THE UNIVERSITY of York

Discussion Papers in Economics

No. 2003/20

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ABSTRACT

Both borrowers and creditors often have an implicit option to extend debt maturity as the debtor approaches financial distress. This implicit "extension option" is associated with the possibility for debtors and creditors to renegotiate the debt contract in the hope that extending debt maturity may allow the debtor to overcome temporary liquidity problems. This paper analyses and evaluates such "extension" option in a time independent setting with constant nominal capital structure and in a time dependent setting with not constant nominal capital structure. The "extension option" is shown to significantly increase the value of equity and has a non-negligible impact on debt credit spreads. The "extension option" can also increase the short-term credit spreads of outstanding debt and, in this respect, it ameliorates the shortcoming typical of structural models of credit risk, i.e. the under-prediction of short term credit spreads. The value of the extension option is very sensitive to different possible exercise

policies. Four such policies are illustrated, encompassing cases in which the debtor extorts concessions (to extend debt maturity) from creditors and cases in which creditors make self interested concessions (to extend debt maturity). In general, when default is triggered by the "worthless equity" condition, the value of the extension option is much higher than when default is triggered by a liquidity shortage. The option to renegotiate debt maturity is of interest because extending debt maturity can decrease debt value even without cutting promised coupon payment, i.e. without giving up part of the tax shield associated with coupon payments.

Keywords: corporate debt, debt maturity, default barrier, renegotiation, credit spreads.

INTRODUCTION

Firms that approach financial distress may renegotiate their debt obligations. Such debt renegotiations can entail extending the original contractual maturity of debt in order to allow the firm to overcome temporary problems such as a temporary lack of liquidity. The re-negotiation of debt maturity as the debtor approaches financial distress has been neglected by the debt valuation literature that adopts a continuous time finance approach. Such literature has instead concentrated on the re-negotiation of contractual coupon payments or of debt principal.

In this paper the problem is the valuation of the firm's debt (and equity) when debt holders and equity holders may have the ability to extend debt maturity in order to avoid default and costly assets liquidation. In such case debt holders and equity holders have an "implicit option to renegotiate and extend debt average maturity".

The results of the analysis are:

- the option available to equity holders to extend the average maturity of debt increases equity value more than it decreases debt value; such option can materially increase equity value, while often causing a non negligible increase or decrease in the yield required by debt holders;
- provided debt maturity is extended before assets liquidation, the different rational "extension option exercise policies" do not alter total firm value, but they significantly affect the extension option values;
- as in Longstaff (1990), sometimes it is possible for both equity holders and debt holders to benefit from the extension of debt maturity as the firm approaches distress;

- different default conditions, either worthless equity or cash flow shortage, can materially affect the values of the extension option and imply different incentives for debt holders and equity holders to re-negotiate and extend debt maturity;
- the presence of an implicit extension option can improve the prediction of the structural model by inflating short-term credit spreads.

The analysis of this paper is split into time dependent and time independent settings. In a time independent setting, single debt issues are continuously refunded as they continuously fall due at maturity. Thus the nominal amount of debt outstanding at any time is constant, which makes the valuation of debt and equity a problem independent of time. Later, instead, the "extension option" is analysed in a time dependent setting in which debt is *not* refunded at maturity, in which the nominal amount of debt outstanding is *not* constant, in which the default probability is lower and in which the extension option is less valuable.

Past literature

Extendible debt was valued by Brennan and Schwartz in 1977 and by Ananthanarayanan and Schwartz in 1980, but these two papers assume that debt is default free. Two other papers deal with debt that is subject to default risk and whose maturity can be extended by equity holders or by debt holders. The first paper is by Franks and Torous (1989) and considers the implicit option for equity holders to file for US Code Chapter 11 reorganisation, which entails suspending all payments of coupons and principal to debt holders. Franks and Torous show that recognising this implicit option to file for chapter 11 makes contingent claims models capable of predicting credit spreads on corporate debt that more closely approximate those observed in the bond market.

The second paper is by Longtaff (1990) and provides closed form solutions for a similar option for equity holders to extend debt maturity. Longstaff also considers the option for debt holders to spontaneously extend debt maturity in order to avoid costly assets liquidation as the debtor defaults.

Both Franks/Torous and Longstaff restrict their attention to a Mertonian setting in which default and extension of debt maturity can take place just on the contractually agreed debt maturity date. Instead, this paper considers the case in which default and extension of debt maturity can take place at any time. Central to this paper is the case in which debt maturity can be renegotiated by equity holders and debt holders. In fact the re-negotiation of debt maturity has been neglected by the debt valuation literature concerned with strategic debt service (e.g. Anderson and Sundaresan (1996), Anderson and Sundaresan and Tychon (1996), Mella-Barral and Perraudin (1997)).

In sections 1 to 3 the analysis of the option to extend debt maturity is carried out in a time independent setting in which default is triggered by cash flow shortage or by worthless equity, whereas in section 4 the same analysis is carried out in a time dependent setting in which default is triggered by worthless equity.

1. THE GENERAL MODEL IN A TIME INDEPENDENT SETTING

1.1) Results from past literature

Now we introduce the notation and some results of past literature. These results are the basis for the subsequent analysis of the option to extend debt maturity.

Let us assume that equity holders and debt holders are risk neutral and have perfect information. V is the value of the firm's assets, whose risk neutral process is a geometric Brownian motion, i.e.:

1) $dV = (r-d)\,V\,dt + s\,V\,dz$,

where:

- s is the volatility of the firm's assets;
- d is the firm's assets pay-out rate;
- r is the default free interest rate, which is assumed constant
- *dz* differential of a Wiener process.

In addition, let us assume that:

- a denotes bankruptcy/liquidation costs proportional to assets value;
- K is the fixed cost of assets liquidation/bankruptcy,
- "tax" is the corporate income tax rate,
- C is the annual coupon,
- P is the face value of debt,
- $c = C / P$,
- m is the fraction of outstanding debt that is retired and substituted with newly issued debt every year (in short m is the debt retirement rate),
- ff(V) is the value of the firm's debt when extension of debt maturity is not possible,
- BC(V) is the value of the firm's bankruptcy costs,
- e(V) is the value of the firm's equity,
- TT(V) is the value of the tax shield,

- V_B is the default the barrier (i.e. the value of the firm's assets at which default occurs).

Following Leland (1998) and Ericsson (2000), but adding fixed bankruptcy costs (K), it is possible to show that

$$2)\ ff(V) = \frac{C + m \cdot P}{r + m} + \left[-\frac{C + m \cdot P}{r + m} + (1 - a) \cdot V_B - K \right] \cdot \left[\frac{V}{V_B} \right]^b$$

3)

$$e(V) = V + tax \cdot \frac{C}{r} \cdot \left\{ 1 - \left[\frac{V}{V_B} \right]^j \right\} - (a \cdot V_B + K) \cdot \left[\frac{V}{V_B} \right]^j - \frac{C + mP}{r + m} \left\{ 1 - \left[\frac{V}{V_B} \right]^b \right\} - \left[(1 - a) \cdot V_B - K \right] \cdot \left[\frac{V}{V_B} \right]^b$$

with

$$4)\ b = \frac{-\left(r - d - \frac{s^2}{2} \right) - \sqrt{\left(r - d - \frac{s^2}{2} \right)^2 + 2 \cdot (r + m) \cdot s^2}}{s^2}$$

$$5)\ j = \frac{-\left(r - d - \frac{s^2}{2} \right) - \sqrt{\left(r - d - \frac{s^2}{2} \right)^2 + 2 \cdot r \cdot s^2}}{s^2}.$$

Total firm value is then equal to

$$6)\ ff(V) + e(V) = V + tax \cdot \frac{C}{r} \cdot \left\{ 1 - \left[\frac{V}{V_B} \right]^j \right\} - (a \cdot V_B + K) \cdot \left[\frac{V}{V_B} \right]^j.$$

The important aspect is that every year a fraction “m” of debt is continuously refunded as it falls due. Then average debt maturity is equal to 1/m years.

1.2) When average debt maturity can be extended

The above results are next modified to account for the possibility that equity holders and debt holders renegotiate the debt contract and extend debt average maturity by rescheduling the payments of debt principal. Re-negotiation may take place in an informal workout or in a formal bankruptcy proceeding. It is important to remark that in what follows it is assumed that all single debt issues comprising the firm's total outstanding debt have their respective time to maturity extended at the same time and by the same proportion. For example, if there are two outstanding debt issues, one with a residual life of 1 year and the other one of 2 years, their respective residual lives are *simultaneously* extended to 2 years and 4 years.

Equity holders and debt holders may want to renegotiate the debt contract and extend debt maturity before default or as soon as default takes place, where default here means missing a payment that is due to creditors. By agreeing with creditors to postpone repayment of debt principal, equity holders may avoid default, insolvency or difficult and costly refunding through issuance of new debt.

On the other hand, also debt holders may be enticed to renegotiate the debt contract as explained later in section 2. An important assumption underlies all the analysis: equity holders always keep paying the contractually agreed coupons to debt holders until debt principal is eventually paid back.

Let us now assume V_R denotes the value of the firm's assets at which debt average maturity is extended. For now we take V_R as given, but in section 2 it will be shown how V_R can be determined. Anyway V_R cannot be lower than V_B, otherwise debt maturity would not be extended and there would be no difference from the analyses of past literature, thus

7) $V_R \geq V_B$.

On the other hand, it is here assumed that

8) $V_R < V_0$,

where V_0 denotes the firm's assets value today. Condition 8) does not imply a great loss in generality as it will become apparent later.

Then $1/m_R$ is debt average maturity after "extension" and $1/m$ is debt average maturity before "extension". The change from m to m_R is irreversible, and we can write:

9) $\infty \geq m \geq m_R \geq 0$.

For simplicity, in all this paper it is assumed that debt average maturity can be extended just once.

