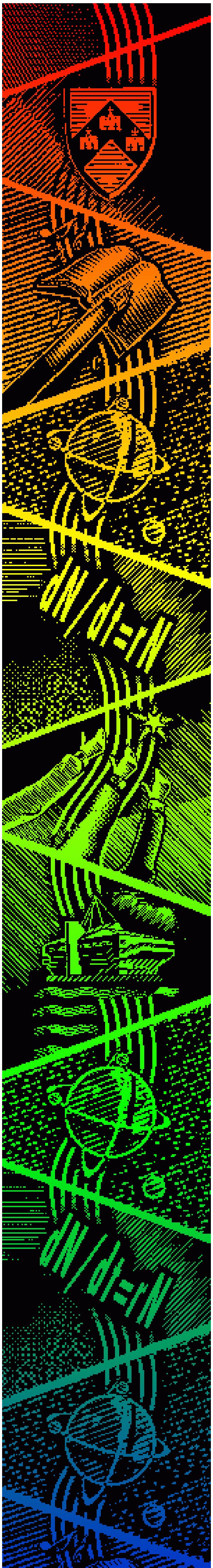

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Consumption Externalities, Production Externalities and Efficient Capital Accumulation under Time Non-separable Preferences

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Abstract

We examine the effects of both consumption and production externalities on capital accumulation and economic performance under time non-separable preferences and a non-scale production technology. We show that a consumption externality in isolation has long-run distortionary effects if and only if labour is supplied elastically. With fixed labour supply, it has only transitional distortionary effects, though it may generate long-run distortions through its interaction with the production externality. Production externalities always generate long-run distortions, irrespective of labour supply. The optimal taxation to correct for the distortions is characterized. Further quantitative insights are obtained by supplementing the theoretical analysis with numerical simulations based on the calibration of a plausible macroeconomic growth model.

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1. Introduction

Externalities have engaged the attention of economists over a long period of time. Broadly speaking, they can be categorized as (i) consumption externalities, and (ii) production externalities.[1] Recently, the former have been extensively studied in the context of models of "keeping up with the Joneses," and their implications for a range of important issues investigated. For example, Abel (1990), Gali (1994), and Campbell and Cochrane (1999) study the effects of consumption externalities on asset pricing and the equity premium. Ljungqvist and Uhlig (2000) analyze the impact of consumption externalities on the effect of short-run macroeconomic stabilization policy. Dupor and Liu (2003) define alternative forms of consumption externalities and explore their implications for equilibrium over-consumption, while Fisher and Hof (2000, 2001) and Liu and Turnovsky (2003) consider their impact on the process of capital accumulation and growth.

Production externalities provide the cornerstone for the endogenous growth model pioneered by Romer (1986). The key feature of this literature is that even though the individual firm's capital stock may be subject to diminishing marginal physical product, the presence of an aggregate production externality enhances its productivity so that in equilibrium the economy is able to sustain a steady growth rate.

Empirical evidence on the importance of externalities is sparse, but overall, the existing evidence provides convincing support for the importance of consumption externalities. Easterlin (1995) discusses a number of studies relating happiness to income growth. Using US, European, and Japanese time series data, he concludes that raising income of all does not increase everyone's happiness, implying the presence of negative externalities. Clark and Oswald (1996) present some direct empirical evidence for British workers, showing that their reported satisfaction levels are inversely related to their comparison wage rates. Frank (1997) provides a comprehensive discussion based on both psychological evidence and the more fragmentary evidence in behavioral economics. He concludes that both these sources support the claim that satisfaction depends upon the agent's

[1] Consumption externalities were emphasized in early work by Veblen (1912) and were first formalized as a determinant of aggregate consumption by Duesenberry (1949) in his development of the "relative income hypothesis". The earliest discussion of production externalities dates back to Marshall (1890).

relative position, again emphasizing the role of the environment and the externalities it generates.

The evidence on production externalities, though less conclusive, is still quite compelling. Caballero and Lyons (1990, 1992) analyze externalities within the context of EU and US manufacturing industries. Although they find evidence of externalities, their analysis has been questioned by Basu and Fernald (1996) and Burnside (1995). On the other hand, using a methodology similar to that employed by Basu and Fernald, Benarroch (1997) finds evidence of externalities at the two-digit industry level in Canada.

A related and equally important issue concerns the specification of preferences themselves. The conventional intertemporal utility function is assumed to be time-separable, with any consumption externality being introduced as *contemporaneous* economy-wide consumption; see e.g. Boskin and Sheshinski (1978), Gali (1994), Harbaugh (1995), Ljungqvist and Uhlig (2000), Dupor and Liu (2003), Fisher and Hof (2000, 2001), Liu and Turnovsky (2003). But a growing body of empirical evidence has confirmed the importance of *time non-separable* preferences, in which utility depends not only upon current consumption, but also on some benchmark or "habit" level of consumption determined from past behavior. In the case that this benchmark is defined in terms of the consumption of an external reference group it introduces a consumption externality (utility interdependence), but one that is tied to *past* consumptions. This type of externality is often termed as "*catching* up with" rather than "*keeping* up with" with the Joneses.[2]

Using panel data for the Netherlands, van de Stadt, Kapteyn, and van de Geer (1985) model both habit formation and utility interdependence. Their results are compatible with the hypothesis that utility depends upon relative consumption, although they cannot exclude the possibility that utility reflects both relative and absolute consumption. Using UK data, Osborn (1988) introduces a consumption specification that allows for seasonal variations and habit persistence, and finds the habit persistent terms to be jointly significant. More recently Fuhrer (2000) uses maximum likelihood to estimate an approximate linear consumption function derived from time non-separable

[2]Utility functions in which the benchmark depends upon the agent's *own* past consumptions the evolving benchmark is not acting as an externality, and such agents are sometimes referred to as being "inward looking", whereas if the benchmark depends upon the consumption of an outside reference group, the agent is referred to as "outward looking" see e.g. Carroll, Overland, and Weil (1997, 2000). Since for plausible parameterization of the model both inward and outward looking models behave similarly [see Carroll. et al. (1997) and Alvarez-Cuadrado et al. (2004)], we shall focus on the outward-looking case.

preferences, strongly rejecting the hypothesis of time separable preferences. In addition, Fuhrer and Klein (1998) present empirical evidence suggesting that habit formation is relevant for the characterization of consumption behavior among the G-7 countries. More indirect evidence is provided by studies of Constantinides (1990) and others who have shown how habit formation can provide a solution to the equity premium puzzle.

In light of these bodies of evidence, the question of how consumption and production externalities affect economic performance under time non-separable preferences is important. To what extent do they introduce distortions into the process of capital accumulation, and if so, what are the appropriate corrective policy responses? Liu and Turnovsky (2003) have addressed this issue employing a standard time-separable utility function, emphasizing particularly the interaction between the two types of externalities. But given the evidence suggesting that preferences are characterized by a high degree of complementarity between consumption over time, it is important to extend the analysis to incorporate these more general preferences.[3]

To do so is the objective of the present paper. More specifically, we introduce time non-separable preferences, as originally specified by Abel (1990) in the context of asset pricing, into the "non-scale growth model" developed by Eicher and Turnovsky (1999a, 199b). The interaction between preferences and technology is important. Previous applications of these preferences by Carroll, Overland, and Weil (1997, 2000), Fisher and Hof (2000), Alonso-Carrera, Caballe, and Raurich (2001a), have typically assumed rigid production conditions of the simplest endogenous growth model.[4] Recently, Alvarez-Cuadrado, Monteiro, and Turnovsky (2004) have highlighted the importance of combining more general preferences with the more flexible production technology of the neoclassical model, in replicating observed behavior.

We consider both consumption and production externalities, comparing their impacts on economic performance, and deriving the appropriate fiscal policy responses. In this respect our analysis is related to Alonso-Carrera et al. (2001b) who consider labor supply to be inelastic. Our

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[3] There is a growing literature, characterized as capturing "the spirit of capitalism" that expresses consumption externalities in terms of relative wealth effects, see e.g. Kurz (1978) and more recently Zou (1995).

[4] There are exceptions, however. Early pioneering work by Ryder and Heal (1973) introduced habit formation into the basic neoclassical growth model Alonso-Carrera et al. (2001b) employ the hybrid neoclassical-AK production function introduced by Jones and Manuelli (1990).

analysis allows for the endogeneity of labor supply, which previous research has been shown to be an important determinant of optimal tax policy in a growing economy; see e.g. Turnovsky (2000b), Liu and Turnovsky (2003).[5]

The paper proceeds in two main stages. The first part develops the general theoretical model. Our results take the form of a series of propositions summarizing how the externalities impinge on the macroeconomic equilibrium and proposing fiscal policies to correct for the distortions they may create. The latter part conducts numerical simulations of both the steady-state equilibrium and the transitional dynamics in response to a specific shock, taken to be an increase in productivity.

One general conclusion is that consumption externalities in isolation will have long-run distortionary effects on the economy if and only if labor supply is endogenous. This is because such externalities affect the marginal valuation of consumption, which, if the leisure decision is endogenous, changes the optimal utility value of the marginal product of labor, thereby influencing long-run capital and output. Thus, with elastic labor supply, a negative consumption externality leads to long-run consumption, capital, labor supply, and output that are all too high relative to their respective optima. If labor supply is fixed, then consumption externalities alone have no long-run distortionary effects. They do, however, distort effects the transitional dynamic path, and they will generate long-run distortions through their interaction with production externalities.

Production externalities alone always generate long-run distortions, irrespective of whether labor supply is fixed or not. Thus a positive production externality leads to a sub-optimally low capital stock with under-production and under-consumption. In addition, a consumption externality will affect the potency of the production externality on long-run activity through its impact on the labor-leisure choice and thus on the marginal product of capital.

We characterize an optimal tax policy to correct for the distortionary effects. It requires capital income to be taxed or subsidized at a constant rate that corrects for the production externality, while consumption should be taxed or subsidized at a time-varying rate that corrects for the divergence between the social and private benefits from the consumption externality as reflected in

[5] Our model differs in several other respects from Alonso-Carrera et al. (2001b). One important difference is that they assume that the reference consumption level in utility is determined by the previous period's consumption, and thus adjusts rapidly. We allow for a much more gradual adjustment, which turns out to be important in reconciling the implications of this model with certain observed empirical phenomena; see Alvarez-Cuadrado et al. (2004)

the evolving relative shadow value of habits to capital. The tax on labor income can be set at an arbitrary constant level, allowing the policy maker to accommodate some other objective. This means that labor income can be untaxed, or alternatively can be taxed at the same rate as capital, thus taxing all income at a constant uniform rate,

The simulations confirm and supplement these theoretical findings in several dimensions. First, the numerical impacts of the two externalities on steady-state equilibrium are assessed. We find that both consumption and production externalities of plausible magnitudes generate substantial long-run deviations of the decentralized economy from the optimum. Whereas, the consumption externality affects key measures of economic activity proportionately, the production externality has substantial differential effects. There are also pronounced asymmetries in the effects of positive and negative externalities of comparable magnitudes. Finally, our results illustrate the role of the labor supply elasticity on the impacts of the externalities on equilibrium.

Second, the long-run and short-run effects of a 50% increase in productivity are considered. Because of the non-scale technology, this shock affects the long-run measures of economic activity in both the decentralized and centrally planned economies by identical proportionate amounts. The differences in proportionate welfare gains between the two economies reflect differences along the transitional paths and are therefore small. However, the actual magnitudes of the welfare differences can be substantial. We also trace out the ratios of key variables in the decentralized to the centrally planned economy to determine the time paths of the various distortionary effects. In most cases there is little change over time, though there are important exceptions. The time path is sensitive to the weight assigned to past consumption in the construction of the benchmark consumption level. We provide a plausible example showing how as this increases, capital may go through phases of over-accumulation, under-accumulation, and then over-accumulation during the transition. Finally, we provide numerical estimates of the optimal tax rates necessary to track the optimal equilibrium.

The rest of the paper proceeds as follows. Section 2 sets out the preferences and technology, together with the assumptions on the consumption and production externalities. Sections 3 and 4 characterize the macrodynamic equilibria in the decentralized and centrally planned (optimal) economies respectively, while Section 5 compares their steady states. First-best optimal tax policy is characterized in Section 6. The numerical analysis of the transitional paths is discussed in Sections 7 and 8. Section 9 provides concluding comments and technical details are relegated to the Appendix.

2. Preferences and Technology: Consumption and Production Externalities

Consider an economy populated by *N* infinitely-lived identical households, where *N* grows at the constant exponential rate, *n*. The agent is endowed with a unit of time, part of which, L_i can be supplied as labor input and the remainder, $l_i \equiv 1 - L_i$, consumed as leisure. We assume that at any instant of time households derive utility not only from their current consumption, C_i, but also from leisure, as well as the current level of a reference consumption stock, H_i, (habit) based on economy-wide consumption, that the agent takes as given. Thus the agent's utility is represented by an iso-elastic function of the type employed by Abel (1990), Carroll, Overland, and Weil (1997, 2000), Ljungqvist and Uhlig (2000), and Alvarez, Monteiro, and Turnovsky (2004):

$$\Omega \equiv \frac{1}{1-\varepsilon}\int_0^\infty \left(C_i H_i^{-\gamma} l_i^\theta\right)^{1-\varepsilon} e^{-\beta t}dt = \frac{1}{1-\varepsilon}\int_0^\infty \left(C_i^{1-\gamma}\left(\frac{C_i}{H_i}\right)^\gamma l_i^\theta\right)^{1-\varepsilon} e^{-\beta t}dt \quad \varepsilon > 0, \theta > 0, 1-(1-\varepsilon)(1+\theta) > 0 \quad (1)$$

From the right hand side of (1), we see that agents derive utility from a geometric weighted average of absolute and relative consumption, these corresponding to $\gamma = 0$, and $\gamma = 1$, respectively. The form of the utility specification (1) raises questions about whether or not the necessary first-order conditions that we derive are in fact optimal. This problem is characteristic of all the literature that employs the utility function in (1). In the case of the representative agent, H_i is an externality and the restrictions imposed on θ, ε ensure that the utility function is jointly concave in the private variables C_i, l_i. Given that the constraints are concave functions, the first order conditions suffice to ensure a maximum. By contrast, for the central planner who chooses H_i together with C_i, the utility function is not jointly concave in both these variables, unless $\gamma < 0$, and thus the first-order conditions may not yield a maximum. In this case the paper by Alonso-Carrera et al. (2001b) argues that the interior solution will ensure utility maximization if one restricts $\varepsilon > 1$, consistent with the empirical evidence, and a restriction that we shall impose.

