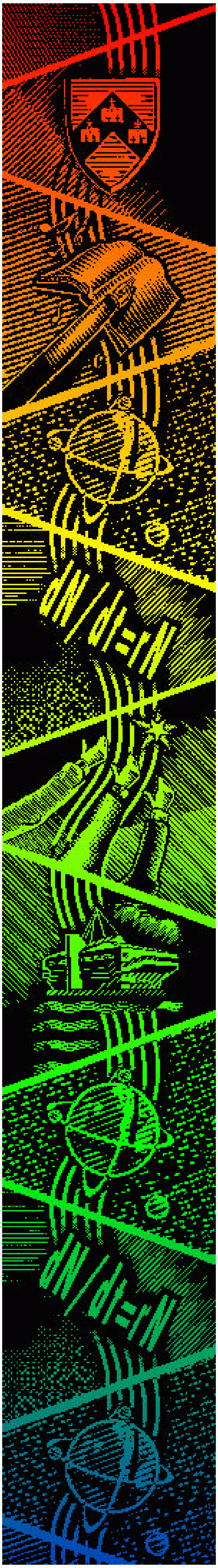

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Entry Dynamics, Capacity Utilisation
and Productivity in a Dynamic Open Economy

by

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Entry dynamics, capacity utilisation and productivity in a dynamic open economy.*

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Abstract

This paper analyses an open economy Ramsey model with an endogenous labour supply without capital. The technology defines an optimal firm size. Changes to the number of firms is subject to adjustment costs, so that the entry dynamics is determined endogenously. We find that there is a short run transitory productivity dynamic introduced when there is imperfect competition due to changes in capacity utilization. We are able to analyze this in different contexts, including demand and technology shocks, both anticipated and unanticipated.

Keywords: entry, capacity utilisation, adjusment costs, Ramsey.

JEL Classification: E2, E6, L1.

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1 Introduction

This paper is a theoretical exploration of the dynamic relationship between entry, capacity utilization and productivity in a monopolistic economy in the presence of changes in underlying technology. By *capacity utilization* we mean the extent to which the output of the firm compares to some standard reference level[1] (the free-entry equilibrium or the technically efficient scale of production)[2]. We employ an entry model found in Das and Das (1996) and developed by Datta and Dixon (2002) in which the cost of entry depends on the flow of entry due to some congestion effect or externality. There is an intertemporal zero-profit condition which equates the cost of entry in each instant to the net present value of incumbency, resulting in the number of firms gradually adjusting towards its long-run value. Output per firm and capacity utilization vary in the short term with output, whilst in the long-run there is a free-entry steady state condition which ties down output per firm and capacity utilization.

We consider an economy of monopolistic firms and find that there is a direct relationship between the level of capacity utilization and productivity which is stronger when monopoly power is greater. There are two elements to the equilibrium: first there is a markup of price over marginal cost resulting from the market power of the firms, $p > MC$; second in the long-run with free entry there will be zero profits so price equals average cost $p = AC$. Putting these two elements together implies that $p = AC > MC$: in long-run equilibrium average cost will be decreasing since it exceeds marginal cost. This is of course the standard Robinson-Chamberlin excess capacity result for free-entry monopolistic equilibrium: if firms have a $U-$shaped average cost curve, then equilibrium output will be on the downward sloping portion of the AC and below the efficient operating level defined by the minimum AC output. Hence there is a direct relationship between productivity and capacity utilization: if capacity utilization varies from the free-entry equilibrium due to a demand or technology shock, there will be a first order effect of capacity utilization on average costs; since average cost is simply the dual of factor productivity, a fall in costs represents an increase in productivity. This effect is absent in a Walrasian setting with free entry: since firms are at optimum scale, the AC curve is locally flat ($P = MC = AC$), with no relationship between capacity utilization and productivity.

The focus of this paper is on the relationship the *underlying technology* and the actual or *measured productivity* of the economy. We define the underlying technology in terms of the long-run or steady state labor productivity (labor is the sole factor of production). We can then compare the behavior of the

[1] Nelson (1989) provides a useful discussion and history of the concept of capacity utilization in this context and its relation to survey data. Since there is no capital in our model, the standard definitions of Cassel (1937) and Klein (1960) coincide, with the reference level of output being minimum average cost.

[2] The term "capacity utilization" is also sometimes used in a different meaning, to distinguish between the flow of capital services and the stock of capital (see Greenwood et al 1988, King and Rebelo 1999). The more accurate term for this is *capital* utilization (Winston 1974, Burnside and Eichenbaum 1996).

actual or measured labor productivity to that of the underlying technology. The reason for a divergence of actual productivity from underlying productivity is that capacity utilization varies from its long-run level, which in turn is due to the divergence of the stock of firms from its long-run level.

Whilst other papers have emphasized the relationship between imperfect competition and increasing returns to scale (Hall 1986,1990, Rotemberg and Woodford 1995, Basu 1996, Devereux et al 1996, Ambler and Cardia 1998, Coto and Dixon 1999, Cook 2001), the contribution of this paper is to set this in the context of an explicit dynamic entry model. Existing papers have tended to assume either that there is instantaneous free entry (the actual or expected profit is driven to zero in each period) or that the number of firms is constant (possibly imposing a long-run zero profit condition as in Hornstein 1993). The dynamic entry model we employ has both instantaneous free-entry and no-entry as limiting cases. Of course, many papers with perfect competition assume that there are global constant returns to scale, so that entry is irrelevant. This is also true in our model in the case of perfect competition, even when the underlying firm technology has varying returns to scale, since in equilibrium there will be locally constant returns when $MC = AC$.

In this paper, we explore the role of entry dynamics in a simple small open economy with a Ramsey consumer. In order to focus on the dynamics induced by entry and exit, we eliminate all other sources of dynamics. There is no capital, only labor; there is an internationally traded bond with world interest rates equalling the household discount rate so that the household is able to perfectly smooth utility. We consider changes in demand and technology, changes which can be either anticipated or unanticipated. The entry dynamics induces an endogenous productivity dynamic through variations in output per firm, capacity utilization. The technology assumed at the firm level allows for the case of $U-$shaped AC curve with increasing MC, and for the cases of globally decreasing AC with either constant or decreasing MC.

In the case of a real technology shift, the transitory dynamic induced by capacity utilization is superimposed on the underlying technology change. This means that the measured productivity changes may not be a good guide to the underlying technology changes except in the long-run: the time profile of measured productivity will differ from that of the underlying technology. For example, in the case of an *anticipated permanent* technological improvement, there will be entry prior to the actual occurrence of the technological change: if there is imperfect competition this entry will increase the degree of capacity underutilization and reduce measured productivity. With an *unanticipated* technological change, the impact effect can either be smaller or larger than the long-run effect. The crucial factor here is the response of the labor supply to the shock, which can take either sign due to the conflict of income and substitution effects. Thus we can either have the case where the capacity utilization effect reinforces the technological shock in the short run, leading to an overshooting of measured productivity over true productivity, or the opposite. Hence, even a simple step change in productivity causes a more complicated time profile in its aftermath (when unanticipated) and possibly before and after (when anticipated).

In the case of a pure demand shock (modelled as a balanced budget fiscal policy), there is no actual technology change, so that all variations in measured labor productivity (such as the Solow residual) are caused by changes in capacity utilization and are transitory. Indeed, the basic insight of the paper is that the output of the economy or industry depends not only on the total level of input (labor in this paper, but also capital[3] and intermediates in general), but also the way that this is divided between firms. Here consider the simplest of a symmetric industry where all firms are identical[4]: the number of firms should be viewed as an additional quasi-input, representing the organization of the industry/economy. In the short-run, the number of firms can adjust only slowly, so that output changes only through changes in labor. At the firm level, this implies that output per firm varies. In the long-run, the number of firms can adjust alongside labor, so that output can change even if output per firm is constant. The key point is that we would expect a completely different relationship between output and employment in these two cases. This reinforces the applied work of Caves D *et al* (1981) and Bendt and Fuss (1986), which emphasized the need to focus on firm level data to understand productivity, not just industry or economy wide data.

Although the model we present is primarily theoretical and not intended to model data, it does have the following features, each of which is supported by empirical evidence:

- **Entry is procyclical.** Frank Portier (Portier 1995) used aggregate data from the French economy[5] and found that the number of firms was correlated with GDP: the correlation of current entry with current GDP using annual data was 53%, with lagged GDP 31% and lead GDP 42%. Ambler and Cardia (1998) used quarterly US data on net business formation (Citibase data series) from 1954-1991. They found that the correlation between output and net firm creation was 58%[6]. Campbell (1998) used US plant data[7] for both entry and exit over the period 1972-1988. He found that the flow of entry is procyclical and exit is countercyclical (*op. cit.* 376).

- **Productivity is procyclical**. Rotemberg and Summers (1990), Basu (1996). Ryan (2000) found not only is productivity procyclical, *but also that the relationship is stronger when the markup is larger.* His study used the NBER Manufacturing productivity database with annual data for 450 four-digit SIC level from 1951-1991.

[3] Brito and Dixon (2000) consider a closed economy model of entry with capital and labor.

[4] In a more general and realistic model, the distribution of firm sizes would also be crucial (see for example Hopenhayn 1992, Ericson and Pakes 1995 and Basu and Fernald 1997)

[5] Portier usedthe *Sirene* database for the period 1977 to 1989, using the number of firms less the self-employed. The correlations are log-deviations from a linear trend.

[6] Ambler and Cardia (1998) used HP filtered data.

[7] The U.S. Bureau of the Census compiles an Annual Survey of Manufacturers to construct a panel.

- **Productivity is correlated with capacity utilization.** Hulten (1986), Zegeye and Rosenblaum (2000), Ryan (2000), Bendt and Fuss (1986), Morrison (1986, 1992, 1993, 1997) find that there is a substantial positive relationship between capacity utilization and productivity.
- **Labor's share is countercyclical.** Ambler and Cardia (1998): the correlation is -71%.

The advantage of our model is that not only does it posses all these features, but also it is able to explain how they are all linked. For example, in the case of a demand shock, increasing returns at the firm level will cause capacity utilization and productivity to increase in the short-run; this will increase profits and reduce labor share; the profits will encourage entry. Of course, whilst our model leaves out many things that are of importance (we have no nominal rigidities in wages or prices and a constant markup, no capital and no uncertainty), we have provided a framework and justification for the analysis of entry which can be developed into richer models.

