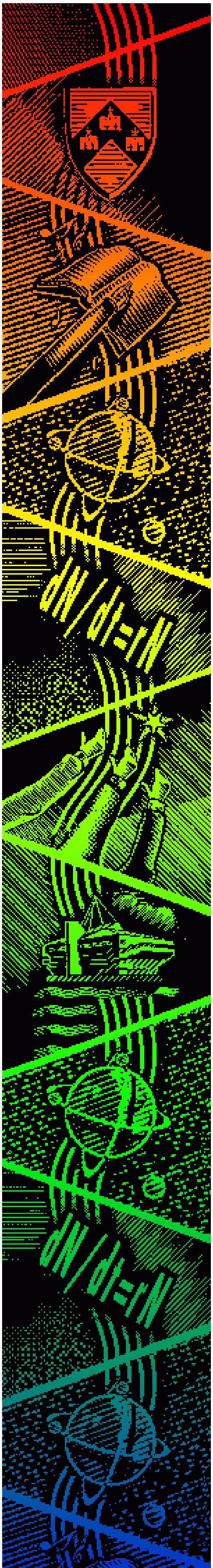

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Income Stratification Across Public and Private Education: The Multi-community Case

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Income stratification across public and private education: The multi-community case

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Abstract

This paper analyses the question of which households opt out of public education in a multi-community economy with local school finance and housing markets. In particular, the objective is to investigate whether perfect income stratification across public and private educational sectors predicted by single jurisdiction models and by multiple jurisdiction ones without housing markets holds in this setting. Nechyba (1999) has shown that the existence of a fixed stock of heterogeneous houses can prevent perfect income stratification from arising in equilibrium. Here we demonstrate that, even with homogeneous housing, perfect income stratification is not assured. On the contrary, it is possible to find equilibria in which households from intermediate income intervals use private schools, while richer ones prefer to send their youths to a local public school of higher quality. The emergence of very high quality public schools that attract students from the best-off households and survive the competition of private schools is therefore possible. The paper identifies a new way whereby housing markets affect how the market for education works.

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1 Introduction

In response to a sizeable decline in students' achievement, the last decade has witnessed the emergence of a hot debate about school finance reform in many countries (e.g. US, UK and Spain). The role economists have played in this debate has been continuously increasing. Theoretical and empirical research on the economics of education has shed light on key issues within it: the impact of decentralisation on public schools productivity (Hoxby, 1999), the role of different inputs into the education production function (Hanushek, 1997) or the consequences of peer group effects on sorting across schools and neighbourhoods (de Bartolome, 1990, Epple and Romano, 1998, 2002) are only three examples. One fascinating branch of research within this field focuses on the co-existence of private and public education institutions. A voluminous literature on this topic has rapidly developed during this time, partly stimulated by private schools vouchers proposals[1].

When private alternatives to public education are made available, multiple questions must be addressed. For instance, it is crucial to know what the consequences over public and total education investment and over the distribution of educational benefits across households are. An important related question is which households exert choice and opt out of the public system. Clearly, the answer to this question has a significant impact on the others. For example, if students attending private schools are the brightest ones or those from the richest households, then the cream skimming effect of private education could worsen the situation of poorer and less able students who are left behind in public schools.

The focus of this paper will be on that question, i.e., on how households choose among public and private schools. This issue has been analysed theoretically by Stiglitz (1974), De Fraja (2001) and Epple and Romano (1996), among others. Let us briefly discuss these models. In all of them, agents differ along a single dimension: ability to earn future income in the first two, income in the last one. The public educational sector faces two constraints: public schooling must be free and all students in the public system must receive education of the same quality. Finally, funding levels for public education are decided through majority voting in Stiglitz (1974) and Epple and Romano (1996), and by a welfare maximiser utilitarian government in De Fraja (2001). A pervasive feature of equilibrium in these models is the

[1] See De Fraja (2001) for an in-depth analysis of this literature.

perfect stratification of households across public and private schools, with the best-off (either the richest or the brightest) choosing (higher quality) private education. When households differ by income, what assures perfect stratification is the normality of educational services. In Stiglitz (1974) and De Fraja (2001), in turn, it is the complementarity of education and ability in the function determining (future) income.

One important exception to the perfect stratification result is in Epple and Romano (1998). In their model, households differ by both income and the student's ability and stratification occurs not only by income but also by ability. Consequently, there are relatively low income households with students of relatively high ability which end up using a private school. Because school quality is determined by the average ability of its students, profit maximising private schools make tuition fees a function of ability, in order to internalise the positive externality generated by high ability peers. Hence, even if demand for education does not increase in ability, private schools charge lower tuition fees to high ability students, making the 2-dimensional stratification arise.

All the models above consider single-jurisdiction economies. This is clearly adequate either if public education is centrally provided or if mobility among jurisdictions can reasonably be ignored, but not if public education is locally provided and mobility across communities is significant. Therefore, choice among public and private schools must also be studied in a multiple-jurisdiction framework with costless mobility.

One distinct feature of these economies is that school choice can be exerted not only among public and private institutions but also among (different quality) local public schools. Because attending the local public school of a particular community requires to reside and pay taxes within it, there exists a link between the quality of public education available for a household and their community of residence. Thus, the choice of residential location becomes a crucial issue, at least for households choosing public schooling. For those opting-out, in turn, school quality is not linked to the choice of community. However, because public schools quality differentials are likely to be capitalised into housing prices, their choice of community is also affected by the market for education. At the same time, political support for public education falls in the community in which they choose to live. The interactions among the market for education and housing markets are therefore strong in these economies.

Nechyba (1999, 2002) and Bearse et al. (2001) have investigated the workings of decentralised systems of public education with mobility and opting-out. Bearse et al. (2001) develop a quite stylised model with no housing markets in which households differ by income in order to carry-out simulations in a dynamic economy. The objective is to investigate the role of private education when comparing central school finance and a vouchers system to a decentralised regime. In their model, the answer to the question posed above is the same in single and multiple jurisdiction economies: there exists perfect income stratification across educational sectors and the rich are the ones choosing private schooling. They opt out in order to receive an education of higher quality than that offered in the public sector.

On the other hand, Nechyba (1999, 2002) develops a rich theoretical model in which each community has a fixed stock of heterogeneous houses, households differ by their endowment of housing wealth and income and by the ability of their student, and peer group effects are an input for producing school quality. The theoretical model, however, is too complex to yield many analytical results and a computational counterpart is therefore analysed. An important assumption in the analysis in Nechyba (1999) is that student ability and income are perfectly correlated. Under such assumption, again, private schools are always used by high income households (with high ability students). Yet, even in that case, perfect income stratification across educational sectors is not assured. In his computational experiments, high quality houses are more abundant in one community, which ends up offering high quality public education. A number of affluent households prefer to reside in that community in order to choose a house of high quality. However, because the local public school is of high quality and the price of housing is very high there, these households decide to continue using public education.

Housing heterogeneity is a first way through which housing markets can prevent perfect income stratification from arising in a multi-community framework. The current paper adopts a theoretical perspective to analyse if perfect income stratification across educational sectors remains in a similar economy with homogeneous housing. In other words, if the rich (and only they) are always the ones who opt out of public education in such setting. The analysis demonstrates that, even with housing homogeneity, housing markets may significantly affect the way households and children sort into communities and schools. In particular, it shows that households from different income intervals may choose private education and that households from the top income interval in the economy may not be among them. Consequently, perfect income stratification across public and private education is not assured even in a single-dimensional characteristics space. Furthermore, the best-off households may prefer to send their youths to a public school of higher quality than the best private school in the market. Therefore, "elite" public schools may emerge and survive the competition of private schools.

The paper is organised as follows. The next section introduces the multi-community model with local school finance, opting-out and housing markets. This model is analysed and discussed in section 3. Section 4 concludes.

2 The basic model

2.1 Communities and housing markets

There exists a finite and exogenous number of communities, J, with fixed boundaries, which may differ in the amount of land contained within their limits. Every community has a competitive local housing market. We adopt a simple specification of housing markets borrowed from Epple and Romano (2002). Houses are homogenous and each household consumes one (and only one) unit of housing at price p_h. Every community i has a backward-L housing supply, horizontal at c (where c is the common construction cost) until community land capacity is reached and vertical at that quantity. Each house requires one lot of land. It is assumed that the economy housing capacity is just enough to house the population.

2.2 Education system and taxation

Education is treated as a private good. Educational services are produced from the numeraire, following a technology of production which exhibits constant returns to scale with respect to the number of students and the quantity produced. The cost function $c(x, n) = x \cdot n$ captures this technology. For simplicity sake, it assumes away the influence of peer effects and other inputs such as student ability and effort in the production of education. Moreover, because efficiency differences between public and private schools are not of interest to the model, the technology of production is assumed to be common for public and private producers of education.

