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Horizontal and Vertical Integration in the Presence of Research Spillovers

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Abstract. We investigate how different types of merger affect input prices, research levels and equilibrium profits in vertical market structures when there is research activity in the upstream market that spills over to the downstream retailers. To do so, we develop a very simple model where three downstream Cournot oligopolists are served by monopolist plant-specific input suppliers. We consider a situation in which both vertical and horizontal integration are feasible and we investigate which equilibrium structures are likely to emerge following an initial merger between two units.

JEL classification: L13, L22, L41

Keywords: Integration, research spillovers, sequential mergers.

1. Introduction

Consider an industry with two vertically related activities, where plant-specific input suppliers sell to their respective downstream retailer, each producing a differentiated product. Firms in the downstream market compete in quantities and benefit from the research activity of their input specific supplier.

This market structure may be justified in the presence of switching costs, arising from sunk investments and asset specificities, that decrease the value of any outside option. A typical example may be a contract between a local farmer and a supermarket that enjoy a certain degree of local market power, because of the concentration of the market. The farmer specialises in the production of the products requested by the supermarket; choosing a different retailer may oblige the farmer to move towards different products and thus to face new sunk investments, whose cost would add to the fixed costs associated with the previous contract. On the other side for the supermarket choosing a new supplier, that is already locked in a contract with another supermarket, may mean a reduction in the degree of differentiation of its product with respect to its competitors. Thus our analysis would apply to situations in which both sides of the vertical structure are sufficiently

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concentrated and the trade between upstream and downstream units is characterised by strong asset specificities. This is a plausible assumption if we consider that concentration is rapidly increasing also at the retail level as the success of some supermarket chains in several countries seems to confirm, Dobson and Waterson, (1999).

The main purpose of our work is to analyse the incentives for firms to merge in this type of setting, when both horizontal and vertical integration are feasible and there are no merging restrictions. To do so, after discussing some related literature in section 2, in section 3 we develop a benchmark scenario where three oligopolist downstream units are served by their input specific suppliers. We then analyse how the equilibrium outcome changes when different types of merger occur.

Starting from a pre-merger situation, we show that there are always incentives for the first two units to merge. If a merger occurs, however, some of the outsiders will be harmed and will have therefore incentives to respond with a countermerger. This may lead to a new merger configuration where the initial merged units may be worse off; thus even if profitable when considered in isolation, an initial merger might not be carried through if it encourages subsequent mergers that have a negative effect on the profitability of the initial merged units.

In sections 4 and 5, we consider two different initial mergers and we investigate which equilibrium market structure is likely to emerge when both horizontal and vertical integration are possible. We don't consider as a possible outcome the case in which all the upstream/downstream units horizontally integrate, because this would lead to excessive concentration.

Suppose that the initial merger is between a downstream unit and its specific input supplier. In section 4, we show that this type of merger is always profitable for the participants, but always unprofitable for the outsiders. Thus there exist incentives for the excluded firms to react by merging. The main purpose of section 6 is to show how an initial vertical merger may trigger successive mergers by the firms left out. If all the

units vertically merge, then the initial merged units are worse off, but if the products are sufficiently differentiated and the externality effects are not too strong, then the profits of the industry as a whole can be shown to increase. We also consider the case, in which, in response to a vertical merger, the downstream outsiders react by merging. This leads to a new merger configuration where mergers are always profitable for the initial vertically integrated unit but likely to be unprofitable for the suppliers to the downstream participants; there are therefore incentives for these firms to merge in turn. If a merger between these units occurs then the initial vertically integrated unit still gains, with respect to a pre-merger situation, but the profitability of the downstream merged units decreases. Anticipating the reactions of their suppliers, the downstream merged units might try to preempt this merger by vertically integrating. This leads to the last merger configuration where all the units are integrated vertically and the initial downstream outsiders are also integrated horizontally. Thus an initial vertical merger could lead to the following new mergers configurations:

a) three vertical mergers: following a vertical merger, the downstream outsiders respond by integrating vertically with their input specific suppliers

b) one vertical merger and one downstream merger: following a vertical merger, the downstream outsiders integrate horizontally

c) vertical integration and downstream and upstream mergers: following a vertical merger, both the downstream and upstream outsiders integrate horizontally

d) downstream mergers and vertical integration: following a vertical merger, the downstream merged units vertically integrate with their input specific suppliers

Suppose alternatively that the initial merger is between two downstream units. In section 5, we show that this type of merger is always profitable for its participants but likely to be unprofitable for their suppliers, who might have therefore incentives to respond by coordinating their price and research decisions. This may lead to a new merger configuration where the initial participants are worse off. When this happens and if there are no

merging restrictions, the initial participants might try to vertically integrate to preempt a merger between their input suppliers. In this new merger configuration, however, the outsiders would be in a position of a clear disadvantage and would have therefore incentives to integrate in turn. Thus an initial horizontal merger between two downstream units could lead to the following new merger configurations:

e) downstream and upstream mergers: following a downstream merger, the input suppliers of the participants respond by merging

f) downstream mergers and vertical integration: the downstream merged units integrate vertically with their input specific suppliers

g) downstream mergers and vertical integration: following a vertical merger between the downstream units with their input specific suppliers, the outsiders react by integrating vertically.

In these contexts, vertical integration gives rise to three different gains:

- it internalises the pricing externality and the research spillover and hence it lowers the cost for the integrated units and induces more research
- it may preempt a horizontal merger between the suppliers of the downstream merged units (only in merger configurations c and f)
- it avoids the losses coming from being non-integrated after a merger between the downstream merged units with their input specific suppliers (in merger configurations g)

A downstream horizontal integration instead is associated with the following gains:

- it allows the merged units to benefit from an additional research/demand induced externality effect. This is modelled by assuming that the research activity of each participant's supplier spills over to the other participant

- it reduces the monopoly power of the input specific suppliers and hence it may involve lower input prices but also lower research for the participants (in merger configuration b)

- it avoids the losses from not being integrated after a vertical merger but may trigger a horizontal merger between input suppliers (in merger configurations b and g respectively)

There are then indirect effects whose nature and intensity vary with the merger configuration. For instance, when only vertical integration is feasible, the internalisation of the research spillover induces more research from the integrated units but the internalisation of the pricing externality involves a cost advantage that modifies the incentives for these firms to decrease research when competition on the final market becomes fiercer. We will show that these incentives are stronger when competition is less fierce and the demand spillovers are larger. This suffices to erode the competitive advantage of the integrated units as the gains from the internalisation of the pricing externality decrease.

The intensity of this effect however depends on the number of units that integrate vertically.

When all the units integrate vertically, the gains from the internalisation of the pricing externality are higher and therefore the incentives to decrease research stronger. When only one unit integrates vertically instead the cost advantage for the integrated unit is determined by the input price faced by the non-integrated units, which is lower. Thus the incentives to decrease research when competition gets fiercer are now smaller and mergers are always profitable for the participants.

When only horizontal integration in the downstream market is feasible, the participants benefit from a merger induced externality effect that reduces the incentives for the outsider's supplier to decrease research (and input prices) when competition on the final market gets fiercer. Thus, the downstream outsider always faces worse cost conditions,

only partially compensated by more intense research activity from its supplier, when the margins to profitably increase final prices are lower. This suffices to give the participants a competitive advantage and to reduce the demand losses for the downstream outsider, who may now gain from a merger. A horizontal merger in the downstream market however tends to decrease the monopoly power of the participants' suppliers; thus, following a downstream merger, the participants' suppliers will reduce research and input prices, unless, as we will see, the products are strongly differentiated. In this case, in fact, stronger merger induced externality effects tend to intensify the research activity of the participants' suppliers thus allowing them to set higher prices for their inputs and to earn higher profits. If however the products are not too differentiated the participants' suppliers are forced to decrease research and input prices with negative effects on their profitability. In this case, they might have incentives to react by merging to recover the lost profits.

The main purpose of section 7 is to show how a horizontal merger in the downstream market may trigger successive mergers by the firms left out. We first investigate the case in which, following a downstream merger, the input specific suppliers respond by coordinating their price and research decisions. This leads to a new merger configuration, where, due to the internalisation of the research spillover, the upstream participants produce more research and set higher input prices, but only if the merger induced externality effect is sufficiently strong or if the products are sufficiently differentiated. When both the degree of product differentiation and the merger induced externality are sufficiently strong, mergers may still be profitable for the downstream participants. When the degree of product differentiation decreases, competition on the final market gets fiercer and all the input suppliers are induced to decrease their research intensity. Since however the downstream participants benefit from an additional demand enhancing effect, the incentives to decrease research are stronger for the upstream participants; thus research and therefore demand tend to decrease more for the downstream participants but without inducing a corresponding decrease in input prices. This obviously contributes negatively

to the profitability of the downstream participants.

If there were no restrictions to merging, both the upstream and downstream participants could earn higher profits by vertically integrating. This type of merger in fact gives rise to two different gains: a demand enhancing gain, induced by the horizontal merger in the downstream market, and the internalisation of the pricing externality, induced by the vertical integration of the downstream participants with their input specific suppliers. As a result, research activity and demand increase for the participants but fall for the downstream outsider. Since however now the outsiders are in a position of clear disadvantage, they might have incentives to respond by integrating vertically. This leads to the last merger configuration where the only element of differentiation between the participants is the merger induced externality effect. Thus the problem reduces to the analysis of the effects of a horizontal merger when there are merger induced gains and all the players face identical cost conditions. It is well known from the literature that, in the absence of spillovers, mergers of this type are always more profitable for the outsiders, Salant et al., (1983), Deneckere and Davidson, (1985). Thus the strength of the research spillover will be decisive in determining which players earn more.

In section 8 we identify which equilibrium merger configurations are likely to emerge: we show that when both the research spillover and the merger induced effect are sufficiently strong, horizontal integration is always more profitable for the initial merged units; when instead the merger induced externality is weak, the initial merged units might earn more by integrating vertically. In both scenarios, however, complete vertical integration is never an equilibrium outcome when both horizontal and vertical integration are feasible.

In section 9 we conclude and we make some suggestions for future research.

2. Related literature

Most of the existing studies on vertical integration assume that upstream units have all the market power and make take-it-or-leave-it offers to downstream firms. For instance, Ga-lor, (1990), and Jansen, (2003), analyse the conditions under which integration and

separation coexist in equilibrium, when there is a limited number of upstream firms and many downstream units with zero reservation payoffs. Ga-lor considers a market of n producers, each producing a differentiated product and facing a linear demand function. Firms compete in prices, and each producer may decide whether to sell directly to consumers or to delegate the sales to an agent, selected from a large population of interested individuals. If the contracting costs are not too high, then the unique equilibrium is for each producer to be represented by an agent. Coexistence of integration and separation is never an equilibrium, since the benefits from contracting with an agent are increasing with the number of producers who choose to be represented. In quantity setting games, instead, such coexistence may occur, Jansen, (2003), if there are no vertical externalities among vertically separated firms.

Hart and Tirole, (1990), O'Brien and Shaffer (1992), McAfee and Schwartz (1993), using general demand functions, show how the adoption of unobservable contracts, when an upstream monopolist sells to many downstream units, gives rise to a commitment problem that reduces the monopoly power of the upstream producer.

The assumption that upstream units have all the market power may be justified when the upstream market is much more concentrated than the downstream market, where the entry barriers would be therefore so low that all the downstream profits would be extracted. Casual empiricism however suggests that concentration is increasing also at the retail level and the success of some supermarket chains in several countries would seem to confirm this tendency, Dobson and Waterson, (1999).

Recent studies have therefore moved in the opposite direction, assuming that retailers have all the market power and may therefore propose contracts that have the effect of reducing competition in the product market, Shaffer, 1991.

All these studies however focus on the welfare implications of different types of contracts when the bargaining power is concentrated on only one side of the vertical structure. We are instead interested in the profitability of different types of mergers when each

downstream unit is engaged in a bilateral monopoly situation with its independent input supplier.

There is an extensive literature identifying the incentives for firms to vertically integrate when both sides of the market are sufficiently concentrated. For instance, Greenhut and Ohta, (1976), show that successive vertical mergers between monopolist input suppliers, with fixed proportion coefficients, and Cournot-oligopolist retailers provide participants with greater profits and costumers with greater output and lower prices. Similar welfare gains are attained in the case of Cournot-oligopolist input suppliers, Greenhut and Ohta, (1979). The typical scenario is that of two vertical related activities, where m oligopolist firms in the upstream market produce a homogenous input, used by n downstream oligopolist units to produce a final good, according to a linear technology. The final market demand is a general decreasing function of the market price and involves negligible cross elasticities of demand. In this context, if $l \leq \min(m, n)$ units vertically integrate, the equilibrium price decreases, while the equilibrium quantity increases and the participants are always better off than any outsider. A relevant question is whether the welfare effects of vertical integration are related to the assumptions on the production technology and market structure. This issue has been investigated by Abiru, (1988), who extends the model of successive Cournot oligopolies to the case of a final market demand with constant elasticity and a C. E. S. production function. He shows that, also in this case, the effect of vertical integration of downstream firms with upstream units is to lower the final product price and therefore to increase the equilibrium quantity.

An interesting issue that emerges from this literature is that of endogenous vertical structures. Greenhut and Ohta (1979) show, for the case of vertically connected Cournot duopolies, that, when there is an equal number of upstream and downstream firms, integration is the equilibrium structure: starting from a pre-merger situation, since vertical integration increases the profits of the first merging units, there is an incentive to integrate vertically. As a result of this merger, however, all the outsiders and the industry

as a whole would suffer from a decrease in profits. This would create new incentives for the excluded firms to merge in order to recover the lost profits. Thus, in the absence of antitrust regulations, the equilibrium outcome would be a situation in which all the firms integrate vertically, the individual profits are lower than in a pre-merger situation and it is not possible to go back to this better state unless all the firms simultaneously and cooperatively disintegrate. When however there are unequal numbers of firms in the two markets, Abiru et al., (1998), show that complete vertical integration is the unique equilibrium structure only when the number of upstream units exceeds the number of downstream units.

