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Book Values and Market Values of Equity and Debt

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BOOK VALUES AND MARKET VALUES OF EQUITY AND DEBT

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Abstract

This paper proposes a contingent claims model to value a firm's debt and equity as functions of observable book values appearing in published financial statements. Equity fair value critically depends on expected earnings, equity book value and earnings volatility, because of the options to default or to voluntarily liquidate the firm. Debt value increases in earnings volatility in the proximity of default. Default is triggered by the erosion of equity due to negative earnings. Debt and equity values are materially affected by the strength of the mean reversion of profitability. Voluntary liquidation before default may be optimal and it entails that a sudden sharp decline in profitability can be less detrimental to creditors than a slower but persistent one.

Key words: book values, mean reverting return on assets, equity valuation, debt valuation, default option, structural models, voluntary liquidation.

JEL classification: G13;G33.

1 Introduction

This paper presents a contingent claims model to value a firm's equity and debt as functions of observable book values that appear in financial statements. This model attempts to make fuller use of information in financial statements than the existing structural models in literature do. This attempt has practical advantages.

The model is based on observable earnings. Market values of assets and equity are driven by expected earnings, which is consistent with the theory and with the way equity analysts value equity. But market values of assets and equity also depend on the equity holders' option to default or to voluntarily liquidate the firm. Multiples like price-earnings and price-to-book are endogenous to the analysis: a feature that can be used in model calibration. Since the model is

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based on book values, it is immediately applicable even to firms whose stock is not traded in the market. The focus on book values permits to model the dividend policy and debt covenants more accurately than in existing structural models. In fact debt covenants are normally expressed in terms of book values. Default is also made to depend on book values. The model is consistent with the way accounting ratios are used to predict bankruptcy. No estimate of the unobservable market value of the firm is required. Rather the unobservable market value of the firm's assets is endogenous to the model. The earnings process is consistent with empirical evidence: return on assets is mean reverting and negative operative earnings are possible.

All these advantages come at the cost of not having closed form solutions for debt and equity values. Instead numerical solutions are necessary, but the computations are affordable.

The model provides theoretical insights. The options to default or liquidate the firm cause equity value to exceed the present value of expected future earnings. This is especially true in the proximity of default. Earnings volatility increases equity value and typically decreases debt value. But debt value tends to increase in earnings volatility in the proximity of default. This somehow challenges the assets-substitution argument, which suggests that near default the risk-shifting incentive is strongest. Equity value materially increases (decreases) with the strength of the mean reversion of profitability, when profitability is below (above) its long term level. Debt value typically rises with the strength of profitability mean reversion.

It may be in the equity holders' interest to voluntarily liquidate the firm before default. This implies that a slow decline in profitability and small persistent losses may be more dangerous for debt holders than a sudden large decline in profitability and sudden large losses. The reason is that sudden losses would trigger voluntary liquidation, while small persistent ones would lead to default.

The paper is organised as follows. The next section reviews the relevant literature. Then the valuation model is presented. Then the effect of possible voluntary liquidation is analysed. The conclusions follow.

2 Literature

Most theoretical structural models, e.g. Leland (1994, 1998), Leland and Toft (1996), Mella-Barral and Perraudin (1997), Anderson and Sundaresan (1996), Goldstein-Ju-Leland (2001), make limited use of the accounting numbers available in published financial statements. Typically they just use the book value of outstanding debt and cash flows or earnings from operations. Instead the building block of the model in this paper is the stochastic process of the return on assets. This starting point is similar to that in Brennan and Schwartz (1984). But the process of the return on assets is here assumed to be mean reverting in keeping with the relevant empirical literature, e.g. Fama and French (2000) and Penman and Nissim (2001).

The model endogenously prices the option to default implicit in equity prices.

This seems important in light of the empirical evidence in Brockman and Turtle (2003), who show that stock prices imply that equity is much better modelled as a down-and-out call option than as a simple European call option. In other words default barriers are economically and statistically significant in equity valuation.

The model of this paper has also been inspired by Davydenko (2005), who provided empirical evidence about the level and determinants of the default barrier. He found that liquidity and access to external finance were important determinants of default. The higher is the liquidity the lower the firm's market value at default. As suggested by his results, also in this paper default can take place at different possible firm values.

The model can also explain the determinants of the voluntary liquidation decision. Voluntary liquidation is a prominent phenomenon even for firms whose stock is publicly traded as reported in a series or empirical papers by Erwin and McConnell (1997), Mehran-Nogler and Schwartz (1998), Sullivan-Crutchley and Johnson (1997), Hite-Owers and Rogers (1987), Kim and Schatzberg (1987). Voluntary liquidation is beneficial to creditors.

3 The valuation model

This section introduces a contingent claims valuation model based on the book values of earnings, equity and debt observable in published financial statements. Hereafter V is the market value and A the book value of the firm's equity. D is the market value and F the face value of the firm's debt. F is assumed to be constant over time. The building block of the model is the annualised rate of return on the book value of assets (ROA), which we denote with R. To fix ideas we denote the present date with t_0 and set equally spaced future times t_i with $i = \{1, .., \infty\}$ and with $t_i - t_{i-1} = \Delta$. Δ could be arbitrarily small. At any time t_i the firm records earnings before interest and taxes (EBIT) equal to $R_{t_i}\left(A_{t_{i-1}} + F\right)\Delta$, since EBIT at t_i is assumed equal to ROA at t_i times the book value of assets at t_{i-1}, which is $A_{t_{i-1}} + F$. At any time t_i the firm has to pay interest equal to $cF\Delta$. R is assumed to follow a continuous process that only becomes observable at times t_i, when earnings are recorded. The real world process for R is assumed to be

$$dR = a\left(b' - R\right)dt + \sigma dz \tag{1}$$

where σ is the volatility parameter and dz is the differential of a Wiener process. This process for R is consistent with the empirical evidence that documents mean reverting profitability and possible operating losses. In fact R can become negative according to equation 1. In the risk-neutral world R follows the process

