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Can Liars Ever Prosper?

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## Can liars ever prosper?<sup>1</sup>

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#### Abstract

Within a costly state verification setting, we derive the optimal financial contract between an entrepreneur, a (potentially financing) supervisor and a pure investor when there is non verifiable and non contractible auditing and limited liability.

We show that in such circumstances diversion of cash flows to the entrepreneur arises as optimal behaviour. In such cases it is crucial to have both the supervisor and the pure investor participating financially into the venture with the role of creating correct reporting and monitoring incentives. This is achieved by setting negatively correlated repayments: the contract uses the pure investors to smooth out the repayments of the entrepreneur optimally. This reduces the entrepreneur's incentive to make false reports and mitigates the supervisor's incentive to monitor. This contract, as well as being Pareto superior to a single investor contract, is shown to be always renegotiation-proof, and, within a set of parameters, collusion-proof.

Keywords: financial contracts, multiple investors, no commitment JEL Nos: D82, D83

#### Introduction

The recent wave of corporate scandals has brought to light quite widespread malpractice in the accounting profession and limited control by the designated auditors. The Worldcom or Enron scandals are only the most dramatic high profile scandals involving major U.S. corporations in recent years. Before Enron and Worldcom slid into bankruptcy, several high-profile companies were investigated for fraud by the SEC, and ultimately paid tens of millions of dollars in fines to settle the charges. Waste Management, Cendant, Sunbeam and MicroStrategy all faced federal scrutiny.

This paper argues that such malpractices can in some circumstances, e.g. under limited liability or under contractual incompleteness, e.g. if audit is insufficiently transparent, arise as optimal behaviour and proposes a role for multiple investors to help limit the extent of such malpractices as well as a more direct involvement of designated auditors in the financing of the venture.

We model a situation in which a firm/entrepreneur seeks funding to finance an investment project whose cash flows are his private information but which can be verified by a supervisor at a cost. The necessary funding can be provided either by a pure investor or by the supervisor, or by both. The supervisor can thus be a pure auditing company or an investor who also performs a monitoring task, but we leave this to be determined endogenously.

It is well known that any contract with repayments which induces the entrepreneur to report its state truthfully has potential problems of commitment to the monitoring policy written into the contract (Hart [1]): if the monitor knows that the contract induces truthful reporting, she has no incentive to undertake costly monitoring ex post. To obviate this problem here, we assume that monitoring is non-contractible. This means that the contract has to regulate two types of incentives: the incentive for the entrepreneur to reveal his private information and the incentive for the supervisor to carry out the monitoring ex post.

As well as non-contractible, we also assume that audits are neither observable nor verifiable to outside parties. The reasons for these assumptions are the following: first, since monitoring involves randomisation by the supervisor, there must be public display of the randomisation (i.e. even though the firm and pure investor can compute the equilibrium probability of monitoring, to ensure the supervisor actually uses this needs a public lottery); second, if the lottery result dictates monitoring, then all three parties must be able to see that the monitoring actually occurs. If either of these is not possible, then a further source of contractual incompleteness arises.

Non-contractibility of monitoring is not a novel issue in the literature: it has been studied by Jost [3] in a moral hazard setting and by Khalil [4] in an adverse selection setting; Persons [9] and Khalil and Parigi [6] have instead focused on the financial contract between an entrepreneur and one or two investors. The general finding is that there are two ways to give correct incentives to the supervisor to carry out the audit ex post. One consists of "rewarding" the supervisor whenever she monitors, i.e. increasing her payoff by the amount of the observation cost, so as to make her indifferent between monitoring and not monitoring and hence inducing the entrepreneur to truthfully reveal his cash flows (Jost [3], Persons [9]). The other consists in having the entrepreneur "misrepresenting" the true state, i.e. diverting cash flows in the high state, with a positive probability, thus using the possibility of collecting a penalty for detected false reporting to give the supervisor an incentive to monitor (Khalil [4], Khalil and Parigi [6], Persons [9]).

When the act of monitoring is not publicly observable, the first of these two alternatives, however, is not implementable. If the contract induces truth-telling and the supervisor gets a premium for monitoring, then the supervisor will always claim to have monitored (and discovered truth-telling) even when she has not. To prevent this, no premium can ever be paid when monitoring detects truth-telling. The incentive to monitor can thus arise only from the possibility of collecting a penalty for detected false reporting, and a contract inducing misrepresentation in equilibrium will arise, i.e. a contract in which there is diversion of cash flows.<sup>1</sup>

Although both Persons [9] and Khalil and Parigi [6] have derived the properties of the misrepresentation contract, they have focused on the bilateral case in which the investor acts also as a monitor. There is widespread evidence, though, that firms do not contract with just a single party, but rather with various parties, and in particular with multiple investors. We attempt to explain this by studying a simplified setting in which there is at least in principle a division of tasks: the entrepreneur has the skills to carry out the project, the pure investor and the supervisor each have the funding to finance it and only the supervisor has the monitoring technology to verify the entrepreneur's report.

We show that among the misrepresentation contracts, those with three parties are always preferable to those with two. In particular a firm-supervisorinvestor contract always dominates a bilateral structure in which the supervisory role coincides with the financing role. However there is no full specialisation of tasks, in the sense that it is still optimal to have the supervisor providing a share of finance and have the "pure" investor providing the balance. Thus, even if the pure investor is wealthy enough to finance the entire project and all parties are risk neutral, it is optimal to ask the supervisor to participate in the financing as this allows the entrepreneur to reduce his incentive to cheat, thus mitigating the supervisor's incentives for monitoring and reducing the overall observation cost. In practical terms this means that auditors should have a financial stake in the firms they audit.

Given our setup, the tool for controlling these incentives is the structure of repayments to the pure investor. It is a standard result in models of this type (e.g. Khalil [4], Khalil and Parigi [6]) that to induce the supervisor to monitor, the repayments she receives must be increasing in the firm's cash flow.

<sup>&</sup>lt;sup>1</sup>The assumption of non verifiability of monitoring is not crucial to get misrepresentation contracts. Even if the act of monitoring were publicly observable and verifiable, the implementation of a truthtelling contract could be shown to rely either on unlimited liability or, when there is limited liability, on low state revenues being sufficiently high to cover the observation cost.

Analogously, to induce the entrepreneur to comply, he must be given a rent for truthful reporting. We show that the repayments to the pure investor can be used to enhance these two incentive effects: by setting them to be decreasing in the firm's cash flows -in the high cash flow state the pure investor gets a repayment lower than the one she gets in the low cash flow state- the entrepreneur can increase the rent he gets for compliance, as well as the premium paid to the monitor for detected misreporting, thus mitigating his own incentive to cheat and consequently the supervisor's incentive to monitor. This results in lower expected monitoring costs compared to either a setting in which there is a single investor who has also a monitoring task (no specialisation) or the scenario in which tasks are separated (full specialisation). We cannot get such a high powered incentive scheme in either of the other cases considered -full specialisation, no specialisation- as the spread of total repayments to the supervisor and the pure investor is always higher than that achieved with a supervisor who also acts as a financier.

We thus argue that when some form of contractual incompleteness (unverifiability of monitoring) prevents the attainment of solutions with truthful revelation, diversion of cash flows arises as optimal behaviour. Under such circumstances, it is crucial to have both the supervisor and the pure investor participating financially in the venture to get the best possible reporting and monitoring incentives. In reality most economies regulate to prevent auditors from having financial stakes in the companies they audit. For example the UK Companies Act 1985 prevents any creditor or equityholder from acting as an auditor of the firm. It follows from our results that this concern may be misguided.

It is worth noting that our rationale for the desirability of multiple parties is solely in terms of improving incentives for truthtelling and monitoring. Since we have assumed that all parties are risk neutral, there are no risk diversification gains from multiple investors; by contrast it is risk aversion which motivates diversification in most of the literature. But even with risk aversion we would expect that these incentive effects will remain.

A last remark concerns the robustness of this contract to collusion and renegotiation possibilities. In particular, costly monitoring raises the issue of whether the parties have an incentive to renegotiate the terms of the contract ex post to save the monitoring costs, where by renegotiation we mean a revision of the repayments and/or the players strategies that leads to a Pareto improvement. We find that there is no room in our setting for such renegotiations. This is in contrast with Krasa and Villamil [8], who on these grounds show the superiority of deterministic contracts over stochastic contracts. We will argue that this has to do with the particular setting analysed by Krasa and Villamil. Moreover generally in the literature it is recognised that truthtelling contracts which are not completely pooling are prone to renegotiation (Persons [9]).

Of course, with more than two parties, a renegotiation of the terms of the contract could also involve two of the parties at the expense of the third one, thus taking the form of a collusive agreement. Due to the particular structure of repayments, we find that this may occur only between the supervisor and the

entrepreneur, but that there is a range of parameters where no such collusive agreement can be reached.

The plan of the paper is to outline the model assumptions in section 1; in section 2 to derive the various forms of second best contract with two investors; in section 3 to derive the implications in terms of collusion and renegotiation; in section 4 to analyse monotonic contracts and in the last section to conclude.

#### 1 The Model Assumptions

The risk neutral entrepreneur seeks funding to finance an investment project of fixed size D which gives him access to a technology in which revenues depend on the state of nature  $f(s) = f_s$  with  $s \in \{H, L\}$  and  $f_H > f_L$ . State s = H occurs with probability p.

Revenues are private information to the entrepreneur and are unobservable to any party. We assume that the supervisor has the technology to monitor, at a cost  $\phi$ , as well as the resources to finance the project, and that the result of an audit is neither observable nor verifiable to any other party. Moreover we assume that the entrepreneur can raise finance from a pure risk neutral investor who has no incentive problems. The shares of funding  $\alpha$  and  $1 - \alpha$  provided by each potential investor are determined endogenously and for each group the opportunity cost of capital is equal to r.

