$$\begin{aligned} T\left(V\right) &= \left(V - \frac{c}{r}\right)\left(t_c + \left(1-t_c\right)t_d\right) + \frac{c_s t_i}{r} \\ &\quad + \left(-\left(V_d - \frac{c}{r}\right)\left(t_c + \left(1-t_c\right)t_d\right) - \frac{c_s t_i}{r} + V_d\left(1-a\right)\left(\left(1-t_c\right)t_d + t_c\right)\right)\left(\frac{V}{V_{dd}}\right)^q. \end{aligned} \tag{37}$$

As for $B\left(V\right)$

$$\frac{\partial^2 B}{\partial V^2}\frac{1}{2}\sigma^2 V^2 + \frac{\partial B}{\partial V} V\left(r-b\right) - rB = 0 \tag{38}$$

$$B\left(V \to \infty\right) \to 0,\ B\left(V^*\right) = B'\left(V^* - x\right)$$

with solution

$$B\left(V\right) = aV_{dd}\left(\frac{V}{V^*}\right)^q. \tag{39}$$

A.2 The model with repayment

This section summasises the valuation of the various claims in case of repayment.

$D\left(V\right)$ satisfies

$$\frac{\partial^2 D}{\partial V^2}\frac{1}{2}\sigma^2 V^2 + \frac{\partial D}{\partial V} V\left(r-b\right) - rD + c = 0 \tag{40}$$

$$D\left(V \to \infty\right) \to \frac{c}{r},\ D\left(V^*\right) = D'\left(V^* - x, \tau_o\right)$$

and has the solution given by equation 23. $T\left(V\right)$ satisfies

$$\frac{\partial^2 T}{\partial V^2}\frac{1}{2}\sigma^2 V^2 + \frac{\partial T}{\partial V} V\left(r-b\right) - rT + \left(Vb - c - c_s\right)\left(t_c + \left(1-t_c\right)t_d\right) + c_s t_i = 0 \tag{41}$$

$$T\left(V \to \infty\right) \to V\left(t_c + \left(1-t_c\right)t_d\right),\ T\left(V^*\right) = T^{'}\left(V^* - x\right) + x\left(1-k\right)\left(t_c + \left(1-t_c\right)t_d\right)$$

and has the solution given by equation 24. $B\left(V\right)$ satisfies

$$\frac{\partial^2 B}{\partial V^2}\frac{1}{2}\sigma^2 V^2 + \frac{\partial B}{\partial V}V\left(r-b\right) - rB = 0 \tag{42}$$

$$B\left(V \to \infty\right) \to 0,\; B\left(V^*\right) = B'\left(V^* - x\right)$$

and has the solution given by equation 25. $L\left(V\right)$ satisfies

$$\frac{\partial^2 L}{\partial V^2}\frac{1}{2}\sigma^2 V^2 + \frac{\partial L}{\partial V}V\left(r-b\right) - rL = 0 \tag{43}$$

$$L\left(V \to \infty\right) \to 0,\; L\left(V^*\right) = xk$$

and has the solution given by equation 26.

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TABLE 1: BASE CASE SCENARIO (BENCHMARK)

	No repayment	With repayment	
(inputs in Italic)		**Before repayment**	**After repayment**
a (bankruptcy costs as fraction of V_d)	*20%*	*20%*	*20%*
r (default risk-free spot interest rate)	*4%*	*4%*	*4%*
(volatility of V)	*20%*	*20%*	*20%*
b (assets total payout to security holders)	*8%*	*8%*	*8%*
(suspect period before bankruptcy filing)		*1.00*	*1.00*
t_c (corporate tax rate)	*33%*	*33%*	*33%*
t_d (witholding tax on dividends)	*12.5%*	*12.5%*	*12.5%*
t_i (tax on interest payments)	*12.5%*	*12.5%*	*12.5%*
k (coefficient for liquidation costs due to repayment)		*0*	
C_s (annual coupon, which is paid continuously)	*1.3*	*1.3*	
Coupon rate: C_s/F_s	*5.0%*	*5.0%*	
C (annual coupon, which is paid continuously)	*1.3*	*1.3*	*1.3*
Coupon rate: C/F	*5.0%*	*5.0%*	*5.0%*
F_s (face value of arm's length debt)	*25.0*	*25.0*	
F (face value of debt)	*25.0*	*25.0*	*25.0*
V (assets value) before or after taxes	*100.0*	*100.0*	*100.00*
V* (repayment barrier)	-	**68.93**	
V_{dd} (default barrier without repayment)	31.25		
V_d (default barrier with repayment)		15.6	15.6
x (liquidated assets)		42.6	
q (endogenous constant)	- 0.6	- 0.6	- 0.6
L(V) (value of expected liquidation costs)		-	
L(V*)		-	
B(V) (value of expected bankruptcy costs)	3.3	1.9	
B(V*)		2.3	
B'(V) (value of bankruptcy costs after repayment)			1.1
T(V) (value of claim tax)	29.5	33.8	
T(V*)		24.3	
T'(V) (value of tax claim after repayment)			32.5
D_s(V) (value of equity holders' loan)	13.1	25.2	
D_s(V*)		24.7	
R(V,t) (claim due to avoiding powers)			-
D(V) (value of arm's length debt)	22.6	17.0	
D(V*)		13.7	
D'(V) (value of arm's length debt after repayment)			22.8
E(V) (value of equity)	31.5	22.1	
E(V*)		3.9	
E'(V) (equity value after repayment)			43.5
Total liabilities	100.0	100.0	100.0
Yield spread on D_s(V): (C_s/D_s-r)	5.53%	0.97%	
Yield spread on D(V): (C/D-r)	1.53%	3.34%	1.48%