Each jurisdiction may impose a proportional property tax, with tax rate t, on the value of housing and use the proceeds to provide public educational services (E). Each community i chooses the pair (E_i, t_i) through a political process, simplified to majority voting. Besides the decentralised public system, there exists a private competitive market for education. Every household can opt out of the public system and acquire any amount of education in the private market at competitive price $p_x = 1$. As it is usual in models of education, we consider public and private alternatives as mutually exclusive. A relevant difference between public and private education is that, while a household can acquire as many units of private education as it desires, regardless its community of residence, it is only allowed to send the children to a community's local public school if it resides and pays taxes in it.

2.3 Households and preferences

The system of jurisdictions is inhabited by a continuum of households, each composed by one adult, the decision-maker, and one school-aged children. The population mass is normalised to one. Households are perfectly mobile between jurisdictions and only differ in their exogenously determined endowment of the numeraire, y (income). Income is thus independent of residential location choices[2]. The income distribution is characterized by a continuous density function, $f(y)$.

There are three commodities: education (x), a private composite good (b), used as the numeraire, and housing (h). Because all houses in the system are homogenous and each family consumes one unit of housing, this good can be ignored in the utility function which represents households' preferences over different bundles in the economy.

Assumption 1 *Households share the same preferences represented by a utility function separable in x and b and increasing in all its arguments: $U(x, b) = u(x) + z(b)$. $u(x)$and $z(b)$are both twice continuously differentiable in (x, b) over all $(x, b) >> 0$. $u(x)$ is strictly quasi-concave, while $z(b)$ is strictly concave.*

Preferences are, therefore, rational, continuous, strictly convex and strictly monotone. An important consequence of assumption 1 is that educational services and the numeraire are normal goods.

[2] This assumption is typical in multi-community models and makes them most accurate for explaining the workings of urban economies with multiple jurisdictions. In such settings the place of work imposes a weak restriction on the choice of location.

Assumption 2 $\lim_{x \to 0} u(x) = -\infty$ and $\lim_{b \to 0} z(b) = -\infty$

Assumption 2 is for technical convenience. It ensures that any strictly positive combination of (x, b) is strictly preferred to any bundle with one of the goods equal to zero.

2.4 Households' decision problem and timing

Each household's adult must adopt the following decisions: (i) choose the community in which to reside; (ii) decide to send her child to the local public school or to a private school somewhere; (iii) vote on the pair (E, t) in the community in which the household resides; and, if her child attends a private school, (iv) allocate income between private education and numeraire consumption. Households are atomistic and, consequently, adults behave as price-takers.

These decisions are made in two stages within a single period. In the first stage, households simultaneously choose communities and schools, taking into account their (correct) expectations over the equilibrium vector of public policies and housing prices $e^* = (E_1, t_1, p_h^1, ..., E_J, t_J, p_h^J)$. In this stage, since the supply of housing is fixed, local housing markets clear. In the second one, once residence and schools decisions are committed, households vote on their community education policy. This sequence of decisions, also found in Nechyba (1999) and Epple and Romano (2002), is essential for solving the non single-peakedness problem that arises in models of public provision of education with opting-out (Stiglitz, 1974). We will come back to this question later on in the paper.

2.5 Definition of equilibrium

In this model, an equilibrium is a partition of households across communities and schools, an allocation (x, b) across households and a vector of community policies and housing prices $e^* = (E_1, t_1, p_h^1, ..., E_J, t_J, p_h^J)$ which satisfy:

1. *Rational choices*: for each household j, the pair (x_j, b_j) associated to the chosen community and school maximises utility among the household feasible set. This implies that no household wants to move to another community or to shift school.

2. *Housing market equilibrium*: housing demand equals housing (fixed) supply in every community.

3. *Majority voting equilibrium*: for all i=1,2,...,J, the pair (E_i, t_i) is majority-preferred by voters in community i, given the partition of households across communities and schools and the price of housing in the community. A pair (E_i, t_i) is majority preferred in community i if the associated pair $(E, p_h^i(1+t_i))$ is on the community Government Possibility Frontier (GPF) as defined in section 3.3 (i.e. if it satisfies the government budget constraint), and if it is preferred by 50 percent or more of community i voters in a binary comparison against any other bundle on the GPF.

3 Who opts out of public education?

In this section, we show that in multi-community economies with local provision of education, opting-out and homogeneous housing a large array of stratification patterns can arise in equilibrium. The analysis leads to some counter-intuitive results. For example, it reveals the existence of equilibria in which households from intermediate income intervals choose private schooling, while households from the richest tail of the income distribution send their youths to a (high quality) local public school.

3.1 Induced preferences

In order to clarify how households sort into communities and schools, we first obtain indirect utility functions corresponding to households choosing public and private education, respectively. From a household point of view, communities are characterised by the pair $(E, p_h(1+t))$, i.e. by the combination of expenditures per student (which ascertains the quality of the local public school) and the gross-of-tax price of housing (which determines the maximum feasible level of private consumption in the community). For this reason, such indirect utility functions are used to depict the indifference map in that space.

On the one hand, a household's decision-maker that sends their children to the local public school cannot acquire any private education and, from strict monotonicity, devotes $y - p_h(1+t)$ to consumption of the private composite commodity. The corresponding indirect utility function is:

$$v(E, y-p) = u(E) + z(y - p_h(1+t)) \tag{1}$$

Let p be equal to $p_h(1+t)$ and $M(E, y-p)$ be the slope of indifference curves in this space. This slope is given by:

$$\frac{dp}{dE}|_{v=\overline{v}} = M(E, y-p) = -\frac{v_E(E, y-p)}{v_p(E, y-p)} = \frac{u'(E)}{z'(y-p)} > 0 \tag{2}$$

It is equal to the marginal benefit of education in terms of the numeraire. This implies that, in response to a marginal increase in E, a household is willing to accept an increase in the gross-of-tax price of housing equal to the marginal benefit it obtains from education[3].

On the other hand, if the household chooses private education, a demand function for private education, $x(y-p)$, must be obtained. The indirect utility function can then be written as:

$$w(y-p) = u(x(y-p)) + z(y - p - x(y-p)) \tag{3}$$

In this case, because the child does not attend the local public school, marginal benefit of public educational services is zero and, therefore, indifference curves in (E, p) space are flat at each level of p.

For a utility-maximizing household choosing between public and private schooling in a given community, the induced utility function is,

$$V(E, y-p) = \max\left[v(E, y-p), w(y-p)\right] \tag{4}$$

The complete indifference map in (E, p) space is in figure 1. $\hat{E}(y-p)$ is the locus of points at which the household is exactly indifferent between public and private schooling. For each pair $(y-p)$, there is only one level of E at which this is satisfied[4]. The indifference map sketched in figure 1 is analogue to that in Epple and Romano (1996)[5]. It shows that, given the

[3] Furthermore, indifference curves of $v(E, y-p)$ in (E, p) space are strictly concave:

$$\frac{d^2p}{dE^2}|_{v=\overline{v}} = \frac{u''(E)z'(y-p)^2 + u'(E)^2 z''(y-p)}{z'(y-p)^3} < 0$$

because strict quasiconcavity ensures that,

$u''(E)z'(y-p)^2 + u'(E)^2 z''(y-p) < 0$.

[4] First of all, note that continuity of $U(\cdot)$ implies continuity of $v(\cdot)$, $w(\cdot)$ and $V(\cdot)$. Assumption 2 implies that, for any p, when $E = 0$ every household in the community consumes some amount of private education. Moreover, for each pair (y, p), there is a level of E above which the household prefers public education (this is clear because at $E = x(y-p) > 0$, strict monotonicity ensures that $v(\cdot) > w(\cdot)$. Thus, because $U(x, b)$ increases continuously with E, there is a unique level of public provision of education in the community at which the household is indifferent between the public and a private alternative. $\hat{E}(y-p)$ is implicitly defined by the equality $v(\hat{E}(y-p), y-p) = w(y-p)$.

[5] The only difference is in the vertical axis. In both cases this axis represents a cost

gross-of-tax price of housing in the community, p, a household with income y prefers private education for low enough levels of public provision ($E < \hat{E}(\cdot)$), is exactly indifferent between the local public school and private schools for $E = \hat{E}(\cdot)$, and prefers public education for large enough amounts of public educational services ($E > \hat{E}(\cdot)$). The flat part of indifference curves corresponds to the range in which the household prefers the private sector and is, thus, indifferent with respect to E. The increasing part corresponds to the range in which the household uses a public school. In such range, an increase in p must be compensated with an increase in E in order to maintain indifference. For any indifference curve, the upper contour set is below it and the lower contour set is above it. Lemma 1 analyses the behaviour of $\hat{E}(y-p)$.