When $V \geq V_R$, debt value before "extension", F(V), must satisfy

$$10)\ \frac{1}{2} \cdot s^2 \cdot V^2 F_{VV} + (r-d) \cdot V \cdot F_V - r \cdot F + C + m \cdot (P - F) = 0,$$

subject to $F(V \to \infty) \to \frac{C + m \cdot P}{r + m}$ and to $F(V_R) = f(V_R)$,

where debt value after "extension", f(V), must satisfy

$$11)\ \frac{1}{2} \cdot s^2 \cdot V^2 f_{VV} + (r-d) \cdot V \cdot f_V - r \cdot f + C + m_R \cdot (P - f) = 0,$$

subject to $f(V \to \infty) \to \frac{C + m_R \cdot P}{r + m_R}$ and to $f(V_{BR}) = (1 - a) \cdot V_{BR} - K$, where V_{BR} denotes the default barrier after debt average maturity has been extended. In general V_{BR} is lower than V_B, since by extending debt maturity default is postponed.

The solutions to equations 10) and 11) are respectively

12)

$$F(V)=\frac{C+mP}{r+m}+\left[-\frac{C+m\cdot P}{r+m}+\frac{C+m_R\cdot P}{r+m_R}+\left[-\frac{C+m_R\cdot P}{r+m_R}+(1-a)\cdot V_{BR}-K\right]\cdot\left[\frac{V_R}{V_{BR}}\right]^h\right]\cdot\left[\frac{V}{V_R}\right]^b$$

13) $$f(V)=\frac{C+m_R\cdot P}{r+m_R}+\left[-\frac{C+m_R\cdot P}{r+m_R}+(1-a)\cdot V_{BR}-K\right]\cdot\left[\frac{V}{V_{BR}}\right]^h,$$

with b as per equation 4) and with

14) $$h=\frac{-\left(r-d-\frac{s^2}{2}\right)-\sqrt{\left(r-d-\frac{s^2}{2}\right)^2+2\cdot\left(r+m_R\right)\cdot s^2}}{s^2}.$$

Then, the value of equity in the presence of the "extension option" [E(V)] is equal to total firm value in the presence of the “extension option”, which is given by equation 17) below, minus the value of debt before “extension”, thus:

15) $$E(V)=V+tax\cdot\frac{C}{r}\cdot\left\{1-\left[\frac{V}{V_{BR}}\right]^j\right\}-\left(a\cdot V_B+K\right)\cdot\left[\frac{V}{V_{BR}}\right]^j-F(V).$$

Instead, the value of equity after the "extension option" has been exercised [ER(V)] is equal to total firm value as per equation 17) below, minus the value of debt after “extension”, thus:

$$16)\ ER(V) = V + tax \cdot \frac{C}{r} \cdot \left\{ 1 - \left[\frac{V}{V_{BR}} \right]^{j} \right\} - \left(a \cdot V_B + K \right) \cdot \left[\frac{V}{V_{BR}} \right]^{j} - f(V).$$

When debt maturity can be extended the above formulas give the values of debt and equity.

1.3) Modigliani and Miller' s proposition 1 and the option to extend debt maturity

Now, since corporate taxes and bankruptcy costs have been assumed, Modigliani - Miller' s proposition 1 cannot hold. This entails that total firm value changes due to the presence of the "extension option": total firm value would no longer be given by equation 6) but by

$$17)\ F(V) + E(V) = V + tax \cdot \frac{C}{r} \cdot \left\{ 1 - \left[\frac{V}{V_{BR}} \right]^{j} \right\} - \left(a \cdot V_B + K \right) \cdot \left[\frac{V}{V_{BR}} \right]^{j}.$$

This equation is the same as equation 6), but for the fact that the default barrier is now lower since debt maturity is extended before or at default. This means that total firm value is now higher than total firm value as per 6), because a lower default barrier entails higher expected value of the debt induced tax shield

[$TT(V) = tax \cdot \frac{C}{r} \cdot \left\{ 1 - \left[\frac{V}{V_{BR}} \right]^{j} \right\}$] and lower expected value of bankruptcy costs

[$BC(V) = \left(a \cdot V_B + K \right) \cdot \left[\frac{V}{V_{BR}} \right]^{j}$].

Now, since longer debt maturity implies higher total firm value, it is not clear why firms should ever be interested in an option to extend debt maturity if they could simply choose to issue debt of longer maturity in the first place. As Leland (1996) puts it, longer-term debt may not be incentive compatible, in other words it is too sensitive to assets substitution or to other agency costs. Anyhow, corporate debt usually does have finite maturity.

Equation 17) also reveals that total firm value does not depend on V_R as long as condition 7 holds. In other words, *given the presence of corporate taxes and/or bankruptcy costs, total firm value only depends on "whether or not" debt maturity is extended not later than default, but not on "when" debt maturity is extended.* Later numerical examples will confirm this statement.

1.4) The payoff and the values of the option to extend debt maturity

The value of debt whose maturity can be extended (F(V)) can be viewed as the value of debt whose maturity cannot be extended (ff(V)) plus a position in the option to extend debt maturity: hereafter the value of this (often short) position is denoted by O(V).

In the same way the value of equity when debt maturity can be extended (E(V)) can be thought of as the value of equity when debt maturity cannot be extended (e(V)) plus a position in the option to extend debt maturity: hereafter the value of this (often long) position is denoted by OE(V).

At this point we can specify the payoffs for OE(V) and O(V) when it is equity holders who decide as to the exercise of the "extension option" and determine V_R :

18) $OE(V_R) = \max\{E(V_R) - e(V_R), 0\}$,

19) $O(V_R) = F(V_R) - ff(V_R)$.

Instead when it is debt holders who exercise the "extension option", which is a possibility as is noticed later on, then[1]:

20) $OE(V_R) = E(V_R) - e(V_R)$,

21) $O(V_R) = \max\{F(V_R) - ff(V_R), 0\}$.

The above allows to derive the expression for O(V) as the difference between F(V) and ff(V) and the expression for OE(V) as the difference between E(V) and e(V):

22)

$$O(V) = \left[-\frac{C + m \cdot P}{r + m} + \frac{C + m_R \cdot P}{r + m_R} + \left[-\frac{C + m_R \cdot P}{r + m_R} + (1 - a) \cdot V_{BR} - K \right] \left[\frac{V_R}{V_{BR}} \right]^h \right] \cdot \left[\frac{V}{V_R} \right]^b$$

$$- \left[-\frac{C + m \cdot P}{r + m} + (1 - a) \cdot V_B - K \right] \left[\frac{V}{V_B} \right]^b .$$

[1]Clearly these payoffs imply that $E[V=V_{R1}] = ER[V=V_{R1}]$ with $F[V=V_{R1}] = f[V=V_{R1}]$.

23)

$$OE(V) = -O(V) + tax \cdot \frac{C}{r} \cdot \left\{ -\left[\frac{V}{V_{BR}}\right]^{j} + \left[\frac{V}{V_{B}}\right]^{j} \right\} + \left\{ -\left(a \cdot V_{BR} + K\right) \cdot \left[\frac{V}{V_{BR}}\right]^{j} + \left(a \cdot V_{B} + K\right) \cdot \left[\frac{V}{V_{B}}\right]^{j} \right\}$$

.

Equation 23) highlights how OE(V) is different from O(V). In the jargon of options: the value of the (often long) position in the "extension option" (OE) is different from the value of the (often short) position in that same "extension option" (O). This unusual asymmetry is again due to the fact that Modigliani and Miller's proposition 1 does not hold, because taxes and bankruptcy costs are assumed to exist.

2. CONDITIONS FOR EXTENSION OF DEBT MATURITY AND FOR DEFAULT

As stated above, given that $V_{BR} \leq V_{B} \leq V_{R}$, when the value of the firm's assets (V) declines down to V_{R} , debt average maturity is extended. Possible ways to determine V_{R} are now discussed and then possible ways to determine V_{B} and V_{BR} are discussed too.

2.1) The conditions for debt maturity to be extended

As for V_{R} , there are at least four possible ways to determine when debt maturity can be extended.

2.1.1) Take-it-or-leave-it offers

Debt maturity can be extended when equity holders make the following "take-it-or-leave-it" hostile offer to debt holders: "If you, debt holders, want us, equity holders, to keep servicing outstanding debt, you must concede that debt average maturity be extended!". This hostile offer is similar in spirit to the “take-it-or-leave-it” offer assumed by that Anderson and Sundaresan (1996). If equity holders stopped servicing debt, debt holders would need to satisfy their claim through costly liquidation of the firm's assets.

We now assume that equity holders make their hostile offer to debt holders when $V = V_{R1}$. Then debt holders will concede an "extension" only if assets recovery value upon immediate liquidation is lower than debt value with extended maturity, i.e.

24) $f(V_{R1}) \geq X(V_{R1})$

where $X(V_{R1})$ is the assets recovery value if default is forced when $V = V_{R1}$ and $f(V_{R1})$ is the value of debt with extended maturity. The assumption about assets recovery is

25) $X(V) = (1-a)\cdot V - K$,

where K denotes the fixed costs of assets liquidation and “a” denotes the proportional costs of liquidation. From 24) and 25) the condition for debt holders to accept the "take-it-or-leave-it" offer by equity holders can be restated as

26) $f(V_{R1}) \geq (1-a)\cdot V_{R1} - K$.

Condition 26) implies that all bargaining power during re-negotiation of the debt contract lies with equity holders.

Then, equity holders will want to have debt average maturity extended just if the extension option (OE(V)) is “in the money”, i.e. if

27) $E(V_R) = ER(V_R) > e(V_R)$.

Equity holders may want to optimally choose V_{R1}, while making sure that condition 26) is met. This means that equity holders would determine V_{R1} as

28) $\max_{V_{R1}} \left\{ E\left[V, V_{R1} \right] \right\}$

subject to conditions 26) and 27). As it will be apparent later, this often implies that equity holders choose V_{R1} as the highest value of the firm's assets (V) at which condition 26) is met. Thus 26) is often a binding constraint.

Condition 26) is more easily met when fixed liquidation costs (K) are high. The same is not always true if proportional liquidation costs are high (i.e. if “a” is high). Condition 26) is more easily met also when V_B is low. If V_B is low, also V_{R1} can be low even without violating constraint 7) (i.e. $V_B \leq V_{R1}$). Then the lower V_{R1} implies the lower $X\left(V_{R1}\right)$ and condition 26) is more likely to hold.