This framework also allows us to analyse the profitability implications of horizontal mergers when input prices are endogenous. While there is an extensive literature on the effects of horizontal mergers when firms face constant marginal costs of production, there has been very little concern on how downstream mergers may influence the pricing behaviour of the input suppliers and therefore the competition between downstream units in oligopolist markets. There are however some remarkable exceptions[1]. For instance, Lommerud et al., (2003), analyse the effects of a horizontal merger in a quantity setting game with three downstream firms producing differentiated products when the input suppliers are organised in three different structures. They distinguish between firm-specific, plant-specific and industry-specific input suppliers and show that, for the case of plant-specific input suppliers, a horizontal merger between two downstream units, by inducing lower input prices, may be profitable for values of the parameters for which, with exogenous prices, it would be unprofitable. This type of merger however tends to reduce the profits of the suppliers to the merged units and may therefore create incentives for these units to

[1] Ziss (2001), using a model of Cournot competition with homogenous products, shows that the profitability of a horizontal merger may be enhanced by delegating the output decision to a manager, with an appropriate incentive scheme. Since the incentive scheme is endogenous and therefore affected by the merger, it may be assimilated to the case of endogenous input prices. Horn and Wolinsky, (1988), consider a bilateral monopoly model to analyse how input prices and profits are affected by different structures of the upstream and downstream market. These studies however focus on the profitability of single mergers considered in isolation and don't investigate the incentives for the firms left out to react with a countermerger.

merge in turn, with negative effects on the profitability of the initial merged units.

Our work is also related to the literature on sequential mergers when there are no vertical relations among firms. For instance, Nilssen and Sorgard, (1998), using a linear Cournot model with an exogenously fixed number of firms, show how an initial merger may trigger or preempt subsequent mergers. Gowrisankaran, (1996), analyses how mergers may affect entry in an industry where merger, entry, exit and production decisions are made in each period.

To our knowledge, the only model studying sequential mergers when both horizontal and vertical integration are possible is Colangelo's, (1995), who analyses preemptive merging in two different game settings: *a*) a model where an upstream monopolist, U_1, sells an input to two downstream firms, D_1 and D_2, each producing a differentiated product; *b*) a model where the same downstream units, D_1 and D_2, are served by an upstream Bertrand duopoly, U_1 and U_2. In the first stage of game *a*, firms U_1 and $D_1(D_2)$ bid for firm $D_2(D_1)$; the outcome of the bid game determines which type of merger will take place (either vertical or horizontal integration). In the second stage of the game, the input supplier sets the input price, while in the third stage the downstream units compete in prices. Game *b* has the same time structure as game *a*, but, because of the presence of two upstream suppliers competing in prices, three possible merger configurations may emerge from the bid game; *i*) one vertical integration; *ii*) one upstream merger; *iii*) one downstream merger. Colangelo finds that vertical mergers always preempt horizontal merger in the first game, but that horizontal mergers prevail over vertical integration in the second game when the products are close substitutes. Further, when the excluded firms may respond to an initial merger with a subsequent countermerger, complete vertical integration may be an equilibrium outcome only when all the firms bid for an upstream unit.

3. A Benchmark

We consider a very simple model in which the final market is served by three firms producing a differentiated product. Let q_i denote the quantity produced and sold by firm

$i = 1, 2, 3$. We assume that all the downstream firms face a constant and identical production cost, equal to C_D. For simplicity, but without loss of generality, we set $C_D = 0$. The demand facing the individual downstream firm is:

$$p_i = A_i - q_i - b\sum_{j=1}^{3} q_j,\ i \neq j,\ b \in (0, 1) \tag{1}$$

with $A_i = A + B_i x_i$, where x_i is the research activity of the plant specific supplier to improve the quality of its input and $0 \leq B_i \leq 1$ is a parameter reflecting the ability of the individual firm to transform the input into a higher quality final product, for which the representative consumer is willing to pay more.

Alternatively we could think of x_i as the advertising effort of each plant specific input supplier which tends to increase the market size of the downstream retailers.

We consider a very simple production function, in which one unit of input is required to produce one unit of output. Let w_i be the cost per unit of input for the i^{th}downstream firm and suppose that the plant specific suppliers bear all the research/advertising costs; these are assumed to be quadratic, x_i^2.

Then the individual downstream firm i chooses quantity to maximise:

$$\pi_{iD} = \left(A_i - q_i - b \sum_{j=1, i\neq j}^{3} q_j - w_i \right) q_i$$

and each plant specific input supplier i, knowing $q_i = q_i(w_i, x_i)$, chooses price w_i and research effort x_i to maximise:

$$\pi_{iU} = w_i q_i - x_i^2$$

To better clarify the configuration of the game, we offer a graphical representation of the relationships between downstream and upstream units, Figure 1.

This is a three-stage game with the following time structure:

in stage 1, the firm specific input suppliers choose independently the research efforts;

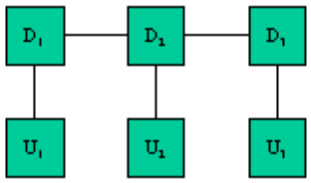

Figure 1: Benchmark

in stage 2, the same units set their input prices;

in the third and last stage, the independent downstream units compete a la Cournot on the final product market. We solve the game using backward induction.

Solving for the non-cooperative Nash equilibrium in the retail market, assuming that all the firms have identical abilities, $B_i = B$ for all $i = 1, 2, 3$, we get the following candidate equilibrium quantities:

$$q_1 = \frac{A(b-2) - bB(x_2 + x_3 - x_1) - b(w_3 + w_2 - w_1) - 2Bx_1 + 2w_1}{b^2 - b - 2}$$

$$q_2 = \frac{A(b-2) - bB(x_1 + x_3 - x_2) - b(w_1 + w_3 - w_2) - 2Bx_2 + 2w_2}{b^2 - b - 2}$$

$$q_3 = \frac{A(b-2) - bB(x_1 + x_2 - x_3) - b(w_1 + w_2 - w_3) - 2Bx_3 + 2w_3}{b^2 - b - 2}$$

Given the equilibrium quantities, the firm specific input suppliers set prices so as to maximise profits;

$$w_1 = \frac{A(2b + 8 - 3b^2) - b^2B(x_1 + x_2 + x_3) + Bx_1(6b + 8) - 2bB(x_2 + x_3)}{4(3b + 4)}$$

$$w_2 = \frac{A(2b+8-3b^2) - b^2B(x_1+x_2+x_3) + Bx_2(6b+8) - 2bB(x_1+x_3)}{4(3b+4)}$$

$$w_3 = \frac{A(2b+8-3b^2) - b^2B(x_1+x_2+x_3) + Bx_3(6b+8) - 2bB(x_1+x_2)}{4(3b+4)}$$

We can now solve for the first stage of the game, where the firm specific input suppliers, given the equilibrium quantities and input prices, set simultaneously and non cooperatively the research effort so as to maximise profits. Solving the first order conditions of the individual input suppliers' profits with respect to x_i, we get the following equilibrium research efforts:

$$x_1 = x_2 = x_3 = -\frac{(b+2)(b^2-6b-8)BA}{(b+2)(b^2-6b-8)B^2 + 32(3b+4)(b+1)}$$

where

$(b+2)(b^2-6b-8)B^2 + 32(3b+4)(b+1) > 0$

hence the equilibrium profits are:

$$\pi_{1D}^C = \pi_{2D}^C = \pi_{3D}^C = 16\frac{(b+2)^2(3b+4)^2A^2}{\left[(b+2)(b^2-6b-8)B^2 + 32(3b+4)(b+1)\right]^2}$$

$$\pi_{1U}^C = \pi_{2U}^C = \pi_{3U}^C = \frac{-A^2(b+2)\left[(b+2)(b^2-6b-8)^2B^2 + 32(b+1)(b-2)(3b+4)^2\right]}{\left[(b+2)(b^2-6b-8)B^2 + 32(3b+4)(b+1)\right]^2}$$

where $(b+2)(b^2-6b-8)^2B^2 + 32(b+1)(b-2)(3b+4)^2 < 0$ for $B \leq 1$

4. One vertical merger

In this section we consider the case in which, starting from a pre-merger situation, one downstream unit integrates vertically with its input specific supplier, Figure 2. We find that this merger is always profitable for the participants and always unprofitable for the

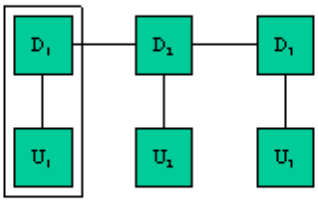

Figure 2: One vertical merger

outsiders, Figure 3. A vertical merger in fact gives rise to two different gains: it internalises the pricing externality and hence it creates a cost advantage for the integrated unit; it also internalises the research spillover and thus induces more research from the integrated unit.

Before discussing the profitability implications of this merger, we briefly describe the structure of the game. Let π_{V1} denote the post-merger profits of the unique vertically integrated unit;

$$\pi_{V1} = p_1 q_1 - x_1^2$$

while the profits of the other downstream units and their input specific suppliers are respectively given by:

$$\pi_{D2} = (p_2 - w_2)q_2$$

$$\pi_{D3} = (p_3 - w_3)q_3$$

$$\pi_{U2} = w_2 q_2 - x_2^2$$

$$\pi_{U3} = w_3 q_3 - x_3^2$$

Starting from the third stage of the game, the vertically integrated unit and the independent downstream firms choose simultaneously and independently quantities so as to maximise their individual profits. Given the candidate equilibrium quantities, in the second stage of the game, the two upstream outsiders choose input prices; in the first stage of the game, the same upstream units and the initial integrated unit, knowing $q_i = q_i(x_i)$ and $w_i = w_i(x_i)$, choose the profit maximising levels of research, x_i.

This merger is profitable for the participants if $\pi_{V1} > \pi_D^C + \pi_U^C$. As can be seen in Figure 3, which represents the locus $\pi_{V1} = \pi_D^C + \pi_U^C$, as long as $B \leq 1$, this merger is always profitable for the participants and always unprofitable for any outsider. This happens because the internalisation of both the pricing and the research externality allows the integrated unit to set a lower final price and to increase research. Thus, research and demand increase for the integrated unit. The independent downstream firms respond by decreasing their prices, but by a smaller amount, since they have still to pay the input price, which however decreases. Lower input prices for the downstream outsiders however imply lower research from their suppliers. Thus research and therefore demand decrease for the downstream outsiders and this obviously contributes negatively to the profitability of both the downstream and upstream outsiders.

4.1. Stackelberg oligopoly. Assume that the firm that vertically integrates with its specific input supplier is able to identify the reaction functions of the non colluding firms and therefore, in the absence of antitrust regulations, to lead them accordingly. For the case of quantity competition, the problem of the merged units is:

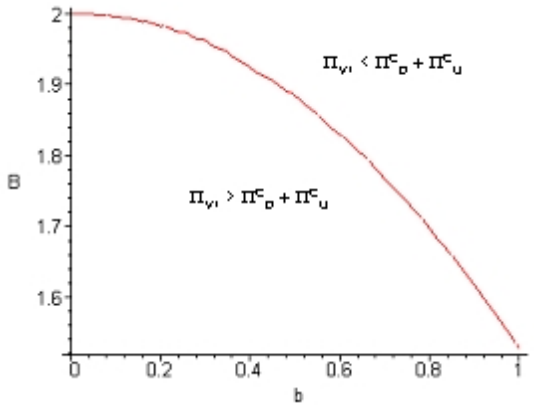

Figure 3: Merger profitability for the unique vertically integrated unit

$$\max_{q_1} \pi_1 = (A + Bx_1 - q_1 - b\,(R_2(q_1) + R_3(q_1)))q_1 - x_1^2$$

where $R_2(q_1)$, $R_3(q_1)$ denote the reaction functions of the non-colluding firms.

The first order condition of the integrated unit determines the candidate equilibrium quantities that may be used to derive input price, research and profit levels in the asymmetric Nash Equilibrium. More specifically, given the candidate equilibrium quantities, the upstream outsiders set the input prices so as to maximise their individual profits; replacing the candidate equilibrium quantities and input prices into the profits of the integrated unit and the upstream outsiders, the Nash equilibrium research levels may be found solving the first order conditions of these units' maximisation problem.

We find that quantity leadership in the downstream market tends to increase (decrease) the incentives for the integrated unit (the upstream suppliers) to do research when competition on the final market gets fiercer. Thus, when both the externality effect and the degree of substitutability are very strong, the competitive advantage of the leader is so strong that the outsiders are driven out of the market. In this case, we have a corner solution where the upstream outsiders don't produce research (see Appendix A.2.). This is shown in Figure 4, where the reaction function of the upstream supplier, R_O, is flatter

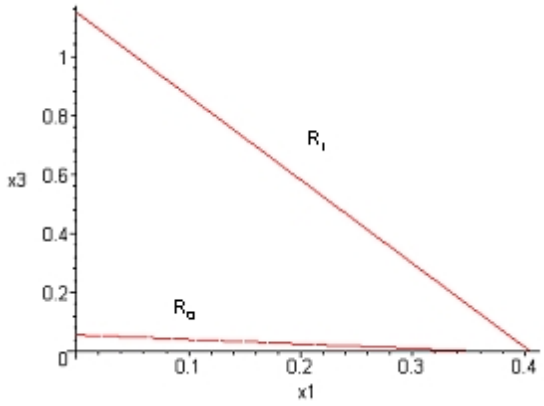

Figure 4: Nash equilibrium research levels when both the externality effect and the degree of substitutability are very strong

and lies below the reaction function of the merged units, R_I. The equilibrium is at the point at which the steeper reaction function cuts the horizontal axis; at this point the independent upstream units don't produce research; this in turn decreases the willingness to pay for the products of the non-colluding downstream units to such a level that their demand falls to zero.

Figure 5 shows the combinations of externality effect and degree of product differentiation for which the non-integrated units are driven out of the market.

In our computations, we assumed that the costs to produce the input are so low with respect to the research cost that they can be set equal to zero. This is however without loss of generality: the results wouldn't change if we assumed that there is a positive constant cost of production per unit of input, (see Appendix A.2.).

Thus, when the integrated unit behaves as a Stackelberg leader with respect to its downstream competitors, no subsequent merger will follow to an initial vertical merger, if the products are close substitutes and the research spillover is sufficiently strong.

