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Minority Programming in Television Markets

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Abstract

This paper studies how a private and a public television broadcaster choose their programming in different market configurations: a private monopoly, a public monopoly and a duopoly competition with one another. The two broadcasters differ in their sources of finance and in their objective functions. The private broadcaster is financed by selling advertising time and is a profit maximiser.

The public one is financed by both advertising and licence fee, and maximises a utilitarian welfare function.

We investigate the availability of programmes for small groups of consumers (minority programming). We find that a private monopolist does not supply any programme for these consumers; a public monopolist supplies programmes for all the consumers; and the effect of the competition with one another is to reduce the amount of minority programming with respect to the public monopoly scenario.

Keywords: Television; programme choice; minority programming; duopoly competition.

JEL Classification: L13, L82.

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1 Introduction

Markets for free television services are often charged of a bias in programme selection against small groups of audiences. The literature on programme choice[1] shows that advertising supported television services exhibit a bias in supplying different types of programmes and this bias is due to structural characteristics of the industry. Some models (Steiner (1952), Rothenberg (1962), Wiles (1963), and Beebe (1977)) show a strong tendency to duplication of those programmes which have large audiences (minority programming). Some others (Spence and Owen (1977), and Wildman and Owen (1985)) find that television markets show a bias against either programmes with small audiences or costly programmes. Most of the papers in the literature about programme choice have dealt mainly with advertising supported television services, however, Wildman and Owen (1985) show a less strong bias against minority programming when broadcasting is directly financed by viewers in the form of Pay-Tv.

Broadcasters supplying free television services are usually either advertising supported (private) or public. In this paper we use a theoretical model to investigate the choices made by private and public broadcasters with respect to the types of programmes they include in their broadcasting schedules. We consider three scenarios: private monopoly, public monopoly and duopoly with the private and the public broadcaster in competition one another.

In this paper we study how private and public broadcasters choose their programming. The private and public broadcasters differ in the access to

[1] A summary review of this literature is in Chapter 3 and Chapter 4 of Owen and Wildman (1992).

sources of finance and in their objective functions. The private broadcaster is a profit maximiser and its revenues come from selling advertising time. The public broadcaster is a welfare maximiser and its revenues come from both selling advertising time and a licence fee. In line with the practice in several European countries, the licence fee is essentially a tax on every home with a television set and it is exogenously fixed.

The main results of this paper are first that the amount of minority programming is reduced in competition with respect to the public monopoly and, second, that the allocation of the channels available between the private and the public broadcaster determines which groups of consumers are worse off.

Our starting point is the monopoly provision of television programmes. We analyse two simple monopoly models in which the only broadcaster is either private or public. The private monopolist maximises its profits: this depends the size of the audience of its programmes since broadcasters can charge more for advertising the more the audience of their programmes. Consequently, the private monopolist, with a limited number of channels, supplies programmes for large audiences and does not offer any programme for minority groups of consumers (minority programming). The public monopolist instead includes the utility of all consumers in its objective function and with as many channels as the private monopolist, his optimal choice is to broadcast programmes for all consumers.

The public monopoly result of our model is interesting both because it is different from the programme selections of a public broadcaster in the literature[2] and because it is an example of a market configuration under which minority groups are supplied.

We compare the selections of programmes in private and public monopolies with the duopoly outcome, when in competition the two broadcasters share the channels available. One of the results of our paper is that with competition all consumers are broadcast their favourite types of programmes. The fact that minority groups of consumers are supplied is due to the presence of the public broadcaster. One of the main result of this paper is that in duopoly the amount of minority programming broadcast is reduced with respect to the public monopoly. Moreover, we find that the allocation of channels between the private and the private broadcaster in duopoly determines which groups of consumers are worse off.

Although our analysis fits into the literature of models of programme choice, it differs from previous works in two important ways. First, this literature has dealt mainly with advertiser supported and Pay-Tv television markets. In this paper we try to fill the gap in the models of programme choices and analyse the case of public and private broadcasters in competition. Second, this literature investigates and compares monopolistically competitive markets and monopoly scenarios, whereas we study the strategic interaction of multi-channel private and public broadcasters.

Other developments of programme choice models include Cancian et al. (1995), Nilssen and Sørgard (1998), and Papandrea (1997). In Cancian et al. (1995) television broadcasters compete on time scheduling of television

[2]Noam (1987) considers the analysis of programme choice and its link with public choice theory. His model predicts the maximisation of government policy goals, if the channel is operated by a government controlled monopolist.

programmes and in the Nilssen and Sørgard (1998) they compete on programme profile and time scheduling. Papandrea (1997) develops an analysis of the impact on social welfare of the biases of television market. Papandrea illustrates the tendency of monopolistically competitive commercial broadcasters to discard intensity of demand of viewers and external benefits; and a tendency to supply programmes that may not maximise social welfare.

The plan of the paper is as follows. We describe the model in Section 2. In Section 3 we present the private monopoly and in Section 4, the public monopoly. Section 5 studies the duopoly competition. In Section 6 we discuss the results and the main conclusions are presented. An Appendix collects all the proofs.