The outline of the paper is as follows. First, in sections 2 and 3 we outline the optimization problem of the household and the firm, introducing the entry model in section 3.2. In section 4 we put these together into a dynamic general equilibrium model, exploring both the steady state and the linearized dynamics. In section 5 we use the model to analyze the relationship between technological change and measured productivity. Section 6 analyzes the effect of demand on measured productivity when there is no change in underlying technology. All proofs are in the appendix.

2 The household

There is a small open economy, with a world capital market interest rate r equal to the discount rate of the Ramsey household. Utility satisfies standard assumptions and depends on aggregate consumption C and leisure $\ell = 1 - L$, where L is the labor supply[8]:

$$U(C, 1-L) = \log C - \frac{1}{\beta} L^{\beta}, \quad \beta > 1 \tag{1}$$

The household earns income from three sources: supplying labor at wage w, receiving interest income from net foreign bonds rb and receiving profit income Π. As is standard, the household treats profit income as a lump sum payment. It solves:

$$\max \quad \int_0^{\infty} U(C, 1-L) e^{-rt} dt$$

[8] The crucial feature of the utility function is that leisure and consumption are additively separable ($U_{C\ell} = 0$): the results in the paper in the case of diminishing marginal productivity go through with this more general case. The explicit functional form is needed when we embrace increasing returns.

subject to

$$\dot{b} = rb + wl + \Pi - C - G \tag{2}$$

Where we assume that the government finances its expenditure G by a lump-sum tax equal to expenditure in each instant[9].

The solution[10] to this is defined by the equations

$$\begin{aligned} U_C - \lambda &= 0 && \text{(3a)} \\ -U_\ell + \lambda w &= 0 && \text{(3b)} \\ \dot{\lambda} &= 0 && \text{(3c)} \end{aligned}$$

along with the Transversality condition (TVC)

$$\underset{t\to\infty}{Lim} \lambda b e^{-rt} = 0 \tag{4}$$

The solution to the households problem is simple. Since $U_{C\ell} = 0$, we can write optimal consumption and labor supply in terms of Frisch demands:

$$\begin{aligned} C &= C(\lambda) = \frac{1}{\lambda} \\ L &= L^F(w, \lambda) = (\lambda w)^{\frac{1}{\beta - 1}} \end{aligned}$$

The presence of international capital markets means that the household can completely smooth its consumption ($\dot{\lambda} = 0$). λ is an index of the level of utility derived from consumption: a high λ means a low level of consumption and *vice versa*. For a given wage, L^F is increasing in λ.

The aggregate consumption good C is either imported or produced domestically by a perfectly competitive industry with a $CRTS$ production function using intermediate inputs which are monopolistically supplied. There is a continuum of possible intermediate products, $i \in [0, \infty)$. At instant t, there is a range of active products defined by $n(t) < \infty$, so that $i \in [0, n(t))$ are *active* and available, whilst $i > n(t)$ are inactive and not produced. Total domestic output Φ is related to inputs y_i by the following technology

$$\Phi = n^{\frac{1}{1-\theta}} \left[\int_0^n y_i^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \tag{5}$$

where $\theta > 1$ is the elasticity of substitution between products. Notice that there is no *Ethier effect* here: an increase in the range of intermediates does not affect the unit cost function. Treating the unit price of the consumption good as the numeraire, the demand for each available product i takes the constant elasticity form $y_i = p_i^{-\theta} \Phi$.

[9] This is for convenience and avoids the need for introducing government bonds. Since Ricardian equivalence holds, the timing of taxation does not matter.

[10] These are the first order conditions for the current value Hamiltonian $H = U(C, 1 - L) + \lambda[wL + \Pi - C]$.

3 Firms: Technology, entry and exit

There is a continuum of potential firms: each firm can produce only one product. the index of firms and products is therefore the same. At time t, firm $i \in [0, n(t))$ uses labor L_i to produce output y_i using the technology[11]

$$y_i = A.L_i^{\gamma} - F \tag{6}$$

where $\gamma > 0$ and $F \geq 0$ is a fixed flow overhead[12] and $A > 0$ is a technology parameter which increases the marginal product of labor. When $\gamma < 1$, $F > 0$ there is an *U-shaped* average cost (AC) with increasing marginal cost (MC); this is compatible with both perfect (Walrasian) and imperfect competition. When $\gamma = 1, F = 0$, we have constant returns to scale: $AC = MC$. When $\gamma = 1, F > 0$, we have constant MC and decreasing AC; when $\gamma > 1$ there is decreasing AC and MC (the extent to which γ can exceed 1 is limited - the upper bounds are stated below). In these last two cases with globally increasing returns to scale, equilibrium can only exist with imperfect competition.

The number of active firms at instant t is denoted $n(t)$: we will drop the time index when it does not lead to ambiguity. Throughout we will be assuming that labor markets function perfectly so that labor is allocated equally across firms, so that $L_i = L/n$. The *aggregate production function*, obtained from (6,5) under symmetry is Homogenous of degree 1 in n and L :

$$\Phi(L, n, A) = AL^{\gamma} n^{(1-\gamma)} - nF \tag{7}$$

Note that the aggregate marginal product of labor equals the firm-level marginal product(this is because labor is allocated equally across firms). Clearly, when $\gamma < 1$ we have $\Phi_{LL} < 0$, $\Phi_{nn} < 0$ and $\Phi_{nL} < 0$. When $\gamma > 1$ the signs are reversed. The number of firms operating influences the level of output in the economy: it increases the level of overhead costs nF and decreases the level of employment per firm. When $\gamma < 1$, an increase in the number of firms initially increases output, but eventually reduces it ($\Phi_{nn} < 0$). When $\gamma \geq 1$, more firms always reduce output. When $\gamma < 1$ additional firms *increase* the marginal product of labor MPL (it reduces employment per firm); with $\gamma = 1$ there is constant MPL so that $\Phi_{nL} = 0$; when $\gamma > 1$ more firms *reduce* MPL.

There is a clear optimal firm size when $\gamma < 1$: the technically efficient level of employment and output *per firm* where $AC = MC$ are

$$\left(\tfrac{L}{n}\right)^e = \left[\tfrac{F}{A(1-\gamma)}\right]^{1/\gamma} \quad ; \quad y^e = \tfrac{\gamma F}{1-\gamma}$$

When $(\gamma = 1, F = 0)$ so that $\Phi_{nn} = \Phi_{nL} = 0$, the number of firms has no effect on aggregate output, there is no optimal firm size. In the cases of $\{\gamma = 1, F > 0\}$

[11] This specification is similar to Hornstein (1993) and Ambler and Cardia (1998), the main difference being that there is no capital here.

[12] In this paper overhead represent foregone output: an alternative is to have the overhead in terms of labor as in Dixon and Lawler (1996).

and $\{\gamma > 1, F \geq 0\}$ there are globally increasing returns to scale, so that $\Phi_n < 0$ and the efficient firm size is unbounded.

In this paper we define $\mathcal{P}$ as *output per unit labor* ($\mathcal{P} = y/L$): since there is only one factor of production, $\mathcal{P}$ can be unambiguously interpreted as *productivity*. When firms are at their optimum size, the average and marginal product of labor are equated, so that for $\gamma < 1$

$$\mathcal{P}^e = w^e = A^{\frac{1}{\gamma}}\gamma\left(\frac{F}{1-\gamma}\right)^{\frac{\gamma-1}{\gamma}}, \quad \gamma < 1$$

When $\gamma = 1$, efficient production implies $\mathcal{P}^e = A$ (this occurs at any output if $F = 0, \infty$ if $F > 0$). If $\gamma > 1$ the efficient production is undefined (productivity is unbounded as L tends to infinity).

3.1 Profits

In this section, we determine the operating profits of an active firm, i.e. a firm that does not incur any entry costs. Due to imperfect competition, the firm maximizes profits given real wage w (using output price as the numeraire) by choosing employment to satisfy

$$w = (1-\mu)\Phi_L \tag{8}$$

Where μ is the Lerner index of monopoly. Now, the aggregate flow of operating profits given w equals $n\pi$, where π is firm level profit. Since Φ is homogenous of degree 1 in $\{n, L\}$ we have

$$\pi = \Phi_n + \mu\Phi_L\frac{L}{n} \tag{9}$$

$$n\pi = \Phi - (1-\mu)\Phi_L L \tag{10}$$

The zero operating-profit[13] condition $\pi = 0$ implies

$$\Phi_n = -\mu\Phi_L\frac{L}{n} \tag{11}$$

Under the functional form assumed, zero-profits implies that employment and output per firm are given by

$$\left(\frac{L}{n}\right)^* = \left(\frac{F}{A(1-\gamma(1-\mu))}\right)^{\frac{1}{\gamma}} \tag{12}$$

$$y^* = \frac{\gamma(1-\mu)}{1-\gamma(1-\mu)}.F \tag{13}$$

For a zero profit equilibrium to exist, we require that $\gamma < \frac{1}{1-\mu}$. This restriction stems from the requirement that for a profit maximizing output MR must cut

[13]Operating profit is the profit earned by firms already in the industry. Firms which enter pay an entry cost. This is dealt with in the next section.

MC from above: a higher μ means a steeper MR which allows a steeper fall in MC. Hence a higher degree of monopoly allows for larger γ: $\gamma < 1$ is compatible with any $\mu \geq 0$. If $\gamma > 1$, then it implies that the market cannot be too competitive: $\mu > \frac{\gamma-1}{\gamma}$. In fact there is a second restriction $\gamma < \beta$: this means that the slope of the labor supply curve in (w, L) space is greater than the slope of the marginal product schedule. Thus we have a clear upper bound on γ given μ, β:

$$\gamma < \min\left[\frac{1}{1-\mu}, \beta\right] \tag{14}$$

Note that whilst an increase in productivity A reduces employment per firm under zero profits, output per firm is unaffected. In the zero profit equilibrium productivity is

$$\mathcal{P}^* = A^{\frac{1}{\gamma}}\gamma(1-\mu)\left(\frac{F}{1-\gamma(1-\mu)}\right)^{\frac{\gamma-1}{\gamma}} \tag{15}$$

The number of firms conditional upon employment is,

$$n = LA^{\frac{1}{\gamma}}\left(\frac{1-\gamma(1-\mu)}{F}\right)^{\frac{1}{\gamma}} \tag{16}$$

Comparing the efficient and the free-entry outcomes with $\gamma < 1$, we can see that in the Walrasian case ($\mu = 0$), the free-entry and zero-profit outcomes are the same. In both cases, firms are operating where $AC = MC$, at the bottom of the $U-$shaped AC curve. When $\mu > 0$ however, $\left(\frac{L}{n}\right)^* < \left(\frac{L}{n}\right)^e$,$y^* < y^e$, $\mathcal{P}^* < \mathcal{P}^e$: this is the standard *excess capacity* result of Chamberlin and Robinson. With monopolistic competition, free entry leads to excess entry and firms operate on the decreasing part of the AC curve (there are locally increasing returns to scale).