A remark is warranted regarding the terminology. We will throughout define the supervisor as the monitoring investor, although at this point this labelling is abusive, as we don't know yet whether the supervisor participates financially in the venture. Nevertheless, this allows us to keep in mind that she has the resources to finance the project. It will in fact turn out that it is optimal to have her providing a share of finance under limited liability.

Participation in the venture pays a non negative return of  $R_s$  to the monitoring investor and  $P_s$  to the pure investor, depending on the state and on whether it is audited. They are thus protected by limited liability. We also require the firm to be solvent in each state:  $f_s - R_s - P_s \ge 0$  so that the firm itself has limited liability.<sup>2</sup>

When there is asymmetric information, the monitoring investor can choose to audit any state report of the firm; and, if she does so, she must pay an observation cost of  $\phi$ . We allow the monitoring investor to randomly monitor the low state report with an endogenous probability m.<sup>3</sup> If it is found that the entrepreneur has falsely declared the low cash flow state instead of the high true state then, as well as paying the returns due in the high state, it is punished by paying an amount that is specified in the contract to the investors.

Risk neutrality of each party and the existence of a perfect capital market imply that there are neither risk sharing issues nor liquidity constraints. Because of this we can measure the relative inefficiency of different contracts by their

<sup>&</sup>lt;sup>2</sup>Otherwise the model would not be financially closed; the firm could always pay anything required perhaps by expost borrowing, which makes the issue investigated unimportant.

<sup>&</sup>lt;sup>3</sup>There is no gain here to monitoring a high state report.

expected observation cost: those with lower expected observation cost are exante preferable, if they enable financing.

We assume that the fixed financing requirement is such that

$$f_L < (1+r)D; \tag{A.1}$$

otherwise the firm could just pay a constant repayment to the investors in each state s and meet their participation constraints, have no reporting incentive problem and face no monitoring. Under this assumption, a feasible contract will require either  $R_s$  or  $P_s$  to vary by state, which in turn induces incentive constraints on the entrepreneur and on the monitoring investor.

Last, in line with the existing literature, we assume that the investment is socially profitable:

$$pf_H + (1-p)f_L - (1+r)D - \phi > 0, \tag{A.2}$$

i.e. the expected return from the project is sufficiently high to cover the observation cost  $\phi.$ 

The entrepreneur offers a contract R to the monitoring investor and a contract P to the pure investor specifying all possible contingent transfers covering all verifiable states of the world. When monitoring occurs, it reveals only to the monitor whether the entrepreneur cheated or whether he told the truth, i.e. whether cash flows are truly low or high. To communicate the result of monitoring credibly to other parties (other investors, courts or the firm) the monitor must be able to produce hard, informative evidence of the firms financial position. When the state is truly low there is no such available evidence - the best the monitor can do is replicate the firms own report. This implies that following a low state report the repayments cannot be made contingent on whether monitoring occurred, i.e. cannot have  $R_{LL} > R_L$ , where  $R_{LL}$  is the payoff the supervisor gets if monitoring detects compliance whereas  $R_L$  is the payoff the supervisor gets if there is no monitoring following a low state report. If so, the supervisor would always claim to have monitored even when she has not and ask for the corresponding repayment  $R_{LL} > R_L$ . Thus non-observability implies that  $R_{LL} = R_L$  and by the same token that  $P_{LL} = P_L$ .

However when the true state is high, there is hard evidence available - despite the low report of the firm, the monitor can show that the firms revenues are actually high.

One way of interpreting this setup is to think of the accounting books as being available to all parties, so that monitoring a true low state report simply confirms what is written in the books and is no proof that an inspection did actually take place. Since the pure investor does not know whom to trust (especially given that repayments following a true low state report contingent on monitoring imply that  $P_{LL} < P_L$ , and thus the pure investor has no reason to trust the monitor), low state repayments cannot be made contingent on monitoring. When the firm cheats instead, the monitor can provide evidence of fraud by showing a discrepancy between the accounts  $(f_L)$  and the true cash flows  $(f_H)$ . It is this discrepancy that proves that monitoring did occur. The structure of repayments is as follows: if the low state occurs the entrepreneur declares it and pays  $R_L$ ,  $P_L$  to the monitoring and to the pure investor respectively independently of whether monitoring occurs or not. If the high state occurs the entrepreneur can either declare it and pay  $R_H$ ,  $P_H$  to the investors or can falsely declare the low state, in which case his report is monitored with probability m. If monitored and caught cheating the entrepreneur pays  $R_{HL} = R_H + \delta_R$  to the monitoring investor, i.e. what she had a right to if a truthful report had been made, plus the penalty for misreporting  $\delta_R \ge 0$ , and  $P_{HL} = P_H + \delta_P$  to the pure investor, i.e. what she had a right to if a truthful report had been made, plus the penalty for misreporting  $\delta_P \ge 0$ ; last, if not monitored, again the entrepreneur pays  $R_L, P_L$ .

Distinguishing between two stages of the game, the time line is the following:

- Ex-ante stage.
  - At time zero a financial contract is offered by the entrepreneur to the monitoring and to the pure investors specifying the repayments due in each monitored or non monitored state.
  - At time one production takes place and at time two the state of nature is realised and observed by the entrepreneur.
- Ex-post stage.
  - At time three the entrepreneur sends a public report to the investors and the monitoring one decides whether to verify the report. These strategies are chosen as mutual best responses.
  - At time four, conditional on the reported state and on the result of monitoring, if any, the relevant transfers are made.

At the ex-post stage the decision variable for the firm is the probability l with which it falsely declares the low state when the high state H has occurred. For the monitoring investor, the decision variable is the probability m with which she will audit the entrepreneur's low state report  $\hat{L}$ . As standard in models of this type (e.g. in Khalil [4]), we assume that the entrepreneur never cheats when the low state occurs; and the investor never monitors when a high state report is received. Then, the expected returns to the two parties are:

$$E\pi_{E}|_{H} = (1-l)(f_{H} - R_{H} - P_{H}) + lm(f_{H} - R_{H} - \delta_{R} - P_{H} - \delta_{P}) + (1)$$
  
$$l(1-m)(f_{H} - R_{L} - P_{L}),$$
  
$$E\pi_{R}|_{\hat{L}} = m[\frac{pl}{1-p+pl}(R_{H} + \delta_{R}) + \frac{1-p}{1-p+pl}R_{L} - \phi] + (1-m)R_{L}, (2)$$

where l, m are selected as mutual best responses

$$l \in \arg\max_{l'} E\pi_E|_H \tag{3}$$

$$m \in \arg\max_{m'} E\pi_R|_{\hat{L}}$$
 (4)

and we have omitted the sunk cost of the loan size made at time zero from the monitoring investors expected return. Recall that  $\delta = \delta_R + \delta_P$  is the penalty for misreporting and thus  $R_H + \delta_R$  and  $P_H + \delta_P$  are the repayments due to the monitor and to the pure investor respectively when a low state report is discovered to be false.

#### 2 The misrepresentation contract

As we have argued in the previous section, the lack of commitment and the non-observability of monitoring imply that a truth-telling contract cannot be implemented. An alternative would be to provide the incentive to monitor outside the contract and in the form of the gains to the monitor from punishing a misreporting firm who is detected. The monitoring incentive is therefore obtained by allowing the supervisor to sometimes collect a punishment repayment from a lying firm. Thus, under non-observable monitoring, the incentive constraints (3) and (4) become indifference conditions associated with a mixed strategy equilibrium:

$$R_{H} + P_{H} - R_{L} - P_{L} = m(R_{H} + \delta_{R} + P_{H} + \delta_{P} - R_{L} - P_{L})$$
(5)

$$\frac{pl}{1-p+pl}(R_H+\delta_R) + \frac{1-p}{1-p+pl}R_L - \phi = R_L.$$
 (6)

and the Nash equilibrium is:

$$l = \frac{(1-p)\phi}{p(R_H + \delta_R - R_L - \phi)}; \tag{7}$$

$$m = \frac{R_H + P_H - R_L - P_L}{R_H + \delta_R + P_H + \delta_P - R_L - P_L}.$$
 (8)

Then 0 < m < 1 requires  $R_H + \delta_R + P_H + \delta_P > R_H + P_H$  and  $R_H + P_H > R_L + P_L$ ; 0 < l < 1 requires  $R_H + \delta_R - R_L - \phi/p > 0$ . If the investor-firm contract does not satisfy these conditions, then no interior mixed strategy Nash equilibrium is possible.

Thus, given a subsequent mixed strategy equilibrium, we can write the contract problem  $C^G$  as one of choosing  $R_s$ ,  $P_s$ ,  $\alpha$  to:

$$\max \ p(f_H - P_H - R_H) + (1 - p)(f_L - P_L - R_L) \tag{9}$$

st 
$$\alpha(1+r)D \le p(1-l)R_H + (1-p+pl)R_L$$
 (10)

$$(1-\alpha)(1+r)D \le (p-pl+plm)P_H + plm\delta_P + (1-p+pl-plm)P_L \quad (11)$$

$$f_H - R_H - \delta_R - P_H - \delta_P \ge 0 \tag{12}$$

$$f_L - R_L - P_L \ge 0 \tag{13}$$

$$R_H + \delta_B > R_L \tag{14}$$

and to (7) and (8).

The constraints have the usual meaning: (10) and (11) are the individual rationality constraints for the monitor and for the pure investor respectively,

(12) and (13) are the feasibility conditions, (7) and (8) are respectively the probability of lying and the probability of monitoring determined at the ex-post stage; last (14) is the supervisor reporting incentive constraint ensuring that when the audit reveals noncompliance the supervisor has no incentive to hide the result of the audit and claim not to have audited.

Solving programme  $C^G$ , we derive the following properties of the optimal contract (the technical details are in the appendix).

