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How to Compare Taylor and Calvo Contracts: A Comment on Michael Kiley

by

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How to compare Taylor and Calvo contracts: a comment on Michael Kiley*

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Abstract

In a recent paper, Michael Kiley argued that the Calvo model of price adjustment is both quantitatively and qualitatively different from the Taylor model. What we show is that Kiley (along with most other people) are choosing the wrong parameterization to compare the two models. In effect they are comparing the average age of Calvo contracts with the completed length of Taylor contracts. When we compare the average age of Taylor contracts with the average of Calvo, the differences become much smaller and easier to understand. We also show that autocorrelation of output can be larger in a Taylor economy than in the age-equivalent Calvo economy.

1 Introduction

> "Comparison of these models is simple. Suppose that the average length of time between price changes for a firm in each model are equal ($N = \pi^{-1}$), which is the standard assumption used throughout the literature"

In a recent thoughtful paper Kiley (2002), Michael Kiley compares the two standard models of price adjustment: Calvo[1] and Taylor. Kiley argues quite forcefully that whilst "previous research suggests that partial adjustment and staggering imply similar dynamics, at least in reduced form models...", he finds otherwise: "the dynamics are both qualitatively and quantitatively different across the two pricing specifications when the models are calibrated with identical frequencies of price adjustment". We show that existing comparisons between Taylor and Calvo are inconsistent and do not compare like with like[2]. Once the correct comparison is made, there are indeed differences between Taylor and Calvo, but they are much less than Kiley argues for.

[1] We exclude Rotemberg's partial adjustment model, since this applies to the aggregate price level directly and is not derived from microfoundations.

[2] See Kiley for a complete list of references.

How should we compare Taylor and Calvo contracts? There are three obvious criteria: the average length of completed contracts (the contract lifetime); the average age of contracts (age since birth); the average frequency of price adjustment. There are also more sophisticated criteria. Calvo contracts are defined by the reset probability π: Taylor contracts by their length N. You can choose some property of a specific macroeconomic model (e.g. autocorrelation in output) and choose π and N so that this property is the same for both models of price-setting. Let us go through each of the three simple criteria and see how the comparison should be made. The notation we use is exactly the same as in Kiley (2002).

1.1 The average age of contracts: A

If we consider a Taylor model with N period contracts, in which there are N equally sized cohorts, the average age of the contracts in cross-section is

$$A^T = \frac{N+1}{2} \tag{1}$$

For example, $2-$period contracts have an average age of $1.5 = (1+2)/2$.

In a Calvo model with reset probability π, the average age of contracts in steady-state is of course well known: it is the reciprocal of the reset probability:

$$A^c = \pi^{-1} \tag{2}$$

So, suppose we want to choose a pair $\{\pi, N\}$ such that the average ages are the same: $A^T = A^c$. The relationship between $\{\pi, N\}$ is

$$\pi = \frac{2}{N+1} \Leftrightarrow N = \frac{2-\pi}{\pi} \tag{3}$$

Thus, for example, suppose we have $2-$period Taylor contracts, then $\pi = 2/3$ will give the same average age (1.5). For $4-$period Taylor contracts the avaerage age is 2.5 and the corresponding reset probability $\pi = 0.4$.

1.2 The Average Lifetime of Contracts: N

What is the average lifetime of a contract in the Calvo and Taylor models? This is simple in the Taylor model: it is defined in terms of completed contract length, N. In an $N-$period contract Taylor model, we know that all contracts last for N periods. In the Calvo model, however, to our knowledge no macroeconomist has derived the distribution of completed contract lengths. We know the *age* distribution: the steady-state proportion of firms having age s is given by α_s

$$\alpha_s = \pi (1-\pi)^{s-1} : \ s = 1...\infty$$

However, for each age s, a proportion $(1-\pi)$ that do not reset prices now will go on to survive at least another another period. In fact, the hazard/reset probability is constant in the Calvo model: however old the contract, it is still expected to live for another π^{-1} periods. Since the average age is π^{-1}, it follows that the expected average lifetime of the population of contracts will be about $2\pi^{-1}$. In fact it turns out that the distribution mean is actually a little less because we have measured time in integers starting with 1. Hence we have (proof in the appendix):