Under symmetry, aggregate free entry output of the consumption good is given by $\Phi = ny^*$, which can be written as a function of L only using (16)

$$\Phi = \left[A^{\frac{1}{\gamma}}.\gamma(1-\mu)\left(\frac{1-\gamma(1-\mu)}{F}\right)^{\frac{1}{\gamma}-1}\right]L \tag{17}$$

Since Φ is homogenous of degree 1 in (n, L), the fixing of the ratio (L/n) in (12) means that Φ becomes proportional to L. Free entry imposes long-run $CRTS$ on the relation between L and Φ irrespective of the value of γ at the firm level.

3.2 The entry decision

What determines the number of firms operating at each instant t? In this paper we employ the model developed from Das and Das (1996) by Dixon (2000) and

Datta and Dixon (2001). At time t, there is a flow cost of entry $q(t)$ for each entrant (entry and exit are symmetric for simplicity, with $-q$ being the cost of exit at time t). The cost of entry is assumed to be increasing in the flow of entry $E = \dot{n}$

$$q = \nu E \tag{18}$$

The total entry costs incurred by entrants are therefore νE^2. The relationship between the flow of entry and the cost of entry is based on the notion that there is a congestion effect: when more firms are being set up, the cost of setting up is higher. We do not model this: however, this might be because of a direct externality in the production of new firms, or due to the fixed supply of some factor involved in the creation of new firms. In Dixon (2000) it is also shown that this model can be derived form a locational setup where firms are situated along a real line representing location in some technological/product or geographical space. If the cost of entry depends on the distance of the new entrant from the nearest incumbent firm, then the same relation between entry flow and cost exists: a higher flow of entry means that firms more distant from $n(t)$ are setting up. Whilst exit and entry are treated symmetrically in this paper, this is not essential. It is possible to model exit differently (e.g. there is a fixed cost of exit, perhaps zero, as in Das and Das 1996 or Hopenhayn 1992), the alternatives being analyzed in Dixon (2000).

The flow of entry in each instant is determined by an *arbitrage condition.* Suppose a firm is inactive: it can either set up in instant t or delay. The firm can either invest in setting up or not. The opportunity cost of funds is given by the return on the bond, r. This must equal the return on investing a dollar in setting up a new firm, given by the LHS of (19)

$$\frac{\pi}{q} + \frac{\dot{q}}{q} = r \tag{19}$$

where π is given by (9). The first LHS term is the number of firms per dollar $(1/q)$ times the flow operating profits the firm will make if it sets up: the second term reflects the change in the cost of entry. If $\dot{q}/q > 0$, then it means that the cost of entry is increasing, so that there is a capital gain associated with entry at time t; if $\dot{q}/q < 0$ it means entry is becoming cheaper, thus discouraging immediate entry. The arbitrage[14] condition equates the return on bonds with setting up a new firm, and is a differential equation in q, which determines the entry flow by (18).

With entry, the total profits are the operating profits of firms less the entry costs paid by the entrants

$$\Pi = n\pi - \nu E^2 = n\Phi_n + \mu \Phi_L L - \nu E^2 \tag{20}$$

[14] The arbitrage equation can be written in a way directly analagous to the user cost of capital:

$$\pi = q(r - \frac{\dot{q}}{q})$$

In equilibrium, $q(t)$ represents the net present value of incumbency[15]: it is the present value of profits earned if you are an incumbent at time t. This arises since the entrants are indifferent between entering and staying out. When $q < 0$, the present value of profits is negative: in equilibrium this is equal to the cost of exit. In steady state, we have $E = q = 0$, so that the entry model implies the zero-profit condition. Entry costs are thus a disequilibrium phenomenon.

Note that our entry model has the standard models as limiting cases: when $\nu = 0$, we have instantaneous free entry so that (19) becomes $\pi = 0$ and there are zero profits each instant; if we have $\nu \to +\infty$, then changes in n become very costly and n moves little if at all which approximates the case of a fixed number of firms.

4 The dynamic system

We are now ready to draw together the different elements of the economy in order to represent the economy as an integrated dynamic system. Since consumption is constant, we have three dynamic equations

$$\left.\begin{array}{c} \dot{n} = E = \frac{q}{\nu} \\ \dot{q} = rq - \left[\Phi_n + \mu\Phi_L \frac{L}{n}\right] = Q(n, \lambda, q) \\ \dot{b} = rb + wL + \Pi - C(\lambda) - G = B(b, n, \lambda, q) \end{array}\right\} \tag{21}$$

where

$$\begin{aligned} w &= (1-\mu)\Phi_L \\ \Pi &= n\Phi_n + \mu\Phi_L L - \nu E^2 \end{aligned}$$

In addition to (21), we also have the household TVC (4). From (21) we have a subsystem in (n, q) which determines the dynamics of the whole system, the bond equation being a residual (see Brock and Turnovsky 1994, Turnovsky 1997, Coto and Dixon 2001). The bond equation along with the TVC then determines the equilibrium value of λ.

In order to understand the dynamics system, we will take two steps. First, we will specify the steady state for a given level of bonds b^*. Secondly, we will linearize and from the dynamics derive an expression for steady state bonds. The steady state depends on the path taken to equilibrium, since the stock of bonds will vary in response to any trade deficit or surplus along the path to equilibrium. This is a standard feature of open economy models (see Turnovsky 1997) which differentiates them from closed economy Ramsey models where the steady-state is not path dependent. As we shall see, the entry process in moving towards equilibrium has a long-run effect through its impact on bonds.

4.1 Steady-state

In steady state we have $\dot{n} = E = \dot{q} = \dot{b} = 0$. Before proceeding further, it is useful to write the labor supply as a function of (n, λ): this is obtained by com-

[15] See Datta and Dixon (2000) for a proof.

bining the Frisch supply $L^F(\lambda, w)$ with the marginal productivity relationship defining w

$$L(\lambda, n, A) = L^F(\lambda, (1-\mu)\Phi_L(L,n)) = (\lambda\gamma(1-\mu).A)^{\frac{1}{\beta-\gamma}}.(n)^{\frac{1-\gamma}{\beta-\gamma}} \quad (22)$$

Whilst $L_\lambda, L_A > 0$ are unambiguous, the effect of n on L depends on $\gamma : L_n > 0$ for $\gamma < 1$; $L_n = 0$ for $\gamma = 1$; $L_n < 0$ for $\gamma > 1$. Entry alters employment per firm: this affects the real wage. With a decreasing MPL, entry increases the real wage and hence labor supply; with increasing MPL the opposite holds.

Let us first consider the steady state conditional on b^*. In this case we have three equations in three unknowns $\{n, \lambda, L\}$

$$\Phi_n(L^*, n^*) = -\mu\Phi_L(L(\lambda^*, n^*), n^*)\left(\frac{L^*}{n}\right) \quad (23a)$$

$$(1-\mu)\Phi_L(L^*, n^*) = \frac{U_\ell(1-L^*)}{U_C(C(\lambda^*))} \quad (23b)$$

$$C(\lambda^*) = w^*L^* + rb^* - G \quad (23c)$$

Equation (23a) implies that there are zero-profits in steady-state. Since Φ is homogeneous of degree 1 in $\{L, n\}$ the free entry condition (23a) determines both the ratio of $(L/n)^*$ and the wage w^*. Equation (23b) means that the wage equates the MRS and the real wage. The final equation (23c) comes from the steady state condition for bonds: $B(b, n, \lambda, q) = 0$. For an arbitrary value of b^*, we can thus solve the equations for the implied values of $\{n, \lambda, L\}$. We can represent the steady state in consumption-leisure space. The Income Expansion Path (IEP) represents the consumption leisure choice given the equilibrium real wage w^*: since both leisure and consumption are assumed normal, it is upward sloping. The steady-state budget constraint (23c) is linear with slope w and intercept[16] $rb^* - G$, the dynamics of the path to steady state being reflected in the divergence between steady state bonds b^* and initial bonds b_0. The steady state equilibrium is then the intersection of the IEP and the budget constraint at point A.

Given the functional forms we have assumed, free-entry condition determines the steady-state real wage and productivity (15) $w^* = \mathcal{P}^*$. Substituting into (3b) yields the reduced form equations for steady-state employment, the number of firms and output conditional on (λ, A)

[16] As depicted, we have $rb^* - G > 0$. Of course, the intercept can be negative if b^* is small or negative.

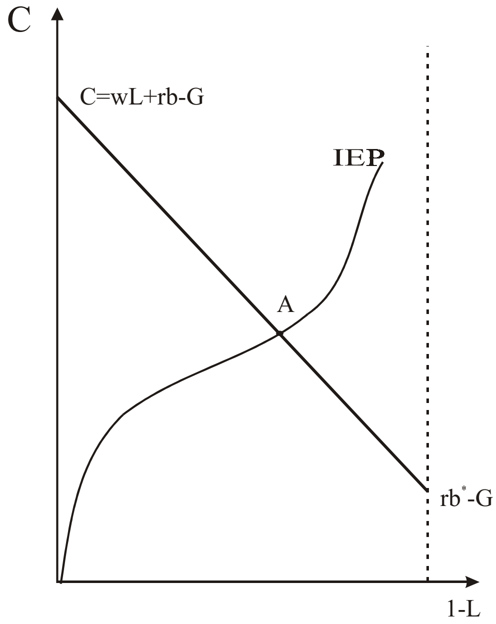

Figure 1: The Steady State in $\{C, 1-L\}$ space.

$$\begin{aligned}
L &= \mathcal{L}(\lambda, A) = (\lambda(1-\mu)\gamma)^{\frac{1}{\beta-1}} A^{\frac{1}{\gamma(\beta-1)}} \left(\frac{1-\gamma(1-\mu)}{F}\right)^{\frac{1-\gamma}{\gamma(\beta-1)}} \\
n &= n(\lambda, A) = (\lambda(1-\mu)\gamma)^{\frac{1}{\beta-1}} A^{\frac{\beta}{\gamma(\beta-1)}} \left(\frac{1-\gamma(1-\mu)}{F}\right)^{\frac{\beta-\gamma}{\gamma(\beta-1)}} \\
\Phi &= \mathcal{Y}(\lambda, A) = (\lambda)^{\frac{1}{\beta-1}} ((1-\mu)\gamma)^{\frac{\beta}{\beta-1}} A^{\frac{\beta}{\gamma(\beta-1)}} \left(\frac{1-\gamma(1-\mu)}{F}\right)^{\frac{\beta(1-\gamma)}{\gamma(\beta-1)}}
\end{aligned}$$

By use of the reduced form equation $\mathcal{L}(\lambda, A)$ we can then determine the equilibrium level of λ^*, conditional on A and b^*, by the output market clearing condition

$$G + C(\lambda^*) - w\mathcal{L}(\lambda^*, A) - rb^* = 0 \tag{24}$$

where (24) can be viewed as an excess demand function for the steady state in terms of the price of marginal utility λ. The first two terms of the expression above, representing the expenditure side, are decreasing in λ, while the income terms, $w\mathcal{L}(\lambda) + rb^*$, are increasing in λ; there exists a $\lambda^* > 0$ such that the economy is at the steady state equilibrium given b^*. We have now defined the steady-state for a given value of the steady-state bonds b^*: we now turn to the dynamics to derive the steady state stock of bonds.