2 The model

We consider a market where television programmes are broadcast free to viewers in a time interval T. Since television broadcasters arrange their broadcasting schedules in slots, the time interval considered here can be any of these slots. There are m channels available to broadcast. This number mainly depends on the means used to broadcast[3] television programmes, and/or it may be fixed by an authority which regulates the access to broadcasting.

We consider two types of television broadcasters: private and public. We assume two sources of finance: revenues from advertising for both broadcasters, and the licence fee, only for the public one.

[3]For technical reasons there are fewer channels available for terrestrial broadcasting than for cable and satellite.

Revenues from advertising come from selling advertising time in the broadcasting schedule. We assume advertising is included into the broadcasting schedule and interrupts programmes. Broadcasters choose the amount of advertising to include in their broadcasting schedule in the size of minutes per amount of programming. Advertising time is the same for all the programmes[4] in a broadcasting schedule and each broadcaster chooses the amount of advertising to insert in it.

Revenues from the licence fee come from consumers. The licence fee can be considered as a direct monetary cost associated with watching television and it is paid by any household with a television set. Consumers have to pay the licence fee only if there is a public broadcaster; they do not pay it in the private monopoly scenario. Moreover, the licence fee is due irrespective of the amount of television watched and of the income of viewers.

We suppose there are n different types of television programmes, i.e. news, sports, soap operas, quiz shows, etc. The number of these types can be rather large and is intended to be exhaustive. As the number of channels available to broadcast is limited, we simply assume that the number of types of programmes is larger than the number of channels: $n > m$.

2.1 The consumers

Consumers derive utility from watching television programmes and have preferences over them. We assume that each consumer has a favourite type of programmes. The time interval T is assumed to be short enough in order

[4] We have tried to allow advertising time to differ across programmes in the broadcasting schedule, but the model becomes analytically complicated.

to have that consumers' preferences are exclusive in the interval considered. If no programme of the most favourite type is available, consumers do not watch any other programme. For example, consumers that like quiz shows most, watch only quiz show programmes, and will not watch any other type of programmes if a quiz show is not broadcast on any of the available channels.

We group consumers according to their preferences over television programmes. For example, all consumers whose favourite type of programmes is quiz show belong to the same group. Then, we get n different groups of consumers corresponding to the n types of television programmes. The number of consumers in each group is denoted with H_i, for $i = 1, \ldots, n$, and we normalise the total number of consumers to one: $\sum_{i=1}^{n} H_i = 1$. Since consumers watch only their favourite programmes, H_i is also the potential audience of the programmes of type i. We assume there are no two groups having the same exact size and we put the groups of consumers in a decreasing order of size with the labelling satisfying: $H_1 > H_2 > \cdots > H_n > 0$.

Television programmes are interrupted by advertising in a way that viewers cannot watch their favourite programmes and avoid it. Consumers dislike advertising. Moreover, consumers have to pay the licence fee $F > 0$ to access to television programmes if there is a public broadcaster. Thus, in the time interval T the utility of viewers of type i is:

$$V(q_i, s) = U(q_i) - N(s) - F, \qquad i = 1, \ldots, n; \tag{1}$$

where q_i is the amount of programmes of type i seen by consumers of type i; s is the amount of advertising inserted into programmes and $N(s)$ represents the utility cost due to advertising; $F \geq 0$ is the licence fee. The utility function of consumers of type i, $i = 1, \ldots, n$, $V(q_i, s)$, is concave by the next assumption.

Assumption 1. *$U(q_i)$ is concave with $U'(q_i) \geq 0$, $U'(q_i)\big|_{q_i=0} \to \infty$ and $U'(q_i)\big|_{q_i \geq T} = 0$. $N(s)$ is convex with $N'(s) > 0$ and $N(0) = 0$*

Consumers have a decreasing positive marginal utility of watching their favourite type of television programmes ($U'(q_i) \geq 0$, and $U''(q_i) < 0$), and they do not get any extra utility from watching television for longer than T. Moreover, the more programmes are interrupted by advertising, the lower the utility of consumers, ($N'(s) > 0$). With the additive formulation of Eq. (1) we are assuming that the marginal utility of watching television is not affected by the amount of either the advertising or the licence fee.

2.2 The broadcasters

2.2.1 Revenues

Both public and private broadcasters are financed by selling the advertising time of their broadcasting schedule. The public broadcaster is also financed by the licence fee. In assuming this we take into account of the fact that public broadcasting on the mainland of Europe is generally financed by a mixture of licence fee and advertising revenues.

In general, what is valuable to advertisers is not only the size of the audience, but also the extent of the programmes in which advertising is included. Since the amount of advertising time is the same for every programme of each broadcaster, the longer a programme is broadcast, the more advertising is watched. Then, we define a weighted measure of the total audience of a broadcasting schedule, as: $\sum_{i=1}^{n} H_i q_i$. This measure captures both the number of consumers watching television, H_i, and the time they spend doing it, q_i.

The next assumption defines the function of the price paid for advertising.