**Proposition 1** With a subsequent mixed strategy, the optimal three-party contract has:

- i. investors getting their reservation utility level;
- ii. maximum punishment:  $\delta_R = f_H R_H P_H$ , with  $\delta_P = 0$ ;
- iii. zero rent to the firm in the low state however it reports  $(f_L = R_L + P_L)$ , but positive rent in the high state with truthful reports  $(f_H R_H P_H > 0)$ .

Result (i) is not surprising: since the firm is writing the contract it has no gain from leaving any rent to either investor: if it did, it could reduce either P or R in a way that leaves the other constraints preserved.

Maximum punishment reduces the incentive of the firm to cheat and so the frequency of low state reports and, for given m, the amount of monitoring that will be undertaken, thus saving on monitoring costs. Put slightly differently, given that there is a mixed strategy equilibrium, the penalty for misreporting  $\delta = \delta_R + \delta_P$  only enters the contract problem via the participation constraints of the investors through l. Raising the punishment reduces l and reduces the impact of the participation constraint.

Similarly, punishment payments are crucial to the monitoring incentive of the supervisor but are only important to the pure investor in meeting her participation constraint. So it is most efficient to give all the reward to the supervisor and set  $\delta_P = 0$ .

Last, binding low state feasibility is a common result in agency problems that assists in ensuring incentive compatibility.

Given maximum punishment, binding low state feasibility and  $\delta_P = 0$ , we can set  $R_H + P_H = f_H - \delta_R$ , and  $R_L = f_L - P_L$ . Using the monitoring investor participation constraint to eliminate  $R_H$ , the problem reduces to choosing  $P_H$ ,  $P_L$  and  $\alpha$  to:

$$\max p \delta_R \tag{15}$$

st 
$$\mu P_H + (1 - \mu)P_L = (1 - \alpha)(1 + r)D$$
 (16)

where  $\mu = p - pl + plm$  and

$$\delta_R = f_H - P_H - f_L + P_L + \frac{(f_L - P_L - \alpha(1+r)D)}{p(1-l)}.$$
 (17)

Note that since  $p\delta_R = p(1-m)(f_H - f_L)$  maximising  $p\delta_R$  is equivalent to minimising the probability of monitoring. We then have:

## **Theorem 1** Any contract which supports a mixed strategy, that has maximum punishment and binding low state feasibility has $P_H < P_L$ .

The formal proof of the theorem is in the appendix but the line of the argument goes as follows. First note that the passive investors participation constraint (16) must bind. Write the objective  $p\delta_R$  and the participation constraint (16) in terms of the three variables  $\Delta P = P_H - P_L$ ,  $P_L$ ,  $\alpha$ . Then as  $P_L$  or  $\alpha$  increase (given  $\Delta P$ ),  $p\delta_R$  falls whilst the passive investors constraint becomes less tight with repayments (the LHS) increasing relative to the outstanding debt (the RHS). Hence if the constraint were slack, a decrease in  $P_L$  (and since  $\Delta P$  is constant, also in  $P_H$ ) would be possible without violating the constraint, but leading to a fall in m and so an increase in the objective.

So we can use the passive investors participation constraint to eliminate  $\alpha$  which we write as a function of  $\Delta P = P_H - P_L$  and  $P_L$ . Using this function for  $\alpha$  the resulting value of m depends only on  $\Delta P$ . The problem then becomes one of choosing  $\Delta P, P_L$  to minimise m under the constraints that 0 < l, m < 1, and feasibility in the form  $f_L = R_L + P_L$ ,  $f_H \ge R_H + P_H$ . At the solution:

l

$$=\frac{(1-p)\phi}{p(f_H - f_L - \Delta P - \phi)}\tag{18}$$

$$m = \frac{((1+r)D - f_L)(f_H - f_L - \Delta P - \phi)}{(p(f_H - f_L) - \phi)(f_H - f_L) - \Delta P[p(f_H - f_L) - (1-p)\phi]}$$
(19)

with

$$\frac{\partial m}{\partial \Delta P} = \frac{((1+r)D - f_L)(1-p)\phi^2}{\left[\Delta P\phi(1-p) + p(f_H - f_L)^2 - (\phi + p\Delta P)(f_H - f_L)\right]^2} > 0.$$
(20)

Since m is increasing in  $\Delta P$  this requires minimising  $\Delta P$ . Hence any mixed strategy optimum must involve  $\Delta P < 0$  or  $P_H < P_L$ .

At first sight this result seems surprising; it arises because the pure investor gets caught up in the incentive problem, in the sense that the repayments she gets can be used to sharpen the incentives that have to be provided in the contract to the entrepreneur and to the supervisor. Partly the repayments she gets affect the incentive compatibility of monitoring; partly they affect the probability with which the high transfer will be made. The monitoring investor receives transfer in the good state higher than the one received in the bad state. The pure investor receives a bad state return higher than that received in the good state. By making the payments negatively correlated with the profits of the firm and with the repayments to the monitoring investor, the spread in the retained surplus of the firm between truthfully reported high states  $(f_H - R_H)$  and non-monitored false reports of low states  $(f_H - f_L)$  is reduced. By reducing the variability of the net profit received across monitored and non-monitored states, the firm reduces its incentive to cheat when the high state occurs and mitigates the investor's incentive to monitor.

With limited liability on both the monitoring investor and the pure investor all repayments should be nonnegative. Then, given the negative correlation result established in Theorem 1, the upper bound on  $P_L$  is  $f_L$  with  $R_L = 0$ . Similarly the lower bound on  $P_H$  is 0 with  $R_H + \delta_R = f_H$ . Hence the optimal contract has  $R_L = P_H = 0$ ,  $R_H > 0^4$  with a subsequent Nash equilibrium in mixed strategies defined by:

$$l = \frac{(1-p)\phi}{p(f_H - \phi)};$$
 (21)

$$m = \frac{(f_H - \phi)((1+r)D - f_L)}{p(f_H - f_L)f_H - \phi(f_H - pf_L)}.$$
(22)

The resulting investment share of the monitor is

$$\alpha = \frac{(pf_H - \phi) \left[ (1+r)D(f_H - f_L)(f_H - \phi) - (1-p)f_L(f_H^2 - f_H f_L + \phi f_L) \right]}{(1+r)D \left[ f_H(p(f_H - f_L) - \phi) + p\phi f_L \right] (f_H - \phi)}.$$
(23)

In the appendix we show that all of these are in the interior of the unit interval, in particular  $0 < \alpha < 1$ . The intuition for this result is the following: given that the contract offer comes from the entrepreneur, non trivial contracts "strengthening" the incentive to monitor require the supervisor to get in either state of nature non-negative transfers which are increasing in the firm's cash flows. This implies that the supervisor is to receive a positive transfer in expected terms, which is costly to the entrepreneur. By giving her a share of finance it is possible to hold her down to her reservation utility while ensuring that she has an incentive to monitor.

#### 2.1 Two special cases of the misrepresentation contract

It is worth comparing these results with those we would obtain in the two extreme cases in which either the project is entirely financed by the supervisor or these functions are separated and assigned to two distinct agents.

The case in which  $\alpha = 0$  can be interpreted as one in which the parties specialise, i.e. one acts as a monitor and the other as an investor.

Since the entrepreneur is not willing to give up any rent to the supervisor, the only contract that he is ready to offer is one which has zero payoff in all states which either are not monitored, or if monitored are found to be true, and a positive payoff only when a false low state report is detected. The supervisor will accept such an offer since, with  $R_H = R_L = 0$ , it trivially satisfies the participation constraint (10) and, so long as (7) holds, i.e.  $pl\delta_R = (1 - p + pl)\phi$ , keeps her indifferent between monitoring and not monitoring. Using the superscript s to denote the separation of tasks, we derive the following properties of the optimal contract (the technical details are in the appendix):

**Proposition 2** The optimal contract with specialisation ( $\alpha = 0$ ) has the following features:

<sup>&</sup>lt;sup>4</sup>The value of  $R_H$  is given in the appendix.

- $\delta_R^s = f_H P_H^s$  and  $\delta_P^s = 0;$
- $R_H^s = R_L^s = 0;$
- $P_H^s > P_L^s = f_L;$

• 
$$l^s = \frac{(1-p)\phi}{p(f_H - P_H^s - \phi)}, \ m^s = \frac{P_H^s - f_L}{f_H - f_L}$$

Thus we see that the supervisor gets a positive repayment only when she detects cheating, in which cases maximum punishment is imposed; in all other states she gets zero payoffs. The investor gets repayments which are increasing in the firm's cash flow,<sup>5</sup> and by the same argument as in Proposition 1 above, she gets no share of the penalty for misreporting, i.e.  $\delta_P = 0$ .

The case in which  $\alpha = 1$  is related to Khalil and Parigi [6] who study the contract between an entrepreneur and a single investor. If  $\alpha = 1$  then with limited liability and the requirement that the pure investor should receive no rent because of binding participation, any contract that is optimal given  $\alpha = 1$  will have  $P_H = P_L = 0$  and then effectively there is a single investor. Adding a superscript 1 to the variables to denote the presence of a single investor/monitor, their main results are as follows:

$$f_{H} = R_{H}^{1} + \delta_{R}^{1} > R_{H}^{1} > f_{L} = R_{LL}^{1} = R_{L}^{1}$$

$$l^{1} = \frac{(1-p)\phi}{p(f_{H} - f_{L} - \phi)}$$
(24)

$$m^{1} = \frac{((1+r)D - f_{L})(f_{H} - f_{L} - \phi)}{[p(f_{H} - f_{L}) - \phi](f_{H} - f_{L})}$$
(25)

In both the above cases the value of  $\alpha$  is chosen exogenously. However from (23) we know that optimally  $0 < \alpha < 1$  and so it must be that a contract in which the supervisor also acts as a financier dominates any other contract in which either there is full specialisation of tasks or there is no specialisation (and the supervisor is the sole investor). In more detail we can see why this so in the following corollary.