4.2 Linearized system

The analysis of the steady-state was conditional on the level of steady state bonds b^*. However, to determine b^* we need to know the path taken to equilibrium. The dynamics of the system will be analyzed by linearizing around the steady state:

$$\begin{bmatrix} \dot{n} \\ \dot{q} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\nu\lambda^*} & 0 \\ \frac{\lambda^*}{n(\lambda^*)}\frac{\gamma(\beta-1)F}{\beta-\gamma} & r & 0 \\ \gamma\left(\frac{F}{1-\gamma(1-\mu)}\right)(\varepsilon^*_{nL}-\mu) & 0 & r \end{bmatrix} \begin{bmatrix} n-n^* \\ q-q^* \\ b-b^* \end{bmatrix} \tag{25}$$

where $q^*=0$ and $\varepsilon^*_{Ln} \equiv L_n\left(\frac{n}{L}\right)^* = \frac{1-\gamma}{\beta-\gamma}$ represents the elasticity of the labor supply to the number of firms evaluated at the steady state. The determinant of the sub-system (25) in $\{n,q\}$ is

$$\Delta = -\frac{1}{n(\lambda^*)}\frac{\gamma(\beta-1)F}{\nu\,(\beta-\gamma)} < 0 \tag{26}$$

which is negative since $\beta > \gamma$. Hence the system is saddle-path stable, with the negative real root $\Gamma < 0$ and a positive root Γ^+

$$\Gamma = \frac{r}{2}\left(-1-\sqrt{1-\frac{4\Delta(\lambda^*)}{r^2}}\right) < 0 < \Gamma^+ = \frac{r}{2}\left(-1+\sqrt{1-\frac{4\Delta(\lambda^*)}{r^2}}\right) \tag{27}$$

The solution to the linearized system is

$$n(t) = n^* + (n_0-n^*)\exp[\Gamma t] \tag{28a}$$
$$q(t) = (n_0-n^*)\Gamma\nu\exp[\Gamma t] \tag{28b}$$
$$b(t) = b^* + \frac{\Omega}{\Gamma_1 - r}(n_0-n^*)\exp[\Gamma t] \tag{28c}$$

where Ω gives the effect of entry on the stock of bonds:

$$\Omega = \gamma\frac{F}{1-\gamma(1-\mu)}(\varepsilon^*_{nL}-\mu) \tag{29}$$

where $sign\ \varepsilon^*_{nL} = sign\ (1-\gamma)$.

Note that the sign of Ω is ambiguous

$$sign\ \Omega = sign\ \left(\left[\frac{1-\gamma}{\beta-\gamma}\right]-\mu\right)$$

in the Walrasian case ($\mu=0, \gamma<1$), $\Omega>0$ and the accumulation of firms leads to a *reduction* in bonds. The main mechanism here is that there is a positive effect of n on labor supply and output ($\Phi_{Ln} > 0$), so that having too few firms means that wages, labor income and home production are below

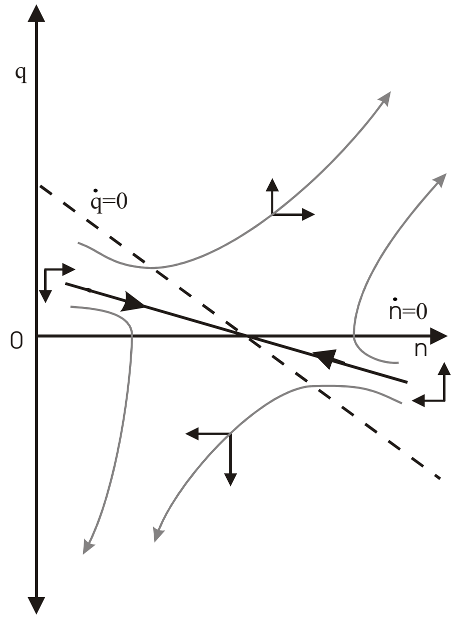

Figure 2: Phase Diagram in in $\{n, q\}$ space.

their steady state level. To maintain consumption, this low level of income is compensated by higher than steady state imports, financed by running down bonds[17]. However, given $\gamma < 1$, if μ is large enough then bonds will increase as firms are accumulated. This is because the level of profits along the path to equilibrium is large: whilst the number of firms is below equilibrium, the extra profits generated are enough to exceed the adjustment costs and lower wage. In addition, there is a capacity effect, so that productivity is higher whilst the number of firms is below equilibrium (for $\mu > 0$, free-entry leads to excessive number of firms in steady-state). In the case of $\gamma \geq 1$, the flow of entry leads to an increase in the stock of bonds: this is because n has a negative effect on wages and profits, so that n below its steady state implies income above the steady state. The phase diagram of the system in $\{n, q\}$ space is depicted in Figure 2. The downward sloping line represents the combinations of $\{n, q\}$ for which $\dot{q} = 0$ and the arbitrage condition is satisfied $\pi = qr$. Above the $\dot{q} = 0$ line, the arbitrage condition implies that $\dot{q} > 0$; below it implies $\dot{q} < 0$. The $\dot{n} = 0$ phase line corresponds to the $n-$axis, since $\dot{n} = 0$ whenever $q = 0$. The saddle-path is downward sloping between the horizontal axis and the arbitrage line. Note that from (28b) the growth (shrinkage) in the marginal cost of entry

[17] It is important to understand that an *increase* in firms per se makes wages higher. However, the number of firms is increasing because it is below the steady-state. The stock of bonds decreases not because the accumulation of firms lowers income, but because entry implies that the initial level of n was low in the first place.

q is given in absolute terms by the stable eigenvalue

$$\left|\frac{\dot{q}}{q}\right| = \Gamma$$

with the sign being determined by whether profits are positive (firms are being accumulated or negative (decumulated).

The linearized dynamics gives an explicit solution for steady state bonds as a function of λ, A and the initial condition n_0.

$$b^* = b(\lambda^*, A) = b_0 - \frac{\Omega}{\Gamma - r}(n_0 - n(\lambda^*, A)) \tag{30}$$

where *sign* $b_\lambda =$ *sign* Ω. We can now rewrite the SS condition for bonds (23c) as a function of λ only

$$w\mathcal{L}(\lambda^*, A) + rb(\lambda^*, A) - C(\lambda^*) - G = 0 \tag{31}$$

Hence we now have three equations (23a,23b,31) to determine the three variables $\{\lambda, n, L\}$ in steady state.

The following is a useful result for what follows.

Lemma $w\mathcal{L}_\lambda + rb_\lambda - C_\lambda > 0$

Similarly to the case analyzed in the previous section, we can interpret the equation (31) as a steady state market clearing condition: the LHS is the excess of income over expenditure. The Lemma shows that this is strictly monotonic in λ. Hence, if a steady-state exists it is a *unique* steady state solution *for* λ^*. Existence follows from the functional forms for $\mathcal{L}(\lambda^*, A)$ and $C(\lambda^*)$,.and the fact that b^* is bounded in (30) since n^* is bounded[18]. When λ is close to zero, L is very small and C is very large, with C unbounded as $\lambda \to 0$: hence there expenditure exceeds income. When λ is very large, C is very small and L is close to 1, so that there is an excess of income over expenditure. Hence for some intermediate value of λ (31) is satisfied.

5 Technological change and labor productivity

We will now consider the effect of a change in technology on the economy. In order to properly understand this, we need to introduce a national income accounting framework. We define total consumption Y to consist of private and public consumption, and classify the expenditure incurred insetting up new firms as investment, $I = \nu E^2$.

- *Gross domestic product*: $GDP = \Phi(L, n, A)$
- *Gross National product*: $GNP = \Phi + rb - \dot{b}$.

[18] n^* is proportional to L^*, which lies in $[0, 1]$.

- *Total Consumption*: $Y = C + G$.

These measures are clearly related: since we are considering a small open economy, all of these measures capture different aspects of the behavior of the economy. In steady state $Y = GNP = GDP + rb$.

We will focus on the fundamental distinction between *measured* labor productivity $\mathcal{P}$ (GDP per unit labor) and the *underlying technology* measured by the long-run or steady-state average product of labor $\mathcal{P}^*$ corresponding to technological parameters $\{A, \gamma, F\}$. The underlying technology is independent of the precise state of the economy: and representing this by $\mathcal{P}^*$ ensures that measured and underlying technology are in the same units and equal in steady-state. Whilst we could explore changes in any of $\{A, \gamma, F\}$, we choose to focus on changes in A, which is directly related to $\mathcal{P}^*$ from (15) and is the variable that directly alters the marginal productivity of labor. Throughout this section, since we are concentrating on the technology, we assume for simplicity that $G = 0$: there is neither government expenditure nor taxation.

5.1 The long run effects of a technological change

To study the long run effect of a permanent unanticipated change in the technology parameter A, note that from the SS bond equation (31) we have

$$\frac{d\lambda^*}{dA} = \frac{-(w_A \mathcal{L} + rb_A)}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} = -\frac{\lambda^*}{A} \frac{\frac{\beta}{\gamma}(w\mathcal{L}_\lambda + rb_\lambda)}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} < 0 \tag{32}$$

where $w_A > 0$ is the derivative of the steady state wage (15) *w.r.t.* A. As we would expect, an increase in productivity reduces the steady state marginal utility: technological progress raises real income and the real wage in the long-run.