5. A downstream merger

In this section we consider the situation in which the initial merger is between two downstream units. More specifically, assume that firms 1 and 2 merge in the downstream

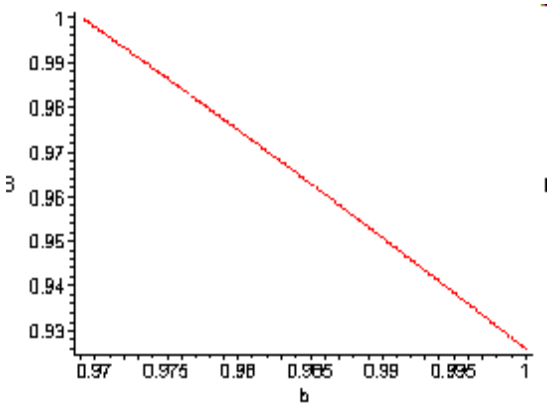

Figure 5: At any point above the curve the non-integrated units are driven out of the market

market; given our assumption of firm-specific input suppliers, the upstream firms serving firms 1 and 2 after the merger will both continue to serve the merged entity.

Assume also that the merger process increases the ability of the individual participants to improve the quality of their final product, because of the presence of learning by doing effects. We model this by assuming that the research effort of each plant-specific input supplier serving the participants spills over to the other participant. Suppose that the magnitude of the spillover effect, a, is the same for both the participants. Thus, the demands facing each individual participant are given by:

$$p_1 = A + Bx_1 + ax_2 - q_1 - b(q_2 + q_3)$$

$$p_2 = A + Bx_2 + ax_1 - q_2 - b(q_1 + q_3)$$

where $0 \leq a \leq 1$

and in the post-merger game, the merged entity chooses q_1 and q_2 to maximise:

$$\pi_{D(1,2)} = (p_1 - w_1)q_1 + (p_2 - w_2)q_2$$

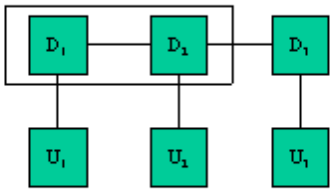

Figure 6: A downstream merger

while the other firms continue to act independently and maximize:

$$\pi_{D3} = (p_3 - w_3)q_3$$

$$\pi_{U1} = w_1 q_1 - x_1^2$$

$$\pi_{U2} = w_2 q_2 - x_2^2$$

$$\pi_{U3} = w_3 q_3 - x_3^2$$

For the sake of clarity, we offer a diagram of the merger configuration, Figure 6.

This is a three-stage game with the following time structure:

- In stage 1, the independent input specific suppliers choose the research effort, x;

- In the second stage, the same firms set the input prices, w;

- In stage 3, the independent downstream unit and the merged entity compete in quantities on the final market.

Solving the non-cooperative game by using backward induction, we find that this merger is always profitable for the participating firms and it is always more beneficial to participate in a merger, rather than being an outsider. This is in accordance with the results found by Lommerud et al., (2003), who also consider a downstream merger between two firms served by firm specific input suppliers but in the absence of externality effects. In their model, this happens because a merger induces the input suppliers of the participants to set lower input prices; since the inputs are complements, also the outsider's input specific supplier responds by setting a lower input price, though higher than for the participants: this means that input prices are always lower for the participants and this obviously contributes positively to the profitability of a merger, thus eliminating any incentive to free-ride. In our model, however, the driving force is the change in research incentives induced by a merger; more specifically, the participants benefit from a demand enhancing externality effect that reduces (increases) the incentives for the outsider's supplier (participants' suppliers) to decrease research (and input prices) when competition on the final market gets fiercer.

Thus a downstream merger tends to induce more research from the supplier to the downstream outsider, unless the products are strongly differentiated when the merger induced externality is sufficiently strong; only in this case, the incentives to do research are stronger for the suppliers to the merger's participants. Higher research from the participants' suppliers will induce, (due to the research spillover and the merger induced externality, a), stronger demand for the participants, thus allowing their input suppliers to set higher prices for their inputs and earn positive profits. Since however the degree of product differentiation is very high, competition on the final market is less fierce, and the participants may compensate for higher costs of production with higher final prices without suffering significant demand losses. Because of the strategic substitutability between research levels, the outsider's input supplier will respond by decreasing its research effort; lower research from the outsider's supplier will in turn induce lower input demand from

its customer thus leading the outsider's supplier to set a lower input price. This means that, when competition on the final market is less fierce, the downstream outsider faces lower input prices but benefits from lower research induced externality effects and this suffices to give the participants a merger advantage.

As the degree of substitutability increases, however, price competition on the final market becomes fiercer, and all the suppliers will have incentives to reduce research, in order to deliver lower input prices to their purchasers. These incentives however are stronger for the participants' suppliers; and the reason is that a reduction in research intensity has a less strong negative impact on the input demand of the participants, who, due to the merger, benefit from an additional demand enhancing externality effect. Thus when the degree of product differentiation is not too strong, a merger will induce the participants' suppliers to decrease research and set lower input prices. Since research levels are substitutes, the outsider's supplier will now respond by increasing both research and input price levels; this implies that when competition gets fiercer the outsider is faced with worse price conditions only partially compensated by more intensive research activity in the upstream market.

We now analyse the profitability of this merger for the outsiders. To do that we have to distinguish between high and low research/demand side spillover effects.

For $0 < B \leq 0.5$, mergers are profitable for the independent downstream firm at any point between the two branches of Figure 7 ; for $0.5 < B \leq 1$, mergers are profitable at any point below the curve in Figure 8. Thus if the horizontal merger induced externality is not too strong, mergers of this kind are always profitable for the downstream outsider only if the products are sufficiently differentiated. This is not surprising if we consider that the outsider faces worse price conditions when competition on the final market becomes fiercer. When the research spillover, B, is not too strong, the incentives for the outsider's supplier to reduce research, as both b and a increase, are weaker; this means that as the degree of product differentiation decreases, the outsider will face higher input prices

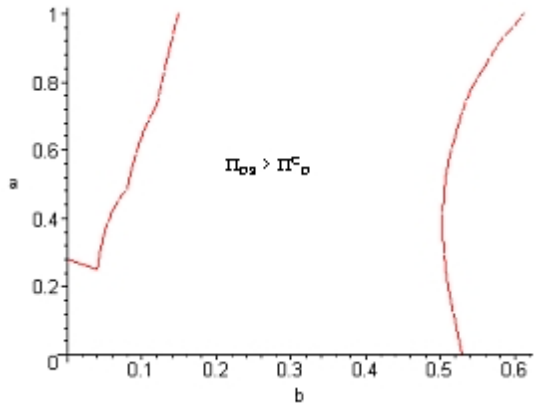

Figure 7: Merger profitability for the independent downstream firm, $B = 0.3$

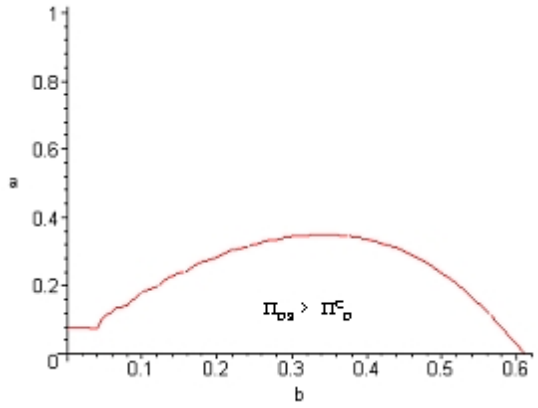

Figure 8: Merger profitability for the independent downstream firm, $B = 1$

but, given the weakness of the research spillover, only modest research induced demand increases.

When the research spillover is sufficiently strong, the incentives for the outsider's supplier to reduce research are stronger; in this case, the outsider faces slightly lower input prices that positively contribute to their profitability when price competition becomes fiercer. Since however now the participants benefit from stronger research induced demand increases, mergers may be profitable for the downstream outsider only if the merger induced externality is sufficiently weak.

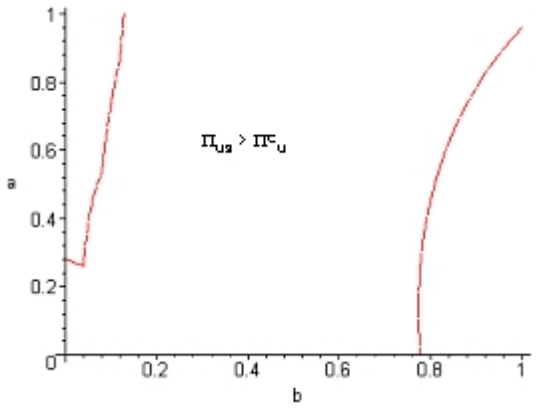

Figure 9: Merger profitability for the upstream outsider serving the independent downstream firm, $B = 0.3$

If we now turn to analyse the profitability of this merger for the firms operating in the upstream market, we can observe that this merger is always profitable for the supplier to the downstream outsider unless the merger induced externality is sufficiently strong when the degree of product differentiation is very low, Figures $9 - 10$; in this case in fact the incentives to increase research for the outsider's supplier are still enough weak. When the products are close substitutes and the merger induced effect is weak, the input price of the outsider's supplier tends to increase while its research activity tends to decrease; this in turn decreases the demand of the downstream outsider and therefore the profitability of the input supplier.

Mergers of this type are profitable for the upstream firms serving the merged entity only if the merger induced externality and the degree of product differentiation are sufficiently strong, Figures 11 and 12. According to the existing literature, with plant-specific input suppliers, mergers of this type are always unprofitable for the suppliers to the merged entity: as a result of a downstream merger in fact the input demands become more price responsive, thus inducing the suppliers to set lower input prices. A downstream merger also induces a reduction of output and therefore of input demand from the merged firms. If the degree of product differentiation however is very strong (thus implying weaker

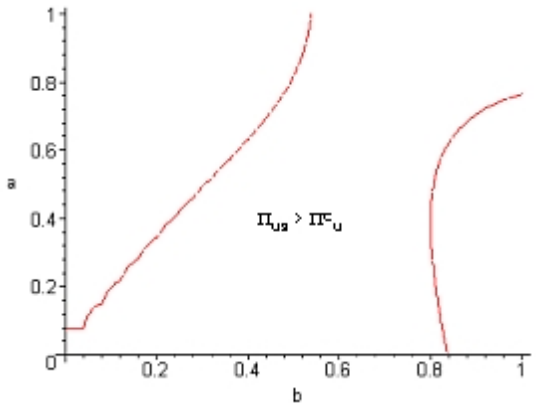

Figure 10: Merger profitability for the upstream outsider serving the independent downstream firm, $B = 1$

responses from the outsiders), when there are positive merger induced externalities (which tend to counteract the merger induced reduction in the output for the merged unit), these effects tend to be softened thus allowing the input suppliers to make gains from the merger. As competition on the final market gets fiercer the incentives to decrease research are stronger for the participants' suppliers; thus demand tends to decrease more for the insiders and only stronger merger induced externalities may counteract this effect.

It may be interesting to observe that under this merger configuration, when the integrated units behave as Stackelberg leaders with respect to their competitor, the outsiders are never driven out of the market. This happens because as competition on the final market becomes fiercer the supplier to the downstream outsider is induced to decrease research less than the participants' suppliers.

6. Other merger configurations

The main purpose of this section is to investigate which alternative merger configurations may emerge in response to an initial vertical merger. We consider four possible scenarios:

- following a vertical merger, all the outsiders respond by integrating vertically. This leads to a new merger configuration in which the initial merged units are worse off.

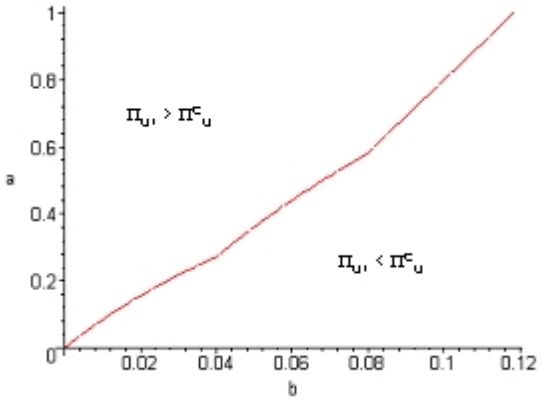

Figure 11: Merger profitability for the upstream outsiders serving the merged unit, $B = 0.3$

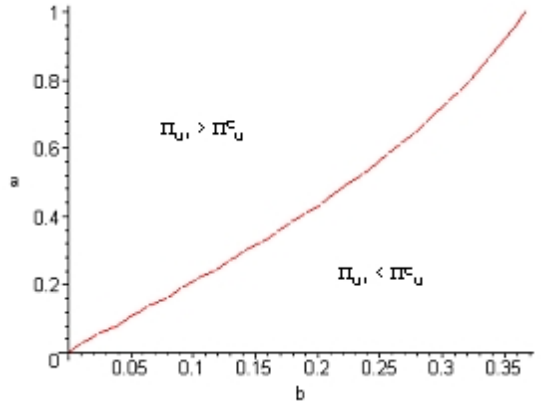

Figure 12: Merger profitability for the upstream outsiders serving the merged firms, $B = 1$

If however the products are sufficiently differentiated and the research spillovers are not too strong, all the units are better off with respect to a pre-merger situation; in this case the initial merged units would still have incentives to integrate vertically

- following a vertical merger, the downstream outsiders respond by merging. In this scenario mergers are always profitable for the initial merged units and likely to be profitable for the downstream participants. Since however this merger configuration may further reduce the profitability of the suppliers to the downstream participants, we also consider the case in which

- following a vertical merger, both the downstream and upstream outsiders respond by horizontally integrating. This new merger configuration while being extremely unfavourable for the downstream participants doesn't significantly improve the profitability of the upstream participants. Thus both the merged units might have incentives to integrate vertically. This leads to a new merger configuration where

- all the units are integrated vertically but the initial downstream outsiders are also integrated horizontally. In this case the only element of differentiation between the participants is the merger induced externality. Thus the problem reduces to the analysis of the effects of a horizontal merger when there are merger induced gains and all the players face identical cost conditions. It is well known from the literature that, in the absence of spillovers, mergers of this type are always more profitable for the outsiders, Salant et al., (1983), Deneckere and Davidson, (1985). Thus the strength of the research spillover will be decisive in determining which players earn more.