2.1.2) When also debt holders gain from extension of debt maturity

Equity holders and debt holders can agree to renegotiate the debt contract and to extend debt maturity even if equity holders do not make the hostile offer implied by condition 26). This is the case when debt holders (as well as equity holders) are better off by extending maturity, i.e. when the value of debt with longer average maturity surpasses the value of debt with shorter average maturity. Then debt maturity would be extended at V_{R2}, where V_{R2} is determined as

29) $\max_{V_{R2}} \left\{ E\left[V, V_{R2} \right] \right\}$

subject to 27) and to

30) $f\left(V_{R2}\right) \geq F\left(V_{R2}\right)$.

2.1.3) Extension of debt maturity upon default

Default can take place without being preceded by the extension of debt maturity. This may be the case whenever equity holders and debt holders cannot renegotiate the debt contract, due for example to the high number of creditors involved or to asymmetric information between debtor and creditors. But, when default takes place, debt holders may spontaneously concede an extension of debt maturity to avoid immediate and costly assets liquidation. Then debt maturity would be extended at V_{R3}, where V_{R3} would be determined by the two simultaneous conditions:

31) $V_{R3} = V_B$,

and again

32) $f(V_{R3}) \geq (1-a) \cdot V_{R3} - K$.

2.1.4) Explicit option to extend debt maturity

Debt holders may be unconditionally subjected to the decision of equity holders as to the extension of debt maturity. This theoretical limit case applies when the debt contract or the bankruptcy code concede an "explicit" option to equity holders to extend debt maturity at any time. The debt indenture may concede one such option in some issues of “extendible debt” giving equity holders the unilateral right to extend debt maturity. A hypothetical bankruptcy code may concede to equity holders the right to voluntary file for an official reorganisation proceeding that, *without the approval of creditors*, would grant a moratorium to the debtor. The moratorium would allow the debtor to temporarily suspend debt payments and thus to stretch the effective maturity of debt.

Such extension options are theoretical limit cases and are explicit in that they are provided by the debt contract or by the code. Instead, the previous extension options are implicit in that debt maturity is extended through re-negotiation or through a unilateral concession by debt holders upon default. Anyway, an explicit extension option would allow equity holders to unilaterally decide to extend debt maturity so as to maximise equity value[2]. Equity holders could then extend debt maturity at V_{R4}, where V_{R4} is such that

33) $\max\limits_{V_{R4}} \left\{ E\left[V, V_{R4} \right] \right\}$.

Usually $V_{R4} \geq V_{R1}$ since condition 26) is not required in this case. For realistic parameters V_{R4} is usually an internal value internal value, i.e.: $V_B \leq V_{R4} \leq V_0$. Equity holders choose $V_{R4} = V_0$ when they want to immediately extend debt maturity. This may be the case especially when assets volatility is high. Instead, when the rate of debt coupons is very high equity holders would never want to extend debt maturity (i.e. $V_B \geq V_{R4}$) as they would want to minimise the number of high coupons to be paid and refinance at cheaper interest rates. Finally, when liquidation costs are exceptionally high, constraint 26) is not binding so that $V_{R4} = V_{R1}$.

Of course $V_{R1}, V_{R2}, V_{R3}, V_{R4}$ all imply rationality and symmetric information for both equity holders and debt holders. V_{R1}, V_{R2} and V_{R4} can be found by numerical algorithms. For V_{R3} also closed form solutions are available as becomes apparent next.

[2] Equity holders are assumed to extend the maturity of all outstanding debt at the same time.

2.2) The default barriers

Now the ways to determine the default barriers V_B and V_{BR} are discussed.

As for V_B, default before debt maturity is extended can take place at different possible values of the firm's assets (V), in particular:

- at V_{BI} when default is triggered by a cash flow shortage that makes the firm insolvent;
- at V_{BE} when default is triggered by equity becoming worthless .

As for V_{BR}, default after debt maturity has been extended can again be determined either by a cash flow shortage or by equity becoming worthless. In the first case default takes place at V_{BIR}, whereas in the second case default takes place at V_{BER}.

Then, V_{BI} would be determined by the following cash flow shortage condition:

34) $d \cdot V_{BI} + m \cdot f(V_{BI}) = d \cdot V_{BI} + m \cdot \lfloor (1 - a) \cdot V_{BI} - K \rfloor = C \cdot (1 - tax) + m \cdot P$,

which implies

35) $V_{BI} = \dfrac{m \cdot (K + P) + C \cdot (1 - tax)}{d + m \cdot (1 - a)}$.

Conditions 34) and 35) are the same as in Ericsson (2000) and state that default occurs when the firm becomes insolvent, i.e. when the instantaneous inflows to the firm are equal to in the instantaneous outflows from the firm. Condition 34) presupposes that debt average maturity cannot be extended. But, if debt average maturity is extended not later than default, i.e. $V_{BI} \le V_R$, then the default barrier becomes

36) $V_{BIR} = \frac{m_R \cdot (K+P) + C \cdot (1-tax)}{d + m_R \cdot (1-a)}$.

Finally, when default is triggered by worthless equity, equity holders are assumed to endogenously determine the default barrier as per Leland (1998). In this case, and if debt maturity cannot be extended, the default barrier is determined by the following conditions

37) $[E]_{V=V_{BE}} = 0$,

38) $[E_V]_{V=V_{BE}} = 0$

that imply

39) $V_{BE} = \frac{\left(tax \cdot \frac{C}{r} + K\right) \cdot j - \frac{C + m \cdot P}{r+m} \cdot b - K \cdot b}{(1 - a \cdot j - (1-a) \cdot b)}$.

If instead debt maturity is extended before default or at default, i.e. $V_{BE} \le V_R$, then

40) $V_{BER} = \frac{\left(tax \cdot \frac{C}{r} + K\right) \cdot l - \frac{C + m_R \cdot P}{r + m_R} \cdot b - K \cdot b}{(1 - a \cdot l - (1-a) \cdot b)}$.

In this section the conditions for debt maturity to be extended and the default barriers have been determined. In the next section such conditions are discussed with reference to a base case scenario in which realistic average parameter values are assumed. Different conditions for extension of debt maturity and different default barriers are shown to heavily affect the values of debt, equity and the extension options.

3. NUMERICAL ANALYSIS IN A TIME INDEPENDENT SETTING WHEN DEFAULT IS TRIGGERED BY CASH FLOW SHORTAGE OR BY WORTHLESS EQUITY

The following analysis builds on a base case scenario, which is of interest since it assumes realistic average parameters. Such parameters are similar *inter alia* to those in Fan and Sundaresan (2001), Ericsson (2000), Leland (1998) and are displayed in italics in Table I. The significant effect of different default conditions and different policies to extend debt maturity is highlighted.

3.1) Base case scenario when default is triggered by a cash flow shortage (liquidity default)

The base case scenario with liquidity default reveals that

$V_{BIR} < V_{BI} < V_{R4} < V_0 < V_{R2}$ ($V_{BI} = 49.8$, $V_{R4} = 82.8$, $V_0 = 100$, $V_{R2} = 123.5$)

where V_0 denotes the value of the firm's assets today. V_{R1} and V_{R3} are non existent since condition 26) is never met when $V \geq V_{BI}$. The fact that V_{R1} and V_{R3} are non-existent means that debt holders will always choose immediate liquidation rather than extension of debt maturity, even if extending debt maturity would in fact postpone default by lowering the default barrier from $V_{BI} = 49.8$ to $V_{BIR} = 44.8$. The fact that $V_{BI} < V_{R4}$ means that, if equity holders can unilaterally decide when to extend debt maturity in an unconstrained fashion, they will do so at V_{R4} before default. On the other hand, the fact that $49.8 = V_{BI} < V_{R2} = 123.5$ reveals that debt holders may accept an offer to extend debt maturity before default when $V \geq V_{R2}$, i.e. when the firm is very far from default. If debt maturity was extended at V_{R2}, then OE(V) = 1.72 and O(V) = 0.

In this scenario, equity holders may want to have an "explicit option" to unilaterally impose an extension of debt maturity to debt holders. Such explicit option would be optimally exercised at $V = V_{R4} = 82.8$, since V_{R4} maximises OE(V) (and E(V)) and minimises O(V) and (F(V)).

Results for $V_R = V_{R4}$ are displayed in Table I Panel A[3]. Total firm value increases and equity value (E(V)) rises by some 4.1% (from 55 to 57.2), while debt value (F(V)) decreases just slightly (from 50.5 to 50). A slight increase in the annual coupon rate (0.24%) would be enough to compensate debt holders for conceding the explicit "extension option" (i.e. c = 6.24% implies ff(V) = F(V)). This case is an example of the result that generally, *given taxes and bankruptcy costs, the "extension option" increases the value of equity well more than it decreases the value debt*.

3.2) When assets volatility is low

Now assets volatility is assumed to be equal to 10% rather than 20% and all other things are equal to the base case scenario. This new scenario implies that now:

$V_{BIR} \leq V_{BI} = V_{R3} < V_{R1} < V_{R2} < V_{R4} < V_0$ ($V_{R3} = V_{BI} = 49.8$, $V_{R1} = 52.4$, $V_{R2} = 52.7$, $V_{R4} = 63.4$, $V_0 = 100$). So, unlike in the base case scenario, V_{R1} and V_{R3} exist since condition 25) can be met even when $V \geq V_{BI}$. The

[3] The Panels of Table I are separately reproduced here below and the entire Table I is displayed also at the end of the paper for direct comparisons across different cases.

reason why condition 26) can now be met is that lower assets volatility makes debt less risky and more valuable and hence the value of debt with extended maturity (f(V)) is now more likely to be higher than the assets recovery value (X(V)).

When condition 26) is met, debt holders will prefer to have debt maturity extended rather than outright assets liquidation. Anyway, since in this case $V_{R1} < V_{R2}$, debt holders may accept to have debt maturity extended even if equity holders do not make the "take-it-or-leave-it" offer mentioned above. In fact, when $V \leq V_{R2} = 52.7$ debt of longer maturity (f(V)) is not less valuable than debt of shorter maturity F(V).

