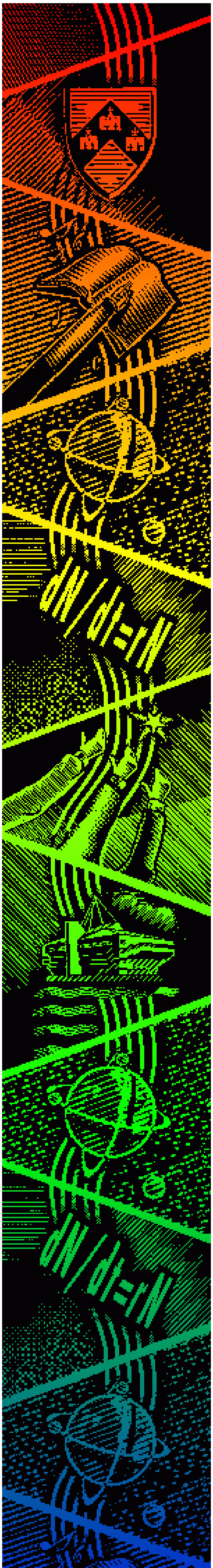

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Multiple Equilibria with Externalities

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Abstract

We analyse the implications of multiple Nash equilibria in consumption patterns. Multiple equilibrium theory has been extensively used in macroeconomics to explain the converge of the system to a low inefficient equilibrium. Its microfoundations appear very abstract and stylised. It has been recognised that a rigourous microfoundation matters above all when policy and dynamic implications are considered. For example, Cahuc and Kempf (1997) show that, just by assuming a restrictive number of players, the standard result of Pareto superior simultaneous solutions on staggered decisions is not confirmed, or that introducing a more specific framework, the general expansionary demand policies do not generate the beneficial effects predicted by general stylised models (see Pagano (1990) for a discussion). The main contribution of this paper is to provide a more specific microfoundation of multiple equilibria in consumption theory.

Keywords: Multiple Equilibria, Consumption Externalities.

JEL Classification: C62, C72, D11.

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1 Introduction

We analyse consumption interaction between individuals when multiple equilibria in individual consumption patterns exist. We study the conditions under which best replay dynamics converge to a stable equilibrium. A discrete dynamic adjustament approach is used for this purpose.

Multiple equilibrium theory has been extensively used since the late 80s as an alternative paradigm to Keynesian rigidities for the explanation of macroeconomic equilibrium inefficiency. Cooper and John (1988) concentrate on strategic complementarities between individual economic interactions as the source of multiple equilibria. Given positive spillovers in the player strategies, all the Nash equilibria are inefficient and Pareto rankable. The economic system can be stuck in a low-activity equilibrium in which no agent has an incentive to deviate from it. These results are obtained from an abstract static symmetric framework, in which the individual payoff depends on the other individual actions and the individual optimal strategies are non cooperatively positively correlated. This abstract game can be applied to analyse different economic situations in which multiple equilibria arise: positive spillovers in the production process (Bryant (1983), Weil (1989), Durlauf (1991)), participation externality in search and matching market (Diamond (1982), Howitt (1985), Howitt and McAfee (1988)), demand externality in multi-sector models of imperfect competition (Hart (1982), Heller (1986)), increasing returns to scale in specialised production with imperfect competition (Weitzman (1982)). Although the static results of all these different models are homogenous (multiple inefficient and Pareto rankable Nash equilibria) the dissimilarities between them are significant once dynamics and policy implications are considered. For example, let us focus on the dynamic implications of multiple equilibria generated by demand externality in imperfectly competitive economy. Three principal results arise in the Cooper and Haltiwanger (1990) dynamic models of imperfect competition with heterogeneous agents: strategic complementarities cause positive comovement in output and employment across sectors, simultaneous non cooperative solutions are preferred to staggering strategy solutions, a specific sector shock is magnified and propagated in all the other sectors. Similar results are confirmed in models of investment decisions with complementarities (Cooper and Haltiwanger (1992) (1993)). But in contrast to the general results of the static framework, these dynamic predictions can not be generalised for the entire class of multiple equilibrium models. Cahuc and Kempf (1997) showed that in a model with strategic complementarities and with only two agents playing Markov strategies, staggered decisions are Pareto superior to simultaneous decisions. The predictive power of these models is strictly related to their microfoundations: restrictions on the set of number of players and committed strategies are sufficient to change the dynamic outcome of these multiple equilibrium games. Moreover, a detailed specification of the structure of the models matters when policy implications are analysed. In highly stylised models aggregate demand expansions are welcome and considered a remedy to push the system away from the unemployment trap (see, for example, Matsuyama (1995) for a survey on this topic). By introducing a more specific framework, these policy prescriptions are disproved. For example, treating explicitly the saving decisions within an overlapping generation model expansionary demand policies are not only beneficial but even generally counterproductive (see Pagano (1990)).

These insights suggest that the general framework of multiple equilibrium models is too highly stylised and abstract. The principal aim of this chapter is to provide a more rigorous microfoundation of multiple equilibria in consumption theory. Cooper and John's model (1988) assuming strategic complementarities in the consumption patterns can be summarised as follows. Suppose two identical individuals who allocate income for the consumption of two goods purchased at constant unit prices p_1, p_2. Suppose that good 1 is the source of consumption externality. The utility maximisation problem is: $\max\limits_{x_{1h}} U^h(x_{1h}, \widehat{x}_{2h}(M_h, p_1, p_2, x_{1h}), x_{1g})$ where $\widehat{x}_{2h}(\cdot) = (M_h - p_1 x_{1h})/p_2$. The solution of this problem defines the reaction curve of the model. If $\frac{\partial U^h(\cdot)}{\partial x_{1g}} > (<) 0$ the game exhibits positive (negative) spillovers. If $\frac{\partial^2 U^h(\cdot)}{\partial x_{1h} \partial x_{1g}} > (<) 0$ the game exhibits strategic complementarities (substitutes). Suppose that S is the set of all the symmetric Nash equilibria (SNE): $S = \left\{ x_1 \in [0, \widetilde{x}_1] \mid \frac{\partial U^h(x_1,x_1)}{\partial x_1} = 0 \right\}$. Assume that $\lim\limits_{x_1 \to 0} \frac{\partial U^h(x_1,x_1)}{\partial x_1} > 0$ and $\lim\limits_{x_{1h} \to \widetilde{x}_1} \frac{\partial U^h(x_1,x_1)}{\partial x_1} < 0$ and that a SNE exists for which the slope is greater than 1: then the game exhibits multiple equilibria. The microfoundation of this game is thus not very detailed: fixing upper and lower bounds on the reaction curve and assuming that an internal symmetric equilibrium exists with a slope greater than one are sufficient to have a multiplicity of equilibria. In this chapter we show that with a more rigorous microfoundation, the properties of the reaction curve are strictly connected to the properties of the marginal utility. Nash corner solution equilibria can arise in our framework: this enriches the economic implications of this class of models. For example, assuming an imperfectly competitive setting, externalities in the production process can influence the market structure. For particular features of the production externality the oligopolistic equilibrium is not stable and the system will approach to an equilibrium in which only one firm remains in the market. Another application can be in the entry games: production externalities can block the entrance of new firms in the monopolistic market. The prediction that shocks are propagated and magnified is not still confirmed. We show counterintuitive examples in which positive shocks don't generate positive comovement and instead negative shocks can lead the system to upper equilibria. The rest of the chapter is organised in four sections. In section II the general framework is presented with a specific microfoundation of the strategic consumption interactions. Different scenarios are considered in section III according to the nature of the individual consumption interactions. For each of them, the convergence to a stable equilibrium is examined assuming a discrete dynamic approach. Comparative static examples are considered in section IV. In the last sections we show that all these results are still valid once the consumption setting is expanded.

2 The symmetric case

Consider two individuals with identical preferences consuming two goods with one externality inducing good. Let us assume that good one is the source of the externality. The utility maximisation problem for each individual h (with $h = A, B$) is stated as follows:

$$\max_{x_{1h}, x_{2h}} \{U^h(x_{1h}, x_{2h}, x_{1g}) \mid p_1 x_{1h} + p_2 x_{2h} = M_h\}$$

where x_{1g} is the quantity of good 1 consumed by the other individual (with $g = A, B$ and $h \neq g$). The utility function of each individual h satisfies the standard conditions of quasiconcavity and continuous differentiability. The two variable constraint Lagrangian is transformed into the following one variable unconstrained problem:

$$\max_{x_{1h}} \{U^h(x_{1h}, \widehat{x}_{2h}(M_h, p_1, p_{2,} x_{1h}), x_{1g})\} \tag{1}$$

where $\widehat{x}_{2h}(M_h, p_1, p_{2,} x_{1h}) = (M_h - p_1 x_{1h}) \backslash p_2$.

Solving (1) ($\partial U^h(\cdot) \backslash \partial x_{1h} = 0$) provides the reaction curve of each individual h for good 1 as a function of prices, individual income and the quantity consumed of good 1 by the other individual ($x_{1A} = f(p, M_A, x_{1B})$, $x_{1B} = f(p, M_B, x_{1A})$ with $p = (p_1, p_2)$). Total differentiation of the first order condition gives the slope of each individual reaction curve:

$$\frac{dx_{1h}}{dx_{1g}} = \frac{\frac{\partial^2 U^h(\cdot)}{\partial x_{1h} \partial x_{1g}} - \frac{p_1}{p_2} \frac{\partial^2 U^h(\cdot)}{\partial \widehat{x}_{2h} \partial x_{1g}}}{-\frac{\partial^2 U^h(\cdot)}{\partial x_{1h}^2} + 2\frac{p_1}{p_2} \frac{\partial^2 U^h(\cdot)}{\partial x_{1h} \partial \widehat{x}_{2h}} - \frac{\partial^2 U^h(x_{1h}, \phi, x_{1g})}{\partial \widehat{x}_{2h}^2} (\frac{p_1}{p_2})^2} \tag{2}$$

The denominator of (2) is the determinant of the Bordered Hessian of the utility maximisation problem for individual h. It gives information about the curvature properties of the utility function when the externality is treated as a parameter. The individual utility maximisation problem has a maximum only if the Bordered Hessian is positive, i.e. the utility function is quasiconcave. The sign of the slope of the reaction curves depends thus only the sign of the numerator. We can introduce the following two definitions.

Definition 1 *: if* $[\partial^2 U^h(\cdot)/\partial x_{1h}\partial x_{1g} - (p_1/p_2)(\partial^2 U^h(\cdot)/\partial \widehat{x}_{2h}\partial x_{1g})] > 0$ *then* $dx_{1h}/dx_{1g} > 0$*, we define the externality effects to be strategic complements.*

The reaction curve of each individual is sloping upward: the optimal strategy of individual h for good 1 is positively associated with the optimal strategy of individual g for the same good. A possible interpretation of this case is that of conformity behaviour in the consumption of the same good. Individuals prefer having similar consumption patterns.