The agent's reference stock or consumption habit is assumed to be specified by

$$H_i(t) = \rho\int_{-\infty}^{t} e^{\rho(\tau - t)}\bar{C}(\tau)\,d\tau \qquad \rho > 0 \tag{2}$$

Thus (2) implies that the agent's reference stock is an exponentially declining weighted average of the economy-wide average consumption $\bar{C} \equiv \sum_{i=1}^{N} C_i / N$. Differentiating (2) with respect to time implies the following rate of adjustment for the reference stock

$$\dot{H}_i = \rho\left(\bar{C} - H_i\right) \tag{3}$$

The speed of adjustment, ρ, parameterizes the relative importance of recent consumption in determining the reference stock; higher values of ρ imply a higher influence of current consumption in the determination of the future reference stock. In particular, as $\rho \to \infty$, $H_i \to \bar{C}$, contemporaneous economy-wide average consumption, as in Gali (1994), Fisher and Hof (2000), Dupor and Liu (2003) and Liu and Turnovsky (2003), which obtains as a limiting case.

With H_i determined by (2) [or (3)] it is clear that economy-wide consumption imposes an externality on the agent. Most of the literature focuses on the case $\gamma \geq 0$, implying that agents derive disutility from a *ceteris paribus* increase in the consumption reference stock. This corresponds to the idea of "catching up to the Joneses", as formulated by Carroll et al (1997, 2000), Ljungqvist and Uhlig (2000) and Alvarez et al (2004). But we shall also entertain the possibility that $\gamma < 0$, so that the agent's utility increases with a *ceteris paribus* increase in the reference consumption level. This describes the notion of altruism. The representative agent feels better off if members of the community around him are successful; see Dupor and Liu (2003).[6]

The specification of utility in terms of (1) and (2) implies that the time non-separability of utility is due entirely to the consumption externality. It is possible for the time non-separability to be internally generated, or to be due to some hybrid, as formulated by Alvarez-Cuadrado et al. (2004).

Analogous to Liu and Turnovsky we shall impose the following restrictions on the size of the consumption externality, to ensure in effect that it is dominated by the direct effect:

$$\gamma < 1 \tag{4a}$$

[6] They distinguish between the notion of "jealousy" that depends upon the first derivative of the externality on utility and "keeping up with the Joneses" which they formulate in terms of appropriate cross derivatives.

$$\varepsilon(1-\gamma)+\gamma>0 \tag{4b}$$

The first of these conditions is the non-satiation condition initially imposed by Ryder and Heal (1973), which asserts that a uniformly sustained increase in consumption level increases utility. The second condition restricts the externality so as to ensure that a uniformly sustained increase in consumption level across agents has diminishing marginal utility.[7]

The household has a production technology that is homogeneous of degree one in its private inputs, capital K_i and labor L_i, with both factors having positive but diminishing marginal physical product. In addition, output depends on the aggregate stock of capital, denoted by $K=\sum_i K_i$. Assuming a Cobb-Douglas production function, individual output is determined by[8]

$$Y_i=\alpha L_i^{\sigma}K_i^{1-\sigma}K^{\eta}; \qquad 0<\sigma<1 \tag{5}$$

The externality generated by aggregate capital, η, may be positive, zero, or negative. In Romer's seminal work (1986) the aggregate capital stock serves as a proxy for the level of knowledge and thus generates a positive production externality. However, $\eta<0$ may reflect adverse congestion effects of aggregate capital on the productivity of private capital; see e.g. Barro and Sala-i-Martin (1992), Eicher and Turnovsky (2000).

Analogous to the restrictions on the consumption externality, the following restrictions on the production externality are assumed

$$\sigma>\eta>-(1-\sigma) \tag{6}$$

The right hand inequality ensures that the externality, if negative, is sufficiently small so that the social marginal product of capital remains positive [see (18c) below]. The left hand inequality imposes an upper limit on any positive externality generated by aggregate capital, in order for a

[7] Expressed in terms of general utility functions these two conditions require: $U_C(C_i,C_i,l_i)+U_H(C_i,C_i,l_i)>0$; $U_{CC}(C_i,C_i,l_i)+U_{CH}(C_i,C_i,l_i)<0$

[8] Our choice of the Cobb-Douglas production function is imposed by the fact we wish to introduce production externalities leading to non-constant returns to scale. The Cobb-Douglas production function is the only technology consistent with a balanced growth path under non-constant returns to scale; see Eicher and Turnovsky (1999b).

uniformly sustained increase in capital stock to have diminishing marginal product.[9]

3. Macrodynamic Equilibrium: Decentralized Economy

The individual in the decentralized economy chooses consumption, labor supply, and rate of capital accumulation to maximize the utility function (1) subject to the capital accumulation equation

$$\dot{K}_i = (r - n - \delta)K_i + wL_i - C_i \tag{7}$$

where r denotes the gross return to capital, w denotes the wage rate, and δ denotes the constant rate of depreciation of capital. In doing so, the agent behaves atomistically, taking the aggregate quantities $\bar{C}$ and K, as well as the evolution of the reference consumption stock, (3), as given.

The first order conditions for an optimum are

$$U_{C_i} \equiv C_i^{-\varepsilon}\left(\frac{l^{\theta}}{H_i^{\gamma}}\right)^{(1-\varepsilon)} = \lambda_i \tag{8a}$$

$$U_{l_i} \equiv \theta l^{\theta(1-\varepsilon)-1}\left(\frac{C_i}{H_i^{\gamma}}\right)^{(1-\varepsilon)} = \lambda_i w \tag{8b}$$

$$r - \delta - n = \beta - \frac{\dot{\lambda}_i}{\lambda_i} \tag{8c}$$

where λ_i denotes the private shadow value to agent *i* of an additional unit of capital, together with the transversality condition

$$\lim_{t\to\infty} \lambda_i K_i e^{-\beta t} = 0 \tag{8d}$$

The interpretations of these equations are standard; (8a) equates the marginal utility of consumption to the *private* shadow value of capital taking into account the fact that utility depends upon current consumption relative to the external benchmark; (8b) equates the marginal utility of leisure to the private opportunity cost, the real wage valued at the shadow value of capital, while (8c) equates the marginal return to capital to the rate of return on consumption, given by the right hand side.

[9] Turnovsky (2000a) derives $\eta < \sigma$ as a necessary and sufficient for stability in the basic one-sector non-scale growth model with conventional utility. It also turns out to be a necessary condition for stability for the present model.

With all individuals being identical, aggregating (5) over the *N* agents, yields the aggregate production function $Y \equiv \sum_i Y_i$,

$$Y = \alpha\left((1-l)N\right)^{\sigma} K^{1-\sigma+\eta} \tag{9}$$

Total returns to scale, $1+\eta$, are decreasing, constant, or increasing, according to whether the spillover from aggregate capital is negative, zero, or positive. The equilibrium gross real return to capital, *r*, and the real wage, *w*, are thus respectively:

$$r = \left.\frac{\partial Y_i}{\partial K_i}\right|_{K_i=K} = (1-\sigma)\frac{Y_i}{K_i} = (1-\sigma)\frac{Y}{K};\; w = \left.\frac{\partial Y_i}{\partial L_i}\right|_{K_i=K} = \sigma\frac{Y_i}{L_i} = \sigma\frac{Y}{NL_i} \tag{10}$$

Substituting (10) into (7), the individual's rate of capital accumulation can be expressed as

$$\dot{K}_i = \alpha L_i^{\sigma} K_i^{1-\sigma} K^{\eta} - C_i - (n+\delta)K_i \tag{7'}$$

We define a balanced growth path as being one along which all variables grow at a constant rate. With capital being accumulated from final output, the only balanced solution is one in which the capital-output ratio, K/Y, remains constant. Taking percentage changes of the aggregate production function, the long-run equilibrium growth rate of output and capital along the balanced growth path, $\hat{Y}$ and $\hat{K}$, is

$$\hat{Y} = \hat{K} = \frac{\sigma}{\sigma-\eta} n \equiv gn \tag{11}$$

Because of the non-scale nature of the production function, the equilibrium growth rate is determined solely by technological factors, together with the population growth rate, and is independent of all demand characteristics, including the consumption externality; see Jones (1995). There is long-run per capita growth if and only if $\eta > 0$.

Following our definition of the balanced growth path, it is convenient to write the system in terms of the following stationary variables $k \equiv K/N^g, y \equiv Y/N^g, h \equiv H/N^g, c \equiv C/N^g$, (where *C*, *H*, also denote aggregate quantities) which we characterize as being "scale-adjusted" per capita quantities, and which under constant returns to scale $(g=1)$ reduce to standard per capita quantities.

Using this notation, the scale adjusted aggregate output (9) can be written as:

$$y = \alpha\left(1-l\right)^{\sigma} k^{1-\sigma+\eta} \tag{12}$$

We will focus on equilibrium paths along which all households are identical, so that $C_i = C = \overline{C},\ K_i = K$. We shall refer to such paths as "symmetric equilibria".

In the Appendix we show how the equilibrium dynamics of the decentralized economy can be expressed by the following system in terms of the redefined stationary variables, l, k, h, c, where * denotes the decentralized economy:

$$\dot{k}^* = \left(1-\frac{c^*}{y^*}\right)y^* - \left(\delta + gn\right)k^* \tag{13a}$$

$$\dot{h}^* = \rho\left(c^* - h^*\right) + \left(1-g\right)nh^* \tag{13b}$$

$$\dot{l}^* = F\left(l^*\right)\left\{\left[(1-\sigma) - \varepsilon(1-\sigma+\eta)\left(1-\frac{c^*}{y^*}\right)\right]\frac{y^*}{k^*} - \rho\gamma\left(1-\varepsilon\right)\left(\frac{c^*}{h^*}-1\right)\right.$$

$$\left. -\left[\beta + \delta\left(1-\varepsilon(1-\sigma+\eta)\right) + n\left[1-\varepsilon\left(1-\sigma\right)\right]\right]\right\} \tag{13c}$$

$$\frac{c^*}{y^*} \equiv \frac{C^*}{Y^*} = \frac{\sigma}{\theta}\frac{l^*}{\left(1-l^*\right)} \tag{13d}$$

and

$$F\left(l^*\right) \equiv \frac{l^*\left(1-l^*\right)}{\varepsilon\left(1-\sigma l^*\right) - \theta\left(1-\varepsilon\right)\left(1-l^*\right)} > 0 \tag{13e}$$

together with the production function (12). [10]

Equation (13d) is obtained by dividing the optimality conditions (8a) and (8b). It reflects the condition that the marginal rate of substitution between consumption and leisure $\left(\theta C/l\right)$, which grows with per capita consumption, must equal the wage rate $\left(\sigma Y/(1-l)\right)$, which grows with per capita income. Notice that the equilibrium consumption-output ratio increases with leisure, this

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[10] Solving (13d) and (12b) yields $c^* = c^*(k^*, l^*),\ y^* = y^*(k^*, l^*)$. Then substituting these solutions into (13a) – (13c) yields an autonomous dynamic system in $\dot{k}^*, \dot{h}^*, \dot{l}^*$. The numerical analysis conducted in Sections 7 and 8 is based on the linearized approximation to this system.

result following from the complementarity between leisure and consumption in utility.

Imposing the steady state condition, $\dot{l}^* = \dot{k}^* = \dot{h}^* = 0$, we can solve (13) for the steady-state values of the relevant variables denoted by tildes, as follows,

$$\left(1-\frac{\tilde{c}^*}{\tilde{y}^*}\right)\frac{\tilde{y}^*}{\tilde{k}^*} = (\delta + gn) \tag{14a}$$

$$\rho\left[\frac{\tilde{c}^*}{\tilde{h}^*}-1\right] = (g-1)n \tag{14b}$$

$$(1-\sigma)\frac{\tilde{y}^*}{\tilde{k}^*} - (n+\delta) = \beta + (g-1)\left[\gamma + \varepsilon(1-\gamma)\right]n \tag{14c}$$

$$\tilde{y}^* = \alpha(1-\tilde{l}^*)^{\sigma}(\tilde{k}^*)^{1-\sigma+\eta} \tag{14d}$$

$$\frac{\tilde{c}^*}{\tilde{y}^*} = \frac{\sigma}{\theta}\frac{\tilde{l}^*}{\left(1-\tilde{l}^*\right)} \tag{14e}$$

These five equations determine the steady-state equilibrium as follows. First (14c) yields the steady-state output-capital ratio, so that the long-run net private return to capital equals the rate of return on consumption, while (14b) determines the equilibrium ratio of consumption to its reference stock. Having determined the output-capital ratio, (14a) determines the consumption-output ratio consistent with generating the growth rate of capital necessary to equip the growing labor force and replace the depreciating capital stock. Given the steady-state consumption to output ratio, (14e) determines the allocation of time to leisure, $\tilde{l}^*$. Given the steady-state values for $\tilde{l}^*$ and $\tilde{y}^*/\tilde{k}^*$, the production function, (14d), then implies the corresponding value of capital, $\tilde{k}^*$ and hence output, $\tilde{y}^*$.

The key parameters include γ, ρ, which reflect the consumption externality, and η, which specifies the production externality. From (14) we observe the following asymmetry between the two externalities. The consumption externality influences steady-state equilibrium in the decentralized economy if and only if there is a production externality (i.e. $\eta \neq 0$). In contrast, the production externality impacts the equilibrium independent of any consumption externality.