TABLE 2: HIGHER INTEREST PAYMENTS ON SHAREHOLDERS' DEBT

	No repayment	**With repayment**	
(inputs in Italic)		**Before repayment**	**After repayment**
a (bankruptcy costs as fraction of V_d)	*20%*	*20%*	*20%*
r (default risk-free spot interest rate)	*4%*	*4%*	*4%*
(volatility of V)	*20%*	*20%*	*20%*
b (assets total payout to security holders)	*8%*	*8%*	*8%*
(suspect period before bankruptcy filing)		*1.00*	*1.00*
t_c (corporate tax rate)	*33%*	*33%*	*33%*
t_d (witholding tax on dividends)	*12.5%*	*12.5%*	*12.5%*
t_i (tax on interest payments)	*12.5%*	*12.5%*	*12.5%*
k (coefficient for liquidation costs due to repayment)		*0*	
C_s (annual coupon, which is paid continuously)	*3.8*	*3.8*	
Coupon rate: C_s/F_s	*15.0%*	*15.0%*	
C (annual coupon, which is paid continuously)	*1.3*	*1.3*	*1.3*
Coupon rate: C/F	*5.0%*	*5.0%*	*5.0%*
F_s (face value of arm's length debt)	*25.0*	*25.0*	
F (face value of debt)	*25.0*	*25.0*	*25.0*
V (assets value) before or after taxes	*100.0*	*100.0*	*100.00*
V* (repayment barrier)	-	**67.42**	
V_{dd} (default barrier without repayment)	62.50		
V_d (default barrier with repayment)		15.6	15.6
x (liquidated assets)		42.6	
q (endogenous constant)	- 0.6	- 0.6	- 0.6
L(V) (value of expected liquidation costs)		-	
L(V*)		-	
B(V) (value of expected bankruptcy costs)	9.6	1.9	
B(V*)		2.4	
B'(V) (value of bankruptcy costs after repayment)			1.1
T(V) (value of claim tax)	28.1	30.3	
T(V*)		23.9	
T'(V) (value of tax claim after repayment)			32.5
D_s(V) (value of equity holders' loan)	22.3	35.8	
D_s(V*)		24.3	
R(V,t) (claim due to avoiding powers)			-
D(V) (value of arm's length debt)	26.4	17.0	
D(V*)		13.5	
D'(V) (value of arm's length debt after repayment)			22.8
E(V) (value of equity)	13.5	15.0	
E(V*)		3.3	
E'(V) (equity value after repayment)			43.5
Total liabilities	100.0	100.0	100.0
Yield spread on D_s(V): (C_s/D_s-r)	12.79%	6.48%	
Yield spread on D(V): (C/D-r)	0.73%	3.35%	1.48%

TABLE 3 : OPTIMAL CAPITAL STRUCTURE AND REPAYMENT

V_d + x (default barrier + sold assets)	46.2	50.2	54.8	59.2	63.3	67.3	71.1	74.8	78.4	78.0	78.8
V* (repayment barrier)	46.6	52.2	58.8	65.3	71.6	77.8	83.9	89.9	95.8	97.6	100.0
F (face value of D(V))	1	5	10	15	20	25	30	35	40	45	50
F_s (face value of D_s(V))	27	28	28	29	30	30	31	31	31	29	28
E(V)+D(V)+D_s(V)	62.21	62.70	63.13	63.37	63.45	63.39	63.19	62.88	62.45	62.57	62.41
E(V) (equity value)	33.82	30.59	26.83	23.35	20.09	17.03	14.15	11.43	8.85	8.61	7.81
D(V) (arm's length debt value)	0.79	3.80	7.26	10.43	13.33	16.00	18.44	20.67	22.70	25.18	27.27
D_s(V) (debt value of equity holders)	27.60	28.32	29.03	29.59	30.03	30.36	30.61	30.78	30.89	28.78	27.33
C_s/F_s (coupon rate of D_s(V))	5%	5%	5%	5%	5%	5%	5%	5%	5%	5%	5%
C/F (coupon rate of D(V))	5%	5%	5%	5%	5%	5%	5%	5%	5%	5%	5%

Figure 1: Value of arm's length credit before repayment as a function of V and V* in the presence of avoiding powers (base case scenario)

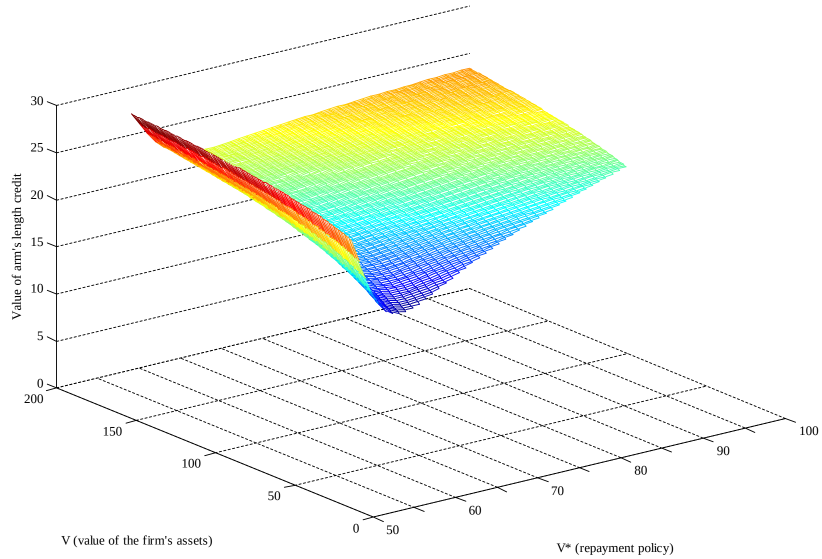

Figure 2: Value of equity holders's loan before repayment as a function of V and V* in the presence of avoiding powers (base case scenario)