Definition 2 *if* $[\partial^2 U^h(\cdot)/\partial x_{1h}\partial x_{1g} - (p_1/p_2)(\partial^2 U^h(\cdot)/\partial \widehat{x}_{2h}\partial x_{1g})] < 0$ *then* $dx_{1h}/dx_{1g} < 0$*, the externality effects are defined to be strategic substitutes*

An increase in individual g's consumption of good 1 decreases the other player's consumption for that good. An example is snobbish behaviour in the consumption patterns.

Information on the curvature of the reaction curves is obtained from totally differentiating (1):

$$\frac{d}{dx_{1g}}\left(\frac{dx_{1h}}{dx_{1g}}\right) = \frac{[2\widetilde{U}^h_{x_{1h},x_{1h},x_{1g}}\widetilde{U}^h_{x_{1h},x_{1g}}\widetilde{U}^h_{x_{1h},x_{1h}} - \widetilde{U}^h_{x_{1h},x_{1g},x_{1g}}\widetilde{U}^h_{x_{1h},x_{1h}} - \widetilde{U}^h_{x_{1h},x_{1g}}\widetilde{U}^h_{x_{1h},x_{1h},x_{1h}}]}{-\widetilde{U}^3_{x_{1h},x_{1h}}} \tag{3}$$

where:

$$\widetilde{U}^h_{x_{1h},x_{1h},x_{1g}} = [\frac{\partial^3 U^h(\cdot)}{\partial x^2_{1h}\partial x_{1g}} - 2\frac{\partial^3 U^h(\cdot)}{\partial \widehat{x}_{2h}\partial x_{1h}\partial x_{1g}}\frac{p_1}{p_2} + \frac{\partial^3 U^h(\cdot)}{\partial \widehat{x}^2_{2h}\partial x_{1g}}(\frac{p_1}{p_2})^2]$$

$$\widetilde{U}^h_{x_{1h},x_{1h}} = [-\frac{\partial^2 U^h(\cdot)}{\partial x^2_{1h}} + 2\frac{\partial^3 U^h(\cdot)}{\partial x_{1h}\partial \widehat{x}^2_{2h}}\frac{p_1}{p_2} - \frac{\partial^2 U^h(\cdot)}{\partial \widehat{x}^2_{2h}}(\frac{p_1}{p_2})^2]$$

$$\widetilde{U}_{x_{1h},x_{1g}} = [-\frac{\partial^2 U^h(\cdot)}{\partial x_{1h}\partial x_{1g}} + \frac{\partial^2 U^h(\cdot)}{\partial \widehat{x}_{2h}\partial x_{1g}}\frac{p_1}{p_2}]; \widetilde{U}_{x_{1h},x_{1g},x_{1g}} = [\frac{\partial^3 U^h(\cdot)}{\partial x_{1h}\partial x^2_{1g}} - \frac{\partial^3 U^h(\cdot)}{\partial \widehat{x}_{2h}\partial^2 x_{1g}}\frac{p_1}{p_2}]$$

$$\widetilde{U}_{x_{1h},x_{1h},x_{1h}} = [\frac{\partial^3 U^h(\cdot)}{\partial x_{1h}^3} - 3\frac{\partial^3 U^h(\cdot)}{\partial \widehat{x}_{2h}\partial x_{1h}\partial x_{1h}}\frac{p_1}{p_2} + 3\frac{\partial^3 U^h(\cdot)}{\partial \widehat{x}_{2h}^2 \partial x_{1g}}(\frac{p_i}{p_2})^2 + \frac{\partial^3 U^h(\cdot)}{\partial \widehat{x}_{2h}^3}(\frac{p_1}{p_2})^3]$$

It is remarkable to notice that the numerator of the curvature condition is equal to the determinant of the following matrix:

$$B_{mu}^h = \begin{bmatrix} 0 & \widetilde{U}^h_{x_{1h},x_{1h}} & \widetilde{U}_{x_{1h},x_{1g}} \\ \widetilde{U}^h_{x_{1h},x_{1h}} & \widetilde{U}_{x_{1h},x_{1h},x_{1h}} & \widetilde{U}^h_{x_{1h},x_{1h},x_{1g}} \\ \widetilde{U}_{x_{1h},x_{1g}} & \widetilde{U}^h_{x_{1h},x_{1h},x_{1g}} & \widetilde{U}_{x_{1h},x_{1g},x_{1g}} \end{bmatrix}$$

The matrix B_{mu}^h is the 3×3 Bordered Hessian of the Marginal utility function $\widetilde{U}^h_{x_{1h}} = \frac{\partial U^h(\cdot)}{\partial x_{1h}} - \frac{\partial U^h(\cdot)}{\partial \widehat{x}_{2h}}\frac{p_1}{p_2}$ of individual h. Thus (3) becomes:

$$\frac{d}{dx_{1g}}(\frac{dx_{1h}}{dx_{1g}}) = \frac{\det(B_{mu}^h)}{-\widetilde{U}^3_{x_{1h},x_{1h}}} \tag{4}$$

The curvature properties of the reaction curves are stated in the following propositions.

Proposition 3 : *If the marginal utility is a quasiconcave function, the reaction curve is convex.*

If the marginal utility is a quasi concave function, $\det(B_{mu}^h)$ is positive (see for example Katzner (1970) p. 211). The denominator is positive from the properties of the Bordered Hessian of the utility function. It follows that (4) is positive.

Proposition 4 *: If the marginal utility is a quasiconvex function, the reaction curve is concave.*

If the marginal utility is a quasi convex function, $\det(B_{mu}^h)$ is negative. It follows that (4) is negative.

In the appendix we show that the properties of the slope and of the curvature of the reaction curve are ordinal properties. They are invariant to any increasing monotonic transformation.

In the next sections different cases are analysed according to the sign and the rate of increase of the slope of the reaction curve. For each case we discuss the conditions for the existence of a unique Nash equilibrium or of multiple equilibria. A discrete dynamic approach is used to analyse the stability conditions of the equilibria. In each period, each individual adjusts his consumption pattern by observing the action of the other player in the previous period. Given arbitrary initial conditions at time $t = 1$ (x_{1A}^1, x_{1B}^1), the adjustment process adopted by each individual in each period is given by: $x_{1h}^t = f(p, M_h, x_{1g}^{t-1})$ with $h, g = A, B$ and $h \neq g$. This adjustment process has been used in different contexts to analyse under which assumptions a system of best reaction curves converges to a stable equilibrium (see Friedman (1977), Moulin (1982), Lippman et al. (1987), Vives (1990), Milgrom and Roberts (1990)). It has been argued that this is dynamically naive (see, for example, Varian (1992) p. 288)) but this adjustment process has been valued for being empirical appealing. In the next section, we analyse which conditions should be imposed to have best reply dynamics converging to stable equilibria. Different scenarios can be depicted according to the sign of the slope, its rate of increase and upper and lower bonds of the reaction curve. We exclude the cases of free goods: the prices are always positive.

2.1 Strategic Complementarities and convex reaction curve

At first, we analyse the consumption interaction between the two individuals assuming reaction curve with positive slope and increasing curvature. Individual optimal strategies are positively correlated and individual marginal rate of substitution of the two goods is increasing with the intensity of the externality effect. To analyse the existence of equilibria, upper and lower bounds of the reaction curves are defined. Let x_{1h}^* solve $\partial U^h(x_{1h}, \widehat{x}_{2h}(M_h, p_1, p_{2,} x_{1h}), 0)/\partial x_{1h} = 0 : x_{1h}^*$ is the optimal level of x_{1h} when $x_{1g} = 0$. Let $\overline{x}_{1g}$ solve $\partial U^h(0, \widehat{x}_{2h}(M_h, p_1, p_{2,} x_{1h}), x_{1g})/\partial x_{1h} = 0 : \overline{x}_{1g}$ is the level of g consumption of good 1 which makes h consume 0 of good 1. Let $\underline{x}_{1h} \leq M_h/p_1$ the optimal level of h's consumption satisfying the utility maximisation problem when $x_{1g} = M_g/p_1$ ($\underline{x}_{1h}$ solves $\partial U^h(x_{1h}, \widehat{x}_{2h}(M_h, p_1, p_{2,} x_{1h}), M_g/p_1)/\partial x_{1h} = 0$. Let $\widetilde{x}_{1g}$

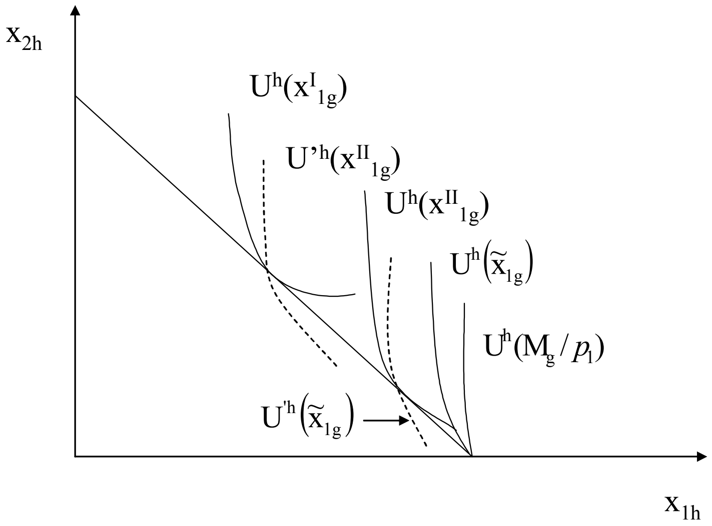

Figure 1: Effect of the externality on the maximisation problem when $\widetilde{x}_{1g} < M_g/p_1$

be the level of the externality that makes individual h consume $x_{1h} = M_h/p_1$ ($\widetilde{x}_{1g}$ solves $\partial U^h(M_h/p_1, \widehat{x}_{2h}(M_h, p_1, p_{2,}x_{1h}), x_{1g})/\partial x_{1h} = 0$).

Proposition 5 *With strategic complementarities and convex reaction curves, the upper bound of individual h's reaction curve is* $\underline{x}_{1h}$

The value of the upper bound and the behaviour of the reaction curve in the neighborhood of this point depend on the effect of the externality on the utility maximisation problem. As the quantity consumed by the other individual increases, the indifference curves get steeper through any point. Suppose for example that the quantity consumed by the other individual increases from x^I_{1g} to x^{II}_{1g}. The new indifference map is steeper than the previous one (see for example the indifference curves $U^{'h}(x^{II}_{1g})$ or $U^h(x^{II}_{1g})$ in Fig. 1). The new optimal bundle of individual h is the point of tangency of the new steeper indifference curve to the budget constraint (it is point in which $U^h(x^{II}_{1g})$ is tangent to the budget constraint). At each externality increase,

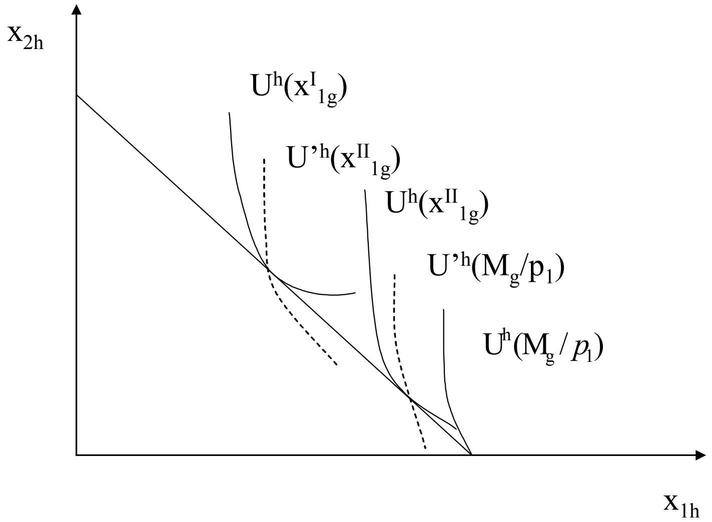

Figure 2: Effect of the externality on the maximisation problem when $\widetilde{x}_{1g} = M_g/p_1$

the new equilibrium point will be never on the LHS of the previous one. Individual h reacts to the externality till individual g reaches his maximum consumption affordable of good 1 ($x_{1g} = M_g/p_1$). To define the upper bound of the reaction curve, three different cases can be distinguished.