Lemma 1 $\hat{E}(y-p)$ *is everywhere increasing in* $y-p$.

Proof. Differentiate $v(\hat{E}(y-p), y-p) = w(y-p)$ with respect to $y-p$:

$$d(y-p)\left(u'(\hat{E}(y-p))\hat{E}'(y-p) + z'(y-p)\right) = d(y-p)\left(z'(y-p-x(y-p))\right).$$

Solve for $\hat{E}'(y-p)$ to obtain:

$$\hat{E}'(y-p) = \frac{z'(y-p-x(y-p) - z'(y-p)}{u'(\hat{E}(\cdot))} > 0 \tag{5}$$

Given that assumption 2 assures a strictly positive demand for private education when private schooling is chosen, the latter inequality is guaranteed by concavity of $z(\cdot)$. ■

From lemma 1 it is immediate to establish:

Corollary 1 *Within-communities perfect income stratification across educational sectors. If for any* (E_j, p_j) *a household with income* y *residing in community* j *weakly prefers private to public education, then all households with income* $y' > y$ $(y'' < y)$ *residing in that community strictly prefer the private (public) sector.*

Proof. Let y be such that, given (E_j, p_j), households with income equal to y in community j are indifferent between private and public education. This entails that $E = \hat{E}(y-p)$. Lemma 1 proves that $E = \hat{E}(\cdot)$ rises monotonically

variable: in Epple and Romano it is the income tax rate, while in this paper it is the gross-of-tax price of housing.

with income. Hence, all households with income $y' > y$, satisfy $E < \hat{E}(y' - p)$, and they strictly prefer the private sector. Similarly, for all households with income $y" < y$, $E > \hat{E}(y'' - p)$, and, therefore, they strictly prefer the public option. ■

Corollary 1 shows that, in equilibrium, mixed communities are characterised by perfect income stratification across public and private education. The poorest households send their children to the local public school and the richest ones opt out of the public system and acquire private education. This result is a standard prediction in the literature (e.g. Epple and Romano, 1996 and Bearse et al., 2001). Lemma 1 has another important implication: for any level of income, the higher the gross-of-tax price of housing the smaller the amount of public educational services above which the household prefers the local public school. That is to say, other things equal, increases in housing prices or in tax rates have a direct negative impact on private school attendance in a given community.

3.2 Rational residential choices

We now turn to the analysis of optimal residential choices of households. In doing so, we take as given the choice among public and private education. For households choosing private education, it is shown that the rational residential location is the community with the lowest gross-of-tax price of housing. For households sending their children to one of the local public schools, we demonstrate some properties that characterise rational residential choices. For expositional convenience and to avoid uninteresting cases, we adopt the following realistic assumption.

Assumption 3 *All communities provide a positive amount of public educational services.*

Assumption 3 restricts attention to empirically relevant equilibria in which all communities are inhabited by (some) households sending their youths to the local public school.

The analysis proceeds as follows. Lemma 2 establishes an ordering of communities which must be satisfied by any vector of public policies and housing prices to be a candidate for equilibrium. Communities are then numbered according to this ordering. Using such ranking and taking school choices as given, propositions 1 and 2 establish conditions that must be satisfied by residential choices of households with children in private and public schools, respectively, to be rational.

Lemma 2 *In equilibrium, for every pair of communities i and j:* $E_j > E_i \Leftrightarrow p_j > p_i$ *and* $E_j = E_i \Leftrightarrow p_j = p_i$.

Proof. An allocation with $E_j > E_i$ and $p_j \leq p_i$ cannot be an equilibrium because in that case $u(E_j) > u(E_i)$, $z(y - p_j) \geq z(y - p_i)$ and, therefore, $v(E_j, y - p_j) > v(E_i, y - p_i)$ for all y. Consequently, all households choosing public education in community i would want to move to community j, which is incompatible with our definition of equilibrium. A similar argument serves to prove the second part of the lemma. ■

Housing prices, therefore, serve as screening mechanisms and differences in E are capitalised to some extent into housing prices. Those communities with higher provision levels also have higher gross-of-tax housing prices and those with identical level of provision have equal gross-of-tax housing prices. Henceforth, for expositional convenience, we shall assume that all communities have different gross-of-tax housing prices. All results below extend to the case in which some communities have the same price just by considering them a community group which is treated as a single community. Let communities be numbered such that $(E_i, p_i) << (E_{i+1}, p_{i+1})$ for all $i = 1, 2, ..., J - 1$.

Proposition 1 *In equilibrium, all households using private schools reside in the community with the lowest gross-of-tax price of housing (community 1).*

Proof. Strong monotonicity of preferences makes $w(y-p)$ to be everywhere increasing in $y - p$. Because $p_1 < p_j$ for all $j = 2, ..., J$, $y - p_1 > y - p_j$ and $w(y - p_1) > w(y - p_j)$ for all $j = 2, ..., J$. ■

Proposition 1 states that, in equilibrium, households who opt out of public education reside in the community with the lowest gross-of-tax housing price. This result is easily deduced from the indifference map in figure 1 and occurs because these households only care about the level of private consumption in each community.

Proposition 2 *In equilibrium:*

(i) Perfect income stratification across public schools. Households using public schools are perfectly stratified by income across communities.

(ii) Ascending bundles. Let $\hat{y}_i^u$ be the income of the richest household in community i consuming public education. All communities satisfy the following ascending bundles condition. If $\hat{y}_j^u > \hat{y}_i^u \Rightarrow (E_j, p_j) >> (E_i, p_i)$.

Proof. (i) The slope of indifference curves corresponding to $v(E, y-p)$ in (E, p) space, $M(E, y-p)$, increases monotonically with y[6]:

$$\frac{\partial M(E, y-p)}{\partial y} = \frac{-u'(E)z''(y-p)}{z'(y-p)^2} > 0 \quad (6)$$

An important consequence of this slope rising in income (SRI) condition is the following single-crossing property: the indifference curve of a household with income y crosses the indifference curve of any other household with different income at most once, and the indifference curve of the wealthier of any two households always cuts that of the poorer from below. The single-crossing property, in turn, implies the following preference ordering, proved in Epple et al. (1993), lemma 1: given $(E_i, p_i) << (E_j, p_j)$,

$$v(E_i, y - p_i) \geq v(E_j, y - p_j) \Rightarrow v(E_i, y' - p_i) > v(E_j, y' - p_j);\ \forall y' < y \quad \text{(7a)}$$

$$v(E_i, y - p_i) \leq v(E_j, y - p_j) \Rightarrow v(E_i, y' - p_i) < v(E_j, y' - p_j);\ \forall y' > y \quad \text{(7b)}$$

(7a) and (7b) entail that, in equilibrium, a community cannot be inhabited by households in the public sector from different income intervals.

(ii) By contradiction. Suppose that in equilibrium $(E_j, p_j) >> (E_i, p_i)$ and $\hat{y}_j^u < \hat{y}_i^u$. In that case, the following conditions must hold (a) $v(E_i, \hat{y}_i^u - p_i) \geq v(E_j, \hat{y}_i^u - p_j)$, and (b) $v(E_j, \hat{y}_j^u - p_j) \geq v(E_i, \hat{y}_j^u - p_i)$. From (7a), however, we know that,

$$v(E_i, \hat{y}_i^u - p_i) \geq v(E_j, \hat{y}_i^u - p_j) \Rightarrow v(E_i, y' - p_i) > v(E_j, y' - p_j);\ \forall y' < \hat{y}_i^u.$$

And, therefore, (b) cannot hold if $\hat{y}_j^u < \hat{y}_i^u$. ■

Corollary 2 *Communities 2 to J show perfect income stratification and are composed by households from single income intervals.*

[6] This slope rising in income condition is usual in multi-community models (e.g. Epple et al., 1984). In our model, it is automatically satisfied due to housing homogeneity. If housing were not restricted to be homogenous, this condition would require income elasticity of marginal benefit of education to be larger than the income elasticity of housing demand (see Ross and Yinger, 1999).