Assumption 2. *The price paid for advertising is a function of the total number of consumers watching television weighted by the time they spend doing it,* $\sum_{i=1}^{n} H_i q_i$*, and of the amount of advertising inserted into programmes,* s*:* $p\big(\sum_{i=1}^{n} H_i q_i, s\big)$*, with:*

$$\frac{\partial p(\cdot)}{\partial q_i} > 0, \quad \frac{\partial^2 p(\cdot)}{\partial q_i^2} = 0 \tag{2}$$

$$\frac{\partial p(\cdot)}{\partial s} < 0, \quad \frac{\partial^2 p(\cdot)}{\partial s^2} \leq 0. \tag{3}$$

The price paid for advertising increases constantly with the amount of programmes and decreases with the advertising. The total revenues is given by the product of the amount of advertising time and its price:

$$R(p, s) = p\Big(\sum_{i=1}^{n} H_i q_i, s\Big)s. \tag{4}$$

Assumption 3. *The revenue function* $R(p, s)$ *has a maximum, with respect to* s*, at the level* $s^* > 0$*.*

Revenues increase with the amount of advertising s, at a non increasing rate for $0 \leq s < s^*$. Moreover, the marginal revenue of advertising increases as the broadcast time of programme of type i, q_i, increases.

The public broadcaster is also financed by the licence fee F and its total revenues are: $R(p,s) = p\big(\sum_{i=1}^{n} H_i q_i, s\big)s + F$.

2.2.2 Costs

We assume that both television broadcasters have access to the same technology in producing programmes, and/or they can buy programmes from competitive suppliers. Hence, they have the same cost function, defined by the following assumption.

Assumption 4. *The total costs of a television broadcaster are the costs of producing and/or buying programmes for its broadcasting schedule:*

$$C(q_1, \ldots, q_n) = c \sum_{i=1}^{n} q_i; \tag{5}$$

with $c \geq 0$. Moreover, we assume the public broadcaster faces the cost of collecting the licence fee F,[5] this is a share α of F, with $0 < \alpha < 1$. Then, total costs of the public broadcaster are: $C(q_1, \ldots, q_n) = c \sum_{i=1}^{n} q_i + \alpha F$.

[5] In the UK, collection of the licence fee is the responsibility of the BBC itself under the *Broadcasting Act 1990*.

2.2.3 Feasibility constraints

The problems of both broadcasters in monopoly and in duopoly will have some common constraints. For the monopoly cases, they are:

$$\sum_{i=1}^{n} q_i \leq mT; \tag{6}$$

$$q_i \leq T, \qquad i = 1, \ldots, n; \tag{7}$$

$$q_i \geq 0, \qquad i = 1, \ldots, n; \tag{8}$$

$$s \geq 0. \tag{9}$$

By constraint (6), it is not feasible to broadcast more than T amount of time of programmes on all the m channels. Since consumers watch only one type of programmes and do not watch television for more than T, it is not reasonable to broadcast more than T of any type of programmes, constraint (7). Negative amount of any television programme and advertising are not feasible, constraint (8), and (9).

In duopoly, constraint (6) changes, whereas constraints (7), (8), (9) are the same but notation changes: $q_i = q_i^\pi$ and $s = s^\pi$ will denote quantities for the private broadcaster and $q_i = q_i^w$ and $s = s^w$, for the public broadcaster.

3 The private monopoly

In this section we assume the private broadcaster to be the only supplier of television programmes for the time interval T on all the m channels available. The aim of the private broadcaster is to maximise its profits: the difference between its revenues from advertising Eq. (4) and its costs Eq. (5). The problem of the private monopolist is then:

$$\max_{q_1,\ldots,q_n,s} \pi = p\Big(\sum_{i=1}^{n} H_i q_i, s\Big)s - c\sum_{i=1}^{n} q_i \tag{10}$$

$$\text{subject to constraints } (6), (7), (8), \text{and } (9).$$

Proposition 1. *The optimal choice of the private monopolist is to broadcasts on the m channels available, the favourite programmes of the m largest groups of consumers:*

$$q_i = \begin{cases} q_i = T & i = 1, \ldots, m; \\ q_i = 0 & i = m+1, \ldots, n. \end{cases}$$

Advertising is set at the level $s = s^*$.

Proof. See Appendix.

The most striking feature of the results presented in Proposition 1 is that, with m channels, the private broadcaster supplies only programmes for the m largest groups of consumers in the time interval T. The private monopolist does not supply any programmes for the $(n-m)$ smallest groups of viewers.

There are two reasons for the result that private monopolist broadcasts programmes only for largest groups of audiences. First the marginal cost of television programmes is constant and it is the same for all the programmes; revenues increase with the audience size, the time programmes are broadcast and the advertising time. Broadcasting $q_i = T$ $(i = 1, \ldots, m)$ allows the private monopolist to obtain a higher price for advertising and then higher revenues which, with constant marginal and average cost, imply higher profits. Second, the number of channels available is limited and smaller than the types of programmes. This determines the number of the largest groups of consumers which are broadcast their favourite types of programmes.