Consider Fig. 1. Individual h utility maximisation problem has an interior solution if $x_{1g} < \widetilde{x}_{1g}$. When $x_{1g} = \widetilde{x}_{1g}$ a boundary optimum occurs: individual h will use all his income to buy only good 1. This maximum quantity is the optimal response of individual h to each externality increase above $\widetilde{x}_{1g}$ till the other individual reaches is maximum affordable quantity. In this case the reaction curve is thus $x_{1h} \geq 0$ for $x_{1g} < \widetilde{x}_{1g}$ and $x_{1h} = M_h/p_1$ for $\widetilde{x}_{1g} \leq x_{1g} \leq M_g/p_1$. The reaction curve has a kink when $x_{1g} = \widetilde{x}_{1g}$.

Suppose instead that individual h maximisation problem has a boundary optimum ($x_{1h} = M_h/p_1$) only when the other individual is consuming the same maximum quantity (see Fig. 2). In this case the value of the upper bound of the reaction curve

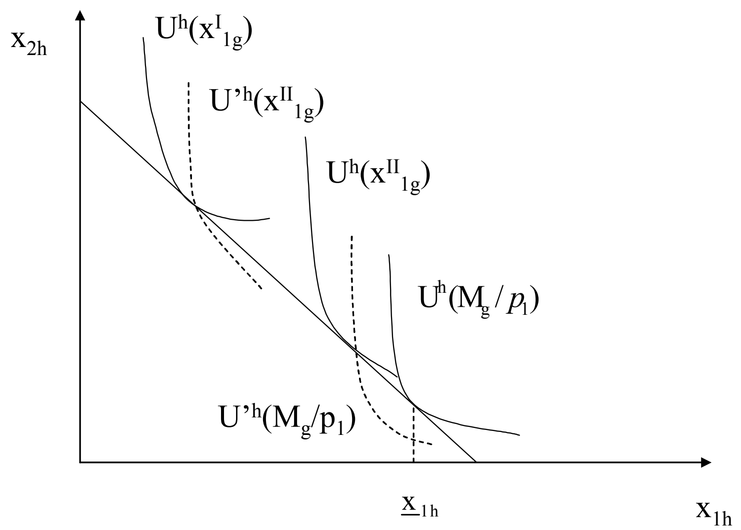

Figure 3: Externality effect on the utility maximisation problem when only interior equilibria occur

is still $\underline{x}_{1h} = M_h/p_1$.

The last case arises when individual h utility maximisation problem has an interior solution when individual g consumes his maximum quantity affordable of good 1. Whatever is the externality effect, individual h will never use all his income to buy only good (see Fig. 3). In this case the upper bound of the reaction curve is $\underline{x}_{1h} < M_h/p_1$. Similarly to the second case considered, the reaction curve is not kinky shaped in the neighborhood of the maximum value, but in this case the individual will never reaches his maximum quantity affordable.

Also for the definition of the lower bound different cases can be distinguished according to the effect of the externality on the individual utility maximisation problem.

Definition 6 *With strategic complementarities and convex reaction curves, if the lower bound is $x_{1h} = 0$ the externality is essential*

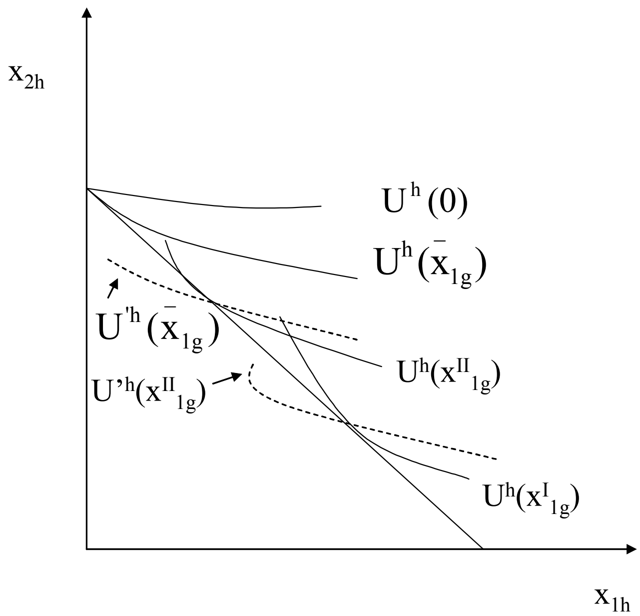

Figure 4: Essential externality effect

The externality is essential if the individual decision to participate in the market depends on the intensity of the externality effect. In this case, for particular level of the externality ($0 \leq x_{1g} \leq \overline{x}_{1g}$) the individual decides to give up the consumption of good 1 and to consume only the other good. In terms of utility maximisation problem, this case can be graphically represented by Fig. 4.

Suppose that the value of the externality decreases from x^I_{1g} to x^{II}_{1g}. The indifferences curves become flatter (see for example $U'^h(x^{II}_{1g})$ or $U^h(x^{II}_{1g})$). The new optimal bundle of individual h is the point in which $U^h(x^{II}_{1g})$ is tangent to the budget constraint. At each externality decrease, the optimal quantity consumed of good 1 by individual h decreases. There is a minimum level of the externality effect ($\overline{x}_{1g}$) for which an interior solution of the utility maximisation problem doesn't exist. For each level of the externality below this minimum level the individual consumes positive quantities only of good 2. In this case, thus the reaction curve is $x_{1h} = 0$ for $0 \leq$

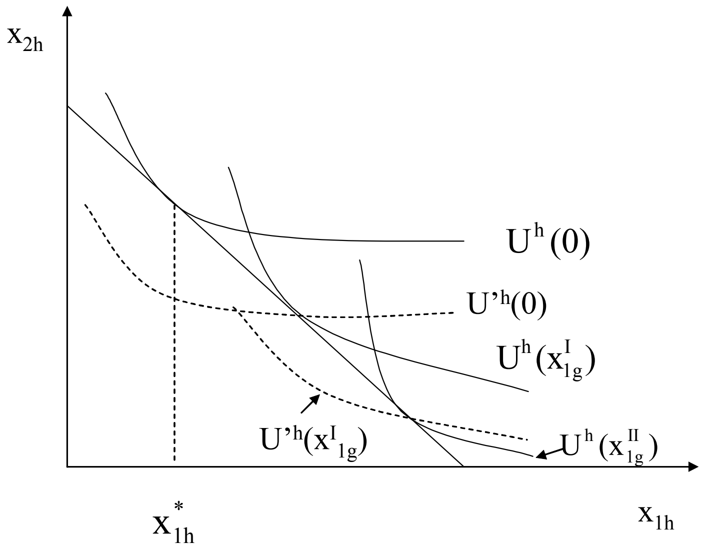

Figure 5: Non essential externality effect

$x_{1g} < \overline{x}_{1g}$ and $x_{1h} > 0$ for $\overline{x}_{1g} < x_{1g} \leq M_g/p_1$.

Definition 7 *With strategic complementarities and convex reaction curves, if the lower bound is x^*_{1h} the externality is not essential*

In this case whichever is the effect of the externality the individual always consumes positive quantities of good 1 (see Fig. 5). The externality doesn't induce the individual to give up the consumption of good 1. In this case the reaction curve has a positive intercept.

We analyse now different cases according to the value of the lower and upper bond of the reaction function. For each of them we study the conditions to have a single equilibrium or multiple equilibria and the stability properties of them.

Proposition 8 *With strategic complementarities and convex reaction curves and non essential externality, if $\underline{x}_{1h} < M_h/p_1$ the symmetric game exhibits only one Nash equilibrium.*

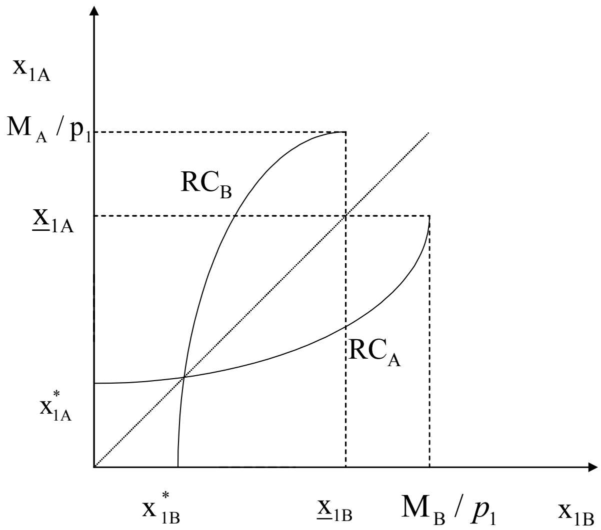

Figure 6: Strategic complements and convex reaction curves with non essential externality when $\underline{x}_{1h} < M_h/p_1$.

If the externality is not essential, the reaction curve has a positive intercept. If at $x_{1g} = M_g/p_1$ the individual h utility maximisation problem has an interior solution, than the system has an unique stable interior Nash equilibrium (see Fig. 6).

Proposition 9 *With strategic complementarities and convex reaction curves and non essential externality, the symmetric game exhibits two Nash equilibria if* $\widetilde{x}_{1h} = \underline{x}_{1h} = M_h/p_1$ *and if there is* x_{1g} *satisfying* $x_{1h}(x_{1g}) < x_{1g}$.

