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Abstract

In this paper we explore the possibility of "partial-multihoming" in a two-sided market where a subset of agents, on one or both side(s), may multihome in equilibrium. We consider a model in which platforms are spatially differentiated and on each side of the market there are two type of agents, low type and high type agents, that differ only by their preferences over the network benefits. We derive under which conditions of network preferences, an equilibrium with partial multihoming on both sides exists. We show that for such an equilibrium to exist, the network benefits of high type agents must be sufficiently higher than transportation costs. Furthermore, the proportions of agents who multihome on both sides must be sufficiently small. Finally, we show that independently of the degree of multihoming on the other side of the market, agents in each group face higher prices when there is partial multihoming on their side than when there is singlehoming.

Keywords: two-sided markets; network externalities; heterogeneous agents
JEL: L13

1 Introduction

A significant number of real-life markets involve two groups of agents who interact via "*platforms*", and where one group's benefit from joining a platform depends on the size of the other group which joins the platform, a property referred to as (indirect) network externalities. Markets with these features are commonly termed two-sided markets (examples include media markets, entertainment platforms, search engines, computer operating systems and shopping malls).[1] The classic example is provided by the credit card services. For instance, cardholders value credit or debit cards only

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[1]In general many markets or platforms are multi-sided. However, most of the literature has focused on two-sided markets for expositional simplicity and also because the insights obtained for two-sided platforms apply more generally to multi-sided ones.

to the extent that these are accepted by the merchants they patronize; the affiliated merchants benefit from a widespread diffusion of cards among consumers. More generally, many if not most markets with network externalities are characterized by the presence of two distinct sides whose ultimate benefit stems from interacting through a common platform.[2] One special difficulty of dealing with platform competition is given by the fact that agents can often join more than one platform. Platform services usually are not exclusive, and users may rely on the services of several platforms.

When an agent chooses to use only one platform it has become common to say the agent is "singlehoming". When an agent uses several platforms he is said to "multihoming".[3] In a number of markets, a fraction of end-users on one or both side(s) may connect to several platforms. As Evans (2002) notes, "*Most two-sided markets we observe in the real world appear to have several competing two-sided firms and at least one side appears to multihome*". Analyses of multihoming markets, however, are complicated by two elements. First some of the costs and benefits are endogenously determined in a market equilibrium. For instance, platforms may use price instruments to attract customers. In doing so, they do not only affect market shares, but also the extent of multihoming behavior. For example, when Visa reduces the (transaction-proportional) charge paid by the merchants, merchants become more tempted to turn down the more costly Amex as long as a large fraction of Amex customers also own a Visa card. Second, agents decisions are interdependent. Consider this trivial example, consumers choosing products of different brands. If all brands are offered in two or more shops (multihoming), each consumer needs to visit only one shop to have the whole range available (singlehoming). It makes a significant difference to outcomes whether groups singlehome or multihome, since prices on one side of the market depend on the extent of multihoming on the other side.

Despite the recent growing literature, two-sided markets with multihoming have been analyzed in a framework that does not allow for partial multihoming on both sides of the markets. In particular, in the standard framework it is assumed that only one side completely multihomes while the opposite side completely singlehomes. This situation is defined as "competitive bottlenecks" after the seminal paper of Armstrong (2005). Examples of papers that analyze multihoming in that standard framework are Armstrong and Wright (2004) and Rochet and Tirole (2004). Gabszewicz and Wauthy (2004) obtain the same structure as an equilibrium outcome without imposing it.[4] However, there are many examples of markets where on one or both side(s) of the market there are only a fraction of agents that multihome. This is also confirmed

[2]See Rochet and Tirole (2003, 2004) and Evans (2003a, b) for further examples of two-sided markets.

[3]For example, many merchants accept both Amex and Visa; furthermore, some consumers have both Amex and Visa cards in their pockets. Readers may subscribe to multiple newspapers or magazines.

[4]Gabszewicz and Wauthy (2004) do not use a "competitive bottleneck" framework in their model. By allowing for heterogenous types of agents, they found that a "competitive bottleneck" structure may arise as an equilibrium of the model.

by a recent empirical paper of Kaiser and Wright (2006) on the magazine industry in Germany. They show that about 8 percent of readers and around 17 percent of advertisers multihome. Thus, allowing for partial multihoming in two-sided markets not only adds a more realistic feature to the analysis but may highlight some more insights into the equilibrium price structures and their relationship with network externalities. An exception in the existing literature on multihoming is the analysis of Doganoglu and Wright (2006), in which they allow for partial multihoming by assuming that the agents are heterogeneous in terms of preferences over network benefits. Although, their analysis mainly concerns one-sided markets, they also consider the case of two-sided markets with restriction on the size of network benefits on both sides.

In this paper we explore the possibility of "partial-multihoming" in two-sided markets where a subset of agents, on one or both side(s), may multihome in equilibrium. In particular, we consider a model in which there are two differentiated platforms that compete over two groups of agents, one on each side of the market. We assume two types of agents in each group: there are low type agents that have a low preference over the network benefits and high type agents that have a high preference over network benefits. When the platforms' services are heterogeneous, parameter values are assumed so that high types will choose to multihome while low types will not.[5] The closest models to our analysis are Doganoglu and Wright (2006) and Armstrong and Wright (2004). However, our analysis differs from theirs in several important aspects. While in Doganoglu and Wright (2006) preferences are heterogeneous within the same group, we assume that there is intra-group heterogeneity as well as inter-group heterogeneity. This means that in our model high type and low type agents on each side of the market can have a different valuation of the network benefits with respect to the agents on the opposite side. Furthermore, while Doganoglu and Wright's analysis mainly focuses on one-sided markets we consider only two-sided markets. This allows us to derive explicitly the incentive to multihome in a particular side of the market and thus to analyze different market structures depending on the degree of partial multihoming. Armstrong and Wright's (2004) analysis is also related to our analysis, since they consider the case in which there is strong product differentiation on each side of the market. However, they assume only inter-group heterogeneity and they do not allow for partial multihoming.

Our aim is to analyze the effects on the equilibrium price structures of the presence of partial multihoming. We consider three basic settings: (I) two-sided singlehoming where all users join a single platform exclusively as the costs of duplicated purchases are higher on both sides of the market than the levels of network benefits; (II) partial multihoming on both sides where a subset of agents join a single platform exclusively (low types), but not for the rest of the group, who join both platforms (high types);[6]

[5] In a two-sided market with strong network benefits, it is natural to consider an equilibrium in which all agents buy from at least one platform.

[6] Very often when mutihoming is considered, it is restricted to one-side of the market. If inter-

(III) partial multihoming on one side where all agents on one side join a single platform exclusively because the costs of joining both is higher than the network benefits. While in the other group there exists a subset of agents joining both platforms (high types); the rest of the group would join only a single platform (low types). We derive under which conditions of network preferences, an equilibrium with partial multihoming on both sides exists. We show that for such an equilibrium to exist, the network benefits of high type agents must be sufficiently higher than transportation costs. Furthermore, the proportions of agents that multihome on both sides must be sufficiently small. In general, platforms tend to charge higher prices to the side that multihomes. By contrast, platforms need to compete for the singlehoming agents, and the high profits generated from the multihoming side are passed to the singlehoming side through low prices (see Proposition 6). Contrary to results of competitive bottlenecks, prices charged to one side may be lower even if that side partially multihome and has an average network benefit higher than the other side.

The rest of the paper proceeds as follows. Section 2 introduces our general framework of a two-sided market. Subsection 2.1 presents the benchmark model of a two-sided market with singlehoming on both sides. Section 3 analyzes partial multihoming on both sides, while Section 4 assumes only one side has partial multihoming. A comparison of multihoming and singlehoming equilibria for this model are presented in Section 5, while Section 6 briefly concludes.