**Corollary 1**  $m^s, m^1 > m$  and  $l^s > l^1 > l$ , that is the two investor misrepresentation contract with limited liability has a lower monitoring probability and a lower cheating probability than either of the specialised cases. Consequently it has lower expected observation costs.

Intuitively the reasons why the separation of functions does not work very well in giving the correct incentives are that:

• for monitoring, the need to keep monitoring low to save on observation costs requires the smallest possible spread in  $P_H - P_L$  (which implies that  $P_L = f_L$ ), but then the participation constraint of the pure investor who is providing all the finance still requires such a large repayment in the high state that it is not possible to keep the chance of monitoring low.

<sup>&</sup>lt;sup>5</sup>The value of  $P_H^s$  is given in the appendix.

• under separation the only endogenous element that affects the incentives for cheating is the size of the punishment repayment  $\delta = \delta_R + \delta_P$ . The higher this is the lower the cheating. But, given limited liability, its upper limit  $f_H - P_H$  is constrained by the need to pay enough  $(P_H)$  to the pure investor to satisfy her participation constraint. If the finance is shared then there is not the need to keep  $P_H$  so high since the pure investor requires a lower return.

The reasons why a single investor/monitor does not work as well as two investors are:

- with just a single investor/monitor there is no need to pay positively correlated returns to the pure investor  $(P_H > P_L > 0)$  as is the case when  $\alpha = 0$ . In particular with a single investor/monitor  $P_H = P_L = 0$ . As a result the punishment for detected misreporting  $\delta_R$  is larger, which reduces cheating:  $l^1 < l^s$ . But it is possible to do still better by bringing in the pure investor and giving her negatively correlated repayments since the effective punishment depends on  $f_H - P_H - R_H$ . By having  $P_H = 0$  the loss from detection after cheating as compared with the gain from getting away with it is smaller. So  $l < l^1 < l^s$ ;
- making a positive transfer to the pure investor in the low state  $(P_L > 0)$ subtracts resources from the supervisor  $(R_L \text{ decreases})$  and, to preserve participation, requires an increase in transfers due in false detected low states  $(R_H + \delta_R \text{ increases})$ . On the other hand, making a lower transfer to the pure investor in the high state  $(P_H < P_L)$  increases the resources retained by the entrepreneur in truthfully reported high state, the only state in which he gets a rent. The higher rent mitigates the firm's incentive to lie (l falls) and along with the rise in  $R_H + \delta_R$  slackens the monitor's incentive constraint (m falls). So  $m < m^1$ .

Here we give an example of how much it is possible to gain by having a monitoring investor with parameters p = 0.52,  $f_H = 10$ ,  $f_L = 2$ ,  $\phi = 2.1$ , (1 + r)D = 3. With  $P_L = f_L$ ,  $P_H = 0$ ,  $\delta_P = 0$ , because of limited liability, we have m = .34, l = .24,  $\alpha = .62$ ,  $R_H = 4.77$ ,  $R_L = 0$ . In this numerical example the functions m and l as a function of the spread  $\Delta P$  are plotted in the accompanying diagram. The value of the objective function is  $p\delta_R = 2.72$  and the expected return to the monitor is  $p(1 - l)R_H = 1.87$ . In the single investor contract in which the project is financed entirely by the supervisor, using the same parameter values we get  $l^1 = .33$ ,  $m^1 = .36$ ,  $R_H^1 = 4.86$  and optimal expected utility for the entrepreneur  $p\delta_R^1 = 2.67$ . In the specialised case instead we get  $l^s = .83$ ,  $m^s = .44$  and optimal expected utility for the entrepreneur  $p\delta_R^s = 2.1$ .

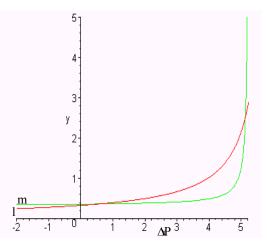


Fig. 1. l, m as a function of  $\Delta P$ 

### 3 Collusion and renegotiation

So far, we have not analysed collusion or renegotiation problems that might arise in this setting. Generally the literature has focused on those arising in a principal-supervisor-agent hierarchy (Tirole [11], Khalil and Lawarrée [5], Kofman and Lawarrée [7], Strausz [10]). Unlike the existing literature, the present paper looks at issues of collusion, as well as renegotiation when the supervisor also acts as a principal by providing funding to the agent. We think of collusion as involving two parties to the contract who agree to a deal between themselves at the expense of the third party at the interim stage once the entrepreneur knows his type. There are then different coalitions that could form: between the two investors, between the monitor and the firm or between the firm and the passive investor. By contrast we think of renegotiation as involving a Pareto improvement for all three parties again at the interim stage, so that for the firm it is in terms of ex-post utilities, since it knows its type, whereas for the investors it is in terms of exante utilities conditional on any offer that is made. Since all these deals are done at the interim stage prior to any report of the firm, once any offer is made all parties can update their information and their best strategies conditional on the offer. Since the strategies are determined simultaneously in the game (reporting and monitoring are simultaneous), this is the most informative time at which an offer can be made.

#### 3.1 Collusion

Collusion will typically involve a renegotiation of the terms of the contract between the two colluding parties leading to outcomes which are at the cost of the third party. What sort of collusions/renegotiations can be made? Recall the information assumptions: the original contract repayments  $R_s$ ,  $P_s$  and  $\alpha$  are common knowledge together with the exogenous parameters such as  $\phi$ , p,  $f_s$ . So all three parties can work out l, m.

Although the act of audit is neither observable, nor verifiable, the structure of repayments is such that if an audit occurs, either their results have no effect on repayments or they become public information. When the low state occurs, monitoring has no effect on repayments, as these are not contingent on monitoring. When the high state occurs and the firm reports low, monitoring requires the supervisor to disclose the result of the audit to get the right to the premium for detected misreporting. Thus all parties, in particular the pure investor who is uninformed on whether there has been an audit, get to know all relevant information.

The firms state at the interim stage is its private information and the actual repayments the firm makes to either investor are private information of the firm and the respective investor. Outsiders to any collusion or renegotiation deal do not know there has been a deal or what it is but have the enforceable right to the repayment due in the original contract after any audit or report by the firm.

A collusive agreement between the non-monitoring investors and the firm is impossible. From the original contract a true high type firm who reports high has an expected return of  $E\pi_E|_H = (f_H - R_H) = (1 - m)(f_H - f_L)$  whilst the passive investor has an expected return of 0. If the firm stays with the original contract and reports the truth it should pay 0 to the passive investor and  $R_H$  to the monitor. Since m is independent of the passive investor there is no room for negotiation between the firm and the passive investor. Conversely if the high type firm cheats and reports low, under the original contract it should pay  $f_L$  to the passive investor and 0 to the monitor if not monitored, or if it is monitored and punished, pay 0 to the passive investor and  $f_H$  to the monitor. The return to the firm is then  $m(0) + (1 - m)(f_H - f_L) = (1 - m)(f_H - f_L)$ . The passive investor does not know the true state of the firm or whether it will be monitored so its expected return after a low report is plm(0)/(1-p+pl) + (1-p+pl(1-p+pl)) + (1-p+pl(1-p+pl(1-p+pl))) + (1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+pl(1-p+p(1-p+p(1-p+pl(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1-p+p(1 $m)f_L/(1-p+pl)$ . No bargaining is possible over  $P_L = f_L$  since changing  $P_L$ affects the two payoffs in different directions. A cut in m would benefit both parties but m is not under their control. A low type firm should pay the passive investor  $P_L = f_L$  so the latter will not agree to colluding with the firm to tell the monitor that it is high type.

Similarly because the repayments to the passive and monitoring investor are negatively correlated the two investors cannot collude. Even if they could force truth-telling from the firm the passive investor would like the firm to cheat whereas the monitor would like the firm to be honest.

So the only possible collusion is between the firm and the monitor and takes the form of giving the firm an incentive to report high for sure which allows a zero repayment to the passive investor.

By following the original contract the high type gets  $(1-l)(f_H - R_H) + l(1-m)(f_H - f_L) = f_H - R_H$  given the Nash equilibrium, while the low type gets zero. On the other hand the expected payoff to the monitor from following this

contract is

$$m[plf_H - (1 - p + pl)\phi] + p(1 - l)R_H$$
  
=  $p(1 - l)R_H$  (26)

since by (21)  $[plf_H - (1 - p + pl)\phi] = 0$ . Thus the collusive offers can either come from the monitor or from the firm and the payoffs are:

- $pR_H(1-l)$  for the monitor
- $f_H R_H$  for the high type firm
- 0 for the low type firm.

To determine which offers will be made and whether they will be accepted, the relative sizes of  $p(1-l)R_H$ ,  $R_H$ ,  $f_L$  are important. We know that  $f_L < R_H$ since

$$f_L - R_H = -\frac{(f_H - f_L)(f_H - \phi)((1+r)D - f_L)}{(pf_H^2 - pf_H f_L - \phi f_H + p\phi f_L)} < 0$$

but the relation of  $p(1-l)R_H$  and  $f_L$  is ambiguous. We therefore consider two possible rankings according to whether

$$p(1-l)R_H \stackrel{<}{\leq} f_L \tag{27}$$

and we derive the following proposition:

**Proposition 3** Collusion between the firm and the monitor at the interim stage before the game is played is impossible if  $f_L \leq p(1-l)R_H$  no matter who initiates the collusion.

The detailed demonstration is in the appendix. The intuition is as follows: suppose it is the monitor who makes a take it or leave it offer to allow the firm to repay b in exchange for no monitoring. If the firm accepts b is paid, the firm reports  $\hat{H}$  and pays  $P_H = 0$  to the passive investor. If the firm rejects, the firm and/or the monitor can revise their cheating/monitoring strategies if they wish, then the firm sends a report, any audit is carried out and the contract payments are made. If  $p(1-l)R_H < f_L$  the monitor can make an offer  $b = f_L - \varepsilon$ that both types of firm will accept. This is the only offer the monitor will be willing to make that would be accepted by either type. On the other hand if  $f_L < p(1-l)R_H$  there is no acceptable offer that the monitor can make-any offer  $b > p(1-l)R_H$  will lead to the high type pooling with the low type.