At the firm level, the specification of the technology means that whilst the free-entry firm output (13) is unaffected, the firm's free-entry employment (12) is *reduced* by technological improvement. First we will consider the long-run effect of a permanent technological improvement.

Proposition 1 *A permanent improvement in technology: the long-run effects.*
(a) $\frac{dC^*}{dA} > 0$; $\frac{dn^*}{dA} > 0$; $\frac{dw^*}{dA} > 0$
(b) $sign\ \frac{dL^*}{dA} = sign\left[b_0 - \frac{\Omega}{\Gamma - r} n_0\right]$
(c) $\frac{d\Phi^*}{dA} > 0$ *if* $\frac{dL^*}{dA} > 0$
(d) $\frac{d\mathcal{P}^*}{dA} > 0$

The effect on the labor supply is ambiguous because there is a conflict of income and substitution effects: the higher wage causes a substitution effect for less leisure and more consumption; the income effect is for more leisure. Which effect dominates depends on the level of initial wealth: $b_0 - \frac{\Omega}{\Gamma - r} n_0$ is the initial value of wealth in terms of bonds ($-\frac{\Omega}{\Gamma - r} n_0$ is the present value of the bonds that would have been decumulated/accumulated if $n_0 = 0$). If $\Omega > 0$, (i.e. $\gamma < 1$

and μ small enough) then a sufficient condition for $\frac{dL^*}{dA} > 0$ is that $b_0 \geq 0$: if $\Omega < 0$, (for which $\gamma \geq 1$ is sufficient) then a sufficient condition for $\frac{dL^*}{dA} < 0$ is $b_0 \leq 0$.

This ambiguity in the labor supply response may in principle carry over to GDP: whilst productivity tends to boost the output given employment, if employment falls enough it might lead to an overall reduction. Only if the labor supply increases can we be sure that GDP also increases. Note the contrast of part (a) and part (c): GNP always increases, irrespective of labor supply, with income and substitution effects working together to increase consumption.

5.2 Impact effect of technological change

In order to find the impact effect of technological change, we hold n fixed. The contrast between the impact and long-run effects depends on the effect of entry which has three elements. First there is the direct effect of increasing fixed cost nF. Second there is the effect of entry on lifetime income (the Ω-effect on the bond stock). Third, there is the capacity utilization effect: on impact GDP varies with employment, so that the changes in capacity utilization will induce changes in productivity.

If we first consider the impact effect on employment, this like the long-run effect, is ambiguous for the same reason (income and substitution effects may clash). However, if we look at the difference between the impact and long-run effect, this turns out to depend on whether there is an increasing or diminishing marginal product of labor. When $\gamma < 1$, on impact there is a negative relationship between the real wage and employment; when $\gamma > 1$ a positive relation; when $\gamma = 1$ no relation. We can thus get undershooting of employment ($\gamma > 1$) or overshooting ($\gamma < 1$) on impact depending on whether entry increases or decreases the marginal product (see Proposition 2 a,c). The change in GDP is also ambiguous on impact for the same reason, and there is no obvious ranking of impact and long-run effects.

Proposition 2 *Impact versus long-term effects of technological change on employment, output and wages.*

(a) $$sign\left[\frac{dL(\infty)}{dA} - \frac{dL(0)}{dA}\right] = sign\ L_n = sign\left[\gamma - 1\right].$$

(b) $$\frac{d\Phi(0)}{dA} = \Phi_A + \Phi_L \frac{dL(0)}{dA}$$

$$\frac{d\Phi(\infty)}{dA} - \frac{d\Phi(0)}{dA} = \Phi_L \frac{dL(\infty)}{dA}\left[1 - \mu - \frac{dL(0)/dA}{dL(\infty)/dA}\right]$$

(c) $$\frac{dw(0)}{dA} = (1-\mu)\Phi_{LL}\frac{dL(0)}{dA} + \frac{w^*}{A\gamma}$$

$$sign\left[\frac{dw(0)}{dA} - \frac{dw(\infty)}{dA}\right] = sign\left[\gamma - 1\right].$$

Lastly, we turn to the impact effect of the technological improvement on measured productivity $\mathcal{P}$. First there is the pure technology effect: with fixed employment and labor supply, an increase in technology boosts output. Secondly there is the *capacity utilization effect*, the impact of changes in employment and output per firm. The *capacity utilization effect* is in general ambiguous in sign because $dL(0)/dA$ can take either sign. When employment increases on impact, we have *overshooting*, so that the short run impact of the technological change exceeds the long-run effect. When employment decreases on impact, we have*undershooting* of underlying measured productivity. A direct comparison of the impact and long-run effect indicates that:

Proposition 3 *Impact versus long-run productivity effects.*

$$\frac{d\mathcal{P}(0)}{dA} - \frac{d\mathcal{P}(\infty)}{dA} = \frac{\mu}{1-\mu}\mathcal{P}^* \frac{dL(0)}{dA}$$

If the impact effect on employment is positive, then productivity overshoots its long-run value, if employment falls on impact, it undershoots. The interesting thing to note is that *this result holds irrespective of whether the marginal product of labor is increasing or decreasing.* This is easy to understand on further reflection. If we start from a free-entry equilibrium, then AC is decreasing (since $P = AC > MC$) for all values of γ. An marginal improvement in technology will still leave the AC curve downward sloping at the existing level of employment. Hence, if employment *falls* on impact, then you move back up the AC curve thus tending to increase productivity; if employment rises on impact, then you move down the AC curve and increase measured productivity. *Hence, it is the fact that there are locally increasing returns to employment that drives the simple relationship between employment and productivity independently of the technology parameter γ.* The degree of increasing returns in free-entry equilibrium is determined by the degree of imperfect competition.

The fact that capacity utilization causes endogenous productivity dynamics is important, since it implies that the time profile of measured productivity will differ from, and may tend to mask, the true changes in underlying technology. For example, if there is productivity overshooting, then a permanent change in productivity leads to an exaggerated instantaneous impact, that dies away to the permanent change. On the other hand, if there is a capacity effect which tends to reduce measured productivity at first, the measured technology increase will adjust more slowly up to the full effect. The deviations of measured productivity form technological change become small when there is near-perfect competition, and in the limiting Walrasian economy they disappear. This indicates that it may be misleading to use measured productivity as a guide to technological change in the short-run unless the economy has almost perfect competition in the output market.

We show the two cases of measured productivity undershooting and overshooting in figure 3. In both cases, we have a permanent step change in technology with A rising to A_1 at time T, so that the corresponding steady states are $\mathcal{P}^*$ and $\mathcal{P}_1^*$ respectively.

With overshooting the capacity effect reinforces the technology effect, leading to an increase in measured productivity that initially exaggerates the underlying increase. With undershooting, the capacity effect counteracts the technology improvement, leading to a gradual increase in measured productivity towards

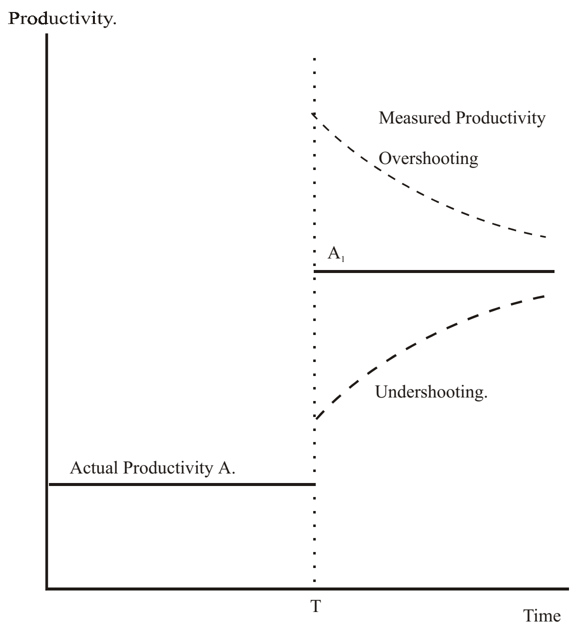

Figure 3: Undershooting and Overshooting of Measured Productivity.

the long-term increase. This reinforces the notion that we have to take into account capacity utilization when we are using productivity measures to understand technological change.

Burnside and Eichenbaum (1996, p.1169) examined the response to a technology shock in the presence of variable factor utilization (factor hoarding). They found that there would be overshooting: the initial shock in technology is amplified by the fact that capital utilization and labor effort both increase in response to a positive technology shock. The mechanism differs: in our setup, the intensity of factor use is fixed, so that the mechanism is one of variation in *capacity* utilization, movements down the firm's average cost function rather than *capital* utilization.

5.3 Anticipated technological change

Lastly, we will briefly consider the impact of an *anticipated* permanent technological change from A to A_1 at T. When the technological advance becomes known, there is a jump in the flow of entry in anticipation of the additional profits to be made. In terms of the phase diagram, the economy follows an unstable path in terms of the initial steady state with the flow of entry increasing through time. When the new technology comes on line, the flow of entry is at its peak: thereafter it follows the new saddlepath to the new steady state.

The time profile of productivity is depicted in Figure 4. In the initial

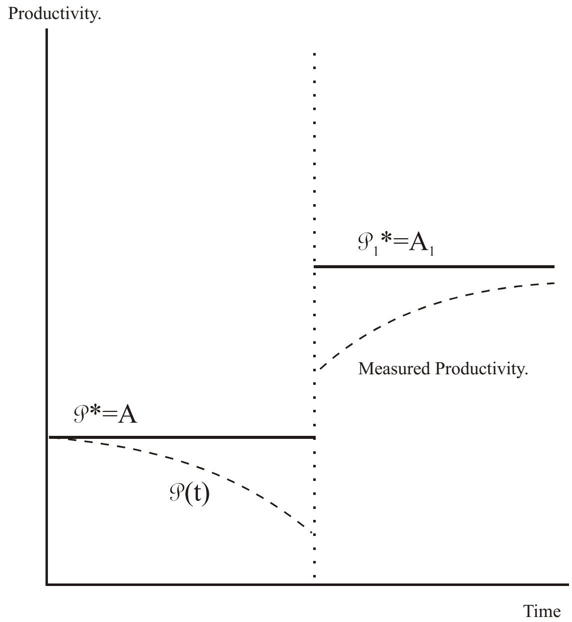

Figure 4: An Anticipated Improvement in Technology.

announcement phase prior to the change, productivity is falling: it then jumps up when the change occurs and gradually converges to the new state. We have depicted the case where there is undershooting, when the initial jump takes it to a point below the new steady state: it is of course possible for overshooting to occur.

