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by

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1 Introduction

This paper develops a model of the UK macroeconomy and the government bond market and uses it to analyse the operation of monetary policy since 1979. Observations on both the economy and the bond market are used to determine the parameters of the model, which assumes that the long run inflation expectation implicit in the bond market is the same as that in the product and factor markets. This approach generates a well-determined macro-based yield model with a relatively large number of factors and provides new insights into the efficacy of UK monetary policy over the last 25 years.

The macromodel is based on the specification originally developed for the US by Svensson (1999); Smets (1999) and others. This model represents the behaviour of the macroeconomy in terms of three variables: the output gap (g_t), inflation (π_t) and the short term interest rate (r_t). This specification is often called the 'central bank model' since it provides a basic dynamic description of an economy in which the central bank implicitly or explicitly targets inflation using a Taylor rule, which determines the policy interest rate in terms of inflation and the output gap. The other two equations of the system then show how inflation and the output gap respond to each other and to the interest rate. This model can be specified as a Vector Auto-Regression (VAR) or as a structural system in which the parameters are restricted in the light of economic theory.

This macro model is combined with an arbitrage-free model of the yield curve. As the name suggests, this type of model specifies bond yields as functions of basic driving variables or 'factors' in a way that removes arbitrage opportunities. The standard model is 'affine,' meaning that yields are linear combinations of the basic factors. This in turn means that if there are N underlying factors, they can be represented by

N bond yields, which can in principle be used to explain yields at other points in the maturity spectrum. This property is exploited by the yield factor approach, which (since Brown and Schaefer (1994)) has been extensively used for testing arbitrage-free specifications (Duffie and Kan (1996),Dai and Singleton (2000)). This property is also used in the hybrid macro-finance specification developed in this paper.

Recent studies have used macroeconomic factors to model the yield curve (Ang and Piazzesi (2003),Dewachter and Lyrio (2003)). However, these 'macro-finance' studies reveal that although macro variables provide a good description of the behaviour of short term yields; they do not describe long term yields very well (Kozicki and Tinsley (2001)). In fact, this finding suggests that the basic central bank model, which is used to represent the macro dynamics in these yield models, suffers from an omitted variable misspecification. That is because in an arbitrage free world, variables can influence (non-defaultable) bond yields if and only if they influence the short term interest rate and the macroeconomy. In order to allow for this phenomenon, macro-finance studies have typically used a Kalman filter to model the additional effect of an latent expectations variables on inflation, the short rate and hence bond yields. This unobservable variable is updated in the light of forecast errors. It is interpreted as an inflation target or expectations variable, but could also reflect changes in the real rate of return.

This approach assumes that the market forms expectations adaptively. However, bond markets are arguably forward looking. They can react immediately to political events and policy announcements, which an adaptive scheme can respond to only gradually. Instead, the specification developed in this paper derives a direct measure of market expectations by exploiting the yield-factor property of the affine yield model. The affine nature of the system means that if the basic macro-yield model were to omit M (possibly unobservable) factors, then this misspecification could be

rectified by introducing M yields into the macromodel as additional explanatory variables.

This immediately means that we can use these yields in Granger causality tests to check for omitted variable misspecification in the central bank model. In this context, it can be shown that long term (17 year[1]) conventional (y_t^*) and index linked (l_t^*) yields Granger-cause the three macro variables[2]. These tests suggest a model in which there are (besides the three macro variables) two unobservable factors, real and nominal, represented by (l_t^*) and (y_t^*). Work by (Grilli and Roubini (1996), Estrella (2005)) and others suggest the use of long conventional yields (or yield spreads) in is macromodels, on the grounds that they act as indicators of inflation and output expectations. In the case of the UK, we can improve upon this specification by putting the long term index linked yield into the VAR as well.

These variables were used initially in a VAR to determine the broad dynamic dimensions of the empirical version of the macro model set out in the next section. Then section 3 shows how the arbitrage free yield specification can be used to back out the unobservable nominal variable, adjusting the long term yield to get an estimate of the inflation expectation implicit in the data. This represents an improvement on other methods (described for example by Anderson and Sleath (2001)) which take the simple difference between nominal and real yields (or forward rates), because it allows for risk premia. This estimate is also determined by the behaviour of the varaibles in the macro model. Section 4 of the paper discusses the results and draws out the implications for monetary policy. Section 5 provides a brief conclusion and suggestions for further work in this area.

[1] This is the longest maturity for which a constent set of data could be obtained - see section 3.

[2] Once these two yields were included in the regression (alongside the spot rate), other yields were insignificant. So in this hybrid macro-yield factor model $M = 2$. Consistent with the theory of the long yield asymptote set out in appendix 1, the macro variables do not Granger-cause the two long rates.

2 The macroeconomic model

For simplicity, in this paper I use the real rate (l_t^*) itself to represent the unobservable real factor[3]. This gives a model with a single (nominal) unobservable variable, acting on the yield curve through the macro system and hence the spot rate (r_t). Because a latent variable is only defined up to a linear transform, this needs to be normalised. In this paper I do this by equating it with the long run inflation expectation (π_t^*), so that long term inflation expectations in the gilt-edged (British Government) bond market coincide with those in the product and labour markets. If the monetary authorities announce a credible inflation target then we should also find that π_t^* is in line with the target. To be consistent with the target, the asymptote of the equation driving the spot rate (r_t^*) should equal π_t^* plus a mark-up representing the real spot rate. This should (in the absence of a real risk or liquidity premium) equal l_t^*, giving: $r_t^* = \pi_t^* + l_t^*$. However, if the markets have to infer π_t^* from say a monetary or exchange rate target (as they did until 1993) or if the inflation target is not credible, then this identity could break down. To allow for this possibility, the general macro model is specified with: $r_t^* = k_r + \rho_r l_t^* + \nu_\pi \pi_t^*$; where the restrictions $k_r = 0, \rho_r = 1, \nu_r = 1$ offer tests for target transparency & credibility and for the presence of real risk premia.

The dynamic specification was explored using $(y_t^* - l_t^*)$ as a proxy for π_t^* in

[3] Indexed yields may contain a risk premium, but their relatively low volatility means that this should be relatively small, allowing the 17 year discount factor rate to provide a good indicator of the medium term real rate of return.

preliminary OLS regressions. This suggested a macro model of the form:

$$m_t = \kappa_m + \Phi'_{31} l_t^* + \Phi'_{32}\ \pi_t^* + \Phi'_{33} m_t + \Phi'_{34} m_{t-1} + w_{m,t} \tag{1}$$

$$where\ :\ m'_t = \{\pi_t, g_t, r_t\}$$

$$\kappa_m = (I - \Phi'_{33} - \Phi'_{34})k;\ \Phi'_{31} = (I - \Phi'_{33} - \Phi'_{34})\rho;\ \Phi'_{32} = (I - \Phi'_{33} - \Phi'_{34})\nu. \tag{2}$$

This has the steady state solution:

$$m_t^* = (k + \rho l_t^* + \upsilon\ \pi_t^*) \tag{3}$$

where $k' = \{0, k_g, k_r\}, \rho' = \{0, \rho_g, \rho_r\}$ and $\nu' = \{1, \nu_g, \nu_r\}$ show the steady state intercepts and the effects of l_t^* and π_t^* on the macrovariables.

The exploratory regressions show that the macro variables do not Granger cause the real rate (l_t^*). I was able to model this yield satisfactorily using a simple mean reverting AR(1) process:

$$l_t^* = \kappa_1 + \phi_{11}\ l_{t-1}^* + w_{l,t}. \tag{4}$$

consistent with the view that long term real interest rates are determined independently of macroeconomic influences. Similarly, using $(y_t^* - l_t^*)$ as a proxy for π_t^* preliminary OLS regressions also suggested an AR(1) model for the inflation objective:

$$\pi_t^* = \kappa_2 + \phi_{22}\pi_{t-1}^* + w_{2,t}. \tag{5}$$

consistent with the view that this objective should be independent of short run mean-reverting macroeconomic considerations. These preliminary results thus suggest a

dynamic model for $z_t' = \{l_t^*, \pi_t^*, m_t'\}$ of the recursive form:

$$
\begin{aligned}
z_t &= \kappa + \Phi_1' z_{t-1} + \Phi_2' m_{t-2} + w_t; \text{ where:} \\
\Phi_1' &= \begin{bmatrix} \phi_{11} & 0 & 0_{13} \\ 0 & \phi_{22} & 0_{13} \\ \Phi_{31}' & \Phi_{32}' & \Phi_{33}' \end{bmatrix}; \ \Phi_2' = \begin{bmatrix} 0_{13} \\ 0_{13} \\ \Phi_{34}' \end{bmatrix}; \\
\kappa' &= \{\kappa_{1,t}, \kappa_{2,t}, \kappa_{m,t}'\}; \\
w_t' &= \{w_{1,t}, w_{2,t}, w_{m,t}'\};
\end{aligned} \tag{6}
$$

and where:

$$
\begin{aligned}
w_t &= C' u_t \sim N(0, \Sigma) \\
\Sigma &= C' D C.
\end{aligned} \tag{7}
$$

u_t is a 5×1 vector of orthogonal i.i.d. normal error terms:

$$
\begin{aligned}
u_t &\sim N(0, D) \\
D &= diag[d_1, d_2, d_3, d_4, d_5].
\end{aligned} \tag{8}
$$

D is an 5×5 diagonal matrix and C' is a lower triangular matrix with unit diagonals. 0_{mn} represents an $m \times n$ null matrix.

This system may be written in the standard first order difference format as:

$$
Z_t = K + \Phi' Z_{t-1} + W_t \tag{9}
$$

with:

$$\Phi' = \begin{bmatrix} \Phi'_1 & \Phi'_2 \\ [0_{32}\ I_3] & 0_{33} \end{bmatrix} \tag{10}$$

$$K' = \{\kappa'_t, 0_{13}\}$$

$$W'_t = \{w'_t, 0_{13}\}$$

where the state vector is defined as $Z'_t = \{l^*_t, \pi^*_t, m'_t, m'_{t-1}\}$ and I_n is an $n \times n$ identity matrix.

