THE UNIVERSITY of York

Discussion Papers in Economics

No. 2003/12

A note on the 'Natural Rate of Subjective Inequality' hypothesis and the approximate relationship between the Gini coefficient and the Atkinson index

by

James Harvey

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

A note on the 'Natural Rate of Subjective Inequality' hypothesis and the approximate relationship between the Gini coefficient and the Atkinson index.

James Harvey*

Department of Economics, University of York, York YO10 5DD, UK.
Frontier Economics, Holborn, London EC1N 2NS, UK.

Abstract

In a recent paper in this journal, Lambert *et al* (2003) sought to establish the Natural Rate of Subjective Inequality (NRSI) hypothesis. In this note, their test of the NRSI hypothesis is critically evaluated and an alternative reason is offered as to why their empirics appeared to support it. The findings, based on simulation, do not overturn the NRSI hypothesis, but indicate the need for deeper and more thorough analysis if this insightful and potentially far-reaching hypothesis is to be established.

Keywords: Natural rate of subjective inequality; inequality indices; simulation

JEL classification: C15, D30, O15.

1. Introduction and Background

The Natural Rate of Subjective Inequality (NRSI) hypothesis introduced by Lambert, Millimet and Slottje (2003) in this journal, henceforth LMS, suggests that different levels of inequality aversion (or state inequality intolerance) explain differences in 'objective' inequality between countries. Countries arrange their fiscal affairs such that the Atkinson index (I_e) equals the NRSI (φ), which implies country-specific values for the inequality aversion parameter (e).

The authors regress country-specific inequality aversion, e_i consistent with $\varphi = 0.1$ on a range of social, economic and political variables, x_i. They repeat the regression, but replace e_i with the Gini coefficient G_i; they find that the same variables are significant, but with coefficients of the opposite sign. This, they contend, supports the NRSI hypothesis.

Suppose, however, that a functional relationship $I_e = f(G,e)$ exists, where $\partial f / \partial G$ >0 and $\partial f / \partial e$ >0. Letting x be any variable influencing G or e, since $\frac{dI_e}{dx} = \frac{dI_e}{df}\frac{df}{dx} = \frac{dI_e}{df}\left[\frac{\partial f}{\partial G}\frac{dG}{dx} + \frac{\partial f}{\partial e}\frac{de}{dx}\right]$, fixing I_e equal to a constant φ means

* *E-mail address*: james.harvey@frontier-economics.com

$dI_e / dx \equiv 0$ which implies that $-\frac{\partial f}{\partial G}\frac{dG}{dx} \equiv \frac{\partial f}{\partial e}\frac{de}{dx}$. It follows that any of the variables that cause e to rise (fall) would have to cause G to fall (rise) in order to satisfy the identity. In a footnote, the authors claim that a functional relationship is unlikely to hold due to the underlying features of the two indices being used. Whilst it is true that an exact relationship does not exist[1], an approximate relationship could. This analysis establishes the existence of an approximate relationship between the Gini coefficient and the Atkinson index, explores the nature of it and examines its implications for the NRSI hypothesis.

2. Simulation Methodology

In order to generate a sufficient number of income vectors, simulation is used. The Singh-Maddala (1976) or Burr type 12 distribution function (hereafter SM) offers a suitable parametric form since it is possible for Lorenz curves to cross and the fit to actual income distribution data is good (see McDonald, 1984). The distribution function for the SM is $F(y) = 1 - \left(1 + (y/b)^a\right)^{-q}$ where all parameters are non-negative and $q>1/a$.[2]

Theorem 1 of Wifling and Kramer (1993) shows that a necessary and sufficient condition for Lorenz (second-order stochastic) dominance of one SM distribution $F_J(y)$ over another SM distribution $F_K(y)$ is that $a_J \geq a_K$ and $a_J q_J \geq a_K q_K$.[3] In the simulation, each distribution has an $a \in [2,3]$ greater than the previous and an $aq \in [3,8]$ less than the previous such that the above condition is continually violated and some (though not all) associated Lorenz curves will intersect. One thousand income vectors are created, each with three thousand observations.[4] G is calculated along with I_e for $e = 0.25$, 0.5, 1, 2, 3, 5 and the Rawlsian case.