6 Fiscal Demand shock

We will consider first a demand shock in terms of a tax financed permanent and *unanticipated* change in government expenditure G. Since we are focussing on measured productivity, we will restrict our attention here primarily to GDP and labor supply. In the following Propositions we track the behavior of these variables, which we then proceed to interpret in terms of measured productivity changes. Throughout this section, the underlying technology is unchanged so that all variations in productivity are due to capacity utilization effects.

Proposition 4 *Long-run multipliers for government expenditure.*

(a) $GDP \quad 1 > \frac{d\Phi^*}{dG} > 0$

(b) $\frac{dL^*}{dG} > 0;\ sign\ \frac{db^*}{dG} = -\ sign\ \Omega.$

The increase in government expenditure has two distinct effects on steady state consumption. First, there is the standard *resource withdrawal effect*: for a given level of steady-state employment, steady-state consumption is reduced by dG. Secondly, there is the *bond effect*: the increase in output and the number of firms causes a reduction (increase) in steady-state bonds when $\Omega < 0$ ($\Omega > 0$), since bond decumulation occurs along the path to the new steady state, represented by the further downward shift in the $LRBC$ by $r(db^*/dG)$.

The instantaneous effects of an increase in G differ from the long-run effects since the number of firms is at its initial value. To analyze this, we use (22) which defines the labor supply conditional on n.

Proposition 5 *Impact compared to long-run multipliers for* $\mu > 0$.
(a) $\frac{dL(\infty)}{dG} > 0,\ \frac{dL(0)}{dG} > 0$
$sign\left[\frac{dL(0)}{dG} - \frac{dL(\infty)}{dG}\right] = sign\,[\gamma - 1]$
(b) $\frac{dw(\infty)}{dG} = 0,\ \ sign\frac{dw(0)}{dG} = sign\,[\gamma - 1]$

The increase in taxes G causes an increase in labor supply and this leads to a reduction in the wage when $\gamma < 1$ and an increase if $\gamma > 1$. In the long run this is reversed as the economy moves to the free entry equilibrium and corresponding real wage w^*. The initial number of firms is below the new steady state. Firms are then accumulated which has the effect of stimulating the labor supply if $\gamma < 1$, or reducing it if $\gamma > 1$.

GDP jumps initially in response to the jump in the labor supply. In the long-run there are two effects at work to modify this: as firms are accumulated it changes the real wage and alters labor supply (increasing it when $\gamma < 1$, reducing it when $\gamma > 1$), whilst the additional firms reduce output due to the additional fixed costs (unless $\mu = 0$). When $\gamma \geq 1$, both these effects work in the same direction, reducing GDP as firms are accumulated. If $\gamma < 1$ and there is a low level of imperfect competition, the first effect will dominate and the long-run multiplier is bigger than the impact. If the markup is large enough the long-run multiplier will be smaller. The Walrasian case is a useful benchmark. When $\mu = 0$, or close to 0, we have the following:

Proposition 6 *GDP impact and long-run effects.*
(a) Let $\gamma < 1$. *There exists* $\bar{\mu} > 0$ *such that for* $\mu < \bar{\mu}$,
$\frac{d\Phi(0)}{dG} < \frac{d\Phi(\infty)}{dG}$.
(b) Let $\gamma \geq 1$. $\frac{d\Phi(0)}{dG} > \frac{d\Phi(\infty)}{dG}$

We are now in a position to examine the effect of an increase in demand on measured productivity $\mathcal{P}$. Before the increase in G, the industry is in free entry equilibrium and operating at normal capacity $y = y^*$, with productivity equal to the real wage, $\mathcal{P}^* = w^*$. The impact effect of an increase in demand is to increase output per firm.

Proposition 7 *Productivity and capacity utilization.*

(a) If $\mu = 0$ *then* $\frac{d\mathcal{P}(0)}{dG} = \frac{d\mathcal{P}(\infty)}{dG} = 0$
(b) If $\mu > 0$ *then* $\frac{d\mathcal{P}(0)}{dG} > \frac{d\mathcal{P}(\infty)}{dG} = 0$

When $\mu > 0$, the effect of an increase in government expenditure is to produce a transitory increase in the average product of labor, i.e. productivity. The long run average product is equal to the steady state real wage which is of course unaffected by changes in government expenditure. The productivity dynamic induced by demand is endogenous. The size of this transitory effect is zero in the Walrasian case of $\mu = 0$, but is increasing in μ and proportional to the growth in employment when $\mu > 0$. *This is exactly as found by Ryan (2000), that the relationship between capacity utilization and productivity is stronger when there is more imperfect competition.*

The behavior of productivity is best understood in terms of the change in *capacity utilization* caused by the change in employment. We can represent the technology in terms of the cost function as in Fig.4, with output on the horizontal axis and cost on the vertical axis: we depict the marginal and average cost functions on the assumption of diminishing marginal productivity of labor to yield the traditional U-shaped average cost function.

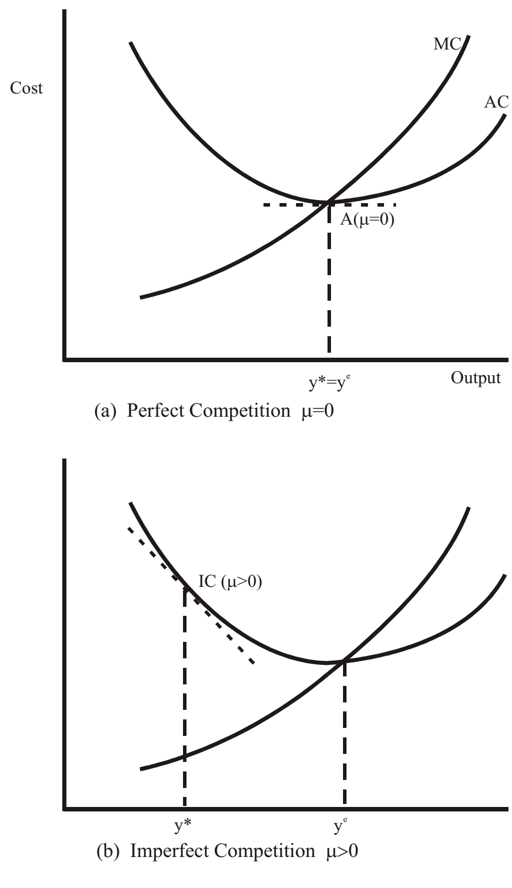

Figure 5: Capacity Utilization and productivity.

In Fig.5(a) we depict the equilibrium in the Walrasian case: the zero-profit long-run equilibrium output per firm is the technically efficient level y^e. Average cost equals marginal cost, so that average cost is flat, there being no first-order effects of capacity utilization on productivity (there is constant returns to scale in the neighborhood of the equilibrium). However, in Figure 5(b) we depict the long-run equilibrium when $\mu > 0$. In this case, we have the standard Robinson-Chamberlin excess capacity in long-run equilibrium. The AC curve is downward sloping, there being increasing returns in the neighborhood of the equilibrium. As output increases, capacity utilization increases towards the efficient level and there is a resultant increase in productivity.

7 Concluding remarks

In this paper we have developed a dynamic model of entry in the context of a simple yet standard open economy macromodel. We have shown how the dynamics of entry is crucial to understanding the behavior of measured productivity in the response to both demand and technology changes. The crucial insight is that variations in output per firm, capacity utilization, has an important role in the short-run. We have developed this insight in an integrated and consistent manner to explore both the long-run and short run effects of changes in demand and technology, both when unanticipated and when anticipated.

One of the most important implications of the paper is that using actual productivity measures to capture technology is only valid in general under one of the following cases:

- There is perfect competition.
- There are constant returns to scale at the firm level.
- There is instantaneous free entry.

If these case do not apply, then the message of this paper is that we need model endogenous productivity effects due to capacity utilization. It is our belief that none of these cases is generally applicable.

8 References

- Ambler S, Cardia E (1998). The cyclical behavior of wages and profits under imperfect competition, *Canadian journal of economics*, 31, 148-164.
- Basu S (1996). Procyclical productivity: Increasing returns or cyclical utilization? *Quarterly journal of economics,* 111, 719-751.
- Basu S and Fernald J (1997). Returns to scale in US production: estimates and implications. Journal of political economy, 105, 249-283.
- Berndt E and Fuss M (1986). Productivity measurement with adjustments for variations in capacity utilization and other forms of temporary equilibrium, *Journal of econometrics*, 33, 7-29.
- Brito P and Dixon H.D. (2000). The dynamics of entry and capital accumulation in a closed economy. Mimeo, University of York.
- Brock, P. L. and S. J.Turnovsky, 1994, The dependent economy model with both traded and nontraded capital goods, *Review of International Economics* 2, 306-325.
- Burnside C, Eichenbaum M. (1996). Factor-hoarding and the propagation of business-cycle shocks, *American Economic Review*, 86, 1154-1174.
- Campbell JR (1997). Entry, Exit, Embodied technology, and business cycles. *Review of economic dynamics*, 1, 371-408.
- Cassel G. (1937). Excess Capacity and Monopolistic competition, *Quarterly Journal of Economics*, 51, 426-443.
- Caves D, Christensen L, Swanson J (1981). Productivity growth, scale economies and capacity utilization in the U.S. railroads 1955-1974, *American economic review*, 71, 994-1002.
- Cook D (2001). Time to enter and business cycles, *Journal of Economic Dynamics and Control*, 25, 1241-1261.
- Coto, J. and Dixon H.D. (1999). Profits, Markups and Entry: Fiscal Policy in an Open Economy, *Journal of Economic Dynamics and Control*, forthcoming.
- Das S and Das S. (1996). Dynamics of entry and exit of firms in the presence of entry adjustment costs. *International journal of industrial organization*, 15, 217-241.
- Datta B and Dixon HD (2002). Technological change, entry and stock market dynamics: an analysis of transition in a monopolisitic industry. *American economic review*, forthcoming.