With essential externality, if the individual reaches the maximum amount when the other individual is consuming his maximum amount and if it exists a point of the reaction curve below the 45 degree line then two equilibria arise: an interior equilibrium and a corner Nash equilibrium in which the individual devote all the income to the purchase of good 1 (see Fig. 7). Using the discrete dynamic approach described in the previous section, only the lower equilibrium will be locally stable. The system converges to it. The individual prefers to decrease the consumption of

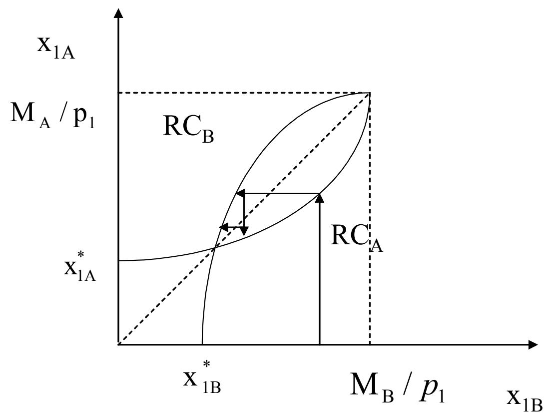

Figure 7: Strategic complements and convex reaction curves with non essential externality when $\widetilde{x}_{1h} = M_h/p_1 = \underline{x}_{1h}$

the good generating the externality effect and substitutes it with the other good. Since the reaction curve is convex, how much more of good 2 the consumer would lose to compensate for the gain of a unit of good 1 to keep the same level of utility is increasing with the level of the externality. If the effect of the externality is intense, the amount of x_{2h} lost to compensate for an increase in x_{1h} is relevant. The system is not converging to the upper equilibrium since it is too costly in terms of the quantity of x_{2h} lost. If there isn't any x_{1g} satisfying $x_{1h}(x_{1g}) < x_{1g}$, then the system will have only the corner stable Nash equilibrium $(M_A/p_1, M_B/p_1)$.

Proposition 10 *With strategic complementarities and convex reaction curves and non essential externality, the symmetric game exhibits three Nash equilibria if and only if* $\widetilde{x}_{1h} < \underline{x}_{1h} = M_h/p_1$ *and if there is* x_{1g} *satisfying* $x_{1h}(x_{1g}) < x_{1g}$.

In this case individual h reaches the maximum amount when the other individual is not consuming his maximum amount. The reaction curve crosses twice the 45

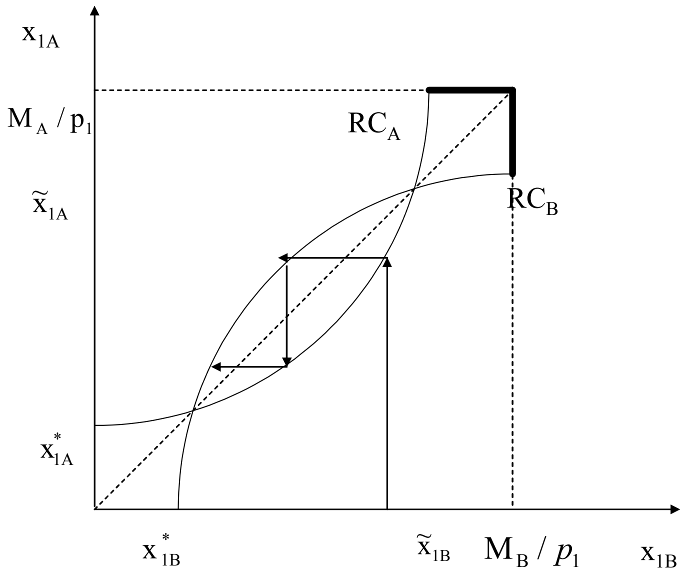

Figure 8: Strategic complements and convex reaction curves with non essential externality when $\widetilde{x}_{1h} < M^h/p_1$

degree line because of the existence of a point of the reaction curve below the 45 degree line. Three equilibria arise: one corner Nash equilibrium and two interior equilibria (see Fig. 8).

The Corner Nash and the lower interior equilibria are stable. The convergence to the system to one of these equilibria depends on the initial conditions. If at the initial conditions the individuals are consuming low quantities of the good the system converges to the lower interior equilibrium. If the individuals consume initially high quantities of the good, then the stable equilibrium is reached with both the individuals consuming the maximum quantities.

Different and interesting outcomes arise in the case in which the game is still characterised by positive correlation in the player strategies and convex reaction curves, but the externality is essential.

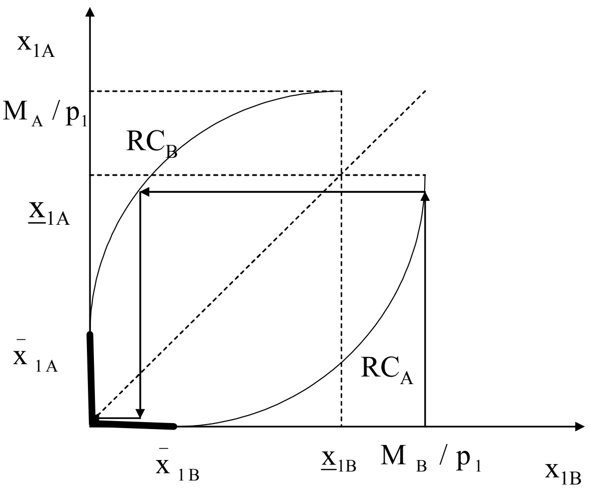

Figure 9: Strategic complements and convex reaction curves with non essential externality when $\underline{x}_{1h} < M_h/p_1$

Proposition 11 *In a symmetric game with strategic consumption complementarities and convex reaction curves with essential externality if* $\underline{x}_{1h} < M_h/p_1$ *there is only one corner Nash equilibrium.*

With essential externality if at $x_{1g} = M_g/p_1$ the individual h utility maximisation problem has an interior solution, then the system will have only one interior corner Nash equilibrium (0,0). This equilibrium is stable (see Fig. 9).

Proposition 12 *In a symmetric game with strategic consumption complementarities and convex reaction curves with essential externality if* $\underline{x}_{1h} = M_h/p_1$ *there are two or three equilibria.*

In this case individual h's reaction curve is $x_{1h} = 0$ if $0 < x_{1g} \leq \overline{x}_{1g}$ and $x_{1h} > 0$ for $x_{1g} > \overline{x}_{1g}$. It has a kink at $\overline{x}_{1g}$. This is the level of the externality that induces individual h to drop out the market.

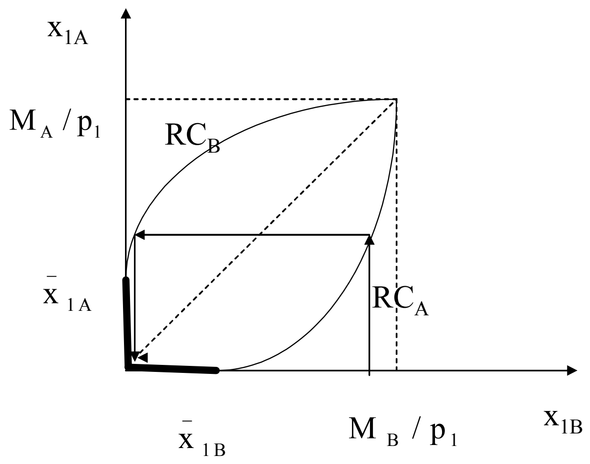

Figure 10: Strategic complements and convex reaction curves with essential externality when $\widetilde{x}_{1h} = \underline{x}_{1h} = M_h/p_1$

The existence of two or three equilibria depend on the behaviour of the reaction curve in the upper equilibrium. If $\widetilde{x}_{1g} = M^g/p_1$ and $\underline{x}_{1h} = M_h/p_1$, then the game always exhibits two corner Nash equilibria (see Fig. 10): the system still converges to the corner Nash equilibrium (0,0). If the reaction curve is kinky shaped also in the upper bound ($\widetilde{x}_{1g} < M^g/p_1$), then the system has always three Nash equilibria (see Fig. 11). The lower and upper corner Nash equilibria are stable, in this case. Assuming essential externality and that the upper bound of the reaction curve is the maximum amount affordable of good 1 are sufficient to guarantee multiple equilibria.

If the externality is essential, the system will converge to corner Nash equilibria. During the adjustment process if $\overline{x}_{1g}$ ($\widetilde{x}_{1g}$) or a point in the flat part of the reaction curve is reached, in the following period both individuals will be in the corner Nash equilibrium.

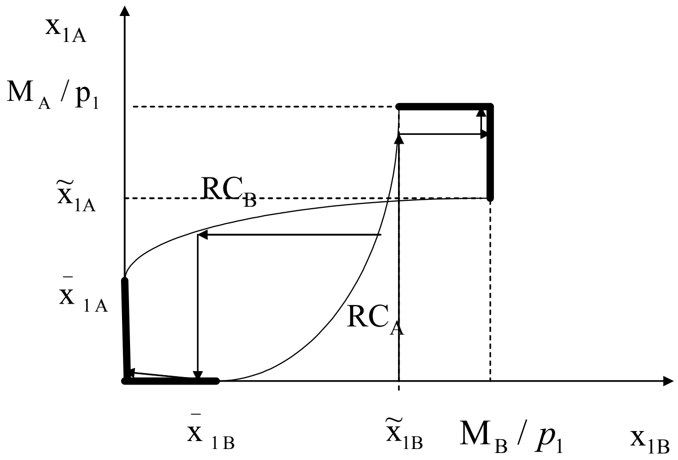

Figure 11: Strategic complements and convex reaction curves with essential externality when $\widetilde{x}_{1h} < \underline{x}_{1h} = M_h/p_1$

2.2 Strategic complements and concave reaction curve

We are still in the case of positive interactions in the players strategies but the individual marginal rate of substitution is now decreasing with the intensity of the externality. The lower and upper bounds of the reaction curves are the same as the previous case (see definition 5-7). Let us consider the case in which the externality effect is essential. The reaction curve is kinky shaped in the lower bound as previously described. In this case the game will always exhibits at least two equilibria.

Proposition 13 *With strategic complementarities and concave reaction and essential externality, the symmetric game exhibits always at least two equilibria if* $\underline{x}_{1h} = M_h/p_1$.

Proposition 14 *With strategic complementarities and concave reaction and essential externality, the symmetric game exhibits always three equilibria if and only if there is* x_{1g} *satisfying* $x_{1h}(x_{1g}) > x_{1g}$.

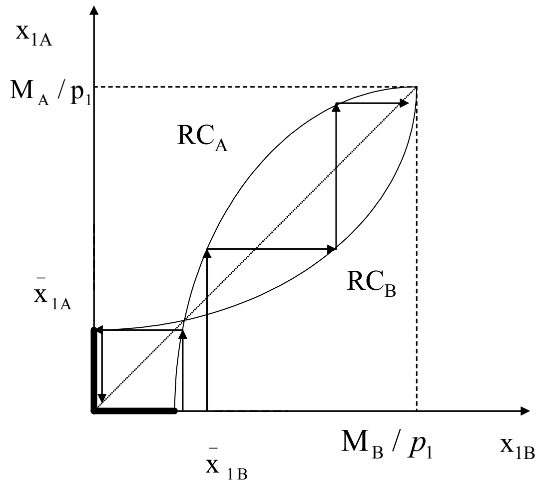

Figure 12: Strategic complements and concave reaction curves with essential externality when $\underline{x}_{1h} = \widetilde{x}_{1h} = M^h/p_1$.