6.1. Three vertical mergers. Consider now a situation in which, following the initial vertical merger, the outsiders respond by vertically integrating. In this case a completely integrated structure emerges, Figure 13, where the initial merged units are worse off. If

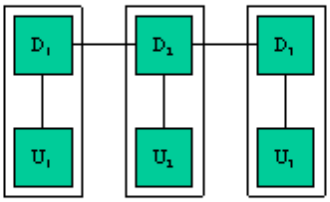

Figure 13: Three vertical mergers

however the products are sufficiently differentiated and the research spillover is not too strong, all the units are better off with respect to a pre-merger situation.

The objective function of the individual post-merger unit in this case is given by:

$$\pi_{iV} = \left(A + Bx_i - q_i - b \sum_{j=1, i \neq j}^{3} q_j \right) q_i - x_i^2$$

for $i = 1, 2, 3$.

Each integrated entity thus chooses quantity and research effort so as to maximise individual profit levels given the quantity and research decisions of the other integrated units. Solving backwards the two stage game, we find that both research and quantity levels are higher than when firms act independently. The internalisation of the pricing externality in fact lowers the cost for the integrated units, who may now set lower prices and devote more resources to research, with positive effects on their final demand.

These mergers are profitable if $\pi_{iV} > \pi_{iD}^C + \pi_{iU}^C$. In Figure 14, we show the locus of the points for which $\pi_{iV} = \pi_{iD}^C + \pi_{iU}^C$. At any point below this locus mergers are profitable. Notice that this happens only for very low degrees of substitutability and if the ability of the firms to transform the intermediate good into a higher quality final product is not

too strong. Notice also that in the absence of research spillovers the profitability of such mergers would always be positive for $b < 0.35$. Thus the presence of research induced externality effects tends to decrease the profitability of vertical integration.

The intuition behind these results is simple. Due to the internalisation of the pricing externality, the integrated units face better cost conditions that decrease their incentives to do research when competition on the final market gets fiercer. Stronger research spillovers tend to increase the competitive advantage of the integrated units and therefore to reinforce these incentives.

We first notice that, when the products are sufficiently differentiated, competition on the final market is less fierce and this allows the input suppliers to set higher input prices and research efforts. When instead the products are less differentiated, input demands are more responsive to input prices and the input suppliers are induced to set lower prices and therefore to decrease research. This means that, as the degree of product differentiation decreases, the gains from the internalisation of the pricing externality tend to decrease and this changes the incentives to do research for the integrated units. When competition is less fierce, the cost advantage of the integrated units is stronger and lower research intensity has a stronger negative impact on the demand of the independent units, who therefore, following an increase in competition, will decrease their research effort by less. When however competition becomes fiercer, the gains from the internalisation of the pricing externality are smaller and this induces the integrated units to respond by increasing their research activity. Thus the incentives to reduce research for the integrated units are stronger when competition on the final market is less fierce and weaker when the gains from the internalisation of the pricing externality get smaller; this means that research and therefore demand decrease more for the integrated units when they face better cost conditions and this suffices to erode their competitive advantage when competition gets fiercer.

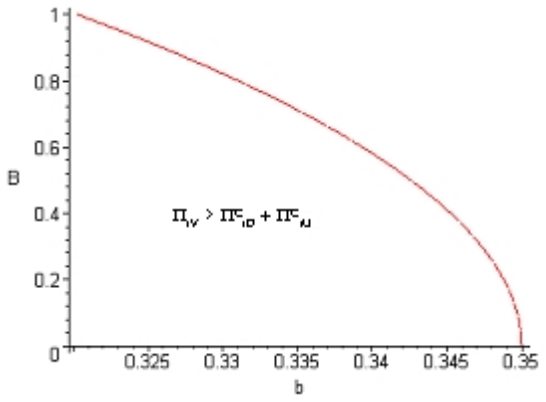

Figure 14: Merger profitability of vertical mergers

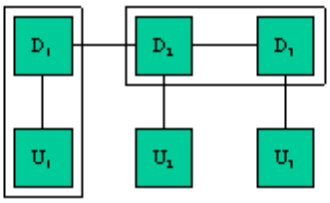

Figure 15: Vertical and downstream horizontal mergers

6.2. Vertical and downstream horizontal mergers. Consider now a situation in which in response to an initial vertical merger, the downstream outsiders respond by merging, Figure 15.

The profits of the post-merger units may be expressed as:

$$\pi_{V1} = (A + Bx_1 - q_1 - b(q_2 + q_3))q_1 - x_1^2$$

$$\pi_{D(2,3)} = (A + Bx_2 + ax_3 - q_2 - b(q_1 + q_3) - w_2)\, q_2 + (A + Bx_3 + ax_2 - q_3 - b(q_1 + q_2) - w_3)\, q_3$$

while the upstream outsiders continue to act independently:

$$\pi_{U2} = w_2 q_2 - x_2^2$$

$$\pi_{U3} = w_3 q_3 - x_3^2$$

In the first stage of the game, the vertically integrated unit and the independent input suppliers set research; in the second stage the suppliers to the downstream merged units set input prices; in the last stage of the game the integrated units compete in quantities.

In this context, the initial merged unit still benefits from the internalisation of both the input price and the research spillover and may therefore increase research with positive effects on its final demand and profitability. The downstream merged units instead benefit from a merger induced externality that, though reinforcing the positive effect of research on their final demand, tends to decrease the incentives for their suppliers to do research when competition on the final market gets fiercer. Thus research and demand tend to decrease for the downstream participants, unless the merger induced externality is sufficiently strong when the products are strongly differentiated. In this case, stronger merger induced externalities induce more research also from the suppliers to the downstream participants and this obviously contributes positively to the profitability not only of the downstream units but also of their input suppliers, who may now set higher input prices.

When however the degree of product differentiation decreases, competition on the final market gets fiercer and the upstream suppliers reduce their research intensity while the vertically integrated unit, internalising the research spillover, intensifies its research activity; thus demand decreases for the downstream participants and increases for the integrated unit. This suffices to make these mergers always profitable for the initial

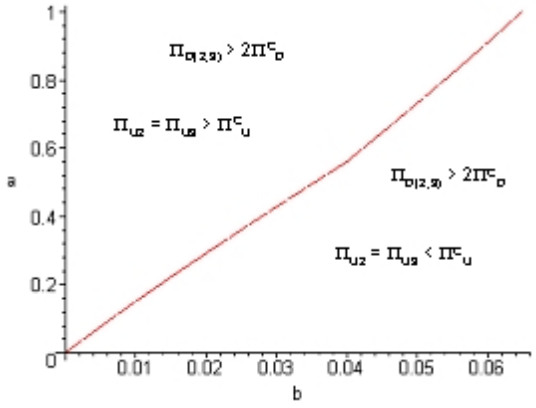

Figure 16: Merger profitability for the downstream participants and their suppliers, $B = 0.3$

integrated unit. Whether the downstream participants earn or lose with respect to a pre-merger situation, it will depend instead upon the strength of the research spillover: if the research spillover is sufficiently weak, the incentives for the upstream suppliers (vertically integrated unit) to decrease (increase) research are weaker and mergers are always profitable for the downstream participants, Figure 16. When instead the research spillover is sufficiently strong, the upstream suppliers (vertically integrated unit) have stronger incentives to decrease (increase) research and mergers may be profitable for the downstream participants only for sufficiently strong horizontal merger induced effects and if the products are sufficiently differentiated, Figure 17.

6.3. Vertical and downstream/upstream mergers. We now consider a situation in which, following an initial vertical merger, both the downstream and upstream outsiders respond by integrating horizontally. This is a reasonable reaction if we consider that a downstream merger is likely to be unprofitable for the suppliers to the downstream participants. In Figure 18, we provide a representation of this new merger configuration.

In this new context, the post merger profits are:

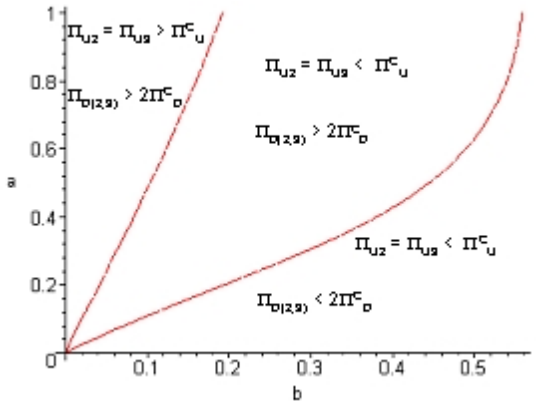

Figure 17: Merger profitability for the downstream participants and their suppliers, $B = 1$

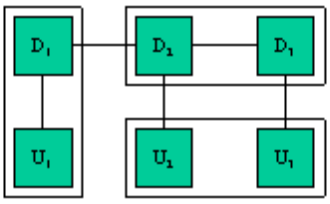

Figure 18: Vertical and downstream/upstream mergers

$$\pi_{V1} = (A + Bx_1 - q_1 - b(q_2 + q_3))q_1 - x_1^2$$

$$\pi_{D(2,3)} = (A + Bx_2 + ax_3 - q_2 - b(q_1 + q_3) - w_2)\, q_2 + (A + Bx_3 + ax_2 - q_3 - b(q_1 + q_2) - w_3)\, q_3$$

$$\pi_{U(2,3)} = w_2 q_2 - x_2^2 + w_3 q_3 - x_3^2$$

In the third stage of the game the vertically integrated unit and the downstream merged units set quantities so as to maximise profits; in the second stage, the upstream merged units, given the candidate equilibrium quantities, set input prices; finally, in the first stage, knowing the candidate equilibrium quantities and input prices, the upstream participants and the vertically integrated unit choose the profit maximising level of research.

In this context, mergers are always profitable for the initial integrated unit. As in the previous scenario, in fact, the internalisation of the pricing externality creates a cost advantage for the vertically integrated units that allows them to profitably increase research. The downstream merged units benefit as before from an additional demand enhancing externality that however reduces the incentives for their now merged suppliers to do research when competition on the final market gets fiercer. An important difference with respect to the previous merger configuration is that now the research activity of the suppliers to the downstream participants is higher; this is due to the internalisation of the research spillover induced by an upstream merger, that tends to increase the research activity of its participants. Since however now the upstream suppliers coordinate their price and research decisions, their downstream customers cannot shift production between their two goods to induce fiercer price competition and therefore lower input prices from their suppliers. Thus input prices tend to increase more than justified by the increase in research and this obviously contributes positively to the profitability of the upstream merged units

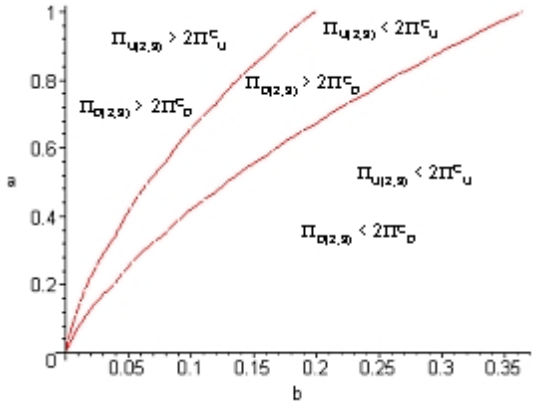

Figure 19: Merger profitability for the downstream and upstream participants, $B = 0.3$

but tends to decrease the profitability of the downstream participants. This is shown in Figures 19 and 20, where, for both weak and strong research spillovers, mergers are profitable for the downstream participants only for sufficiently strong values of both the merger induced externality and the degree of product differentiation. It may be also interesting to observe that in contrast to the previous merger configuration, stronger research spillovers tend now to increase the profitability of both the downstream and upstream participants, Figure 20; this happens because, by internalising the research spillover, an upstream merger induces its participants to increase research as the research spillover gets stronger.

6.4. Vertical and horizontal integration. We now consider the case in which in response to a vertical merger, the downstream merged units vertically integrate with their specific input suppliers.

The merger configuration is given in Figure 21. With respect to the previous scenario, where the downstream merged units were in a position of clear disadvantage, when all the units integrate vertically, the only element of differentiation is the merger induced externality. Thus the problem reduces to the analysis of the effects of a horizontal merger when there are merger induced gains and when all the players face identical cost conditions.

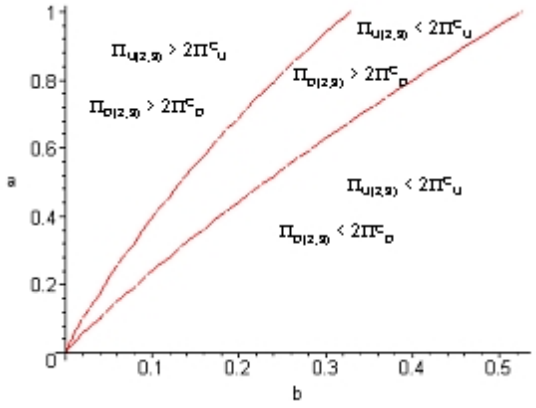

Figure 20: Merger profitability for the downstream and upstream participants, $B = 1$

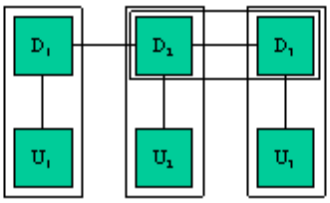

Figure 21: Vertical and horizontal integration

It is well known from the literature that when firms compete in quantities and there are no efficiency gains, horizontal mergers are always profitable for the outsiders and may be profitable for the participants only if the products are sufficiently differentiated. We will show that the presence of a merger induced externality effect, by limiting the supply response of the outsider, may make these mergers more profitable for the participants.