Then, in this scenario debt holders are interested in spontaneously extending debt maturity at default, i.e. at $V = V_{BI} = V_{R3}$, in order to avoid assets liquidation (this case is illustrated in Table I Panel B). This important point is similar to that of Longtaff (1990), who assumes that, as the firm defaults, debt holders may prefer to extend debt maturity rather than costly liquidation of the firm's assets. Though the analysis of Longstaff is limited to the classic Mertonian setting: a zero coupon bond is the only debt and default cannot occur before debt maturity. So, when $V_{BI} = V_{R3}$ *the analysis by Longstaff is being extended to a time independent setting in which the firm has multiple debt issues that are continuously refunded at maturity and in which default can take place at any time. As in Longstaff, even in this setting debt holders can prefer extension of debt maturity to liquidation.*

3.3) Base case scenario when default is triggered by worthless equity

Base case scenario parameters now imply

that: $V_{BER} \leq V_{BE} = V_{R3} < V_{R2} < V_{R1} < V_{R4} < V_0$ ($V_{BER} = 30.4$, $V_{BE} = V_{R3} =$

35.5, $V_{R2} = 44.5$, $V_{R1} = 50.5$, $V_{R4} = 76.5$, $V_0 = 100$). Table I Panels C, D and E illustrate this scenario when debt maturity is extended at V_{R1} or V_{R2} or V_{R3}. Thus, unlike when default is triggered by a cash flow shortage, when default is triggered by worthless equity V_{R3} and V_{R1} exist even with base case scenario parameters. In fact, when default is triggered by worthless equity, default takes place at lower values of the firm's assets ($V_{BER} \leq V_{BIR}$ and $V_{BE} \leq V_{BI}$) so that condition 26) is more likely to obtain before or at default. In other words, *as the firm approaches default, debt holders are more likely to prefer to concede an extension of debt maturity (rather than assets liquidation) when default is triggered by worthless equity than when default is triggered by a cash flow shortage*. Moreover, optimal leverage is higher when the explicit option to extend maturity is present as opposed to when such option is absent, and the higher the firm leverage is, the stronger is the incentive for debt holders to renegotiate and concede a maturity extension.

Figure 1 shows the values of debt and equity, assuming base case scenario parameters, when debt maturity is extended at $V_{R1} = 50.5$. V_{R1} is such that $f(V_{R1}) = X(V_{R1})$ and is the highest value of V at which condition 26) is met before the firm defaults, i.e. before the value of equity in the absence of the extension option drops to 0: in this case $V_B \leq V_{R1} = V_R$. Debt maturity can be extended only if the recovery value of assets (X(V)) is not greater than debt value after exercise of the extension option (f(V)) and only if equity (e(V)) has not yet become worthless.

For this same case, Figure 2 displays the values of $OE(V, V_{R1})$ and $O(V, V_{R1})$[4] and their respective payoffs (ER(V)-e(V), f(V)-ff(V)). Before "extension" we can see that $\lfloor ER(V, V_{R1}) - e(V, V_{R1}) \rfloor > OE(V, V_{R1})$, so it is clear that $OE(V, V_{R1})$ is not optimally exercised. In fact, equity holders can extend debt maturity only when condition 26) is met. Figure 2 also shows the case in which, ceteris paribus, debt of extended maturity is a perpetuity so that $m_R = 0$: the longer the "extension" is, the more valuable $OE(V, V_{R1})$ is and the less valuable $O(V, V_{R1})$ is.

Panels C, D and E of Table I show the effect of different exercise policies of the extension option with base case parameters and when default is triggered by worthless equity. In particular:

- $OE(V = 100, V_{R1}) = 171$ and $O(V = 100, V_{R1}) = -0.09$;
- $OE(V = 100, V_{R2}) = 161$ and $O(V = 100, V_{R2}) = 0$;
- $OE(V = 100, V_{R3}) = 144$ and $O(V = 100, V_{R3}) = 0.17$.

$OE(V = 100, V_{R1})$ denotes the value of the extension option when debt maturity is extended at V_{R1}. It is then clear that equity holders have an incentive to exercise their bargaining power by renegotiating debt maturity as soon as condition 26) is met, i.e. at V_{R1}, since this increases the extension option value. On the other hand $O(V = 100, V_{R1})$ is negative, which means that the detriment of debt holders if debt maturity were extended at V_{R1}.

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[4] $O(V, V_{R1})$ and $OE(V, V_{R1})$ denote the option values when debt maturity is extended at V_{R1}.

If debt maturity was extended at V_{R2}, debt holders would neither lose nor gain, so the extension option would be worthless for debt holders in such case:

$O(V = 100, V_{R2}) = 0$.

On the other hand, when maturity is extended at $V < V_{R2} = 44.5$, debt holders too would gain from an extension of debt maturity and indeed they would gain the most if maturity were extended just at default, i.e. at $V_{BE} = V_{R3}$.

In these cases equity holders would have to share with debt holders the benefit of having debt maturity extended (i.e. the increase in total firm value). Panels C, D and E of Table I show that:

$$OE(V, V_{R1}) + O(V, V_{R1}) = OE(V, V_{R2}) + O(V, V_{R2}) = OE(V, V_{R3}) + O(V, V_{R3}).$$

Finally Figure 3 displays how higher assets volatility does not necessarily increase the option values $OE(V, V_{R3})$ and $O(V, V_{R3})$. Higher volatility increases $O(V, V_{R3})$ when default is far, moreover it can decrease $OE(V, V_{R3})$ since $OE(V, V_{R3})$ becomes a locally concave function of V as default nears. Higher volatility implies a lower default barrier. Figure 3 shows the values of the extension option, $OE(V, V_{R3})$ and $O(V, V_{R3})$, in the base case scenario with default triggered by worthless equity and maturity extended just upon default: $V_{R3} = V_{BE}$, volatility is equal either to 10% or to 20%.

3.4 When debt holders gain from having debt maturity extended

It is here reminded that V_{R2} is the value of assets at which debt holders would be indifferent as whether to have debt maturity extended or not, because V_{R2} is such that

$F(V_{R2}) = f(V_{R2})$. It may appear surprising that V_{R2} equals 44.5 in the base case scenario when default is triggered by worthless equity, given V_{R2} equals 123.5 when default is triggered by a cash flow shortage condition (see above). The reason for this difference is that, when default is triggered by worthless equity, there are in reality two values of the firm's assets that make debt of shorter maturity of equal value to debt of longer maturity in this base case scenario. So there are two values for V_{R2}: one is 44.5 and the other one is 106.6. More precisely, when V > 106.6 debt of longer maturity is

more valuable than debt of shorter maturity (f(V) > F(V)): this is because when V grows, debt becomes safer and the contractual coupon over-remunerates the risk of default of debt (debt value rises above par). When this is the case, debt holders will want to extend debt maturity in order to get such over-remuneration for a longer period. In the base case when default is due to a cash flow shortage this happened when V > 123.5 rather than when V > 106.6.

Then, when 44.5 < V < 106.6 debt of longer maturity is less valuable than debt of shorter maturity (f(V) < F(V)): this is because when V falls below 106.6, the risk of default increases in such a way that the contractual coupon under-remunerates the risk of default of debt. In this case debt holders will not want to extend debt maturity in order to limit the period in which they are under-remunerated.

Then again, when V < 44.5, debt of longer maturity becomes again more valuable than debt of shorter maturity: this is because longer maturity postpones default by implying a lower default barrier and hence a lower probability of default. In the base case when default is due to a cash flow shortage this never happened since, when V < 123.5, debt of shorter maturity was always more valuable than debt of longer maturity, even if debt of shorter maturity implied a higher default barrier.

The above analysis has covered a time independent setting. The following analysis covers a time dependent setting.

4. A TIME DEPENDENT SETTING

The main new assumption in this setting is that debt is not continuously refunded so as to keep the nominal capital structure constant and independent of time as in the previous section. Rather, the (continuous) payment of principal is funded by assets generated cash flows and/or by issuance of new equity. Now time is an explicit independent variable. In a time dependent setting closed form solutions for the values of debt, equity and the option to extend debt maturity are no longer possible, so explicit finite differences are employed to provide numerical solutions to the relevant valuation equations.

4.1) The model in a time dependent setting

Some more notation before proceeding:

- P(t) is the face value of debt outstanding at time t;
- c is the annual coupon rate on debt;
- C(t) is the instantaneous coupon payment at time t, C(t) = c P(t);
- t denotes time; to highlight time dependence the notation changes to $V_R(t)$, $V_B(t)$, $V_{BR}(t), e(V,t), E(V,t), ER(V,t), f(V,t), ff(V,t), F(V,t)$;
- without loss of generality today's date is set equal to t = 0, e.g. P(0) denotes today's outstanding debt;
- T is the contractually agreed time at which debt amortisation is completed;

- T*(>T) is the time at which debt amortisation is completed after debt maturity has been extended;
- M is the rate at which debt principal is continuously amortised, so that P(T) = P(0) - M T =0; unlike in Appendix 2, it is here assumed that P(T) = 0, so that debt principal is completely paid back by time T.

Now the problem of valuing equity and debt whose maturity can be extended is reformulated in a time dependent setting. Before debt maturity is extended, when $V \geq V_R(t)$, debt value before "extension" (F(V, t)) must satisfy

$$41)\ F_t + \frac{1}{2} \cdot s^2 \cdot V^2 \cdot F_{VV} + (r-d) \cdot V \cdot F_V - r \cdot F + c \cdot [P(0) - M \cdot t] + M = 0,$$

with $F(V,T) = 0$, with P(T) = 0, with $F(V_R(t), t) = f(V_R(t), t)$ and with

$$42)\ F(V \to \infty) \to \int_0^T e^{-r \cdot t} \{M + c \cdot [P - M \cdot t]\} dt =$$

$$= \frac{-e^{-rT} \cdot r \cdot (M + c \cdot P) + e^{-rT} \cdot c \cdot M \cdot (r \cdot T + 1) + (M + c \cdot P) \cdot r - c \cdot M}{r^2}.$$

Condition 42) states that, as V grows infinitely, debt value approaches the value of a default free debt that promises the same cash flows, i.e. {M + c [P - M t}dt in every small period "dt". Then t* is the first time at which V reaches $V_R(t)$ from above. t* is a random variable that depends on the future path of V. For every $0 \leq t^* \leq T$, debt value after the re-negotiation, f(V, t), must satisfy

43)

$$f_t + \frac{1}{2} \cdot s^2 \cdot V^2 \cdot f_{VV} + (r-d) \cdot V \cdot f_V - r \cdot f + c \cdot [P(t^*) - M_R \cdot (t - t^*)] + M_R = 0,$$

with $f(V, T^*) = 0$, with P(T*) = 0, with $T^* = t^* + \frac{P(t^*)}{M_R}$, with

44) $f\left(V_{BR}(t),t\right)=(1-a)\cdot V_{BR}(t)-K$,

and with

45)

$$f(V\to\infty,t)\to\int_{t^*}^{T^*} e^{-r\cdot(t-t^*)}\cdot\left[M_R+c\cdot P(t^*)-c\cdot M_R\cdot(t-t^*)\right]dt=$$

$$=\frac{-e^{-r\cdot(T^*-t^*)}\cdot r\cdot\left(M_R+c\cdot P\right)+e^{-r\cdot(T^*-t^*)}\cdot c\cdot M_R\left(r\cdot(T^*-t^*)+1\right)+\left(M_R+c\cdot P\right)\cdot r-c\cdot M_R}{r^2}$$

.