Proof. Proposition 1 proves that no household in communities 2 to J acquires private educational services. Proposition 2 demonstrates that households in the public sector are perfectly stratified across communities. Hence, communities 2 to J show perfect income stratification and are inhabited by households from single income intervals. ■

3.3 The voting problem

Some features of the voting problem in the model deserve further explanation. At the voting stage, the government budget constraint of any community i is given by:

$$E_i(t_i) = t_i p_h^i \frac{N_i}{n_i}; \; \forall t_i \geq 0 \tag{8}$$

where n_i is the mass of households sending their child to the local public school in community i and N_i is the community population capacity. Note that voting takes place once community and schooling choices are committed. For this reason, voting outcomes do not change the community composition, the price of housing and private school enrolment. We can then define the Government Possibility Frontier (GPF) of community i in $(E_i, p_h^i(1+t_i))$ space as:

$$p_h^i(1+t_i) = E_i \frac{n_i}{N_i} + p_h^i \tag{9}$$

The GPF of community i shows the maximum level of expenditures per student in the local public school, given the community housing price and the proportion of households using that school in the community (see figure 2). Voters are assumed to know the government budget constraint, the identity among net and gross of tax price of housing $p = p_h(1+t)$.They also know that voting occurs at the final stage, once the choice of community and school has been made. Under these assumptions voters know which alternatives are on the GPF when voting.

Moreover, under the sequence of decisions in our model, preferences over tax rates are single-peaked at the voting stage (as in Nechyba, 1999). The reason is again that residential and schooling choices are already committed when voting takes place. Thus, the sequence of decisions in the model allows us to avoid the non single-peakedness problem which arises in previous models of public provision of education when private alternatives are available (Epple and Romano, 1996; Bearse et al., 2001).

Proposition 3 *Given a partition of households across communities and schools and a vector of housing prices* $(p_h^1, ..., p_h^J)$, *there exists a unique majority voting equilibrium and the median voter is decisive in every community. Moreover, (i) In community 1, the income of the decisive voter,* $\widetilde{y}_1$, *satisfies:*

$$F(\hat{y}_1^u) - F(\widetilde{y}_1) = \frac{N_1}{2} \quad (10)$$

Where $\hat{y}_1^u$ *is the income of the richest household choosing the local public school in community 1. (ii) In communities 2 to J the community median income voter is decisive.*

Proof. At the voting stage, preferences over tax rates are single-peaked because the choices of community and school are already committed. For all households who have chosen a private school the peak is at $t = 0$. This is because a zero tax rate maximises their level of private consumption and because they do not benefit from expenditures on public education. Households with income y using the local public school in community i reach their peak at the unique t that solves:

$$\max_t \; u(E_i) + z(y - p_h^i(1 + t_i))$$

$$\text{s.t. } E_i = t_i p_h^i \frac{N_i}{n_i}$$

Consequently, by the median voter theorem (Black, 1958), a unique majority voting equilibrium exists in every community and the median voter is pivotal. We now prove (i) and (ii) in the proposition. The proofs follow Epple and Romer (1991) and Epple and Romano (1996).

(i) Consider the case of community 1 and the voter with income $\widetilde{y}_1$. By axiom 3, this voter sends their children to the local public school and thus, by axiom 2, votes for a strictly positive pair $(E, t) >> 0$. Suppose that their most preferred pair on the GPF is given by $(E^*, p_h(1 + t^*))$ in figure 2. This point is determined by the tangency between the community GPF and an indifference curve corresponding to $v(E, \widetilde{y}_1 - p_h(1 + t))$ in the figure. Proposition 3 states that this is the unique majority voting equilibrium in community 1. To prove it, first consider any point on the GPF to the left of $(E^*, p_h(1 + t^*))$. From the SRI property of indifference curves shown in the proof to proposition 2, all households with income $y > \widetilde{y}_1$and with their offspring attending the local public school (e.g. households with income y' in figure 3) prefer $(E^*, p_h(1+t^*))$ to any of these points. Given the definition of $\widetilde{y}_1$, these households are half the population in community 1. Therefore, at least half the electorate prefers $(E^*, p_h(1 + t^*))$ to any larger bundle on the

GPF and it defeats all of these alternatives. Consider now any point on the GPF to the right of $(E^*, p_h(1+t^*))$. From the SRI property, all households with income $y < \widetilde{y}_1$prefer $(E^*, p_h(1+t^*))$ to any of these points. Moreover, from proposition 1, households sending their youths to a private school have the same preference. Since all these households are half the electorate in community 1, $(E^*, p_h(1+t^*))$ also defeats any smaller bundle on the GPF.

(ii) In any community $i = 2, ..., J$ all households use the local public school. This makes the median income voter most preferred bundle on the GPF to be the unique majority voting equilibrium. To see why, first note that at least half the population (households with income $y > \widetilde{y}_1$) prefer it to any alternative on the GPF located to the left of it; and, second, that at least half the electorate (voters from households with income $y < \widetilde{y}_1$) prefer it to any alternative on the GPF located to the right of it. Consequently, this bundle defeats all alternatives on the community GPF. ■

3.4 Income stratification across public and private education

We now turn to the question of which households use public schools and which decide to opt out of the public system and acquire private education. In their model, Bearse et al. (2001) obtain that, in equilibria with no empty communities, rich and poor households mix in the community with the lowest income tax rate and level of provision. This community shows perfect income stratification across schools, with the rich acquiring education in the private market and the poor using the local public school. Middle income households choose the other community and send their youths to that community local public school. This result does not change the basic intuition in Epple and Romano (1996): the richest fraction of households in the economy send their children to private schools because they do not find a public school of high enough quality.

An implication of this result is that, in such equilibria, if households with income y are indifferent between "community 1-private education" and "community 2-public education", then households with income $y' > y$ strictly prefer the former alternative, while households with income $y" < y$ strictly prefer the latter. We begin the analysis proving in Lemma 3 and corollary 3 that this property does not hold in general.

For expositional convenience, and without loss of generality, we set $J = 2$ for all the analysis below. Define $\hat{E}_2(y, p_1, p_2)$ as community 2 level of provision at which households with income y are just indifferent between private education at community 1 and public education in community 2. $\hat{E}_2(y, p_1, p_2)$ is a function, i.e. for each (y, p_1, p_2) there exists a unique $\hat{E}_2$ for which $v(\hat{E}_2, y - p_2) = w(y - p_1)$[7]. In (E, p) space it coincides with the indifference curve corresponding to $V(E, y - p)$ and to a utility level equal to $w(y - p)$ (see figure 2). Given the actual level of provision in community 2, E_2, households with income y such that $\hat{E}_2(y, p_1, p_2) > E_2$ strictly prefer private education at community 1, while households with income y such that $\hat{E}_2(y, p_1, p_2) < E_2$ strictly prefer living in community 2 and sending their children to the local public school there.

Lemma 3 *$\hat{E}_2(y, p_1, p_2)$ is increasing (decreasing) in y if $p_2 - p_1 < x(y - p_1)$ ($p_2 - p_1 < x(y - p_1)$).*

Proof. $\hat{E}_2(y, p_1, p_2)$ is implicitly defined by $v(\hat{E}_2(y, p_1, p_2), y - p_2) = w(y - p_1)$. Differentiate this expression with respect to y:

$$dy\left(u'(\hat{E}_2(\cdot))\frac{\partial \hat{E}_2(\cdot)}{\partial y} + z'(y - p_2)\right) = dy\left(z'(y - p_1 - x(y - p_1)\right)$$

Again, for differentiating the right hand side we use assumption 2, which guarantees an interior solution for the utility maximisation problem of households using private schools. Solve for $\frac{\partial \hat{E}_2(\cdot)}{\partial y}$ to obtain:

$$\frac{\partial \hat{E}_2(\cdot)}{\partial y} = \frac{z'(y - p_1 - x(y - p_1) - z'(y - p_2)}{u'(\hat{E}_2(\cdot))} \tag{11}$$

Now, given strict concavity of $z(\cdot)$, this derivative will be positive (negative) if $y - p_1 - x(y - p_1) < y - p_2$ ($y - p_1 - x(y - p_1) > y - p_2$), i.e. if $p_2 - p_1 < x(y - p_1)$ ($p_2 - p_1 > x(y - p_1)$). ■

Corollary 3 *Let ε be an arbitrarily small positive number. If for given (E_2, p_1, p_2) households with income y are indifferent between "private education-community 1" and "public education-community 2", then:*

(i) households with income $y + \varepsilon$ ($y - \varepsilon$) strictly prefer "public education-community 2" ("private education-community 1") if $p_2 - p_1 > x(y - p_1)$.

[7] See footnote 3.

(ii) households with income $y+\varepsilon$ $(y-\varepsilon)$ strictly prefer "private education-community 1" ("public education-community 2") if $p_2 - p_1 < x(y - p_1)$.