In this context, the integrated units set research and quantity levels to maximise their individual profits; these are given by:

$$\pi_{V1} = p_1 q_1 - x_1^2$$

$$\pi_{V(2,3)} = p_2 q_2 + p_3 q_3 - x_2^2 - x_3^2$$

This is a two stage game between two vertically integrated entities, which presents the following time structure: in stage 1, the two merged units choose simultaneously and independently the research levels, x_1, x_2 and x_3; in stage 2, the two merged units choose simultaneously and independently the quantity levels, $q_1$and q_2, q_3.

Solving the model by backward induction, we find that, for sufficiently strong research spillovers, $B \geq B^*$and $a \neq a^*$, we have a corner solution where the initial integrated unit doesn't produce research and is driven out of the market, (see Appendix A.7.).

For $B \leq B^*$, all the units produce research and the relative merger profitability depends on the strength of the merger induced effect, a.

Figure 22 shows that, in the absence of research spillovers, these mergers are profitable for the new merged units only if the products are sufficiently differentiated, if $b \leq 0.363$. For $b > 0.363$, these mergers may be profitable for the new merged units only if the merger induced externality effect is sufficiently strong; for stronger research spillovers, however, lower merger induced externalities may guarantee a positive profitability, Figure 23.

In the absence of research spillovers, mergers of this type are always profitable for the initial vertically integrated unit; when there are research induced spillovers instead these mergers are profitable only if the merger induced externality is sufficiently weak, Figure 24. Notice also that stronger research spillovers must be compensated by weaker merger induced effects, for this merger to be profitable for the initial integrated unit, Figure 25.

Thus, mergers of this type are more profitable for the new merged units if the merger induced externality and the ability of firms to transform the input into a higher quality product are sufficiently strong. The intuition behind these results is very simple: due to

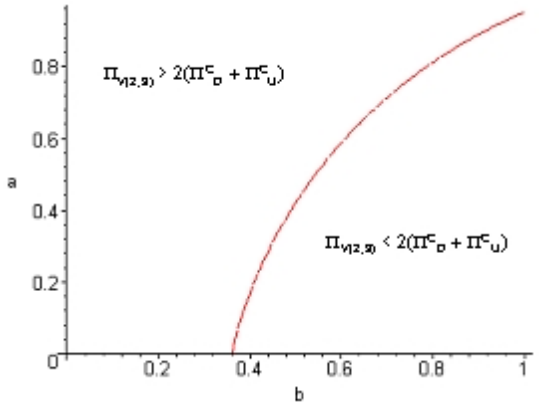

Figure 22: Merger profitability for the new merged units, $B = 0.3$

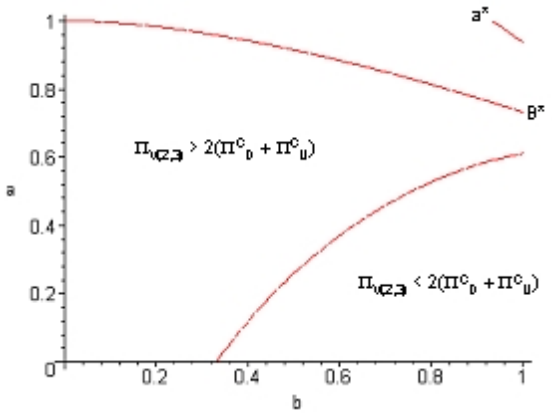

Figure 23: Merger profitability for the new merged units, $B = 1$

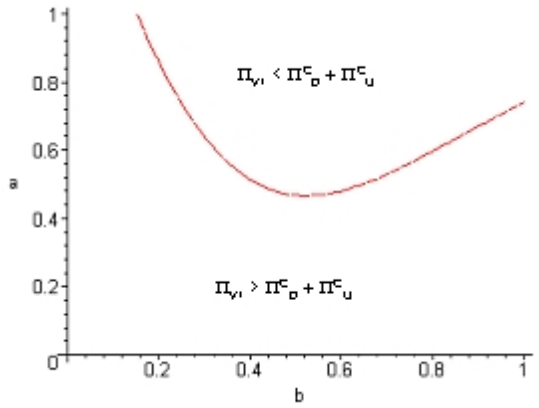

Figure 24: Merger profitability for the initial integrated unit, $B = 0.3$

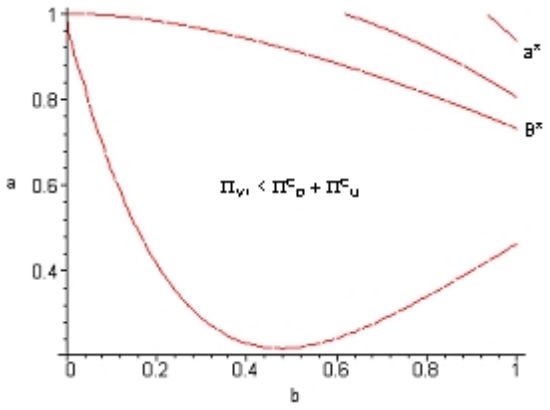

Figure 25: Merger profitability for the initial integrated unit, $B = 1$

the internalisation of the research spillover induced by vertical integration, both the units increase research. Since however the horizontally integrated units also benefit from an additional demand enhancing externality, the incentives for these units to do research are different.

For very low values of the horizontal merger induced externality, the horizontally integrated units produce more research than the initial integrated unit only if the products are sufficiently differentiated; for stronger values of the merger induced effect, instead, research levels are always higher for the new merged units. As the degree of substitutability increases, however, the incentives to reduce research are always stronger for the new merged units; thus demand tends to decrease more for these units and only stronger merger induced spillovers may counteract this effect. Stronger research spillovers, B, tend to induce more research and thus are more beneficial to the new integrated units, while an increase in the degree of substitutability induces less research from the horizontal merged units, thus resulting more beneficial to the initial integrated unit. Further, for sufficiently strong externality effects, the initial integrated unit may be driven out of the market when the products are close substitutes.

7. SUBSEQUENT MERGERS

In this section we investigate which merger configurations are likely to emerge in response to an initial horizontal merger in the downstream market.

As shown in section 5, a downstream merger may be unprofitable for the participants' suppliers, who may therefore have incentives to react by coordinating their price and research decisions. This would lead to a new merger configuration, in which the downstream participants may be worse off. Anticipating this, the downstream participants might try to integrate vertically to preempt a horizontal merger between their input specific suppliers. A merger of this type however would reduce the profitability of the outsiders and provide them too with incentives to integrate vertically. In this section, we briefly analyse these new merger configurations.

7.1. Downstream and upstream mergers. We consider a situation in which, in response to a downstream merger, the input specific suppliers of the merged entity react by merging. We will show that this type of merger reduces the profitability of the downstream participants and may provide both the upstream and the downstream merged units with incentives to integrate vertically. In Figure 26, we provide a representation of this merger configuration.

In this situation, the post-merger downstream and upstream units respectively maximise:

$$\pi_{D(1,2)} = (p_1 - w_1)q_1 + (p_2 - w_2)q_2$$

$$\pi_{U(1,2)} = w_1 q_1 + w_2 q_2 - x_1^2 - x_2^2$$

while the other firms continue to act independently and maximise:

$$\pi_{D3} = (p_3 - w_3)q_3$$

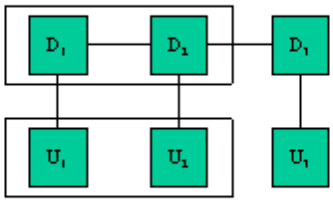

Figure 26: Downstream and upstream mergers

$$\pi_{U3} = w_3 q_3 - x_3^2$$

This is a three stage game with the following time structure:

In stage 1, the upstream units, the merged units and the independent unit, choose simultaneously and independently the research levels, x_1, x_2 and x_3.

In stage 2, the same units choose simultaneously and independently the input prices, w_1, w_2 and w_3.

In stage 3, the downstream units, the merged entity and the independent unit, choose simultaneously and independently quantity levels, q_1, q_2 and q_3.

Solving the model by backward induction, we find that these mergers are profitable for the downstream participants and their suppliers only if both the externality effects and the degree of product differentiation are sufficiently strong, Figures 27 and 28 .

The intuition for this result may be better understood by considering how this merger affects the research (and price) decisions of the suppliers to the downstream participants.

We first notice that because of the internalisation of the research spillover induced by an upstream merger, research for the upstream participants is now always higher than for the case of only one downstream merger. With respect to a pre-merger situation, however,

the incentives for the upstream participants to increase research depend on the strength of the merger induced externality, a. For low values of a, the upstream participants produce more research only when the products are sufficiently differentiated. In this case, lower degrees of product differentiation makes competition on the final market fiercer and this leads all the input suppliers to decrease research. Since however the downstream participants benefit from an additional demand enhancing externality effect, the incentives to reduce research are stronger for their suppliers; thus research and therefore demand decrease more for the downstream participants and this obviously negatively contributes to their profitability. Stronger merger induced externalities tend to increase the research effort of the upstream participants, with respect to a pre-merger situation, but don't alter the incentives for these firms to decrease research when competition on the final market gets fiercer; thus, also in this case, the supplier to the downstream outsider will have weaker incentives to reduce research. The change in research incentives induced by a merger has also important implications on the input prices. Following a merger, equilibrium input prices increase for both the insiders and the outsider, but the downstream merged units face higher input prices than their rival. The interesting feature is that even if the upstream participants have stronger incentives to reduce research when competition becomes fiercer, lower research from these firms doesn't induce a corresponding decrease in input prices; thus input prices tend to decrease less than research and this negatively contributes to the profitability of the downstream participants. The intuition behind this result is very simple: now the upstream suppliers are coordinating their price decisions and therefore the downstream participants cannot shift production between their two goods to induce lower input prices from their suppliers.

We have seen that with respect to the situation in which only two downstream firms merge, the upstream firms are better off if they respond by merging; this reaction however is likely to reduce the profitability of the downstream units, while not significantly improving the profitability of their suppliers. We therefore wonder whether it might be

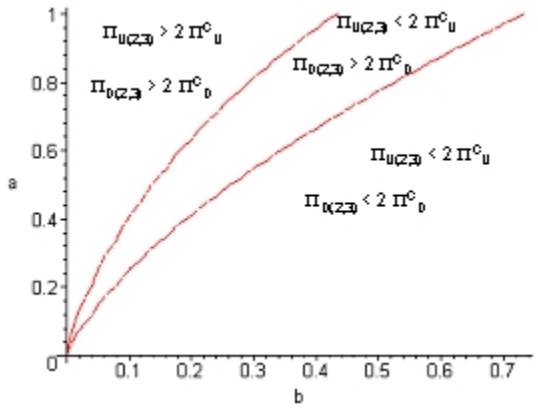

Figure 27: Merger profitability for the downstream and upstream participants, $B = 0.3$

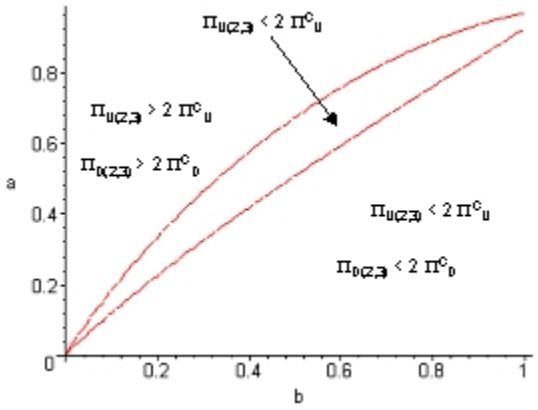

Figure 28: Merger profitability for the upstream and downstream participants, $B = 1$

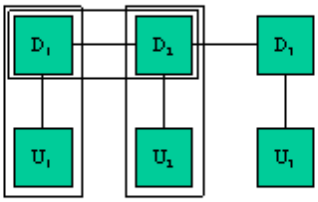

Figure 29: Downstream mergers and vertical integration

profitable for the two separated merged entities to vertically integrate.

7.2. Downstream mergers and vertical integration. In this section we consider the case in which the downstream merged units integrate vertically with their input specific suppliers. A graphical representation of this merger configuration is offered in Figure 29. In section 4, we have shown that one vertical merger between one downstream unit and its input specific supplier is always profitable for the participants but always unprofitable for the firms left out. We will now show that when both horizontal and vertical integration are feasible, these results are reinforced: not only mergers are always profitable for the participants, but if the externality effects are sufficiently strong when the products are close substitutes the outsiders may be driven out of the market. In this new context, in fact, mergers give rise to two different gains: a demand enhancing effect, induced by the merger in the downstream market, and the internalisation of the pricing externality, induced by the vertical integration of the downstream merged units with their suppliers.

The problem of the post-merger entity is to maximise:

$$\pi_{V(1,2)} = p_1 q_1 + p_2 q_2 - x_1^2 - x_2^2$$

where:

$$p_1 = A + Bx_1 + ax_2 - q_1 - b(q_2 + q_3) \tag{2}$$

$$p_2 = A + Bx_2 + ax_1 - q_2 - b(q_1 + q_3) \tag{3}$$

while the other firms continue to act independently. The independent downstream firm maximises:

$$\pi_{D3} = (p_3 - w_3)q_3$$

with

$$p_3 = A + Bx_3 - q_3 - b(q_1 + q_2) \tag{4}$$

while its input specific supplier maximises:

$$\pi_{U3} = w_3 q_3 - x_3^2$$

This is a three stage game with the following time structure:

In the first stage, the independent upstream firm and the integrated unit choose simultaneously and independently the level of research so as to maximise profits; in the second stage, the independent upstream firm chooses the input price for its purchaser; and finally, in the third stage, the integrated unit and the independent downstream firm set quantities.

Solving the model by backward induction, we find that only the equilibrium variables of the merged entity are increasing in the merger-induced externality, a. The effect of an increase in B is more ambiguous: research activity increases for both the units, but more for the participants; this has a positive effect on the individual demands but a negative effect on the input price of the independent unit.