Next suppose it is the firm who makes the offer which has the form: "I'll pay you b and I'll report high". The low type can only afford to offer  $b < f_L$ , while the high type can offer  $b < R_H$ . So long as  $b > f_L$ , the offer will be type revealing. If the monitor accepts, b is paid, a high report is made and  $P_H = 0$  is paid to the passive investor. If the monitor rejects, both the firm and the monitor can revise their cheating/audit strategies if they wish, the game is

played and the contract payments are made. If  $p(1-l)R_H < f_L < R_H$  both types of firm can gain from an offer  $b = p(1-l)R_H + \epsilon$  which is accepted by the monitor. However if  $f_L < p(1-l)R_H$  an acceptable offer will reveal that the firm is of high type which will lead the monitor to reject and revise her monitoring strategy. But then the high type will not make this offer.

The numerical example underlying Fig. 1 is not actually collusion proof since the expected return to the monitor (1.87) is below the low state revenue (2.0). However if we just change p to 0.6 but keep the other parameters unchanged the expected return to the monitor becomes 2.044 and is then above the low state revenue so that the contract is then collusion proof.

#### 3.2 Renegotiation

By renegotiation we mean a revision of the repayments and/or the cheating and audit strategies that leads to a Pareto improvement at the interim stage, i.e. none of the three parties lose out. If we take the situation after the firm knows its type but before the report and audit, then as before the high type expects to get  $R_H$ , the low type 0, by (26) the monitor  $p(1-l)R_H$  from the original contract and the passive investor expects  $(1-p+pl-plm)f_L$ . Since the interests of the low type and the passive investor are diametrically opposed, offers can only involve either changes in l or in  $R_H$  or in m.

Suppose it is the monitor who makes a renegotiation offer, in particular, suppose that in exchange for m' = 0 the monitor offers e, P': to be acceptable to a low type the offer should satisfy  $e \ge p(1-l)R_H$ ;  $e+P' \le 0$ ,  $P' \ge (1-p+pl)f_L$  leading to  $p(1-l)R_H + (1-p+pl)f_L \le 0$  which is impossible. On the other hand, offers will only be acceptable to a high type firm if  $e \ge p(1-l)R_H$ ;  $e+P' \le R_H$ ,  $P' \ge (1-p+pl)f_L$  requiring  $p(1-l)R_H + (1-p+pl)f_L \le R_H$  or  $f_L \le R_H$ . The firm would accept such an offer. The firm's acceptance or rejection of the offer signals that the firm is truly low, the high type firm would want to mimic the low type one in order not to be monitored and thus would also reject the offer. The two types would be indistinguishable to the monitor and no renegotiation offer would be made.

Suppose it is the firm who makes a renegotiation offer. First note that the low type cannot afford and has no interest in making any acceptable offer to either one or both investors as the firm's resources  $f_L$  are entirely paid out in  $R_L + P_L = f_L$ .

Thus only the high type firm can make offers, which are therefore type revealing. Unless the firm offers  $e > R_H$ , the monitor will never accept and will revise her monitoring strategies to m' = 1, thus leading to l' = 0 by the firm. However the firm will never wish to make this offer as it is better off staying with the original contract.

So we have shown:

**Proposition 4** There are no deviations from the original contract that can be made and that can lead to a Pareto improvement.

This result is interesting as it contrasts with the one found by Krasa and Villamil [8], who show the non optimality of stochastic contracts when there is limited commitment. This is partly a result of the timing of offers and partly due to Krasa and Villamil's assumption of imperfect monitoring technology that allows the entrepreneur to hide part of the income, in spite of the audit, and thus to always have the resources to propose a revision of the terms of the contract which is beneficial for all. In their model monitoring is written into the contract ex-ante and after learning its type the firm reports - hence it is most informative to have offers after the report. With noncooperative simultaneous determination of reporting and monitoring this is not possible. In addition in our case the entrepreneur has no way to hide income and is deprived of all resources in low state cash flows, which implies that he can never make a renegotiation offer if he is low type. A renegotiation offer can thus come only from the high type, but we have seen there to be no acceptable renegotiation offer coming from the high type. The assumption of imperfect monitoring technology seems thus to be a crucial one to get the result of optimality of deterministic contracts.

#### 4 The optimal monotonic contract

We have seen in the collusion section that there is a set of parameters with which the contract fails to be collusion proof, in particular for parameter values such that  $f_L > p(1-l)R_H$  the firm and the monitor could collude at the expense of the passive investor declaring that the high state has occurred. So then the limited liability contract would not be collusion-proof. To restore collusion-proofness would thus require addition of the constraint that  $P_H \ge P_L$ .

If we add this constraint to the two investor contract problem it follows that the optimal contract will have  $P_H = P_L = P$ . But then the passive investors participation constraint reduces to  $P = (1 - \alpha)(1 + r)D$  and the passive investor plays no role in shaping the incentives for the firm to cheat or the monitor to audit. Then the probabilities of audit and cheating in the single investor and two investor cases coincide and so does the level of welfare in the two scenarios. Thus in this case there is no gain from having two investors. For any investment share of the passive investor there is an associated P giving the passive investor their reservation level and the welfare of the contract is independent of this share.

The need for a contract with non-decreasing repayments when the set of parameters leads to  $f_L > p(1-l)R_H$  at first sight appears to be analogous to the analysis of Innes ([2]), who, in a moral hazard setting with observable revenues, discusses the properties of the two-party contract between an entrepreneur and an investor. One of his results is that the contract that maximises the incentive for the entrepreneur to exert effort is a "live-or-die" contract, i.e. a non monotonic contract with repayments that are decreasing in the entrepreneur's profits. A problem with such contracts is that the entrepreneur could inflate revenues and pretend that the state is good: it could for example, borrow from outside sources, and use the savings in repayment partly to repay the outside financing source and partly to make a profit. However, this kind of objection has no bite in our case as our contract is in fact monotonic in cash flows:  $f_H > R_H + P_H = R_H > R_L + P_L = f_L$ . Therefore, if the firm borrowed money to pretend that state is  $\hat{H}$ , it would save  $f_L$  but it would have to pay  $R_H > f_L$  to the monitor. The only possibility of exploiting this particular structure of repayments is then for the firm and the monitor to play against the passive investor, which we have seen to be possible only when the parameter set is such that  $f_L > p(1-l)R_H$ .

#### Conclusions

Within a costly state verification setting, we have derived the optimal limited liability financial contract between an entrepreneur, a (potentially financing) supervisor and a pure investor when monitoring is non verifiable and non contractible.

It turns out that because of non-observability and non-verifiability of monitoring, a truth-telling contract in which the incentive to monitor is provided by paying a premium to the supervisor when she monitors is not always feasible. As a consequence diversion of cash flows can arise as an equilibrium strategy.

In such cases we show that to ameliorate the incentive problem it is crucial to have both the supervisor and the pure investor participating financially in the venture to create the best possible reporting and monitoring incentives. The share provided by the supervisor ensures that she wants to carry out the monitoring; the share provided by the pure investor allows the entrepreneur to design a higher powered incentive scheme. This is achieved by giving the pure investor repayments which are negatively correlated with the firm's true cash flows. This allows the entrepreneur to better self-police his incentive to cheat, and hence the amount of state observation cost that he must ultimately bear from his residual profit, and in turn mitigates the supervisor's incentive to monitor.<sup>6</sup> As a consequence of the smoothing out of repayments, we also find that the two investor scenario is Pareto superior to a single investor contract with false reporting, with the extent of the gain in welfare depending on the investors' degree of limited liability.

Last, allowing for an agreement to a change in the terms of the contract once the firm has learned its true cash flow state, we find that the two-investor contract is always renegotiation-proof, irrespective of who makes the renegotiation offer. Moreover, we show that the particular structure of repayments also limits the scope for collusion. In particular, we find that this can only arise when the monitor and the low type of entrepreneur collude against the passive investor and falsely report high. But this is never optimal if the low state revenues are below the expected return of the monitor in the original contract. Hence

<sup>&</sup>lt;sup>6</sup>It also discourages pure investors from ever wishing to monitor: in the event of false report by the entrepreneur, she would have no incentive to overturn a low state report since she would have received a higher return from this than from a high state report.

under these circumstances the two investor misrepresentation contract is both collusion and renegotiation proof.

One last remark concerns the choice of the party who writes the contract. This is in fact a crucial issue in the problem of giving the right incentives to the firm when the uninformed party cannot commit to an audit policy. Intuitively, if the uninformed party writes the contract, she will try and extract all the surplus from the informed one thus maximising his incentive to cheat. Conversely, by designing the contract, the informed party can set the repayments so as to keep a rent for correct reporting (i.e. impose a sure loss for misreporting), thus making cheating less attractive.

Various open questions remain. In this paper we have assumed that the act of audit is neither observable, nor verifiable. An obvious next step would be to consider the case in which both the report of the firm and the results of audit are public knowledge. Then the repayments  $R_s, P_s$  could be conditioned on the audit result. The question is then what form the low state repayments to the monitor take. Imposing equality of repayments when a low state report is made and either this is monitored and found to be truthful or it is not monitored, i.e.  $R_L = R_{LL}$ , we have seen that commitment to the monitoring policy is entirely secured by allowing the monitor to have the gains from maximum punishment. If  $R_{LL} > R_L$ , instead, then commitment to monitoring is partly secured through paying a premium for audited truthful low reports. Notice that a truth-telling solution in which  $R_{LL} - R_L = \phi$  is still impossible when there is limited liability and  $f_L < \phi$ , and so a mixed strategy is still necessary. A related case to study is that in which exogenously  $\alpha = 0$  so that there is a separation of function between the principals. Limited liability and the monitors participation constraint imply that  $R_H = R_L = 0$  but there is still the question of the optimal mix of payments via  $R_{LL}$  or via  $\delta_R$  to secure commitment to monitoring.