Due to the symmetric structure of the game, if the externality is essential, the game will always have the two symmetric Nash equilibria if no point of the individual reaction curve is above the 45 degree line and if $\underline{x}_{1h} = M_h/p_1$. Under the same conditions if $\underline{x}_{1h} < M_h/p_1$, the game has only one equilibrium. In this case the system converges to the corner solution Nash equilibrium (0,0). If instead a point on the reaction curve below the 45 degree line exists, then the game will have always three equilibria (see Fig. 12, Fig. 13 and Fig. 14).

If $\underline{x}_{1h} = M_h/p_1$, despite of the behaviour of the reaction curve in the neighborhood of the upper bound, both the two corner Nash equilibria are stable. The value of the initial conditions will drive the system to one of these equilibria. If instead $\underline{x}_{1h} < M_h/p_1$ the system will still have three equilibria, but in the upper one the individual is not using all the income to buy good 1 (see Fig. 14).

We are now analysing the case in which still the marginal rate of substitution is decreasing with the intensity of the externality effect, but the individual is always consuming a positive quantity of the good, whichever is the effect of the externality.

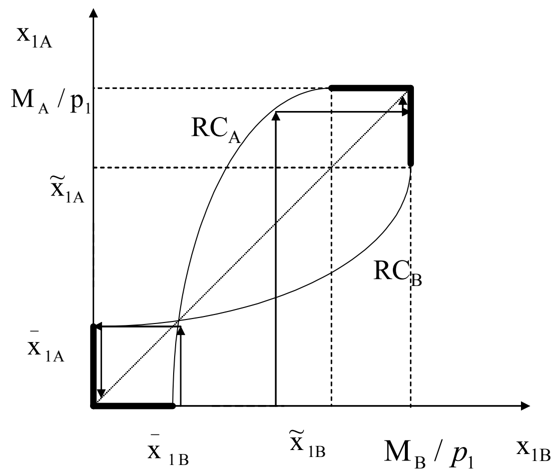

Figure 13: Strategic complements and concave reaction curves with essential externality when $\widetilde{x}_{1h} < \underline{x}_{1h} = M_h/p_1$

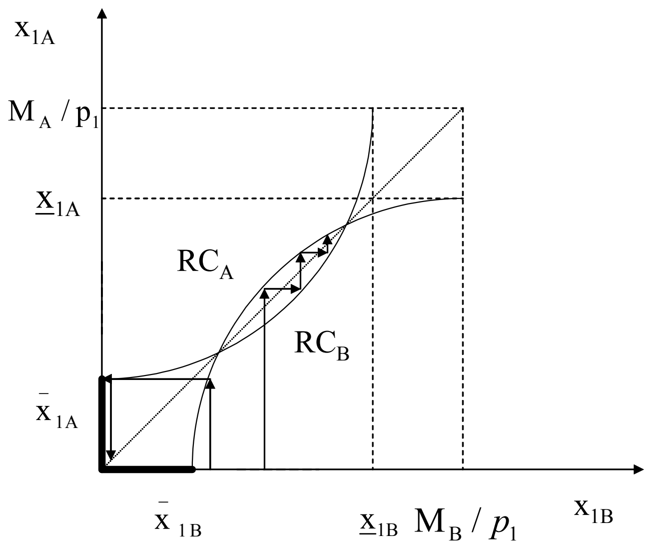

Figure 14: Strategic complements and concave reaction curves with essential externality when $\underline{x}_{1h} < M^h/p_1$.

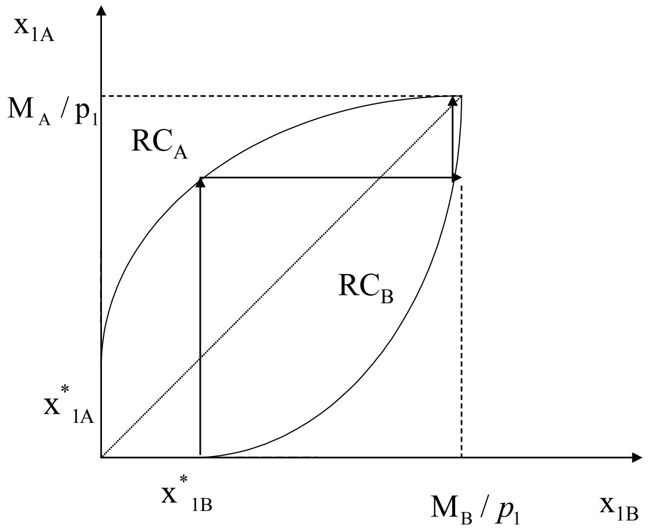

Figure 15: Strategic complements and concave reaction curves with essential externality when $\widetilde{x}_{1h} < \underline{x}_{1h} = M_h/p_1$

In this case, thus the externality is not essential: the reaction curve will always have a positive intercept.

Proposition 15 *With strategic complementarities and concave reaction and non essential externality, the symmetric game exhibits always an equilibrium.*

If the player strategies are positively correlated and if the reaction curve is concave with a positive intercept, then the system will always have the stable corner Nash equilibrium $(M^A/p_1, M^B/p_1)$ if $\underline{x}_{1h} = M_h/p_1$ (see Fig. 15 and Fig. 16).

If instead under the same conditions but with $\underline{x}_{1h} < M_h/p_1$ then the system will approach to a symmetric equilibrium with positive but not maximum quantities consumed of good 1(see Fig.17).

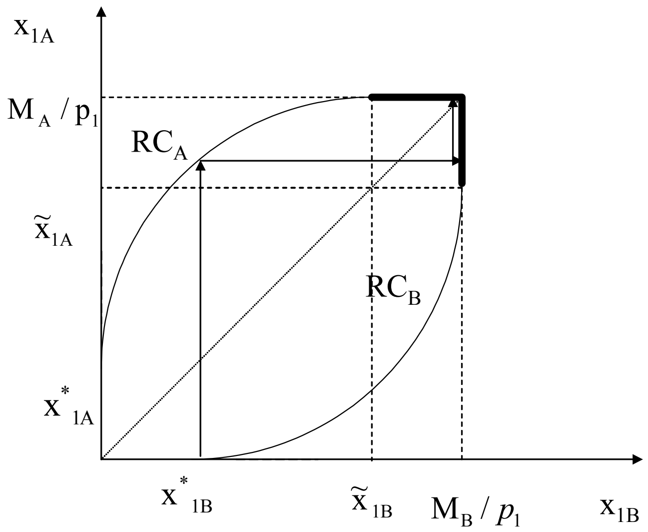

Figure 16: Strategic complements and concave reaction curves with non essential externality when $\widetilde{x}_{1h} < \underline{x}_{1h} = M_h/p_1$

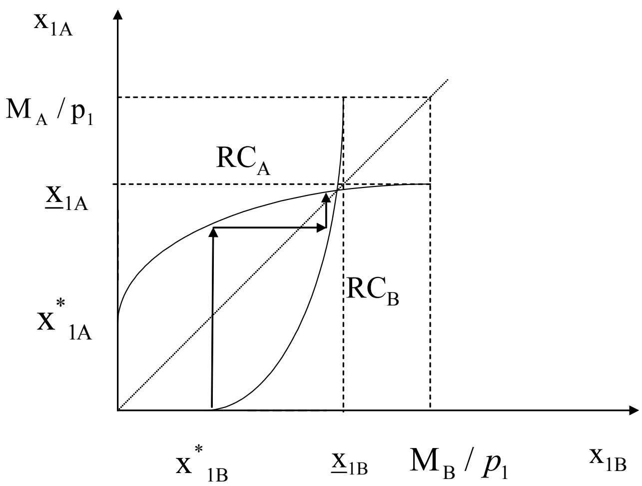

Figure 17: Strategic complements and concave reaction curves with non essential externality when $\underline{x}_{1h} < M_h/p_1$

2.3 Strategic substitutes and convex reaction curve

We are now considering the case in which the individual optimal strategies are negatively correlated. The consumption activity of each individual is affected by negative spillovers. Each consumer reacts negatively to the consumption action of the other individual. The marginal rate of substitution is increasing with the intensity of the externality effect: when the level of the externality effect increases, increasing quantities of good 2 are required to compensate the individual for a loss of a unity of good 1 to keep the same level of utility. With strategic substitutes the indifference curves become steeper at each externality decrease. Suppose that initially individual $h's$ maximisation problem has an interior equilibrium for a positive given level of the externality. Suppose that individual g decreases the consumption of good 1. Individual h indifference curve gets steeper: the new optimal quantity chosen by individual h of good 1 increases. At each externality decrease, individual h will increase his optimal quantity of good 1 till his maximum amount affordable is reached. Individual h reacts to the optimal strategy of individual g till $x_{1g} = 0$. The value of the quantity consumed by individual h when the effect of the externality is null defines the upper bound of the individual reaction curve with strategic substitutes.

Definition 16 *With strategic substitutes and convex reaction curves, the upper bond is* x_{1h}^*

We have previously defined x_{1h}^* as the optimal quantity of the good when $x_{1g} = 0$. Also in this case the value of the upper bound of the reaction curve depends on the effect of the externality on individual h utility maximisation problem. Suppose that at $x_{1g} = 0$ the utility maximisation problem has an interior equilibrium then $x_{1h}^* < M^h/p_1$. If instead at $x_{1g} = 0$ the individual maximisation problem has a boundary optimum, then the value of the upper bound is given by $x_{1h}^* = M^h/p_1$. This value of the optimal quantity can be reached when the other individual is consuming a positive quantity of the good ($\widetilde{x}_{1g}$): the reaction curve in this case is flat for each level of the externality between this critical level and $x_{1g} = 0$. Above this externality level, the individual decreases the consumption of the good at an increasing rate with the intensity of the externality effect. If the externality effect reaches the critical level $\overline{x}_{1g} < M^h/p_1$ the optimal strategy of the other individual is to consume only good 2. No unity of good 1 is consumed for each level of the externality above this critical level ($x_{1h} = 0$ for $\overline{x}_{1g} \leq x_{1g} \leq M^h/p_1$). When $\overline{x}_{1g}$ exists, the lower bound of the reaction curve is thus defined by a null consumption of good 1. This is the case of essential externality: the individual decision to participate in the market depends on the intensity of the externality effect.

Definition 17 *With strategic substitutes and convex reaction curves, if the lower bound is* $x_{1h} = 0$ *the externality is essential*

By contrast with the previous case, the lower bound is reached when the other individual is consuming a quantity approaching to the maximum level affordable.

There can be the case in which the individual consumes always positive quantities, despite of the intensity of the externality effect. Whichever is the level of the externality effect, the individual will never devote his income to buy only good 2 ($\underline{x}_{1h} > 0$): in this case the externality is not essential.

Definition 18 *With strategic substitutes and convex reaction curves, if the lower bound is* $\underline{x}_{1h}$ *the externality is non essential*

As in the strategic complement case, different scenarios can be distinguished according to the value of the upper and lower bound of the reaction curve and to the shape of the reaction curve in the neighborhood of these points.