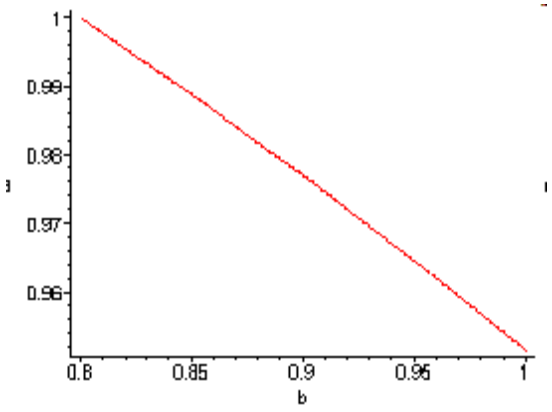

Figure 30: At any point above the curve, the outsiders are driven out of the market, $B \geq 0.9$

This merger has two positive effects on the profitability of the participants: the internalisation of the input cost and the externality effect arising from the horizontal merger. As a result, research activity and demand increase for the participants and fall for the outsider. Thus it is not surprising to find that mergers of this type are always profitable for the participants and always unprofitable for the outsiders, regardless of the externality effect and of the ability of the downstream units to transform the input into a higher quality product.

Further if the research spillover is sufficiently strong, we have a corner solution where the upstream outsider doesn't produce research and both the upstream and the downstream outsiders are driven out of the market. In Figure 30, we show that this happens only if both the research spillover and the merger induced externality are very strong when the products are close substitutes; for the special case in which $B = a = 1$, instead, the outsiders are always driven out of the market, Figure 31.

Since however now the outsiders are in a position of clear disadvantage, they might have incentives to respond by integrating vertically. This leads to the merger configuration analysed in section 6.4, where the relative profitability for the participants depends on the strength of the horizontal merger induced externality effect.

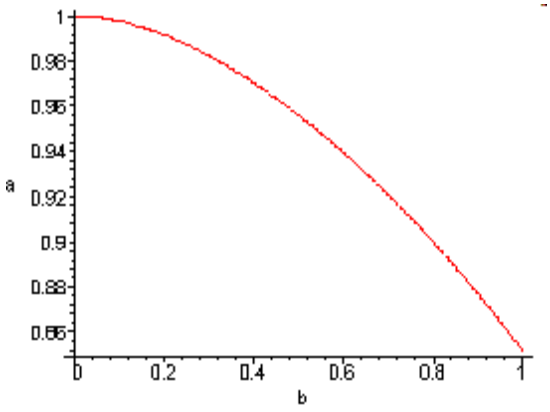

Figure 31: At any point above the curve, the outsiders are driven out of the market, $B = 1$

8. Equilibrium merger configurations

Following an initial vertical merger, two possible merger configurations might emerge: a situation where all the units integrate vertically or a situation where all the units internalise the pricing externality but two downstream units also integrate horizontally. This second merger configuration is likely to emerge also in response to an initial downstream merger.

Anticipating the reactions of the non-merging firms, would the first downstream participant prefer to vertically integrate with its input supplier or to horizontally integrate with another downstream unit?

In tables 1, 2 and 3, we show, for different values of the parameters of the model, the individual equilibrium profits for the benchmark model, π_U^C and π_D^C, and the two possible equilibrium merger configurations: more specifically, $\pi_{V(i,j)}$, with $i, j = 1, 2, 3$, $i \neq j$, denotes the profit of the horizontally and vertically integrated downstream units while π_{Vk}, with $k = 1, 2, 3$ and $k \neq i, j$, reflects the profit of the unique vertically integrated unit; finally π_V denotes the individual equilibrium profit for the case of complete vertical integration. The values in brackets reflect the average merger profitability for each possible merger configuration.

When the products are sufficiently differentiated our results are that:

- for weak externality effects, vertical integration is more profitable for the initial merged units. In this case the outsiders would respond by internalising the pricing externality and integrating horizontally in the downstream market. Thus complete vertical integration would never be an equilibrium outcome
- for strong research spillovers, horizontal integration is more profitable for the initial merged units. In this case the outsiders would be always harmed by the initial merger and for sufficiently strong merger induced externality effects they would be driven out of the market.

When instead the products are close substitutes, we find that:

- for low externality effects, vertical integration is always better for the initial merged units; in this scenario, however, the outsiders would respond by integrating vertically and all the firms would be worse off with respect to a pre-merger situation. In this context the initial merger might not be carried through
- for very strong research spillovers and weak merger induced effects, vertical integration should prevail over horizontal integration; in this case, the outsiders would respond by internalising the pricing externality and by horizontally integrating in the downstream market. If however the initial merged units were able to identify the reaction functions of the outsiders and to lead them accordingly, no further merger would follow. In this case, in fact, the outsiders would be driven out of the market
- for strong research spillovers and merger induced effects, horizontal integration is more profitable for the initial merged units; in this scenario, the outsiders would be harmed or driven out of the market

For intermediate values of the degree of substitutability, our results are that:

- for low externality effects, vertical integration is always better for the initial merged units; in this case, however, the outsiders to minimise their losses would respond by horizontally integrating in the downstream market and by internalising the pricing externality
- for very strong research spillovers, horizontal integration is always more profitable for the initial merged units; in this context, however, the outsiders would be always harmed.

Thus when both horizontal and vertical integration are feasible, complete vertical integration should never be an equilibrium outcome.

Table 1 - Equilibrium merger configurations, $b = 0.3$

b	B	a	π_U^C	π_D^C	$\pi_{V(i,j)}$	π_{Vk}	π_V
0.3	0.3	0.1	.095	.050	.306 (.004)	.160 (.007)	.150 (.002)
0.3	0.3	0.3	.095	.050	.320 (.008)	.156 (.006)	.150 (.002)
0.3	1.0	0.3	.103	.062	.419 (.022)	.164 (-.001)	.169 (.002)
0.3	1.0	0.6	.103	.062	.587 (.064)	.114 (-.025)	.169 (.002)
0.3	1.0	0.9	.103	.062	1.14 (.202)	.015 (-.075)	.169 (.002)

Table 2 - Equilibrium merger configurations, $b = 0.9$

b	B	a	π_U^C	π_D^C	$\pi_{V(i,j)}$	π_{Vk}	π_V
0.9	0.3	0.1	.052	.037	.129 (-.012)	.112 (.011)	.068 (-.010)
0.9	0.3	0.3	.052	.037	.135 (-.011)	.107 (.009)	.068 (-.010)
0.9	1.0	0.3	.054	.044	.119 (-.019)	.116 (.009)	.054 (-.022)
0.9	1.0	0.6	.054	.044	.206 (.003)	.046 (-.026)	.054 (-.022)
0.9	1.0	0.9	.054	.044	.252 (.014)	.000 (.000)	.054 (-.022)

Table 3 - Equilibrium merger configurations, $b = 0.5$

b	B	a	π_U^C	π_D^C	$\pi_{V(i,j)}$	π_{Vk}	π_V
0.5	0.3	0.1	.079	.044	.228 (-.004)	.132 (.005)	.112 (-.005)
0.5	0.3	0.3	.079	.044	.237 (-.002)	.128 (.003)	.112 (-.005)
0.5	1.0	0.3	.084	.054	.285 (.002)	.128 (-.005)	.116 (-.011)
0.5	1.0	0.6	.084	.054	.401 (.031)	.076 (-.031)	.116 (-.011)
0.5	1.0	0.9	.084	.054	.777 (.125)	.001 (-.068)	.116 (-.011)

9. CONCLUSIONS

We have examined how the presence of research activity in the upstream market may affect the profitability of different types of mergers. We have also investigated how an initial merger may create incentives for the non-merging firms to respond with a countermerger. Starting from a pre-merger situation, we considered two initial mergers between two units

and we showed that, when considered in isolation, these mergers are always profitable for the participants. Thus there are always incentives for the first two units to merge. When the initial merger is between one downstream unit and its input specific supplier, all the outsiders are harmed and have therefore incentives to react by merging in turn. If all the units integrate vertically, then the initial participants are worse off but, if the products are sufficiently differentiated and the research spillover is not too strong, the profits of the industry as a whole increase and all the units are better off with respect to a pre-merger situation. When however both vertical and horizontal integration are feasible, the outsiders might consider to respond with alternative mergers; more specifically, we have shown that the outsiders would be better off by both internalising the pricing externality and integrating horizontally in the downstream market, unless the products are close substitutes when the externality effects are weak. In this case however complete vertical integration would be unprofitable for all the units and the initial merger might not be carried through.

We have also shown that an initial merger between two downstream competitors, while being profitable for its participants, is likely to create incentives for their suppliers to respond with a countermerger. This new merger, while reducing the profitability of the downstream participants, doesn't improve significantly the profitability of their suppliers. Anticipating this, the downstream participants might try to integrate vertically to preempt a merger between their suppliers. In this new merger configuration, however, the outsiders would be in a position of clear disadvantage and would therefore have incentives to respond by vertically integrating. This leads to the last merger configuration where all the units internalise the pricing externality but the initial participants also benefit from an additional merger induced effect. For sufficiently strong externalities, the initial participants always earn more and horizontal integration should prevail over vertical integration.

Thus when both the research spillover and the merger induced effect are sufficiently

strong, horizontal integration is always more profitable for the initial merged units; when instead the merger induced externality is weak, the initial merged units might earn more by integrating vertically. In both scenarios, however, complete vertical integration is never an equilibrium outcome when both horizontal and vertical integration are feasible.

It might also be interesting to investigate which equilibrium structures are likely to emerge when firms in each period may decide whether to form new links or break existing links. An attempt in this direction has been made by Gowrisankaran, (1996), who, using numerical simulations, examines how a merger decision may affect subsequent entry decisions in a model where merging, entry/exit and production choices are made in each period.

We feel that this issue needs further investigation and this is part of our future research agenda.

To check the robustness of our results we repeated the same analysis for the case of price competition on the final market. While our main results still hold, there are however some interesting differences related to the presence of an additional strategic effect: when firms compete in prices, the rivals' suppliers research activity has a direct negative effect on the individual final demands, which tends to intensify the research activity of all the input suppliers when competition on the final market gets fiercer. Since the incentives to increase research are stronger for the suppliers to the outsiders, mergers are profitable for the participants for a lower set of parameter values.

Another relevant question is whether our results are related to the assumption of linear production technologies and demands. For the case of vertical integration the question has been investigated by Abiru, (1988), who shows how the welfare effects of vertical integration do not change with different assumptions on production technology and market structure. For the case of horizontal integration, instead, the problem is far from being fully investigated, maybe because very little may be said on the issue unless specific functional forms are used. Most of the earlier work, however, Salant et al., (1983),

Deneckere and Davidson, (1985), Perry and Porter, (1985), Lommerud et al., (2003), is based on linear demand functions.

A. APPENDIX: EQUILIBRIUM OUTCOMES

A.1. One vertical merger. Solving the first order conditions of the profits with respect to the quantity level, the candidate equilibrium quantities are given by:

$$q_{V1} = \frac{1}{2}\frac{A(2-b) - bB(x_2+x_3) + B(b+2)x_1 + b(w_2+w_3)}{b+2-b^2}$$

$$q_{D2} = \frac{1}{2}\frac{A(2-b) - bB(x_1+x_3) + B(b+2)x_2 + bw_3 - w_2(b+2)}{b+2-b^2}$$

$$q_{D3} = \frac{1}{2}\frac{A(2-b) - bB(x_1+x_2) + B(b+2)x_3 + bw_2 - w_3(b+2)}{b+2-b^2}$$

Given the equilibrium quantities, the independent upstream units choose their input prices so as to maximise their individual profits; these are given by:

$$w_2 = \frac{A(2b+8-3b^2) + B(b^2+8b+8)x_3 - bB(b+2)x_2 - bB(3b+4)x_1}{3b^2+16b+16}$$

$$w_3 = \frac{A(2b+8-3b^2) + B(b^2+8b+8)x_2 - bB(b+2)x_3 - bB(3b+4)x_1}{3b^2+16b+16}$$

Finally by substituting the candidate equilibrium quantities and input prices into the profits of the integrated unit and of the independent suppliers, the equilibrium research efforts may be found solving the first order conditions with respect to the research activity:

$$x_{V1} = -AB(b^2-6b-8)\left[(b+2)(b^2+8b+8)B^2 + (b+4)(b-2)(3b+4)^2\right]/G$$

$$x_{U2} = x_{U3} = -AB(b+2)(b^2+8b+8)\left[(b^2-6b-8)B^2 + 2(b+4)(b+1)(b-2)^2\right]/G$$

where

$$\begin{aligned} G &= (b+2)(b^2+8b+8)(b^2-6b-8)B^4 - 4(3b+4)(b+1)^2(b-2)^2(b+4)^3 + \\ &\quad -(b+4)(b^5+68b^4+36b^3-480b^2-832b-384)B^2 \end{aligned}$$

is always negative for $B \leq 1$

Hence the equilibrium quantity, input price and profit levels are:

$$q_{V1} = -2A(b+4)(b-2)(b+1)\left[(b+2)(b^2+8b+8)B^2 + (b+4)(b-2)(3b+4)^2\right]/G$$

$$q_{D2} = q_{D3} = -A(b+4)(3b+4)(b+2)\left[(b^2-6b-8)B^2 + 2(b+4)(b+1)(b-2)^2\right]/G$$

$$w_2 = w_3 = 2A(b-2)(3b+4)(b+4)(b+1)\left[(b^2-6b-8)B^2 + 2(b+4)(b+1)(b-2)^2\right]/G$$

$$\pi_{V1} = -A^2G_1^2\left[(b^2-6b-8)B + 2(b+4)(b-2)(b+1)\right]\left[(6b-b^2+8)B + 2(b+4)(b-2)(b+1)\right]/G^2$$

$$\pi_{D2} = \pi_{D3} = A^2(b+4)^2(3b+4)^2(b+2)^2\left[(b^2-6b-8)B^2 + 2(b+4)(b+1)(b-2)^2\right]^2/G^2$$

$$\pi_{U2} = \pi_{U3} = -A^2G_2(b+2)\left[(b^2-6b-8)B^2 + 2(b+4)(b+1)(b-2)^2\right]^2/G^2$$

where:

$$G_1 = (b+2)(b^2+8b+8)B^2 + (b+4)(b-2)(3b+4)^2$$

$$G_2 = (b+2)(b^2+8b+8)^2B^2 + 2(b+1)(b-2)(b+4)^2(3b+4)^2 < 0$$

A.2. Stackelberg Oligopoly. The problem of the integrated unit is to maximise

$$\max_{q_1} \pi_1 = (A + Bx_1 - q_1 - b\,(R_2(q_1) + R_3(q_1)))q_1 - x_1^2 \tag{5}$$

where $R_2(q_1)$, $R_3(q_1)$ denote the reaction functions of the non-colluding firms

$$R_2(q_1) = \frac{(2x_2 - bx_3)B + (2-b)A + (b^2-2b)q_1 - 2w_2 + bw_3}{4-b^2} \tag{6}$$

$$R_3(q_1) = \frac{(2x_3 - bx_2)B + (2-b)A + (b^2-2b)q_1 + bw_2 - 2w_3}{4-b^2} \tag{7}$$