2 The Model

We start with a standard Hotelling model of competition with network effects similar to that of Doganoglu and Wright (2006) and Armstrong and Wright (2004). We allow agents to be heterogeneous in terms of their marginal valuations of network benefit so that multihoming can arise in equilibrium. We assume that there are two platforms that offer a differentiated product (for example: newspapers, TV networks, shopping centres etc.). The two platforms are denoted by k ($k = 1, 2$). Platforms deal with two distinct groups of agents, denoted by i ($i = A, B$), each of which wishes to interact on a common platform. Platforms coordinate the possible matches between the two groups. Group i agent can subscribe to a service from either platform 1, platform 2 (singlehome), or both platforms if this is possible (multihome). The utility of an agent on one side of the market, among other variables, depends on the number of other agents on the opposite side that he/she can access through a platform (or through both platforms, if multihoming). For example, subscribing to a service gives group A (group B) agents network benefits that are linear in the number of other agents (e.g.

acting with the other side is the primary reason for an agent to join a platform, Armstrong (2005) claims that both groups multihoming might not be expected to be very common — if each member of group B joins all platforms, there is no need for any member of group A to join more than one platform — and so this configuration is not analyzed in his paper. However, this is not the case of partial multihoming.

group B (group A)), that the group A (group B) can access through the service.[7] Thus, there is a cross-group network externality.

Platforms provide a service to agents of group i ($i = A, B$) at a constant marginal cost f that is assumed to be small.[8] In order to allow for partial multihoming (a subset of agents on each side will join only one platform, while the remaining agents in each side will join both platforms), we need to assume that agents on both sides of the market are heterogeneous in terms of preferences over the network benefits (or costs in the case of negative externality) denoted by α and β depending on which group of agents we are referring to. We assume that the total number of agents in each group is equal to unity. Using the framework of Doganoglu and Wright (2006), we assume that a fraction λ_A of agents in group A value the network benefits highly (high types) and $\alpha = \alpha_H > 0$. The remaining consumers, a fraction of $1 - \lambda_A$, do not value the network benefits highly (low types), and have $\alpha = \alpha_L > 0$, with $\alpha_H > \alpha_L$ and $0 \leq \lambda_A < 1$.

For group B, there is a fraction of agents, λ_B, that has high value for the network benefits, $\beta = \beta_H > 0$, while a fraction $1 - \lambda_B$ has a low value for the network benefits, $\beta = \beta_L > 0$, with $\beta_H > \beta_L$ and $0 \leq \lambda_B < 1$.[9] The utility of group A agents from joining platform k ($k = 1, 2$) is given by:[10]

$$U_{Ak} = v_A - p_{Ak} - t(x) + \alpha n_{Bk} \tag{1}$$

where v_A is the intrinsic benefit from the service (or the reservation utility level) regardless of whether they subscribe to a single platform or both platforms, p_{Ak} is the price that group A must pay to join platform k, and $t(x)$ is the transportation cost faced by the agent located at x in the unit interval in order to reach platform k. We assume prices must be non-negative.[11] Meanwhile n_{Bk} is the number of group B agents that can be reached by group A if he/she joins platform k. We will assume that the value of v is sufficiently high such that all agents of group A will always wish to join at least one platform.[12]

The utility for group A agents that multihome is given by:

[7] A member of one group cares only about the number of the other group who join the same platform. For simplicity, we ignore the possibility that agents care also about the number of the same group who join the platform.

[8] Armstrong (2005) claim that it makes little sense to discuss price discrimination if the costs are significantly different for the two groups.

[9] We focus on the case where the network externality is positive from one side of the market to the other. However, the case with negative network externality can be easily handled in our framework.

[10] Our utility specification implies that platform charges are levied as a lump-sum fee, as in Armstrong (2005) and Doganoglu and Wright (2006). For the case in which agents pay a per-transaction fee for each agent on the platform from the other side, see Rochet and Tirole (2003).

[11] Armstrong and Wright (2004), Gabszewicz and Sonnac (2001), and McCabe and Snyder (2004) make a similar restriction.

[12] Following Armstrong and Wright (2004), multihoming arises for network reasons, so we allow agents to obtain intrinsic benefits of the platform only once. Agents will only multihome if doing so allows them to connect with more agents of the opposite type.

$$U_A = v_A - p_{A1} - p_{A2} - t(x) - t(1-x) + \alpha(n_{A2} + n_{B2}) \tag{2}$$

Notice that equation (2) can be simplified further by noticing that $t(x)+t(1-x) = t$ since the total distance of travelling to both firms is always unity. Thus, the utility from multihoming does not depend on x. Moreover, given that the total number of agents on each side of the market is normalized to unity, we have $n_{A2} + n_{B2} = 1$.

The utility of group B agents is defined in a similar way. For an agent that decides to singlehome:

$$U_{Bk} = v_B - p_{Bk} - t(x) + \beta n_{Ak} \tag{3}$$

While for an agent that multihomes:

$$U_B = v_B - p_{B1} - p_{B2} - t(x) - t(1-x) + \beta(n_{A1} + n_{A2}) \tag{4}$$

where v_B is the intrinsic benefit from the service that we assume to be sufficiently high such that all agents in group B will always wish to join at least one platform. Notice that we assume that the intrinsic utility is symmetric between the types in each group and that the cost of transportation is the same in each side of the market.[13]

2.1 Singlehoming on Both Sides of the Market

In this section we consider the benchmark case where two platforms that are differentiated based on the Hotelling model compete for each group, but all agents of both groups join a single platform exclusively.

Let $\overline{\alpha} = \alpha_L(1-\lambda_A) + \alpha_H\lambda_A$ and $\overline{\beta} = \beta_L(1-\lambda_B) + \beta_H\lambda_B$ denote the average values of the network benefit for groups A and B respectively. We assume that the costs of transportation are higher than these average benefit values:

Assumption A1: $t > \overline{\alpha} = \alpha_L(1-\lambda_A) + \alpha_H\lambda_A$ *and* $t > \overline{\beta} = \beta_L(1-\lambda_B) + \beta_H\lambda_B$, with $\lambda_A \in [0,1)$ and $\lambda_B \in [0,1)$.

Assumption $A1$ implies that product differentiation is stronger than average network benefits. It also implies that $t > \alpha_L$ and $t > \beta_L$. This fact will rule out the possibility of corner solutions in which low type agents, independently of their location, will all choose the same platform. As a result, low type agents will never multihome for non-negative equilibrium prices. Thus, only high value type agents can have an incentive to multihome, as in Doganoglu and Wright (2006).

Assumptions $A1$ does not rule out the possibility that high type agents may have an incentive to multihome, since the transportation cost can be much lower than their valuation of network benefits. The next proposition states under which condition high type agents may not have an incentive to multihome:

[13] The assumption of equal transportation costs does not affect qualitatively the results of our analysis. The case where transportation costs may differ from one side of the market to the other is considered in Armstrong and Wright (2004).

Proposition 1 *High type agents have no incentive to multihome only if* $\alpha_H < t + p_{A1} + p_{A2}$ *and* $\beta_H < t + p_{B1} + p_{B2}$.