M_R is such that $M_R < M$ and is the rate at which debt principal is repaid after debt maturity has been extended. In appendix 1 the problem is reformulated for the case in which $M_R = 0$. Condition 45) states that, as V grows infinitely, debt value approaches the value of a default free debt that promises the cash flows equal to $\left[M_R+c\cdot P(t^*)-c\cdot M_R(t-t^*)\right]dt$ in every small period "dt" after t*.

Then, as in section 2, we are left with the problem of determining $V_R(t), V_B(t)$ and $V_{BR}(t)$, where it is again assumed that $V_0 \geq V_R(t) \geq V_B(t)$[5]. Such problem is solved by valuing the firm's equity, which is done next. Hereafter E(V,t) denotes equity value before debt maturity has been extended and ER(V,t) denotes equity value after debt maturity has been extended. Then:

46)

$$E_t+\frac{1}{2}\cdot s^2\cdot V^2\cdot E_{VV}+(r-d)\cdot V\cdot E_V-r\cdot E+V\cdot d-(1-tax)\cdot c\cdot[P-M\cdot t]-M=0$$

,

[5] V_0 denotes the value of the firm's assets today.

with $E(V \to \infty, t) \to V$, with $E(V,T) = V - P(T) = V$ (since P(T) = P(0) – M T = 0),

with

47) $E(V_R(t), t) = ER(V_R(t), t)$,

for $\forall t, 0 \le t \le T$ max $V_R(t)$, subject to

48) $ER(V_R(t), t) \ge E(V_R(t), t)$,

48.1) $V_0 \ge V_R(t) \ge V_B(t)$,

49) $f(V_R(t), t) \ge (1-a) \cdot V_R(t) - K$.

50)

$$ER_t + \frac{1}{2} \cdot s^2 \cdot V^2 \cdot ER_{VV} + (r-d) \cdot V \cdot ER_V - r \cdot ER + V \cdot d - (1 - tax) \cdot c \cdot [P(t^*) - M_R \cdot (t - t^*)] - M_R = 0$$

with $ER(V \to \infty, t) \to V$, with $ER(V, T^*) = V$, with

51) $ER(V_{BR}(t), t) = 0$,

52) $[ER(V,t)]_{V=V_{BR}(t)} = 0$.

Explicit finite differences allow to solve equations 41) to 52) simultaneously. $V_R(t)$ is determined for every time "t" as the highest assets value at which conditions 48) 48.1 and 49) are all satisfied: these conditions ensure that debt maturity is extended in such a way that equity value, E(V), is maximised subject to condition 49) and provided default has not yet taken place. Condition 49) is similar to condition 26) and must hold if the option to extend debt maturity is implicit in the possibility of debt re-negotiation. This is the case we focus on below and the maturity extension policy [$V_R(t)$] is comparable to V_{R1} in the time dependent setting of the previous sections.

For some parameter values, there is no $V_R(t)$ satisfying conditions 48), 48.1) and 49); in such case debt maturity cannot be extended and E(V,t) is equal to e[V,t] as defined below.

Conditions 51) and 52) grant that equity value is always non-negative and that it be maximised by the choice of $V_B(t)$. Conditions 51) and 52) are similar to conditions 37), 38) and to condition 17) in Mello and Parsons (1992) at page 1891.

Assuming it is equity holders who decide as to the exercise of the extension option, the payoff to equity holders is:

53) $OE(V,t^*) = \max\{ER[V_R(t^*),t^*] - e[V_R(t^*),t^*],0\}$,

with $ER[V_R(t^*),t^*] = E[V_R(t^*),t^*]$,

with $e[V_R(t^*),t^*]$ denoting the value of equity deprived of the extension option.

Then e[V,t] must satisfy the same equation as E(V,t), but the lower boundary condition is the default condition (since debt maturity cannot be extended before default):

54)

$$e_t + \frac{1}{2}\cdot s^2 \cdot V^2 \cdot e_{VV} + (r-d)\cdot V\cdot e_V - r\cdot e + V\cdot d - (1-tax)\cdot c\cdot[P(0) - M\cdot t] - M = 0$$

,

with $e(V\to\infty,t)\to V$, $e(V,T) = \mathrm{Max}[V-(P(0)-M\cdot T),0]$ and with

55) $[e(V,t)_V]_{V=V_B(t)} = 0$,

56) $[e(V,t)]_{V=V_B(t)} = 0$.

Moreover, upon extension the payoff to debt holders is:

57) $O(V,t^*) = \{f[V_R(t^*),t^*] - ff[V_R(t^*),t^*]\}$,

with $f\left[V_R(t^*), t^*\right] = F\left[V_R(t^*), t^*\right]$ and with $ff\left[V_R(t^*), t^*\right]$ denoting the value of debt in the absence of the "extension option". Then ff[V,t] must satisfy the same equation as F(V,t), but the lower boundary condition is the payoff upon default (since debt maturity cannot be extended before default):

$$58)\ ff_t + \frac{1}{2}\cdot s^2 \cdot V^2 \cdot ff_{VV} + (r-d)\cdot V \cdot ff_V - r\cdot ff + c\cdot\left[P(0) - M\cdot t\right] + M = 0,$$

with $ff(V,T) = 0$ since P(T) = 0, with $ff(V_B(t), t) = (1-a)\cdot V_B(t) - K$

and with

$$59)\ ff(V\to\infty, t) \to \int_t^T e^{-rt}\cdot\left[\,M + c\cdot P(0) - c\cdot M\cdot t\right] dt =$$

$$= \frac{-e^{-rT}\cdot r\cdot(M + c\cdot P) + e^{-rT}\cdot c\cdot M\cdot(r\cdot T + 1) + (M + c\cdot P)\cdot r - c\cdot M}{r^2}.$$

We have formulated the time dependent model. Next numerical results with base case parameters are examined.

4.2) Base case scenario in a time dependent setting when default is triggered by worthless equity

The base case scenario parameters assumed in section 3 are here employed again[6]. Here again ER(V,t=0) is greater than or equal to E(V,t=0) for every value of the firm's assets (V). In fact extending debt maturity increases equity value by increasing the value of the tax shield, since more coupons must be paid if debt maturity is

[6] In the previous time independent settings debt average maturity "1/m" was doubled upon exercise of the "extension option": similarly in the base case scenario of this time dependent setting the average maturity of debt is doubled at t*, so that $M_R = \frac{M}{2}$. Moreover, T is now chosen so that the initial debt average maturity (T/2) is such that T/ 2 = (1/m) = 5.

extended. Equity value increases also because equity is here similar to some sort of compound call option that is continuously exercised as debt is continuously serviced: thus extending debt maturity increases equity value also by increasing the time value of the equity compound call option. The endogenous default barrier drops from V_B (t), for t < t*, to V_{BR} (t) for t > t*.

Since ER(V,t=0) is greater than or equal to E(V,t=0), equity holders will have an incentive to renegotiate debt maturity as soon as condition 49) is met. If condition 49) is satisfied, debt holders have incentives to voluntarily concede extensions of debt maturity before default. The values of the model parameters determine whether or not condition 49) is satisfied.

Then, the base case scenario in this setting reveals that debt both before and after default is more valuable (F(V=100, t=0) = 52.02 and f(V=100, t=0) = 52.82) than debt before and after default as per the base case scenario of section 3 (respectively F(V=100) = 50.54 and f(V=100) = 50.52). This is mainly due to the fact that the probability of default is now lower since assets pay-outs are mainly used to pay back debt principal, whereas in section 3 debt was refunded and a greater share of assets pay-outs could be destined to be distributed as dividends rather than to repaying debt principal.

Since debt is now more valuable, the extension option is much less valuable for equity holders than in the base case of section 3. In fact, *the riskier debt is, the more valuable the extension option for equity holders is*. The base case scenario now gives OE(V=100, t=0) = 0.08 instead of OE(V=100) = 1.71, and O(V=100, t=0) = -0.03 instead of O(V=100) = -0.09. Figure 4 displays the values of the extension option OE(V, t=0) and O(V, t=0) assuming base case scenario parameters in the present time dependent setting: due to constraint 49), [ER(V, t) - e(V, t)] > OE(V, t=0). Unlike in the time independent setting, now nominal outstanding debt is not constant and the time at which debt maturity is extended affects total firm value. An explicit finite differences scheme is employed with asset-step = 4 and time-step < (1 year /100). See appendix 3 for the case in which debt maturity is extended at $V_{R2}(t)$.

4.3) The term structure of credit spreads

In a time dependent setting the term structure of credit spreads can be analysed. Figure 5 displays the differential credit spreads due to an implicit option to re-negotiate and extend debt maturity when bankruptcy costs are high (K=10, a=15%) and debt is not amortised (M=0). It is interesting that the implicit extension option causes a significant increase in short-term credit spreads (lower assets values entail a more accentuated increase). In fact it is precisely such short-term credit spreads that traditional structural models, which do not account for debt re-negotiation, systematically understate. So, *these results suggest that structural models may understate short-term credit spreads because they neglect the presence of the implicit option to extend debt maturity*.