3 The yield model

To translate this into a model with observable variables I use the relationship between the long yield y^*_t and the unobservable variable π^*_t implied by the no arbitrage assumption. Appendix 1 shows that under this assumption, when the system is described by (9) and (10) every nominal bond yield must be described by a linear function of the state vector Z_t. The solution for the negative of the logarithm of the $m-$maturity discount bond price is:

$$-p_{m,t} = \gamma_m + \Psi'_m Z_t \tag{11}$$

where the slope coefficients are:

$$\Psi'_m = j'(I - \tilde{\Phi}'^m)(I - \tilde{\Phi}')^{-1} \tag{12}$$

$$where\ :\ \tilde{\Phi}' = \Phi' - \Sigma\Lambda'_1.$$

and the intercept follows by recursion of (28) and (24). These coefficients depend upon the dynamics under the risk adjusted measure ($\tilde{\kappa}, \tilde{\Phi}$). They differ from those

estimated under the observed measure for the macromodel dynamics (κ, Φ) by additive terms which depend upon the error variances and parameters which reflect the price of risk (25) and determine the risk premia in this type of model (30).

Dividing across (11) by maturity gives discount yield:

$$\begin{aligned} y_{m,t} &= \gamma_m/m + \Psi'_m Z_t/m \\ &= \alpha_m(\Lambda, K, \Phi, C, D) + \beta_m(\Lambda, K, \Phi) Z_t \end{aligned} \tag{13}$$

For the long yield $y_{17} = y_t^*$ we may partition $\beta^*(\Lambda, K, \Phi) = \beta_{17}(\Lambda, K, \Phi)$ to get:

$$y_t^* = \alpha^*(\Lambda, K, \Phi, C, D) + \beta_1^*(\Lambda, K, \Phi) l_t^* + \beta_2^*(\Lambda, K, \Phi) \pi_t^* + \beta_3^*(\Lambda, K, \Phi)' m_t + \beta_4^*(\Lambda, K, \Phi)' m_{t-1} \tag{14}$$

where Λ is a vector of auxiliary parameters defining the risk aversion of the market and the α and β parameters are defined by (12), (28) and (24) & (13). Since all of the other variables in this equation are observable, we can use this to infer π_t^* :

$$\pi_t^* = (y_t^* - \alpha^*(\Lambda, K, \Phi) - \beta_1^L(\Lambda, K, \Phi) l_t^* - \beta_3^*(\Lambda, K, \Phi)' m_t - \beta_4^*(\Lambda, K, \Phi)' m_{t-1}) / \beta_2^*(\Lambda, K, \Phi) \tag{15}$$

giving the transforms:

$$X_t = \Theta + \Xi' Z_t \ ; \tag{16}$$

$$Z_t = \Xi'^{-1} \ (X_t - \Theta);$$

$$where \ : \ \Xi' = \begin{bmatrix} \Xi'_1 \ \Xi'_2 \\ 0_{35} \ I_3 \end{bmatrix} \ ; \ (\Xi')^{-1} = \begin{bmatrix} (\Xi')_1^{-1} \ (\Xi')_2^{-1} \\ 0_{35} \qquad I_3 \end{bmatrix}$$

$$\Xi'_1 = \begin{bmatrix} 1 & 0 & 0_{13} \\ \beta_1^* & \beta_2^* & \beta_3^* \\ 0 & 0 & I_3 \end{bmatrix} ; \Xi'_2 = \begin{bmatrix} 0_{13} \\ \beta_4^* \\ 0_{33} \end{bmatrix} ;$$

$$(\Xi')_1^{-1} = \begin{bmatrix} 1 & 0 & 0_{13} \\ -\beta_1^*/\beta_2^* & 1/\beta_2^* & -\beta_3^*/\beta_2^* \\ 0 & 0 & I_3 \end{bmatrix} ; (\Xi')_2^{-1} = \begin{bmatrix} 0_{13} \\ -\beta_4^*/\beta_2^* \\ 0_{33} \end{bmatrix}$$

$$\Theta' = \{\theta', 0_{13}\}$$

$$\theta' = \{0, -\alpha^*/\beta_2^*, 0_{13}\}$$

where $X'_t = \{l_t^*, y_t^*, m'_t, m'_{t-1}\}$ and similarly $x'_t = \{l_t^*, y_t^*, m'_t\}$. Substituting (16) into (9) allows the macromodel to be written in terms of this observable vector as:

$$\begin{aligned} X_t &= \Theta + \Xi[K + \Phi' Z_{t-1} + W_t] \\ &= \Theta + \Xi[K + \Phi' \Xi'^{-1} \ (X_{t-1} - \Theta) + W_t] \end{aligned} \tag{17}$$

Dropping the identity represented by the last three lines of this system and simplifying

gives the empirical version of the macromodel:

$$x_t = \theta + \Xi_1'\kappa + [\Xi_1' \;\; \Xi_2'] \begin{bmatrix} \Phi_1' & \Phi_2' \\ [0_{32} \; I_3] & 0_{33} \end{bmatrix} \begin{bmatrix} (\Xi')_1^{-1} & (\Xi')_2^{-1} \\ 0_{35} & I_3 \end{bmatrix} \begin{bmatrix} x_{t-1} - \theta \\ m_{t-2} \end{bmatrix} + \Xi_1' w_t$$

$$= \theta + \Xi_1'\kappa + [[\Xi_1'\Phi_1' + \Xi_2'[0_{32} \; I_3]] \; \Xi_2'\Phi_2'] \begin{bmatrix} (\Xi')_1^{-1} & (\Xi')_2^{-1} \\ 0_{35} & I_3 \end{bmatrix} \begin{bmatrix} x_{t-1} - \theta \\ m_{t-2} \end{bmatrix} + \Xi_1' C' u_t$$

$$= \Xi_1'\kappa + [I_5 - \{[\Xi_1'\Phi_1' + \Xi_2'[0_{32} \; I_3]](\Xi')_1^{-1}\}]\theta + \{[\Xi_1'\Phi_1' + \Xi_2'[0_{32} \; I_3]](\Xi')_1^{-1}\}x_{t-1}$$

$$+ \; \{[\Xi_1'\Phi_1' + \Xi_2'[0_{32} \; I_3]](\Xi')_2^{-1} + \Xi_2'\Phi_2'\}m_{t-2} + \Xi_1' C' u_t$$

$$= k + F' x_{t-1} + G' m_{t-2} + \eta_t$$

where:

$$k = \Xi_1'\kappa + [I_5 - \{[\Xi_1'\Phi_1' + \Xi_2'[0_{32} \; I_3]](\Xi')_1^{-1}\}]\theta; \tag{18}$$

$$F' = [\Xi_1'\Phi_1' + \Xi_2'[0_{32} \; I_3]](\Xi')_1^{-1}$$

$$G' = \; [\Xi_1'\Phi_1' + \Xi_2'[0_{32} \; I_3]](\Xi')_2^{-1} + \Xi_2'\Phi_2'$$

$$\eta_t = \Xi_1' C' u_t$$

Similarly, let y_t be a vector containing yields on the 1,2,3,5,7 and 10 year maturities modelled in this study. Appendix 1 shows that this may be represented as:

$$y_t = \alpha(\Lambda, K, \Phi, C, D) + \beta(\Lambda, K, \Phi)' Z_t + e_t$$

$$e_t \sim N(0, \Delta);$$

$$\Delta = diag[\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6].$$

which we may write using the transform (16) as:

$$y_t = \alpha(\Lambda, K, \Phi, C, D) + \beta(\Lambda, K, \Phi)'\Theta + \beta(\Lambda, K, \Phi)'\Xi X_t + e_t \tag{19}$$

The residuals e_t in these relationships are conventionally interpreted as measurement errors and assumed to be independent of the errors w_t in the dynamic system (9). Appendix 2 derives the likelihood function for the joint model (18) and (19).

4 The empirical model

The algebra of the previous section provides an arbitrage-free model of the macroeconomy and the bond market, but can it provide a plausible description of the data generating process? After all, VAR-type analysis, which eschews the restrictions implied by structural models of monetary policy and the macroeconomy often generate puzzling dynamic responses (Grilli and Roubini (1996)). This section describes the dataset and the empirical results.

4.1 The data

In this paper I use the Retail Price Index excluding mortgage interest payments (RPIX) to measure inflation (π_t). This was the policy objective (with a target rate of 2.5 %) between November 1992 and April 2004. As in previous macro-finance studies, inflation is measured on an annual basis. The three month Treasury Bill rate is used to represent the spot rate (r_t). Both of these series were taken from Datastream. Quarterly estimates of the GDP output gap (g_t) were kindly provided by Oxford Economic Forecasts. This measure is based on the production function approach, building up potential GDP from estimates of the capital stock, labour force and productivity, and then subtracting GDP to obtain the estimate of the output

gap[4]. These data dictated the use of a quarterly time frame. The macro data are shown in chart 1[5].

The gilt-edged yield data were taken from the Banks of England's website and are derived using the methodology discussed in Anderson and Sleath (2001). These data are available monthly from the beginning of 1979, which determines the starting point of my estimation period (1979 q1-2004q2). The 17 year indexed (l^*) and conventional (y^*) discount gilt yields (the longest maturity for which a continuous series are available from the Bank of England) influence the macro variables in this model and are shown in chart 2. The former is only available since the first quarter of 1985 and so the equation for (l^*) is only fitted over the period 1985q1-2004q2. This indexed yield is used as a lagged dependent variable in the other equations. These were estimated over the period 1979 q1-2004q2 assuming that l^* remained unchanged at its value before that date[6]. The yield model is fitted to conventional yields for the 1,2,3,5,7 and 10 year maturities (y_t), shown in chart 3.

Table 3 shows the means; standard deviations and first order autocorrelation coefficients of these data. It also shows ADF test results (the 95% critical value of the ADF test statistic is 2.83). Consistent with the preliminary OLS results mentioned above, the ADF test suggest that we reject the null hypothesis of non-stationarity in the case of the output and inflation variables. The results for the spot rate are not decisive, but the null hypothesis of non-stationarity for the two long rates cannot be rejected.