Proof. y satisfies $\hat{E}_2(y, p_1, p_2) = E_2$. Lemma 3 proves that when $p_2 - p_1 > x(y - p_1)$, $\hat{E}_2$ is decreasing in income. This implies $\hat{E}_2(y+\varepsilon, p_1, p_2) < E_2$ and $\hat{E}_2(y-\varepsilon, p_1, p_2) > E_2$, implying statement (i). To demonstrate statement (ii) just change the direction of all inequalities. ■

Lemma 3 stems from the fact that, for any alternative, marginal utility of income is larger the lower the consumption of the hicksian commodity. Corollary 3 shows that for such reason, if households which are indifferent between "private education-community 1" and "public education-community 2" have a demand for private education in community 1 smaller than the difference in gross-of-tax housing prices between community 2 and community 1 (i.e., if for these households consumption of the hicksian commodity is larger in the former alternative), then richer (poorer) households strictly prefer the latter (former) of both alternatives. Note that if this occurs, it seems possible to find equilibria in which households from intermediate income intervals opt out of the public system, while poorer and richer households continue using public schools.

The result in Lemma 3 is not new. It is implicit in the single jurisdiction case (Epple and Romano, 1996) and an equivalent condition can be obtained for the two-community model in Bearse et al. (2001). In the model with central provision, nevertheless, consumption of the hicksian commodity is always larger in the public alternative, precluding situations of the type just described to be an equilibrium. In the model with income taxation and without housing markets in Bearse et al. (2001), in turn, whenever marginal utility of income is larger in the alternative "public education-community 2" than in "private education-community 1", the former option provides a strictly lower utility than the latter, for all $y > 0$[8].

As it will be shown below, what it is new in our model is that it is indeed possible to find equilibria in which intermediate income households choose a private school. As it will be explained below, the presence of housing markets is what makes this possible. Moreover, note that demand for private education increases with income. Consequently, $\hat{E}_2(y, p_1, p_2)$ decreases with income for levels of income satisfying $p_2 - p_1 > x(y - p_1)$, reaches a minimum at y satisfying $p_2 - p_1 = x(y - p_1)$ and then increases for levels of income

[8] I thank G. Glomm, P. Bearse and B. Ravikumar for showing me this point.

for which $p_2 - p_1 < x(y - p_1)$. Thus, it seems also possible to find equilibria in which $\hat{E}_2(y, p_1, p_2) > E_2$ holds for two income intervals and, therefore, in which households from those income intervals prefer a private alternative. Through a set of examples, proposition 4 reveals that the introduction of housing markets and property taxation into the picture generates existence of this type of equilibria.

Consider an economy corresponding to the model in section 2. This economy has a population mass N normalised to 1. It is composed by two communities, 1 and 2, with capacity equal to N_1 and N_2, respectively. Households preferences are captured by the following utility function, borrowed from Bearse et al. (2001):

$$U(x, b) = \frac{1}{1-\sigma}\left[b^{1-\sigma} + \delta x^{1-\sigma}\right];\ \sigma, \delta > 0 \qquad (12)$$

This utility function is separable in (b, x) and strictly concave for $\sigma, \delta > 0$. It violates assumption 2 but this is inconsequential for the examples in proposition 3. Parameters of the utility function are set at levels in Bearse et al. (2001): $\sigma = 2.23$ and $\delta = 0.0032$. The income cumulative distribution function is given by $F(y)$ with lower and upper bounds $\underline{y}$ and $\overline{y}$. Finally, the cost of producing a unit of housing is c and equals the net-of-tax price of housing in community 1, p_h^1.

Proposition 4 *Examples 1, 2 and 3 below are equilibria of the above described economy.*

Proof. See appendix A2. ■

Example 1 *Communities capacity:* $N_1 = 0.75$; $N_2 = 0.25$.

Income cumulative distribution function (figure 4):

$F(y) = -0.2232 + 3.6261 \cdot 10^{-2}y - 1.2669 \cdot 10^{-3}y^2 + 3.0696 \cdot 10^{-5}y^3 - 3.8289 \cdot 10^{-7}y^4$

$+2.2992 \cdot 10^{-9}y^5 - 5.2834 \cdot 10^{-12}y^6$; $\underline{y} = 8$; $\overline{y} = 125$.

Vector of public policies and housing prices:

$e^* = (E_1, t_1, p_h^1, E_2, t_2, p_h^2) = (2, 0.2712, 5.9, 6.2, 0.8857, 7)$

Median voters: $(\widetilde{y}_1, \widetilde{y}_2) = (23.3679, 94.7001)$

Partition of households across communities and schools: households with income $y \in [8, 59.88)$ *(with a mass equal to 0.6) choose public education-community 1; households with income* $y \in [59.88, 71.96)$ *(with a mass equal* *to 0.1) choose private education-community 1; households with income* $y \in [71.96, 117.47]$ *(with a mass equal to 0.25) choose public education-community 2; households with income* $y \in [117.47, 125)$ *(with a mass equal to 0.05) choose private education-community 1.*

Example 2 *Communities capacity:* $N_1 = 0.8$*;* $N_2 = 0.2$.

Income cumulative distribution function (figure 5):

$F(y) = -.2133 + 1.8200 \cdot 10^{-2} y + 4.4177 \cdot 10^{-4} y^2 - 1.4110 \cdot 10^{-5} y^3 + 1.2776 \cdot 10^{-7} y^4 - 3.7943 \cdot 10^{-10} y^5$*;* $\underline{y} = 10$*;* $\overline{y} = 120$

Vector of public policies and housing prices:

$e^* = (E_1, t_1, p_h^1, E_2, t_2, p_h^2) = (1.5, 0.1753, 7.4875, 6.6, 0.8684, 7.6)$

Median voters: $(\widetilde{y}_1, \widetilde{y}_2) = (22.9718, 100.9582)$

Partition of households across communities and schools: households with income $y \in [10, 48.09)$ *(with a mass equal to 0.7) choose public education-community 1; households with income* $y \in [48.09, 64.57)$ *(with a mass equal to 0.1) choose private education-community 1; households with income* $y \in [64.57, 120]$ *(with a mass equal to 0.2) choose public education-community 2.*

Example 3 *Communities capacity:* $N_1 = 0.8$*;* $N_2 = 0.2$.

Income cumulative distribution function (figure 6):

$F(y) = -3.6344 \cdot 10^{-2} - 2.1132 \cdot 10^{-3} y + 9.8265 \cdot 10^{-4} y^2 - 2.0059 \cdot 10^{-5} y^3$
$+1.5755 \cdot 10^{-7} y^4 - 4.3713 \cdot 10^{-10} y^5$*;* $\underline{y} = 8$*;* $\overline{y} = 125$

Vector of public policies and housing prices:

$e^* = (E_1, t_1, p_h^1, E_2, t_2, p_h^2) = (2, 0.2892, 6.05, 4.5, 0.6716, 6.7)$

Median voters: $(\widetilde{y}_1, \widetilde{y}_2) = (26.6958, 70.3533)$

Partition of households across communities and schools: households with income $y \in [8, 54.52)$ *(with a mass equal to 0.7) choose public education-community 1; households with income* $y \in [54.52, 99.18)$ *(with a mass equal to 0.2) choose public education-community 2; households with income* $y \in [99.18, 125]$ *(with a mass equal to 0.1) choose private education- community 1.*

3.5 Discussion

In a model with homogenous housing, households choosing private schooling are those that do not find a community with the mix of price of housing, tax rate and per-student expenditures which leads to the combination of numeraire consumption and school quality within their (non-convex) feasible set providing the maximum utility. For these households, then, it is worth spending some extra money in order to acquire private educational services. In the single jurisdiction case, these households are always the rich ones, whose demand for education is larger than the common provision level. Therefore, the distribution of households across public and private education shows perfect income stratification. As examples 1 and 2 above demonstrate, in a multi-community setting this result does not necessarily hold.

Figures 7, 8 and 9 show, respectively for examples 1, 2 and 3 the level of education received by households from each level of income, given their choice of community and school. In examples 1 and 2 households from an intermediate income interval are not satisfied by the mix of public education and housing prices offered by communities 1 and 2. Consequently, they exit the public system and send their youths to private schools. These schools are of higher quality than the public school in community 1 but of lower quality than the public school in community 2. That is to say, for these households, $E_1 < x_1(\cdot) < E_2$, as it can be checked in figures 7 and 8. On the other hand, their level of numeraire consumption is larger than that feasible in community 2 and obviously smaller than the amount they could purchase were they using the local public school in community 1. The reason why they choose private schooling is that they prefer an intermediate combination of school quality and numeraire consumption to the alternatives offered by communities 1 and 2. Clearly, the convexity property satisfied by the preference relation adopted in the model is key for this result to hold. For these households, the quality of the local public school of community 1 is too low given the level of disposable income available for them there $(y - p_1)$. But also, because the price of housing is very high in community 2, the level of disposable income available for them in this community $(y - p_2)$ is too low, given the quality of its local public school.