$$dR = a\left(b - R\right)dt + \sigma dz. \tag{2}$$

To clarify this assumption, we can assume that the R-risk premium is a constant λ such that

$$b' = \frac{ab - \sigma\lambda}{a}. \tag{3}$$

Hereafter we are just concerned with the risk-neutral process of R for valuation purposes. At any time t_i equity holders receive an amount of dividends equal to

$$d_{t_i} = \Delta \cdot \max\left(R_{t_i}\left(A_{t_{i-1}} + F\right) - cF, 0\right) \cdot 1_{A_{t_{i-1}} > A^*} \tag{4}$$

where $1_{A_{t_{i-1}} > A^*}$ is the indicator function of the event $A_{t_{i-1}} > A^*$. This equation states that all EBIT net of interest, if positive, is distributed as dividends, provided that the book value of equity $A_{t_{i-1}}$ is higher than the level A^*. A^* may be determined by a debt covenant requiring a minimum level of retained earnings or by the company law requirement that the firm maintain a minimum share capital level as a pre-condition for the firm to be able to distribute dividends. At any time t_i the change in the book value of equity is

$$A_{t_i} = A_{t_{i-1}} + \Delta \cdot \begin{pmatrix} \max\left(R_{t_i}\left(A_{t_{i-1}} + F\right) - cF, 0\right) \cdot 1_{A_{t_{i-1}} \leq A^*} \\ + \min\left(R_{t_i}\left(A_{t_{i-1}} + F\right) - cF, 0\right) \end{pmatrix}. \tag{5}$$

This equation states that when the book value of equity is lower than A^*, dividends cannot be distributed and all positive net earnings are retained. Negative earnings erode the book value of equity. Equity book value represents a buffer of reserves that enables the firm to absorb losses. It is worth stressing that equation 5 implies that negative earnings erode the book value of the firm's assets and hence the scale operations. Instead positive earnings, in so far as they are retained, increase assets and hence the scale of operations.

Equations 4 and 5 imply that at any time t_i equity holders receive a payoff Q_{t_i} equal to

$$Q_{t_i} = d_{t_i} + V\left(R_{t_i}, A_{t_i}, t_i\right) - V\left(R_{t_i}, A_{t_{i-1}}, t_i\right). \tag{6}$$

$V\left(R_{t_i}, A_{t_i}, t_i\right) - V\left(R_{t_i}, A_{t_{i-1}}, t_i\right)$ is the change in equity value due the change in the book value of equity at time t_i. Notice that when $d_{t_i} = 0$, $V\left(R_{t_i}, A_{t_i}, t_i\right) - V\left(R_{t_i}, A_{t_{i-1}}, t_i\right) \neq 0$ and when $V\left(R_{t_i}, A_{t_i}, t_i\right) - V\left(R_{t_i}, A_{t_{i-1}}, t_i\right) = 0$, $d_{t_i} \neq 0$.

The firm may default at any time t_i. The majority of the literature on structural models assumes that equity holders inject funds into the firm and that default it triggered when equity holders stop such injections as the market value of equity V drops to 0. Although a tractable assumption, this does not seem the way default is actually triggered, especially in the case of firms whose stock is traded in the market. Thus here we abstract from equity injections. If equity holders do not inject funds into the firm, V cannot become negative. Thus we assume that default is triggered when losses have eroded the book value of equity down to the level A^{**}. For now we set $A^{**} = 0$, so that default takes place as soon as

$$A_{t_i} \leq 0. \tag{7}$$

This condition is roughly consistent with the provisions of the Belgian, Italian and Swedish bankruptcy codes, which envisage the erosion of share capital as a condition for triggering bankruptcy or liquidation and is also somewhat consistent with the UK, German, French and Swiss codes, which refer to the excess of liabilities over assets as a condition to trigger bankruptcy (see Altman and Hotckiss (2006) at chapter 2). Condition 7 is also consistent with positive net worth debt covenants, which are discussed for example in Leland (1994). At default the firm's assets are liquidated and the recovery value of debt is

$$D_{t_i}^{**} = \min\left(\left(A_{t_i} + F\right)(1-\alpha), F\right) \tag{8}$$

where α denotes proportional bankruptcy costs. Liquidation proceeds net of liquidation costs are $\left(A_{t_i} + F\right)(1-\alpha)$. So liquidation proceeds depend on the book value of assets and are used to pay debt at the face value F. The absolute priority rule is applied.

In the present setting the absence of arbitrage opportunities implies that $V(R, A, t)$ satisfies the following equation and conditions

$$\frac{\partial V}{\partial t} + \frac{\partial^2 V}{\partial R^2}\frac{1}{2}\sigma^2 + \frac{\partial V}{\partial R}a(b-R) - rV + \delta(t-t_i)\cdot Q_{t_i} = 0 \tag{9}$$

$$\lim_{R\to-\infty} V(R, A, t) \to 0 \tag{10}$$

$$\lim_{R\to\infty} V(R, A, t) \to \infty \tag{11}$$

$$V(R, A=0, t_i) = 0 \tag{12}$$

$$V(R, A, t_i = 100) = \max\left(\left(A_{t_i} + F\right)(1-\alpha) - F, 0\right) \tag{13}$$

where $\delta(t-t_i)$ denotes Dirac's delta function. The first condition states that, because of limited liability, the market value of equity tends to 0 as the return on assets becomes infinitely negative. The second condition states that the market value of equity tends to infinity as the return on assets tends to infinity. The third condition states that the market value of equity vanishes to 0 when its book value is completely eroded and default is triggered. The firm can be liquidated only on the dates t_i, since just on such dates earnings are recorded as well as the erosion of the book value of equity. The final condition states that after 100 years equity holders liquidate the firm's assets and pay F to debt holders, in so far as liquidation proceeds suffice. This terminal condition seems a reasonable approximation of the fact that equity is typically a perpetuity. By similar no arbitrage arguments the market value of debt D satisfies