[4] This was used in preference to the OECD measure based on the trend filtering approach, since this indicates that output was above trend in 2004, in contrast to the impression given by the behaviour of inflation and other macroeconomic variables.

[5] These are annual rates in percentages. In the empirical model these were appropriately converted to quarterly decimal fractions by dividing by 400.

[6] The indexed market was very thin during the early 1980s but there is little evidence from the prices that are available that long term real rates of return shifted significantly over this period.

4.2 The empirical model

The model of section 2 has a total of 78 parameters. However, 30 of these are used for the risk adjustments in λ_0 and Λ_1. As explained, these parameters drive a wedge between the coefficients estimated under the observed measure for the macromodel dynamics and those estimated under the risk adjusted measure for the yield coefficients. In the general specification the two sets of parameters are largely separate[7] and reflecting this, there is a degree of parameter indeterminacy in that specification. However, starting with this and testing down showed that the last three columns of the matrix Λ_1 could be set to zero, leaving a specification in which the time variation in the risk premia depends upon the two long rates, but not inflation, output or spot rates. This reduces the number of parameters by 15, to give a model with 63 parameters, which are in the main well determined statistically[8]. This demonstrates the benefit of using the macro dynamics to help inform the parameters of the yield curve model and *vice versa*.

Table 2 reports the basic goodness of fit statistics for the 11 equations of this model. The first row shows that the model explains 99% of the variance of the one year yield, falling to a minimum of 98.3% for the 3 year, then increasing to 99.6% in the 10 year area. The second row reports the Root Mean Square Errors (RMSE), which show a similar (inverse) pattern. Tables 3(a)-(c) report the dynamic (κ,Φ); stochastic (C, D, Δ) and risk (Λ) parameters of the model. These are generally well determined, although as is usual in a VAR, some are insignificant at the 5% level.

[7] Note however, that because m_{t-1} cannot affect the price of risk $\tilde{\Phi}_2' = \Phi_2'$.

[8] Comparing this with the general model gave a loglikelihood ratio test statistic of 17.5, well below the 95% critical value of 25.0 ($p = 0.29$).

4.3 Steady state properties

The long run properties of the model are determined by the normalisation described in the previous section and the estimates of ρ_g, ρ_r, k_g, k_r and ν_g, ν_π shown in Table 3(a). The first of these parameters is not significant, but the second estimate shows that a change in l^* increases the spot rate on a point for point basis, consistent with the assumption that the risk premium at the long end of the indexed market is negligible. The estimate for ν_π is insignificantly different from unity, suggesting a high degree of consistency between the inflation and spot rate equations. The estimate for ν_g presents more of a puzzle, suggesting that there may be a relationship between long run inflation (π^*) and unemployment (g). However, this effect is not very well determined statistically and may reflect measurement error in the output gap indicator.

One of the novel features of this model is the expectation variable (π^*) for the inflation rate and (adding l^*) the yield curve. This is backed out of the data using (15). The empirical version is:

$$\pi_t^* = 4.661 + 0.949229 y_t^* - 0.738557 l_t^* + 0.12605 \pi_t + 0.14000\, g_t - 0.27830 r_t \quad (20)$$

$$\begin{array}{cccccc} (43.33) & (233.33) & (\text{-}135.49) & (9.55) & (14.99) & (\text{-}18.43) \end{array}$$

$$+9.619\,9 \times 10^{-3} \pi_{t-1} - 4.293\,3 \times 10^{-2} g_{t-1} + 2.500\,2 \times 10^{-2} r_t$$

$$\begin{array}{ccc} (-) & (-) & (-) \end{array}$$

(The figures in parentheses show asymptotic t-ratios, which are calculated numerically.) This equation shows that (π^*) is essentially the difference between the conventional and real yields used in the preliminary regressions, but that significant

adjustments should to be made for the current (though not the lagged) values of the macro variables. This variable is primarily an indicator of long run expectations in product and factor markets. However, it provides an important test of the credibility of monetary policy since if policy is credible (π^*) should be related to the inflation objective (at least if this is expressed as an explicit target). Also, the deviation from target ($\pi_t - \pi_t^*$) should help the model to explain the interest rate responses of the authorities.

The near-unit value of the estimate of ν_π suggests a close alignment between monetary policy and the markets' inflation expectations. Time variations in π_t^* throw more light on this issue, suggesting that the precise expression and implementation of the policy framework is important in influencing expectations. The implicit values of (π^*) are shown (as the bold line) in Chart 4 alongside the simple proxy ($(y_t^* - l_t^*)$, continuous line) and RPIX inflation itself (broken line). Allowing for the impact caused by disequilibrium in the macroeconomy, π_t^* gives an indicator which is less variable than the simple indicator. However, the economy has been relatively stable since 1992 and the difference between the two indicators since then has been small. The π_t^* measure is remarkably stable, but suggests that inflationary expectations fell back between 1982 and 1986. This was associated with favourable developments in the world economy, but a more interesting explanation lies in the Falklands War of 1982 and the re-election of the Thatcher government May 1983. This shift probably reflects a move to a more credible monetary policy rather than a change in the policy objective, but it is hard to be sure since the inflation objective was not made explicit at the time.

There is another downward movement in π_t^* in 1997-98. This is more instructive because the 2.5% target for RPIX inflation was announced earlier: late in 1992 after the pound was forced out of the ERM. Chart 4 shows that the introduction

of the target had little effect initially. Indeed, there is evidence of an increase in inflationary expectations in September 1993 (π_t^*, increased by 133 basis points in Q3). Although inflation remained close to the target, π_t^* remained well above 2.5%. However, the handover of monetary control to the Bank of England following the May 1997 election reduced both indicators, suggesting that the credibility of the framework depends upon the operational arrangements for its implementation as well as its precise expression. These indicators have been remarkably stable since 1998, broadly consistent with the 2.5 % target for the RPIX.

The π_t^* indicator also gives a plausible account of policy reactions to deviations from target. In particular, the large deviations $(\pi_t - \pi_t^*)$ which Chart 4 shows in 1979-82 and 1988-92 coincide nicely with the episodes of high policy interest rates shown in chart 1, reflecting the use of this indicator in the macromodel. The next section looks at the monetary and macro responses in more detail.

4.4 Dynamic properties

These dynamic responses are dominated by the diagonal elements of Φ_1 which are relatively large and significant (Table 3(a)). Table 4 reports the eigenvalues. The first two roots are provided by the autoregressive coefficients for the long bond yields (in the order ϕ_{22} and ϕ_{11}) which imply a very high degree of persistence. The other eigenvalues are associated with the macro variables. These are oscillatory, but the imaginary (and real) components are relatively small, meaning that cyclical behavior is weak. Consequently, the dynamics are dominated by exponential rather than cyclical behaviour, which is is heavily damped.

This can be seen more clearly from the impulse responses of the macrosystem, which show the effect of shocks to the five driving variables. Because innovations in these variables (w_t) are correlated it is unrealistic to vary them independently.

Instead it is usual to work with the orthogonalised shocks (u_t) which are determined by the factorisation (7). Appendix 3 gives the technical detail.

At this stage the ordering of the macro variables in the vector z_t becomes important. Because any non-negative definite matrix has a triangular factorisation, the ordering of the variables in z_t does not affect the estimate of Σ, only the parameters of the matrices C and D used to make the factorisation (34). Since C' is a lower triangular matrix, this means that shocks to the first variable (u_1) disturb all five driving variables contemporaneously, shocks to the second (u_2) affect the remaining four variables but not the first and so on. This makes it important to order the variables in terms of their degree of exogeneity or sensitivity to contemporaneous shocks. In this model, l^* and π^* are ordered first on the view that macroeconomic developments should not affect the steady state real rate of return or the long run policy objective[9]. Next, I follow (Hamilton (1994)) and order prices before output. There are various ways to justify this, but the negative contemporaneous correlation between output and inflation means that with this arrangement, positive inflation shocks coincide with negative output shocks. This allows u_3 to be interpreted as a negative supply shock. In turn, u_4 increases output without any immediate impact on inflation and is interpreted as a demand shock. Interest rates are sequenced after these variables on the view that output and prices do not react immediately to monetary policy. Thus the variable ordering is: long bond yields; inflation, output gap then the spot rate.

These orthogonal shocks (u_t) determine the innovations (w_t) in the five driving variables via (7). This may be interpreted as a system of linear regression equations,

[9] The ordering of this pair does not matter because I assume that shocks to these variables are also independent.

with the regression betas:

$$C' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ c_{31} & c_{32} & 1 & 0 & 0 \\ c_{41} & c_{42} & c_{43} & 1 & 0 \\ c_{51} & c_{52} & c_{53} & c_{54} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0.32008 & 0.48864 & 1 & 0 & 0 \\ 0.29910 & 0.31058 & -0.11681 & 1 & 0 \\ 0.98520 & 0.50953 & 0.53893 & 0.53209 & 1 \end{bmatrix}$$

These coefficients determine the immediate response of the driving variables to the shocks. So a long rate shock is reflected immediately in the spot rate on (approximately) a point-for-point basis. In contrast, c_{32} indicates a relatively small adjustment in the spot rate in response to a change in the inflation expectation (u_2). A unit supply shock (u_3) puts up inflation by one point, depresses output by just over a tenth of a point initially, and increases the spot rate by just over half a point.

Charts 5(a)-(e) show the dynamic responses (in percentage points) of inflation, output and interest rates to shocks in the structural errors (u). Since l^* and π^* are driven by simple autoregressions, their responses are not illustrated. For diagnostic simplicity, shocks to l^* and π^* are assumed to be permanent, while the other three error terms are perturbed for just one period, after which they are set back to zero, allowing the dynamics effects to work through. These tables show the effects over the first 40 quarters (10 years) and the horizontal axis is calibrated in terms of

quarters. The effects of the macro shocks are generally quick and cyclical, reflecting the eigenvalues discussed earlier, while the effect of shocks to the long bond yields persist. The tables show the effect of these shocks on inflation (dotted line), output (broken line) and interest rates (unbroken line).