Proposition 19 *With strategic substitutes and convex reaction curves and essential externality, than the symmetric game will always have three equilibria if and only if* $x^*_{1h} > \overline{x}_{1h}$

When the externality is essential, then the system will always have three equilibria if and only if $x^*_{1h} > \overline{x}_{1h}$. When this condition is satisfies the system will have an interior equilibrium and two corner Nash equilibria $((x^*_{1A}, 0),\ (0,\ x^*_{1B}))$. The value of x^*_{1h} depends on the externality effect on the utility maximisation problem as previously described.

In Figs 18 and 19, we have considered the special case in which $x^*_{1h} = M^h/p_1$: in this case the other individual is devoting all his income to buy good 1 when $x_{1g} = 0$. In Fig. 19, the boundary optimum is reached when $\overline{x}_{1g} < M^g/p_1$. If instead at $x_{1g} = 0$ the individual maximisation problem has an interior equilibrium, then the value upper bound will be a positive but never the maximum amount. Despite the value of the upper bound if $x^*_{1h} > \overline{x}_{1h}$ the system will always have the three equilibria previously defined and only the corner equilibria are stable. The convergence to one of the corner equilibrium depends on the initial conditions of the system: if initially the quantity consumed by individual g is low (high), the system converges to $(x^*_{1A}, 0)$. If instead individual h reaches the minimum when the other individual reaches the maximum ($\overline{x}_{1g} = M^g/p_1$ and $x^*_{1h} = \overline{x}_{1h}$) than two corner Nash equilibria arise: the system converges to the Nash equilibrium (0, x^*_{1B}).

Proposition 20 *With strategic substitutes and convex reaction curves and essential externality the symmetric game has always two equilibria if and only if* $x^*_{1h} = \overline{x}_{1h}$.

Under these restrictions, the system will have only the two corner Nash equilibria. The equilibrium (0, x^*_{1B}) is the stable one. In Fig. 20 we have represented the special case in which $x^*_{1h} = \overline{x}_{1h} = M^h/p_{1.}$

Proposition 21 *With strategic substitutes and convex reaction curves and essential externalities than the symmetric game has always one equilibrium if and only if* $x^*_{1h} < \overline{x}_{1h}$

If and only if the optimal quantity that individual $h's$ consumes when the externality is null is greater than the quantity that makes the other individual drop out from the market, then the strategic game will have only an interior stable equilibrium solution in which the quantity consumed by individual B is greater than the quantity consumed by individual A (see Fig. 21).

Different outcomes arise when the externality is non essential.

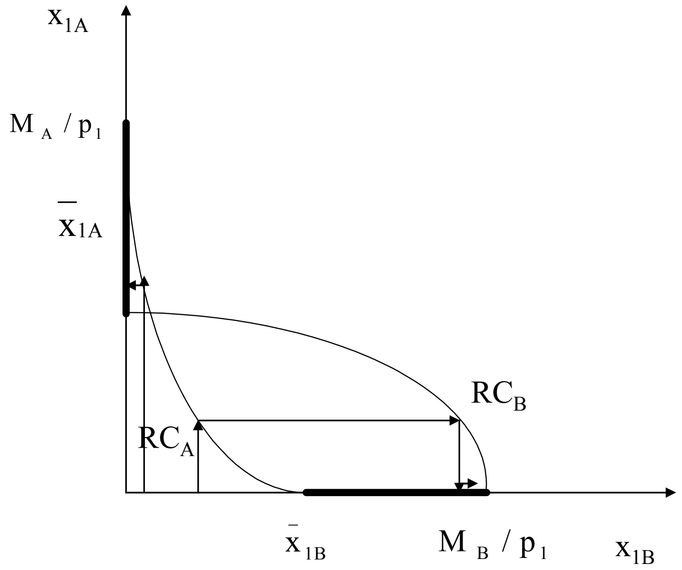

Figure 18: Strategic substitutes and convex reaction curves with essential externality when $\widetilde{x}_{1g} = 0$ and $x^*_{1h} > \overline{x}_{1h}$.

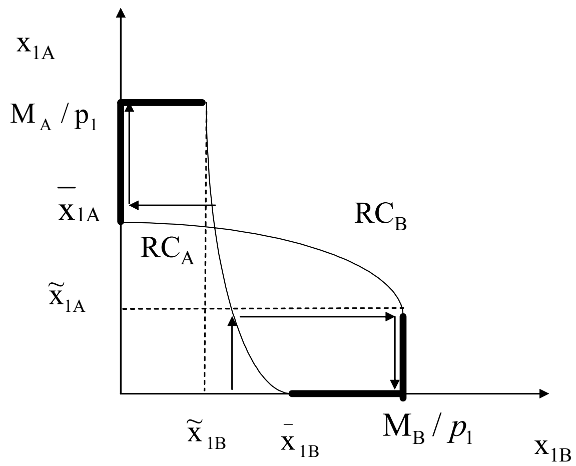

Figure 19: Strategic substitutes and convex reaction curves with essential externality when $\widetilde{x}_{1g} > 0$ and $x^*_{1h} > \overline{x}_{1h}$.

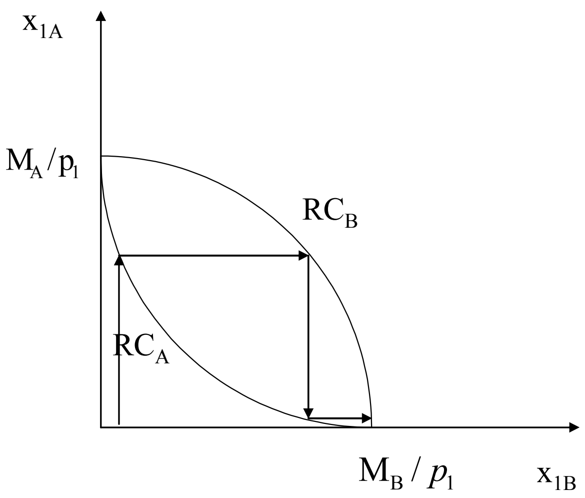

Figure 20: Strategic substitutes and convex reaction curves with essential externality when $x_{1h}^{*} = \overline{x}_{1h} = M^h/p_1$.

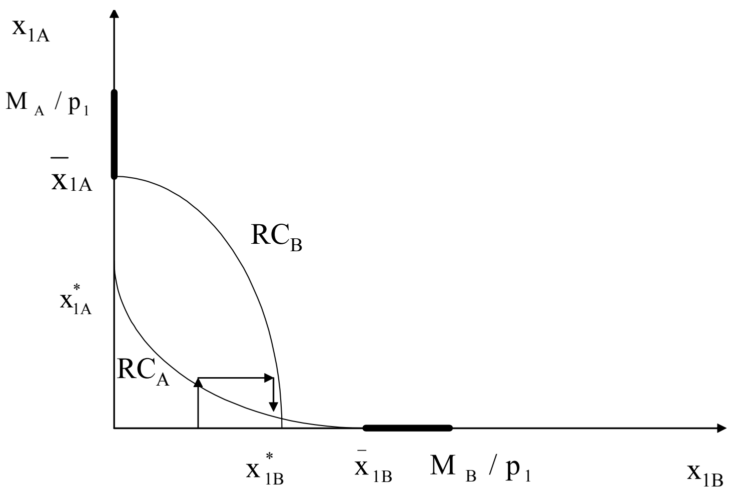

Figure 21: Strategic substitutes and convex reaction curves with essential externality when $x_{1h}^{*} < \overline{x}_{1h}$.

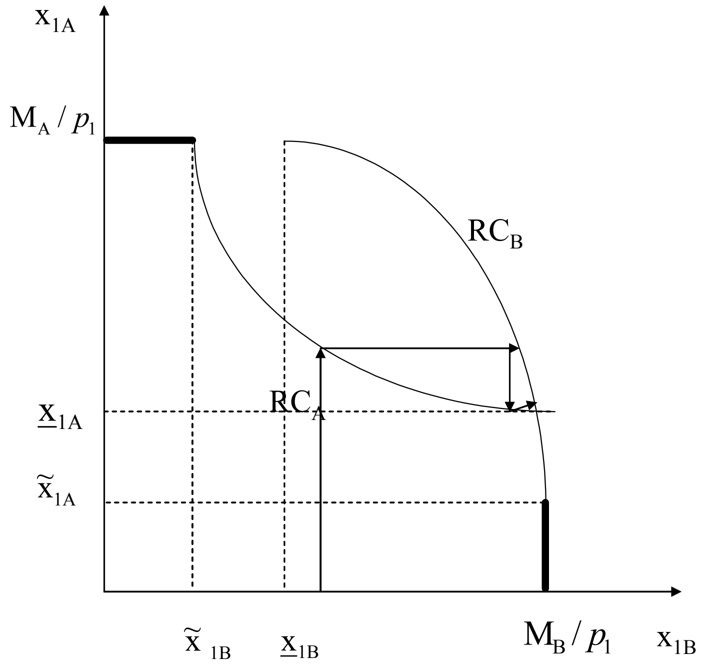

Figure 22: Strategic substitutes and convex reaction curves with essential externality when $\widetilde{x}_{1g} > \underline{x}_{1g}$

Proposition 22 *With strategic substitutes and convex reaction curves with non essential externality the symmetric game will always have at maximum two equilibria*

If the externality is non essential and $\widetilde{x}_{1g} < \underline{x}_{1g}$, the game has a unique interior stable equilibrium: the individual with the steeper reaction curve consumes more than the other individual (see Fig.22). In Fig. 23 it is consider the special case in which $\widetilde{x}_{1g} = 0$. If instead $\widetilde{x}_{1g} \geq \underline{x}_{1g}$, the system has two equilibria.

The system still converges to the equilibrium in which individual B consumes more than individual A (see, for example, Fig. 24 in which $\widetilde{x}_{1g} = \underline{x}_{1g}$).