If $\eta = 0$ (i.e. $g = 1$), the parameters γ, ρ, pertaining to the consumption externality become irrelevant. In that case (14b) implies $\tilde{c}^* = \tilde{h}^*$, so that the stationary current and reference consumption levels coincide. Equation (14c) reduces to the standard modified golden rule condition, consistent with the early result of Ryder and Heal (1973) and the overall steady-state equilibrium reduces to that of conventional time-separable utility. If $\eta \neq 0$, then ρ affects only the consumption-habits ratio, while γ has more pervasive effects. If $\eta > 0$, and assuming $\varepsilon > 1$, a larger weight to the consumption habit, γ, reduces the long-run rate of return on consumption [the right hand side of (14c)], and thus the output-capital ratio. For goods market equilibrium to prevail, the consumption-output ratio must then decline, inducing more leisure.

In Section 8 below, we shall illustrate the dynamic response of the economy by introducing an increase in productivity, α. From (14), we see that the long-run responses are

$$\frac{d\tilde{k}^*}{\tilde{k}^*} = \frac{d\tilde{y}^*}{\tilde{y}^*} = \frac{d\tilde{c}^*}{\tilde{c}^*} = \frac{1}{\sigma - \eta}\frac{d\alpha}{\alpha} \tag{15a}$$

$$\frac{d\left(\tilde{y}^*/\tilde{k}^*\right)}{\tilde{y}^*/\tilde{k}^*} = \frac{d\left(\tilde{c}^*/\tilde{y}^*\right)}{\tilde{c}^*/\tilde{y}^*} = \frac{d\tilde{l}^*}{\tilde{l}^*} = 0 \tag{15b}$$

The capital stock, output, and consumption change proportionately, with leisure (labor) remaining unchanged. That these results are independent of the consumption externality, but depend upon the production, is a manifestation of the non-scale production technology.

4. Macrodynamic Equilibrium: Centrally Planned Economy

In deriving his optimum, the individual agent neglects the externalities present in both consumption and production. As a consequence, the macroeconomic equilibrium generated by the decentralized economy may diverge from the social optimum. To derive the optimal resource allocation of the economy, we consider a social planner who chooses quantities directly to maximize the intertemporal utility of the representative agent, (1), while taking both externalities into account.

Specifically, the central planner internalizes the aggregation relationship $K = NK_i$, thus perceiving the individual's resource constraint (7') as

$$\dot{K}_i = \alpha L_i^{\sigma} K_i^{1-\sigma+\eta} N^{\eta} - C_i - (n+\delta)K_i \tag{7''}$$

In addition, he perceives that the consumption reference stock depends upon the economy-wide average of consumption, which is just equal to the consumption of the representative agent, and thus fully internalizes the impact of the agent's current decisions on the future evolution of the reference stock, in accordance with

$$\dot{H}_i = \rho(C_i - H_i) \tag{3'}$$

Performing the maximization, the optimality conditions become

$$C_i^{-\varepsilon}\left(\frac{l^{\theta}}{H_i^{\gamma}}\right)^{(1-\varepsilon)} + \rho\lambda_{2i} = \lambda_{1i} \tag{16a}$$

$$U_l \equiv \theta l^{\theta(1-\varepsilon)-1}\left(\frac{C_i}{H_i^{\gamma}}\right)^{(1-\varepsilon)} = \lambda_{1i}\sigma\frac{Y_i}{(1-l)} \tag{16b}$$

$$(1-\sigma+\eta)\frac{Y}{K} - \delta - n = \beta - \frac{\dot{\lambda}_{1i}}{\lambda_{1i}} \tag{16c}$$

$$-\frac{\gamma}{\lambda_{2i}}\left(\frac{C_i}{H_i}\right)\left[C_i^{-\varepsilon}\left(\frac{l^{\theta}}{H_i^{\gamma}}\right)^{(1-\varepsilon)}\right] - \rho = \beta - \frac{\dot{\lambda}_{2i}}{\lambda_{2i}} \tag{16d}$$

where, λ_{1i}, λ_{2i}, denote the social shadow values associated with the capital stock, K_i, and the reference consumption stock, H_i, respectively, together with the transversality conditions

$$\lim_{t\to\infty} e^{-\beta t}\lambda_{1i}K_i = \lim_{t\to\infty} e^{-\beta t}\lambda_{2i}H_i = 0 \tag{16e}$$

There are several key differences from the corresponding conditions for the decentralized economy. First, (16a) equates the utility of an additional unit of consumption, adjusted by its impact on the future reference stock, to the social shadow value of capital. Second, (16c) equates the social rate of return to capital to the social rate of return on consumption. Third, (16d) is an intertemporal allocation condition equating at the margin, the rate of return on habits – which consists of its direct utility benefits less costs through its impact on future accumulation – to the rate of return on consumption, both evaluated in terms of the shadow value of habits.

Thus the optimization problem confronting the central planner requires the monitoring of two state variables. Letting $q_i \equiv \lambda_{2i}/\lambda_{1i}$ denote the relative price of consumption habit to physical capital, after summing across households we can express the macrodynamic equilibrium of the centrally planned economy by the following fourth order system of the scale-adjusted variables:

$$\dot{k}^o = \left(1-\frac{c^o}{y^o}\right)y^o - \left(\delta + gn\right)k^o \tag{17a}$$

$$\dot{h}^o = \rho\left(c^o - h^o\right) + \left(1-g\right)nh^o \tag{17b}$$

$$\dot{l}^o = F\left(l^o\right)\left\{(1-\sigma+\eta)\left[1-\varepsilon\left(1-\frac{c^o}{y^o}\right)\right]\left(\frac{y^o}{k^o}\right) - \gamma\rho\left(1-\varepsilon\right)\left(\frac{c}{h}-1\right)\right.$$

$$-\left[\beta + \delta\left(1-\varepsilon(1-\sigma+\eta)\right) + n\left(1-\varepsilon(1-\sigma)\right)\right]$$

$$\left. + \frac{\rho q^o(1-\varepsilon)}{1-\rho q^o}\left((1-\sigma+\eta)\frac{y^o}{k^o} + \gamma\frac{c^o}{h^o}\left(\frac{1-\rho q^o}{q^o}\right) + \rho - \delta - n\right)\right\} \tag{17c}$$

$$\dot{q}^o = q^o\left\{(1-\sigma+\eta)\frac{y^o}{k^o} + \gamma\frac{c^o}{h^o}\left(\frac{1-\rho q^o}{q^o}\right) + \rho - \delta - n\right\} \tag{17d}$$

where

$$\frac{c^o}{y^o} \equiv \frac{C^o}{Y^o} = \frac{\sigma}{\theta}\left[\frac{l^o}{\left(1-l^o\right)}\right]\left(\frac{1}{1-\rho q^o}\right) \tag{17e}$$

o denotes the "optimum" or equilibrium in the centrally planned economy, *F* is defined as in (13e), and the production function is given by (12).

Imposing the stationary conditions, $\dot{l}^o = \dot{h}^o = \dot{q}^o = \dot{k}^o = 0$, together with the production function, (12), and (17e), we can determine the steady-state values as follows: [11]

$$\left(1-\frac{\tilde{c}^o}{\tilde{y}^o}\right)\frac{\tilde{y}^o}{\tilde{k}^o} = \delta + gn \tag{18a}$$

$$\rho\left(\frac{\tilde{c}^o}{\tilde{h}^o}-1\right) = \left(g-1\right)n \tag{18b}$$

[11] The derivations actually involve some details that are spelled out in the Appendix.

$$\left(1-\sigma+\eta\right)\frac{\tilde{y}^o}{\tilde{k}^o}-\left(n+\delta\right)=\beta+\left(g-1\right)\left[\gamma(1-\varepsilon)+\varepsilon\right]n \tag{18c}$$

$$\tilde{y}^o=\alpha(1-\tilde{l}^o)^{\sigma}(\tilde{k}^o)^{1-\sigma+\eta} \tag{18d}$$

$$\frac{\tilde{c}^o}{\tilde{y}^o}=\frac{\sigma}{\theta}\left(\frac{\tilde{l}^o}{1-\tilde{l}^o}\right)\left(\frac{1}{1-\rho\tilde{q}^o}\right) \tag{18e}$$

$$\tilde{q}^o=\frac{-\gamma\left[1+(g-1)n/\rho\right]}{\beta+(1-\gamma)\left[\rho+(g-1)\varepsilon n\right]} \tag{18f}$$

The parallels between these six equations and (14a) – (14e) for the decentralized economy are clear, and indeed, equations (18a), (18b), and (18d) remain unchanged. An interesting difference arises with regard to (18c), where the left hand side includes the effect of the productive externality and thus measures the social rate of return to capital, while the right hand side is the same as that of (14c) and equals the private rate of return on consumption. In other words, the steady-state social and private rates of return on consumption coincide. The role of the consumption externality is reflected in (18e). In contrast to the representative agent who evaluates the consumption-leisure choice in terms of the private marginal rate of substitution, (14e), the central planner also takes into account the social marginal value of the reference consumption stock through the term $1/(1-\rho\tilde{q}^o)$. Equation (18f) implies that unless there is an implausibly large negative production externality, $\tilde{q}^o<0$ if and only if the consumption externality is negative ($\gamma>0$). In that case, an increase in the level of the reference stock, given current consumption, is welfare-reducing, so that its shadow value is negative. The opposite argument applies in the case of altruism.

One important difference in (18) from (14) is that in the centrally planned economy the consumption externality γ,ρ has effects even in the absence of a production externality. This occurs through the social marginal rate of substitution, (18e). Setting $\eta=0$, $\tilde{y}^o/\tilde{k}^o$, $\tilde{c}^o/\tilde{y}^o$ remain independent of the consumption externality. But an increase in γ makes the shadow value of the reference stock $\tilde{q}^o$ more negative, reducing the relative valuation, $1/(1-\rho\tilde{q}^o)$. Given $\tilde{c}^o/\tilde{y}^o$, this raises leisure (reduces labor supply). Given $\tilde{y}^o/\tilde{k}^o$, the reduction in labor supply reduces capital, output, and consumption. The same applies as ρ increases. One further point is that the long-run responses in the centrally planned economy to an increase in productivity remain given by (15).

As we will show, the significance of consumption externalities depends crucially upon whether labor supply is elastic or fixed. In the latter case, the optimality conditions for the labor/leisure decision [(14e) and (18e)] drop out, while the remaining equations are unchanged, with l^*, l^o set at their inelastically fixed levels. In that case, as in the decentralized economy, the consumption elasticity will impact the equilibrium if and only if there is a production externality.

Our objective is to determine how closely the decentralized economy tracks the time path of the optimal centrally planned economy and to propose tax policies to correct for the distortions that may arise. Because of the complexity of the model we need to conduct the analysis of the dynamics numerically, but to aid in our understanding of these numerical simulations it is useful to examine first the steady states.

5. Comparison of Steady-State Equilibria

We begin with the simple, but important, case where labor is supplied inelastically.

5.1 Inelastic labor supply

In the decentralized economy the steady-state equilibrium values of $\tilde{k}^*, \tilde{c}^*, \tilde{h}^*, \tilde{y}^*$ are determined by (14a) – (14d), while in the centrally planned economy, the corresponding steady-state values, $\tilde{k}^o, \tilde{c}^o, \tilde{h}^o, \tilde{y}^o$, are determined by (18a) – (18d). With labor supplied inelastically at a common level, $\bar{l}$, in the two economies, it is immediately seen that in the absence of a production externality $\eta = 0$, these two sets of equations coincide, so that the steady-state equilibria in the two economies exactly coincide. Consumption externalities, γ, alone then have no effect on the steady state in either economy.

In the absence of a consumption externality, $\gamma = 0$, the crucial difference is in the presence of the production externality in (18c). Assuming this to be positive, (18c) and (14c) together imply

$$\frac{\tilde{y}^*}{\tilde{k}^*} = \frac{1-\sigma+\eta}{1-\sigma}\frac{\tilde{y}^o}{\tilde{k}^o} > \frac{\tilde{y}^o}{\tilde{k}^o} \tag{19a}$$

which together with the production functions (14d), (18d) implies

$$\frac{\tilde{k}^*}{\tilde{k}^o}=\left(\frac{1-\sigma}{1-\sigma+\eta}\right)^{1/(\sigma-\eta)}<1;\quad \frac{\tilde{y}^*}{\tilde{y}^o}=\left(\frac{1-\sigma}{1-\sigma+\eta}\right)^{(1-\sigma+\eta)/(\sigma-\eta)}<1 \qquad (19b)$$

Combining (19a) with (14a) and (18a) implies $\tilde{c}^*/\tilde{y}^* > \tilde{c}^o/\tilde{y}^o$ and if in addition, we assume that the economy is dynamically efficient, we further obtain $\tilde{c}^* < \tilde{c}^o$.[12] We may summarize these results in

> **Proposition 1:** *In a decentralized economy with inelastic labor supply and a positive production externality, the steady-state equilibrium capital stock and output are below their respective optimal levels, while the equilibrium output-capital ratio is too high. The consumption-output ratio is also too high, although if the economy is dynamically efficient, the consumption level is too low. These comparisons are reversed if the production externality is negative. In the absence of any production externality, a consumption externality causes no long-run distortionary effects.*

Although, consumption externalities alone play no role in determining the steady state, they do interact with production externalities in a growing economy. This can be seen from (14c), (18c), together with (19b). If $\eta > 0, n > 0$, an increase in γ will raise $\tilde{y}^*, \tilde{y}^o$ proportionately. It will also raise $\tilde{k}^*, \tilde{k}^o$ by a larger proportionate amount, thus reducing the output-capital ratios in the two economies proportionately. The distortions in output and capital increase in absolute size.