Chart 5(a) shows the effect of permanent shock to the long term rate of return (u_1). This has (approximately) a point-for-point impact upon the spot rate, reflecting the coefficients discussed earlier. The initial effect on inflation is positive, but the long run impact is restricted to be zero. Chart 5(b) shows the effect of permanent shock to the inflation expectation (u_2). To interpret this, I now reverse the signs on these effects, to show the effect of a *reduction* in π^* (which is more realistic over this period). The spot rate lags inflation slightly, so real rates increase temporarily, helping to push the system to a lower inflation target. Output falls initially, but this fall is then reversed as a result of the effect of lower inflation on output.

The effect of inflation on output - one of the noteworthy features of this model - is shown more clearly in Chart 4 (c). This shows the effect of a negative supply side shock (u_3). Recall that the impact effect is to depress output (c_{43}). This is reinforced by the negative dynamic effects of inflation on output ($\phi_{33,21}$ and $\phi_{34,21}$). These probably reflect real balance and other inflationary effects on consumption which are a well known feature of the UK economy (Davidson and Yeo (1978)). The initial responses of inflation and interest rates to the supply shock are positive, but Chart 5 (c) reveals that these effects are quickly countered by the fall in output. As we would expect, Chart 5(d) shows that the output gap (representing a demand shock) has a positive effect on inflation and interest rates. The spot rate responds immediately, but adjusts back towards its initial level quickly. This swift policy response is effective, bringing output (and inflation) back down again over a five year timespan, without inducing oscillations in their behaviour. In this respect monetary

policy appears to have been very timely.

Finally, Chart 5(d) shows the effect of innovations in interest rates on the economy. The effect on the spot rate itself decays away exponentially, without policy reversals. Inflation falls in response to higher interest rates and although the effects may look small, the price level is 3.8% lower after 10 years. Output falls initially, but this effect is soon countered by the favourable real balance effect. There is no evidence here of the 'price puzzle' that has dogged previous VAR-based studies of monetary policy[10]. Taken together, these responses give a plausible description of the macro dynamics, with π^* acting as a lead indicator of output and inflation. Shocks to l^* and π^* are assumed to be persistent, but the system is back close to its initial values after a five year period following supply and demand side shocks.

4.5 Yield curve responses

The impulse response patterns for the bond yields are determined by (35) and thus depend upon the sensitivity of the macro factors to shocks (given by the impulse responses of the previous section) and the sensitivity of yields to the macro factors (the beta coefficients β_m or factor loadings). The factor loadings are reproduced for all maturities up to 10 years in Chart 6. The separate panels show the loadings on the factors, plotted as a function of maturity expressed in quarters, shown on the horizontal axis. These loadings depend upon the risk adjusted dynamics, reflected in the eigenvalues of the matrix $\tilde{\Phi}$ reported in Table 4.

The first panel shows the loadings on l^* (broken line) and π^* (continuous line). The slow-moving nature of these variables means that these loadings increase with

[10] The price puzzle was defined by (Grilli and Roubini (1996)) as follows: 'When monetary policy is identified with innovations in interest rates, the response of the price level is wrong as monetary tightening is associated with a permanent increase in the price level rather than a decrease'. This problem goes back to Sims (1992). My preliminary work on UK macro VARs revealed similar problems - if long term yields are not included in the VAR then inreases in short term interest rates apparently increase the price level. However, the inclusion of long term yields - which better anticipate inflationary developments - seems to solve this problem.

maturity over this range. The next panel shows the loadings on π (dotted line), g (broken line) and r (continuous line). The identity between the one quarter yield and the spot rate means that the contemporaneous spot rate has a unit coefficient at a maturity of one quarter and other factors have a zero loading (24). The interest rate loadings tend to decline monotonically with maturity, while those of the other macro factors exhibit a humped shape.

The impulse response patterns for the yield model are shown in Charts 7 (a)-(d). These report the effects on the 1 - 10 year yields of the inflation, output and interest rates shocks described in the previous section. Although the model is fitted using only 6 yield observations in each period, in principle it can be used to compute the yield response at any maturity. The loading pattern means that the impulse response patterns for the short maturity yields are similar to those for the spot rate. Consistent with the earlier results, the effects of macro shocks disappear very quickly, while those of the unobservable variable underpinning the long bond yield are very persistent.

Finally, Chart 8 shows the holding period risk premia (annualised one-period ahead expected excess returns) implied by the model. In an arbitrage-free model, the risk premium on an $m-$period bond (31) is equal to the covariance between the $(m-1)$-period bond price and the nominal SDF (which is related to the marginal rate of intertemporal substitution in a utility-based model). Over this period, this covariance tends to fall with the degree of macroeconomic disequilibrium, explaining the fall in the premia shown in the chart. Appendix 1 shows (30), that these premia depend upon differences between the observed and risk neutral probability measures, as well as the factor loadings at each maturity. The risk premia tend to increase over the maturity range shown in the chart, largely as a consequence of the increase in the factor loadings on l^* and π^*.

5 Conclusion

The model reported in this paper is based on the idea that we can use the markets' inflation expectations to represent the UK monetary authorities' medium inflation objective, allowing for time variation in this aspiration. This variable (π_t^*) takes the long conventional gilt yield and adjusts it to allow for the risk premium, as determined by a macro-finance model of the yield curve. This adjustment reflects the disequilibrium in the UK economy, which was particularly significant before 1992. Because π_t^* drives the macro dynamics in this model, it is informed by the behaviour of the macroeconomy as well as the bond market.

The empirical model provides an interesting account of UK monetary policy since 1979, as well as furnishing a useful description of the gilt-edged yield curve. The π_t^* indicator is surprisingly stable, more so than a simple difference of indexed and conventional yields, but being forward-looking, can shift abruptly in response to political events like wars, general elections and changes in the monetary regime. It reveals a progressive downward movement in expectations between 1980 and 1998, followed by a stabilisation. The movements in this variable since 1992 are particularly revealing, suggesting that the announcement of an inflation target may not be credible unless it is supported by appropriate operational arrangements. The π_t^* indicator acts as a leading indicator of activity and inflation while $(\pi_t - \pi_t^*)$ also gives a good explanation of monetary policy reactions to the gap between inflation and the policy objective.

The model has plausible dynamic responses, in contrast to many VAR-type results (Grilli and Roubini (1996)). Its use of long term yields as explanatory variables seems to solve the notorious price puzzle - the tendency for increases policy interest rates to anticipate inflationary developments and apparently cause inflation. The hypothesis

of a unit root cannot be rejected in the case of the two long term yields and the behaviour of these variables dominates the dynamics of the model, suggesting that expectations are highly persistent. However, the responses of the macro variables to deviations between inflation and the policy objective ($\pi_t - \pi_t^*$) are surprisingly rapid. Remarkably, they are exponential in nature, suggesting that monetary policy has been very effective, securing the objective quickly, without policy reversals or cycles. As in previous UK studies, the model reveals a strong real balance effect of inflation on activity.

The research presented here is able to identify the private sector's medium term inflation expectation and explore its relationship with the objectives of monetary policy, but can say very little about the formation of short run expectations. However, there is now an increasing amount of data for the index linked market across a range of maturities, which could in principle be used to calibrate expectations at shorter maturities, including the two year maturity. This would allow a more precise analysis of the relationship between private sector expectations and the inflation target announced since 1992, which was been expressed as a two-year objective. The significance of any risk premia could in principle be checked a real discount function developed along the lines set out in appendix 1 for the nominal one. This research might also lend itself to a structural analysis of private sector pricing decisions, but is beyond the scope of the present paper and remains on the agenda for future work.

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6 Appendix 1: A macro-based yield curve model

Suppose the spot rate $r_t = j'Z_t$ where j' is a selection vector and the state vector is described by (9). Absent arbitrage, the price $P_{m,t}$ of an m-period discount bond is the stochastically-discounted value of its value $P_{m-1,t+1}$ in the next period. This is given by the pricing kernel:

$$P_{m,t} = E_t[N_{t+1}P_{m-1,t+1}].$$

where N_{t+1} is the nominal Stochastic Discount Factor (SDF), with logarithm:

$$-n_{t+1} = \delta_t + r_t + \lambda_t' w_{t+1} \tag{21}$$

and where λ_t reflects the correlation between consumption and w_t (which determines the price of risk and hence the risk premia). Conditional lognormality extends to bond prices, allowing us to use the standard formula for the expectation of a lognormally distributed variable to get:

$$p_{m,t} = E_t[n_{t+1} + p_{m-1,t+1}] + \frac{1}{2}V_t[n_{t+1} + p_{m-1,t+1}] \tag{22}$$

(using lower case symbols to represent logarithms and E_t and V_t the conditional expectation and variance). In the special case $m = 1$, with $r_t = -p_{1,t}$ and $p_0 = 0$:

$$\begin{aligned} p_{1,t} &= E_t[n_{t+1}] + \frac{1}{2}V_t[n_{t+1}] \\ &= \frac{1}{2}\lambda_t{}'\Sigma\lambda_t - (r_t + \delta_t) \\ &= -r_t \end{aligned}$$

which implies:

$$\delta_t = \lambda_t' \Sigma \lambda_t / 2. \tag{23}$$

Now we adopt the affine log price trial solution (11). For $m = 1$:

$$r_t = -p_{1,t} = \gamma_1 + \Psi_1' Z_t = j' Z_t.$$

which gives the initial conditions:

$$\gamma_1 = 0; \ \Psi_1' = j'. \tag{24}$$

I follow Ang and Piazzesi (2003) and others in assuming that the price of risk depend upon linearly upon the state variables:

$$\lambda_t = \lambda_0 + \Lambda_1{}' Z_t \tag{25}$$

Substituting (11) and (23) into (22):

$$-p_{m,t} = j' Z_t + \gamma_{m-1} + \Psi_{m-1}'(K + \Phi Z_t) - \Psi_{m-1}{}' \Sigma(\lambda_0 + \Lambda_1{}' Z_t) - \frac{1}{2} \Psi_{m-1}{}' \Sigma \Psi_{m-1}. \tag{26}$$