Differentiating (5) with respect to q_1 and solving the first order conditions yield the candidate equilibrium quantity for the merged units:

$$q_1 = \frac{1}{2}\frac{((x_1 - x_2 - x_3)B - A + w_2 + w_3)b + 2Bx_1 + 2A}{2 - 2b^2 + b} \tag{8}$$

Substituting (8) into (6) and (7), we get the candidate equilibrium quantities for the non-colluding firms:

$$q_2 = \frac{(b-2)(3b^2-4)A + b(3b^2-4)(Bx_3 - w_3) + (8-b^3-6b^2+4b)(Bx_2 - w_2) - bB(4-b^2)x_1}{(2b^2-b-2)(b-2)(b+2)} \tag{9}$$

$$q_3 = \frac{(b-2)(3b^2-4)A + b(3b^2-4)(Bx_2 - w_2) + (8-b^3-6b^2+4b)(Bx_3 - w_3) - bB(4-b^2)x_1}{(2b^2-b-2)(b-2)(b+2)} \tag{10}$$

Replacing for (8), (9) and (10) into the input suppliers' profits we find the candidate input prices:

$$w_2 = \frac{(b-2)(3b^2-4)AH + B(b^2-4b-4)H_1x_2 + bB(3b^2-4)H_2x_3 + bB(b^2-4)Hx_1}{(5b^3+12b^2-12b-16)(b^3-12b^2+4b+16)}$$

$$w_3 = \frac{(b-2)(3b^2-4)AH + B(b^2-4b-4)H_1x_3 + bB(3b^2-4)H_2x_2 + bB(b^2-4)Hx_1}{(5b^3+12b^2-12b-16)(b^3-12b^2+4b+16)}$$

$H = (5b^3+12b^2-12b-16)$
$H_1 = (7b^4+4b^3-36b^2+32)$
$H_2 = (b^3+6b^2-4b-8)$

We can now solve for the first stage of the game, where the independent upstream units and the merged firms set simultaneously and non-cooperatively research to maximise profits; the first order conditions for this stage yield the equilibrium research efforts:

$$x_1 = \frac{-AB(b^2-4b-4)\left[(3b-4)(b+2)(b^2-4b-4)B^2H_1H_2 + (b^2-4)(3b-4)D_1H^2\right]}{(3b-4)(b+2)H_1H_2(b^2-4b-4)^2B^4 + (b^2-4b-4)DD_1B^2 + 4(b+2)H(2b^2-b-2)D_1^3}$$

$$x_2 = x_3 = \frac{-BA(b^2-4b-4)H_1H_2\left[(3b-4)(b+2)(b^2-4b-4)B^2 + 2(3b^2-4)D_1\right]}{(3b-4)(b+2)H_1H_2(b^2-4b-4)^2B^4 + (b^2-4b-4)DD_1B^2 + 4(b+2)H(2b^2-b-2)D_1^3}$$

where:

$$D = (73b^9+144b^8-1436b^7-2272b^6+6960b^5+8320b^4-12352b^3-11776b^2+7168b+6144)$$

$D_1 = (b^3-12b^2+4b+16)$
for

$$B < \frac{\left[2(4-3b)(b+2)(b^2-4b-4)(3b^2-4)(b^3-12b^2+4b+16)\right]^{1/2}}{(3b-4)(b+2)(b^2-4b-4)}$$

otherwise:

$$x_1 = \frac{AB(2-b)(3b-4)(b^2-4b-4)(5b^3+12b^2-12b-16)}{(b+2)(3b-4)^2(b^2-4b-4)^2B^2+4(2b^2-b-2)(b^3-12b^2+4b+16)^2}$$

and

$$x_2 = x_3 = 0$$

Replacing back into the expressions for the equilibrium input prices:

$$w_2 = w_3 = \frac{2A(b-2)(2b^2-b-2)\left[(3b-4)(b+2)(b^2-4b-4)B^2+2(3b^2-4)D_1\right]}{(b+2)(3b-4)^2(b^2-4b-4)^2B^2+4(2b^2-b-2)D_1^2} \leq 0$$

for

$$B \geq \frac{\left[2(4-3b)(b+2)(b^2-4b-4)(3b^2-4)(b^3-12b^2+4b+16)\right]^{1/2}}{(3b-4)(b+2)(b^2-4b-4)}$$

and substituting into the equilibrium quantities:

$$q_1 = q_2 = \frac{A(b^3-12b^2+4b+16)\left[(3b-4)(b+2)(b^2-4b-4)B^2+2(3b^2-4)D_1\right]}{(b+2)\left[(b+2)(3b-4)^2(b^2-4b-4)^2B^2+4(2b^2-b-2){D_1}^2\right]} \leq 0$$

the competitive advantage of the merged units is so strong that the independent units are driven out of the market.

If we assume that there is a positive constant cost of production per unit of input, C, for

$$B \geq \frac{\left[2(4-3b)(b+2)(b^2-4b-4)(3b^2-4)(b^3-12b^2+4b+16)\right]^{1/2}}{(3b-4)(b+2)(b^2-4b-4)}$$

we would still have a corner solution, where the non-integrated units don't produce research:

$$x_1 = \frac{(A-C)B(2-b)(3b-4)(b^2-4b-4)(5b^3+12b^2-12b-16)}{(b+2)(3b-4)^2(b^2-4b-4)^2B^2+4(2b^2-b-2)(b^3-12b^2+4b+16)^2}$$

$$x_2 = x_3 = 0$$

And substituting into the expressions for the candidate equilibrium input prices:

$$w_2 = w_3 = \frac{(3b-4)(b^2-4b-4)(b+2)B^2\left[2(b-2)(2b^2-b-2)A-CH_2\right]}{(b+2)(3b-4)^2(b^2-4b-4)^2B^2+4(2b^2-b-2)(b^3-12b^2+4b+16)^2} + \frac{(2b^2-b-2)D_1\left[4(b-2)(3b^2-4)A-8C(b^3+3b^2-4b-4)\right]}{(b+2)(3b-4)^2(b^2-4b-4)^2B^2+4(2b^2-b-2)(b^3-12b^2+4b+16)^2}$$

which is always less than or equal to C.

Substituting $w_1 = w_2 = C$ into the expressions for the equilibrium quantities, we get:

$$q_1 = q_2 = \frac{(A-C)(b^3-12b^2+4b+16)\left[(3b-4)(b+2)(b^2-4b-4)B^2+2(3b^2-4)D_1\right]}{(b+2)\left[(b+2)(3b-4)^2(b^2-4b-4)^2B^2+4(2b^2-b-2)D_1^2\right]} \leq 0$$

and the outsiders are driven out of the market.

A.3. A downstream merger. Solving the game by backward induction, the non-cooperative Nash equilibrium yields the following expressions for the equilibrium research levels:

$$x_1 = x_2 = \frac{-A(b^2-4)\left[D_2-4(2b^2-3b-4)(3b^2-2b-4)\right]\left[(b^2-4)(3b^2-7b-8)ab+BD_1\right]}{D}$$

$$\begin{aligned} x_3 &= \frac{-AB(b^2-4)D2}{D}\left[\frac{b(3b^2-7b-8)(b^2-4)a^2}{B^2}+\frac{2a(b-1)(b^2-4b-4)(3b^2-4b-8)}{B}+D_1\right]+ \\ &+\frac{8A(3b^2-4b-8)(3b^2-2b-4)(2b^3-7b^2+8)D_2}{BD} \end{aligned}$$

$$\begin{aligned} D &= -4(b^2-2)(3b^2-2b-4)(9b^7-129b^6+508b^5-96b^4-1792b^3+224b^2+2304b+1024)B^2+ \\ &+(b^2-4)(b+1)D_1(b^3-7b^2+4b+8)B^4-128(3b^2-4b-8)(b^2-2b-2)(3b^2-2b-4)^3+ \\ &+2B(b-1)(b^2-4)(3b^2-4b-8)(b^2-4b-4)\left[D_2-4(b^2-4b-4)(3b^2-2b-4)\right]a+ \\ &+b(3b^2-7b-8)(b^2-4)^2\left[D_2-4(b^2-4b-4)(3b^2-2b-4)\right]a^2 \end{aligned}$$

with $D > 0$

$D_1 = (3b^5-31b^4+44b^3+76b^2-64b-64) < 0$

$D_2 = (b+1)(b^3-7b^2+4b+8)B^2 > 0$

Hence, the equilibrium input price, quantity and profit levels are:

$$w_1 = w_2 = \frac{-16A(b-1)(3b^2-2b-4)(b^2-2b-2)(3b^2-4b-8)\left[D_2-4(2b^2-3b-4)(3b^2-2b-4)\right]}{D}$$

$$w_3 = \frac{-4A(b^2-2b-2)(3b^2-2b-4)D_4}{D}$$

$$q_1 = q_2 = \frac{A(b^2-4)(-2b+3b^2-4)(3b^2-4b-8)\left[D_2-4(2b^2-3b-4)(3b^2-2b-4)\right]}{D}$$

$$q_3 = \frac{4(b+1)(-2b+3b^2-4)AD_4}{D}$$

$$\pi_{D(1,2)} = \frac{32A^2(b+1)(b^2-4)^2(3b^2-4b-8)^2(-2b+3b^2-4)^2\left[D_2-4(2b^2-3b-4)(3b^2-2b-4)\right]^2}{D^2}$$

$$\pi_3 = \frac{16A^2(b+1)^2(-2b+3b^2-4)^2D_4^2}{D^2}$$

$$\pi_{U1} = \pi_{U2} = \frac{-A^2(b^2-4)\left[D_2-4(2b^2-3b-4)(3b^2-2b-4)\right]^2 D_3}{D^2}$$

$$\pi_{U3} = \frac{-A^2(b+1)\left[(b+1)(b^3-7b^2+4b+8)^2B^2+16(-2-2b+b^2)(3b^2-2b-4)^2\right]D_4^2}{D^2}$$

where

$$\begin{aligned} D_3 &= ab(3b^2-7b-8)^2(b^2-4)^2\left[(b^2-4)ab+2BD_1\right]+(b^2-4)D_1^2B^2+ \\ &\quad 64(b-1)(b^2-2b-2)(-2b+3b^2-4)^2(3b^2-4b-8)^2 \end{aligned}$$

$$\begin{aligned} D_4 &= b(3b^2-7b-8)(b^2-4)^2a^2+2B(b-1)(b^2-4)(b^2-4b-4)(3b^2-4b-8)a+ \\ &\quad (b^2-4)D_1B^2-8(3b^2-4b-8)(3b^2-2b-4)(2b^3-7b^2+8) \end{aligned}$$

with $D_3 > 0$ and $D_4 < 0$

A.4. Three vertical mergers. Solving the model by backward induction, the non-cooperative Nash equilibrium research, quantity and profits levels for the three vertically integrated units are:

$$x_{1V} = x_{2V} = x_{3V} = \frac{(b+2)BA}{4b^3-12b+bB^2+2B^2-8}$$

$$q_{1V} = q_{2V} = q_{3V} = 2\frac{(b+1)(b-2)A}{4b^3-12b+bB^2+2B^2-8}$$

$$\pi_{1V} = \pi_{2V} = \pi_{3V} = \frac{(2b^2+bB-2b+2B-4)(2b^2-bB-2b-2B-4)A^2}{(4b^3-12b+bB^2+2B^2-8)^2}$$

A.5. Vertical and downstream horizontal mergers. Using backward induction, the non-cooperative Nash equilibrium research levels are:

$$x_1 = \frac{D_2\left[-ba^2(b-4)(b^2-4)^2 + 4B(b-1)(b^2-4)(3b^2-4b-8)a + (b^2-4)B^2D_1 + 2D_3\right]AB}{D}$$

$$x_2 = x_3 = \frac{(b^2-4)\left[D_2B^2 + b^5 - 22b^3 + 44b^2 - 32\right]\left[D_1B - b(b^2-4)(b-4)a\right]A}{D}$$

where

$$\begin{aligned} D &= -2(b^2+4b-8)(2b^8-52b^7+273b^6-260b^5-880b^4+1152b^3+1152b^2-1024b-768)B^2 + \\ &\quad +(b-1)(b^2-4)(3b^2-4b-8)\left[-4D_2B^2+8(b^2+4b-8)(b^2-2b-2)\right]aB + \\ &\quad -(b^2-4)D_2D_1B^4 + 2(3b^2-4b-8)(b^2-2b-2)^2(b^2+4b-8)^3 + \\ &\quad +(b-4)(b^2-4)^2b\left[D_2B^2 - 2(b^2-2b-2)(b^2+4b-8)\right]a^2 \end{aligned}$$

$D_1 = b^4 + 8b^3 - 32b^2 + 32$
$D_2 = (b^3 - 7b^2 + 4b + 8)$
$D_3 = (b^2+4b-8)(3b^2-4b-8)(2b^3-7b^2+8)$
with $D_1, D_2, D_3, D > 0$