In this scenario, more groups of consumers will be supplied if the number of channels available increases.

4 The public monopoly

In many countries, when television services began, broadcasting was a public monopoly. In this section we consider the case in which the only broadcaster is public. We assume that the public monopolist is a welfare maximiser and we define its objective function as the weighted sum of the net benefit of all consumers: $\sum_{i=1}^{n} H_i V(q_i, s) = \sum_{i=1}^{n} H_i U(q_i) - N(s) - F$. The problem of the public monopolist is therefore:

$$\max_{q_1,\ldots,q_n,s} \qquad W = \sum_{i=1}^{n} H_i U(q_i) - N(s) - F \tag{11}$$

$$\text{subject to} \qquad c\sum_{i=1}^{n} q_i - p\Big(\sum_{i=1}^{n} H_i q_i, s\Big)s \leq (1-\alpha)F, \tag{12}$$

and constraints (6), (7), (8), and (9).

The public monopolist maximises the weighted net benefit of all types of consumers subject to the constraint that the difference between the total costs and the total revenues from advertising are not larger than the net revenues from the collection of the licence fee Eq. (12); and the feasibility constraints (6), (7), (8), and (9).

Proposition 2. *The solution of the public monopolist's problem is such as:*

$$T = q_1 > q_2 > \cdots > q_n > 0.$$

The advertising time is set to a level $0 < s < s^*$*, such as constraint (12) binds.*

Proof. See Appendix.

The feature of the results presented in Proposition 2 is that the public monopolist supplies all the n types of programmes on the m channels available and the amount broadcast of each type of programmes broadcast follows the ordering of the groups of consumers who like them most. This result is due to a balancing between the revenues from advertising and the utility of consumers. At the optimum, the public monopolist makes equal across types the sum of the marginal revenue from advertising and the marginal utility of consumers weighted by the audience size $\big(H_i\big(U'(q_i) + \frac{\partial p(\cdot)}{\partial q_i}\gamma s\big)\big)$. This result holds only for interior solutions.

Moreover, the public monopolist does not make any profit from supplying television programmes. Due to the licence fee and to the fact that the disutility of advertising suffered by consumers is considered in its objective function, the amount of advertising inserted into programmes is positive and such as the budget constraint binds, but smaller than the revenue maximising level, s^*.

5 Duopoly

In the duopoly scenario private and public broadcasters share the m channels available to broadcast in the following way: βm channels to the private broadcaster and $(1-\beta)m$ channels to the public; with $0 < \beta < 1$. Since the number of channels allocated to private and public broadcasters is usually decided by an authority, we assume β exogenously given.

We translate the duopoly competition problem into a normal-form game of complete information.

- The players are the private and the public broadcaster.
- The strategies available to each broadcaster are the amount of each type of programmes to be broadcast and the amount of advertising to be inserted into them.

  For each broadcaster, a strategy, denoted q_i^π for the private and q_i^w for the public is a quantity choice of the amount of programmes of type i to broadcast $q_i \in [0, T]$, for $i = 1, \ldots n$; and a strategy denoted s^π for the private and s^w for the public is a quantity choice of advertising to insert into programmes $s \geq 0$. The strategies of the private broadcaster are: $(q_1^\pi, \ldots, q_n^\pi, s^\pi)$. The strategies of the public broadcaster are: $(q_1^w, \ldots, q_n^w, s^w)$.
- The payoffs of each broadcaster are written considering the strategies of both players. The revenues from advertising and the utility of each type of consumers depend on the amount of programmes broadcast by both broadcasters $(q_i^w + q_i^\pi)$.

– The objective function of the private broadcaster becomes:

$$\pi = p\Big(\sum_{i=1}^{n} H_i(q_i^\pi + q_i^w), s^\pi\Big)s^\pi - c\sum_{i=1}^{n} q_i^\pi.$$

– and the objective function of the public broadcaster becomes:

$$W = \sum_{i=1}^{n} H_i U(q_i^\pi + q_i^w) - N(s^w) - F$$

- The two players take their decisions simultaneously.

We are interested in a Nash equilibrium in pure strategies of this game. Therefore, the problem of the private broadcaster is:

$$\max_{q_1^\pi,\dots,q_n^\pi,s^\pi} \quad \pi = p\Big(\sum_{i=1}^{n} H_i(q_i^\pi + q_i^w), s^\pi\Big)s^\pi - c\sum_{i=1}^{n} q_i^\pi, \tag{13}$$

$$\text{subject to} \quad \sum_{i=1}^{n} q_i^\pi \le \beta m T, \tag{14}$$

and constraints (7), (8), and (9).

The problem of the public broadcaster is:

$$\max_{q_1^w,\dots,q_n^w,s^w} \quad W = \sum_{i=1}^{n} H_i U(q_i^\pi + q_i^w) - N(s^w) - F \tag{15}$$

$$\text{subject to} \quad c\sum_{i=1}^{n} q_i^w + p\Big(\sum_{i=1}^{n} H_i(q_i^\pi + q_i^w), s^w\Big)s^w \le (1-\alpha)F, \tag{16}$$

$$\sum_{i=1}^{n} q_i^w \le (1-\beta) m T, \tag{17}$$

and constraints (7), (8), and (9).