The nonverifiability of monitoring force a mixed strategy solution in this paper. A natural extension is then to examine the losses of the misrepresentation contract as compared with the two investor truthtelling contract when there is unlimited liability on investors, so that repayments may be negative. One might conjecture that since negative correlation of  $P_s$  and  $f_s$  reduces these losses, the optimal solution with unlimited liability is to increase the spread  $P_L - P_H$  without bound and that doing so could reduce the incentive of the firm to cheat to any level desired.

We have assumed that only the supervisor has access to the monitoring technology. However if monitoring is nonverifiable, the pure investor may also have an incentive to try to access the monitoring technology if it is not too costly to do so.

Last there are two special features of the monitoring technology which should be relaxed. Firstly we have assumed a fixed monitoring cost independent of any effort of the monitor. Instead suppose that monitoring is imperfect so that there is a chance q that an audit reveals the true state and a chance 1 - q that it reveals no new information. q could be exogenous but if the monitor could vary q by varying costly monitoring effort then, if she has a big stake in the project, she will have an incentive to monitor more carefully. Given the way the contract problem is written this would be reflected in higher optimal repayments  $R_s$  when the optimal  $\alpha$  is higher. This is then a further argument against the separation of function implied by  $\alpha = 0$ . A second point is that, in common with most of the literature, we have assumed only two states. With more than two states the monitor may have a moral hazard problem: unless the result of the audit is public, she has to decide on which reported state to announce what the audit has revealed. The contract would then require additional incentive constraints on the monitor.

#### A Appendix

**Proof of Proposition 1.** The proof proceeds as follows: we first prove that there is maximum punishment; then that the entrepreneur gets a positive rent in high state and zero rent in low state. Instead of working with  $R_{HL}$ ,  $P_{HL}$  as a variable, we use the punishment  $\delta_R = R_{HL} - R_H$  and  $\delta_P = P_{HL} - P_H$ . Setting up the Lagrangean, we have:

$$p(f_H - P_H - R_H) + (1 - p)(f_L - P_L - R_L) + \lambda_1[p(1 - l)R_H + (1 - p + pl)R_L - \alpha(1 + r)D] + \lambda_2[\mu P_H + plm\delta_P + (1 - \mu)P_L - (1 - \alpha)(1 + r)D] + \lambda_3(f_H - P_H - R_H - \delta_R - \delta_P) + \lambda_4(f_L - R_L - P_L)$$

where  $\mu = p(1-l) + plm$ ,  $l = \frac{(1-p)\phi}{p(R_H + \delta_R - R_L - \phi)}$ ,  $m = \frac{R_H + P_H - R_L - P_L}{R_H + \delta_R - R_L + P_H + \delta_P - P_L}$ and  $\alpha$  is the share of capital provided by the monitoring investor. We then use the *FOC*'s:

$$\frac{\partial L}{\partial R_H} = -p + \lambda_1 \left(p - pl + \frac{pl(R_H - R_L)}{R_H + \delta_R - R_L - \phi}\right) + \lambda_2 \left(P_H - P_L\right) \left[\frac{pl(1 - m)}{R_H + \delta_R - R_L - \phi} + \frac{pl(1 - m)}{R_H + \delta_R - R_L + P_H + \delta_P - P_L}\right] - \lambda_3 = 0$$

$$\frac{\partial L}{\partial R_L} = -(1-p) + \lambda_1 (1-p+pl - \frac{pl(R_H - R_L)}{R_H + \delta_R - R_L - \phi}) - \lambda_2 (P_H - P_L) [\frac{pl(1-m)}{R_H + \delta_R - R_L - \phi} + \frac{pl(1-m)}{R_H + \delta_R - R_L + P_H + \delta_P - P_L}] - \lambda_4 = 0$$

$$\frac{\partial L}{\partial \delta_R} = \lambda_1 \frac{pl(R_H - R_L)}{R_H + \delta_R - R_L - \phi} + \lambda_2 (P_H - P_L) [\frac{pl(1 - m)}{R_H + \delta_R - R_L - \phi} - \frac{plm}{R_H + \delta_R - R_L + P_H + \delta_P - P_L}] - \lambda_3 \le 0$$

$$\frac{\partial L}{\partial \delta_P} = \lambda_2 \left[ plm - \frac{plm \left( P_H - P_L \right)}{R_H + \delta_R - R_L + P_H + \delta_P - P_L} \right] - \lambda_3 \le 0$$

$$\frac{\partial L}{\partial P_H} = -p + \lambda_2 \left[\mu + \frac{pl(1-m)(P_H - P_L)}{R_H + \delta_R - R_L + P_H + \delta_P - P_L}\right] - \lambda_3 = 0$$

$$\frac{\partial L}{\partial P_L} = -(1-p) + \lambda_2 \left[(1-\mu) - \frac{pl(1-m)(P_H - P_L)}{R_H + \delta_R - R_L + P_H + \delta_P - P_L}\right] - \lambda_4 = 0$$
• From  $\frac{\partial L}{\partial R_H} + \frac{\partial L}{\partial R_L}$  and  $\frac{\partial L}{\partial P_H} + \frac{\partial L}{\partial P_L}$ :  
 $1 + \lambda_3 + \lambda_4 = \lambda_1$  (28)  
 $= \lambda_2$  (29)

whence we deduce that  $\lambda_1 = \lambda_2 > 1$ .

- For m < 1,  $\delta_R + \delta_P > 0$  and either  $\delta_R$  or  $\delta_P$  or both are positive. Hence,  $\frac{\partial L}{\partial \delta_R} \leq 0$  and  $\frac{\partial L}{\partial \delta_P} \leq 0$ .
- To prove maximum punishment  $(\lambda_3 > 0)$ , we use  $\lambda_1 = \lambda_2$  from (28) and (29) in the *FOC* on  $\delta_R$ , and get:

$$\lambda_{3} = \lambda_{1} \{ \frac{pl(R_{H} - R_{L})}{R_{H} + \delta_{R} - R_{L} - \phi} + (P_{H} - P_{L}) [\frac{pl(1 - m)}{R_{H} + \delta_{R} - R_{L} - \phi} - \frac{plm}{R_{H} + \delta_{R} - R_{L} + P_{H} + \delta_{P} - P_{L}} ] \}$$

$$\lambda_{3} = \lambda_{1} p l \{ \frac{R_{H} - R_{L} + P_{H} - P_{L}}{R_{H} + \delta_{R} - R_{L} - \phi} - m(P_{H} - P_{L}) [\frac{1}{R_{H} + \delta_{R} - R_{L} - \phi} + \frac{1}{R_{H} + \delta_{R} - R_{L} + P_{H} + \delta_{P} - P_{L}} ] \}$$

$$\lambda_{3} = \lambda_{1} \frac{pl}{R_{H} + \delta_{R} - R_{L} - \phi} \{ R_{H} - R_{L} + P_{H} - P_{L} - m(P_{H} - P_{L})(1 + \frac{R_{H} + \delta_{R} - R_{L} - \phi}{R_{H} + \delta_{R} - R_{L} + P_{H} + \delta_{P} - P_{L}}) \}$$

$$\lambda_{3} = \lambda_{1} \frac{plm(R_{H} + \delta_{R} - R_{L})}{R_{H} + \delta_{R} - R_{L} - \phi} \left[1 + \frac{R_{H} + \delta_{R} - R_{L} - \phi}{R_{H} + \delta_{R} - R_{L} + P_{H} + \delta_{P} - P_{L}}\right]$$

which is certainly positive. Hence  $f_H - R_H - \delta_R - P_H - \delta_P = 0$ .

• The next step is to prove that the low state feasibility constraint binds:  $R_L + P_L = f_L \ (\lambda_4 > 0)$ . Using the FOC on  $P_L$ :

$$\lambda_4 = -(1-p) + \lambda_2 [(1-p+pl-plm) - \frac{pl(1-m)(P_H - P_L)}{R_H + \delta_R - R_L + P_H + \delta_P - P_L}]$$

This can also be rearranged to:

$$\lambda_4 = (\lambda_2 - 1)(1 - p) + \lambda_2 \frac{pl(1 - m)(R_H + \delta_R - R_L)}{R_H + \delta_R - R_L + P_H + \delta_P - P_L},$$

which, using (28), is certainly positive.

• Last we prove that all the punishment is paid out to the monitor, i.e.  $\delta_P = 0$ . Using maximum punishment and binding low state feasibility, m and l can be written as:

$$m = \frac{R_H + P_H - f_L}{f_H - f_L}$$
$$l = \frac{(1-p)\phi}{p(R_H + \delta_R - R_L - \phi)},$$

and the expected observation cost is  $E[MC] = m\phi(1 - p + pl)$ . Then

$$\frac{\partial E[MC]}{\partial \delta_R} = -\frac{plm\phi}{R_H + \delta_R - R_L - \phi} < 0$$

the expected observation cost is decreasing in  $\delta_R$ . Thus it is possible to minimise the expected observation cost by maximising  $\delta_R$ , which implies that  $\delta_P = 0$ .

**Proof of Theorem 1.** The reduced form problem is:

$$\max_{\alpha, P_L, P_H} p \delta_R \tag{30}$$

st 
$$(p - pl + plm)P_H + (1 - p + pl - plm)P_L = (1 - \alpha)(1 + r)D$$
 (31)

where

$$\delta_R = f_H - P_H - f_L + P_L + \frac{f_L - P_L - \alpha(1+r)D}{p(1-l)}.$$
(32)

• The passive investors participation constraint must bind otherwise either  $\alpha$  or the repayments could be reduced.