Equating this with (11) gives the recursion:

$$\Psi_m' = \Psi_{m-1}' \tilde{\Phi}' + j' \tag{27}$$

$$\gamma_m = \gamma_{m-1} + \Psi_{m-1}' \tilde{K} - \frac{1}{2} \Psi_{m-1}{}' \Sigma \Psi_{m-1}. \tag{28}$$

where $\tilde{K} = K - \Sigma\lambda_0$ and $\tilde{\Phi}' = \Phi' - \Sigma\Lambda_1'$ represent the dynamics under the risk neutral pricing measure, obtained by substituting these for K and Φ in (9):

$$Z_t = \tilde{K} + \tilde{\Phi}' Z_{t-1} + W_t. \tag{29}$$

The recurrence relationship (27) gives an elementary solution for the slope parameters in the yield model, making it easy to see the relationships between these coefficients and the macro dynamics: (12). The time variation in the yield curve depends upon the eigenvalues of this matrix, which for the empirical model are shown in the final column of table 4. The behaviour at the long end is dominated by the eigenvector or composite variable associated with the largest eigenvalue (the slowest speed of adjustment). In the empirical model these are associated with l^* and π^*. The remaining eigenvalues reflect the speed of adjustment of the macro variables, which is much faster. Consequently, the coefficients of π^* and l^* in the 17 year discount yield y^* equation (20) are much larger than those of the macrovariables, which is why we can use ($y^* - l^*$) as a proxy for π^* in preliminary specification tests without worrying unduly about the distortion caused by short term macroeconomic fluctuations.

The risk premia in the arbitrage free pricing model are determined by differences between the parameters defining the dynamics under the observed (κ, Φ) and risk neutral $(\tilde{\kappa}, \tilde{\Phi})$ measures. To see this, first evaluate the price of today's $m-$period bond expected next period using (11):

$$\begin{aligned} E_t[P_{m-1,t+1}] &= \exp[E_t[p_{m-1,t+1}] + \frac{1}{2}V_t[p_{m-1,t+1}]] \\ &= \exp[-\gamma_{m-1} - \Psi_{m-1}'[\kappa + \Phi Z_t] + \frac{1}{2}\Psi_{m-1}{}'\Sigma\Psi_{m-1}]. \end{aligned}$$

Dividing this by the current price and substituting this using (26) gives the expected gross return:

$$\frac{E_t[P_{m-1,t+1}]}{P_{m,t}} = \exp[r_t - {\Psi_{m-1}}'\Sigma(\lambda_0 + {\Lambda_1}'Z_t)]$$

Taking the logarithm expresses this as a percentage and subtracting the spot rate then gives the expected excess return or risk premium:

$$\begin{aligned} \log\{E_t[P_{m-1,t+1}]\} - p_{m,t} - r_t &= -{\Psi_{m-1}}'\Sigma(\lambda_0 + {\Lambda_1}'Z_t) \\ &= {\Psi_{m-1}}' \left[(\tilde{\kappa} - \kappa) + (\tilde{\Phi}' - \Phi)Z_t\right] \\ where : \tilde{\Phi}' = \Phi' - \Sigma\Lambda_1'; \quad \tilde{\kappa} &= \kappa - \Sigma\lambda_0. \end{aligned} \tag{30}$$

Substituting (25) into the first line of this representation shows that the risk premium is equal to the covariance between the nominal SDF and the $(m-1)$-period bond price:

$$\log\{E_t[P_{m-1,t+1}]\} - p_{m,t} - r_t = -{\Psi_{m-1}}'\Sigma\lambda_t. \tag{31}$$

7 Appendix 2: The likelihood function

This appendix derives the Likelihood function and describes the numerical optimisation procedure.

Because the macro and yield errors are assumed to be orthogonal, the likelihood of the joint model is the sum of macro and yield components. First, consider the macro component of the likelihood function. Note from (6) and (18) that $\eta_t \sim N(0, \Sigma)$ where: $\Sigma = \Xi_1' C' DC \Xi_1$. This means that the log likelihood for period t can be written as:

$$LM_t = -(N/2)\ln(2\pi) - \ln(|\Sigma|)/2 - \eta_t'(\Sigma)^{-1}\eta_t/2 \tag{32}$$
$$= -(N/2)\ln(2\pi) - \sum_{n=1}^{N}\ln(d_n)/2 - \ln(\beta_2) - u_t{}'D^{-1}u_t/2$$
$$where\ :\ u_t = (\Xi_1'C')^{-1}\eta_t = (\Xi_1'C')^{-1}[x_t - k - F'x_{t-1} - G'm_{t-2}]$$

(using $|D| = \Pi_{n=1}^{N} d_n$, $|\Xi_1| = \beta_2$ and $|C'| = 1$). Because the first (l^*) equation is dummied out of this system (i.e. the errors $\eta_{1,t} = 0$) over the first $T_1 = 20$ periods the likelihood for these periods reduces to

$$LM_t^- = -((N-1)/2)\ln(2\pi) - \sum_{n=2}^{N}\ln(d_n)/2 - \ln(\beta_2) - u_t{}'D_t^{-1}u_t/2$$

(19) can be used to represent the likelihood of the yield observation y_t:

$$Ly_t = -(M/2)\ln(2\pi) - \sum_{m=1}^{M}\ln(\delta_m)/2 - v_t{}'\Delta^{-1}v_t/2;$$
$$where\ :$$
$$v_t = y_t - \alpha + \beta(\Lambda, K, \Phi)'\Theta + \beta(\Lambda, K, \Phi)'\Xi X_t.$$

Summing over T periods gives the loglikelihood for the full dataset:

$$LM = -(T(N+M)/2 - T_1/2)\ln(2\pi) - T\sum_{n=1}^{N}\ln(d_n)/2 + T_1\ln(d_1)/2 - T\ln(\beta_2)$$
$$-T\sum_{m=1}^{M}\ln(\delta_m)/2 - \sum_{t=1}^{T}\{u_t{}'D^{-1}u_t/2 - v_t{}'\Delta^{-1}v_t/2\}.$$

8 Appendix 3. The impulse responses

To derive the impulse responses I use the lag operator L (where $Z_{t-1} = LZ_t$) to rewrite (9) as $(I - \Phi' L)Z_t = W_t$, omitting the intercept constants K. Since its eigenvalues are less than unity in absolute value, this system can be inverted to give the Wald (MA) representation:

$$\begin{aligned} Z_t &= (I - \Phi' L)^{-1} W_t \\ &= \sum_{i=0}^{\infty} \Phi'^{i} W_{t-i} \\ &= \sum_{i=0}^{\infty} \Phi'^{i} A' U_{t-i}. \end{aligned} \tag{33}$$

U_t' is a set of orthogonal disturbances: $U_t' = (u_t', 0_{13})$ obtained by factorising W_t using a relationship similar to (7):

$$W_t = A' U_t \tag{34}$$

where:

$$A = \begin{bmatrix} C' & 0_{53} \\ 0_{35} & 0_{33} \end{bmatrix};$$

Similarly, using (19) and omitting the intercept constants:

$$\begin{aligned} y_t &= \beta(\Lambda, K, \Phi) Z_t \\ &= \beta(\Lambda, K, \Phi) \sum_{i=0}^{\infty} \Phi'^{i} A' U_{t-i}. \end{aligned} \tag{35}$$