Proof. Consider a high type agent from group A. The utility from multihoming of that agent is $U_A = v_A - p_{A1} - p_{A2} - t + \alpha_H$. The lowest utility level that agent can obtain from singlehoming is $U_{i1}(x) = U_{i2}(x)$. Solving that equality for x gives us: $x = \frac{p_{A2} - p_{A1} + \alpha_H(n_{B1} - n_{B2}) + t}{2t}$. Evaluating the utility of a high type agent that is singlehoming at x, we obtain: $U_{A1} = v_A - p_{A1} - t\left[\frac{p_{A2} - p_{A1} + \alpha_H(n_{B1} - n_{B2}) + t}{2t}\right] + \alpha_H n_{B1}$. We consider the case when the difference between U_A and U_{A1} is negative: this will give us the result for α_H. In order to find the result for β_H we apply the same procedure to high type agents of group B. ■

Proposition 1 implies that the network valuation parameters of high type agents may be greater than the transportation costs. However, the difference between the network benefits and the transportation costs is bounded by the sum of the prices. If this is the case, complete singlehoming may arise in equilibrium even if agents in each group are heterogeneous in terms of their network benefits valuation. To analyze singlehoming, we assume that the results in Proposition 1 hold.

We define the proportions of high type and low type agents of group i ($i = A, B$) that join platform k as h_{ik} and l_{ik} respectively. Thus, the total number of people from group A joining platform k is given by $n_{Ak} = \lambda_A h_{Ak} + (1 - \lambda_A) l_{Ak}$, while the number of agents B joining platform k is given by $n_{Bk} = \lambda_B h_{Bk} + (1 - \lambda_B) l_{Bk}$ with $k = 1, 2$. The demand function for agents of group A that singlehome solves the problem of the indifferent between the two platforms of low type agent: $U_{A1}(v_A, p_{A1}, t(x), n_{B1}, \alpha_L) = U_{A2}(v_A, p_{A2}, t(1-x), n_{B2}, \alpha_L)$ for $x = l_{A1}$. The demand functions of high type agents can be found in a similar way by solving: $U_{A1}(v_A, p_{A1}, t(x), n_{B1}, \alpha_H) = U_{A2}(v_A, p_{A2}, t(1-x), n_{B2}, \alpha_H)$ for $x = h_{A1}$. Similar expressions are found for low and high type agents in group B.

The solutions for the demand functions, h_{Ak}, l_{Ak}, h_{Bk} and l_{Bk} with $k = 1, 2$ are derived in Appendix A. The profit functions of the two platforms are:

$$\pi_1 = (p_{A1} - f)(\lambda_A h_{A1} + (1 - \lambda_A) l_{A1}) + (p_{B1} - f)(\lambda_B h_{B1} + (1 - \lambda_B) l_{B1}) \quad (5)$$

$$\pi_2 = (p_{A2} - f)(\lambda_A h_{B2} + (1 - \lambda_A) l_{A2}) + (p_{B2} - f)(\lambda_B h_{B2} + (1 - \lambda_B) l_{B2}) \quad (6)$$

The result from the maximization of the profit functions (5) and (6) is summarized in the following proposition:

Proposition 2 *Assume A1 and that* $t^2 > \overline{\beta}\overline{\alpha}$. *Then, there exists a complete (symmetric) singlehoming equilibrium in which prices are given by:*

$$p_{Ak} = t + f - [\beta_L(1 - \lambda_B) + \beta_H \lambda_B]$$

$$p_{Bk} = t + f - [\alpha_L(1 - \lambda_A) + \alpha_H \lambda_A]$$

with $k = 1, 2$.

From Proposition 2 we can see that $p_{Ak} = p_{Bk}$ only if $\beta_L = \alpha_L$, $\beta_H = \alpha_H$ and $\lambda_A = \lambda_B$.[14] In all the other cases the price charged to one side of the market will differ from the one charged on the opposite side. Note that in a Hotelling model without cross-group externalities, the equilibrium price for group A would be $p_{Ak} = t + f$. In this two-sided market the price is adjusted downwards by the factor $[\beta_L(1 - \lambda_B) + \beta_H \lambda_B]$, which represents the average values of the network benefit of group A to B. It is group A's benefit to the other group that determines group A's price, not how much group A benefits from the presence of group B. Note that $p_{Ak} - p_{Bk} = \overline{\alpha} - \overline{\beta}$, our results are consistent with the conventional wisdom that if group A values group B less than group B values group A $\left(\overline{\alpha} < \overline{\beta}\right)$, group A may be subsidized its prices charged, and platforms make their profit from group B $(p_{Ak} < p_{Bk})$. Of course, group B contributes to profits indirectly by raising demand from group A, this is why group B may be subsidized in equilibrium.[15] Given the level of transportation costs and the marginal cost, the prices charged to group A (B) are decreasing in λ_B (λ_A). If $\lambda_B > \lambda_A$ then group B will be charged more than group A, reflecting that there is a greater fraction of high types in group B who care more about connecting with group A than vice versa, other thing equal. Given the previous results, we know that prices are higher where the average value of the network benefit is higher.

Under the symmetric equilibrium stated in Proposition 1, the demand functions faced by the two platforms are given by: $n_{Ak} = \frac{1}{2}[\lambda_A + (1 - \lambda_A)]$ and $n_{Bk} = \frac{1}{2}[\lambda_B + (1 - \lambda_B)]$, with $k = 1, 2$. The corresponding positive profit functions are:

$$\pi_k = \left[t - \overline{\beta}\right]\left[\frac{1}{2}\left(\lambda_A + (1 - \lambda_A)\right)\right] + [t - \overline{\alpha}]\left[\frac{1}{2}(\lambda_B + (1 - \lambda_B))\right] \tag{7}$$

3 Partial Multihoming on Both Sides

In this section we consider the case in which partial multihoming may arise in equilibrium. For example some people read two newspapers and some firms put the same ad in both newspapers. Another example is the credit cards market, in which some customers own two credit cards and there are some shops that accept both credit cards as well. To analyze the case of partial multihoming on both sides, we follow assumption $A1$, where low type agents will always singlehome. We also assume that:

[14] This is the case analyzed by Doganoglu and Wright (2006) in section 3 of their paper. They also consider the case where $\beta_L = \alpha_L$, $\beta_H = \gamma_H$ but with $\lambda_A \neq \lambda_B$ in their technical appendix.

[15] Kaiser and Wright (2006) show that in a magazine industry, for example, advertisers gain more from interacting with readers than vice versa, and that as a result; magazines subsidize cover prices, and make their profit from advertisers.

Assumption A2: $\alpha_H \geq t + p_{A1} + p_{A2}$ and $\beta_H \geq t + p_{B1} + p_{B2}$.

Assumption $A2$ means that high type agents in each group may have an incentive to multihome.

Given the fact that only high types will have an incentive to multihome, we have $h_{Ak} = h_{Bk} = 1$. Thus, the total number of group A agents that join platform k is given by $n_{Ak} = \lambda_A + (1-\lambda_A)l_{Ak}$, while the total number of group B agents that join platform k is given by $n_{Bk} = \lambda_B + (1-\lambda_B)l_{Bk}$. Thus, in this case λ_A and λ_B represent the proportions of agents that multihome in group A and group B respectively.

In order to find the demand functions for agents of group A, we need to solve the problem of the indifferent between the two platforms of low type agent, that is, $U_{A1}(v_A, p_{A1}, t(x), n_{B1}, \alpha_L) = U_{A2}(v_A, p_{A2}, t(1-x), n_{B2}, \alpha_L)$ for $x = l_{A1}$. Similar expressions are found for low and high type agents in group B. The demand functions l_{A1}, l_{A2}, l_{B1} and l_{B2} are derived in Appendix B. Using the results in Appendix B, the total demand from group A agents faced by platform 1 is given by: $n_{A1} = \lambda_A + (1-\lambda_A)\left[\frac{1}{2} + \frac{t(p_{A2}-p_{A1})}{2(t^2-\alpha_L\beta_L g)} + \frac{\alpha_L(p_{B2}-p_{B1})(1-\lambda_B)}{2(t^2-\alpha_L\beta_L g)}\right]$, while the total demand from group B agents is given by: $n_{B1} = \lambda_B + (1-\lambda_B)\left[\frac{1}{2} + \frac{t(p_{B2}-p_{B1})}{2(t^2-\alpha_L\beta_L g)} + \frac{\beta_L(p_{A2}-p_{A1})(1-\lambda_A)}{2(t^2-\alpha_L\beta_L g)}\right]$. Similar expressions are defined for platform 2.