Housing markets play a key role for the existence of equilibria of this type. The reason is that a large enough difference in (gross-of-tax) housing prices among both communities is what makes intermediate income households to have too high a level of disposable income in community 1 to choose the local public school there and, at the same time, too low a level of disposable income in community 2 to prefer the public school there.

Furthermore, in the first example the households from the top income interval in the economy, choose a private school which is of higher quality than the public one in community 2. In the second one, in turn, they reside in community 2 and use the local public school there. In this example, the rich have enough mass to dominate the political process and establish a local public school of high enough quality there. In this case, they are willing to outbid middle income households from that community in order to use this "elite" public school.

It is remarkable that in equilibria of this type, public support for a high quality public school does not necessarily fall in the rich community. This example proves that it is indeed possible to find a public school of higher quality than would be available were private schools not allowed to enter the market. To show this, we compute the equilibrium which would arise in the economy in example 2 if private alternatives were prohibited. In that case, the allocation of households to communities would not change and all households would use the local public school in their community of residence. Moreover, because all voters would vote for a positive tax rate, the median income household in each community would be decisive. The equilibrium vector of community policies and housing prices is $e^* = (E_1, t_1, p_h^1, E_2, t_2, p_h^2) = (1.43, 0.19, 7.49, 6.32, 0.54, 11.6)$[9]. Thus, expenditures per student in community 2 rise from 6.32 to 6.6. once private education is allowed for. An important message is therefore that very high quality public schools can arise in rich communities and survive the competition of private schools.

Example 3 serves to prove that equilibria showing perfect income stratification are, of course, possible in our model. In this example, the differences among communities 1 and 2 in public education quality and gross-of-tax housing prices are relatively small and middle income households do not opt out. Rich households, in turn, are not satisfied by public education and choose private schooling (see figure 9). Similar to the single jurisdiction case, in this equilibrium the quality of the local public school in community 2 is too low for them and they prefer to sacrifice some private consumption in

[9]The proof that this vector is an equilibrium of the economy in example 2 if private education is prohibited is available from the author upon request.

order to receive higher quality education.

4 Concluding remarks

In this paper we have analysed the question of which households opt out of public education in a multi-community economy with housing markets. In particular, the objective has been to investigate whether perfect income stratification across public and private educational sectors predicted by single jurisdiction models (Epple and Romano, 1996) and by multiple jurisdiction ones without housing markets (Bearse et al., 2001) is guaranteed once housing markets are introduced into the picture. Using a very rich computational model, Nechyba (1999) has shown that housing heterogeneity is a first way whereby housing markets can prevent the perfect stratification result from arising in multi-community economies. Here we demonstrate that, even with housing homogeneity, perfect stratification is not assured in a single-dimensional characteristics space. On the contrary, it is possible to find equilibria in which households from intermediate income intervals use private schools, while richer ones prefer to send their youths to a public school of higher quality. It is also possible to find equilibria in which households from the top income interval in the economy end up using a public school of higher quality than the best private alternative available in the market. The analysis in this paper then reveals that very high quality public schools can arise when private schooling is allowed for and survive the competition of private schools.

An important question for further research is under which circumstances each type of equilibrium is more likely to arise. The analysis suggests that the more polarised the income distribution and the stronger the preferences for education the more likely are equilibria with intermediate income households using a private school or with rich households sending their youths to a local public school. Intuitively, this occurs because differences in public education quality and, consequently, in housing prices among communities are usually larger under such circumstances. The introduction of peer group effects into the model (for example, with student ability being perfectly correlated to household income) would also make such equilibria more frequent. The reason is the same as before: differences in the quality of public schooling and in housing prices among communities would be larger. The number of communities can also be a relevant variable. In general, the larger the number of communities, the larger the menu of public alternatives and the smaller the number of households (from whatever income interval) choosing private education.

Last but not least, it should be stressed that this paper highlights, once again, how profoundly housing markets affect the market for education. In our model, a simple specification of housing markets serves to show how difficult it is to predict the way in which households and their children sort into communities and schools, and the key role housing markets play in this process. From our point of view, models which let us capture the impact of housing markets into the market for education will be an important instrument in future research, mainly, on topics related to school choice and competition between private and public educational institutions.

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Appendix A1. Figures

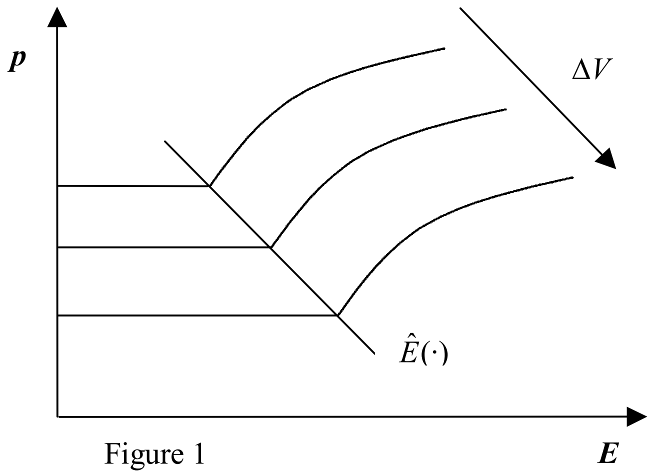

Figure 1

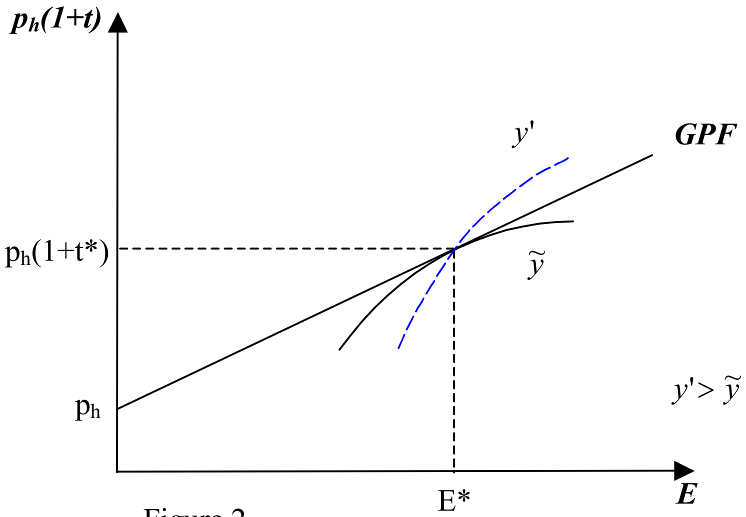

Figure 2

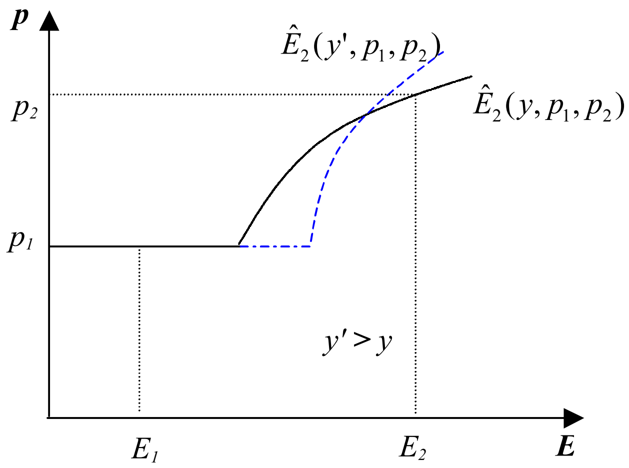

Figure 3

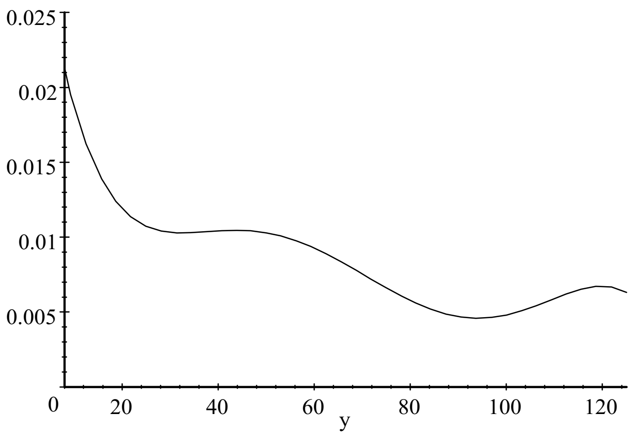

Figure 4. Income distribution density function (example 1)