Let q_i^π and q_i^w be the quantities chosen in equilibrium by both broadcasters. If $q_i^\pi + q_i^w \leq T$, consumers of type i $(i = 1, \ldots, n)$ watch $q_i = q_i^\pi + q_i^w$. If $q_i^\pi + q_i^w > T$, consumers of type i may watch either the programmes broadcast by the private or the programmes broadcast by the public. Which programmes consumers watch depends on the relative size of s^π and s^w. Since consumers dislike advertising, for any programme with different level of advertising, consumers watch first the one with less advertising and then, if time is left, those with more advertising.

If there is less advertising on the programmes of the public broadcaster, $s^\pi > s^w$, consumers watch:

$$q_i = \begin{cases} q_i^w & \text{from the public broadcaster,} \\ T - q_i^w & \text{from the private broadcaster.} \end{cases}$$

If the amount of advertising is the same for both broadcasters $s^\pi = s^w$, consumers watch some programmes from the private and some from the public broadcaster. The amount of time watched may be proportional to the amount broadcast, for example:

$$q_i = \begin{cases} \dfrac{q_i^w}{q_i^\pi + q_i^w} T & \text{from the public broadcaster,} \\ \dfrac{q_i^\pi}{q_i^\pi + q_i^w} T & \text{from the private broadcaster.} \end{cases}$$

If there is less advertising on the private broadcaster's programmes $s^\pi < s^w$, consumers watch:

$$q_i = \begin{cases} q_i^\pi & \text{from the private broadcaster,} \\ T - q_i^\pi & \text{from the public broadcaster.} \end{cases}$$

Proposition 3. *The optimal amount of advertising chosen by the public broadcaster is smaller than the one chosen by the private broadcaster:* $0 < s^w < s^\pi = s^*$.

Proof. See Appendix.

In equilibrium, private broadcaster chooses the amount of advertising corresponding to the profit maximising level and the public broadcaster chooses a smaller amount of advertising. Therefore, in duopoly, the public broadcaster inserts less advertising time in its programming than the private one.

Given Proposition 3, consumers of type i ($i = 1, \ldots, n$) will watch programmes broadcast by both broadcasters if the amount of programmes of type i broadcast is not greater than T, otherwise, they will watch the programmes of the public broadcaster first and then the ones of the private:

$$q_i = \begin{cases} q_i^\pi + q_i^w & \text{if } q_i^\pi + q_i^w \leq T \\ (T - q_i^w) + q_i^w & \text{if } q_i^\pi + q_i^w > T, \end{cases}$$

Proposition 4. *At the optimum, the total amount of programmes of type i broadcast is:* $q_i^\pi + q_i^w \leq T$, *for* $i = 1, \ldots, n$, *with:*

$q_i^\pi = T$ *and* $q_i^w = 0, \qquad i = 1, \ldots, \beta m;$

$q_i^\pi = 0$ *and* $q_i^w \in (0, T), \quad i = \beta m + 1, \ldots, n.$

Proof. See Appendix.

In duopoly the private broadcaster supplies on its βm channels the types of programmes $i = 1, \ldots, \beta m$ to the βm largest groups of consumers, and the public one broadcasts on its $(1-\beta)m$ channels the types of programmes $i = \beta m + 1, \ldots, n$ to the $(n - \beta m)$ smallest groups of consumers.

Proposition 4 suggests that the two broadcasters split the market between them. The private broadcaster provides programmes for the largest groups of consumers and the public one broadcasts programmes for the smallest groups. In particular, in duopoly the private and the public broadcasters supply the same programme patterns they would have broadcast in their respective monopoly scenarios. Therefore, the allocation of channels between the private and the public broadcaster, the value of β, determines which groups of consumers are supplied by the private and by the public broadcasters. If we relax the assumption $0 < \beta < 1$, in the extreme cases we will have the private monopoly $(\beta = 0)$ and the private monopoly $(\beta = 1)$ outcomes.

Lemma 1. *The optimal choice of the public broadcaster in duopoly is equivalent to the solution of the problem (18), for* $i = \beta m + 1, \ldots, n$.

Proof. See Appendix.

Let the superscript B denote the optimal amount broadcast by the public monopolist q_i^B, for $i = 1, \ldots, n$. Therefore the problem of the public broadcaster in duopoly can be rewritten as:

$$\max_{q_1,\dots,q_n,s} \quad W = \sum_{i=1}^{n} H_i U(q_i) - N(s) - F \tag{18}$$

$$\text{subject to} \quad c\sum_{i=1}^{n} q_i - p\Big(\sum_{i=1}^{n} H_i q_i, s\Big)s \leq (1-\alpha)F, \tag{19}$$

$$\sum_{i=\beta m+1}^{n} q_i \leq (1-\beta)mT + \sum_{i=1}^{\beta m} q_i^B - \beta mT, \tag{20}$$

$$q_i = 0, \quad i = 1,\dots,\beta m, \tag{21}$$

and constraints (7), (8), and (9).