$$\frac{\partial D}{\partial t} + \frac{\partial^2 D}{\partial R^2}\frac{1}{2}\sigma^2 + \frac{\partial D}{\partial R}a\left(b-R\right) - rD + \delta\left(t-t_i\right)\cdot\Delta\cdot cF = 0 \tag{14}$$

$$\lim_{R\to-\infty} D\left(R,A,t\right) \to 0 \tag{15}$$

$$\lim_{R\to\infty} D\left(R,A,t\right) \to \frac{cF}{r}\left(1-e^{-r(100-t)}\right) \tag{16}$$

$$D\left(R,A=0,t_i\right) = F\left(1-\alpha\right) \tag{17}$$

$$D\left(R,A,t_i=100\right) = \min\left(\left(A_{t_i}+F\right)\left(1-\alpha\right),F\right) \tag{18}$$

The first condition states that when the return on assets approaches minus infinity, the market value of debt approaches 0 because the liquidation value of assets is completely eroded. The second condition states that debt becomes default free as the return on assets approaches infinity. The third condition states that debt value is equal to the recovery value of assets when the book value of equity becomes 0. The final condition states that after 100 years the firm is liquidated and debt holders recover the face value of debt if the liquidation proceeds are sufficient.

There exists no known closed form solution to equations 9, 14 and to the respective conditions. Thus we need to resort to the implicit finite difference method to solve the equations numerically. Next the predictions of the numerical solutions to above equations are illustrated.

3.1 Model predictions

Simulations using the above model provide theoretical insights, which are now examined. Figure 1 shows the value of equity V in a base case scenario that assumes $\sigma = 0.05$, $a = 0.1$, $b = 0.1$, $r = 0.05$, $F = 1$, $A^* = 0.5$, $\alpha = 0.2$, $c = 0.06$. Choosing 120 time steps per year (i.e. $\Delta = 1/120$) in an implicit finite difference grid guarantees the numerical stability of the solution. In fact, although the finite difference algorithm is unconditionally stable, rounding errors may cause instability if fewer time steps are used. Figure 1 shows that the market value of equity V rises with the return on assets R and with the book value of equity A. The primary reason is that expected future earnings rise in R and A. Expected earnings rise in A, since default becomes less likely. Of course the probability of default decreases in R and A.

V rises with the long-term level b of expected future earnings. It is worth mentioning that b depends not only on real world long term earnings expectations, but also on λ, the risk premium demanded by the market for exposure to R-induced risk. Generally V rises with the variance of expected earnings, since V is convex in R. The variance of earnings rises in the parameter σ and decreases in the mean reversion parameter a. Thus V rises in σ, while the effect of a on V mainly depends on the effect of a on the level of expected earnings. As Figure 1.b suggests, V will typically decrease (increase) in a when R is greater (less) than the long term level of profitability b.

Figure 1: Market value of equity V as a function of return on assets R and equity book value A.

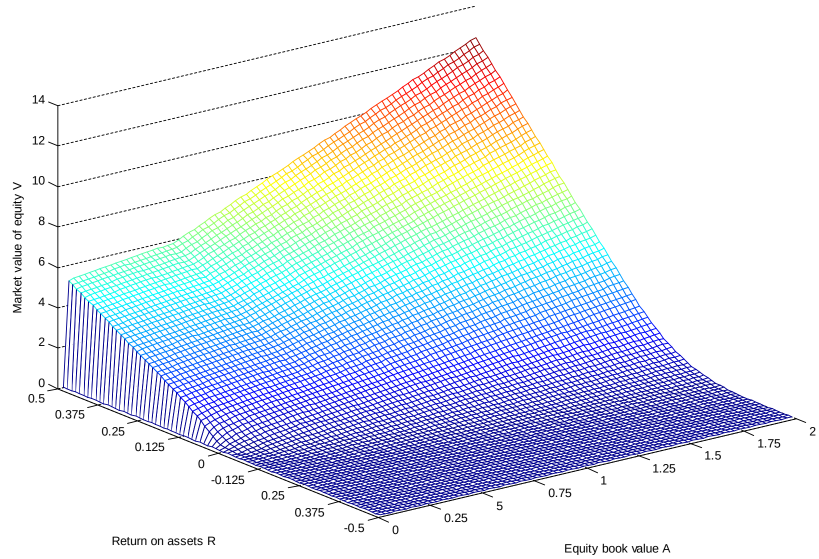


Figure 1.b: Market value of equity V as a function of profitability R and book value of equity A (mean reversion parameter a = 0.5)