The profit function of platform 1 is given by:

$$\pi_1 = (p_{A1} - f)n_{A1} + (p_{B1} - f)n_{B1} \tag{8}$$

while for platform 2 we have:

$$\pi_2 = (p_{A2} - f)n_{A2} + (p_{B2} - f)n_{B2} \tag{9}$$

Proposition 3 *Assume A1, A2 and $t^2 > \alpha_L\beta_L g$, there exists a symmetric equilibrium where $p_{A1} = p_{A2}$ and $p_{B1} = p_{B2}$ in which prices are given by:*

$$p_{Ak} = \frac{t(1+\lambda_A)}{1-\lambda_A} + f - \beta_L(1+\lambda_B)$$

$$p_{Bk} = \frac{t(1+\lambda_B)}{1-\lambda_B} + f - \alpha_L(1+\lambda_A)$$

with $k = 1, 2$.

Equilibrium prices are decreasing in the network benefit parameters. Prices in Proposition 3 are positive if the following conditions hold:

$$t\frac{(1+\lambda_A)}{(1-\lambda_A)} + f > \beta_L(1+\lambda_B) \tag{10}$$

$$t\frac{(1+\lambda_B)}{(1-\lambda_B)} + f > \alpha_L(1+\lambda_A) \tag{11}$$

In the extreme case of $\lambda_A = \lambda_B = 0$, in which there are no high type agents and multihoming is not possible, the equilibrium prices are given by $p_{Ak} = t + f - \beta_L$ for group A, and $p_{Bk} = t + f - \alpha_L$ for group B, which is the result obtained by Armstrong and Wright (2004, Proposition 1). We cannot say a *priori* which side will face higher prices. Indeed the difference between prices is given by:

$$p_{Ak} - p_{Bk} = \frac{2t(\lambda_A - \lambda_B)}{(1-\lambda_A)(1-\lambda_B)} + \alpha_L(1+\lambda_A) - \beta_L(1+\lambda_B)$$

Given the level of t, the sign of $(p_{Ak} - p_{Bk})$ is determined by the relative size of λ_A to λ_B and the relative size of α_L to β_L. Consider the case where $\lambda_A = \lambda_B = \lambda$, then $p_{Ak} - p_{Bk} = (1+\lambda)(\alpha_L - \beta_L)$; prices will be higher for the side for whom the network benefits are more highly valued by the low types. This was also a property of the equilibrium price structure without multihoming, implying that the side which enjoys greater cross-group externalities will be charged more. However, when $\lambda_A < \lambda_B$ and $\alpha_L(1+\lambda_A) > \beta_L(1+\lambda_B)$, we have $\alpha_L > \beta_L$, then $p_{Ak} - p_{Bk} \gtreqless 0$, depending on the level of t. If transportation cost is sufficiently high, then the prices charged to group A may be lower than the prices charged to group B $(p_{Ak} < p_{Bk})$, even if low type agents in the former group have higher network benefits than agents in the latter $(\alpha_L > \beta_L)$.

Here we report some comparative static properties of the equilibrium prices in Proposition 3, with $k = 1, 2$:

$$\frac{\partial p_{Ak}}{\partial \lambda_B} = -\beta_L; \quad \frac{\partial p_{Bk}}{\partial \lambda_A} = -\alpha_L \tag{12}$$

which is negative under the assumption that the network externality parameters are positive. The basic intuition is that: everything else constant, an increase in the proportion of multihoming agents on one side of the market will increase the average valuation of the network benefits on that side *relatively* to those on the other side. Thus, platform will decrease prices on the side where the average network valuation is lower.

For a similar reason we have the following results for the effect of a change in the proportion of agents that multihome on one side and the prices charged on the same side:

$$\begin{aligned} \frac{\partial p_{Ak}}{\partial \lambda_A} &= \frac{2t}{(\lambda_A - 1)^2} > 0 \\ \frac{\partial p_{Bk}}{\partial \lambda_B} &= \frac{2(t - \alpha_L(1-\lambda_A))}{(\lambda_A - 1)^2} > 0; \ t > \alpha_L. \end{aligned} \tag{13}$$

Using the result in Proposition 3, we can express the incentive to multihome of the high type agents as a function of the parameters of the model:

Proposition 4 *Given the equilibrium prices defined in Proposition* 3*, high type agents have an incentive to multihome only if:*
$\alpha_H \geq \frac{2\beta_L(\lambda_A-1)(1+\lambda_B)+t(3+\lambda_A)+2f(1-\lambda_A)}{(\lambda_A-1)(\lambda_B-1)}$ *and* $\beta_H \geq \frac{2\alpha_L(\lambda_B-1)(1+\lambda_A)+t(3+\lambda_B)+2f(1-\lambda_B)}{(\lambda_A-1)(\lambda_B-1)}$

Proof. The proof follows directly from Proposition 1. The high type agents have an incentive to multihome if $\alpha_H \geq t + p_{A1} + p_{A2}$ and $\beta_H \geq t + p_{B1} + p_{B2}$ for groups A and group B respectively. Substituting the price expressions from Proposition 3 into the previous two inequalities gives us the result. ■

The result in Proposition 4 states that the valuation of the network externality (α_H and β_H) by a high type agent must be sufficiently larger than the cost of transportation (and thus larger than α_L and β_L) for that agent to multihome. Note that $\alpha_H = \beta_H$ only under perfect symmetry, that is when $\alpha_L = \beta_L$ and $\lambda_B = \lambda_A \neq 1$.[16] In this case we have perfect symmetry in prices as well, that is $p_{Ak} = p_{Bk}$, with $k = 1, 2$.

Proposition 4 shows the effect of a change in the proportion of agents that multihome on one side on the incentive to multihome for the agents on the other side of the market. Following the same process as in the proof of Proposition 1 we can identify the difference between U_A and U_{A1} and between U_B and U_{B1} in equilibrium. Using the results in Proposition 3 we have:

$$U_A - U_{A1} = \frac{(\lambda_A - 1)\left[2\beta_L(\lambda_B + 1) + \alpha_H(1 - \lambda_B)\right] + t(3 + \lambda_A) + 2f(1 - \lambda_A)}{2(\lambda_A - 1)} \tag{14}$$

and

$$U_B - U_{B1} = \frac{(\lambda_B - 1)\left[2\alpha_L(\lambda_A + 1) + \beta_H(1 - \lambda_A)\right] + t(3 + \lambda_B) + 2f(1 - \lambda_B)}{2(\lambda_B - 1)} \tag{15}$$

Then, it can be easily checked that:

$$\frac{\partial(U_A - U_{A1})}{\partial\lambda_B} = \beta_L - \frac{1}{2}\alpha_H < 0 \tag{16}$$

and

$$\frac{\partial(U_B - U_{B1})}{\partial\lambda_A} = \alpha_L - \frac{1}{2}\beta_H < 0 \tag{17}$$

Under the results in Proposition 4, those derivatives are negative. Thus, under a partial multihoming equilibrium where $U_A - U_{A1} > 0$ and $U_B - U_{B1} > 0$, an increase in the proportion of agents that multihome on one side of the market will decrease the incentive to multihome for the agents on the other side. If each member of group B joins both platforms, there is no need for any member of group A to join more than one platform.