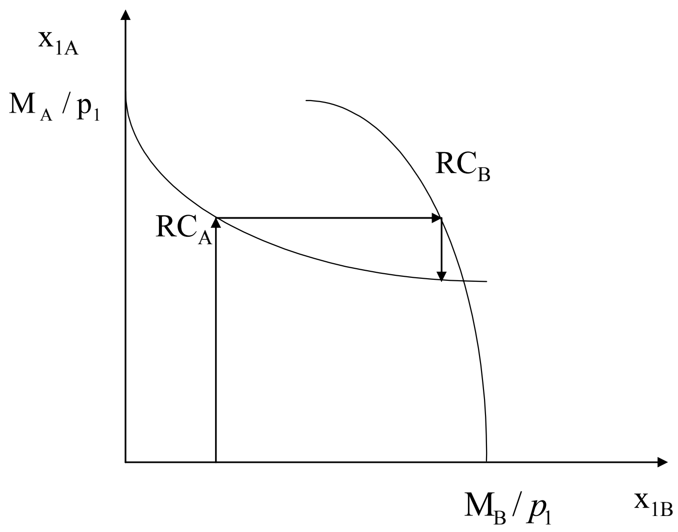

Figure 23: Strategic substitutes and convex reaction curves with non essential externality when $\widetilde{x}_{1g} = 0$

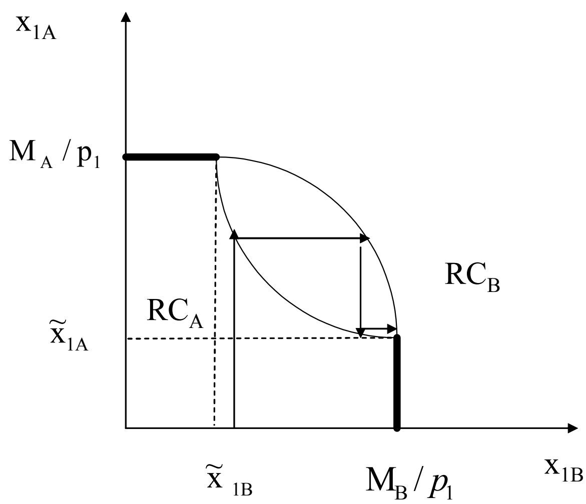

Figure 24: Strategic substitutes and convex reaction curves with non essential externality when $\widetilde{x}_{1g} = \underline{x}_{1h}$

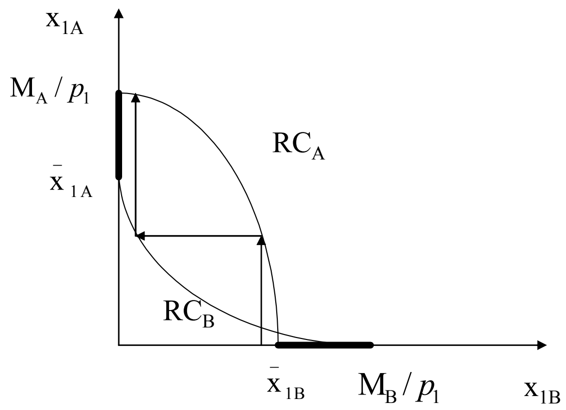

Figure 25: Strategic substitutes and concave reaction curves with essential externality when $\widetilde{x}_{1g} = 0$

2.4 Strategic substitutes and concave reaction curves

We are still in the case in which the individual is decreasing its consumption with the intensity of the externality effect. The reaction curve has still a negative slope but the marginal rate of substitution between the two goods is decreasing with the intensity of the externality effect.

The analysis of the existence and stability of the equilibria is similar to the previous case. The reaction curve are similarly upper and lower bounded. Also in this case, if the externality is essential three equilibria occur if and only if $x^*_{1h} > \overline{x}_{1h}$ and the initial conditions drive the system to one of the corner solution equilibrium (see Fig. 25 and Fig. 26). If $x^*_{1h} = \overline{x}_{1h}$ than the system will approach to the corner solution equilibrium in which only individual A consumes the maximum quantity of good 1 $(x^*_{1h}, 0)$ (in Fig. 27 it is consider the special case in which $\overline{x}_{1h} = M^h/p_1$). If instead $x^*_{1h} < \overline{x}_{1h}$ then the system will have an unique stable equilibrium in which the individual with the flatter reaction curve consumes more.

In the case in which the externality is non essential, instead, if $\widetilde{x}_{1g} > \underline{x}_{1g}$ the

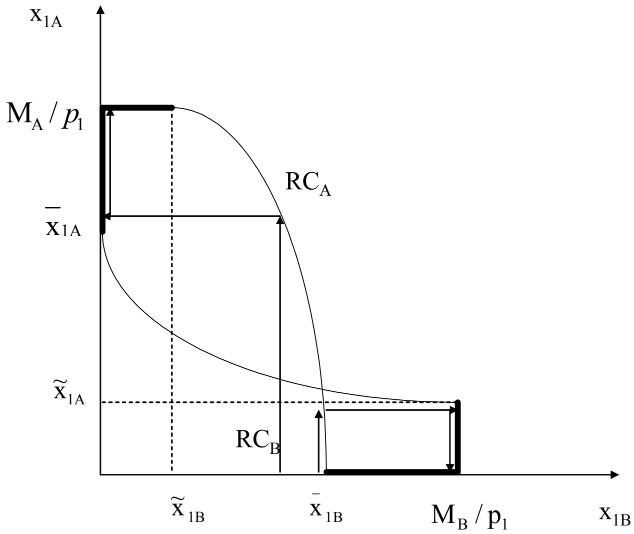

Figure 26: Strategic substitutes and concave reaction curves with essential externality when $\widetilde{x}_{1g} > 0$

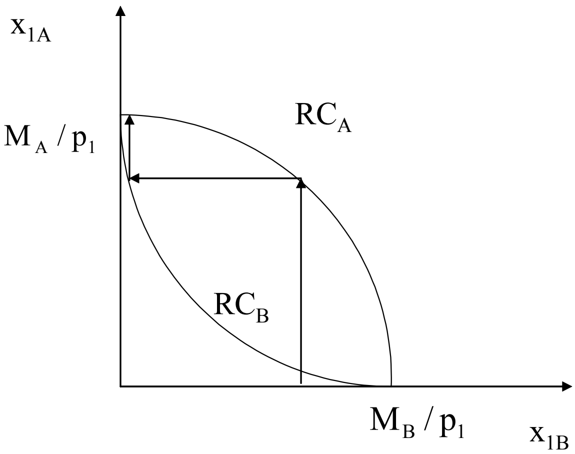

Figure 27: Strategic substitutes and concave reaction curves with essential externality when $x^{*}_{1h} = \overline{x}_{1h} = M^{h}/p_{1}$.

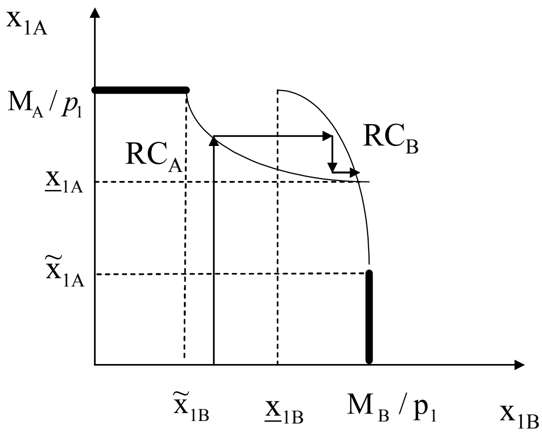

Figure 28: Strategic substitutes and concave reaction curves with essential externality when $\widetilde{x}_{1g} > \underline{x}_{1g}$

system will have a unique stable equilibrium in which the individual with the flatter reaction curve is consuming more than the other individual (see Fig. 28 and Fig. 29).

If $\widetilde{x}_{1g} = \underline{x}_{1g}$ the system will have two equilibria but still the system will approach to the one in which individual A is consuming a higher quantity with regard to individual B (see Fig. 30).

3 Two Comparative static examples

It has been argued in the literature that in a economic system with strategic complementarities positive shocks are magnified and propagates. We show in this section one counterintuitive case in which instead this virtuous process doesn't occur. We start by focusing on the case with strategic complements with essential and concave reaction curves. It has been previously shown that in this case the system has two

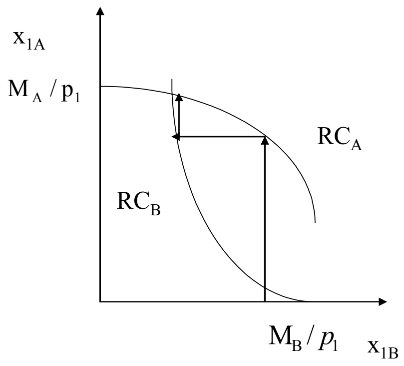

Figure 29: Strategic substitutes and concave reaction curves with non essential externality when $\widetilde{x}_{1g} = 0$

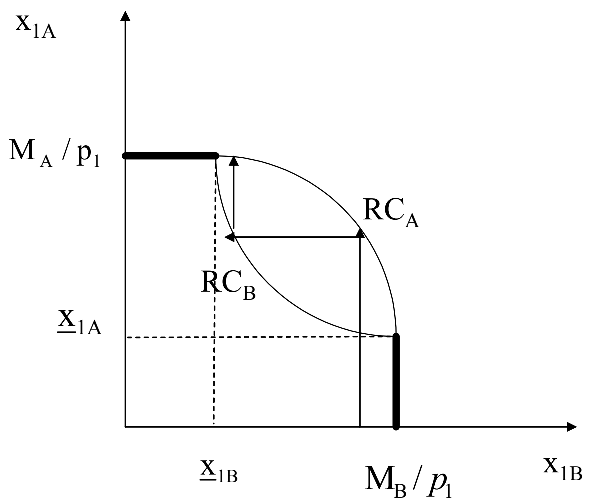

Figure 30: Strategic substitutes and concave reaction curves with non essential externality when $\widetilde{x}_{1g} = \underline{x}_{1h}$

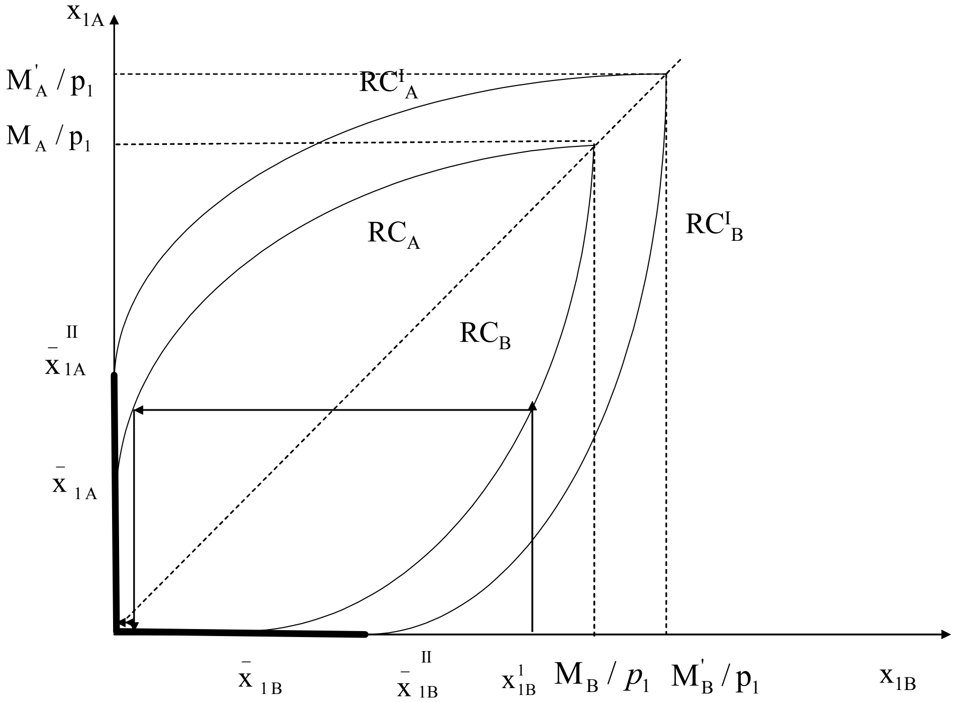

Figure 31: Positive shocks with essential strategic complementarities and convex reaction curves

equilibria and converge to the corner solution equilibria in which both the individual don't buy the good. We now consider the case of a positive symmetric shock that increases the income level of the consumer (see Fig. 31).