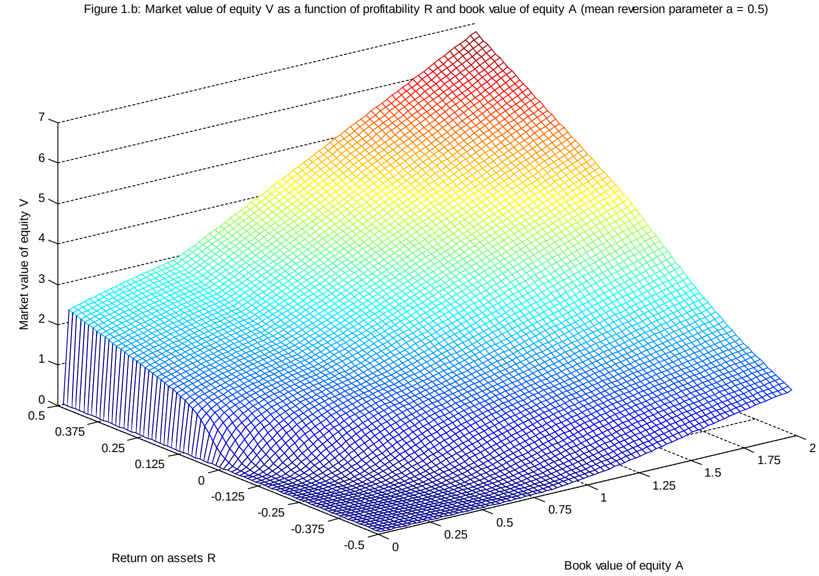
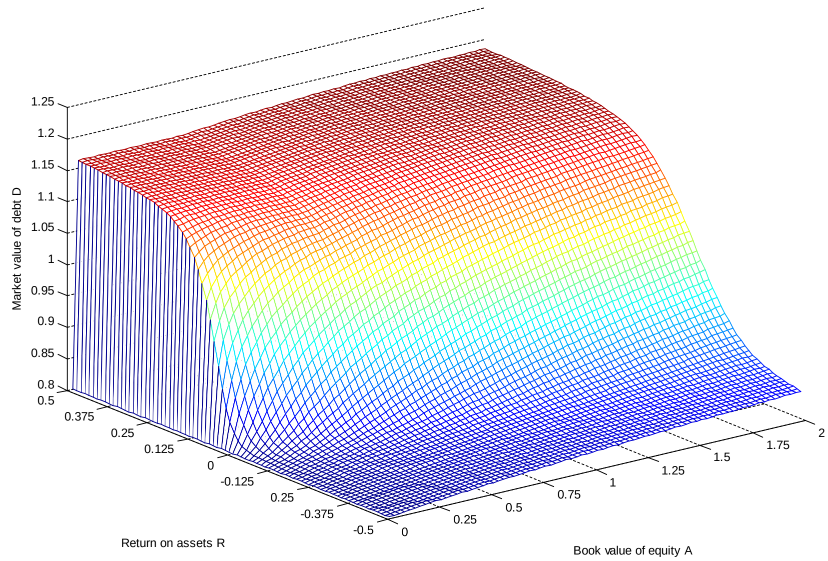


Figure 2: Market value of debt D as a function of return on assets R and book value of equity A.



As expected, V decreases in the interest rate r. The parameter A^* captures the level of A below which the firm does will not distribute any dividends. Equity value V typically decreases in A^*, because the constraint on the dividend policy becomes tighter as A^* rises. Overall Figure 1 suggests that the price-to-book ratio V/A, a popular measure to characterise stocks, should be carefully interpreted in the light of the model parameters. The same can be said for the price-earnings ratio $V/\left(R_{t_i}\left(F+A_{t_{i-1}}\right)-cF\right)$.

Figure 2 displays the results for debt market value D in the base case scenario. D rises with profitability as measured by R and with the size of the equity cushion A. D rises in b, since expected future earnings rise in b, and decreases in the bankruptcy cost parameter α. D rises in A^*, since the dividend policy is less generous as A^* rises and generally decreases in the variance of earnings, i.e. in σ. This is the case in the region where D is concave in R and A, which is when default is a relatively remote prospect. But D can also increase in σ, especially when default is imminent. This is confirmed by Figure 2, which shows how D becomes convex in R when R and/or A are low and default becomes a more likely prospect. It is such convexity that causes debt D to increase in σ. Convexity is caused by the fact that the firm does not default as soon as its earnings turn negative, but only after its equity reserves have been completely eroded by losses. It follows that the risk-shifting argument in the proximity of default, suggested by Jensen and Meckling (1976), is not really convincing in this setting and its importance may have been overstated in the past theoretical literature.

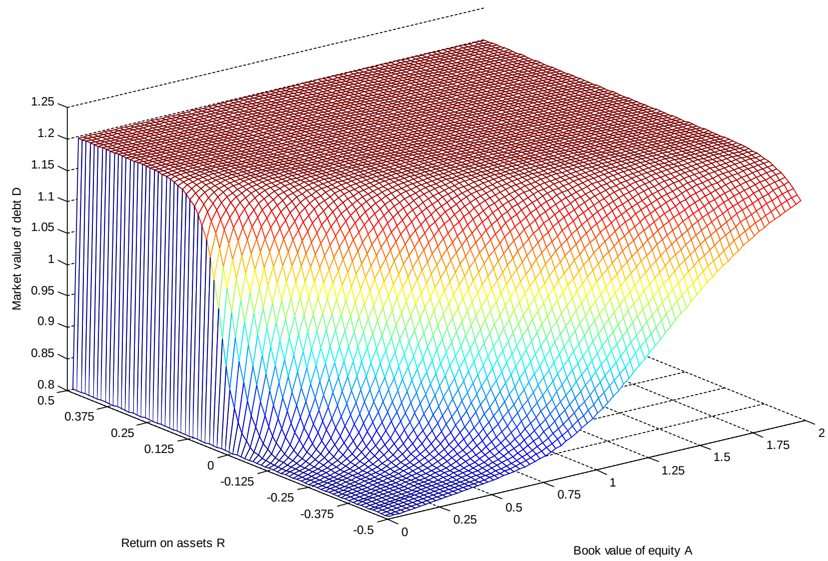


Figure 2.b: Market value of debt D as a function of profitability R and book value of equity A (mean reversion parameter a = 0.5).

The mean reversion parameter a affects debt value D because it affects expected earnings as well as the volatility of future earnings. As shown in Figure 2.b, generally D tends to rise in a, since expected future earnings would increase and earnings volatility would decrease.