[16] This is the case analyzed in Doganoglu and Wright (2006), Section 3. See footnote 14.

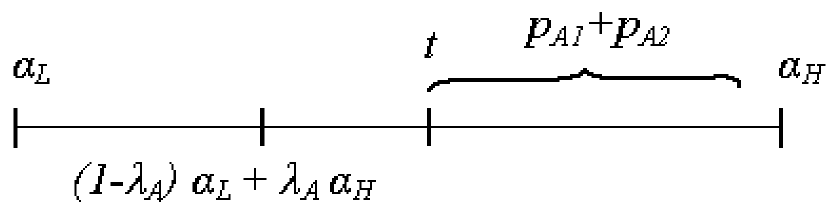

Figure 1: Assumptions $A1$ and $A2$.

Assumptions $A1$ and $A2$, together with the results in Proposition 3 clearly create a constraint on the level of λ_A and λ_B in order for those proportions to be consistent in equilibrium. Indeed, assumption $A2$ implies that the network benefit valuation of high type agents must be much higher than the transportation cost; however, from assumption $A1$ the transportation cost must be higher than the average network benefits. In order for those two assumptions to hold simultaneously the values for λ_A and λ_B must be sufficiently small. This situation is depicted in Figure 1 for the case of group A agents.

The next proposition provides a formalization of the result in Figure 1:

Proposition 5 *Assumptions* $A1$ *and* $A2$ *together with the results in Proposition* 3 *imply that in equilibrium proportions* λ_A *and* λ_B *must be sufficiently small.*

From assumption $A2$ and Proposition 3 we have $\lambda_A \leq \frac{\alpha_H-3t-2f+2\beta_L(1+\lambda_B)}{\alpha_H+t-2f+2\beta_L(1+\lambda_B)}$ and $\lambda_B \leq \frac{\beta_H-3t-2f+2\alpha_L(1+\lambda_A)}{\beta_H+t-2f+2\alpha_L(1+\lambda_A)}$, while from assumption $A1$ we have $\lambda_A < \frac{t-\alpha_L}{\alpha_H-\alpha_L}$ and $\lambda_B < \frac{t-\beta_L}{\beta_H-\beta_L}$. These four conditions induce a subset in the $[0,1)\times[0,1)$ space in which λ_A and λ_B must lie in order to be consistent with the equilibrium prices. We can notice that the conditions derived from assumption $A2$ are two straight lines with positively slopes in the $(\lambda_A,\lambda_B)-$space. It is analytically difficult to determine the domain for λ_A and λ_B from the four conditions derived above. However, we are able to use a numerical analysis to obtain such domains. An example of such a domain is depicted in Figure 2.[17]

The result in Proposition 5 comes directly from equations (16) and (17). The higher the proportion of agents that multihome on one side the lower the benefits from multihoming on the opposite side. Thus, in order to have partial multihoming on both sides, the proportions of multihoming agents must not be too large such that the benefits from multihoming on one side of the market are higher than the negative effect implied by the presence of multihoming agents on the other side. A direct

[17]Figure 2 is derived using the following parametrization: $\alpha_H=20$, $\beta_H=23$, $\alpha_L=2$, $\beta_L=3$, $t=5$, $f=0.5$.

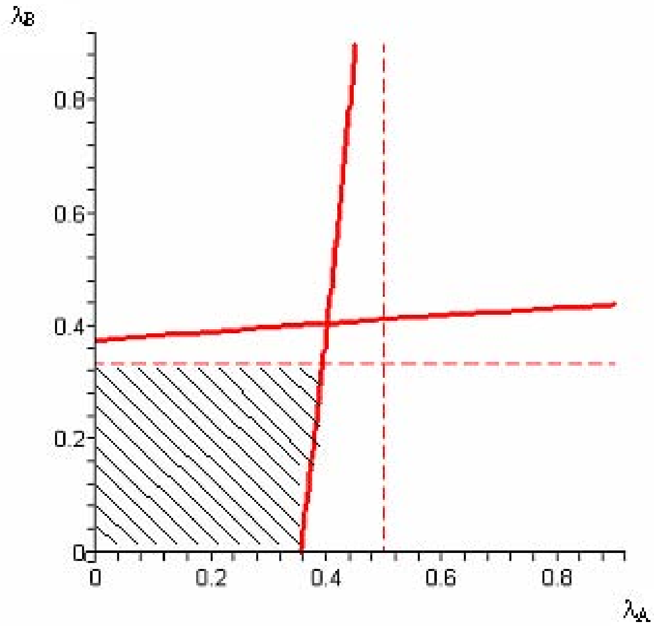

Figure 2: Example of a domain of λ_1 and λ_2 consistent with assumptions $A1$ and $A2$.

implication of Proposition 5 is that equilibrium prices must not be very high. The level of both λ_A and λ_B depends mainly on the difference between the network benefits of high type agents and the transportation cost, that is, the sum of equilibrium prices. Given the values of t, f and the network benefits of low type agents (β_L and α_L), prices are higher (lower) if λ_A and λ_B are closer to one (closer to zero). Thus, higher values for λ_A and λ_B imply higher prices, but this may violate $A1$. Note that λ_A and λ_B must be sufficiently small in equilibrium, prices must be relatively low as well. The fact that λ_A and λ_B must be small in a partial multihoming equilibrium is coherent with the empirical analysis of Kaiser and Wright (2006).[18] Using data on the magazine industry in Germany, they found that the proportion of advertisers that multihome is approximately 0.17 while the proportions of readers that multihome is approximately 0.08.

Under the symmetric equilibrium prices of Proposition 3, the demand functions faced by the two platform in each side of the market are given by:

$$n_{Ak} = \frac{1}{2}\lambda_A + \frac{1}{2} \tag{18}$$

[18]Their analysis mainly focuses on the singlehoming case. However, in their appendix, they extend the analysis to the partial multihoming case.

$$n_{Bk} = \frac{1}{2}\lambda_B + \frac{1}{2} \tag{19}$$

Note that the demand faced by the platforms is now higher by the factor $\lambda/2$ with respect to the case of pure singlehoming analyzed previously. In equilibrium the (positive) profit functions of each platform k are given by:

$$\pi_k = \left[\frac{t(1+\lambda_A)}{1-\lambda_A} - \beta_L(1+\lambda_B)\right]\left[\frac{1}{2}\lambda_A + \frac{1}{2}\right] + \left[\frac{t(1+\lambda_B)}{1-\lambda_B} - \alpha_L(1+\lambda_A)\right]\left[\frac{1}{2}\lambda_B + \frac{1}{2}\right] \tag{20}$$

with $k = 1, 2$.

4 Partial Multihoming on One Side Only

This section encompasses the two previous cases. In particular, we assume that only high type agents on one side of the market have an incentive to multihome. We assume that partial multihoming may arise only on group B's side of the market. This is summarized in the following assumption:

Assumption A3: $\alpha_H < t + p_{A1} + p_{A2}$ *and* $\beta_H \geq t + p_{B1} + p_{B2}$.