But it may not be apparent why short-term credit spreads should increase more than long-term credit spreads when an implicit extension option is recognised. The reason is that, for high leverage, debt market value (f(V, t)) is below debt face value (P), but as debt maturity approaches, debt market value is "pulled to par" if the debtor is solvent. This means that, when V is low, the payoff of the extension option (O(V, t*) = {f[V_R (t*), t*] - ff[V_R (t*), t*]} = {F[V_R (t*), t*] - ff[V_R (t*), t*]}) increases as t* approaches original debt maturity (T): exercising late implies a higher option payoff. Thus, as maturity draws near, O(V, t) becomes more valuable and its presence implies a higher increase in short-term credit spreads. On the other hand, if it is a few months before maturity and V is high enough, the implicit extension option is going to expire out of the money as the probability of the recovery value of assets dropping below ff[V_R (t), t] gradually vanishes. So immediately before maturity O(V, t) is too low to imply any significant increase in credit spreads. These arguments explain the shape in Figure 5 of the increase in the short-term credit spreads due to the presence of an implicit extension option.

CONCLUSIONS

This paper has focused on the value of debt given an option to renegotiate and/or extend debt maturity before default or just at default. The analysis has covered a time independent setting in which the firm keeps a constant nominal capital structure and a time dependent setting in which the firm’s nominal capital structure is not constant.

The main result in a time independent setting is that an implicit or explicit extension option increases equity value more than it decreases debt value. Such option can cause a material increase in the value of equity and may also cause a non-negligible increase or decrease in the yield required by debt holders when the firm is far from default.

Under some conditions, equity holders and debt holders can both be better off by re-negotiating and extending debt maturity, which extends a previous result by Longstaff in a simple Mertonian setting. This may often be the case when debt maturity is extended soon before or just at default in order to avoid costly liquidation of the firm's assets.

In a time independent setting it has also been shown that different default conditions heavily affect the value of the implicit option to re-negotiate debt maturity and the incentive for debt holders to accept re-negotiation: when default is triggered by cash flow insolvency the implicit option to extend debt maturity may easily be worthless if debt holders are not enticed to accept re-negotiation by the threat of high bankruptcy costs.

Finally, in a time dependent setting it has been shown that when the firm does not refund debt with new debt, the probability of default decreases making debt more valuable and the extension option less valuable. Moreover, *in a time dependent setting the presence of the implicit "extension option" boosts the short-term credit spreads on the firm's debt thus partially overcoming the typical problem of structural models predicting too low short-term credit spreads*.

Future research could extend the above analysis and valuation of "extension options" to the case in which default free interest rates are stochastic. Future research may also consider:

1. the impact of the option to extend debt maturity on the choice of optimal capital structure;
2. the case in which the extended maturity of debt is endogenously determined so as to maximise equity value rather than being exogenous as it has been assumed in this paper.

APPENDIX I: THE OPTION TO EXTEND MATURITY AND CREDIT SPREADS

The presence of the option to extend debt maturity (O(V)) implies a change in debt credit spread (dY), where

$$60)\ dY = \frac{C + m \cdot [P - F(V)]}{F(V)} - \frac{C + m \cdot [P - ff(V)]}{ff(V)}$$

and where F(V) is debt value (as per equation 12) when the extension option is present and ff(V) is debt value (as per equation 2) when the extension option is absent. The expressions $m \cdot [P - F(V)]$ and $m \cdot [P - ff(V)]$ denote the cash flows to and from debt holders due to continuously rolling debt over.

Equity holders may compensate debt holders for the option to renegotiate and extend debt maturity by promising a higher coupon (C^R) that would make F(V) = ff(V). Substituting for F(V) and ff(V) from equations 2) and 12), this gives:

61)

$$\frac{C^R + mP}{r + m} + \left[-\frac{C^R + mP}{r + m} + \frac{C^R + m_R P}{r + m_R} + \left[-\frac{C^R + m_R P}{r + m_R} + (1 - a) \cdot V_{BR} - K \right] \cdot \left[\frac{V_R}{V_{BR}} \right]^h \right] \cdot \left[\frac{V}{V_R} \right]^b$$

$$= \frac{C + mP}{r + m} + \left[-\frac{C + mP}{r + m} + (1 - a) \cdot V_B - K \right] \cdot \left[\frac{V}{V_B} \right]^b$$

Then root finding numerical algorithms can easily find C^R by solving 61).

APPENDIX II: ANOTHER CONDITION TO EXTEND MATURITY

Debt in the time dependent setting of section 4 is safer and more valuable than in the previous time dependent settings, so a coupon rate of 6% (i.e. 1% credit spread) over-remunerates debt holders for the risk of default they bear in the base case. Then debt holders will want this over-remuneration to last as long as possible. Then debt holders will want, at some point, to extend the maturity of debt that pays such generous coupons. In particular, they will desire to have maturity extended whenever

62) $f[V(t),t] \geq F[V(t),t]$.

This condition can be satisfied at two points for every time t:

$V_{R2.1}(t) \leq V_0 \leq V_{R2.2}(t)$.

Then, if conditions 48) 48.1) and 49) are substituted, by the following

for $\forall t, 0 \leq t \leq T$ max $V_{R2.1}(t)$, subject to

48.a) $ER(V_{R2.1}(t),t) \geq E(V_{R2.1}(t),t)$,

48.1.a) $V_0 \geq V_{R2.1}(t) \geq V_B(t)$,

49.a) $f(V_{R2.1}(t),t) \geq (1-a)\cdot V_{R2.1}(t) - K$

for $\forall t, 0 \leq t \leq T$ min $V_{R2.2}(t)$, subject to

48.b) $ER(V_{R2.2}(t),t) \geq E(V_{R2.2}(t),t)$,

48.1.b) $V_0 \leq V_{R2.2}(t)$,

49.b) $f(V_{R2.2}(t),t) \geq (1-a)\cdot V_{R2.2}(t) - K$,

and if all other equations are the same as in the system of equations 41) to 52), we can find the values of equity and debt given that debt maturity is extended as soon as it is advantageous for both the debtor and the creditors to do so. The values $V_{R2.1}(t) \leq V_{R2.2}(t)$ make debt holders indifferent between holding debt of shorter or longer average maturity. Then, as in section 3, equity holders could convincingly propose to debt holders to have debt maturity extended as soon as $V(t) \leq V_{R2.1}(t)$ or $V(t) \geq V_{R2.1}(t)$. Though, in section 4 $V_{R2.1}(t=0)$ is about 48 and $V_{R2.2}(t=0)$ is about 80 as opposed to respectively 44.5 and 106.6 in the time independent setting of section 3 with base case parameters.

APPENDIX III: WHEN DEBT AMORTISATION STOPS

Given the time dependent setting of section 4, if $M_R = 0$ then the continuous amortisation of debt principal stops at t = t* and all debt principal still outstanding mat be repaid at T through a single "balloon" payment. Both when $M_R = \frac{M}{2}$ and when $M_R = 0$ with repayment at T, the average maturity of debt still outstanding at time t* is effectively double as long as when $M = M_R$. When $M_R = \frac{M}{2}$ the rate at which debt principal is amortised is halved, when $M_R = 0$ the repayment of debt principal is suspended until T.

When $M_R = 0$, the conditions for equation 43) change, since debt holders receive P(t*) at T and coupons at a rate $c \cdot P(t^*) \cdot dt$ between t* and T. Hence, condition 45) is substituted by

63)

$$f(V \to \infty, t) \to \int_t^T e^{-r \cdot (t - t^*)} c \cdot P(t^*) \cdot dt + e^{-r \cdot (T - t)} \cdot P(t^*) = \frac{1 - e^{-r(T - t^*)}}{r} c \cdot P(t^*) + e^{-r(T - t)} \cdot P(t^*)$$

and the final condition is no longer $f(V, T^*) = 0$, but

64) $f(V, T) = P(t^*)$ if V(T)> P(t*), or

65) $f(V, T) = \min[P(t^*), V \cdot (1 - a)]$ if V(T)< P(t*).

Then, if $M_R = 0$ and P(t*) is due at T, the final condition for equation 50) is no longer $ER(V, T^*) = V$, but

66) $ER(V, T) = \mathrm{Max}\{V - P(t^*), 0\}$.

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Figure 1: Base case scenario with default triggered by worthless equity

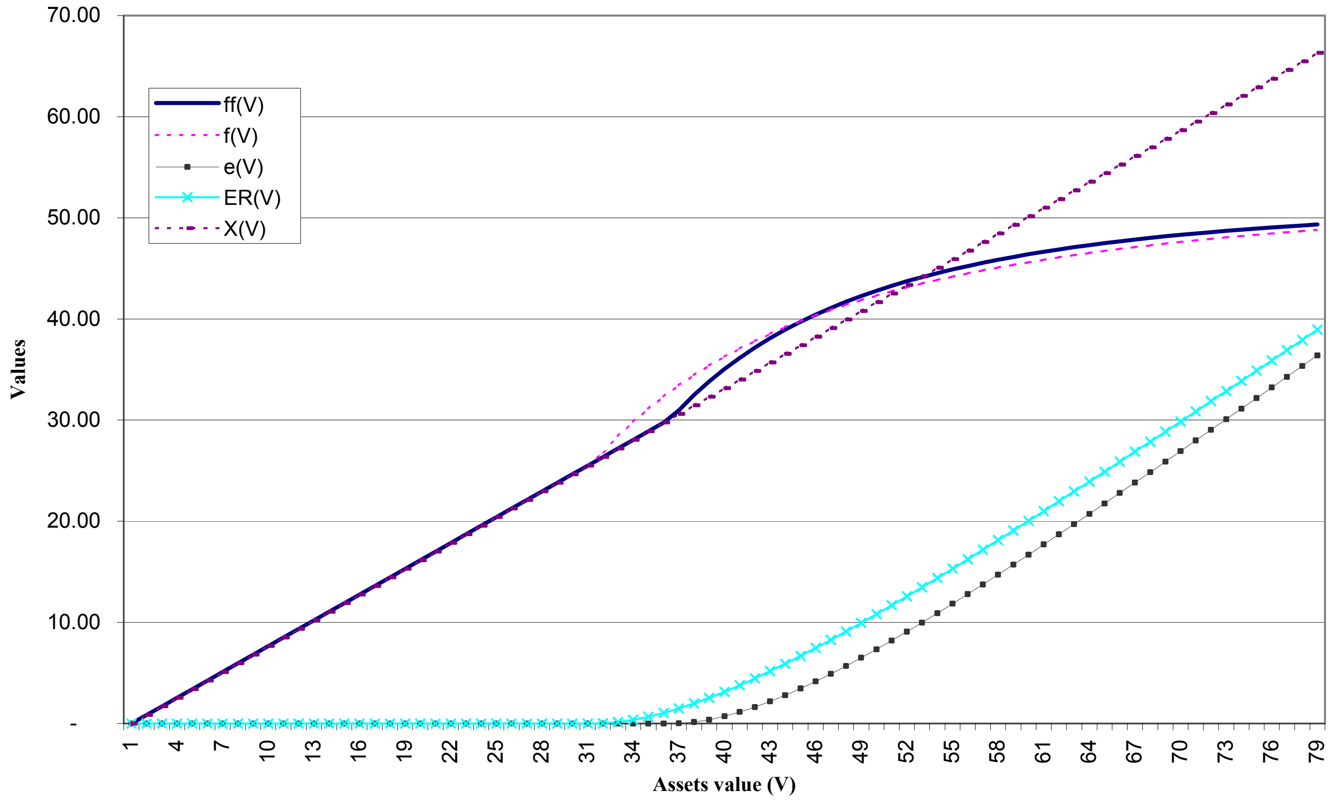

Figure 2: Values of the extension option in the base case scenario with default triggered by worthless equity