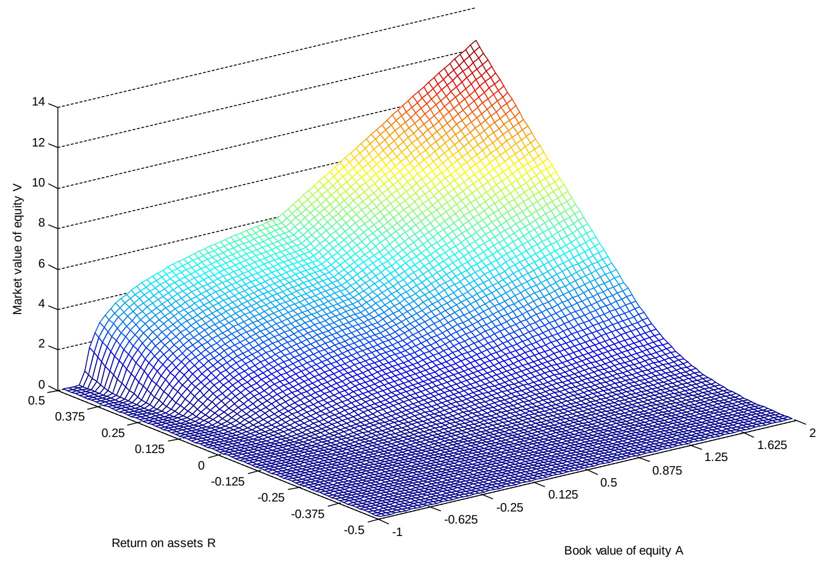
The simulations of this section highlight the material impact on debt and equity values of mean reversion in profitability.

3.2 When the book value of equity can turn negative

Up to know we have maintained the assumption that bankruptcy is triggered as soon as the book value of equity turns negative. In some jurisdictions, notably in the U.S., firms can keep trading even though the book value of equity has turned negative. Now we consider this case. As a benchmark, it is instructive to consider the extreme case whereby default and bankruptcy can only take place when the firm's assets have been totally depleted. This would be the case of a firm whose assets can all be quickly turned into cash and the cash can be used to honour debt service and absorb the operating losses. Although not such an extreme case, L.A.Gear's bankruptcy is a case in point as illustrated by DeAngelo-DeAngleo and Wruck (2002). L.A.Gear was able to stave off bankruptcy for years because it could liquidate its assets and at the time of bankruptcy creditors were left with very few assets.

Figures 3 and 4 show the results for equity and debt values assuming base case scenario parameters but for the fact that bankruptcy is triggered as soon

Figure 3: Market value of equity V as a function of profitability R and book value of equity A. A can be negative.



as the book value of assets drops to the level A^{**} such that $A^{**} + F = 0$. As before V is guaranteed to be non-negative if equity holders do not inject funds into the firm to stave-off distress.

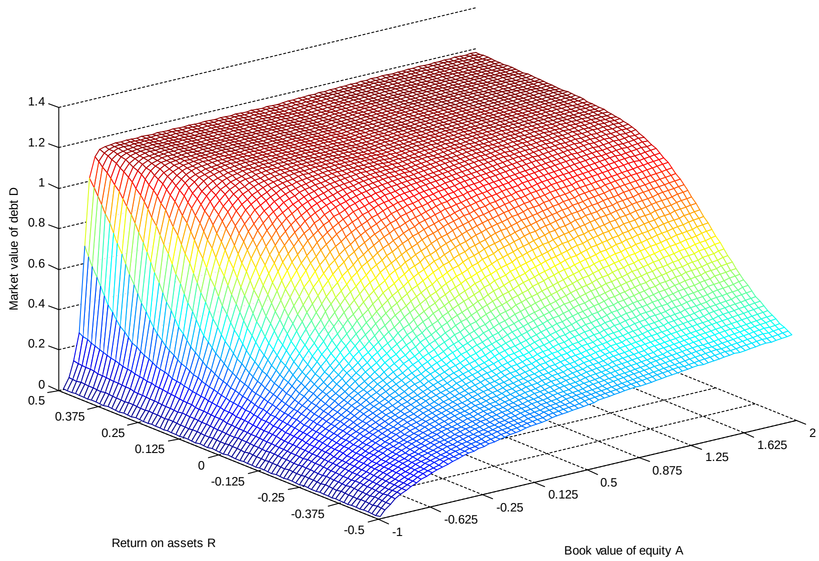
Figure 3 shows how equity value V becomes concave in A when $A < 0$ and R is relative high. In fact, when the book value of equity is negative because of accumulated losses, but profitability has recovered so that R is high, equity is still valuable because of the positive earnings that are expected in the future. In this situation an increase in the variance of earnings is detrimental to equity holders as is underscored by the local concavity of V in A. In fact higher earnings variance makes it more likely that new losses be incurred that would further erode A and trigger bankruptcy, which would make equity worthless.

Figure 4 displays debt value as a function of R and A. When $A = A^{**} = -F$, the firm's assets drop to 0 and debt becomes worthless. When $A < 0$ debt value can still be very high if profitability as measured by R is high. Again this is the case if previous losses have eroded the book value of equity but current profitability has sharply recovered.

In a less extreme default scenario than that of Figures 3 and 4, A^{**} is greater than $-F$ and is determined by the drying up of the firm's liquid assets such as cash and receivables. In this respect Davydenko (2005) highlights how liquidity as measured by the quick ratio is an important determinant of the default event.

Indeed equity holders have all the interest to default as late as possible if they do not inject new funds into the firm. This is the case even when the book value of equity has turned negative. This may lead to excessive continuation

Figure 4: Market value of debt as a function of profitability R and equity book value A. A can be negative.



of the firm's trading, which entails agency costs of the type already described in Mella-Barral and Perraudin (1997): leverage may lead equity holders, and especially unscrupulous equity holders, to follow a non-firm value maximising liquidation policy.

Of course in reality continuation until complete depletion of the firm's assets is not observed. To prevent excessive continuation, among other things, are the hopefully high ethical standard of equity holders or the fear of being accused of wrongful trading.