Chart 1: Macroeconomic variables

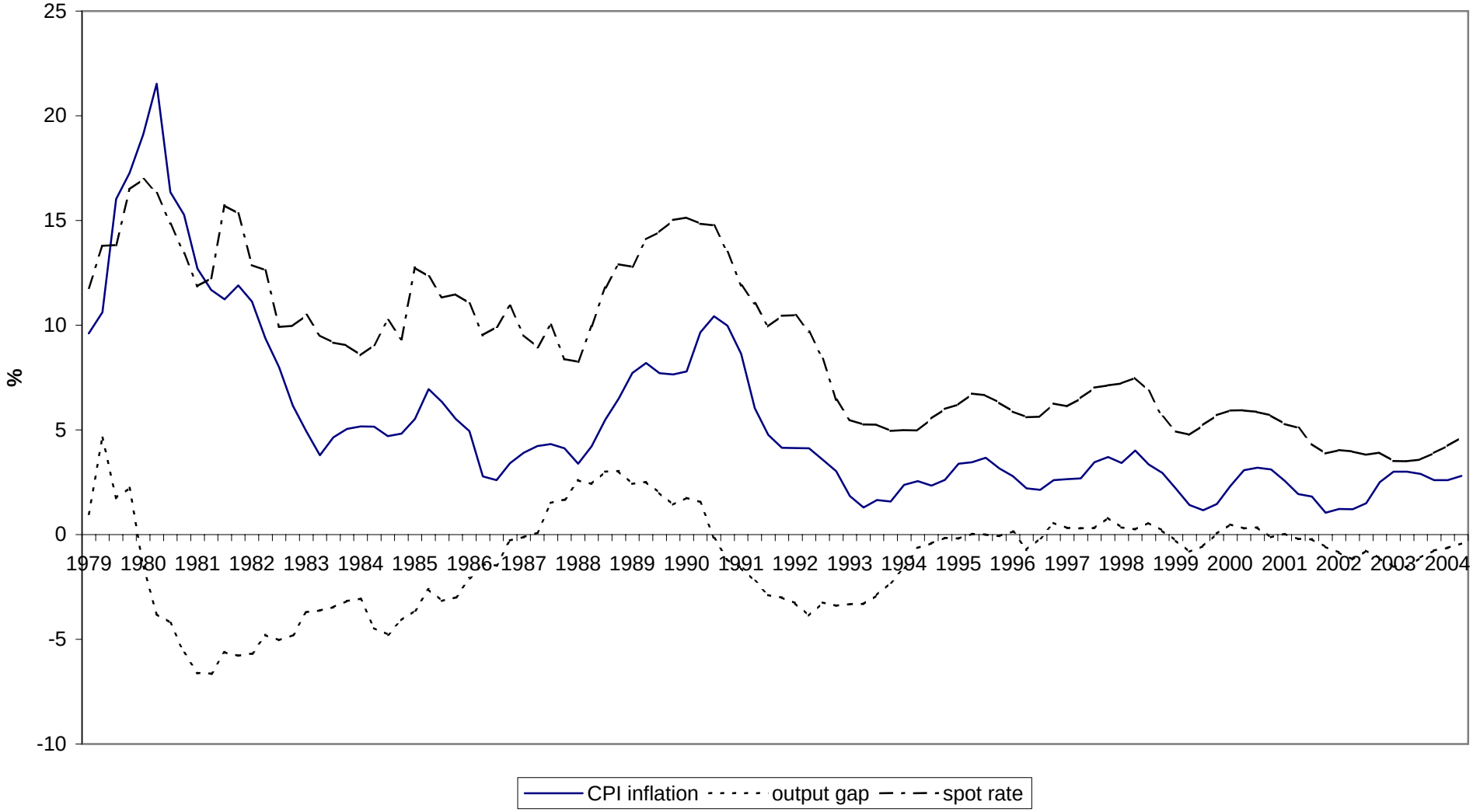


Chart 2: 17 year gilt yields

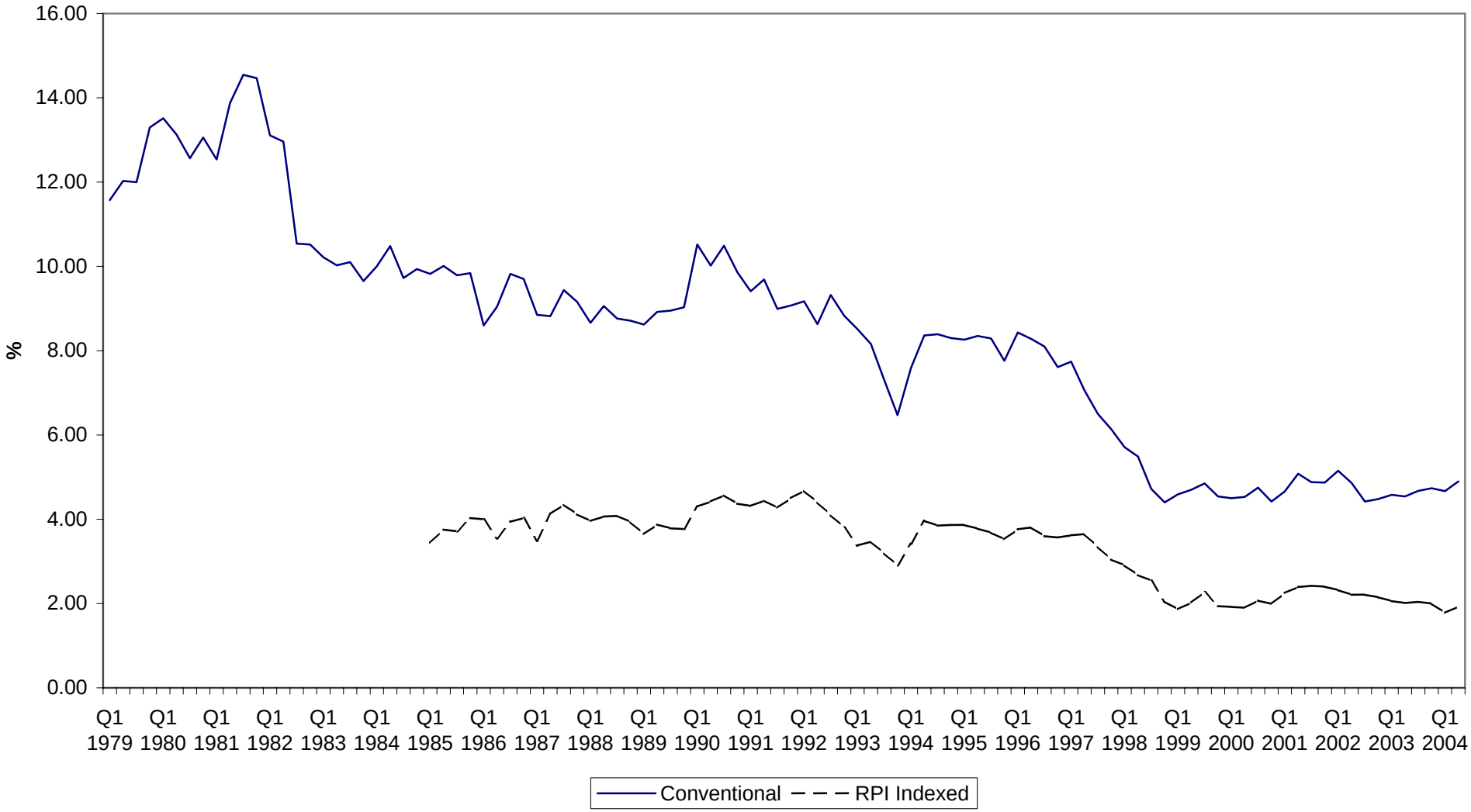


Chart 3: Gilt edged discount yields

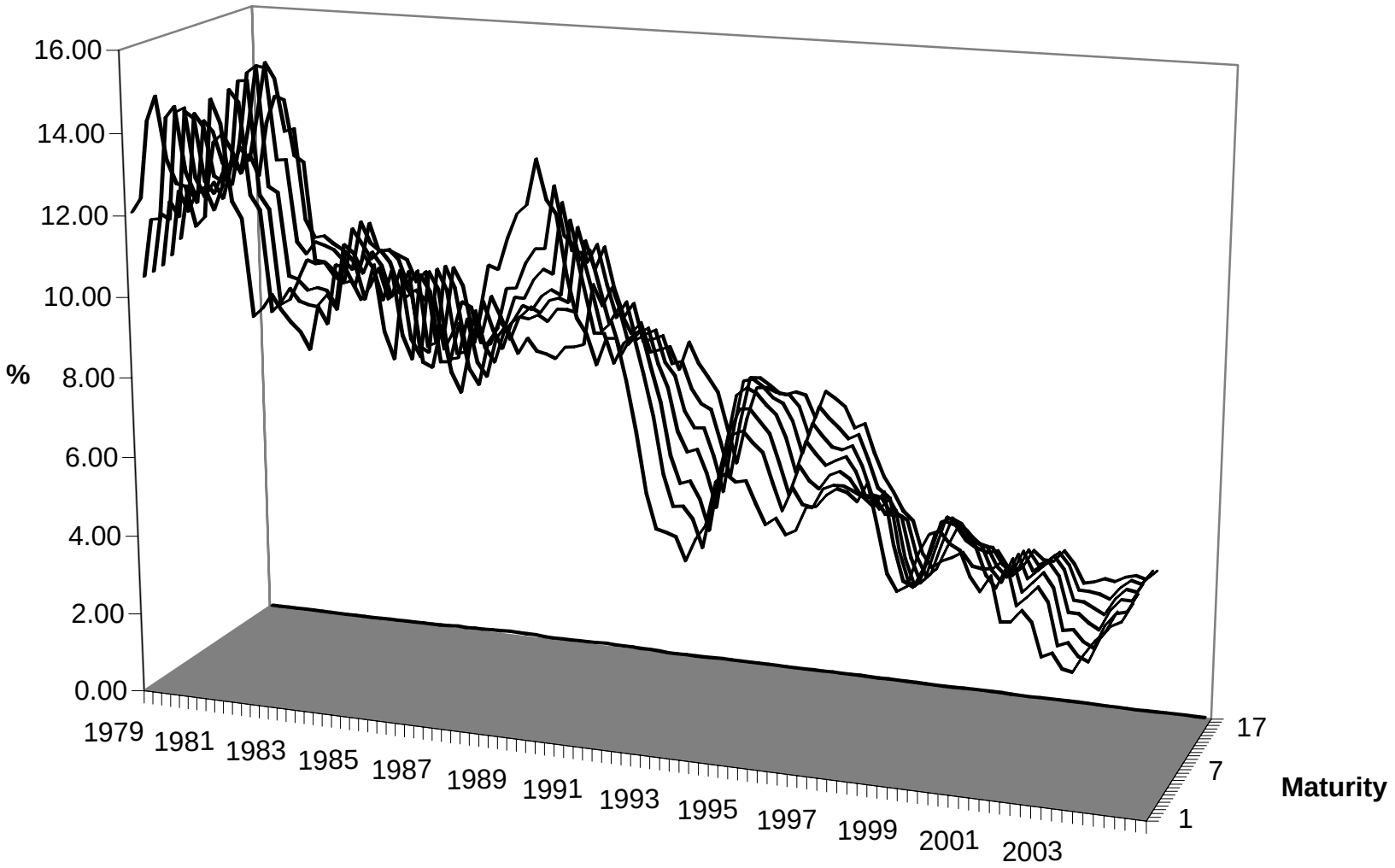


Chart 4: Inflation and measures of the policy objective

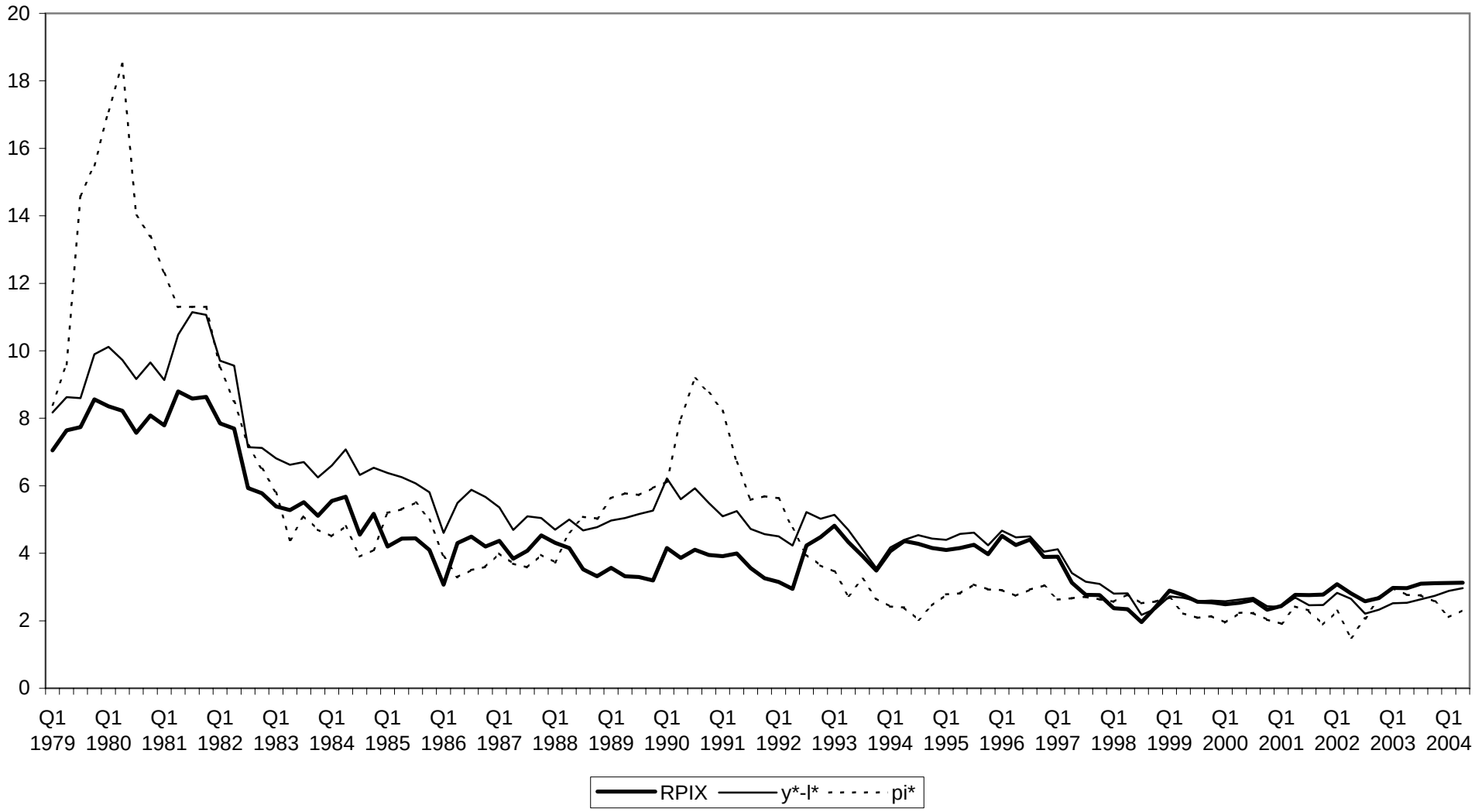
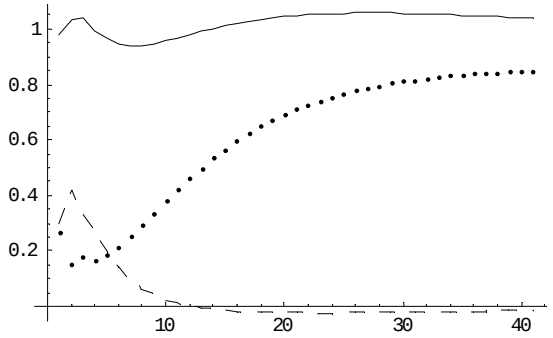
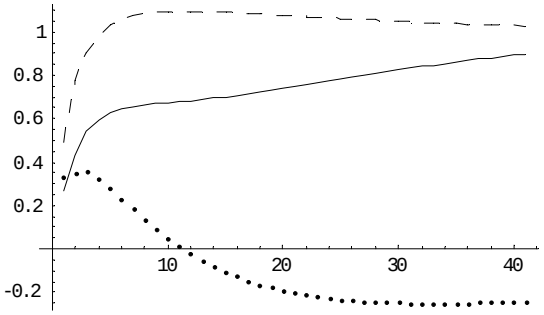


Chart 5: The macroeconomic impulse responses



(a) Effect of increase in l^*



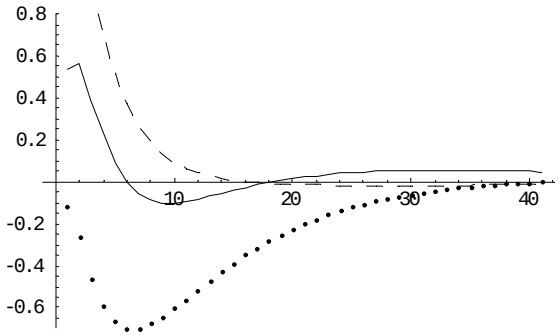
(a) Effect of increase in π^*

Key - effects on:

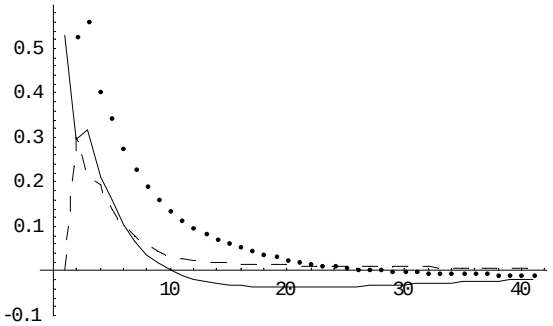