Proposition 1 *With a constant hazard rate π, the steady-state distribution of completed contract lengths N is given by:*

$$\alpha_N^C = \pi^2 N \left(1-\pi\right)^{N-1} \quad :N = 1...\infty \tag{4}$$

which has mean $N^C = \frac{2-\pi}{\pi}$

So, the mean contract lifetime in a Calvo model is about twice the average age. This makes perfect sense. In steady state you will (on average) tend to observe contracts half way through their life. Note that with Taylor contracts, there is exactly the same relationship:

$$N = 2A - 1$$

For example, with two period Taylor contracts, $N = 2$ and $A^T = 1.5$. With Calvo, $A^C = \pi^{-1}$, $N^C = 2\pi^{-1} - 1$. Thus, if we seek to compare Taylor and Calvo in terms of either average age or completed lifetimes, we have exactly the same criterion: (3) above. Most researchers have wrongly compared the average age of Calvo contracts with the lifetime of Taylor contracts, which is clearly inconsistent and almost 100% off in order of magnitude.

1.3 The Average Frequency of Price Adjustment

The third criterion of comparison is the average frequency of price adjustment. We would argue that this is the least satisfactory. The key factor here is that the average frequency of price adjustment is very much affected by the *distribution* of contract lengths and their timing. In the Calvo model, the average frequency of price adjustment is a defining parameter of the model: π. With the Taylor model, the average frequency of price adjustment is N^{-1}. Clearly, if we set $\pi = N^{-1}$, then we are in effect comparing a Calvo economy with the average age of contracts π^{-1} equal to the average lifetime of the contracts in the Taylor economy. Put another way, the average lifetime in the Calvo economy N^C will be almost twice the average lifetime N in the "corresponding" Taylor economy.

$$\pi = N^{-1} \Rightarrow N^C = 2N - 1$$

How can this be possible? The explanation is simple. There is a distribution of contract lengths in the Calvo economy given by (2). Now, on average if we look at the contracts that will last for N periods, N^{-1} will reset their prices each period in steady state. That is, the longer living contracts will turn up less often because they will reset their prices less often. If a contract lasts 100 periods, over its lifetime it will have changed its price on average with a frequency 1/100.

Thus if we compare the Calvo and the Taylor economy, the average frequency of price adjustment means something very different. In the Taylor economy, all contracts have the same lifetime. In the Calvo there is a distribution, with plenty of longer contracts surviving. These longer term contracts only change price infrequently, and do not turn up in the data as changing price. Comparing the average frequency of price changes when the two cases have such different distributions is a largely meaningless exercise.

2 Explaining Kiley's Results

There are two lessons we want the reader to take from this paper:

Lesson 1 When you compare a Taylor and a Calvo economy, you must compare like with like. Choose $\{\pi, N\}$ according to (3), so that both the average age and the average lifetime are the same.

Lesson 2 If you compare Taylor and Calvo economies that have the same average age/lifetime, the differences between the two still exist, but are smaller. Calvo will tend to be more persistent. That is because although the average age/lifetime is the same, there is a distribution. As we show in Dixon and Kara (2004), for a given mean, if there are longer contracts this tends to make the economy behave with more persistence.

Taken together, these lessons support Kiley's original point: even when you get the right comparison between the two types of contract (the same average age), there is still a difference. Although Kiley is wrong in magnitude, he is right in his qualitative analysis of the differences.

2.1 Relative Price Distortions

Kiley quite rightly points out that relative price distortions are greater in the Calvo model. He derives the steady state welfare loss in Taylor and Calvo in equations (16) and (17) on page 290. He then asks the question: in this model, what values of $\{\pi, N\}$ will give the same welfare loss? He gives one pair: we also provide the corresponding average ages and lifetimes (to 2 s.f.)

$$\begin{gathered}\{\pi = 0.58, N = 4\} \\ \left\{A^C = 1.72, A^T = 2.5\right\} \\ \left\{N^C = 2.5, N = 4\right\}\end{gathered} \tag{5}$$

Now, we can see that the welfare loss is equated with a reset probability that implies a lower average age and average lifetime, but far less than Kiley thinks. This is because, along with the rest of the literature he thinks of Calvo in terms of age and Taylor in terms of lifetimes. Thus $\pi = 0.25$ gives an average age of 4 quarters, but average lifetime of 7. To get an average lifetime of 4 requires $\pi = 0.4$. Thus, Kiley is correct, but the difference between the two models is less than he states.