Anyway the present analysis suggests the higher A^{**} is, the safer and more valuable debt is. Thus there may be a case in favour of company law provisions that require a minimum level of the book value of equity in order for the firm to keep trading. Such provisions effectively set $A^{*} \geq 0$. In the absence of these provisions equity holders may be tempted to incur losses in so far as the firm's assets can be liquidated to postpone the state of insolvency.

3.3 Equity valuation: comparison with the practice

The above model suggests a way to use financial statements to value a firm's stock. Also financial analysts use financial statements to values stocks, but they employ simpler models. For example Penman (2003) suggests that, in its most basic version, a firm's equity value at time t_0 is given by the formula

$$V_{t_0} = A_{t_0} + \frac{(ROE - \rho)}{\rho} \tag{19}$$

where ROE is constant and stands for the return on the book value of equity and ρ is the required return on equity in equilibrium. This formula poses the well known problem of estimating ρ and ignores the equity holders's option to default. By contrast the model in this paper assumes a mean reverting stochastic process for the firm's profitability, employs risk-neutral valuation and takes the valuable default option into account. Thus the old problem of estimating ρ is substituted by the new problem of estimating the risk-neutral dynamics of assets profitability as measured by R. To contrast this valuation approach with the traditional one, we can invoke the Feynman-Kac theorem. This tells us that, if for simplicity we set $A^* = A^{**}$, the equity valuation problem of this section can be re-stated as

$$V_{t_0} = E_{t_0}\left[\sum_{i=1}^{\infty} e^{-r(t_i - t_0)} \left(R_{t_i}\left(F + A_{t_{i-1}}\right) - cF\right) \cdot 1_{\tau > t_i}\right] \tag{20}$$

where the time t_0 expectation $E_{t_0}[..]$ is with respect to the risk-neutral measure and τ is the time of default. This risk-neutral valuation approach has the main advantage to permit us to value the default option.

3.4 Distress prediction

The proposed model can also be used to predict default and bankruptcy. More precisely the model can be used to estimate default probabilities over any time horizon. Of course such probabilities would be calculated numerically and would be based on the real rather than on the risk-neutral process of R. The model can map default probabilities to the values of accounting ratios that can predict default. For example, Zmijewski (1983) suggested that the ratios most useful, if taken individually, to predict bankruptcy are:

- profitability as measured by return on equity, which in our model can be captured by $\frac{R_{t_i}\left(F+A_{t_{i-1}}\right)-cF}{A_{t_i}}$;

- volatility of profitability measured by the standard deviation of return on equity, which is our model is driven by a and σ as well as leverage;

- financial leverage measured using the market value of equity; in our model this measure of leverage corresponds to $V/(F+V)$.

3.5 Calibration, estimation and multiples

The calibration of the above model to market data is facilitated by the fact that the model depends on observable book values. R, A, F are observable from published financial statements. a, b and σ can be estimated from the time series of earnings of the same or of similar companies. More difficult it is to estimate these parameters from the time series of the firm's stock price, mainly because the valuation equations can only be solved numerically. The risk premium λ can be set to match the observed price-to-book or price-earnings multiples of the same firm or of similar firms. Calibration to observed multiples is possible because debt and equity values are expressed in terms of book values, in other words because the model explains and predicts V/A and $V/\left(R_{t_i}\left(F + A_{t_i}\right) - cF\right)$. Calibration to multiples seems an advantage of the above model over existing structural models in the literature.

It is worth highlighting that the predicted price-to-book ratio V/A critically depends on the equity holders' options to default. Ignoring such option leads to undervaluing V and hence the price-to-book ratio V/A. This is especially the case when present or expected future earnings are low and default is likely. Other multiples can also be used in calibration. Moreover multiples have the convenient property of usually following stationary processes over time.

3.6 Debt covenants

The proposed model is particularly suitable to estimate the impact of debt covenants on the values of debt and equity. In fact the model is based on book values and debt covenants are typically expressed in terms of book values. Covenants intend to protect debt holders and are more frequent and stringent in the case of loans than in the case of public bonds. Typical covenants require that the interest cover ratio ($\frac{cF}{R_{t_i}\left(A_{t_{i-1}}+F\right)}$) or the book value of equity (A_{t_i}) be above some minimum levels or that the book value of gearing ($\frac{F}{A_{t_{i-1}}+F}$) be below some maximum level. The violation of such protective covenants technically is a default event and gives the creditor the right to demand immediate repayment of principal and accrued interest. But the breach of a loan covenant normally leads to debt renegotiation rather than bankruptcy filing.

As for minimum net worth covenants, also company law may also impose a minimum net worth level A^{**}. For example the firm may have to stop trading if its net assets fall persistently below the share capital level.

Often other debt covenants constrain dividend distributions on part of equity holders. For example Smith and Warner (1979) document how covenants often require that dividends not be distributed when $A < A^*$, where A^* is a level of the book value of equity set in the covenant. The higher A^* is, the safer debt is and the less valuable equity is. Typically company law imposes that share capital cannot be distributed as dividends. Thus, even in the absence of a covenant setting A^*, company law would set A^* equal to the amount of share capital.

As is apparent from the above discussion, common covenants can be easily modelled in terms of the variables appearing in the model proposed above.

4 Voluntary liquidation and default

The proposed model can distinguish voluntary liquidations, whereby equity holders or the board decide to liquidate the business, and defaults, which are followed by forced liquidations. The previous section did not consider voluntary liquidation, but this section does. The omission of voluntary liquidation may correspond to the case whereby managers do not act so as to maximise equity value. In fact Erwin and McConnell (1997) provide evidence that publicly traded firms are typically voluntarily liquidated only if liquidation increases equity value and managers share in the benefit of liquidation through a significant equity ownership stake.