- - - inflation

..... output

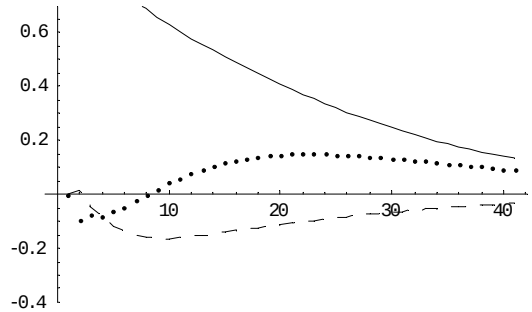
___ spot rate



(c) Effect of increase in π

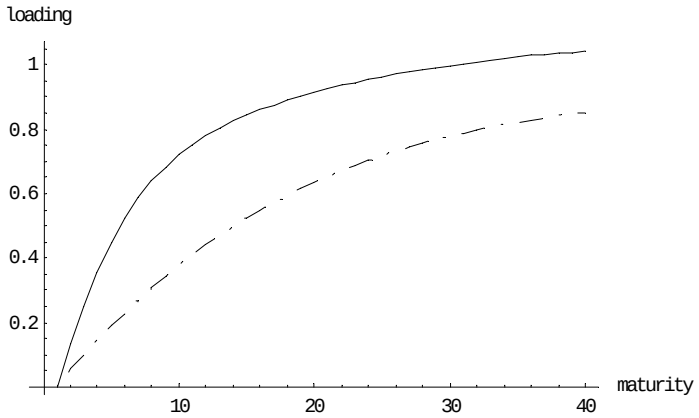


(d) Effect of increase in g

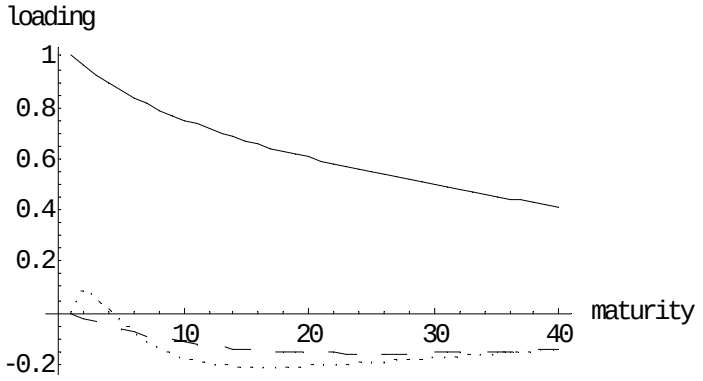


(e) Effect of increase in r

Chart 6: The yield factor loadings

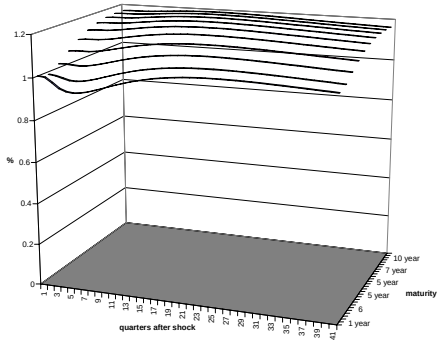


Loadings on l^*(broken line) and π^*(continuous line)

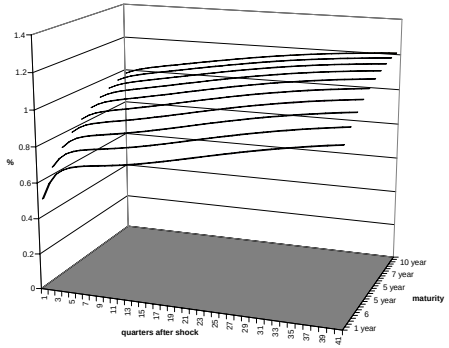


Loadings on π (dotted line), g (broken line) and r (continuous line)

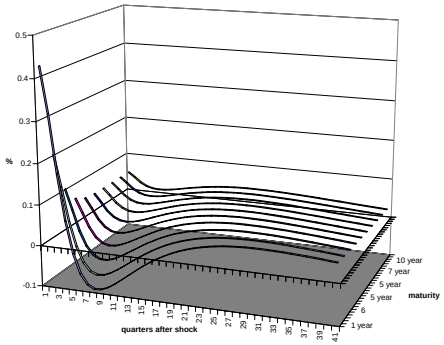
Chart 7: The bond market impulse responses