In fact, things become even clearer when you correctly compare the models by equating average ages. If we take Kiley's measures of the welfare loss, (15) and (16), these are the variances of prices under Calvo and Taylor.

Kiley states: " For...$N = 2$, the welfare losses under the Calvo model are eight times the losses under the Taylor specification; for $N = 3$, the losses under Calvo are nine times those under the Taylor specification, and as N increases further, the gap implied in welfare losses increases" (Kiley 2002, p. 290-291). In fact, the magnitude of the relative welfare loss is much smaller, and does not vary with N at all. Furthermore, we can find a simple relationship between the contract lengths in Taylor and Calvo that equate the welfare loss.

Proposition 2 *For all $N = 1...\infty$, The welfare loss in the Calvo specification is three times the welfare loss in the Taylor specification when we choose π to equate average age of contracts. The average lifetime N^C in Calvo that exactly equates the welfare loss for a given $N-$Taylor economy is*

$$N^C = \sqrt[+]{3N^2 - 2}$$

Proof. The welfare loss is given by[3]

$$\begin{aligned} \text{Taylor}(N) &= \frac{(N^2 - 1)}{12} \\ \text{Calvo}(\pi) &= \frac{(1 - \pi)}{\pi^2} \end{aligned}$$

[3] Without loss of generality, we set $\mu^2 = 1$.

Now, if we choose $\pi = \frac{2}{N+1}$ and substitute out π in Calvo(π), the Calvo welfare loss is defined in terms of lifetime contract length N:

$$Calvo(N) = \frac{N^2 - 1}{4}$$

which is precisely three times as large as Taylor for all N. We can then chooses N^C so that Calvo(N^C) =Taylor(N) to equate welfare losses. ■

2.2 Dynamic Response to Monetary Shocks

Kiley next goes on to look at the persistence of output and price responses to monetary shocks. He chooses the pair $\{\pi = 0.5, N = 2\}$, "the standard calibration in a calvo model that would typically be compared to a two-period staggering model". Lesson 1 tells us that this is confusing age an lifetimes. The correct comparison is the pair $\{\pi = (2/3), N = 2\}$, which gives the same average age and average life across the two models. In this case, we show Fig 1 which corresponds to Kiley's Figure 1

Figure 1 here.

The Calvo model with $\pi = 2/3$ is indeed more persistent than the Taylor model with $N = 2$, but the difference is less marked (particularly for smaller values of b). As we argue in Dixon and Kara (2004), the reason for this is that the longer term contracts present in the Calvo distribution tend to make the economy behave more persistently.

Figure 2 here

If we look at Figure 2, Kiley calculated for each Taylor model with $N = 2...8$ the value of π^{-1} that gives the same autocorrelation[4] in output, for different values of b,the elasticity of output with respect to marginal cost[5]. We have simply superimposed the equation (3) onto Kiley's figure 2. We also depict the "spurious" relation $N = \pi^{-1}$. The important point is that once you realize where the true age-equivalent line lies, all of the three functions lie close to it. They are all a long way from the $N = \pi^{-1}$ line. Furthermore, for values of $b = 2$ and $b = 1$, the difference between the autocorrelation at empirically relevant magnitudes ($N \simeq 4$) is not at all great. When $b = 0.25$, the autocorrelation is always equated with a younger Calvo economy, which is about 33% younger than the Taylor economy. So again, Kiley has got the right qualitative result, but the magnitudes are different.

[4]Note that although the autocorrelation is equated, when $b \geq 1$, in the Taylor model the output response is negative at some point. This shape of the time path makes the Taylor model less plausible when $b \geq 1$.

[5]In fact the lines were incorrectly labelled in the original article: we have corrected this.