If voluntary liquidation is possible and is decided so as to maximise equity value, equity holders or the board liquidate the firm at any time t_i as soon as

$$V\left(R_{t_i}, A_{t_i}, t_i\right) < \max\left(\left(A_{t_i} + F\right)(1-\alpha) - F, 0\right) \tag{21}$$

where the right hand side of this inequality is the payoff to equity holders in case of liquidation at t_i and the left hand side is the market value of equity in case of continuation. Again the assumption is that liquidation proceeds are used to pay debt at the face value F and that the absolute priority rule is strictly enforced. Again equity holders do not inject funds into the firm, so that V is always non-negative. Notice that default is still possible since it may well be the case that $V\left(R_{t_i}, A_{t_i}, t_i\right) \geq \max\left(\left(A_{t_i} + F\right)(1-\alpha) - F, 0\right)$ up to a point where $A_{t_i} \leq 0$. As soon as $A_{t_i} \leq 0$, again default and forced liquidation are triggered. Voluntary liquidation introduces the following condition for the purposes of debt valuation

$$1_{V(R,A,t_i)<\max\left(\left(A_{t_i}+F\right)(1-\alpha)-F,0\right)} \cdot \left[D\left(R, A, t_i\right) = \min\left(\left(A_{t_i} + F\right)(1-\alpha), F\right)\right]. \tag{22}$$

This condition states that when equity holders voluntarily liquidate the firm's assets, debt holders recover the minimum between the face value of their credit and the net liquidation proceeds.

Figure 5 shows the joint effect on equity value V of the options to default and to liquidate the firm voluntarily. In particular it may be rational for equity holders to voluntarily liquidate the firm before default. The probability of voluntary liquidation and the liquidation payoff increase in A and decrease with the size of liquidation costs. Notice that, to improve exposition, Figures 5 and 6 assume that $\alpha = 0.4$, while all other parameters as the same as in the base case of the previous section.

4.1 Path to default and to voluntary liquidation

The model paradoxically predicts that, when equity book value A is high, debt holders should fear a slow decline in profitability more than a fast decline, because a slow decline increases creditors' expected default loss. More precisely, small persistent operative losses can harm creditors more than sudden huge operative losses when the book value of equity is high. In fact, if A is high and earnings rapidly plunge and turn significantly negative, equity holders will want to liquidate the firm immediately, whereupon creditors would receive the full face value $F = \min\left((1-\alpha)(A+F), F\right)$. But if, A being high, R decreases slowly, equity holders will incur only moderate losses, which would not justify immediate liquidation. Such moderate losses would progressively erode equity book value A. A long period of moderate losses, during which it would be rational for equity holders not to voluntarily liquidate the business, could reduce the

Figure 5: Market value of equity V as a function of profitability R and the book value of equity A. Voluntary liquidation is possible.

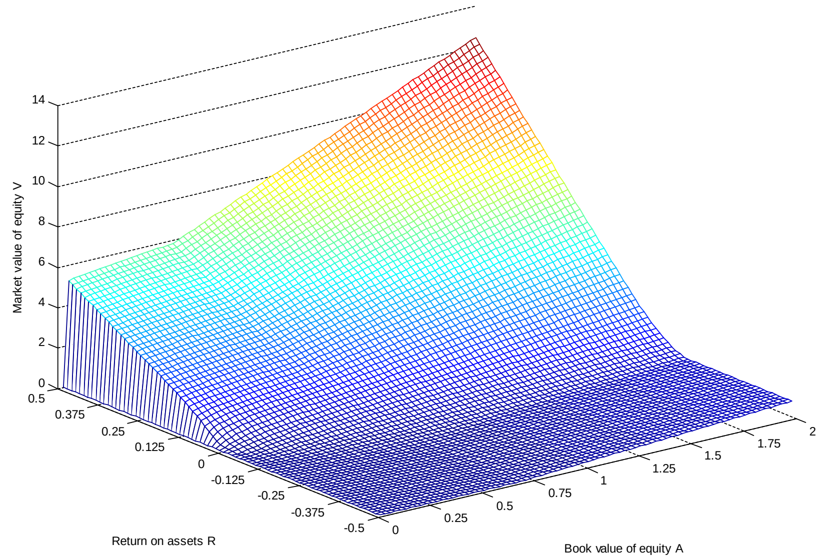
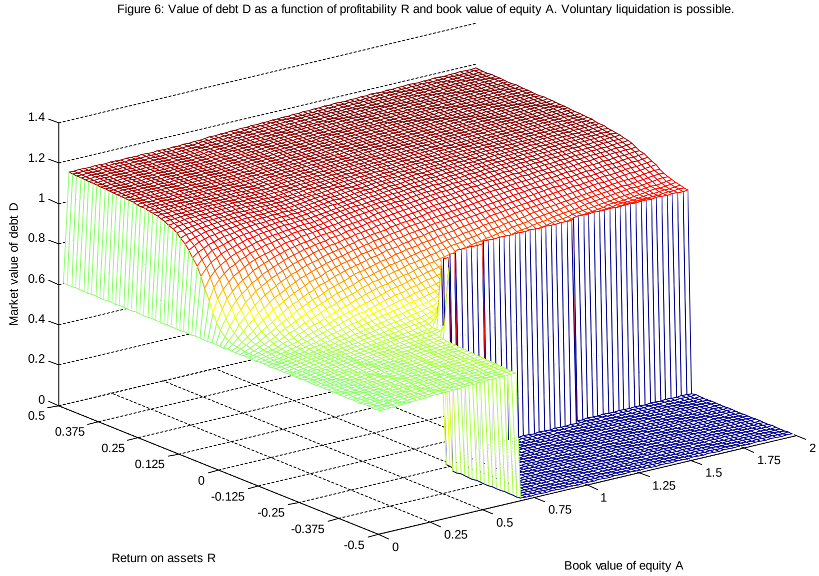


Figure 6: Value of debt D as a function of profitability R and book value of equity A. Voluntary liquidation is possible.



assets recovery value $(1-\alpha)(A+F)$ below the debt face value F. Eventually protracted losses could lead to default, forced liquidation and to a loss for debt holders. This is why, when equity book value is high, a slow persistent decline in the return on assets R can be more damaging to debt holders than a fast decline. Instead, if A is low, immediate voluntary liquidation may not fetch anything to equity holders, so that equity holders will not opt for voluntary liquidation. In this case a rapid profitability decline will mainly increase the probability of default. This whole argument is implied by Figure 6.