5.2 Elastic labor supply

With endogenous labor supply, the consumption externality will now affect the steady state even in the absence of any production externality. This is because it affects the marginal valuation of consumption, which in turn changes the optimal utility value of the marginal product of labor. Thus, consumption distortion results in labor distortion and therefore creates production inefficiency.

The comparison now involves the complete sets of equations (14) and (18) and leads to the following. Consider first the absence of a production externality. In this case comparing (14c) and (18c), (14b) and (18b), (14a) and (18a) implies

[12]This can be formally demonstrated by expanding the decentralized equilibrium around the optimum, as in Liu and Turnovsky (2003).

$$\frac{\tilde{y}^*}{\tilde{k}^*} = \frac{\tilde{y}^o}{\tilde{k}^o}; \quad \frac{\tilde{c}^*}{\tilde{y}^*} = \frac{\tilde{c}^o}{\tilde{y}^o}; \quad \frac{\tilde{h}^*}{\tilde{y}^*} = \frac{\tilde{h}^o}{\tilde{y}^o} \tag{20}$$

If the consumption externality is negative, ($\gamma > 0$) then $\tilde{q}^o < 0$ and equating (14e) to (18e) implies $\tilde{l}^* < \tilde{l}^o$ or equivalently, $\tilde{L}^* > \tilde{L}^o$. Equations (14d) and (18e) then imply $\tilde{k}^*/\tilde{k}^o = \left(\tilde{L}^*/\tilde{L}^o\right)^{\sigma/(\sigma-\eta)} > 1$, and hence $\tilde{y}^* > \tilde{y}^o$, and thus from (20) we obtain $\tilde{c}^* > \tilde{c}^o$. The reverse applies if $\gamma < 0$.[13] In the case of a positive production externality but no consumption externality, $(\eta > 0, \gamma = 0)$, the comparison made above in the case of inelastic labor supply continues to apply. In addition the fact that $\tilde{c}^*/\tilde{y}^* > \tilde{c}^o/\tilde{y}^o$ implies $\tilde{l}^* > \tilde{l}^o$, or equivalently $\tilde{L}^* < \tilde{L}^o$, with the reverse applying if $\eta < 0$.

These results may be summarized by the following proposition relating the actual and socially optimal equilibria in the presence of consumption and production externalities.

Proposition 2: *In an economy with endogenous labor supply, the steady-state equilibrium has the following properties.*

1. *In the case of a negative consumption externality ($\gamma > 0$), equilibrium consumption, capital stock, and labor supply are all greater than their respective long-run optimal values, ($\tilde{c}^* > \tilde{c}^o$, $\tilde{k}^* > \tilde{k}^o$, $\tilde{L}^* > \tilde{L}^o$, $\tilde{y}^* > \tilde{y}^o$). In the case of altruism these relationships are reversed.*

2. *If the production technology exhibits a positive aggregate capital externality $(\eta > 0)$, equilibrium consumption, capital stock, and labor supply are all below their respective long-run optimal values, ($\tilde{c}^* < \tilde{c}^o$, $\tilde{k}^* < \tilde{k}^o$, $\tilde{L}^* < \tilde{L}^o$, $\tilde{y}^* < \tilde{y}^o$). In the case of a negative externality these relationships are reversed.*

To understand these results, we first consider the effect of a consumption externality. The first order condition in the household's maximization problem requires the marginal rate of substitution between consumption and leisure to equal the marginal product of labor. In the case of jealousy, equilibrium consumption is over-valued. This results in the equilibrium consumption being too high compared to the social optimum and leisure being too low, (or equivalently, labor

[13] As ρ increases, the magnitudes of these deviations from their respective optimum increase. It thus follows that the deviations under time non-separable preferences are smaller in magnitude than those obtained when the consumption externality is contemporaneous.

supply being too high). Because capital and labor are complements, and because in steady state the marginal product of capital is fixed, a higher equilibrium labor input implies a higher equilibrium capital stock. The argument is reversed if preferences exhibit altruism.[14]

In the case of a positive production externality the marginal product of capital is undervalued by the private agent. Since the marginal product of capital is diminishing, and the steady state requires the marginal product of capital to be identical in the two economies, less capital is needed in the decentralized economy compared to the socially optimal economy to achieve the equilibrium condition on the marginal product of capital. Thus less output will be produced and equilibrium consumption reduced correspondingly. The higher output-capital ratio in the decentralized economy implies a higher consumption-output ratio. The positive relationship between the latter and leisure in utility leads to a higher proportion of time devoted to leisure, i.e. a reduction in labor supply.

6. Optimal Tax Policy

The fact that consumption and production externalities create distortions in resource allocation provides an opportunity for government tax policies to improve efficiency. Consider again the decentralized economy populated by identical agents. Let τ_k be the tax rate on the return to private capital, τ_w the tax rate on labor income, τ_c the tax rate on consumption, and T_i lump-sum transfers (taxes). The representative agent maximizes the utility function (1), subject to the budget constraint, now modified to:

$$\dot{K}_i = \left[(1-\tau_k)r - n - \delta\right]K_i + (1-\tau_w)wL_i - (1+\tau_c)C_i + T_i \tag{21}$$

The government maintains a balanced budget, rebating all tax revenues as lump sum transfers:

$$\tau_k rK_i + \tau_w wL^i + \tau_c C_i = T_i \tag{22}$$

The first order optimality conditions of the decentralized economy now become:

[14] Frank (1999) describes the over-consumption phenomenon and argues that people tend to work too long, due to negative consumption externalities, consistent with the result of Proposition 2. However, contrary to what he argues, Proposition 2 shows that negative consumption externalities do not imply under-saving. In fact, with negative consumption externalities, the level of steady-state capital is too high, not too low.

$$U_{c_i} \equiv C_i^{-\varepsilon}\left(\frac{l^{\theta}}{H_i^{\gamma}}\right)^{(1-\varepsilon)} = \lambda_i\left(1+\tau_c\right) \tag{8a'}$$

$$U_{l_i} \equiv \theta l^{\theta(1-\varepsilon)-1}\left(\frac{C_i}{H_i^{\gamma}}\right)^{(1-\varepsilon)} = \lambda_i\left(1-\tau_w\right)w \tag{8b'}$$

$$\alpha\left(1-\tau_k\right)r-\delta-n=\beta-\frac{\dot{\lambda}_i}{\lambda_i} \tag{8c'}$$

together with (8d). The equilibrium dynamics in the presence of taxes are modified to:

$$\dot{k}^* = \left(1-\frac{c^*}{y^*}\right)y^* - \left(\delta+gn\right)k^* \tag{13a}$$

$$\dot{h}^* = \rho\left(c^*-h^*\right)+\left(1-g\right)nh^* \tag{13b}$$

$$\dot{l}^* = F\left(l^*\right)\left\{\left[(1-\sigma)(1-\tau_k)-\varepsilon(1-\sigma+\eta)\left(1-\frac{c^*}{y^*}\right)\right]\frac{y^*}{k^*}-\rho\gamma\left(1-\varepsilon\right)\left(\frac{c^*}{h^*}-1\right)\right.$$
$$\left.-\left[\beta+\delta\left(1-\varepsilon(1-\sigma+\eta)\right)+n\left[1-\varepsilon\left(1-\sigma\right)\right]\right]+\varepsilon\frac{\dot{\tau}_w}{1-\tau_w}-(1-\varepsilon)\frac{\dot{\tau}_c}{1+\tau_c}\right\} \tag{13c'}$$

$$\frac{c^*}{y^*}=\frac{\sigma}{\theta}\frac{l^*}{\left(1-l^*\right)}\left(\frac{1-\tau_w}{1+\tau_c}\right) \tag{13d'}$$

$$y^*=\alpha\left(1-l^*\right)^{\sigma}k^{*1-\sigma+\eta} \tag{13e}$$

together with the transversality condition (8d).

The objective is to characterize a tax structure such that the decentralized economy mimics the dynamic equilibrium path of the centrally planned economy, (17a) – (17f). Two relationships are subject to distortions; the consumption-output ratio, (13d'), and the evolution of leisure, (13c'). In principle, the optimal time path can be replicated by the use of two, possibly time-varying, tax rates, which in fact can be chosen in several different ways. This choice proceeds as follows.

First, in order for the consumption-output ratio (13d') to match (17e) we require

$$\frac{1+\tau_c}{1-\tau_w}=1-\rho q^o \tag{23}$$

implying

$$\frac{\dot{\tau}_c}{1+\tau_c}+\frac{\dot{\tau}_w}{1-\tau_w}=-\frac{\rho q^o}{1-\rho q^o}\left(\frac{\dot{q}^o}{q^o}\right) \tag{24}$$

Combining (13c'), (17d), and (24) we see that the optimal path for labor, (17c), can be mimicked by setting $\dot{\tau}_w \equiv 0$ -- so that the wage tax remains fixed at the arbitrary constant level,. $\tau_w = \overline{\tau}_w$ -- and setting the tax on capital

$$(1-\sigma)(1-\tau_k)=(1-\sigma+\eta) \text{ i.e. } \tau_k=-\frac{\eta}{1-\sigma} \tag{25}$$

Setting the tax rates in accordance with (23) and (25) with $\tau_w = \overline{\tau}_w$ ensures the replication of the entire transitional path followed by the first-best equilibrium.

The optimal tax on capital income corrects the resource distortion resulting from the production externality. The government should subsidize capital investment if there is a positive capital externality, and should levy a positive capital income tax if there is a negative capital externality.[15] Given the Cobb-Douglas production function being assumed here, this tax is constant over time, although it would become time-varying for more general production functions; see Liu and Turnovsky (2003). In contrast, the consumption tax corrects for the distortion caused by the consumption externality. This depends upon q^o and is therefore time-varying, as q^o evolves in accordance with (17d), and can be seen to depend upon the production externality, η. Although both externalities generate distortions in labor supply, no active labor income tax is needed once the source of distortions is rectified. The constant labor income tax can thus be set to zero. If $\overline{\tau}_w = 0$, private consumption should be subsidized if there is a positive consumption externality and should be taxed if there is a negative consumption externality.[16]

Noting (18f), the consumption and labor income tax ratio converges to

[15] This result is a familiar one associated with the Romer technology; see e.g. Turnovsky (1996). In the absence of the externality it implies that capital income should be untaxed, a result familiar from early work by Chamley (1986) and Judd (1985).
[16] Other tax structures may also attain the first-best optimum. There are two distortions, one involving consumption, the other production, and therefore two tax rates -- one for each distortion -- are required for their correction.

$$\frac{1+\hat{\tau}_c}{1-\overline{\tau}_w}=1-\rho\tilde{q}^o=\frac{\rho+\beta+(g-1)\big(\varepsilon+\gamma(1-\varepsilon)\big)n}{\rho(1-\gamma)+\beta+\varepsilon(g-1)(1-\gamma)n} \tag{26}$$

In the absence of the production externality, $g=1$, this expression reduces to:

$$\frac{1+\hat{\tau}_c}{1-\overline{\tau}_w}=\frac{\rho+\beta}{\rho(1-\gamma)+\beta} \tag{26'}$$

In the absence of any consumption externality, $q^o\equiv 0$. In that case (23) reduces to $(1+\hat{\tau}_c)/(1-\overline{\tau}_w)=1$, or equivalently $\hat{\tau}_c=-\overline{\tau}_w$, which is constant over time.[17]

It is interesting to compare this result with the corresponding steady-state optimal tax result of Liu and Turnovsky (2003). Using a general utility function and with the externality being due to contemporaneous consumption (i.e. $\rho\to\infty$), they obtain

$$\frac{1+\hat{\tau}_c}{1-\overline{\tau}_w}=\frac{U_C(C,C,l)}{U_C(C,C,l)+U_H(C,C,l)} \tag{27}$$

Using this notation, (26) can be written as

$$\frac{1+\hat{\tau}_c}{1-\overline{\tau}_w}=\frac{U_C(C,H,l)}{U_C(C,H,l)+\dfrac{\rho}{\rho+\beta}U_H(C,H,l)} \tag{27'}$$

The difference is due to the fact that in the present model the reference consumption level is a stock derived as an exponentially declining average of all past consumptions, whereas in the Liu-Turnovsky model it depends upon current consumption. As a result since the consumption reference stock is adjusting over-time, we need to take into consideration the rate of time preference. Therefore, the marginal utility of the reference stock is weighted by $\rho/(\rho+\beta)$. Letting $\rho\to\infty$ and assuming zero growth (27') coincides with (27). For the constant elasticity utility function (1), the optimal consumption tax, (27), reduces to $1/(1-\gamma)$, which is constant over time.

The same optimal structure is derived if labor supply is inelastic, enabling us to summarize our results with the following optimal taxation proposition.

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[17] Interpreting the tax on wage income as a negative tax on leisure, $\overline{\tau}_w=-\hat{\tau}_c$ requires that the two utility enhancing goods, consumption and leisure, should be taxed uniformly. This result can be viewed as an intertemporal application of the Ramsey principle of optimal taxation; see e.g. Deaton (1981). If the utility function is multiplicatively separable in consumption and leisure, as we are assuming here, then their uniform taxation is optimal.

Proposition 3: *The decentralized economy will exactly replicate the entire time path of the optimal (centrally planned) economy if taxes at each instant of time are set in accordance with*

$$\hat{\tau}_k = -\frac{\eta}{1-\sigma}, \; \tau_w = \overline{\tau}_w, \text{ and } \frac{1+\hat{\tau}_c(t)}{1-\overline{\tau}_w} = 1-\rho q^o(t)$$

where $q^o(t)$ *evolves in accordance with (17d). In the limiting case* $\rho \to \infty$*, when the reference stock is contemporaneous,* $\left((1+\hat{\tau}_c)/(1-\overline{\tau}_w)\right) = \left(1/(1-\gamma)\right)$*, and the consumption tax is also constant over time.*

7. Numerical analysis of some transitional paths.

To study the transitional dynamics we calibrate the model to reproduce some key features of actual economies.[18] Table 1 summarizes the parameters upon which our simulations are based. Most of these are standard and non-controversial. In this regard, $\sigma = 0.65$, implying a labor share of income of 65%, the rate of time preference $\beta = 0.04$, the instantaneous intertemporal elasticity of substitution, $1/\varepsilon = 0.4$, population growth rate $n = 0.015$, and depreciation rate, $\delta = 0.05$ are well documented, while being a non-scale model, the normalization $\alpha = 1$ is unimportant.