3 Conclusion

In this paper, we argue for a consistent basis for comparing Taylor and Calvo contracts. We argue that the correct criterion is that the average age across the two should be equated, which his equivalent to equating the average lifetime. This is different to the current and almost universal norm, where the average frequency of price adjustment is used. This leads to a model where the average age of a Calvo contract is equated to the average lifetime of a Taylor contract. Yet, the average age will be almost half the average lifetime in both models. Whilst Taylor and Calvo are very different models at the conceptual level and lead to different sorts of behavior, the differences are much less than appear when you proceed from the wrong starting point.

Overall, Kiley argued that other authors were wrong to think that Calvo and Taylor are similar. In this he is certainly right: the Calvo model has a distribution of contract lengths which makes it very different form the Taylor economy where all contracts have the same length. However, he is using the standard practice of comparing the average age of the Calvo economy with the average lifetime of the Taylor economy. This makes the differences look much larger than they actually are. The conlusion we would draw for empirical work is that the Calvo model cannot be used as a simple approximation to the Taylor model, even when you choose π to equate ages. The presence of long contracts can lead to more presistence and price dispersion.

4 Bibliography

- Dixon, Huw and Kara, Engin (2004). "Persistence and Nominal Inertia in a Generalized Taylor Economy: How Longer Contracts Dominate Shorter Contracts." Mimeo.

- Kiley, Michael (2002). "Price adjustment and Staggered Price-Setting." Journal of Money, Credit and Banking 34, 283-298.

5 Appendix: Proof of Proposition 1

Proof. Suppose we want to find the proportion of contracts that last exactly N periods. Clearly, we can exclude all contracts that have lasted more than T. Consider the steady-state Calvo distribution of durations s : $\alpha_i^s = \pi (1-\pi)^{s-1} .s = 1...\infty$ The corresponding distribution of periods. For each duration s less than or equal to N, the proportion that will survive to exactly N is

$$\alpha_i^s \pi (1-\pi)^{N-s} = \pi^2 (1-\pi)^{N-1}$$

summing this over all durations $s \leq N$ yields

$$\alpha_N = \pi^2 N (1-\pi)^{N-1}$$

The mean completed contract length is

$$\begin{aligned}
\bar{N} &= \pi^2 \sum_{N=1}^{\infty} N^2 (1-\pi)^{N-1} \\
&= \pi^2 \left[1 + 4(1-\pi) + 9(1-\pi)^2 + ...T^2(1-\pi)^{N-1}...\right] \\
&= \pi^2 \left[\sum_{N=1}^{\infty} (1-\pi)^{T-1} + 3(1-\pi) \sum_{N=1}^{\infty} (1-\pi)^{N-1} + ... \right. \\
&\quad \left. + \left(N^k - N^{k-1}\right)(1-\pi)^{k-1} \sum_{N=1}^{\infty} (1-\pi)^{T-1}\right] \\
&= \pi \left[1 + 3(1-\pi) + 5(1-\pi)^2 + ... \left(2k+1\right)(1-\pi)^k ....\right)\right] \\
&= \pi \left[1 + (1-\pi) + (1-\pi)^2 + ... (1-\pi)^k ....\right] + \pi \left[\sum_{k=1}^{\infty} 2k(1-\pi)^k\right] \\
&= 1 + 2\pi(1-\pi) \sum_{k=1}^{\infty} k(1-\pi)^{k-1} = 1 + \frac{2(1-\pi)}{\pi} \\
&= \frac{2-\pi}{\pi}
\end{aligned}$$