For simplicity it is assumed that the income affects only the intercept of the reaction curve. This is the case, for example, for Cobb Douglas preferences with a linear quadratic externality effect. As each income increases, the reaction curve shifts upward: the maximum quantity consumed by each individual increases. The movement of $\overline{x}_{1g}$ depends on the nature of the good. If the good is inferior, this point will move upward. If it is normal this point will move downward. If the good

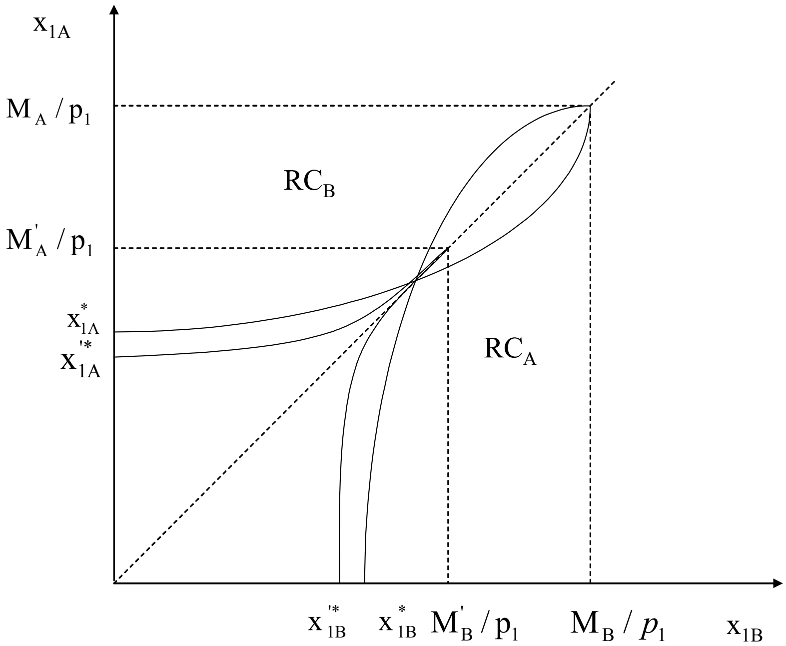

Figure 32: Negative shocks with strategic complementarities, convex reaction curves and essential externality

is inferior whichever is the intensity of the shock, the system will have always two equilibria and the system will converge always to the corner equilibrium (0,0). If the good is normal, this result is still true in the case of essential externality.

Suppose now that the symmetric game is characterised by essential externality and convex reaction curves (see Fig. 32). Suppose that the good is normal and that the consumption activity is affected by a symmetric negative shock. Suppose that the new maximum income level is $x_{1h}^{'M}$ and the new intercept of the reaction curve is $x_{1h}^{'*}$. After the shock, the system has the corner Nash equilibrium $(M_A^{'}/p_1, M_B^{'}/p_1)$, and the system will approach to it. Despite of the negative shock, the two individual will use all their income to buy good 1.

Even if these two cases presented are very specific, they can disprove the general prediction that positive (negative) shocks can drive the system to upper (lower equilibria)

Other counterintuitive cases can be found for shocks in the price level. Extreme cases can occur in which the price change can even switch the sign of the interaction in the players strategies. It is beyond the scope of this thesis to analyse in details all these case.

4 Assuming n goods

The analysis carried out so far can be extended to the n good case adopting the same procedure used in footnote 3 of Chapter 4. Suppose that the individual is maximising his utility on the consumption of n goods:

$$\max_{x_{ih}}\{U^h(x_{1h}, x_{2h}, ..., x_{nh}, x_{1g}) \mid p_1x_{1h} + ... + p_nx_{nh} = M_h\} \tag{5}$$

This maximisation problem can be performed into two steps:

- Maximisation of (5) regarding to the $n-1$ goods excluding good 1, obtaining the following indirect utility function:

$$V^h(x_{1h}, x_{1g}, M_h, p) = \max_{x_{ih}>1}\{u_h(x_h, x_{1k})| \sum_{i=2} p_ix_{ih} \leq m_h - p_1x_{1h}\}$$

- Maximisation of the indirect utility function with respect to good 1:

$$\max_{x_{1h}} V^h(x_{1h}, x_{1g}, M_h, p)$$

The solution of this maximisation problem provides the reaction curve of individual h for good 1. The properties of the reaction curve can be derived using the same methodology adopted in the two good case. The same results in terms of existence and stability of the equilibria in each different scenario previously examined are still valid even if the consumption set has a n vector dimension.

5 Conclusion

"Coordination failure" models have been extensively used in macroeconomics for the explanation of low-activity equilibrium. The microfoundation of this class of model appears to highly stylised and general. In this chapter we propose a more detailed microfoundation when strategic interactions occur in individual consumption patterns. We link the existence of multiple equilibria to particular properties of the slope and rate of increase of the reaction curves. These ordinal properties are strictly connected to the properties of the marginal utility. This detailed analysis enriches and clarify the economic implications of this class of models. Firstly, corner solution equilibria can arise both with strategic complementarities and with strategic substitutes. The individual decision to participate in the market is strictly connected to the feature of the externality effect. For particular level of the externality, it may be thus privately efficient for the agent not to participate to the market. Secondly, a switch in the reaction curves (Cooper and John (1988)) can occur only if the effect of the externality on the marginal rate of substitution changes sign. Multiple equilibria can be generated in the system without imposing switches in the reaction curves. Thirdly, counterintuitive comparative statics arise. Positive exogenous shocks doesn't drive the system always to upper Pareto rankable equilibria. The general prediction that shocks are propagated over time in all sectors is not generally valid when a more precise structure is analysed. Even if the chapter focuses on symmetric non cooperative solutions, the analysis can be easily extended to non symmetric equilibria. The application of this model to an imperfect competitive framework will be the object of future research.

Appendix A: Ordinality of Reaction Curve Properties

Define the utility function as:

$$U^h(x_{1h}, \frac{M_h - p_1 x_{1h}}{p_2}, x_{1g}) = V^h(x_{1h}, x_{1g}, \theta)$$

Consider a monotonic transformation of the original function:

$$\overline{V}^h = F(V^h(x_{1h}, x_{1g}, \theta))$$

The reaction curve is given implicitly by:

$$\overline{V}^h_{x_{1h}} = F' V^h_{1h} = 0 \tag{6}$$

The slope of individual h reaction curve is given by:

$$\frac{dx_{1h}}{dx_{1g}} = -\frac{\overline{V}^h_{x_{1h}x_{1g}}}{\overline{V}^h_{x_{1h}x_{1h}}} - \frac{F^{''}(V^h_{1h}V^h_{1g}) + F^{'}V^h_{1h1g}}{F^{''}(V^h_{1h})^2 + F^{'}V^h_{1h1h}} \tag{7}$$

Since (6) holds, (7) becomes:

$$\frac{dx_{1h}}{dx_{1g}} = -\frac{F^{'}V^h_{1h1g}}{F^{'}V^h_{1h1h}} = \frac{V^h_{1h1g}}{V^h_{1h1h}}$$

The slope property is invariant to any monotonic transformation, i.e. it is an ordinal property.

Consider now the other derivatives:

$$\overline{V}^h_{x_{1h}x_{1h}x_{1g}} = F^{'''}(V^h_{1h})^2 V^h_{1g} + 2F^{''}V^h_{1h}V^h_{1h1g} + F^{''}V^h_{1g}V^h_{1h1h} + F^{'}V^h_{1h1h1g}$$

$$\overline{V}^h_{x_{1h}x_{1h}x_{1h}} = F^{'''}(V^h_{1h})^3 + 3F^{''}V^h_{1h}V^h_{1h1h} + F^{'}V^h_{1h1h1h}$$

$$\overline{V}^h_{x_{1h}x_{1g}} = F^{''}V^h_{1h}V^h_{1g} + F^{'}V^h_{1h1g}$$

$$\overline{V}^h_{x_{1h}x_{1g}x_{1g}} = F^{'''}(V^h_{1g})^2V^h_{1h} + F^{''}V^h_{1g1g}V^h_{1h} + 2F^{''}V^h_{1h1g}V^h_{1g} + F^{'}V^h_{1h1g1g}$$

Since (6) holds, we get:

$$\overline{V}^h_{x_{1h}x_{1h}x_{1g}} = F^{''}V^h_{1g}V^h_{1h1h} + F^{'}V^h_{1h1h1g}$$

$$\overline{V}^h_{x_{1h}x_{1h}x_{1h}} = F^{'}V^h_{1h1h1h}$$

$$\overline{V}^h_{x_{1h}x_{1g}} = F^{'}V^h_{1h1g}$$

$$\overline{V}^h_{x_{1h}x_{1g}x_{1g}} = 2F^{''}V^h_{1h1g}V^h_{1g} + F^{'}V^h_{1h1g1g}$$

The curvature of the reaction curve is given by:

$$
\begin{aligned}
\frac{d^2x_{1h}}{dx_{1g}^2} &= \frac{2\overline{V}^h_{x_{1h}x_{1h}x_{1g}}\overline{V}^h_{x_{1h}x_{1g}}\overline{V}^h_{x_{1h}x_{1h}} - \overline{V}^h_{x_{1h}x_{1h}x_{1h}}(\overline{V}^h_{x_{1h}x_{1g}})^2 - \overline{V}^h_{x_{1h}x_{1g}x_{1g}}(\overline{V}^h_{x_{1h}x_{1h}})^2}{(-\overline{V}^h_{x_{1h}x_{1h}})^3} = \\
&= \frac{2(F^{''}V^h_{1g}V^h_{1h1h} + F^{'}V^h_{1h1h1g})F^{'}V^h_{1h1g}F^{'}V^h_{1h1h} - F^{'}V^h_{1h1h1h}(F^{'}V^h_{1h1g})^2 -}{(-F^{'}V^h_{1h1h})^3} \\
&\quad \frac{(2F^{''}V^h_{1h1g}V^h_{1g} + F^{'}V^h_{1h1g1g})(F^{'}V^h_{1h1h})^2}{(-F^{'}V^h_{1h1h})^3} \\
&= \frac{2F^{'}V^h_{1h1h1g}F^{'}V^h_{1h1g}F^{'}V^h_{1h1h} - F^{'}V^h_{1h1h1h}(F^{'}V^h_{1h1g})^2 - F^{'}V^h_{1h1g1g}(F^{'}V^h_{1h1h})^2}{(-F^{'}V^h_{1h1h})^3} = \\
&= \frac{2V^h_{1h1h1g}V^h_{1h1g}V^h_{1h1h} - V^h_{1h1h1h}(V^h_{1h1g})^2 - V^h_{1h1g1g}(V^h_{1h1h})^2}{(-V^h_{1h1h})^3}
\end{aligned}
$$

Also this property is ordinal.

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