An important parameter is θ, which describes the degree of substitution between leisure and consumption in utility. The chosen benchmark value $\theta = 1.75$ corresponds to the value chosen in the business cycle literature and implies equilibrium fractions of time devoted to leisure of around 0.7, consistent with the empirical evidence; see e.g. Cooley (1995). To illustrate the importance of the elasticity of labor supply, we also report some numerical results when labor supply is inelastic.

The two key parameters upon which we wish to focus pertain to the consumption externality, γ, and the production externality, η. The absence of both externalities $\gamma = \eta = 0$ serves as a natural benchmark. For the negative consumption externality, we consider $\gamma = 0.5$, the value taken by Carroll et al. (1997) as their benchmark. For symmetry, we choose $\gamma = -0.5$ as the magnitude of an equivalent positive consumption externality. We set the speed of adjustment of the reference stock,

[18] The dynamics are studied by linearizing the relevant dynamic system, (13) or (17) about its respective steady state, (14) and (18). The details of this are cumbersome, but standard, and are available from the authors on request.

at $\rho = 0.2$, also the benchmark value of Carroll et al.[19] For the production externality, we consider the positive externality $(\eta = 0.2)$ [increasing returns] and negative externality $(\eta = -0.2)$ [decreasing returns], respectively.

Table 2a presents the steady-state equilibrium and the corresponding optimal equilibrium for the base economy in the case where labor supply is elastic, with $\theta = 1.75$. The benchmark externality-free economy $(\gamma = \eta = 0)$ is indicated in bold face. As can be seen the decentralized economy coincides with the first best optimum. The chosen parameter values imply an output-capital ratio of 0.3, consumption output ratio of around 78%, and fraction of time devoted to labor of around 32% [68% devoted to leisure]. From Table 2a, the following patterns of responses all which are consistent with Proposition 2, can be observed

1. In the absence of both the production and the consumption externality the steady states of the decentralized and centrally planned economies will coincide; see Panel B, row 2.

2. **Consumption externality only**. In this case:

(i) The steady-state equilibrium of the **decentralized** economy is unaffected.

(ii) If the externality is **positive** $(\gamma = -0.5)$, then $\tilde{k}, \tilde{L}, \tilde{y}, \tilde{c}$ in the **centrally planned** economy are all proportionately larger [relative to the benchmark, $\gamma = 0$] by 24.9%; if is **negative** $(\gamma = 0.5)$ $\tilde{k}, \tilde{L}, \tilde{y}, \tilde{c}$ are all proportionately smaller by 32.6%. The output-capital and consumption-output ratios are equal in the two economies and are independent of γ, remaining at 0.300 and 0.783, respectively.

(iii) As a result, $\tilde{k}, \tilde{L}, \tilde{y}, \tilde{c}$ in the decentralized economy are all 20% **below** their respective optima if $\gamma = -0.5$, while they are all 48.5% **above** if $\gamma = 0.5$.

3. **Production externality only**.

(i) If the externality is **positive** $(\eta = 0.2)$, then $\tilde{k}, \tilde{L}, \tilde{y}, \tilde{c}$ in the **centrally planned**

[19]These were also the benchmark values considered by Alvarez-Cuadrado et al. (2004). Because information on both γ, ρ is sparse we have conducted some sensitivity analysis on these parameters. For example, we have performed some simulations using the value $\gamma = 0.8$, estimated by Fuhrer (2000). We have also chosen larger values of ρ, as suggested by some of the applications of these models to the equity premium puzzle problem, as well as smaller values, which yield more plausible speeds of convergence. Our main qualitative conclusions are robust to the parameter choice.

economy increase although each by differential amounts, relative to the benchmark, $\eta = 0$. These range from a 212% increase in $\tilde{k}$ to just a 10.2% increase in $\tilde{L}$, implying a 130% increase in $\tilde{y}$, while $\tilde{c}$ increases by 98%. The $\tilde{y}/\tilde{k}$ and $\tilde{c}/\tilde{y}$ ratios decline by 8 and nearly 11 percentage points respectively. If it is **negative**, $(\eta = -0.2)$ $\tilde{k}, \tilde{L}, \tilde{y}, \tilde{c}$ decline by 68%, 9.6%, 32%, and 21%, respectively, while the $\tilde{y}/\tilde{k}$ and $\tilde{c}/\tilde{y}$ ratios increase by over 34 and nearly 12 percentage points respectively. The output-capital and consumption-output ratios thus decrease uniformly with η.

(ii) The **decentralized** economy responds much more mildly to the production externality. Output and consumption increase by just 13% and 15%, while the capital stock and employment decline by 2% and 1%, respectively to a positive externality; in the case of a negative externality the corresponding changes are -14%, -14.7%, -5.8%, +0.6%. The output-capital and consumption-output ratios thus increase uniformly with η.

(iii) The output-capital and consumption-output ratios in the decentralized economy exceed their optima for $\eta = 0.2$, by 12.7 and 11.8 percentage points. In that case $\tilde{k}, \tilde{L}, \tilde{y}, \tilde{c}$ are all sub-optimally small, being 68.5%, 10.1%, 50.6% and 42.1%, below their respective optima. For $\eta = -0.2$, $\tilde{y}/\tilde{k}$, $\tilde{c}/\tilde{y}$ are 36.6 and 12.8 percentage points below their optima implying that $\tilde{k}, \tilde{L}, \tilde{y}, \tilde{c}$ are all 194%, 11.3%, 25.9%, 7.9% too large.

4. Pronounced asymmetries exist with respect to the effects of positive versus negative externalities of comparable sizes on the deviations of the decentralized from the centrally planned economy.

(i) For the pure **consumption** externality the deviation is uniformly 48.5% for a positive externality but only 20% for a comparable negative externality.

(ii) For the pure **production** externality the deviations are substantially larger (in magnitude) for $\eta = 0.2$, though not necessarily in percentage terms, due to the significant increase in size of the centrally planned economy in that case.

The reason for these differential responses can be traced to (14e) vs. (18e) and (18f) for the consumption externality, and (14c) vs. (18c) for the production externality. In the former case, the key element is the adjustment for the consumption externality, $\tilde{\varphi} \equiv \left(1/1 - \rho\tilde{q}^{o}\right)$, which for

$\gamma = 0.5, -0.5$ implies $\tilde{\varphi} \equiv 0.583,\ 1.417$. Since the decentralized economy ignores this term, the proportionate error being made is larger when $\gamma = 0.5$ than when $\gamma = -0.5$, thus accounting for the greater distortion in that case.

Setting $\gamma = 0$ in (18c) we see that the production externality η impacts positively upon both the return to consumption and the productivity of capital. For the chosen parameterization, the latter effect dominates, so that ($\tilde{y}/\tilde{k}$) declines with η for the central planner. The decentralized economy ignores this effect [see (14c)] and so ($\tilde{y}/\tilde{k}$) increases with η in that economy. The asymmetry in the responses with respect to $\eta = \pm 0.2$ follows from the fact that these imply values of $1 - \sigma + \eta$ equal to 0.55 vs. 0.15, respectively.

5. Distortions arising from positive production externalities are generally reduced in the presence of negative consumption externalities and increased in the presence of positive consumption externalities. Distortions with respect to negative production externalities are generally increased in the presence of negative consumption externalities, but only slightly in the presence of positive consumption externalities.

6. In most cases $\tilde{L}, \tilde{k}$ are either both too large or both too small, relative to the optimum, implying the same for output, $\tilde{y}$. But in some instances -- e.g. a negative consumption externality, $\gamma = 0.5$, accompanied with a positive production externality, $\eta = 0.2$, -- $\tilde{k}$ in the decentralized economy is too low, while $\tilde{L}$ is too high, with the former dominating the latter leading to sub-optimally low output.

7. The employment of capital and labor decline (increase) slightly as the consumption externality declines (becomes less positive) in the decentralized economy, in the presence of a negative (positive) production externality. They both decline much more rapidly in a centrally planned economy, irrespective of the production externality.

8. Employment declines gradually in the decentralized economy as the production externality increases, the response becoming weaker as the consumption externality increases, in the centrally planned economy, the more positive production externality stimulates employment.

Table 2b provides some comparisons in the case where labor is supplied inelastically, yielding some significant differences from Table 2a.[20]

1. In the absence of the production externality, the decentralized and centrally planned economies coincide, irrespective of the consumption externality, consistent with Proposition 1.

2. The implications of Proposition 1 regarding productive externalities are supported. For $\eta > 0$ capital and output are sub-optimally low, while the y/k ratio is too high.

3. With employment fixed, capital in the decentralized economy increases strongly with the production externality, irrespective of the consumption externality.

4. The variations in output and consumption arising from both positive and negative production externalities are much less sensitive to variations in the consumption externality, than when labor is elastically supplied.

5. Output and consumption in the centrally planned economy increase with γ in the presence of a positive production externality, in contrast to when labor is elastically supplied.

8. Increase in Productivity

Tables 3 and 4 summarize some of the steady-state and short-run effects of a 50% increase in productivity, α.

8.1 Long-run Effects

From Table 3 we see that the higher productivity leads to equal long-run increases in output, capital stock, and consumption that are independent of the consumption externality γ, and coincide in the decentralized and centrally planned economies, consistent with (15). The table also includes measures of welfare loss. These measure the loss in the representative agent's optimized utility function Ω [given in (1)], in the two economies (decentralized economy and centrally planned) in

[20]Note that for the cases $\gamma = 0, \eta = -0.2$, $\gamma = -0.5, \eta = -0.2$, $\tilde{c}^* < \tilde{c}^o$ even though $\tilde{k}^* > \tilde{k}^o$, implying that these are two cases of "dynamic inefficiency".

response to the shocks. The welfare gains reported are equivalent variation measures, calculated as the percentage change in the permanent flow of consumption in the respective economies, necessary to equate the levels of welfare to what they would be following the increase in productivity. Details of this calculation are provided in the Appendix.

In contrast to the various measures of economic activity, the changes in welfare do depend upon the consumption externality, γ. This is because with sluggish adjustment, current consumption exceeds the reference level during the transition. Accordingly, if $\gamma > 0$, the "keeping up with the Joneses" individual derives additional utility, relative to a conventional individual having time separable utility, due to the fact that he is doing better than his peers. For example, if $\gamma = \eta = 0$, the individual's intertemporal welfare increases by 69.7%, whereas if $\gamma > 0$, the gain in welfare is increased to 82.4%. An altruistic individual, however, has only a 64.2% welfare gain.

Several related features stand out in Table 3. First, the percentage utility *gains* are similar in magnitude for the decentralized and centrally planned economies with identical externalities, though generally somewhat larger for the latter. But since the decentralized economy usually has a non-optimal production and consumption structure, its initial welfare *level* is substantially below that of the centrally planned economy, so that the absolute utility gains are significantly smaller. Second, in some cases, utility gains are proportionately larger in the decentralized economy, though its welfare remains well below the optimum. Thus, if $\gamma = 0.5,\ \eta = 0.2$, welfare increases by 120% in the decentralized economy and only by 116% in the centrally planned economy. This represents a situation where the welfare of the decentralized economy improves from 69.2% of the optimal economy in terms of consumption flow to 70.4%. Third, the differences between economies in which labor supply is elastic and inelastic, respectively, are small. This is because the accumulated changes in consumption between the various structures are identical while even when labor is supplied elastically there is no change in long-run employment. Intertemporal utility differences therefore reflect only timing changes along the transitional paths and these are presumably small.

It is also of interest to compare these long-run responses to those obtained in the more familiar case of "keeping up with the Joneses" where the consumption externality enters utility contemporaneously. Since the long-run responses are independent of ρ, the long-run changes in capital, output, and other variables are unchanged from those in Table 3. However, the utility gains are different. In general, we find that the utility gains for the time non-separable utility function exceed (are less than) those for conventional utility according to whether the consumption externality, is negative, (positive). If $\gamma > 0$, for example, the agent derives additional utility from having current consumption exceed the economy-wide reference level, with this extra utility gain being enhanced with the gradual adjustment of the reference level in the present case.

8.2 Short-Run Effects

Tables 4a and 4b report the short-run responses to the productivity increase. We shall focus on the interaction between the negative consumption externality $(\gamma = 0.5)$ and the positive production externality $(\eta = 0.2)$ summarized by Rows 2 and 3 in Panels B and C.

Consider first the benchmark case where there are no externalities, $\gamma = \eta = 0$. In this case, the decentralized economy exactly mimics the centrally planned economy throughout the entire transitional path. A productivity increase of 50% immediately raises employment in both economies by 0.95 percentage points, raising output by 52.9% and consumption by 46.4%. The change in instantaneous welfare, expressed in terms of an equivalent variation in consumption is defined in the Appendix. With $\gamma = 0$, the increase in consumption raises this by 46.4%, but this is offset by the higher employment (decline in leisure) so that the short-run rise in welfare is only 42.8%.

Negative consumption externality: Now suppose that there is a negative consumption externality ($\gamma = 0.5$) but no production externality ($\eta = 0$) [see Panel C, row 2]. In this case, the decentralized economy starts out from an initial steady state in which consumption, capital, output, and employment are all 48.3% higher than in the corresponding centrally planned economy. The 50% increase in α raises output directly in both economies, and will raise consumption, though by a lesser amount, due an increase in the savings ratio. As a result, the consumption-output ratio falls initially in both economies. Since the representative agent in the decentralized economy ignores the negative consumption externality, he *overvalues* consumption, relative to its optimum, so that consumption increases more in the decentralized economy, causing the consumption-output ratio to decline by less in that economy. At the same time, by impacting directing on final output, the productivity shock increases the ultimate scarcity of habit relative to capital, thereby making q more negative, so that the social value of the consumption-output ratio, $(c/y)(1-\rho q)$ actually declines less. As a result, leisure in the centrally planned economy declines less [c.f. (13d) and (17e)] so that labor supply increases more in the decentralized economy, raising output by more in that economy, as well. The effect of these non-optimal adjustments is that welfare increases in the decentralized economy by 71.5%, rather than 77.6%, in the optimal economy.