3. Results

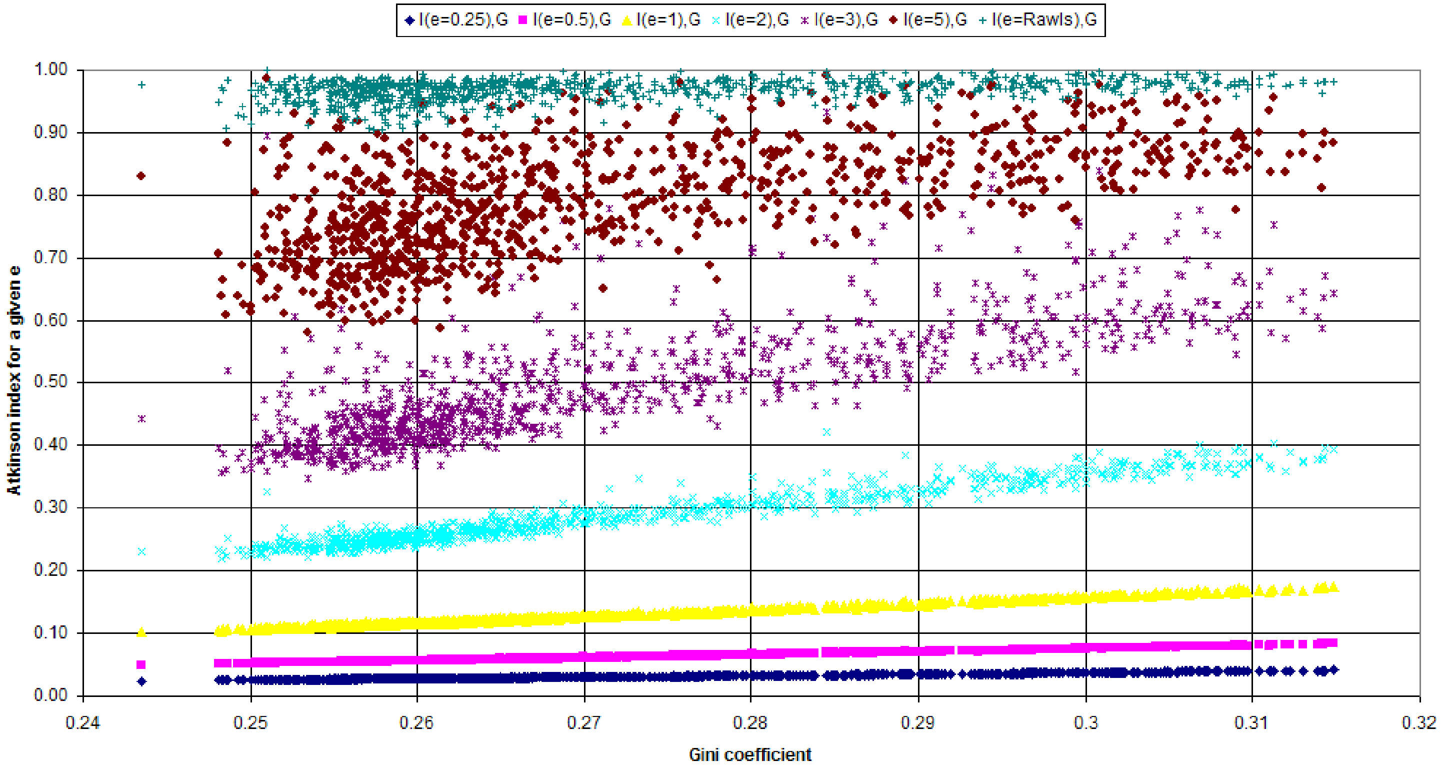
The choice of tools for analysing the relationship between the Atkinson index and the Gini coefficient is motivated by figure 1 which plots G against I_e for a given e.

[1] When the Lorenz curves associated with a pair of incomes distributions cross, the indices can give a different ordinal ranking of inequality.

[2] An additional attractive feature of the SM is that a simple closed-form solution for the inverse function exists: $F^{-1}(y) = b\left((1-y)^{-1/q} - 1\right)^{1/a}$. In order to create the distributions, sample data is randomly drawn from a uniform [0,1] population and substituted into the inverse function.

[3] In their paper the condition for Lorenz dominance of J over K is as above, but with the inequalities reversed, which is incorrect.

[4] This quantity is sufficient for approximate continuity of the distribution and therefore facilitates the use of the Gini calculation procedure advocated by Lerman and Yitzhaki (1984).



I(e=0.25),G
I(e=0.5),G
I(e=1),G
I(e=2),G
I(e=3),G
I(e=5),G
I(e=Rawls),G
Atkinson index for a given e
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
0.24
0.25
0.26
0.27
0.28
0.29
0.3
0.31
0.32
Gini coefficient

Figure 1

The figure highlights the following points:[5]

- As $e \to 0$ there is an almost perfect linear correlation between the indices, which decreases as e increases (and does not weaken until $e>2$). See table 1.
- The range of G values consistent with a single I_e value increases with e.
- The number of possible e values consistent with a single I_e value increases with I_e.