Writing  $p\delta_R$  and the participation constraint in terms of the three variables  $\Delta P = P_H - P_L$ ,  $P_L$ ,  $\alpha$  gives

$$p\delta_R = p(f_H - f_L - \Delta P) + \frac{f_L - P_L - \alpha(1+r)D}{(1-l)}$$
(33)

so that

$$\frac{\partial p \delta_R}{\partial P_L} = -\frac{1}{1-l} \tag{34}$$

$$\frac{\partial p\delta_R}{\partial \alpha} = -\frac{(1+r)D}{1-l} \tag{35}$$

We can define the participation constraint of the passive investor as

$$PC = (p - pl + plm)\Delta P + P_L \ge (1 - \alpha)(1 + r)D$$

and then replace l, m to get

$$\left[p - \frac{(1-p)\phi\left(f_H - f_L - \Delta P + \frac{f_L - P_L - \alpha(1+r)D}{p(1-l)}\right)}{(f_H - f_L - \Delta P - \phi)(f_H - f_L)}\right]\Delta P + P_L \ge (1-\alpha)(1+r)D$$

so that

$$\frac{\partial PC}{\partial P_L} = \frac{(1-p)\phi\Delta P}{p(1-l)(f_H - f_L - \Delta P - \phi)(f_H - f_L)} + 1 > 0$$
(36)

$$\frac{\partial PC}{\partial \alpha} = \frac{(1-p)\phi\Delta P(1+r)D}{p(1-l)(f_H - f_L - \Delta P - \phi)(f_H - f_L)} + (1+r)D > 0 \quad (37)$$

As  $P_L$  or  $\alpha$  increase (given  $\Delta P$ ),  $p\delta_R$  falls whilst the passive investors constraint becomes more slack with repayments (the LHS) increasing relative to the outstanding debt (the RHS). Hence if the constraint were slack, a decrease in  $P_L$  (and since  $\Delta P$  is constant, also in  $P_H$ ) is possible without violating the constraint but leads to a fall in m and so an increase in the objective.

• For arbitrary  $P_H, P_L$  we can solve PC for  $\alpha$  yielding

$$\alpha = \frac{(1+r)D - [p - (1-p)\phi(f_H - \Delta P - f_L + (f_L - P_L)/(p(1-l))]\Delta P - P_L}{(1+r)D\left[1 + \frac{(1-p)\phi\Delta P}{p(1-l)(f_H - \Delta P - f_L - \phi)(f_H - f_L)}\right]}$$

which we can rewrite as

$$\alpha = \frac{\Delta P \left[ p - \frac{pl(f_H - f_L - \Delta P)}{f_H - f_L} - \frac{lf_L}{(1 - l)(f_H - f_L)} \right] - (1 + r)D}{(1 + r)D \left[ 1 + \frac{l\Delta P}{(1 - l)(f_H - f_L)} \right]} - \frac{P_L}{(1 + r)D}.$$
(38)

Replacing this in m yields

$$m = \frac{((1+r)D - f_L)(f_H - f_L - \Delta P - \phi)}{[p(f_H - f_L) - \phi][f_H - f_L] - \Delta P[p(f_H - f_L) - \phi(1-p)]}$$

with

$$\frac{\partial m}{\partial \Delta P} = \frac{((1+r)D - f_L)(1-p)\phi^2}{\left[\Delta P\phi(1-p) + p(f_H - f_L)^2 - (\phi + p\Delta P)(f_H - f_L)\right]^2} > 0.$$
(39)

Writing the objective function as  $p\delta_R = p(1-m)(f_H - f_L)$ , the firm's problem is then to minimise m over  $\Delta P$  with the constraint that  $0 \leq m, l \leq 1$ . Since m is increasing in  $\Delta P$  this requires minimising  $\Delta P$ . With no constraints on repayments if there is a minimising  $\Delta P$  we can then choose  $P_L$  at this value of  $\Delta P$  to give any value of  $\alpha$  that is desired.

With limited liability on the investors so that  $R_s \ge 0, P_s \ge 0$  for s = H, L, the optimal values of repayments to the passive monitor are  $P_H = 0, P_L = f_L$  (implying  $R_L = 0$ ),  $R_H = \frac{(f_H - f_L)(1+r)D(f_H - \phi) + (1-p)f_L((f_H - \phi)f_L - f_H^2)}{p(f_H - f_L)f_H - \phi(f_H - pf_L)}$ , so long as these ensure that  $0 \le m \le 1$ . When  $P_H = 0, P_L = f_L$ 

$$m = \frac{(f_H - \phi)((1 + r)D - f_L)}{p(f_H - f_L)f_H - \phi(f_H - pf_L)}$$

Since  $p(f_H - f_L)f_H - \phi(f_H - pf_L) = [p(f_H - f_L) - \phi(f_H - pf_L)/f_H]f_H > [p(f_H - f_L) - \phi]f_H > 0$  using assumption (A.2) certainly m > 0 at  $P_H = 0$ ,  $P_L = f_L$ . Also m < 1 at  $P_H = 0$ ,  $P_L = f_L$  since for example then

$$\delta_R = \frac{(f_H - f_L)(p(f_H - f_L)f_H - \phi(f_H - pf_L))}{f_H(p(f_H - f_L) - \phi) + pf_L\phi} > 0$$

and  $m = 1 - \frac{\delta_R}{(f_H - f_L)}$  whilst  $l = \frac{(1-p)\phi}{p(f_H - \phi)}$ .

Finally the value of  $\alpha$  which ensures the participation constraint of the passive investor binds at  $P_H = 0, P_L = f_L$  is

$$\alpha = \frac{(pf_H - \phi) \left[ (1+r)D(f_H - f_L)(f_H - \phi) - (1-p)f_L(f_H^2 - f_H f_L + \phi f_L) \right]}{(1+r)D \left[ f_H(p(f_H - f_L) - \phi) + p\phi f_L \right] (f_H - \phi)}$$

This value of  $\alpha$  satisfies the passive investors participation constraint which has the form

$$(1-\mu)f_L = (1-\alpha)(1+r)D$$

so that  $0 < 1 - \alpha = \frac{(1 - \mu)f_L}{(1 + r)D} < 1.$ 

**Proof of Proposition 2.** By setting  $R_H^s = R_L^s = 0$ , the contract problem is reduced to choose  $P_H^s, P_L^s, \delta_R^s$  and  $\delta_P^s$  to

$$\max \ p(f_H - P_H^s) + (1 - p)(f_L - P_L^s) \tag{40}$$

st 
$$(1+r)D \le (p-pl^s+pl^sm^s)P_H^s + pl^sm^s\delta_P^s + (1-p+pl^s-pl^sm^s)P_L^s$$
(41)

$$f_H - \delta_R^s - P_H^s - \delta_P^s \ge 0 \tag{42}$$

$$f_L - P_L^s \ge 0 \tag{43}$$

with the ex post probabilities of lying and monitoring defined as:

$$l^s = \frac{(1-p)\phi}{p(\delta_R^s - \phi)}; \tag{44}$$

$$m^{s} = \frac{P_{H}^{s} - P_{L}^{s}}{\delta_{R}^{s} + P_{H}^{s} + \delta_{P}^{s} - P_{L}^{s}}.$$
(45)

Since the problem has the same structure as that in Proposition 1, we can deduce by the same argument that the investor gets no share of the penalty for misreporting. We can thus set  $\delta_P^s = 0$ , which gives (41) as

$$(1+r)D \le (p-pl^s+pl^sm^s)P_H^s + (1-p+pl^s-pl^sm^s)P_L^s.$$
(46)

>From (46) we see that the solution must involve  $P_H^s > P_L^s$ . A contract with  $P_H^s \leq P_L^s$  would require the low state repayment to at least equal  $f_L$  to meet the investor reservation utility, but it would violate assumption (A.1).

To solve this problem, we rewrite the contract problem defining  $\Delta P^s = P^s_H - P^s_L$  :

$$\max pf_H + (1-p)f_L - P_L^s - p\Delta P^s \tag{47}$$

$$\operatorname{st} (1+r)D \le (p-pl^s+pl^sm^s)\Delta P^s - P_L^s$$
(48)

$$f_H - \delta_R^s - P_L^s - \Delta P^s \ge 0 \tag{49}$$

$$f_L - P_L^s \ge 0 \tag{50}$$

with  $m^s = \frac{\Delta P^s}{\delta_R^s + \Delta P^s}$ . From the participation constraint, we solve for  $P_L^s = (1+r)D - (p-pl^s + pl^sm^s)\Delta P^s$  and substitute out into the objective, getting S:

$$\max_{\delta_R^s, \Delta P^s} S = pf_H + (1-p)f_L - (1+r)D - pl^s(1-m^s)\Delta P^s$$

which is increasing in  $\delta_R^s$  and decreasing in  $\Delta P^s$ :

$$\begin{aligned} \frac{\partial S}{\partial \delta_R^s} &= \frac{(1-p)\phi((\delta_R^s)^2 + \Delta P^s \phi)\Delta P^s}{(\delta_R^s - \phi)(\delta_R^s + \Delta P^s)^2} > 0;\\ \frac{\partial S}{\partial \Delta P^s} &= -\frac{(1-p)\phi(\delta_R^s)^2}{(\delta_R^s - \phi)(\delta_R^s + \Delta P^s)^2} < 0; \end{aligned}$$

so that not surprisingly, there is maximum punishment:  $\delta_R^s = f_H - P_L^s - \Delta P^s$ . Moreover the entrepreneur would like to lower  $\Delta P^s$  as much as possible so as to better shape the incentives. However, to ensure participation, the repayments to the investor are constrained to be increasing in the firm's cash flow. Thus the lowest possible spread between repayments can be obtained by setting  $P_L^s$  at the highest possible level, i.e.  $f_L$ , and  $P_H^s$  to the minimum level that satisfies investor's participation.<sup>7</sup>

**Proof of Corollary.** We have for the general case with a monitor which can also act as an investor:  $m^2 = \frac{(f_H - \phi)((1+r)D - f_L)}{p(f_H - f_L)f_H - \phi(f_H - pf_L)}, l^2 = \frac{(1-p)\phi}{p(f_H - \phi)};$ for the single investor/monitor case  $m^s = \frac{P_H^s - f_L}{f_H - f_L}, l^s = \frac{(1-p)\phi}{p(f_H - P_H^s - \phi)};$ for the specialised case  $m^1 = \frac{(f_H - f_L - \phi)((1+r)D - f_L)}{(p(f_H - f_L) - \phi)(f_H - f_L)}, l^1 = \frac{(1-p)\phi}{p(f_H - f_L - \phi)}.$ From the participation constraint of the pure investor when  $\alpha = 0$  we have  $P_H^s = [(1+r)D - (1-\mu^s)f_L]/\mu^s$  so that  $P_H^s - f_L = [(1+r)D - f_L]/\mu^s$  and  $(1+r)D - f_L$ . Using this we can show that  $m^s \ge m^2$  by first showing

 $m^s = \frac{(1+r)D - f_L}{\mu^s(f_H - f_L)}$ . Using this we can show that  $m^s > m^2$  by first showing that  $m^s > m^1$ , and then that  $m^1 > m^2$ .