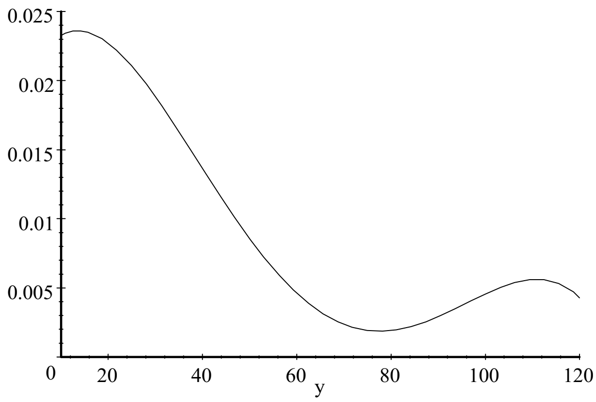

Figure 5. Income distribution
density function (example 2)

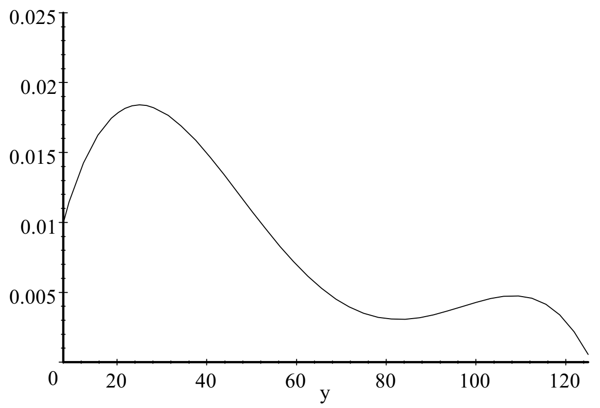

Figure 6. Income distribution
density function (example 3)

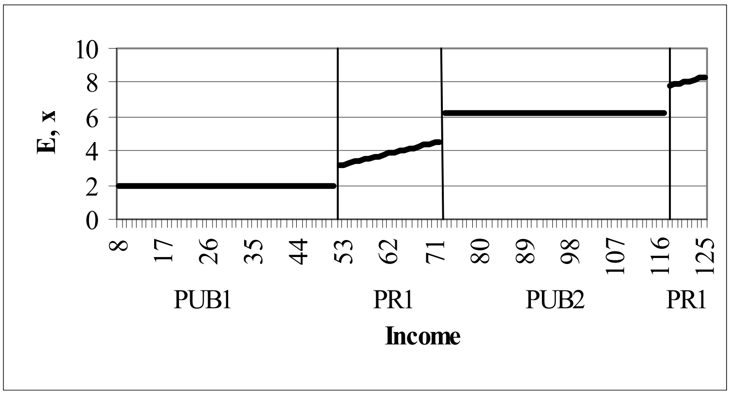

Figure 7. (Example 1)

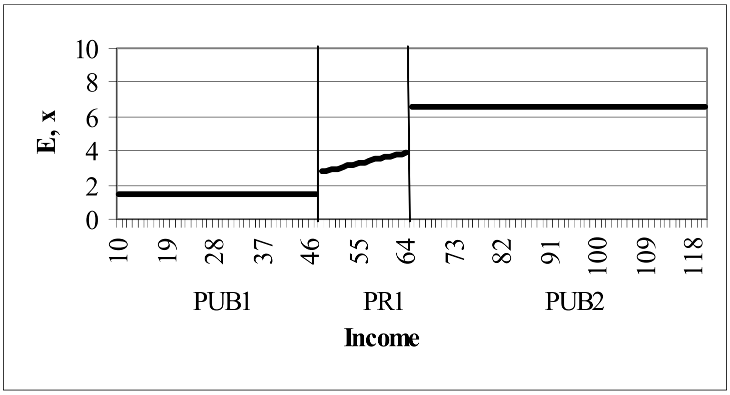

Figure 8. (Example 2)

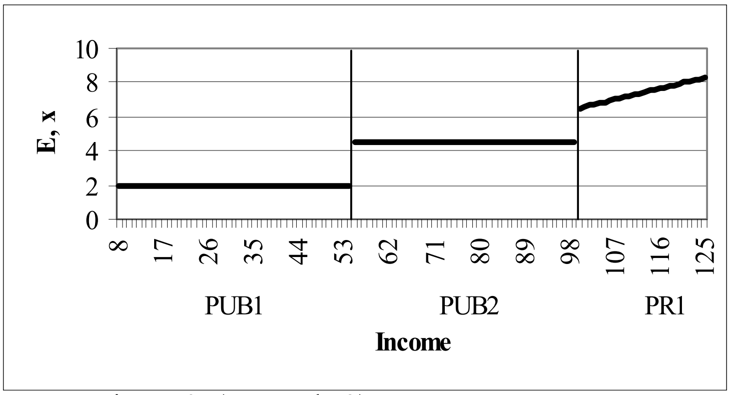

Figure 9. (Example 3)

Appendix A2. Proof of proposition 4:

From proposition 1, for any $p_2>p_1$ no household chooses the option private education-community 2. Thus, we only need to consider the alternatives public education-community 1 (PUB1), public education-community 2 (PUB2) and private education-community 1 (PR1). Given the equilibrium vector $e^*= (E_1,p^1_h,t_1,E_2,p^2_h,t_2)$, and with some abuse of notation let $v_1(y)$ be $v(E_1,y-p_1)$, $v_2(y)$ be $v(E_2,y-p_2)$ and $w_1(y)$ be $w(y-p_1)$, where $p_i=p^i_h(1+t_i)$. From (12) we obtain:

$$v_i(y) = \frac{1}{1-\sigma}\left[(y-p_i)^{1-\sigma} + \delta E_i^{\,1-\sigma}\right], \tag{A1}$$

$$x(y-p_1) = \frac{(y-p_1)\delta^{1/\sigma}}{1+\delta^{1/\sigma}}, \text{ and} \tag{A2}$$

$$w_1(y) = \frac{1}{1-\sigma}(y-p_1)^{1-\sigma}\left(1+\delta^{1/\sigma}\right)^{\sigma} \tag{A3}$$

Example 1:

(i) *Rational choices.*

(a) Consider alternatives PUB1 and PUB2. For an income level y=62.885, the following can be verified: $v_1(62.885)=v_2(62.885)$. By proposition 2, this implies that all households with income y<62.885 strictly prefer PUB1 to PUB2, while all households with income y>62.885 strictly prefer PUB2 to PUB1.

(b) Consider alternatives PUB1 and PR1. For y=59.88, v_1 and v_2 satisfy: $v_1(59.88)=w_1(59.88)$. By corollary 1, this implies that all households with income y<59.88 strictly prefer PUB1 to PR1, while all households with income y>59.88 strictly prefer PR1 to PUB1.

(c) Consider alternatives PR1 and PUB2. It can be verified that $w_1(71.96)=v_2(71.96)$, which implies $E_2 = \hat{E}_2(71.96,\cdot)$. By lemma 3 and given that $p_2-p_1=13.2-7.5=5.7>4.557=x(71.96-7.5)$ and that educational services are normal, all households with income y<71.96 satisfy $E_2 < \hat{E}_2(y',\cdot)$ and, hence, they strictly prefer PR1 to PUB2.

(d) Consider again alternatives PR1 and PUB2. Due to normality of educational services, the sign of $\partial \hat{E}_2(\cdot)/\partial y$ becomes negative for all y>88.128 (because x(88.128-7.5)=5.7). Moreover, for y=117.47, $w_1(117.47)= v_2(117.47)$ and, therefore, $E_2 = \hat{E}_2(117.47,\cdot)$. This result and (c) imply that households with income $y \in [71.96,117.47]$, strictly prefer PUB2 to PR1 (because for them $E_2 > \hat{E}_2(\cdot)$); while households with income *y>117.47* strictly prefer PR1 to PUB2 (because for them $E_2 < \hat{E}_2(\cdot)$).

From (a) and (b), households with income $y \in [8,59.88]$ maximise their utility choosing public education-community 1, while households with income $y \in [59.88,62.885]$ do so choosing private education-community 1. (b) and (c) establish that the optimal choice for households with income $y \in [62.885,71.96]$ is private education-community 1. Finally, (b) and (d) imply that the optimal choices for households with income $y \in [71.96,117.47]$ and $y \in [117.47,125]$ are, respectively, public education-community 2 and private education-community 1.

(ii) *Housing market equilibrium*:

The housing market is in equilibrium if all houses in every community are occupied. Community 1 capacity is 0.75. It is inhabited by households with income $y \in [8,71.96]$ and $y \in [117.47,125]$. It can be verified that:

$F_1(125)-F_1(117.47)=0.05$

$F_1(71.96)-F_1(8)=0.7$

which adds up to 0.75.