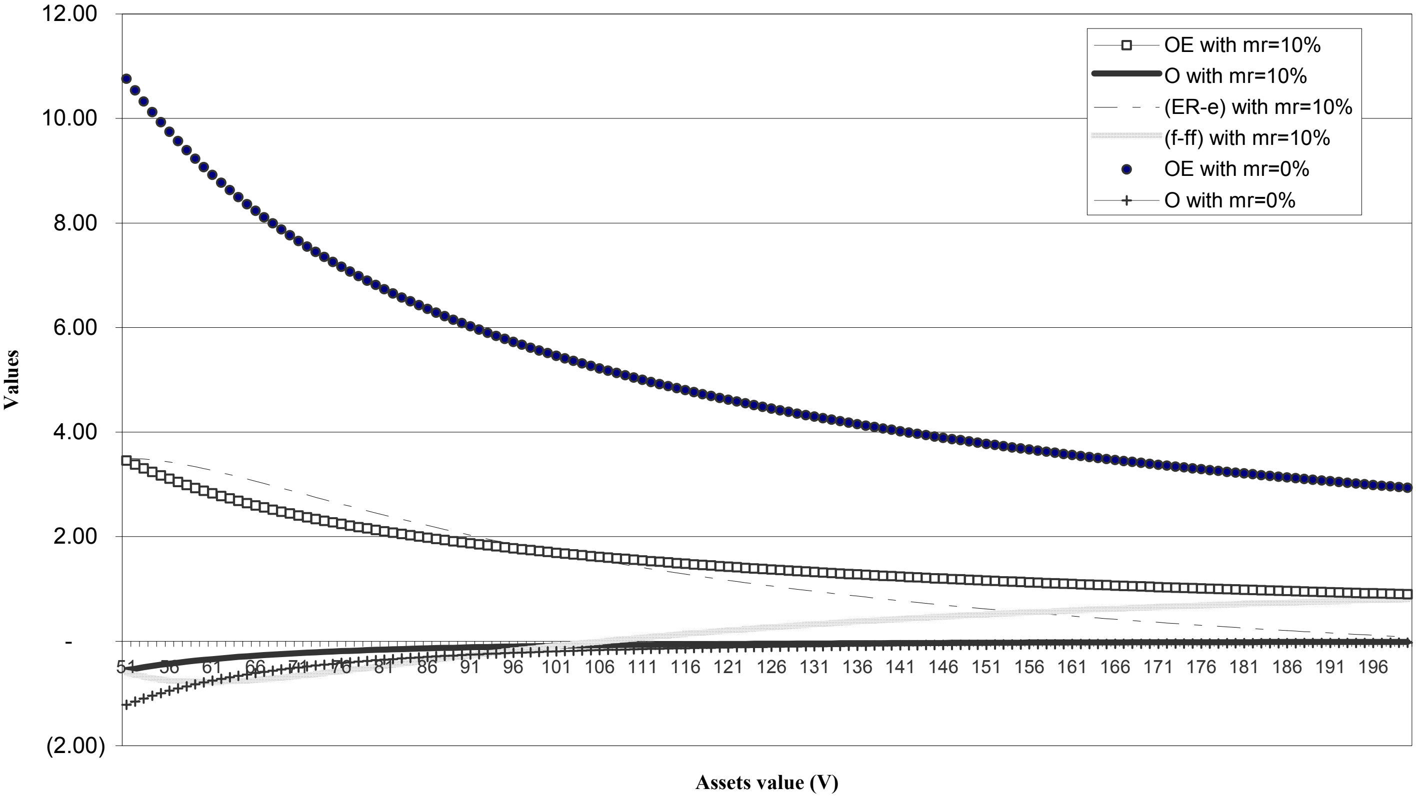

Figure 3: Values of the extension option in the base case scenario with default triggered by worthless equity and maturity extended just upon default

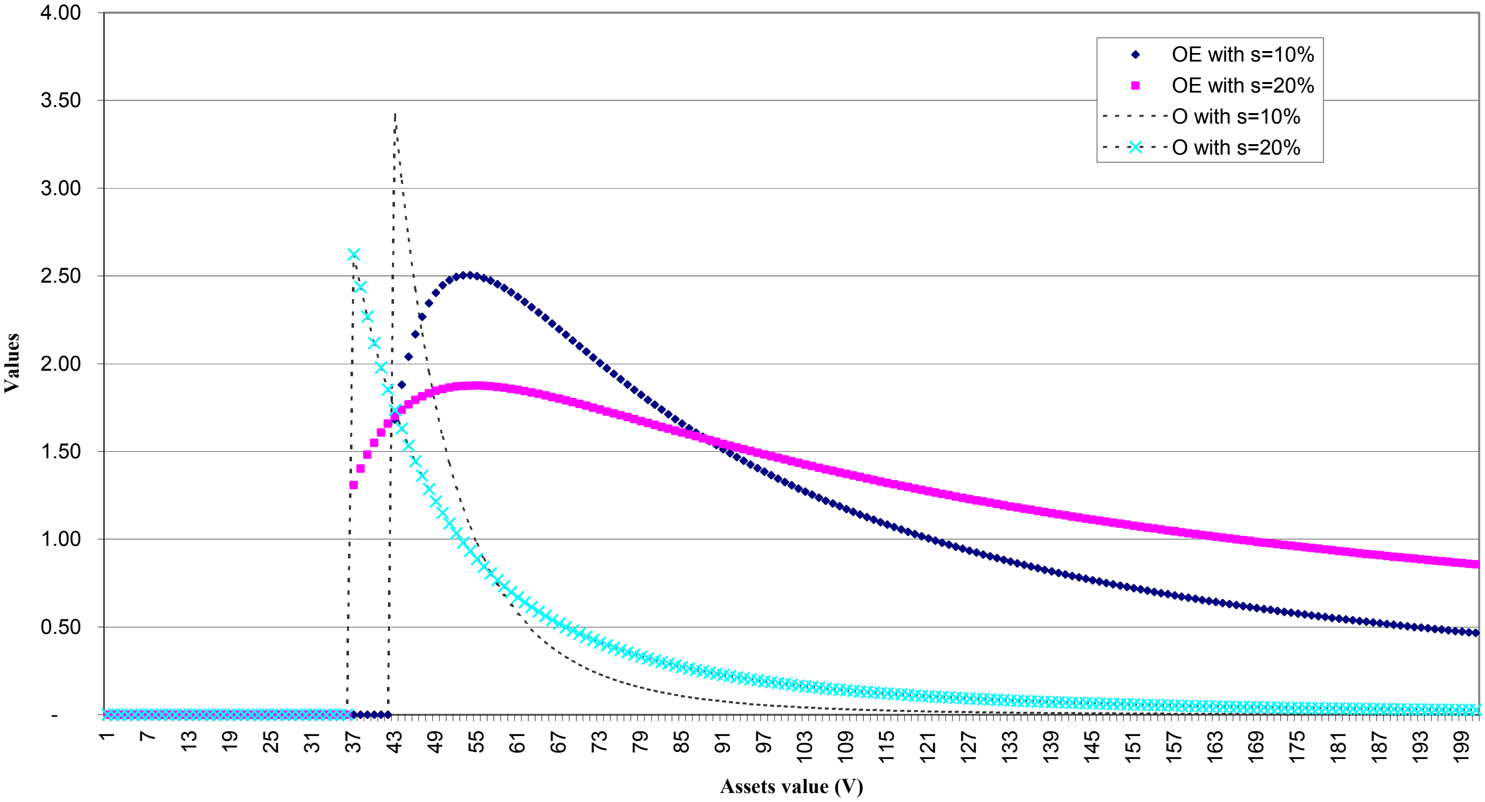

Figure 4: Values of the extension option with base case scenario parameters in the time dependent setting