Proposition 5. *Let q_i^B and q_i^D denote the optimal amount of programmes of type i broadcast respectively in public monopoly and in duopoly, we have $0 < q_i^D < q_i^B < T$, for $i = \beta m + 1, \dots, n$.*

Proof. See Appendix.

Proposition 5 states the main result of our paper: the amount of programmes broadcast for the groups of consumers $i = \beta m + 1, \dots, n$ is reduced in duopoly with respect to the public monopoly. This means that the amount of programmes for minority groups of consumers ($i = m, \dots, n$) is also reduced with respect to the public monopoly scenario.

6 Discussion and conclusions

In this paper we have investigated how television broadcasters select the types of programmes to include in any of the slots of a daily broadcasting schedule. We do this for a television market where the suppliers are: a private monopolist; a public monopolist; private and public broadcasters in competition. The comparison of these three scenarios deals with a bias in the selection of programmes for minority groups of consumers.

The main feature of the private monopoly outcome is the fact that minority programming is not supplied. The private monopolist supplies only programmes for large groups of audience because it is financed by the advertising inserted into its programmes whose price is positively linked with the size of the audiences. We assume constant marginal costs of producing programmes, then, the higher the price for advertising, the higher the profits of the private broadcaster. Therefore, with a limited number of channels, the private monopolist offers only programmes for large groups of consumers and sets advertising at the revenue maximising level. However, in a private monopoly scenario, minority programming will be offered only if the number of channels available to broadcast is large enough.

In public monopoly all the types of television programmes are broadcast. The characteristic of the public monopoly outcome is that the ordering of the amount of any type of television programmes reflects the ordering of the size of the groups of consumers. So the larger the audience of a type of programmes, the longer this type of programmes is broadcast. Moreover, the public monopolist broadcasts in its programming less advertising with respect to the private broadcaster. This is due to the disutility of advertising suffered by consumers, considered in the objective function of the public monopolist, and to the licence fee as a further source of finance.

In duopoly all types of programmes are broadcast. The feature of the duopoly outcome is that the groups of consumers $i = 1, \ldots, \beta m$ are broadcast the same amount of programmes as in the private monopoly. They can see the programmes they like in the time interval considered, T. For these groups of consumers, the amount of programmes broadcast in the public monopoly (except for the largest group, $i = 1$) is smaller. Then, these groups of consumers are better off with a private monopoly and with a duopoly rather than with a public monopolist. Groups $i = \beta m + 1, \ldots, n$ are the ones which have different allocations of their favourite types of programmes in the three different market configurations.

Since $0 < \beta < 1$, we can split these groups of consumers in two: groups $(\beta m + 1, \ldots, m)$ and groups $(m, \ldots, n)$. Groups $(\beta m + 1, \ldots, m)$ are supplied an amount T of their favourite programmes by a private monopolist, a smaller amount by a public monopolist and a further smaller amount in duopoly. Therefore, groups $i = \beta m + 1, \ldots, m$ are better off with the private monopoly. Groups $(m, \ldots, n)$, the smallest groups of consumers, are surely worse off with the private monopolist because they are not offered any programming, but they are worse off in duopoly as well, with respect to the public monopoly scenario. In fact, these groups are broadcast a smaller amount of their favourite types of programmes in duopoly with respect to the public monopoly. This is so because the public monopolist reduces the amount of programmes for the largest groups and reallocates time to broadcast minority programming. This ability of the public broadcaster to reallocate time in its broadcasting schedules for the m channels is limited in duopoly as the number of channels available is reduced.

The results of this paper have some strong policy implications. First of all, with a limited number of channels, we show the importance of public broadcasting. In fact, the existence of a public broadcaster guarantees minority programming which would not be offered if all the channels available are operated by a private broadcaster.

Second, in duopoly minority groups are worse off with respect to a public monopoly: the amount of minority programming broadcast in duopoly is reduced. The effect of having the welfare maximiser broadcaster competing for the audience with a profit maximiser is to worsen off minority groups of consumers with respect to a public monopoly scenario. There are, anyway, some groups (the large ones, $i = 1, \ldots, \beta m$) which are better off in duopoly with respect to the public monopoly.

Third, it is crucial the way in which channels are allocated between the two types of broadcasters, the value of β, since this determines the extent of the large groups of consumers who are better off. In particular, in duopoly, the more channels are operated by the private broadcaster (the larger β), the greater the number of groups which are better off. This may explain the evidence that in some countries, in the market for free television services, the number of channels operated by public broadcasters is generally smaller than the number of channels operated by private broadcasters.