Following their initial jumps, output, and consumption in both the decentralized and centrally planned economies increase monotonically toward their new steady states levels, while the capital stocks increase gradually from their respective initial levels. Since in the long-run labor allocations do not change, labor supply declines monotonically in both economies, back toward their respective original steady-state values.

Since our concern is to focus on the relative deviations of the decentralized economy from the optimum, Fig. 1.A plots the time paths of the ratios y^*/y^o, c^*/c^o, k^*/k^o, L^*/L^o. Fig 1.A(i) illustrates the present case $\gamma = 0.5, \eta = 0$. All four variables start out at the common ratio 1.483 and eventually converge back to that ratio after the shock. On impact, c^*/c^o rises to around 1.51, L^*/L^o to 1.495, y^*/y^o to 1.490, while the capital stock remains fixed instantaneously. The amount of over-consumption, over-production, and over-employment in the decentralized economy all increase. The over-consumption in the decentralized economy means that capital begins to accumulate at a slower rate in the decentralized economy, so that k^*/k^o initially declines, although eventually it is restored back to its equilibrium. The interesting feature of these paths is that throughout the transition all four variables exceed their respective optima, with some deviations in the degree during the early stages. These divergences in the paths reflect the fact that agents in the decentralized economy, by ignoring the impact of current consumption on the reference level generate a faster speed of convergence; see Alvarez-Cuadrado, et. al. (2004).

Positive production externality: Now suppose that there is a positive production externality ($\eta = 0.2$) but no consumption externality ($\gamma = 0$) [see Panel B, row 3]. In this case the decentralized economy is one in which capital, labor, output, and consumption are all less than their respective optima, but to vastly different degrees ($\tilde{k}^*/\tilde{k}^o = 31\%, \tilde{L}^*/\tilde{L}^o = 90\%$, $\tilde{y}^*/\tilde{y}^o = 49\%$, $c^*/c^o = 58\%$).

Since the productivity shock affects all variables proportionately, and since the positive production externality has little effect on the relative speeds of convergence of the two economies, Fig. 1.A (ii) illustrates how the deviations from the optimum remain almost constant along the transitional paths.

Positive production externality and negative consumption externality: We now combine cases (i) and (ii) by assuming $\eta = 0.2,\ \gamma = 0.5$ [see Panel C, row 3]. This combines elements of the two previous cases. Whereas the capital stock, output, and consumption are all initially less than their respective optima ($\tilde{k}^*/\tilde{k}^o = 53\%,\ \tilde{y}^*/\tilde{y}^o = 83\%,\ c^*/c^o = 99\%$), labor exceeds its optimum ($\tilde{L}^*/\tilde{L}^o = 129\%$). On the one hand the negative consumption externality leads to over-employment, over-production, and over-consumption, on the other, the production externality has the opposite effect. As in Fig. 1.A(ii), there is almost no change in the deviation of the decentralized economy relative to its optimum over time. The most interesting feature about Fig. 1.A(iii) is that the effect of the negative consumption externality is to cause initial over-consumption by about 2%, which gradually declines over time and after about 20 years returns to under-consumption.

Table 4b provides the analogous effects in the case where labor supply is perfectly inelastic, with Fig. 1.B illustrating the dynamic adjustments. The same generally qualitative responses can be seen, although there are some differences. With labor inelastic and capital fixed instantaneously the 50% increases in productivity immediately translates to a 50% increase in output, irrespective of other structural parameters.[21] The dynamic adjustments have generally the same qualitative properties as when labor supply is elastic, except that they are shifted up by the production externality and down by the consumption externality. The most interesting aspect is the negative consumption externality in Fig 1.B(i). With inelastic labor supply the consumption externality has no long-run distortionary effects, so that all three ratios, y^*/y^o, c^*/c^o, k^*/k^o, illustrated in that figure, begin and converge to unity, consistent with Proposition 1. During the transition they diverge, and in fact during the first few years of the transition there is over-consumption in the decentralized economy, after which there is under-consumption.

[21]Our results for the transitional path provide an interesting contrast from the result obtained by Liu and Turnovsky (2003) based on the conventional time separable utility function. For the constant elasticity specification being assumed here they find that the consumption externality causes no distortionary effects along the transitional path. That is not the case here, although it does obtain in the limiting case $\rho \to \infty$, when the two utility specifications converge.

8.3 Optimal Taxation

We briefly characterize the optimal tax policy in response to this shock. First, the optimal tax on capital is constant, given by (25). For the chosen parameter values $\hat{\tau}_k = 57.1\%,\ 0,\ -57.1\%$, corresponding to $\eta = -0.20,\ 0,\ 0.20$, respectively. This is identical to the optimal capital tax in the Romer model, requiring that capital income should be taxed or subsidized depending upon whether the externality if negative or positive. The only point worth adding is that the tax depends only upon the technology and is independent of the specific structural change, in this case a 50% increase in α.

Fig. 2 plots the optimal consumption/wage tax ratio, $(1+\hat{\tau}^c)/(1-\overline{\tau}_w)$, as it tracks the time path of $q(t)^o$ generated by the productivity shock. The case of a positive consumption externality is illustrated in the upper figure. The optimal ratio $(1+\hat{\tau}^c)/(1-\overline{\tau}_w)$ begins at its steady-state value 1.714 [see (26')]. Following the jump in the shadow value q^o generated by the productivity increase and it jumps to 1.778, after which it converges back toward 1.714, as consumption habits adjust and q^o declines in magnitude. In the case of a negative externality, the ratio begins at 0.706, drops down to 0.69, then gradually rises after the shock, reaching 0.712 after around 10 years before declining slightly back toward its steady-state equilibrium value 0.706. In contrast to the tax on capital, the optimal consumption/wage tax ratio depends upon the specific shock to the economy.

8.4 Slower convergence

The dynamic adjustments illustrated in Fig. 1 show relatively little variation in the deviations of the decentralized economy from the optimum over time. For the pure consumption externality the maximum deviations during the initial phase are of the order of 1.5% if labor supply is elastic and less if $\theta = 0$. While the deviations are sustained in the presence of a production externality, they are very uniform over time.

One characteristic of our chosen parameterization is that it implies an asymptotic speed of convergence of around 10%. While this is consistent with some estimates [e.g. Caselli et al. (1996)] it is generally above the consensus values, which now range up to about 6%.[22] Calibration of this

[22]Early studies by Barro (1991), Mankiw, Romer, and Weil (1992) established a benchmark convergence rate of 2-3%. Subsequent research suggest that the convergence rates are more variable and sensitive to time periods and the set of

aspect of the model can be improved by increasing the consumption externality γ to 0.9 [closer to the value of 0.8 estimated by Fuhrer (2000)], while slowing its adjustment to $\rho = 0.1$. Figure 3A plots the deviations of the decentralized economy from the optimum in response to a 50% productivity increase in the absence of any production externality and inelastic labor supply. The capital stock in particular shows an interesting time path, although this is mirrored in the other variables. During the first few years there is over-accumulation of capital by about 0.5%, during the next 85 years there is under-accumulation of capital, which reaches 5% below its optimum after around 40 years, and finally after about 95 years there is over-accumulation of capital. The main point is that with slower convergence, the deviations become substantial in magnitude.

Fig. 3B plots the time path for the optimal consumption/wage income tax ratio. It too shows relatively larger variations and non-monotonic behavior, reflecting the more complex dynamics of the underlying economy as the speed of convergence declines, and in particular $q(t)^o$.

9. Conclusions

The theoretical and empirical importance of both consumption and production externalities are well documented. In addition, a growing body of empirical evidence supports the importance of time non-separable preferences as an alternative to the conventional time separable utility function. With this motivation, this paper has examined the effects of both types of externalities on economic performance assuming this more general specification of preferences and discussed the appropriate corrective taxes. In the light of previous research emphasizing the importance of the interaction between preferences and technology, our analysis has employed the more flexible non-scale production technology. The approach we have taken is to combine the theoretical analysis with numerical simulations based on the calibration of a plausible macroeconomic growth model.

We have drawn three main sets of theoretical conclusions. First, a consumption externality in isolation has long-run distortionary effects if and only if labor is supplied elastically. This is because it impinges on the long-run equilibrium through the consumption-leisure tradeoff in utility,

countries than originally suggested, and a wider range of estimates have been obtained. For example, Islam (1995) estimates the rate of convergence to be 4.7% for non-oil countries and 9.7% for OECD economies. Evans (1997) obtains estimates of the convergence rate of around 6% per annum. Alvarez-Cuadrado et al. (2004) provide a detailed analysis of convergence speeds for the time non-separable utility function.

together with the interaction between labor and capital in production. The elasticity of labor supply thus becomes an important determinant of the empirical significance of consumption externalities. Although with fixed labor supply, consumption externalities alone have no long-run effects, they still have transitional distortionary effects and they will generate long-run distortions through their interaction with production externalities. With elastic labor supply, a negative consumption externality leads to sub-optimally large long-run capital, labor supply, output, and consumption.

Second, production externalities always generate long-run distortions, irrespective of whether or not labor supply is fixed. Thus a positive production externality leads to a sub-optimally low capital stock with under-production and under-consumption. In addition, a consumption externality will affect the potency of the production externality on long-run activity through its impact on the labor-leisure choice and thus on the marginal product of capital.

Third, we have provided a simple characterization of optimal tax policy that enables the replication of the entire optimal path. It requires that capital income be taxed or subsidized at a constant rate that corrects for the production externality, while consumption should be taxed or subsidized at a time-varying rate that corrects for the divergence between the social and private benefits from the consumption externality.

The simulations supplement these theoretical findings with important quantitative insights. Perhaps the most striking finding is the sharp contrast between the effects of the consumption and production externalities on the deviations of the decentralized economy from the optimum along the transitional paths. Finally, at several points we have compared our results with those obtained under conventional time-separable preferences. While we have found some similarities, particularly across steady states, there are also important differences, both qualitative and quantitative, particularly during transitions.

Table 1
Benchmark parameters

Production parameters	$\alpha = 1, \sigma = 0.65, \eta = -0.2, 0, 0.2,$
	$\delta = 0.05$
Preference parameters	$\beta = 0.04, \varepsilon = 2.5, \theta = 1.75, 0$
	$\rho = 0.2, 0.8, 2; \gamma = -0.5, 0.5, 0.8$
Population growth	$n = 0.015$

Table 2a: Impact of Externalities on Equilibrium: Elastic labor supply ($\theta = 1.75$)

A. Positive consumption externality ($\gamma = -0.5$)

	Decentralized Economy						Optimum					
η	k	L	y	c	y/k	c/y	k	L	y	c	y/k	c/y
-0.2	2.00	0.325	0.535	0.411	0.267	0.770	0.816	0.371	0.509	0.459	0.624	0.901
0	2.05	0.322	0.615	0.482	0.300	0.783	2.56	0.402	0.768	0.602	0.300	0.783
0.2	1.82	0.317	0.657	0.527	0.362	0.802	7.70	0.430	1.774	1.222	0.230	0.689

B. Zero consumption externality ($\gamma = 0$)

	Decentralized Economy						Optimum					
η	k	L	y	c	y/k	c/y	k	L	y	c	y/k	c/y
-0.2	1.93	0.324	0.530	0.411	0.275	0.776	0.657	0.291	0.421	0.381	0.641	0.904
0	**2.05**	**0.322**	**0.615**	**0.482**	**0.300**	**0.783**	**2.05**	**0.322**	**0.615**	**0.482**	**0.300**	**0.783**
0.2	2.01	0.319	0.698	0.554	0.348	0.794	6.39	0.355	1.414	0.956	0.221	0.676

C. Negative consumption externality ($\gamma = 0.5$)

	Decentralized Economy						Optimum					
η	k	L	y	c	y/k	c/y	k	L	y	c	y/k	c/y
-0.2	1.86	0.322	0.525	0.411	0.282	0.782	0.462	0.192	0.305	0.276	0.659	0.907
0	2.05	0.322	0.615	0.482	0.300	0.783	1.38	0.217	0.414	0.325	0.300	0.783
0.2	2.23	0.321	0.742	0.583	0.333	0.785	4.20	0.248	0.890	0.590	0.212	0.662

Table 2b: Impact of Externalities on Equilibrium: Inelastic labor supply($\theta = 0$)

A. Positive consumption externality ($\gamma = -0.5$)

	Decentralized Economy					Optimum				
η	k	y	c	y/k	c/y	k	y	c	y/k	c/y
-0.2	4.72	1.26	0.972	0.267	0.770	1.74	1.09	0.980	0.624	0.901
0	6.37	1.91	1.50	0.300	0.783	6.37	1.91	1.50	0.300	0.783
0.2	9.57	3.46	2.78	0.362	0.802	26.1	6.02	4.14	0.230	0.689

B. Zero consumption externality ($\gamma = 0$)

	Decentralized Economy					Optimum				
η	k	y	c	y/k	c/y	k	y	c	y/k	c/y
-0.2	4.57	1.26	0.975	0.275	0.776	1.69	1.08	0.978	0.642	0.904
0	**6.37**	**1.91**	**1.50**	**0.300**	**0.783**	**6.37**	**1.91**	**1.50**	**0.300**	**0.783**
0.2	10.46	3.64	2.89	0.348	0.794	28.6	6.32	4.27	0.221	0.676