- Devereux M, Head A, Lapham B (1996). Monopolistic competition, increasing returns, and the effects of government spending, *Journal of money credit and banking*, 28, 233-254.
- Dixon H.D. and Lawler P. (1996). Imperfect competition and the Fiscal Multiplier, *Scandinavian journal of Economics*, 98, 219-231.
- Dixit and Pindyk (1994). *Investment under uncertainty*, Princeton University Press, NJ.
- Ericson R and Pakes A. (1995). Markov-perfect industry dynamics: a framework for empirical work, *Review of economic studies*, 62, 53-82.
- Greenwood J, Hercowitz Z, Huffman G (1988). Investment, Capacity Utilization and the Real Business cycle, *American economic review*, 78, 402-417.
- Hall, R.E. (1986). Market structure and macroeconomic activity, *Brookings papers on economic activity*, No. 2, 285-322.
- Hall, R.E. (1990). Invariance properties of Solow's productivity residual, Diamond P (ed.) *Growth productivity and unemployment*, Cambridge: MIT press, chapter 5, 71-112.
- Hopenhayn H. (1992). Entry, Exit, and firm Dynamics in Long Run Equilibrium, *Econometrica*, 60, 1127-1150.
- Hornstein A (1993). Monopolistic competition, increasing returns to scale, and the importance of productivity shocks, *Journal of monetary economics*, 31, 299-316.
- Hulten CR (1986). Productivity change, capacity utilization and the sources of efficiency growth, *Journal of econometrics*, 33, 31-50.
- King R and Rebelo S. (1999). Handbook of Macroeconomics, Volume 1B, chapter 14. North-Holland: Amsterdam.
- Klein L. (1960). Some theoretical issues in the measurement of capacity, *Econometrica*, 28, 272-286.
- Morrison C.J. (1986). Productivity measurement with non-static expectations and varying capacity utilization: an integrated approach. *Journal of econometrics*, 33, 51-74.
- Morrison CJ (1997). Economic performance, cost economies and pricing behaviour in the US and Australian meat products industries, *Australian journal of agricultural and resource economics*, 41, 361-383.
- Morrison CJ (1992). Unraveling the productivity growth slowdown in the United-States, Canada and Japan - the effects of subequilibrium, scale economies and markups, *Review of economics and statistics*, 74, 381-393.

- Morrison CJ (1993). *A Microeconomic approach to the measurement of economic performance: productivity growth, capacity utilization and related performance indicators.* New York: Springer Verlag.
- Nelson R (1989). On the measurement of capacity utilization, *Journal of industrial economics*, 37, 273-286.
- Portier F. (1995). Business formation and cyclical markups in the French business cycle, *Annales d'economie et de statistique*, 37/38, 411-440.
- Ryan DJ (2000). Cyclical behaviour of productivity growth and price-cost margins: asymmetry and market power effects, *Applied economics letters*, 7,297-300.
- Rotemberg J, Summers L (1990). Inflexible prices and procyclical productivity, *Quarterly journal of economics*,105, 851-874.
- Rotemberg, J. and M. Woodford (1995), Dynamic general equilibrium models with imperfectly competitive product markets, in: F. T. Cooley, ed., *Frontiers of Business Cycle Research*, (Princeton University Press, Princeton) 243-293.
- Turnovsky, S.J., 1997, *International macroeconomic dynamics* (MIT press, Cambridge, Massachusetts).
- Winston G. (1974). The theory of capital utilization and idleness, *Journal of economic literature*, 12, 1301-1320.
- Zegeye A and Rosenblum L (2000). Measuring productivity in an imperfect world, *Applied Economics*, 32, 91-106.

9 Appendix: Proofs

9.1 Proof of Lemma

Proof. Since $C_\lambda < 0$, it suffices to show that $w\mathcal{L}_\lambda + rb_\lambda > 0$.

$$w\mathcal{L}_\lambda + rb_\lambda = \Phi_L \mathcal{L}_\lambda \left[\frac{\Gamma(1-\mu) - r(\frac{\beta-1}{\beta-\gamma})}{\Gamma - r} \right] > 0$$

■

9.2 Proof of Proposition 1

Proof. (a)

$$\begin{aligned}
\frac{dC^*}{dA} &= C_\lambda \frac{d\lambda^*}{dA} = \frac{\beta}{\lambda^* A\gamma} \frac{(w\mathcal{L}_\lambda + rb_\lambda)}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} > 0 \\
\frac{dn^*}{dA} &= \left[n_A + n_\lambda \frac{d\lambda^*}{dA} \right] = n_A \left(\frac{-C_\lambda}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} \right) > 0 \\
\frac{dw^*}{dA} &= \frac{w^*}{\gamma A} > 0.
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{dL^*}{dA} &= \mathcal{L}_A \left(1 - \frac{\beta \left(w\mathcal{L}_\lambda + rb_\lambda \right)}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} \right) \\
&= \frac{\mathcal{L}_A}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} \left((1-\beta) \left(w\mathcal{L}_\lambda + rb_\lambda \right) - C_\lambda \right) \\
&= \frac{\mathcal{L}_A}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} \frac{1}{\lambda^*} \left(C - wL - r\frac{\Omega}{\Gamma - r} n \right)
\end{aligned}$$

However, since in steady state we have

$$C - wL = rb$$

$$\frac{dL^*}{dA} = \frac{\mathcal{L}_A}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} \frac{1}{\lambda^*} \left(rb - r\frac{\Omega}{\Gamma - r} n \right)$$

From (28c) $b - \frac{\Omega}{\Gamma - r} n = b_0 - \frac{\Omega}{\Gamma - r} n_0$

$$\frac{dL^*}{dA} = \frac{\mathcal{L}_A}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} \frac{r}{\lambda^*} \left(b_0 - \frac{\Omega}{\Gamma - r} n_0 \right)$$

Clearly, since $n_0 > 0$, if $\Omega > 0$, then the term in brackets is positive if initial bonds holdings are non-negative, so that employment must rise. Likewise if $\Omega < 0$, $b_0 \leq 0$ ensures that employment falls.

(c) From the zero-profit equilibrium expression for Φ (17), productivity has a direct effect on GDP, Φ_A, and an indirect effect on labor supply via λ and A:

$$\frac{d\Phi^*}{dA} = \frac{\Phi^*}{\gamma A} + w^* \frac{dL^*}{dA}$$

The first term is clearly positive, so an increase in the labor supply is sufficient for GDP to increase. However, if there is a reduction in the labor supply *and* this is sufficient to outweigh the positive effect then the overall effect might be negative.

(d) From (15)

$$\frac{d\mathcal{P}^*}{dA} = \frac{1}{\gamma}\frac{\mathcal{P}^*}{A} > 0$$

■

9.3 Proof of Proposition 2

Proof. (a) Totally differentiating $L = L(\lambda, n, A)$ keeping n fixed yields

$$\begin{aligned} \frac{dL(0)}{dA} &= L_\lambda \frac{d\lambda}{dA} + L_A \\ &= -L_A \left[\frac{(\beta - \gamma)(w\mathcal{L}_\lambda + rb_\lambda) - \gamma C_\lambda}{\gamma\left(w\mathcal{L}_\lambda + rb_\lambda - C_\lambda\right)} \right] \end{aligned}$$

As in the long-run case, the income and substitution effects of a technological improvement work in opposite directions.

The difference between the long-run and impact multiplier is accounted for by the effect of entry, so that

$$\frac{dL(\infty)}{dA} - \frac{dL(0)}{dA} = L_n n_\lambda \frac{d\lambda}{dA} + L_n n_A = L_n n_A \left[\frac{-C_\lambda}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} \right]$$

$$sign \left[\frac{dL(\infty)}{dA} - \frac{dL(0)}{dA} \right] = sign\ L_n = sign\ [1 - \gamma]$$

In the case of diminishing MPL ($\gamma < 1$), the short-run response of employment is less than the long-run response; the opposite is true with increasing MPL.

(b) The impact effect follows directly from (7) holding n constant. The long-run effect from the fact that n is given by (16).

(c)

$$\frac{dw(0)}{dA} = (1 - \mu)\Phi_{LL} \frac{dL(0)}{dA} + \frac{w^*}{A\gamma}$$

Hence

$$\begin{aligned} \frac{dw(0)}{dA} - \frac{dw(\infty)}{dA} &= (1-\mu)\Phi_{LL}\frac{dL(0)}{dA} \\ sign\left[\frac{dw(0)}{dA} - \frac{dw(\infty)}{dA}\right] &= sign\ [\gamma - 1] \end{aligned}$$

The difference between the long-run and short run wage effect depends on whether an increase in employment increases the MPL ($\gamma > 1, \Phi_{LL} > 0$), or decreases it ($\gamma < 1$, $\Phi_{LL} < 0$). ■

9.4 Proof of Proposition 3

Proof.

$$\begin{aligned} \frac{d\mathcal{P}(0)}{dA} &= (1-\mu)\left(\frac{n}{L}\right)^{(1-\gamma)} + (\gamma-1)\frac{A}{L}\left[\frac{n}{L}\right]^{1-\gamma}\frac{dL(0)}{dA} + \left[\frac{n}{L}\right]\frac{F}{L}\frac{dL(0)}{dA} \\ &= \frac{\mathcal{P}^*}{\gamma A} + \frac{\mu}{1-\mu}\mathcal{P}^*\frac{dL(0)}{dA} \\ &= \frac{d\mathcal{P}^*}{dA} + \frac{\mu}{1-\mu}\mathcal{P}^*\frac{dL(0)}{dA} \end{aligned}$$

■

9.5 Proof of Proposition 4

Proof. From the steady state market clearing condition

$$G + C(\lambda^*, A) - w\mathcal{L}(\lambda^*, A) - rb(\lambda^*, A) = 0$$

hence from the Lemma

$$\frac{d\lambda^*}{dG} = [w\mathcal{L}_\lambda + rb_\lambda - C_\lambda]^{-1} > 0$$

(a) For GDP the multiplier is

$$\begin{aligned} \Phi^* &= \mathcal{Y}(\lambda^*, A) \\ &= \Phi\left(\mathcal{L}(\lambda^*, A), \left(\frac{n}{L}\right)^* \mathcal{L}(\lambda^*, A)\right) \\ \frac{d\Phi}{dG} &= \left[\Phi_L + \Phi_n\left(\frac{n}{L}\right)^*\right]\mathcal{L}_\lambda\frac{d\lambda^*}{dG} \\ &= \frac{w\mathcal{L}_\lambda}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} \end{aligned}$$