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A Proofs of Results

Proof of Proposition 1. The Lagrangian of the problem of the private monopolist is:

$$\mathcal{L} = p\Big(\sum_{i=1}^{n} H_i q_i, s\Big)s - c\sum_{i=1}^{n} q_i - \delta\Big(\sum_{i=1}^{n} q_i - mT\Big) \\ - \sum_{i=1}^{n}\lambda_i(q_i - T) + \sum_{i=1}^{n}\mu_i q_i + \rho s. \quad (22)$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial q_i} = H_i \frac{\partial p\big(\sum_{i=1}^{n} H_i q_i, s\big)}{\partial q_i} s - c - \delta - \lambda_i + \mu_i = 0, \quad i = 1, \dots, n; \quad (23)$$

$$\frac{\partial \mathcal{L}}{\partial s} = \frac{\partial p\big(\sum_{i=1}^{n} H_i q_i, s\big)}{\partial s} s + p\Big(\sum_{i=1}^{n} H_i q_i, s\Big) + \rho = 0. \quad (24)$$

We can write Eq. (24) as:

$$\frac{\partial R(p,s)}{\partial s} + \rho = 0. \quad (25)$$

$\rho > 0$ implies $s = 0$, from Assumption 3, when $s = 0$ we have $\frac{\partial R(p,s)}{\partial s} > 0$. This results in a positive LHS of Eq. (25), that contradicts the assumption $\rho > 0$. Then, with $\rho = 0$ and $s > 0$ we rewrite Eq. (25) as:

$$\frac{\partial R(p,s)}{\partial s} = 0.$$

This is the condition for $s = s^*$.

We write Eq. (23), for any $q_i > 0$, and $q_j > 0$, with $i \neq j$, then the value of the following multipliers are: $\mu_i = \mu_j = 0$. We have:

$$H_i \frac{\partial p}{\partial q_i} s - c - \delta = \lambda_i, \quad (26)$$

$$H_j \frac{\partial p}{\partial q_j} s - c - \delta = \lambda_j. \quad (27)$$

From Assumption 2 we have that $p(\cdot)$ is linear in q, then: $\frac{\partial p}{\partial q_i} = \frac{\partial p}{\partial q_j}$. Subtracting Eq. (27) from Eq. (26) we get:

$$(H_i - H_j)\frac{\partial p}{\partial q_i} s = \lambda_i - \lambda_j. \tag{28}$$

$\frac{\partial p}{\partial q_i} > 0$, from Assumption 2; we proved that $s > 0$; groups of consumers are ordered such as $H_1 > H_2 > \cdots > H_n > 0$, then for $H_i > H_j$, the LHS of Eq. (28) is positive. This requires the RHS to be positive, that means: $\lambda_i > \lambda_j$, with $\lambda_j \geq 0$, it follows $\lambda_i > 0$, that implies $q_i = T$. Given constraint (6), $q_i = T$ is feasible only for $i = 1, \ldots, m$. Then the solutions of the problem of the private monopolist are: $q_i = T$, $i = 1, \ldots, m$, and $q_i = 0$, $i = m+1, \ldots, n$. Q.E.D.

Proof of Proposition 2. The Lagrangian of the problem of the public monopolist is:

$$\begin{aligned}\mathcal{L} = \sum_{i=1}^{n} H_i U(q_i) - N(s) - F - \gamma\Big(c\sum_{i=1}^{n} q_i - p\Big(\sum_{i=1}^{n} H_i q_i, s\Big)s - (1-\alpha)F\Big) \\ - \delta\Big(\sum_{i=1}^{n} q_i - mT\Big) - \sum_{i=1}^{n} \lambda_i (q_i - T) + \sum_{i=1}^{n} \mu_i q_i + \rho s. \end{aligned} \tag{29}$$

The first order conditions are:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial q_i} &= H_i U'(q_i) - \gamma c + \gamma H_i \frac{\partial p\big(\sum_{i=1}^{n} H_i q_i, s\big)}{\partial q_i} s - \delta - \lambda_i + \mu_i = 0, \\ &\quad i = 1, \ldots, n. \end{aligned} \tag{30}$$

$$\frac{\partial \mathcal{L}}{\partial s} = -N'(s) + \gamma\left[\frac{\partial p\big(\sum_{i=1}^{n} H_i q_i, s\big)}{\partial s} s + p\Big(\sum_{i=1}^{n} H_i q_i, s\Big)\right] + \rho = 0. \tag{31}$$

We can write Eq. (31) as:

$$-N'(s) + \gamma \frac{\partial R(p,s)}{\partial s} + \rho = 0 \tag{32}$$

If $\rho > 0$, we have $s = 0$, it follows: $-N'(0) = 0$, for Assumption 1. Then Eq. (32) becomes:

$$\gamma \frac{\partial R(p,s)}{\partial s} + \rho = 0. \tag{33}$$

We have: $\rho > 0$ for assumption, this implies $s = 0$, and it follows $\frac{\partial R(p,s)}{\partial s}\Big|_{s=0} > 0$, from Assumption 3; and $\gamma \geq 0$. We get that the LHS of Eq. (33) is positive, this is a contradiction to the assumption $\rho > 0$. Then we have $\rho = 0$ and $s > 0$; Eq. (32) is rewritten as:

$$-N'(s) + \gamma \frac{\partial R(p,s)}{\partial s} = 0. \tag{34}$$

Given that $s > 0$, we have $-N'(s) < 0$, this implies both $\gamma > 0$ and $\frac{\partial R(p,s)}{\partial s} > 0$. Therefore, constraint (12) binds because $\gamma > 0$, and $0 < s < s^*$, because $\frac{\partial R(\cdot)}{\partial s} > 0$.