Hence the equilibrium input prices, quantity and profit levels are:

$$w_2 = w_3 = \frac{-4A(b-1)(b^2-2b-2)(b^2+4b-8)(3b^2-4b-8)\left[D_2B^2 + b^5 - 22b^3 + 44b^2 - 32\right]}{D}$$

$$\begin{aligned} q_1 &= \frac{(b^2-2b-2)(b^2+4b-8)\left[4B(b-1)(b^2-4)(3b^2-4b-8)a + (b^2-4)B^2D_1 + 2D_3\right]AB}{D} + \\ &\quad -\frac{ba^2(b^2-2b-2)(b^2+4b-8)(b-4)(b^2-4)^2AB}{D} \end{aligned}$$

$$q_2 = q_3 = \frac{(b^2-4)(b^2+4b-8)(3b^2-4b-8)\left[D_2B^2 + b^5 - 22b^3 + 44b^2 - 32\right]}{D}$$

$$\pi_{V1} = \frac{A^2D_4\left[-ba^2(b-4)(b^2-4)^2 + 4B(b-1)(b^2-4)(3b^2-4b-8)a + (b^2-4)B^2D_1 + 2D_3\right]^2}{D^2}$$

$$\pi_{D(2,3)} = \frac{2A^2(b+1)(b^2-4)^2(b^2+4b-8)^2(3b^2-4b-8)^2\left(D_2B^2+b^5-22b^3+44b^2-32\right)^2}{D^2}$$

$$\pi_{U2} = \pi_{U3} = \frac{-A^2(b^2-4)\left(D_2B^2+b^5-22b^3+44b^2-32\right)^2 D_5}{D^2}$$

where

$$D_4 = (b^2+4b-8)^2(b^2-2b-2)^2 - (b^3-7b^2+4b+8)^2B^2$$

$$\begin{aligned} D_5 \quad = \quad & (b^2-4)(b^4+8b^3-32b^2+32)^2B^2 + 4(b-1)(b^2-2b-2)(b^2+4b-8)^2(3b^2-4b-8)^2 + \\ & +b^2(b-4)^2(b^2-4)^3a^2 - 2bB(b-4)(b^4+8b^3-32b^2+32)(b^2-4)^2a \end{aligned}$$

with D_4,$D_5 > 0$

A.6. Vertical and downstream/upstream mergers. Solving the game by backward induction, the non-cooperative Nash equilibrium yields the following expressions for the equilibrium variables:

$$x_1 = \frac{AB(4+4b-b^2)(2b^2-4b+a^2+B^2+2aB-8)}{D}$$

$$x_2 = x_3 = \frac{-A(4b^3+b^2B^2-16b^2-4bB^2+8b+16-4B^2)(a+B)}{D}$$

$$w_2 = w_3 = \frac{2A(b^2-2b-2)(4b^3+b^2B^2-16b^2-4bB^2+8b+16-4B^2)}{D}$$

$$q_1 = \frac{-4(b^2-2b-2)(2b^2-4b+a^2+B^2+2aB-8)A}{D}$$

$$q_2 = q_3 = \frac{-2(4b^3+b^2B^2-16b^2-4bB^2+8b+16-4B^2)A}{D}$$

$$\pi_{V1} = \frac{-A^2(b^2B+4b^2-4bB-8b-4B-8)(b^2B-4b^2-4bB+8b-4B+8)D_1}{D^2}$$

$$\pi_{D(2,3)} = \frac{8A^2(b+1)(4b^3+b^2B^2-16b^2-4bB^2+8b-4B^2+16)^2}{D^2}$$

$$\pi_{U(2,3)} = \frac{-2A^2(4b^3+b^2B^2-16b^2-4bB^2+8b-4B^2+16)^2(4b^2-8b+2aB-8+a^2+B^2)}{D^2}$$

where:

$$0 > D = (b^2B^2 - 4bB^2 + 16b - 8b^2 - 4B^2 + 16)a^2 + 2(b^2B^2 - 4bB^2 + 16b - 8b^2 - 4B^2 + 16)Ba + (b^2 - 4 - 4b)B^4 + (2b^4 + 48 - 16b^3 + 80b + 8b^2)B^2 - 32(b^2 - 2b - 2)^2$$

$$D_1 = (2b^2 - 4b + a^2 + B^2 + 2aB - 8)^2$$

A.7. Vertical and horizontal integration. Solving the model by backward induction, we find that, for

$$B \leq \frac{ba(b^3 - 4b^2 + 8 + 4b) + 2\left[2(1-b)(b^2 - 2b - 2)^2(b^4 - 4b^3 - 4b^2 + 8b - 2ba^2 + 8 - 2a^2)\right]^{1/2}}{-4b^2 + 8b + b^4 - 4b^3 + 8}$$

the second order conditions are satisfied, a Nash equilibrium exists and, for $B < B^*$, with:

$$B^* = \frac{1}{2}\frac{-2ba - 2a + 2(4 - 2b^3 + 2b^2 + 8b)^{1/2}}{1 + b}$$

it is characterised by:

$$x_1 = \frac{AB(b+1)(2b^2 - 4b - 4 + ba^2 + a^2 + 2abB + 2Ba + bB^2 + B^2)}{K_2}$$

$$x_2 = x_3 = \frac{-A(B+a)(b+1)(b^3 - 4b^2 - bB^2 + 2b - B^2 + 4)}{K_2}$$

$$q_1 = \frac{-A(b^2 - 2b - 2)(2b^2 - 4b + ba^2 + 2Bba + bB^2 - 4 + a^2 + 2Ba + B^2)}{K_2}$$

$$q_2 = q_3 = \frac{A(b^2 - 2b - 2)(b^3 - 4b^3 - bB^2 + 2b + 4 - B^2)}{K_2}$$

$$\pi_{1V} = \frac{A^2(b^2 + bB - 2b + B - 2)(b^2 - bB - 2b - B - 2)K}{K_2^2}$$

$$\pi_{2V} = \pi_{3V} = -\frac{2A^2(b+1)(bB^2 + B^2 + 2Bba + 2Ba - b^4 + 4b^3 - 8b + ba^2 - 4 + a^2)K_1}{K_2^2}$$

where

$$K = (2b^2 - 4b + ba^2 + 2Bba + bB^2 - 4 + a^2 + 2Ba + B^2)^2$$

$$K_1 = (b^3 - 4b^2 - bB^2 + 2b + 4 - B^2)^2$$

$$K_2 = 2(b^2-2b-2)^3 - 2(b+2)(b+1)(b^2-2b-2)B^2 - (b+1)^4 - \\ +(b+1)(2b^2+bB^2-4b+B^2-4)(2Ba+1)$$

and $K_2 < 0$, for $a < a^*$,
where

$$a^* = -\frac{B(b+1)G + [2(b+1)(b^2-2b-2)(-bB-B-2-2b+b^2)(bB+B-2-2b+b^2)G]^{1/2}}{(b+1)G}$$

and $G = (bB^2+B^2-4-4b+2b^2)$

For $B \geq B^*$and $a \neq a^*$, Figure 23, we have a corner solution where

$$x_1 = 0$$

$$x_2 = x_3 = \frac{1}{2}\frac{A(b+1)(2-b)(a+B)}{b^4-4b^3+8b-2Bba-ba^2-bB^2-B^2-a^2+4-2Ba}$$

Replacing back into the expressions for the equilibrium quantities:

$$q_2 = q_3 = \frac{1}{2}\frac{(b-2)(b^2-2b-2)A}{b^4-4b^3+8b-2Bba-ba^2-bB^2-B^2-a^2+4-2Ba} > 0$$

for

$$B < \frac{1}{2}\frac{-2a-2ba+2(b^5-3b^4+8b^2+12b-4b^3+4)^{1/2}}{b+1}$$

and

$$q_1 = \frac{1}{2}\frac{a(a+2B)(b+1)+2b^2+(B-2)(B+2)(b+1)}{b^4-4b^3+8b-2Bba-ba^2-bB^2-B^2-a^2+4-2Ba} \leq 0$$

for $B \geq B^*$
and the initial integrated unit is therefore driven out of the market.

A.8. Downstream and upstream mergers. Solving the model by backward induction, we find the equilibrium research levels:

$$x_1 = x_2 = \frac{-(a+B)A(b^2-4b-4)\left[(b+1)(b^2-4b-4)B^2+(2b^2-3b-4)(b^2-8b-8)\right]}{D}$$

$$x_3 = \frac{-BA(b+1)(b^2-4b-4)\left[(b^2-4b-4)(a+B)^2+(b^2-8b-8)(b^2-2b-4)\right]}{D}$$

where

$$\begin{aligned} D &= (b+1)(b^2-4b-4)^2(B+2a)B^3+(b^2-4b-4)^2(b^3-6b^2+a^2b-24b-16+a^2)B^2- \\ &\quad 2a(b^2-8b-8)(b^2-4b-4)^2B+(b^2-8b-8)\left[(b^2-4b-4)^2a^2+(b^2-2b-2)(b^2-8b-8)^2\right] \end{aligned}$$

is always positive

Hence the equilibrium input price, quantity and profit levels are:

$$w_1 = w_2 = \frac{A(b^2-2b-2)(b^2-8b-8)\left[(b+1)(-4-4b+b^2)B^2+(b^2-8b-8)(2b^2-3b-4)\right]}{D}$$

$$w_3 = \frac{A(b^2-8b-8)(b^2-2b-2)\left[(-4-4b+b^2)(a+B)^2+(b^2-2b-4)(b^2-8b-8)\right]}{D}$$

$$q_1 = q_2 = \frac{-A(b^2-8b-8)\left[(b+1)(-4-4b+b^2)B^2+(b^2-8b-8)(2b^2-3b-4)\right]}{D}$$

$$q_3 = \frac{-A(b+1)(b^2-8b-8)\left[(-4-4b+b^2)(a+B)^2+(b^2-2b-4)(b^2-8b-8)\right]}{D}$$

$$\pi_{D(1,2)} = \frac{2A^2(b+1)(b^2-8b-8)^2\left[(b+1)(-4-4b+b^2)B^2+(b^2-8b-8)(2b^2-3b-4)\right]^2}{D^2}$$

$$\pi_{D3} = \frac{A^2(b+1)^2(b^2-8b-8)^2\left[(-4-4b+b^2)(a+B)^2+(b^2-2b-4)(b^2-8b-8)\right]^2}{D^2}$$

$$\pi_{U(1,2)} = \frac{-2A^2\left[(b+1)(-4-4b+b^2)B^2+(b^2-8b-8)(2b^2-3b-4)\right]^2 D_1}{D^2}$$

$$\pi_{U3} = \frac{-A^2(b+1)\left[(b+1)(b^2-4b-4)^2B^2+(b^2-2b-2)(b^2-8b-8)^2\right]D_2^2}{D^2}$$

where:

$$D_1 = (-4-4b+b^2)^2(a+B)^2+(b^2-2b-2)(b^2-8b-8)^2 < 0$$

$$D_2 = (b^2-4b-4)(a+B)^2-10b^3+48b+4b^2+32$$

A.9. Downstream mergers and vertical integration. Solving the model by backward induction, we find that for

$$B < \frac{-a(b^2-4b-4)-2\left[2(b^2-4b-4)(b^2-2b-2)\right]^{1/2}}{(b^2-4b-4)}$$

the non-cooperative Nash equilibrium is characterised by the following positive values of research, input price, output and profit levels:

$$x_1 = x_2 = \frac{A(b^2-4b-4)(4b^2+bB^2-6b-8+B^2)(B+a)}{K}$$

$$x_3 = AB\frac{(1+b)(a^2-8+2Ba+B^2)b^2-4(B+a+2)(B+a-2)(b+1)^2}{K}$$

$$w_3 = \frac{-2A(b^2-2b-2)((a^2-8+2Ba+B^2)b^2-4(B+a+2)(B+a-2)(b+1))}{K}$$

$$q_1 = q_2 = \frac{4(b^2-2b-2)(4b^2+bB^2-6b+B^2-8)A}{K}$$

$$q_3 = \frac{2A(b+1)((a^2-8+2Ba+B^2)b^2-4(B+a+2)(B+a-2)(b+1))}{K}$$

$$\pi_1 = \pi_2 = \frac{-2A^2(4b^2-6b+bB^2-8+B^2)^2K_1}{K^2}$$

$$\pi_{D3} = \frac{4A^2(b+1)^2((-8+2Ba+a^2+B^2)b^2-4(B+a+2)(B+a-2)(b+1))^2}{K^2}$$

$$\pi_{U3} = \frac{A^2(b+1)(8-B^2+8b-bB^2-4b^2)((-8+2Ba+a^2+B^2)b^2-4(B+a+2)(B+a-2)(b+1))^2}{K^2}$$

where

$$\begin{aligned} K &= 32(b+1)(b^2-2b-2)^2-(48+112b+56b^2-16b^3-6b^4)B^2-(b+1)(b^2-4b-4)B^4 \\ &\quad -a(a+2B)(b^2-4b-4)(2b^2+bB^2-8b+B^2-8) \end{aligned}$$

is always positive and

$$K_1 = (b^2-4b-4)(a^2+2Ba+B^2)-16(b+1)(b^2-2b-2)^2 < 0$$

Further if the research spillover is sufficiently strong

$$B \geq \frac{-a(b^2-4b-4)-2\left[2(b^2-4b-4)(b^2-2b-2)\right]^{1/2}}{(b^2-4b-4)}$$

we have a corner solution where

$$x_3 = 0$$

and

$$x_1 = x_2 = \frac{-A(b^2-4b-4)(2b^2-3b-4)(B+a)}{a(b^2-4b-4)(a+2B)\left[(b^2-4b-4)^2B^2-16(b+1)(b^2-2b-2)^2\right]}$$

Replacing back into the expression for the equilibrium input price, we get:

$$w_3 = \frac{A(b^2-2b-2)\left[a(b^2-4b-4)(a+2B)+(b^2-4b-4)^2B^2-8b^2+16+16b\right]}{a(b^2-4b-4)(a+2B)\left[(b^2-4b-4)^2B^2-16(b+1)(b^2-2b-2)^2\right]} \leq 0$$

for

$$B \geq \frac{-a(b^2-4b-4)-2\left[2(b^2-4b-4)(b^2-2b-2)\right]^{1/2}}{(b^2-4b-4)}$$

and substituting into the expression for the equilibrium quantity of the downstream outsider:

$$q_3 = \frac{-2A(b+1)\left[a(b^2-4b-4)(a+2B)+(b^2-4b-4)^2B^2-8b^2+16+16b\right]}{a(b^2-4b-4)(a+2B)\left[(b^2-4b-4)^2B^2-16(b+1)(b^2-2b-2)^2\right]} \leq 0$$

and both the upstream and the downstream outsiders are driven out of the market.

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