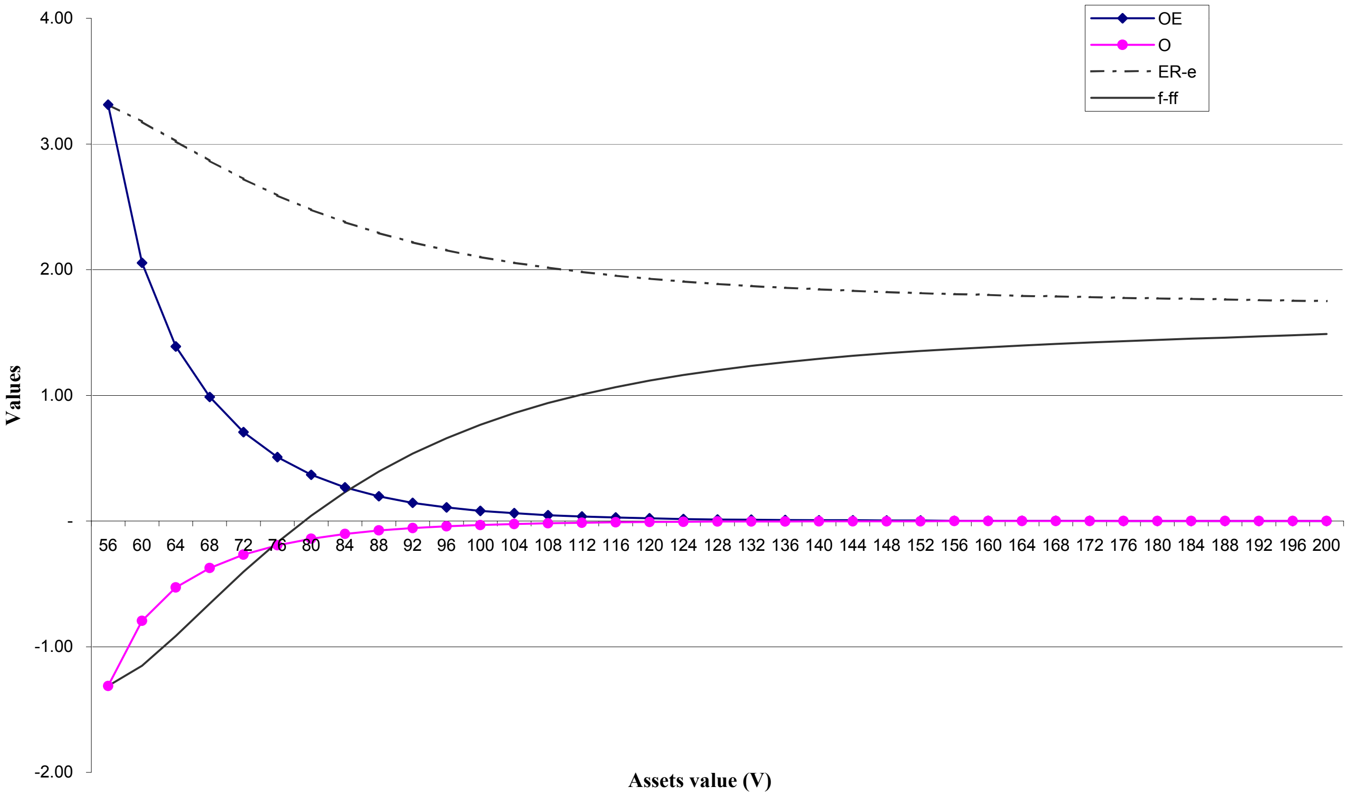

Figure 5: Increase in short term credit spreads due to the "implicit" extension option

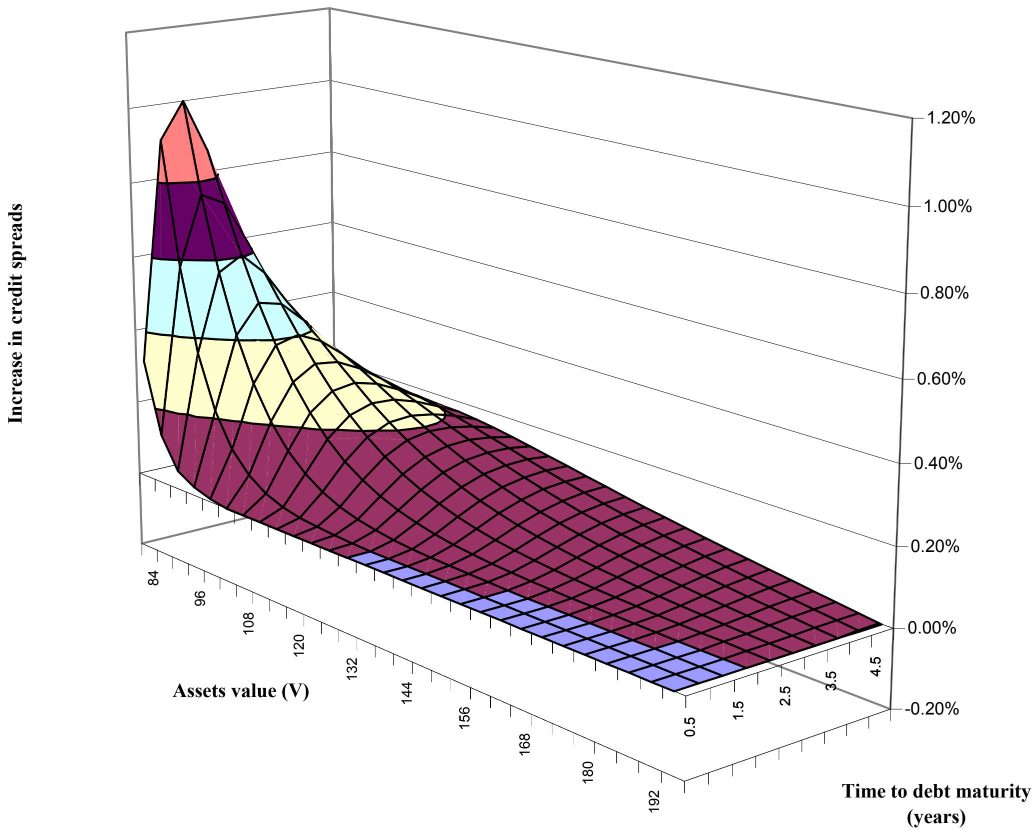

TABLE I: SUMMARY OF THE EFFECTS OF THE PRESENCE OF THE EXTENSION OPTION			
PANEL A: Base case with irreversible extension of debt maturity and cash flow shortage default			
Input data in italics	**No extension option**	**Ante extension**	**Post extension**
a (bankruptcy costs as fraction of V)	*15%*	*15%*	*15%*
r (default risk-free interest rate)	*5%*	*5%*	*5%*
s (volatility of V)	*20%*	*20%*	*20%*
d (assets total payout to security holders)	*7.0%*	*7.0%*	*7.0%*
tax (tax rate)	*35%*	*35%*	*35%*
K (fixed liquidation costs)	*0*	*0*	*0*
C (annual coupon, which is paid coutinuously)	*3.00*	*3.00*	*3.00*
Coupon rate (c = C/P)	*6.00%*	*6.00%*	*6.00%*
m (percentage of P that is refinanced every year)	***20%***	***20%***	***10%***
P (face value of debt)	*50.0*	*50.0*	*50.0*
V_0 (today's assets value)	*100.0*	*100.0*	*100.0*
$V_R = V_{R4}$ (value of asset at which debt maturity is extended)		82.8	
V_{BI} and V_{BIR} (value of assets triggering default)	49.8	44.8	44.8
OE (value of the "extension option" for equity holders)		**2.26**	
E (value of equity)	55.0	57.2	57.2
O (value of the "extension option" for debt holders)		**-0.53**	
F (value of debt ante extension)	50.5	50.0	
f (value of debt post extension)		48.6	50.1
$X(V_{R4})$ (assets recovery value at V_{R4})		70.4	
Credit spread: [C+m(P-F)]/F-r or [C+m(P-f)]/f-r	0.74%	1.02%	0.98%
PANEL B: All as in panel A exept for assets volatility and $Vr = Vr_3$ rather than $Vr = Vr_4$			
s (volatility of V)	*10%*	*10%*	*10%*
$V_R = V_{R3}$ (value of asset at which debt maturity is extended)		49.8	
V_{BI} and V_{BIR} (value of assets triggering default)	49.8	44.8	44.8
OE (value of the "extension option" for equity holders)		**1.65**	
E (value of equity)	59.5	61.2	60.5
O (value of the "extension option" for debt holders)		**0.01**	
F (value of debt ante extension)	51.7	51.7	
f (value of debt post extension)		42.8	52.4
$X(V_{R3})$ (assets recovery value at V_{R3})		42.3	
Credit spread: [C+m(P-F)]/F-r or [C+m(P-f)]/f-r	0.14%	0.14%	0.26%
PANEL C: All as in panel A exept for default when equity is worthless and $Vr = Vr_1$			
$V_R = V_{R1}$ (value of asset at which debt maturity is extended)		50.5	
V_{BE} and V_{BER} (value of assets triggering default)	35.5	**30.4**	30.4
OE (value of the "extension option" for equity holders)		**1.71**	
E (value of equity)	59.7	61.4	61.4
O (value of the "extension option" for debt holders)		**-0.09**	
F (value of debt ante extension)	50.6	50.5	
f (value of debt post extension)		42.9	50.5
$X(V_{R1})$ (assets recovery value at V_{R1})		42.9	
Credit spread: [C+m(P-F)]/F-r or [C+m(P-f)]/f-r	0.68%	0.72%	0.83%
PANEL D: All as in panel A exept for default when equity is worthless and $Vr = Vr_2$			
$V_R = V_{R2}$ (value of asset at which debt maturity is extended)		44.5	
V_{BE} and V_{BER} (value of assets triggering default)	35.5	**30.4**	30.4
OE (value of the "extension option" for equity holders)		**1.61**	
E (value of equity)	59.7	61.3	61.4
O (value of the "extension option" for debt holders)		**0.00**	
F (value of debt ante extension)	50.6	50.6	
f (value of debt post extension)		40.1	50.5
$X(V_{R2})$ (assets recovery value at V_{R2})		37.8	
Credit spread: [C+m(P-F)]/F-r or [C+m(P-f)]/f-r	0.68%	0.68%	0.83%
PANEL E: All as in panel A exept for default when equity is worthless and $Vr = Vr_3$			
$V_R = V_{R3}$ (value of asset at which debt maturity is extended)		35.5	
V_{BE} and V_{BER} (value of assets triggering default)	35.5	**30.4**	30.4
OE (value of the "extension option" for equity holders)		**1.44**	
E (value of equity)	59.7	61.1	61.4
O (value of the "extension option" for debt holders)		**0.17**	
F (value of debt ante extension)	50.6	50.8	
f (value of debt post extension)		32.9	50.5
$X(V_{R3})$ (assets recovery value at V_{R3})		30.2	
Credit spread: [C+m(P-F)]/F-r or [C+m(P-f)]/f-r	0.68%	0.59%	0.83%

Note to Table I: The table shows debt when debt maturity can be extended (a time independent setting). The firm's assets value is normalised at 100 and the face value of debt is assumed to be equal to 50. Panel A (and in the same way the other panels) is to be interpreted as follows: if debt average maturity is extended at V_{R4} (from a 5 years to a 10 years), equity (E(V)) rises from 55 to 57.2 and debt (F(V)) drops from 50.5 to 50. Extending debt maturity decreases default barrier from $V_{BI} = 49.8$ to $V_{BIR} = 44.8$, increases total firm value and the expected value of the tax shield (from TT(V)=13 to TT(V)=13.7) and decrease the expected value of bankruptcy costs (from BC(V)=2.9 to BC(V)=2.3).