Using the analysis developed in the previous two sections, we know that the demand function of a low type agents in group A is given by the solution of the following equation for $x = l_{A1}$:

$$v_A - p_{A1} - tx + \alpha_L(\lambda_B + (1-\lambda_B)l_{B1}) = v_A - p_{A2} - t(1-x) + \alpha_L(\lambda_B + (1-\lambda_B)(1-l_{B1}) \tag{21}$$

Note that $l_{B2} = 1 - l_{B1}$. While for a high type agent we have:

$$v_A - p_{A1} - tx + \alpha_H(\lambda_B + (1-\lambda_B)l_{B1}) = v_A - p_{A2} - t(1-x) + \alpha_H(\lambda_B + (1-\lambda_B)(1-l_{B1}) \tag{22}$$

with $x = h_{A1}$. On the other side of the market, given the fact that high type agents may have an incentive to multihome, we have $h_{B1} = h_{B2} = 1$. Thus, the demand function for a low type agent is given by the solution of equation $(a3)$ in Appendix A.

The total demand of group A faced by platform 1 is given by: $n_{A1} = \lambda_A h_{A1} + (1-\lambda_A)l_{A1}$, while the total demand of group B is given by: $n_{B1} = \lambda_B + (1-\lambda_B)l_{B1}$. For platform 2 we have $n_{A2} = \lambda_A h_{A2} + (1-\lambda_A)l_{A2}$ and $n_{B2} = \lambda_B + (1-\lambda_B)l_{B2}$ from groups A and B respectively.

Solving simultaneously equations (21), (22) and $(a3)$ we obtain:

$$l_{A1} = \frac{1}{2} + \frac{(p_{A2} - p_{A1})\left[t + \beta_L \lambda_A (1 - \lambda_B)(\alpha_L - \alpha_H)\right] + (1 - \lambda_B)\alpha_L (p_{B2} - p_{B1})}{2t(t^2 - \overline{\alpha}\beta_L (1 - \lambda_B))} \tag{23}$$

$$h_{A1} = \frac{1}{2} + \frac{(p_{A2} - p_{A1})\left[t + \beta_L (1 - \lambda_A)(1 - \lambda_B)(\alpha_H - \alpha_L)\right] + (1 - \lambda_B)\alpha_H (p_{B2} - p_{B1})}{2t(t^2 - \overline{\alpha}\beta_L (1 - \lambda_B))} \tag{24}$$

$$l_{B1} = \frac{1}{2} + \frac{t(p_{B2} - p_{B1}) + \beta_L (p_{A2} - p_{A1})}{2(t^2 - \overline{\alpha}\beta_L (1 - \lambda_B))} \tag{25}$$

where we assume that $t^2 > \overline{\alpha}\beta_L(1 - \lambda_B)$. The demand functions for platform 2 are simply $l_{A2} = 1 - l_{A1}, h_{A2} = 1 - h_{A1}$ and $l_{B2} = 1 - l_{B1}$.

The profit functions for the two platforms are given by:

$$\pi_1 = (p_{A1} - f)(\lambda_A h_{A1} + (1 - \lambda_A) l_{A1}) + (p_{B1} - f)(\lambda_B + (1 - \lambda_B) l_{B1}) \tag{26}$$

$$\pi_2 = (p_{A2} - f)(\lambda_A h_{B2} + (1 - \lambda_A) l_{A2}) + (p_{B2} - f)(\lambda_B + (1 - \lambda_B) l_{B2}) \tag{27}$$

The maximization of the profit functions above leads to the following result:

Proposition 6 *Assume A1, A3 and* $t^2 > \overline{\alpha}\beta_L(1-\lambda_B)$, *then there exists a symmetric equilibrium in which prices are given by:*

$$p_{Ak} = t + f - \beta_L(1 + \lambda_B)$$

$$p_{Bk} = \frac{t(1 + \lambda_B)}{1 - \lambda_B} + f - \overline{\alpha}$$

with $k = 1, 2$.

From Proposition 6 we can notice that the difference between p_{Bk} and p_{Ak} is given by

$$p_{Bk} - p_{Ak} = 2t\left(\frac{\lambda_B}{1 - \lambda_B}\right) + \beta_L(1 + \lambda_B) - \overline{\alpha} \tag{28}$$

The sign of $(p_{Bk} - p_{Ak})$ may be positive or negative depending on the specific values of the parameters.

As in the previous section the imposition of assumptions $A1$ and $A3$ induces a natural constraint on λ_B in the interval $[0, 1)$, while λ_A is constrained only by $A1$. Thus, also in the case of partial multihoming on one side, the proportion of multihoming agents must be sufficiently small. In particular, from assumption $A1$ we know $\lambda_B < \frac{t-\beta_L}{\beta_H-\beta_L}$ while from $A2$ we have that $\lambda_B \leq \frac{\beta_H-3t-2f+2\alpha_L(1-\lambda_A)+2\lambda_A\alpha_H}{\beta_H+t-2f+2\alpha_L(1-\lambda_A)+2\lambda_A\alpha_H}$.

Which of those two conditions will be binding for λ_B will depend on the particular values of the (other) parameters of the model. For example, consider the following parametrization: $t = 5$, $\alpha_L = 1$, $\alpha_H = 3$, $\beta_L = 1$, $\beta_H = 20$, $f = 1$ and $\lambda_A = 0.5$. From assumption $A1$ note that $\lambda_B < 0.21$, while, from assumption $A2$ we have $\lambda_B \leq 0.26$. Thus, in this particular case, the first condition is the binding one.

Given the contraints on the value of λ_B now we can see when $(p_{Bk} - p_{Ak})$ may be positive or negative. It is definitely positive if λ_B is not very close to zero, for given values of t, β_L and $\overline{\alpha}$. For example, consider the same parametrization: $t = 5$, $\alpha_L = 1$, $\alpha_H = 3$, $\beta_L = 1$, $\beta_H = 20$, $f = 1$ and $\lambda_A = 0.5$, we know that $\lambda_B < 0.21$ for equilibrium to exist. Let assume $\lambda_B = 0.2$, substituting the values of the relevant parameters into (28) we obtain that $p_{Bk} > p_{Ak}$ for $\overline{\beta} > \overline{\alpha}$. Intuitively, the higher the proportion of agents that multihome and also the higher the average network benefits, the higher will be the price these agents will face. However, if $\lambda_B = 0.06$, it may be possible that $p_{Bk} < p_{Ak}$ even if $\overline{\beta} > \overline{\alpha}$. When the the proportion of agents that multihome is relatively small, the network benefit of low type agents will dominate their average network benefit effect and then $p_{Bk} < p_{Ak}$ for $\beta_L < \overline{\alpha}$. Thus, differently from the standard results of "competitive bottlenecks", if the proportion of agents that multihome is small the side with partial multihoming may face lower prices than the one with singlehoming even if agents on that side have an average network benefit higher than the other side. Howeve, in order to obtain such result, the value of the proportion λ_B must be close to zero.

Under the equilibrium defined in Proposition 6, the demand functions faced by the firms are:

$$n_{Ak} = \frac{1}{2} \tag{29}$$

$$n_{Bk} = \frac{1}{2}\lambda_B + \frac{1}{2} \tag{30}$$

while the profit functions are:

$$\pi_k = \frac{1}{2}\left[\frac{t(1+\lambda_A)}{1-\lambda_A} - \beta_L(1+\lambda_B)\right] + \left[\frac{t(1+\lambda_B)}{1-\lambda_B} - \overline{\alpha}_L\right]\left[\frac{1}{2}\lambda_B + \frac{1}{2}\right] \tag{31}$$

with $k = 1, 2$.