■

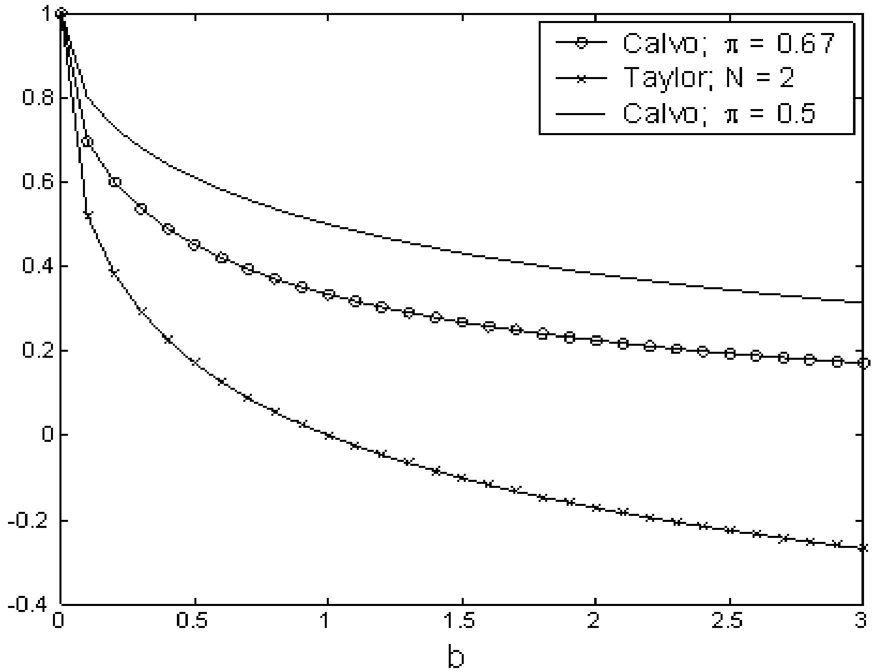

Figure 1: Persistence under Calvo and Taylor

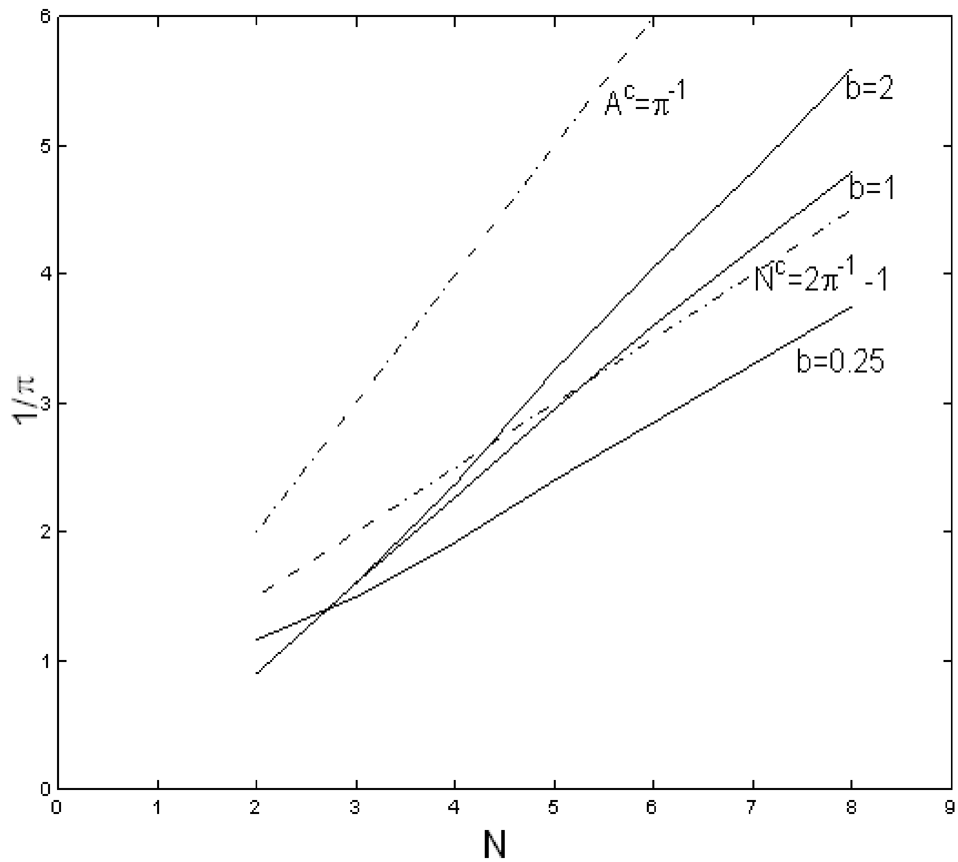

Figure 2: Correspondence between π^{-1} and Contract Length