(b)

$$\begin{aligned} \frac{dL^*}{dG} &= \mathcal{L}_\lambda.\frac{d\lambda^*}{dG} > 0 \\ \frac{db^*}{dG} &= \frac{\Omega}{\Gamma - r}\left(\frac{n}{L}\right)^* \mathcal{L}_\lambda\frac{d\lambda^*}{dG} \end{aligned}$$

■

9.6 Proof of Proposition 5

Proof. (a) The impact effect of fiscal policy is differentiated form the long-run effect by the fact that the number of firms is unchanged. Hence for labor supply

$$\begin{aligned}
\frac{dL(0)}{dG} &= L_\lambda(n,\lambda,A)\frac{d\lambda^*}{dG} > 0 \\
\frac{dL(\infty)}{dG} &= L_\lambda(n,\lambda,A)\frac{d\lambda^*}{dG} + L_n\frac{dn}{d\lambda}\frac{d\lambda^*}{dG} \\
&= \mathcal{L}_\lambda\frac{d\lambda^*}{dG} > 0 \\
sign\ \left[\frac{dL(\infty)}{dG} - \frac{dL(0)}{dG}\right] &= sign\ \left[L_n\frac{dn}{d\lambda}\frac{d\lambda^*}{dG}\right] = sign\ \ [1-\gamma]
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{dw(0)}{dG} &= \Phi_{LL}L_\lambda(n,\lambda)\frac{d\lambda^*}{dG} \\
sign\,[\Phi_{LL}] &= sign\,[\gamma-1]
\end{aligned}$$

■

9.7 Proof of Proposition 6

Proof.

$$\begin{aligned}
\frac{d\Phi(0)}{dG} &= \Phi_L\frac{dL(0)}{dG} = \frac{w^*}{1-\mu}\frac{dL(0)}{dG} > 0 \\
\frac{d\Phi(\infty)}{dG} &= \Phi_L\frac{dL(\infty)}{dG} + \Phi_n\left(\frac{n}{L}\right)^*\frac{dL(\infty)}{dG} \\
&= w^*\frac{dL(\infty)}{dG} \\
\frac{d\Phi(\infty)}{dG} - \frac{d\Phi(0)}{dG} &= w^*\left[1 - \frac{1}{1-\mu}\frac{dL(0)/dG}{dL(\infty)/dG}\right]
\end{aligned}$$

(a) When $\mu = 0$,

$$\frac{d\Phi(\infty)}{dG} - \frac{d\Phi(0)}{dG} = w^*\frac{dL(\infty)}{dG}\left[\frac{dL(\infty)}{dG} - \frac{dL(0)}{dG}\right] > 0.$$

By continuity, there exists $\bar{\mu} > 0$ such that for $\mu < \bar{\mu}$ the inequality is maintained.

(b) when $\gamma \geq 1$, $\frac{dL(0)/dG}{dL(\infty)/dG} \geq 1$. ■

9.8 Proof of Proposition 7.

Proof. From the definition of productivity, we have

$$\frac{d\mathcal{P}}{dL} = \frac{d(\Phi/L)}{dL} = \frac{L\Phi_L - \Phi}{L^2} = \frac{\mu\Phi_L}{L} = \frac{\mu}{1-\mu}\frac{w^*}{L}$$

Hence

$$\frac{d\mathcal{P}(0)}{dG} = \frac{d\mathcal{P}}{dL}\frac{dL}{d\lambda}\frac{d\lambda^*}{dG} = \frac{\mu}{(1-\mu)}\frac{w^*}{L}L_\lambda(n,\lambda)\frac{d\lambda^*}{dG}$$

This is strictly positive if $\mu > 0$, zero if $\mu = 0$. Since $\Phi(L, n)$ is homogeneous of degree 1 in (L, n) and the ratio L/n fixed by free entry, we have $\frac{d\mathcal{P}(\infty)}{dG} = 0$. ■

10 Analysis of anticipated changes

The analysis here follows Datta and Dixon (2002) and uses standard techniques (e.g. Turnovsky 1997, pages 94-98). We will therefore just sketch the solution method taking the case of an anticipated step change, announced at time 0 to occur at time T. Once the change has occurred, the economy will follow the saddle-path to the new steady state. The initial (pre-announcement) stock of firms is in a steady state: we denote the initial steady state stock of firms as n_1. The eventual post-announcement steady state number of firms is n_2. Note that in this model, the eigenvalues depend on the steady state number of firms (26,27): the negative eigenvalues corresponding steady states are denoted Γ_i, $i = 1, 2$: the positive eigenvalue for the initial steady state is denoted Γ_1^+. Of course, the steady-state $q_i = 0$.

First, we describe the path over the initial phase over $t \in [0, T]$. When the "announcement" is made, before the actual improvement in technology, the economy follows an unstable path relative to the initial equilibrium. Since the pre-announcement the economy is in equilibrium, $n(0) = n_1$, $b(0) = b_1$. Hence

$$\begin{aligned}
n(t) &= n_1 + A_1 e^{\Gamma_1 t} + A_2 e^{\Gamma_1^+ t} && \text{(33a)}\\
q(t) &= A_1 \nu \Gamma_1 e^{\Gamma_1 t} + A_2 \nu \Gamma_1^+ e^{\Gamma_1^+ t} && \text{(33b)}\\
b(t) &= b_1 + \frac{\Omega_1}{\Gamma_1 - r} A_1 e^{\Gamma_1 t} + \frac{\Omega_1}{\Gamma_1^+ - r} A_2 e^{\Gamma_1^+ t} && \text{(33c)}\\
&\quad - \left[\frac{\Omega_1}{\Gamma_1 - r}A_1 + \frac{\Omega_1}{\Gamma_1^+ - r}A_2\right] e^{rt}
\end{aligned}$$

where Ω_1 is as in (29) evaluated at n_1. Note that setting $t = 0$, we have $A_1 = A_2$.

After the technological improvement occurs, $t \in [T, \infty)$, the economy follows a stable path to the new steady state

$$\begin{aligned}
n(t) &= n_2 + A_1' e^{\Gamma_2 t} && \text{(34a)}\\
q(t) &= A_1' \nu \Gamma_2 e^{\Gamma_2 t} && \text{(34b)}\\
b(t) &= b_2 + \frac{\Omega_2}{\Gamma_2 - r} A_1' e^{\Gamma_2 t} && \text{(34c)}
\end{aligned}$$

Since both $\{n, q\}$ are continuous at T, we have the two equalities in two unknowns $\{A_1, A_1'\}$.

$$A_1\Gamma_1 e^{\Gamma_1 T} - A_1\Gamma_1^+ e^{\Gamma_1^+ T} - A_1'\Gamma_2 e^{\Gamma_2 T} = 0$$
$$A_1 e^{\Gamma_1 T} - A_1 e^{\Gamma_1^+ T} - A_1' e^{\Gamma_2 T} = n_2 - n_1$$

Now, simple substitution determines $\{A_1, A_1'\}$ conditional on n_2.

$$\begin{aligned}
A_1' &= A_1\left[\frac{\Gamma_1 e^{\Gamma_1 T} - \Gamma_1^+ e^{\Gamma_1^+ T}}{\Gamma_2 e^{\Gamma_2 T}}\right] \\
A_1 &= (n_2 - n_1)\left[e^{\Gamma_1 T} - e^{\Gamma_1^+ T} - \left(\frac{\Gamma_1 e^{\Gamma_1 T} - \Gamma_1^+ e^{\Gamma_1^+ T}}{\Gamma_2}\right)\right]^{-1} \\
&= (n_2 - n_1)\left[\frac{\Gamma_2}{\Gamma_2\left(e^{\Gamma_1 T} - e^{\Gamma_1^+ T}\right) - \left(\Gamma_1 e^{\Gamma_1 T} - \Gamma_1^+ e^{\Gamma_1^+ T}\right)}\right] \\
&= (n_2 - n_1)\left[\frac{\Gamma_2}{e^{\Gamma_1 T}(\Gamma_2 - \Gamma_1) + e^{\Gamma_1^+ T}(\Gamma_1^+ - \Gamma_2)}\right] > 0
\end{aligned}$$

Since $n_2 - n_1 > 0$ implies $\Gamma_2 - \Gamma_1 > 0$. Furthermore, both $\{A_1, A_1'\}$ can be seen as continuous functions of n_2 for $n_2 \geq n_1$.

The new steady state number of firms is determined by λ and comes from the dynamics for $b(t)$. Recall that we know b_1: hence from (33c), $b(T)$ is

$$b(T) = b_1 + A_1\left[\frac{\Omega_1}{\Gamma_1 - r}e^{\Gamma_1 T} - \frac{\Omega_1}{\Gamma_1^+ - r}e^{\Gamma_1^+ T} - \left[\frac{\Omega_1}{\Gamma_1 - r} - \frac{\Omega_1}{\Gamma_1^+ - r}\right]e^{rT}\right] \quad (35)$$

where on the RHS of (35) $\{\Omega_1, \Gamma_1, \Gamma_1^+\}$ are known, only A_1 needs to be determined. Turning to b_2, from (34c), we have

$$b_2 = b(T) + \frac{\Omega_2}{\Gamma_2 - r}A_1' \quad (36)$$

where $\{\Omega_2, \Gamma_2\}$ are functions of n_2 and hence b_2.

The two equations (35,36) give us a relationship between $\{A_1, A_1', n_2\}$ and b_2. We thus have an additional equation to determine n_2, since (23c) gives us the level of λ_2 given b_2, and hence n_2. In effect, we can conceive of the following algorithm: assume an arbitrary level of n_2: this then ties down $\{\Omega_2, \Gamma_2, A_1, A_1'\}$: we can then from (36) determine b_2 and hence λ. If the implied level of n_2 equals the initial value, then we have found the equilibrium value and the full solution to the model.

Does such a solution exist? First, note that in effect we have a mapping from n_2 onto itself. The constants $\{\Omega_2, \Gamma_2, A_1, A_1'\}$ are continuous functions of n_2; b_2 is a continuous function of these variables and the known values $\{b_1, \Omega_1, \Gamma_1, \Gamma_1^+\}$ from (36), and n_2 is a continuous function of b_2 from (23c). Second, note that

n_2 belongs to a compact convex set: we have the lower bound n_1 and the upper bound $(L/n)^*$ (the number of firms in the economy when $L = 1$): $n_2 \in [n_1, (L/n)^*]$. Hence we have a continuous mapping of a compact convex set onto itself, which must posses a fixed point.