Community 2 capacity is 0.25. It is populated by households with income $y \in [71.96,117.47]$. This income interval satisfies:

$F_1(117.47)-F_1(71.96)=0.25$

(iii) *Majority voting equilibrium*:

(a) *Community 1*: The income of the pivotal voter is implicitly defined by $F_1(59.88) - F_1(\tilde{y}_1) = 0.375$, which yields $\tilde{y}_1$=23.3679. Because households with this income choose public schooling, from (8), the median voter most preferred tax rate t_1^* is:

$$t_1^* \equiv \arg\max \frac{1}{1-\sigma}\left[\left(\tilde{y}_1 - p_1^h(1+t_1)\right)^{1-\sigma} + \delta\left(\frac{t_1 p_1^h N_1}{n_1}\right)^{1-\sigma}\right] =$$

$$= \frac{\tilde{y}_1 - p_1^h}{p_1^h\left(1+\left(\frac{\delta N_1}{n_1}\right)^{-1/\sigma}\left(\frac{N_1}{n_1}\right)\right)} \tag{A4}$$

which yields t_1^*=0.2712. The corresponding level of provision is:

$$E_1^* = \frac{t_1^* p_1^h 0.75}{0.6} = 2$$

(b) *Community 2*: The median (income) voter has income $\tilde{y}_2$=94.7001, which satisfies $F_1(117.47) - F_1(94.7001) = 0.125$. Since all households in community 2 choose public schooling, the median voter most preferred tax rate t_2^* is:

$$t_2^* \equiv \arg\max \frac{1}{1-\sigma}\left[\left(\tilde{y}_2 - p_2^h(1+t_2)\right)^{1-\sigma} + \delta\left(t_2 p_2^h\right)^{1-\sigma}\right] = \frac{\tilde{y}_2 - p_2^h}{p_2^h\left(1+\delta^{-1/\sigma}\right)} \tag{A5}$$

which leads to $t_2^* = 0.8857$. The associated level of provision is $E_2^* = t_2^* p_2^h = 6.2$

Example 2:

(i) *Rational choices*.

(a) Consider alternatives PUB1 and PUB2. For an income level y=53.18, the following can be verified: v_1(53.18)=v_2(53.18). By proposition 2, this implies that all households with income y<53.18 strictly prefer PUB1 to PUB2, while all households with income y>53.18 strictly prefer PUB2 to PUB1.

(b) Consider alternatives PUB1 and PR1. For y=48.09, v_1 and v_2 satisfy: v_1(48.09)=w_1(48.09). By corollary 1, this implies that all households with income y<48.09 strictly prefer PUB1 to PR1, while all households with income y>48.09 strictly prefer PR1 to PUB1.

(c) Consider alternatives PR1 and PUB2. It can be verified that w_1(64.57)= v_2(64.57), which implies $E_2 = \hat{E}_2(64.57,\cdot)$. By lemma 3 and given that p_2-p_1=14.2-8.8= 5.4>3.94= x(64.57-8.8) and that educational services are normal, all households with income y<64.57 satisfy $E_2 < \hat{E}_2(y,\cdot)$ and, hence, they strictly prefer PR1 to PUB2. For the same reason, households with income levels marginally larger than 64.57 satisfy $E_2 > \hat{E}_2(y,\cdot)$ and they prefer PUB2 to PR1. Moreover, for households with the highest income it can be checked that w_1(120)<v_2(120).

From (a) and (b), households with income $y \in [10, 48.09]$ maximise their utility choosing public education-community 1, while households with income $y \in [48.09, 53.18]$ do so choosing private education-community 1. (b) and (c) entail that the optimal choice for households with income

$y \in [53.18, 64.57]$ is private education-community 1. Finally, (b) and (c) imply that the optimal choices for households with income $y \in [64.57, 120]$ is public education-community 2.

(ii) *Housing market equilibrium*:

The housing market is in equilibrium if all houses in every community are occupied. Community 1 capacity is 0.8. It is inhabited by households with income $y \in [10, 64.57]$. It can be verified that:

$F_2(64.57)$-$F_2(10)$=0.8

Community 2 capacity is 0.2. It is populated by households with income $y \in [64.57, 120]$. This income interval satisfies:

$F_2(120)$-$F_2(64.57)$=0.2

(iii) *Majority voting equilibrium*:

(a) *Community 1*: The income of the pivotal voter is implicitly defined by $F_2(48.09) - F_2(\tilde{y}_1) = 0.4$, which yields $\tilde{y}_1$=22.9718. Because households with this income choose public schooling, the median voter most preferred solves (A4), yielding t_1^*=0.1753. This tax rate has an associated level of provision equal to:

$$E_1^* = \frac{t_1^* p_1^h 0.8}{0.7} = 1.5$$

(b) *Community 2*: The median (income) voter has income $\tilde{y}_2$=100.9582, satisfying $F_2(120) - F_2(100.9582) = 0.1$. Since all households in community 2 choose public schooling, the median voter most preferred tax rate t_2^* is given by (A5). This gives an equilibrium tax rate equal to 0.8, whose associated level of provision is $E_2^* = t_2^* p_2^h = 6.2$.

Example 3:

(i) *Rational choices*.

(a) Consider alternatives PUB1 and PUB2. For an income level y=54.52, the following can be verified: $v_1(54.52)=v_2(54.52)$. By proposition 2, this implies that all households with income y<54.52 strictly prefer PUB1 to PUB2, while all households with income y>54.52 strictly prefer PUB2 to PUB1.

(b) Consider alternatives PUB1 and PR1. For y=60.18, v_1 and v_2 satisfy: $v_1(60.18)=w_1(60.18)$. By corollary 1, this implies that all households with income y<60.18 strictly prefer PUB1 to PR1, while all households with income y>60.18 strictly prefer PR1 to PUB1.

(c) Consider alternatives PR1 and PUB2. It can be verified that $w_1(99.18)=v_2(99.18)$, which implies $E_2 = \hat{E}_2(99.18, \cdot)$. Given that p_2-p_1=11.2-7.8=3.4<6.46=x(99.18-7.5), lemma 3 implies that all households with income y>99.18 satisfy $E_2 < \hat{E}_2(y', \cdot)$ and, hence, that they strictly prefer PR1 to PUB2.

(d) Consider again alternatives PR1 and PUB2. The sign of $\partial \hat{E}_2(\cdot)/\partial y$ is positive for all y>55.9 (because x(55.9-7.8)=3.4). This result and (c) imply that households with income $y \in [55.9,99.18]$, strictly prefer PUB2 to PR1 (because for them $E_2 > \hat{E}_2(\cdot)$).

(a) and (b) imply that households with income $y \in [8,54.52]$ maximise their utility choosing public education-community 1 and that for households with income $y \in [54.52,60.18]$ it is rational to choose public education-community 2. (b) and (c) establish that the optimal choice for households with income $y \in [99.18,125]$ is private education-community 1. Finally, (b) and (d) imply that the optimal choices for households with income $y \in [60.18,99.18]$ is public education-community 2.

(ii) *Housing market equilibrium*:

Community 1 capacity is 0.8. This community is populated by households with income $y \in [8,54.52]$ and $y \in [99.18,125]$. It can be checked that the cumulative income distribution function $F_3(y)$ satisfies:

$F_3(125)-F_3(99.18)=0.1$

$F_3(54.52)-F_3(8)=0.7$

which adds up to 0.8.

Community 2 capacity is 0.2. It is populated by households with income $y \in [54.52,99.18]$. This income interval satisfies:

$F_3(99.18)-F_3(54.52)=0.2$

(iii) *Majority voting equilibrium*:

(a) *Community 1*: The income of the pivotal voter is implicitly defined by $F_3(54.52) - F_3(\tilde{y}_1) = 0.4$, which yields $\tilde{y}_1$=26.6958. Because households with this income choose public schooling, the median voter most preferred tax rate solves (A4), yielding t_1^*=0.2892. This tax rate has an associated level of provision equal to:

$$E_1^* = \frac{t_1^* p_1^h 0.8}{0.7} = 2$$

(b) *Community 2*: The median (income) voter has income $\tilde{y}_2$=70.3533, which satisfies $F_3(99.18) - F_3(54.52) = 0.1$. Households with this level of income choose public schooling and thus the median voter most preferred tax rate t_2^* is given by (A5). This leads to an equilibrium tax rate equal to 0.6716 and to a level of provision equal to $E_2^* = t_2^* p_2^h = 4.5$.