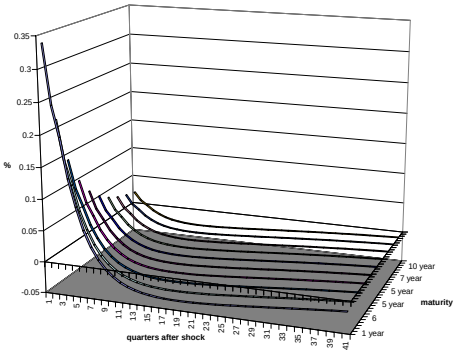
(a) Effect of increase in l^*



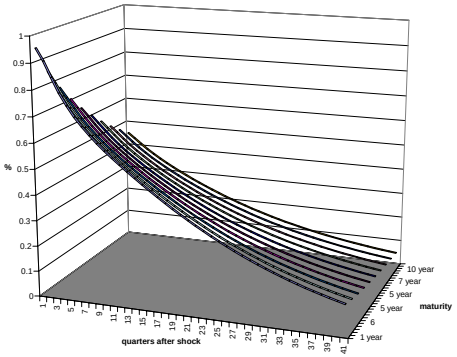
(b) Effect of increase in π^*



(c) Effect of increase in π



(d) Effect of increase in g



(e) Effect of increase in r

Chart 8: Risk Premia

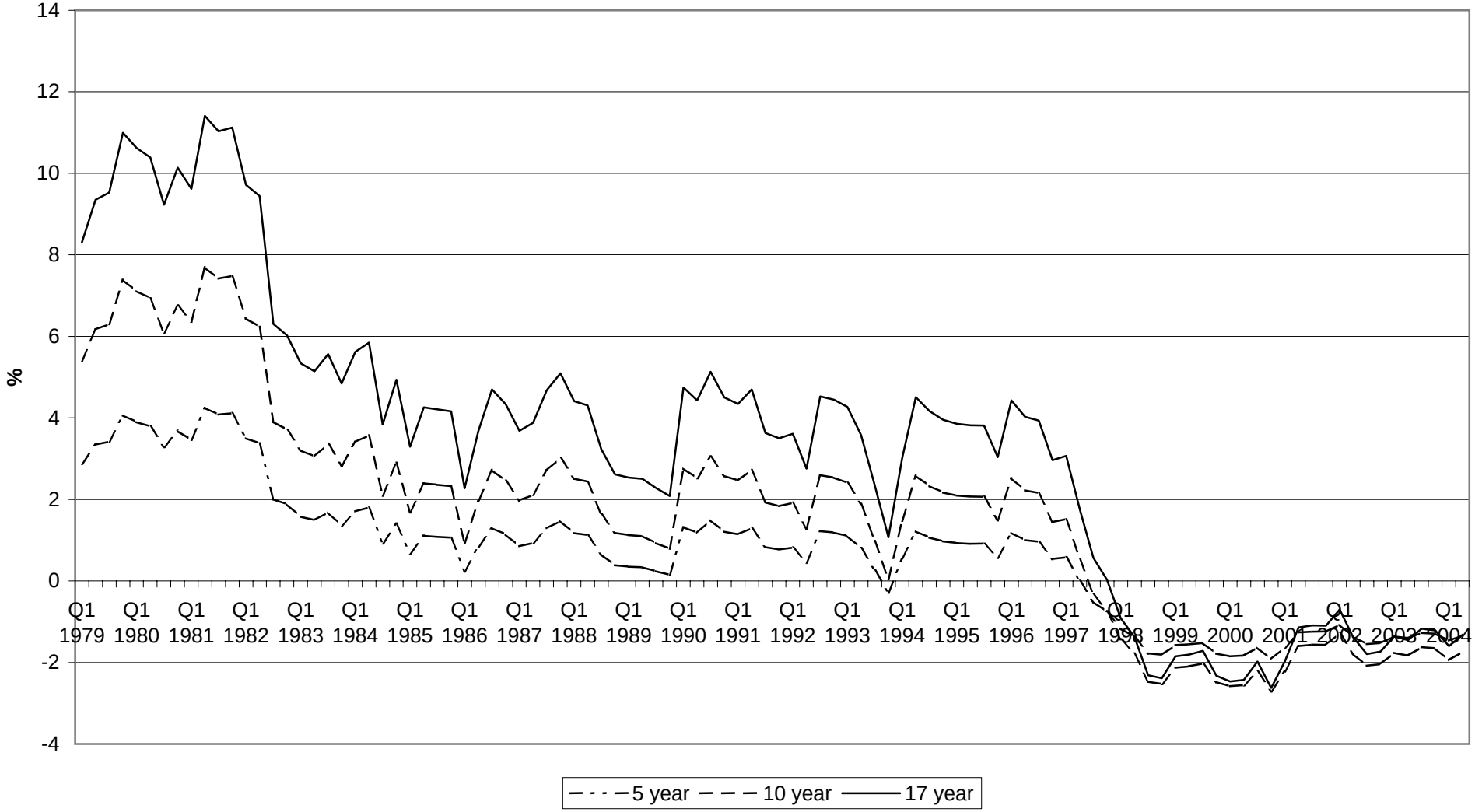


Table 1: Data Summary Statistics (1979Q4-2004Q2)

	l^*	y^*	π	g	r	y_1	y_2	y_3	y_5	y_7	y_{10}
Mean	3.3001	8.8089	5.5441	-2.2863	8. 809	8.5154	8.5856	8.6437	8.7401	8.8052	8.8027
S. D.	0.8826	2.6991	3.8221	2.3485	3.7069	3.1393	2.9732	2.9008	2.8695	2.8913	2.9008
ADF.	-2.211	-1.344	-4.129	-3.1735	-2.304	-2.199	-1.997	-1.773	-1.368	-1.375	-1.356

Output gap (g) is from Oxford Economic Forecasts; RPIX Inflation (π) and 3 month Treasury bill rate (r) are from Datastream. Yield data are UK Gilt edged discount bond equivalent data compiled by the Bank of England. Mean denotes sample arithmetic mean expressed as percentage p.a.; S. D. standard deviation and ADF the Augmented Dickey-Fuller test for non-stationarity.

Table 2: Residual Error Statistics (1979Q4-2004Q2)

	l^*	y^*	π	g	r	y_1	y_2	y_3	y_5	y_7	y_{10}
R^2	0.986253	0.963816	0.735918	0.63205	0.878529	0.990632	0.989038	0.98314	0.984166	0.990275	0.996042
RMSE	0.104804	0.513436	1.83419	1.42462	1.29196	0.303847	0.311298	0.376658	0.361075	0.285129	0.182487

The first row reports the R^2 and the second the Root Mean Square Error (RMSE).

Table 3a: The dynamic structure

			Φ_{33}			Φ_{34}		
parameter	estimate	t-value	parameter	estimate	t-value	parameter	estimate	t-value
ρ_g	-0.16107	-0.857	$\phi_{33,11}$	-0.19359	-34.352	$\phi_{34,11}$	1.09362	188.786
ρ_r	0.99832	7.385	$\phi_{33,12}$	-0.24894	-42.131	$\phi_{34,12}$	0.29054	47.385
ν_g	0.6891	2.289	$\phi_{33,13}$	-0.04742	-9.683	$\phi_{34,13}$	0.01559	3.155
ν_r	1.0541	14.556	$\phi_{33,21}$	-0.11153	-18.630	$\phi_{34,21}$	-0.14373	-24.458
k_g	-0.14111	0.776	$\phi_{33,22}$	0.286189	42.970	$\phi_{34,22}$	0.57689	84.209
k_r	1.09113	1.334	$\phi_{33,23}$	0.074191	15.376	$\phi_{34,23}$	-0.09671	5.221
ϕ_{11}	0.92582	56.666	$\phi_{33,31}$	-0.17856	-75.337	$\phi_{34,31}$	0.04689	1.999
ϕ_{22}	0.97511	686.020	$\phi_{33,32}$	0.185231	68.294	$\phi_{34,32}$	-0.23612	-86.588
κ_1	0.22253	34.213	$\phi_{33,33}$	-0.07334	-28.530	$\phi_{34,33}$	0.99972	327.969
κ_2	0.30101	47.285						

Table 3b: The error structure

D			C		
parameter	estimate	t-value	parameter	estimate	t-value
d_1	0.000521	41.024	c_{31}	0.320082	6.1851
d_2	0.001128	11.732	c_{32}	0.4886426	19.930
d_3	0.001988	35.422	c_{41}	0.299102	0.557
d_4	0.001751	9.105	c_{42}	0.310582	2.731
d_5	0.001862	9.288	c_{43}	-0.11681	-5.574
Δ			c_{51}	0.985205	12.145
δ_1	1.0267×10^{-3}	7.099	c_{52}	0.509531	3.182
δ_2	1.0737×10^{-3}	6.916	c_{53}	0.538939	60.793
δ_3	1.1730×10^{-3}	6.902	c_{54}	0.532091	10.488
δ_5	1.1343×10^{-3}	6.887			
δ_7	1.0053×10^{-3}	6.890			
δ_{10}	8.0589×10^{-4}	6.625			

Table 3c: The price of risk

λ_0			Λ_1^1			Λ_1^2		
parameter	estimate	t-value	parameter	estimate	t-value	parameter	estimate	t-value
$\lambda_{0,1}$	-446.931	-9.126	$\Lambda_{1,11}$	-2923.66	-59.245	$\Lambda_{2,11}$	71.5884	1.122
$\lambda_{0,2}$	-147.386	-22.206	$\Lambda_{1,12}$	-53.9719	-1.777	$\Lambda_{2,12}$	-196.809	-20.775
$\lambda_{0,3}$	257.183	37.127	$\Lambda_{1,13}$	213.466	6.506	$\Lambda_{2,13}$	109.42	12.632
$\lambda_{0,4}$	-60.250	-5.822	$\Lambda_{1,14}$	-81.9515	-3.681	$\Lambda_{2,14}$	186.51	15.320
$\lambda_{0,5}$	0.577307	0.297	$\Lambda_{1,15}$	90.3719	0.508	$\Lambda_{2,15}$	-145.704	-9.181

Table 4: Eigenvalues of dynamic responses under the observed (Φ) and risk neutral ($\tilde{\Phi}$) measures
(in order of absolute value)

Φ	$\tilde{\Phi}$
0.975117	0.988713
0.925823	0.986828
$0.921874 \pm 0.00825223I$	$0.921874 \pm 0.00825223I$
0.682797	0.682797
0.408807	0.408807
-0.393002	-0.393002
0.111896	0.111896