5 Comparing Partial Multihoming and Singlehoming Equilibria

In this section we compare the cases analyzed so far. However, the three cases considered in the paper are not directly comparable given the fact that each configuration is based on different assumptions about the level of the network benefits of the high type agents. For example, we cannot compare the price structure in the pure singlehoming equilibrium with the one in the partial multihoming one, simply because those two equilibria are derived under different and incompatible values of α_H and β_H. However, we can compare the prices faced by agents in group A under partial multihoming on both sides and under partial multihoming on one side. The prices faced by group B under partial multihoming on one side can also be compared with the ones they faced under pure singlehoming.[19]

Another possible comparison is the utility levels in equilibrium of low type agents in each market configuration when those utility levels do not depend on network parameters of high type agents, because in this case the parameters involved satisfy the same assumptions. For example, under the assumption that the location is the same in each configuration, we can compare the utility level of a low type agent in group B under pure singlehoming and the one of the same type of agent under partial multihoming on one side. The utility of a low type agent in group A under partial multihoming on both sides can be compared with the one of the same type of agent under partial multihoming on one side. The problem of comparability is not present in Doganoglu and Wright (2006), since they do not derive explicitly the incentives to multihome for high type agents. They assume that independently of the level of the network benefits of high type agents, those agents exogenously singlehome or multihome.

The subscript SH defined the complete singlehoming case, while PM denoted the partial multihoming on both sides and PS denoted the case with partial multihoming on one side and singlehoming on the other side. We assume the following assumptions for comparisons. The first one is that the network benefit parameters of low type agents are equal in all configurations; that is, we assume that: $\alpha_L^{PS} = \alpha_L^{SH} = \alpha_L^{PM} = \alpha$ and $\beta_L^{PM} = \beta_L^{SH} = \beta_L^{PS} = \beta$. Thus, we have $\overline{\alpha}^{PS} = \overline{\alpha}^{SH} = \overline{\alpha}$. The second assumption concerns the proportions λ_A and λ_B, since in each configuration they are constrained by different assumptions. For example, in the case of SH, λ_A and λ_B need to satisfy only assumption $A1$, while in the case of PM, they need to satisfy also assumption $A2$. However, if λ_A and λ_B are sufficiently small, they will satisfy assumptions $A1$, $A2$ and $A3$ at the same time. This means that if the proportions λ_A and λ_B are sufficiently small, we can assume that they are equal among the different configurations.

The next proposition gives us the result relating to the comparison of p_A^{PM} with p_A^{PS} and the one of p_B^{PS} and p_B^{SH}:[20]

Proposition 7 *Independently of the degree of multihoming (singlehoming or partial multihoming) on the other side of the market, agents in each group face higher prices when there is partial multihoming on their side than when there is singlehoming.*

[19] As assumed in Section 4, partial multihoming on one side arises only on group B's side of the market, while group A's side of the market is singlehoming.

[20] We suppressed the subscript relative to the platform since prices are intra-groups symmetric.

From Propositions 2 and 6 we have $p_B^{PS} - p_B^{SH} = \frac{t(1+\lambda_B)}{1-\lambda_B} - t$, which is always positive, thus equilibrium prices faced by group B are higher with multihoming. Similarly, from Propositions 3 and 6 we have $p_A^{PM} - p_A^{PS} = \frac{t(1+\lambda_A)}{1-\lambda_A} - t$, which is always positive. The prices charged on group A are lower in the absence of multihoming. Multihoming agents' benefits are assumed so high that agents make a decision to join one platform independently from its decision to join the other. In this sense, there is no competition between platforms to attract multihoming agents, while platforms tend to price more aggressively on the singlehoming side.[21]

Looking at the utility levels of low type agents, we have the following possible comparisons: U_B^{PS} with U_B^{SH} and U_A^{PM} with U_A^{PS}. The next proposition states the result of these comparisons:

Proposition 8 *Independently of the degree of multihoming (singlehoming or partial multihoming) on the other side of the market, the utility level of a low type agent in each group is higher under singlehoming than under partial multihoming.*

From equation (3), $n_A^{SH} = \frac{1}{2}\left[\lambda_A + (1-\lambda_A)\right]$, $n_A^{PS} = \frac{1}{2}$ and the results in Propositions 2 and 6, we have $U_B^{SH} - U_B^{PS} = p_B^{PS} - p_B^{SH} > 0$. From equation (1), $n_B^{PM} = \frac{1}{2}\lambda_B + \frac{1}{2}$, $n_B^{PS} = \frac{1}{2}\lambda_B + \frac{1}{2}$ and the results in Propositions 3 and 6, we have $U_A^{PM} - U_A^{PS} = p_A^{PS} - p_A^{PM} < 0$. Proposition 8 says that the utility of a low type is always higher if on his side of the market there is singlehoming. This is because, by assumption, the price effect for a low type agent is higher than the network effect. If there is partial multihoming on his side of the market, the low type agent will face higher prices than in the case in which on his side there is singlehoming.

6 Conclusion

This paper analyzes imperfect competition between platforms with indirect network externalities, with a particular emphasis on partial-multihoming. We derive sufficient conditions for the existence of the partial-multihoming equilibrium. We assume there is a strong product differentiation and agents are heterogeneous in terms of their preferences over the network benefits. We considered a framework in which there are two distinct groups of agents, one on each side of the market. In each group there are two types of agents, according to their marginal valuation of the network size, low types and high types. The model presented here extends the model of Armstrong and Wright (2004) and Doganoglu and Wright (2006). To a certain extent the present paper complements theirs.

[21] Armstrong (2005) identifies an equilibrium with similar features, with the multihoming side being exploited and the singlehoming side being targeted aggressively. A key difference with us is that he assumes the competitive bottleneck structure (singlehoming on one side and multihoming on the other).

We characterized three particular equilibria as a function of the degree of partial multihoming on each side of the market. The paper focused on symmetric equilibria, in which the prices charged by platforms to the same group are equal. After the derivation of the equilibrium in the case in which no multihoming is possible, we derived under which conditions of network preferences, an equilibrium with partial multihoming on both sides exists. We showed that for such an equilibrium to exist, the network benefits of high type agents must be sufficiently higher than transportation costs. The main constraint for the existence of the equilibrium is that the proportions of agents that multihome on both sides must be sufficiently small, which is consistent with the empirical findings of Kaiser and Wright (2006). The reason is that the higher the proportion of agents that multihome on one side the lowers the benefits to those who multihome on the other side. Thus, in order to have partial multihoming on both sides, the proportions of multihoming agents must be significantly low such that the benefits from multihoming on one side of the market are greater than the negative effect implied by the presence of multihoming agents on the other side. We considered also the case in which there is partial multihoming only on one side of the market. We showed that the proportion of agents that multihome must be sufficiently small in equilibrium. With partial-multihoming on one side, the prices faced by the side with partial multihoming may be lower than the side with singlehoming if the proportion of agents multihoming is particularly low.

Finally, the paper compared some elements of the three different configurations. We showed that, compared to the case of complete singlehoming, if there is partial multihoming on one side of the market, agents on that particular side will face high prices. Comparing the case of complete singlehoming on both sides to the partial multihoming on one side of the market, singlehoming agents with partial multihoming on the other side will face higher price. The utility level of low type agents is always higher under complete singlehoming than under partial multihoming (on one side and two sides). This is due to the fact that the price effect always dominate the network effect for a low type agent, and that prices are higher under partial multihoming.

Appendix

Appendix A

In order to find the demand functions l_{Ak}, h_{Ak}, h_{Bk} and l_{Bk}, we need to solve the following equations $(a1)$ and $(a2)$ that arise from the problem of the indifferent consumer.

For a low type agent in group A, we have:

$$v_A - p_{A1} - tx + \alpha_L(\lambda_B h_{B1} + (1-\lambda_B)l_{B1}) = v_A - p_{A2} - t(1-x) + \alpha_L(\lambda_B(1-h_{B1}) + (1-\lambda_B)(1-l_{B1}) \tag{a1}$$

where we used the facts that $l_{B2} = 1 - l_{B1}$ and $h_{B2} = 1 - h_{B1}$. The above equation must be solved for $x = l_{A1}$.