C. Negative consumption externality ($\gamma = 0.5$)

	Decentralized Economy					Optimum				
η	k	y	c	y/k	c/y	k	y	c	y/k	c/y
-0.2	4.43	1.25	0.978	0.282	0.782	1.63	1.08	0.976	0.659	0.907
0	6.37	1.91	1.50	0.300	0.783	6.37	1.91	1.50	0.300	0.783
0.2	11.49	3.83	3.01	0.333	0.785	31.4	6.65	4.41	0.212	0.662

Table 3: 50% increase in Productivity α Steady-state

A. Elastic labor supply ($\theta = 1.75$)

	Decentralized Economy			Optimal		
	$\frac{\Delta\tilde{y}}{\tilde{y}} = \frac{\Delta\tilde{k}}{\tilde{k}} = \frac{\Delta\tilde{c}}{\tilde{c}}$	$\%\Delta$ in Intertemporal Welfare		$\frac{\Delta\tilde{y}}{\tilde{y}} = \frac{\Delta\tilde{k}}{\tilde{k}} = \frac{\Delta\tilde{c}}{\tilde{c}}$	$\%\Delta$ in Intertemporal Welfare	
$\eta = -0.20$	61.1	$\gamma = -0.5$ $\gamma = 0$ $\gamma = 0.5$	53.0 54.7 60.9	61.1	$\gamma = -0.5$ $\gamma = 0$ $\gamma = 0.5$	53.9 57.1 68.2
$\eta = 0$	86.6	$\gamma = -0.5$ $\gamma = 0$ $\gamma = 0.5$	64.2 69.7 82.4	86.6	$\gamma = -0.5$ $\gamma = 0$ $\gamma = 0.5$	64.2 69.7 85.7
$\eta = 0.20$	146.2	$\gamma = -0.5$ $\gamma = 0$ $\gamma = 0.5$	74.3 89.3 119.7	146.2	$\gamma = -0.5$ $\gamma = 0$ $\gamma = 0.5$	76.5 87.8 116.0

B. Inelastic labor supply ($\theta = 0$)

	Decentralized Economy			Optimal		
	$\frac{\Delta\tilde{y}}{\tilde{y}} = \frac{\Delta\tilde{k}}{\tilde{k}} = \frac{\Delta\tilde{c}}{\tilde{c}}$	$\%\Delta$ in Intertemporal Welfare		$\frac{\Delta\tilde{y}}{\tilde{y}} = \frac{\Delta\tilde{k}}{\tilde{k}} = \frac{\Delta\tilde{c}}{\tilde{c}}$	$\%\Delta$ in Intertemporal Welfare	
$\eta = -0.20$	61.1	$\gamma = -0.5$ $\gamma = 0$ $\gamma = 0.5$	52.9 54.9 63.8	61.1	$\gamma = -0.5$ $\gamma = 0$ $\gamma = 0.5$	53.9 57.1 68.1
$\eta = 0$	86.6	$\gamma = -0.5$ $\gamma = 0$ $\gamma = 0.5$	64.2 69.6 85.2	86.6	$\gamma = -0.5$ $\gamma = 0$ $\gamma = 0.5$	64.2 69.6 85.6
$\eta = 0.20$	146.2	$\gamma = -0.5$ $\gamma = 0$ $\gamma = 0.5$	74.0 89.0 121.2	146.2	$\gamma = -0.5$ $\gamma = 0$ $\gamma = 0.5$	76.7 87.6 115.3

Table 4a Short-run Effect of 50% Increase in α : Elastic labor supply ($\theta = 1.75$)

A. Positive consumption externality ($\gamma = -0.5$)

	Decentralized Economy						Optimum					
η	%pt ΔL	Δy	Δc	$\%\Delta$ y	% c	$\%\Delta$ *Welf*	%pt ΔL	Δy	Δc	$\%\Delta$ y	% c	$\%\Delta$ *Welf*
-0.2	-0.04	0.267	0.206	49.9	50.2	31.3	0.15	0.257	0.247	50.4	53.8	32.8
0	-0.71	0.295	0.254	47.9	52.7	34.2	-0.33	0.378	0.333	49.2	55.3	35.0
0.2	-0.87	0.311	0.281	47.3	53.4	35.0	-0.86	0.851	0.695	48.0	56.9	37.4

B. Zero consumption externality ($\gamma = 0$)

	Decentralized Economy						Optimum					
η	%pt ΔL	Δy	Δc	$\%\Delta$ y	% c	$\%\Delta$ *Welf*	%pt ΔL	Δy	Δc	$\%\Delta$ y	% c	$\%\Delta$ *Welf*
-0.2	1.82	0.294	0.178	55.4	43.2	36.5	1.04	0.225	0.175	53.5	46.0	42.2
0	0.95	0.325	0.224	52.9	46.4	42.8	0.95	0.325	0.224	52.9	46.4	42.8
0.2	0.29	0.355	0.271	50.9	48.9	47.7	0.49	0.725	0.460	51.3	48.1	46.2

C. Negative consumption externality ($\gamma = 0.5$)

	Decentralized Economy						Optimum					
η	%pt ΔL	Δy	Δc	$\%\Delta$ y	% c	$\%\Delta$ *Welf*	%pt ΔL	Δy	Δc	$\%\Delta$ y	% c	$\%\Delta$ *Welf*
-0.2	3.56	0.318	0.152	60.6	37.0	55.3	0.87	0.166	0.104	54.4	37.5	82.0
0	2.62	0.355	0.194	57.8	40.3	71.5	1.60	0.236	0.124	57.1	38.2	77.6
0.2	1.61	0.407	0.257	54.8	44.0	90.5	1.69	0.504	0.237	56.6	40.1	81.3

Table 4b Short-run Effect of 50% Increase in α : Inelastic labor supply ($\theta = 0$)

A. Positive consumption externality ($\gamma = -0.5$)

	Decentralized Economy					Optimum				
η	Δy	Δc	$\%\Delta$ y	$\%\Delta$ c	$\%\Delta$ *welf*	Δy	Δc	$\%\Delta$ y	$\%\Delta$ c	$\%\Delta$ *welf*
-0.2	0.630	0.486	50.0	50.0	31.1	0.545	0.522	50.0	53.3	32.6
0	0.955	0.816	50.0	54.4	33.6	0.955	0.843	50.0	56.2	34.6
0.2	1.730	1.551	50.0	55.8	34.4	3.010	2.457	50.0	59.3	36.4

B. Zero consumption externality ($\gamma = 0$)

	Decentralized Economy					Optimum				
η	Δy	Δc	$\%\Delta$ y	$\%\Delta$ c	$\%\Delta$ *welf*	Δy	Δc	$\%\Delta$ y	$\%\Delta$ c	$\%\Delta$ *welf*
-0.2	0.630	0.378	50.0	38.8	38.8	0.540	0.391	50.0	43.2	43.2
0	0.955	0.660	50.0	44.0	44.0	0.955	0.660	50.0	44.0	44.0
0.2	1.820	1.390	50.0	48.1	48.0	3.160	2.003	50.0	46.9	46.9

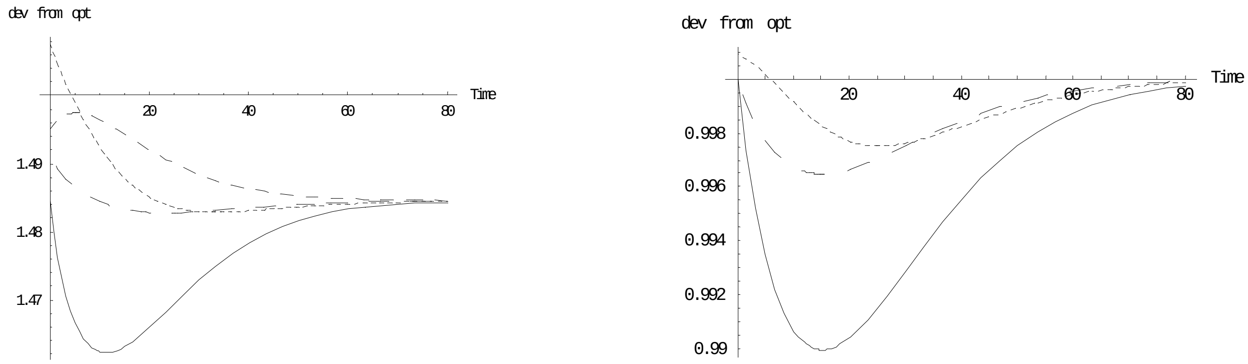
C. Negative consumption externality ($\gamma = 0.5$)

	Decentralized Economy					Optimum				
η	Δy	Δc	$\%\Delta$ y	$\%\Delta$ c	$\%\Delta$ *welf*	Δy	Δc	$\%\Delta$ y	$\%\Delta$ c	$\%\Delta$ *welf*
-0.2	0.625	0.282	50.0	28.8	66.0	0.540	0.348	50.0	35.7	84.0
0	0.955	0.515	50.0	34.3	80.4	0.955	0.513	50.0	34.2	80.1
0.2	1.505	1.207	50.0	40.1	96.3	3.325	1.588	50.0	36.0	85.0

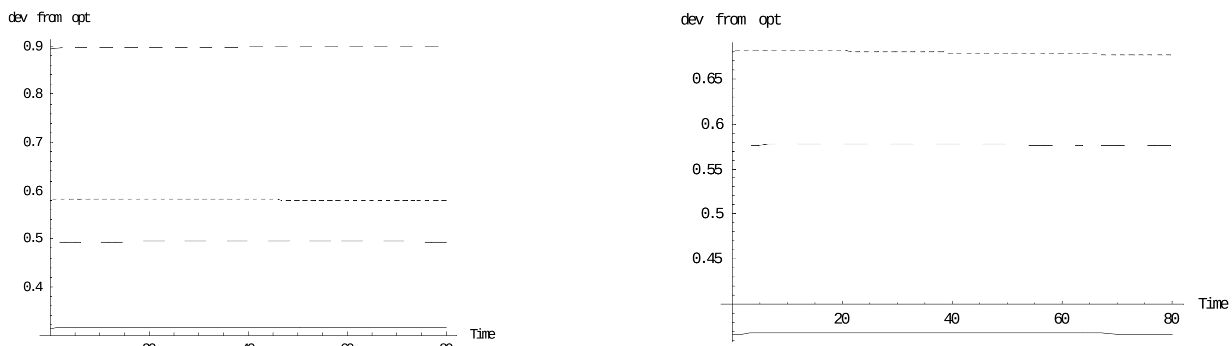
Figure 1: Increase in Productivity α

A. Elastic labor supply ($\theta = 1.75$)

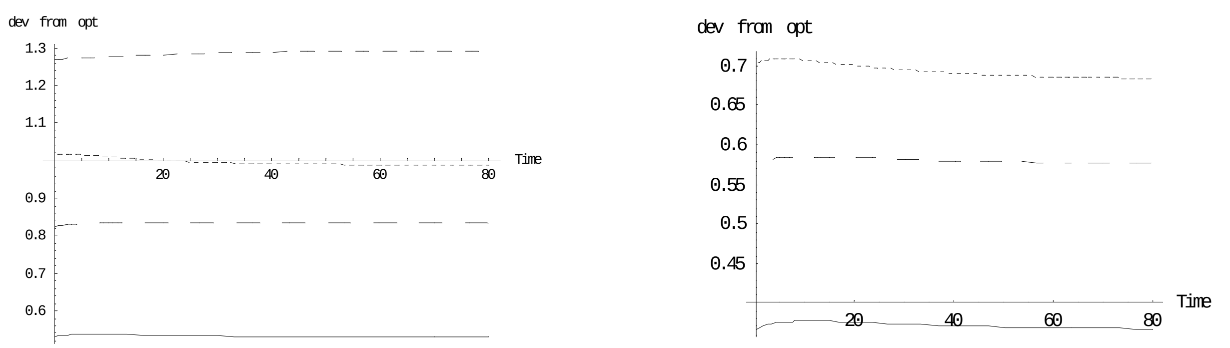
B. Inelastic labor supply ($\theta = 0$)