5 Conclusions

This paper has presented a debt and equity valuation model based on book values of assets, liabilities and earnings observable in published financial statements. The valuation model provides a number of insights. Equity and debt values critically depend on current and expected profitability, on the book value of equity and on the volatility of future earnings. Earnings volatility increases equity value and typically decreases debt value. But debt value tends to increase in earnings volatility in the proximity of default. This fact challenges the traditional assets-substitution and risk-shifting argument, which predicts that near default the risk-shifting incentive is strongest. Equity value materially increases (decreases) with the strength of profitability mean reversion, when profitability is below (above) its long term level. Debt value typically rises with the strength of profitability mean reversion.

The proposed equity valuation model is based on book values and expected earnings, but also accounts for the options to voluntarily liquidate the firm or to default, which seem important to value distressed equity. It may be in the equity holders' interest to voluntarily liquidate the firm before default. Considering voluntary liquidation entails, surprisingly, that a slow decline in profitability and small persistent losses may be more dangerous for debt holders than a sudden large decline in profitability and sudden large losses. Sudden losses would trigger voluntary liquidation, while small persistent ones would lead to default, which is more detrimental to debt holders than voluntary liquidation.

References

[1] Altman E. and Hotchkiss E., 2006, "Corporate financial distress and bankruptcy", third edition, Wiley.

[2] Anderson R., and Sundaresan S., 1996, "Design and valuation of debt contracts", Review of financial studies 9, n.1, 37-68.

[3] Black F. and Scholes M., 1973, "The pricing of options and corporate liabilities", Journal of political economy 637-659.

[4] Brennan M. and Schwartz E., 1977, "Convertible bonds: valuation and optimal strategies for call and conversion", Journal of finance 32, n.5, 1699-1715.

[5] Brockman P. and Turtle H. J., 2003, "A barrier option framework for corporate security valuation", Journal of Financial Economics 67, n. 3, March, 511-529.

[6] Davydenko S., 2005, "When do firms default? A study of the default boundary", working paper, London Business School.

[7] DeAngelo H., DeAngelo L. and Wruck K., "Asset liquidity, debt covenants, and managerial discretion in financial distress: the collapse of L.A. Gear", Journal of Financial Economics, vol. 64, n.1, April 2002, 3-34.

[8] Ericsson J. and Reneby J., 1998, "A framework for pricing corporate securities", SSE/EFI working paper series in economics and finance n. 314.

[9] Erwin G.R. and McConnell J.J., 1997, "To Live or Let Die: An Empirical Analysis of Piecemeal Voluntary Corporate Liquidations, 1971-1991", Journal of Applied Corporate Finance.

[10] Fama E. and French K., 2000, "Forecasting profitability and earnings", Journal of Business 73, n.2, 161-175.

[11] Fan H. and Sundaresan S., 2000, "Debt valuation, renegotiation and optimal dividend policy", Review of financial studies 13, n.4, 1057-1099.

[12] Goldstein R., Ju N. and Leland H., 2001, "An EBIT-Based Model of Dynamic Capital Structure", Journal of Business 74, n.4.

[13] Harrison J.M. and Kreps D.M., 1979, "Martingales and arbitrage in multiperiod securities markets", Journal of economic theory, 381-408.

[14] Hite G., Owers J. and Rogers R., 1987, "The market for interfirm asset sales: partial sell-offs and total liquidations", Journal of Financial Economics 18, 229-252.

[15] Jensen M. and Meckling W., 1976, "Theory of the firm, managerial behaviour, agency costs and ownership structure", Journal of financial economics 3, Autumn, 305-360.

[16] Kim J., Ramaswamy K. and Sundaresan S., 1993, "Does default risk in coupons affect the valuation of corporate bonds?: A contingent claims model", Financial Management 117-131.

[17] Kim H. and Schatzberg G., 1987, "Voluntary corporate liquidations", Journal of Financial Economics 19, 311-328.

[18] Leland H., 1994a, "Corporate debt value, bond covenants and optimal capital structure", Journal of finance 49, n.4, 1213-1252.

[19] Leland H., 1998, "Agency costs, risk management and capital structure", Journal of finance 53, n.4, 1213-1243.

[20] Longstaff F. and Schwartz E., 1995, "A simple approach to valuing risky fixed and floating rate debt", Journal of finance 50, n.3, 789-819.

[21] Mehran H., Nogler G. and K. Schwartz, 1998, "CEO plans and corporate liquidation policy", Journal of Financial Economics 50, 319-349.

[22] Mella-Barral P. and Perraudin W., 1997, "Strategic debt service", Journal of finance 52, n.2, 531-556.

[23] Mella-Barral P. and Tychon P., 1999, "Default Risk in Asset Pricing", Forthcoming in Finance.

[24] Merton R., 1974, "On the pricing of corporate debt: the risk structure of interest rates", Journal of finance 29, 449-470.

[25] Penman S. and Nissim D., 2001, "Ratio analysis and equity valuation: from research to practice", Review of Accounting studies, March, 109-154.

[26] Smith C.W. jr and J.Warner, 1979, "On financial contracting: an analysis of bond covenants", Journal of financial economics 7, 117-161.

[27] Sullivan M., Crutchley C. and Johnson D., 1997, "Motivation for voluntary corporate liquidations: distress, agency conflicts and shareholder gains", Quarterly Journal of Business and Economics 36, n.2, 3-18.

[28] Zmijewski M., "Predicting corporate bankruptcy: an empirical comparison of the extant financial distress models", working paper, SUNY at Buffalo, 1983.