For a high type agent in group A we have:

$$v_A - p_{A1} - tx + \alpha_H(\lambda_B h_{B1} + (1-\lambda_B)l_{B1}) = v_A - p_{A2} - t(1-x) + \alpha_H(\lambda_B(1-h_{B1}) + (1-\lambda_B)(1-l_{B1}) \tag{a2}$$

with $x = h_{A1}$.

Looking at agents in group B we have:

$$v_B - p_{B1} - tx + \beta_L(\lambda_A h_{A1} + (1-\lambda_A)l_{A1}) = v_B - p_{B2} - t(1-x) + \beta_L(\lambda_A(1-h_{A1}) + (1-\lambda_A)(1-l_{A1}) \tag{a3}$$

$$v_B - p_{B1} - tx + \beta_H(\lambda_A h_{A1} + (1-\lambda_A)l_{A1}) = v_B - p_{B2} - t(1-x) + \beta_H(\lambda_A(1-h_{A1}) + (1-\lambda_A)(1-l_{A1}) \tag{a4}$$

for the low type and the high type respectively. Equations $(a3)$ and $(a4)$ must be solved for $x = l_{B1}$ and $x = h_{B1}$ respectively. The solutions of the system of equations $(a3)$ and $(a4)$ are the following:

$$l_{A1} = \frac{1}{2} + \frac{t(p_{A2} - p_{A1}) + \alpha_L(p_{B2} - p_{B1})}{2(t^2 - \overline{\beta}\overline{\alpha})} + \frac{\lambda_A\overline{\beta}(\alpha_H - \alpha_L)(p_{A2} - p_{A1})}{2t(t^2 - \overline{\beta}\overline{\alpha})} \tag{a5}$$

$$l_{B1} = \frac{1}{2} + \frac{t(p_{B2} - p_{B1}) + \beta_L(p_{A2} - p_{A1})}{2(t^2 - \overline{\beta}\overline{\alpha})} + \frac{\lambda_B\overline{\alpha}(\beta_H - \beta_L)(p_{B2} - p_{B1})}{2t(t^2 - \overline{\beta}\overline{\alpha})} \tag{a6}$$

$$h_{A1} = \frac{1}{2} + \frac{t(p_{A2} - p_{A1}) + \alpha_H(p_{B2} - p_{B1})}{2(t^2 - \overline{\beta}\overline{\alpha})} + \frac{(1-\lambda_A)\overline{\beta}(\alpha_L - \alpha_H)(p_{A2} - p_{A1})}{2t(t^2 - \overline{\beta}\overline{\alpha})} \tag{a7}$$

$$h_{B1} = \frac{1}{2} + \frac{t(p_{B2} - p_{B1}) + \beta_H(p_{B2} - p_{B1})}{2(t^2 - \overline{\beta}\overline{\alpha})} + \frac{(1-\lambda_B)\overline{\alpha}(\beta_L - \beta_H)(p_{B2} - p_{B1})}{2t(t^2 - \overline{\beta}\overline{\alpha})} \tag{a8}$$

Using the definitions: $l_{B2} = 1 - l_{B1}, h_{B2} = 1 - h_{B1}, l_{A2} = 1 - l_{A1}$ and $h_{B2} = 1 - h_{B1}$ we obtain:

$$l_{A2} = \frac{1}{2} + \frac{t(p_{A1} - p_{A2}) + \alpha_L(p_{B1} - p_{B2})}{2(t^2 - \overline{\beta}\overline{\alpha})} + \frac{\lambda_A\overline{\beta}(\alpha_H - \alpha_L)(p_{A1} - p_{A2})}{2t(t^2 - \overline{\beta}\overline{\alpha})} \tag{a9}$$

$$l_{B2} = \frac{1}{2} + \frac{t(p_{B1} - p_{B2}) + \beta_L(p_{A1} - p_{A2})}{2(t^2 - \overline{\beta}\overline{\alpha})} + \frac{\lambda_B\overline{\alpha}(\beta_H - \beta_L)(p_{B1} - p_{B2})}{2t(t^2 - \overline{\beta}\overline{\alpha})} \tag{a10}$$

$$h_{A2} = \frac{1}{2} + \frac{t(p_{A1} - p_{A2}) + \alpha_H(p_{B1} - p_{B2})}{2(t^2 - \overline{\beta}\overline{\alpha})} + \frac{(1 - \lambda_A)\overline{\beta}(\alpha_L - \alpha_H)(p_{A1} - p_{A2})}{2t(t^2 - \overline{\beta}\overline{\alpha})} \tag{a11}$$

$$h_{B2} = \frac{1}{2} + \frac{t(p_{B1} - p_{B2}) + \beta_H(p_{B1} - p_{B2})}{2(t^2 - \overline{\beta}\overline{\alpha})} + \frac{(1 - \lambda_B)\overline{\alpha}(\beta_L - \beta_H)(p_{B1} - p_{B2})}{2t(t^2 - \overline{\beta}\overline{\alpha})} \tag{a12}$$

where under assumption $A1$, we have $t^2 > \overline{\beta}\overline{\alpha}$, implying that the demand functions are well defined.

Appendix B

Since high type agents will multihome, we need to solve for the demand functions of the low type agents only. From the problem of the indifferent consumer we obtain the following equations:

For a low type agent in group A, we have:

$$v_A - p_{A1} - tx + \alpha_L(\lambda_B + (1 - \lambda_B)l_{B1}) = v_A - p_{A2} - t(1 - x) + \alpha_L(\lambda_B + (1 - \lambda_B)(1 - l_{B1}) \tag{a13}$$

that has to be solved for $x = l_{A1}$. For a low type agent in group B, we have:

$$v_B - p_{B1} - tx + \beta_L(\lambda_A + (1 - \lambda_A)l_{A1}) = v_B - p_{B2} - t(1 - x) + \beta_L(\lambda_A + (1 - \lambda_A)(1 - l_{A1}) \tag{a14}$$

The solution of that problem gives us:

$$l_{A1} = \frac{1}{2} + \frac{t(p_{A2} - p_{A1})}{2(t^2 - \alpha_L\beta_L g)} + \frac{\alpha_L(p_{B2} - p_{B1})(1 - \lambda_B)}{2(t^2 - \alpha_L\beta_L g)} \tag{a15}$$

$$l_{B1} = \frac{1}{2} + \frac{t(p_{B2} - p_{B1})}{2(t^2 - \alpha_L\beta_L g)} + \frac{\beta_L(p_{A2} - p_{A1})(1 - \lambda_A)}{2(t^2 - \alpha_L\beta_L g)} \tag{a16}$$

Where $g = (\lambda_A - 1)(\lambda_B - 1)$ and $g \in (0, 1]$. Furthermore, since $l_{A2} = 1 - l_{A1}$ and $l_{B2} = 1 - l_{B1}$, we have:

$$l_{A2} = \frac{1}{2} + \frac{t(p_{A1} - p_{A2})}{2(t^2 - \alpha_L \beta_L g)} + \frac{\alpha_L (p_{B1} - p_{B2})(1 - \lambda_B)}{2(t^2 - \alpha_L \beta_L g)} \tag{a17}$$

$$l_{B2} = \frac{1}{2} + \frac{t(p_{B1} - p_{B2})}{2(t^2 - \alpha_L \beta_L g)} + \frac{\beta_L (p_{A1} - p_{A2})(1 - \lambda_A)}{2(t^2 - \alpha_L \beta_L g)} \tag{a18}$$

In order for the demand functions to be well-defined we need to assume that $t^2 > \alpha_L \beta_L g$.

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