In particular, choosing a lower value for the NRSI φ (less than 0.4 say) generally implies a lower value of e; choosing such a value for φ is therefore more likely to be consistent with a close relationship between I_e and G. This is confirmed by the correlation coefficient presented in table 1, which, of course, would equal one if $I_e = \alpha + \beta G$ exactly (where $\beta>0$).[6]

e	**Correlation coefficient between I_e and G**	**% of disassociation**
0.25	0.9989	0.0965
0.5	0.9984	0.1789
1	0.9956	0.5206
2	0.9637	2.4284
3	0.7752	7.8013
5	0.6054	44.5288
Rawlsian	0.3813	23.1227

Table 1

It is possible to exploit a different measurement approach from the taxation literature (see Dardanoni and Lambert, 2001) to measure the degree of disassociation between the Atkinson index and the Gini coefficient. Consider the movement from I_e to G (where the vector of G Lorenz dominates the vector of I_e) or the movement from G to I_e (where I_e Lorenz dominates G). One component of this movement arises from the disassociation between the two indices. The extent to which the Gini coefficient gives a different ordinal ranking to the Atkinson index in the sequence of income distribution inequality pair-wise comparisons is presented as a proportion in the final column of table 1.

The disassociation according to this measure also increases with inequality aversion over the plausible range (in fact, up to the Rawlsian case, where I_e becomes more stable and close to one; this is like the linear model but with $\beta \to 0$).

[5] Whilst the range of Gini values is between approximately 0.24 and 0.32, there is no *a priori* reason to suppose that the behaviour of the relationship will alter outside of this range.
[6] Of course, a more complex non-linear relationship may exist.

4. Implications for the NRSI

It is clear from our simulation and the observations above, that whilst the NRSI hypothesis of LMS is an attractive one (and could be very important for the analysis of convergence), their current empirical approach seems somewhat flawed. The first reason stems from our finding that there is always some linear association between the two inequality indices used. Regardless of the level of inequality aversion, an approximate relationship seems to exist. They are thus unable to prove or disprove their theory. The second reason, also highlighted by our findings, is a clear sensitivity of the degree of association to their arbitrary choice of φ, which is important if it is not possible to establish what φ actually is. Choosing a lower level of φ will yield a lower value of e and a lower value of e generates a higher degree of association between I_e and G, regardless of the measurement device used.[7] Indeed, LMS perform their regressions with $\varphi = 0.1$, which yields values for e no greater than 2.03.

5. Conclusions

The approximate functional relationship between inequality indices is an area which requires further investigation. In the meantime, LMS could rerun their regressions with multiple (higher) values of φ which might indicate whether the parameter estimates and their significance are sensitive to this choice.[8] The use of different 'subjective' and 'objective' measures of inequality could also be enlightening. If the results of LMS are not robust to these changes, their methodology will need reviewing; otherwise analysts must continue to guess at φ or e.

Acknowledgements

I thank Peter Lambert for his excellent supervision of the MSc dissertation from which this paper stems and Caroline Joll of Cardiff University for her helpful comments. Financial support from the Economic and Social Research Council through grant number K42200113017 is gratefully acknowledged.

References

Atkinson, A.B. (1970). On the measurement of inequality. Journal of Economic Theory, vol. 2, pp. 244-263.

Dardanoni, V and P. J. Lambert (2001). Horizontal inequity comparisons. Social Choice and Welfare, vol. 18, pp. 799-816.

---

[7] Note that the sensitivity of the arbitrary choice of φ was only tested by LMS up to a value of 0.4.

[8] Of course, finding that the significance of the results declines does not necessarily imply that the NRSI hypothesis is incorrect, it may simply mean that the choice of φ is wrong.

Lambert, P.J. (2001). The Distribution and Redistribution of Income, 3rd edition. Manchester University Press.

Lambert, P.J., D.L. Millimet and D. Slottje (2003). Inequality aversion and the natural rate of subjective inequality. Journal of Public Economics, vol. 87/5-6, pp. 1061-1090.

Lerman, R. and S. Yitzhaki (1984). A note on the calculation and interpretation of the Gini coefficient. Economics Letters, vol. 15, pp. 363-368.

McDonald, J.B. (1984). Some generalised functions for the size distribution of income. Econometrica, vol. 52, pp. 647-663.

Singh, S and G.S. Maddala (1976). A function for the size distribution of incomes. Econometrica, vol. 44, pp. 963-970.

Wifling, B and W. Kramer (1993). The Lorenz-ordering of Singh-Maddala income distributions. Economics Letters, vol. 43, pp. 53-57.