(i) $(\eta = 0, \gamma = 0.5)$



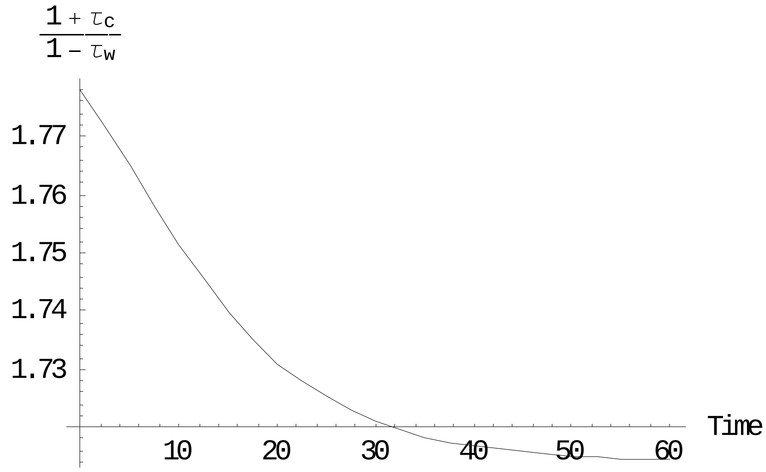
(ii) $(\eta = 0.2, \gamma = 0)$



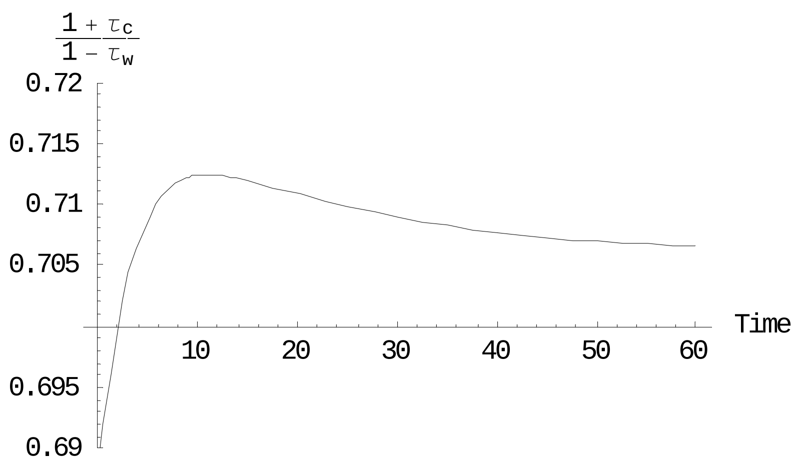
(iii) $(\eta = 0.2, \gamma = 0.5)$

——— Capital	– – – – Labor	——— Output	------- Consumption

Figure 2
Optimal Consumption/Wage Tax $(\theta = 1.75)$



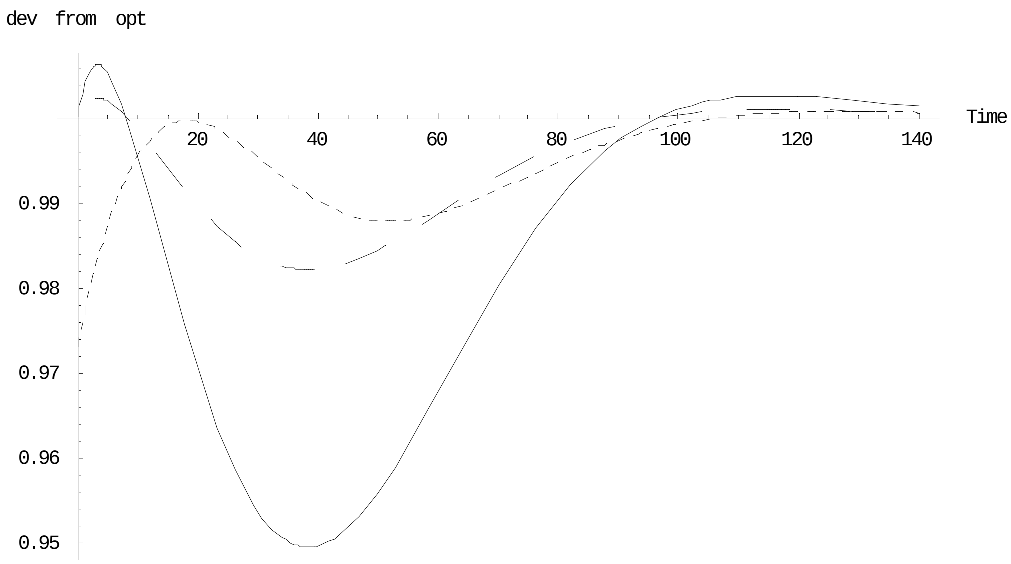
$(\eta = 0, \gamma = 0.5)$



$(\eta = 0, \gamma = -0.5)$

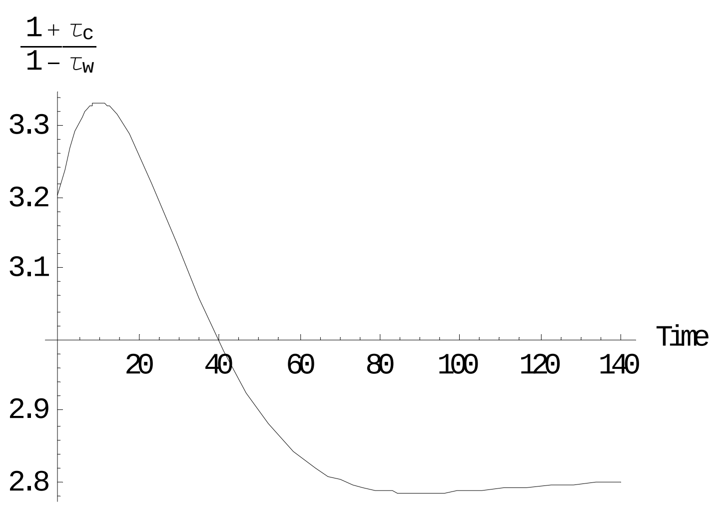
Figure 3:

A. Slower convergence



$(\gamma = 0.9, \rho = 0.1, \theta = 0)$

B. Optimal consumption/wage income tax



——— Capital	——— Output	-------- Consumption

Appendix

A.1 Derivation of Equilibrium System (13)

Using the first order condition (8a) and (8b), together with (10) we can eliminate λ and obtain the marginal rate of substitution condition, (13d). Taking the time derivative of the optimality condition (8a), the aggregate production function (9), the definitions of C_i and K_i, and the marginal rate of substitution condition (13d), yields:

$$\frac{\dot{\lambda}}{\lambda}=-\varepsilon\frac{\dot{C}_i}{C_i}+\theta(1-\varepsilon)\frac{\dot{l}}{l}-\gamma(1-\varepsilon)\frac{\dot{H}_i}{H} \tag{A.1}$$

$$\frac{\dot{Y}}{Y}=(1-\sigma+\mu)\frac{\dot{K}}{K}+\sigma n-\sigma\frac{\dot{l}}{(1-l)} \tag{A.2}$$

$$\frac{\dot{C}_i}{C_i}=\frac{\dot{C}}{C}-n \tag{A.3}$$

$$\frac{\dot{K}_i}{K_i}=\frac{\dot{K}}{K}-n \tag{A.4}$$

$$\frac{\dot{C}}{C}=\frac{\dot{Y}}{Y}+\frac{\dot{l}}{l(1-l)} \tag{A.5}$$

From these five equations, together with (8c), (3) (10), and (7) we are able to eliminate $\dot{\lambda}/\lambda$, $\dot{C}_i/C_i$, $\dot{C}/C$, $\dot{Y}/Y$, $\dot{K}/K$ and H_i/H_i reducing these equations to a unique equation in $\dot{l}$, (13c). Differentiating k and h and using (7') and (3) we obtain (13a) and (13b).

A.2 Elimination of Non-Optimal Output-Capital Equilibrium ratio

We show how there are potentially two steady-state solutions for the output-capital ratio in the centrally planned economy, although one is shown to violate the transversality condition and can thus be eliminated. We first note that setting $\dot{l}^o=0=\dot{q}^o$, the steady-state solutions for (17c) and (17d) reduce to:

$$(1-\sigma+\eta)\left[1-\varepsilon\left(1-\frac{\tilde{c}^{o}}{\tilde{y}^{o}}\right)\right]\left(\frac{\tilde{y}^{o}}{\tilde{k}^{o}}\right)-\gamma\rho\left(1-\varepsilon\right)\left(\frac{\tilde{c}}{\tilde{h}}-1\right)$$

$$-\left[\beta+\delta\left(1-\varepsilon(1-\sigma+\eta)\right)+n\left(1-\varepsilon(1-\sigma)\right)\right]=0 \qquad \text{(A.6a)}$$

$$(1-\sigma+\eta)\frac{\tilde{y}^{o}}{\tilde{k}^{o}}+\gamma\frac{\tilde{c}^{o}}{\tilde{h}^{o}}\left(\frac{1-\rho q^{o}}{q^{o}}\right)+\rho-\delta-n=0 \qquad \text{(A.6b)}$$

provided $(1-\rho\tilde{q}^{o})\neq 0$. Substituting (18a) and (18b) into (A.6a), this equation reduces to (18c) of the text, namely

$$\left(1-\sigma+\eta\right)\left(\frac{\tilde{y}^{o}}{\tilde{k}^{o}}\right)_{1}-\left(n+\delta\right)=\beta+\left(g-1\right)\left[\gamma(1-\varepsilon)+\varepsilon\right]n \qquad \text{(A.7a))}$$

Combining this with (A.6b) and (18b) can then be shown to yield (18f) of the text. Alternatively, if $(1-\rho\tilde{q}^{o})=0$, (A.6b) implies an alternative solution

$$(1-\sigma+\eta)\left(\frac{\tilde{y}^{o}}{\tilde{k}^{o}}\right)_{2}=n+\delta-\rho \qquad \text{(A.7b)}$$

We now consider the transversality condition (16e), and note that it is equivalent to

$$\hat{\lambda}_{1}+\hat{K}_{i}-\beta<0 \qquad \text{(A.8)}$$

where $\hat{\lambda}_{1},\hat{K}_{i}$ are steady-state growth rates, and are given by:

$$\left(\frac{\dot{\lambda}_{1}}{\lambda_{1}}\right)\equiv\hat{\lambda}_{1}=\beta+n+\delta-\left(1-\sigma+\eta\right)\frac{y}{k} \qquad \text{(A.9a)}$$

$$\hat{K}_{i}=(g-1)n \qquad \text{(A.9b)}$$

Substituting (A.9a), (A9b) and (A.7b) into (A8) we find $\hat{\lambda}_{1}+\hat{K}_{i}-\beta=\rho+(g-1)n$, which violates the (16e) under any plausible conditions. We can therefore we can eliminate solution (A.7b). Substituting (A.7a) $\hat{\lambda}_{1}+\hat{K}_{i}-\beta=(g-1)(1-\varepsilon)(1-\gamma)n-\beta$, which satisfies (16e) in general.

A.3. Welfare Changes as Measured by Equivalent Variations in Income Flows

We assume that the economy is initially on a balanced growth path, (indexed by *b)* which is growing at the equilibrium growth rate, gn. Recalling the definitions of the scale-adjusted variables, the corresponding level of base welfare is given by

$$\frac{1}{1-\varepsilon}\int_0^\infty \left(C_{i,b} l_{i,b}^{\theta} H_{i,b}^{-\gamma}\right)^{1-\varepsilon} e^{-\beta t} dt = \frac{c_b^{1-\varepsilon} l_b^{\theta(1-\varepsilon)} h_b^{-\gamma(1-\varepsilon)} \left(N_0\right)^{(1-\gamma)(1-\varepsilon)}}{1-\varepsilon}\int_0^\infty e^{[(1-\varepsilon)(g-1)(1-\gamma)n-\beta]t} dt \tag{A.7}$$

where c_b, l_b, h_b are the constant ratios along the initial balanced growth path, and N_0 serves as a convenient scale for the initial levels of income and consumption. Evaluating (A.7) yields the base level of intertemporal welfare

$$\frac{c_b^{1-\varepsilon} l_b^{\theta(1-\varepsilon)} h_b^{-\gamma(1-\varepsilon)} \phi^{-\gamma(1-\varepsilon)} C_0^{(1-\varepsilon)(1-\gamma)}}{(1-\varepsilon)\left[\beta-(1-\varepsilon)(g-1)(1-\gamma)n\right]} \equiv W(c_b, l_b, h_b; N_0) \equiv W_b \tag{A.8}$$

Intertemporal welfare along an equilibrium path is given by

$$\frac{1}{1-\varepsilon}\int_0^\infty \left(C_i l_i^{\theta} H_i^{-\gamma}\right)^{1-\varepsilon} e^{-\beta t} dt = \frac{\phi^{-\gamma(1-\varepsilon)} N_0^{(1-\varepsilon)(1-\gamma)}}{1-\varepsilon}\int_0^\infty c_a(t) l_a(t)^{\theta} h_a(t)^{-\gamma} e^{[(1-\varepsilon)(g-1)(1-\gamma)n-\beta]t} dt$$
$$\equiv W(c_a, l_a, h_a; N_0) \equiv W_a \tag{A.9}$$

where c_a, l_a, h_a denote the time-varying trajectories along the resulting transitional path.

As a means of comparing these two levels of utility, we determine the percentage change in the initial consumption level, N_0, and therefore in the consumption flow over the entire base path, such that the agent is indifferent between c_b, l_b, h_b and c_a, l_a, h_a. That is, we seek to find ζ such that

$$W(c_b, l_b, h_b; \zeta N_0) = W(c_a, l_a, h_a; N_0) = W_a \tag{A.10}$$

Performing this calculation yields

$$\frac{c_b^{1-\varepsilon} l_b^{\theta(1-\varepsilon)} h_b^{-\gamma(1-\varepsilon)} \left(\zeta N_0\right)^{(1-\varepsilon)(1-\gamma)}}{(1-\varepsilon)\left[\beta-(1-\varepsilon)(g-1)(1-\gamma)n\right]} \equiv \zeta^{(1-\varepsilon)(1-\gamma)} W_b = W_a$$

and hence

$$\zeta - 1 = \left(W_a / W_b\right)^{1/(1-\varepsilon)(1-\gamma)} - 1 \tag{A.11}$$

(A.11) determines the change in the base consumption (income) level, and thus in the consumption level at all points of time that will enable the agent's base level of intertemporal welfare to equal that following some structural change.

The relative welfare gain at any instant of time *t* along the transitional path (over the base level at the corresponding time) is calculated analogously, by

$$\xi - 1 = \left(Z_a / Z_b\right)^{1/(1-\varepsilon)(1-\gamma)} - 1 \tag{A.12}$$

where $Z_b(t) \equiv \left(c_b l_b^{\theta} h_b^{-\gamma}\right)^{1-\varepsilon}$, $Z_a(t) \equiv \left(c(t) l(t)^{\theta} h(t)^{-\gamma}\right)^{1-\varepsilon}$, so that

$$\zeta - 1 = \left(\left(\frac{c(t)}{c_b(t)}\right)\left(\frac{l(t)}{l_b(t)}\right)^{\theta}\left(\frac{h(t)}{h_b(t)}\right)^{-\gamma}\right)^{1/(1-\gamma)} - 1 \tag{A.13}$$

The change in instantaneous utility, (A.13), can also be written as

$$\zeta - 1 = \left(\frac{c(t)}{c_b(t)}\right)\left(\frac{l(t)}{l_b(t)}\right)^{\theta/(1-\gamma)}\left(\frac{\left(c(t)/h(t)\right)}{\left(c_b(t)/h_b(t)\right)}\right)^{\gamma/(1-\gamma)} - 1$$

which brings out the fact that as long as $\gamma \neq 0$, welfare differentials depend upon absolute as well as relative consumption (in addition to leisure).

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