From  $f_H > \phi$  we derive  $(f_H - \phi)(f_H - P_H^s) > f_H(f_H - P_H^s - \phi) = f_H(\delta_R - \phi)$  which gives us

$$p - \mu^s = pl^s(1 - m^s) = \frac{\phi(1 - p)(f_H - P_H^s)}{(\delta_R - \phi)(f_H - f_L)} > \frac{f_H \phi(1 - p)}{(f_H - \phi)(f_H - f_L)}$$

or

$$\frac{p(f_H - f_L)f_H - \phi(f_H - pf_L)}{(f_H - \phi)(f_H - f_L)} = p - \frac{f_H\phi(1 - p)}{(f_H - \phi)(f_H - f_L)} > \mu^s,$$

leading to

$$\frac{1}{\mu^s(f_H - f_L)} > \frac{f_H - \phi}{p(f_H - f_L)f_H - \phi(f_H - pf_L)}$$

which is equivalent to  $m^s > m^1$ . Then we compare  $m^1$  with  $m^2$ :

$$m^{1} - m^{2} = \frac{((1+r)D - f_{L})\phi^{2}f_{L}(1-p)}{(p(f_{H} - f_{L}) - \phi)(f_{H} - f_{L})(p(f_{H} - f_{L})f_{H} - \phi(f_{H} - f_{L}))}$$

which is positive.

We can also show  $l^s > l^1$ . We have

$$l^{s} = \frac{(1-p)\phi\mu^{s}}{p[\mu^{s}(f_{H} - f_{L}) - (1+r)D + f_{L} - \mu^{s}\phi]} > \frac{(1-p)\phi}{p[f_{H} - f_{L} - \phi]}$$

 $\hline \hline & \overline{ (1-p)(1-5p) + p^2 ) f_H^2 f_H^2 + (1-p)f_L^2 - f_L(f_H + \phi(1-2p)) + (1+r)D(f_H - f_L) - \phi f_H \\ - sqrt[-(f_H - f_L)^2 \left( ((1+r)D)^2 - 2\phi \right) + (1-3p)2f_H^2 f_L + (f_H - f_L)(1-p)4\phi^2 - (1-p)2f_L^3 + (3p-2)2f_H f_L^2 + 2pf_H^3 \right) \\ (1+r)D + (2(p-1)f_H f_L + (3-4p)f_L^2 - f_H^2)\phi^2 + 2(1-p)(1-2p)f_H f_L^3 + ((3p-2)f_H f_L^2 + pf_H^3 - (1-3p)f_H^2 f_L - (1-p)f_L^3)2\phi - 2(1-2p)f_H^3 f_L - (1-p)^2 f_L^4 - p^2 f_H^4 - ((1-p)(1-5p) + p^2) f_H^2 f_L^2 \right) \\ + (2p+2)f_H^2 f_L^2 + pf_H^3 - (1-2p)f_H^2 f_L^2 - (1-p)f_L^3)2\phi - 2(1-2p)f_H^3 f_L - (1-p)^2 f_L^4 - p^2 f_H^4 - ((1-p)(1-5p) + p^2) f_H^2 f_L^2 \right) \\ + (2p+2)f_H^2 f_L^2 + pf_H^3 - (1-2p)f_L^2 + pf_H^3 - (1-2p)f_L^2 - pf_L^2 - pf_H^4 - p^2 f_H^4 - p$ 

as  $(1+r)D > f_L$ . Then we can directly compare  $l^1$  with  $l^2$ :

$$l^{1} - l^{2} = \frac{(1 - p)\phi f_{L}}{p(f_{H} - f_{L} - \phi)(f_{H} - \phi)} > 0$$
(51)

We thus have the rankings:  $l^s > l^1 > l^2$ ;  $m^s > m^2$ ,  $m^1 > m^2$ .

**Proof of Proposition 3.** The possibility of collusive agreements depends on whether  $p(1-l)R_H \leq f_L$ .<sup>8</sup>

**A** The monitor makes a collusive offer.

According to the sign of the inequality (27), we have the following cases:

**A.1**  $p(1-l)R_H < f_L < R_H$ 

if  $R_H < b$ : both the high type and the low type reject so the original game, report/auditing and repayments follow. This would give the monitor  $p(1-l)R_H$ ;

if  $f_L < b < R_H$ , the low type rejects and reports low. If the high type accepts, the type is revealed to the monitor: she now knows that the firm is of the high type but this is irrelevant since the firm has accepted. However the high type could also reject and then report low, pooling with the low type. By doing this its payoff would be  $f_H - f_L$ . If  $f_L < R_H$  the high type will also reject. The monitor's best offer in this range (if the high type accepts) is  $b = R_H - \epsilon$  and her expected return is  $p(R_H - \epsilon)$ ;

if  $p(1-l)R_H < b < f_L$ , both types accept and report high. The monitor's best offer in this range is  $b = f_L - \epsilon$  and her expected return  $f_L - \epsilon$ ;

last  $b < p(1-l)R_H$  will never be offered by the monitor.

Of these possible collusive offers, the monitor is best off either with the second one with  $b = R_H - \epsilon$  if  $p(R_H - \epsilon) > f_L - \epsilon$  and  $f_L < R_H$  (which are inconsistent), or with the third one if  $p(R_H - \epsilon) < f_L - \epsilon$ , since for small  $\epsilon \ p(R_H - \epsilon)$  is higher than  $p(1 - l)R_H$ . So the outcome is that if  $p(1 - l)R_H < f_L < R_H$ , the monitor will offer  $f_L - \epsilon$  and both types will report high.

#### **A.2** $f_L < p(1-l)R_H < R_H$

if  $R_H < b$ : both types reject and the original game is played.

if  $p(1-l)R_H < b < R_H$ , the low type rejects. If the high type accepts and reports high he pays  $R_H - \epsilon$  but by pooling with the low type he would pay  $f_L$ . So again if  $f_L < R_H - \epsilon$  both types will reject.

 $b < p(1-l)R_H$  will never be offered by the monitor.

<sup>&</sup>lt;sup>8</sup> The condition  $f_L \leq p(1-l)R_H$  can be written as:

 $<sup>\</sup>begin{array}{l} ((1+r)D + (p-2)f_L)pf_H^3 + (p(2-p)f_L^2 - p(1+r)Df_L - (1+p)(1+r)D\phi + 2\phi f_L)f_H^2 + \\ ((1+p)(1+r)D\phi f_L - \phi^2 f_L + (p(p-2)-1)\phi f_L^2 + (1+r)D\phi^2)f_H - \phi^2 f_L((1+r)D - f_L) \ge 0. \\ \text{Since the coefficient of } f_H^3 \text{ can be positive, the whole expression can be positive for large } f_H. \end{array}$ 

Of these if  $b = R_H - \epsilon$ , the monitor gets an expected return of  $p(R_H - \epsilon)$ if  $f_L > R_H - \epsilon$ . For small  $\epsilon$  this is higher than  $p(1-l)R_H$  and so the monitor will make this offer, it will be accepted by the high type, who will then report high, and there will be no monitoring. But if  $f_L < R_H - \epsilon$ there is no acceptable collusion.

**B.** The firm makes a collusive offer

Again we have the following cases:

**B.1**  $p(1-l)R_H < f_L < R_H$ 

an offer  $b > R_H$  will not be made by either type;

if  $f_L < b < R_H$ , the type is revealed to the monitor: she knows that the firm is high and she sets m' = 1, while the firm sets l' = 0. So by continuing the monitor gets  $R_H > b = R_H - \epsilon$  and she will reject;

an offer  $p(1-l)R_H < b < f_L$  could be made by both types and would thus be unrevealing to the monitor. So the monitor accepts.

Of these, both types will offer  $p(1-l)R_H < b < f_L$  and it will be accepted. The result is that if  $p(1-l)R_H < f_L < R_H$  what actually happens when the firm makes the offer is that  $b = p(1-l)R_H + \epsilon$  is offered by the firm and accepted by the monitor.

#### **B.2** $f_L < p(1-l)R_H < R_H$

an offer  $b > R_H$  will not be made by either type;

if  $p(1-l)R_H < b < R_H$ , the type is revealed and the monitor will monitor with probability one, m' = 1, and the firm will never want to lie, l' = 0.

Since playing this game gives the monitor  $R_H$ , she rejects, thus implying that no offer will be made;

if  $f_L < b < p(1-l)R_H$ , again it is revealed to the monitor that the firm is of the high type. By the same argument given in the previous point the monitor rejects;

last an offer  $b < f_L$  is not information revealing but the monitor can do better continuing to get  $p(1-l)R_H < b < f_L$ .

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