From Assumption 1, we have $U'(q_i)\big|_{q_i=0} \to \infty$, then $\mu_i = 0$, for every i. We write Eq. (30) for types i and j, with $i \neq j$:

$$H_i U'(q_i) + \gamma H_i \frac{\partial p}{\partial q_i} s - \gamma c - \delta = \lambda_i, \tag{35}$$

$$H_j U'(q_j) + \gamma H_j \frac{\partial p}{\partial q_j} s - \gamma c - \delta = \lambda_j. \tag{36}$$

We consider two interior solutions, $q_i \in (0, T)$ and $q_j \in (0, T)$, it follows: $\lambda_i = \lambda_j = 0$. We examine the case $q_j > q_i$. From Assumption 2 we have that $p(\cdot)$ is linear in q, then : $\frac{\partial p}{\partial q_i} = \frac{\partial p}{\partial q_j}$. We assume by contradiction that $H_i > H_j$. We subtract Eq. (36) from Eq. (35) and we get:

$$H_i U'(q_i) - H_j U'(q_j) + \gamma (H_i - H_j) \frac{\partial p}{\partial q_i} s = 0. \tag{37}$$

We have: $H_i > H_j$ by assumption; $U'(q_i) > U'(q_j)$, for Assumption 1; $\frac{\partial p}{\partial q_i} > 0$, for Assumption 2; $\gamma > 0$; and $s > 0$. Then, the LHS of Eq. (37) is positive, and the RHS is zero. This contradicts the assumption $H_i > H_j$. Then, for two interior solutions, $q_j > q_i$, the size of the relative groups is $H_j > H_i$.

We consider now one interior solution $q_i \in (0, T)$ and one corner solution $q_j = T$, this requires $\lambda_i = 0$ and $\lambda_j > 0$. We follow the same procedure used above, we assume by contradiction $H_i > H_j$ and we get:

$$H_i U'(q_i) - H_j U'(T) + \gamma (H_i - H_j) \frac{\partial p}{\partial q_i} s = -\lambda_j. \tag{38}$$

For the same reasons shown above for Eq. (37), the LHS of Eq. (38) is positive, and the RHS is negative. This contradicts the assumption $H_i > H_j$. Then, for the two solutions, $T = q_j > q_i$, the size of the relative groups is $H_j > H_i$.

Therefore, the solutions of the problem of public monopolist can be ordered following the ordering of the size of the groups of consumers: $T = q_1 > q_2 > \cdots > q_n > 0$. Q.E.D.

Proof of Proposition 3. The Lagrangian of the problem (13) of the private broadcaster is:

$$\mathcal{L}^\pi = p\Big(\sum_{i=1}^n H_i(q_i^\pi + q_i^w), s^\pi\Big)s^\pi - c\sum_{i=1}^n q_i^\pi - \delta^\pi\Big(\sum_{i=1}^n q_i^\pi - \beta mT\Big) \\ - \sum_{i=1}^n \lambda_i^\pi(q_i^\pi - T) + \sum_{i=1}^n \mu_i^\pi q_i^\pi + \rho^\pi s^\pi. \quad (39)$$

The first order conditions are:

$$\frac{\partial \mathcal{L}^\pi}{\partial q_i^\pi} = H_i \frac{\partial p\big(\sum_{i=1}^n H_i(q_i^\pi + q_i^w), s^\pi\big)}{\partial q_i^\pi} s^\pi - c - \delta^\pi - \lambda_i^\pi + \mu_i^\pi = 0, \\ i = 1, \ldots, n; \quad (40)$$

$$\frac{\partial \mathcal{L}^\pi}{\partial s^\pi} = \frac{\partial p\big(\sum_{i=1}^n H_i(q_i^\pi + q_i^w), s^\pi\big)}{\partial s^\pi} s^\pi + p\Big(\sum_{i=1}^n H_i(q^\pi + q_i^w)_i, s^\pi\Big) + \rho^\pi = 0. \quad (41)$$

We can write Eq. (41) as:

$$\frac{\partial R(p, s^\pi)}{\partial s^\pi} + \rho^\pi = 0. \quad (42)$$

If $\rho^\pi > 0$, Eq. (42) requires $\frac{\partial R(p,s^\pi)}{\partial s^\pi} < 0$. From Assumption 3, when $s = 0$ we have $\frac{\partial R(p,s)}{\partial s} > 0$. This results in a positive LHS of Eq. (42), that contradicts the assumption $\rho^\pi > 0$. Then, with $\rho^\pi = 0$ and $s^\pi > 0$ we rewrite Eq. (42) as:

$$\frac{\partial R(p, s^\pi)}{\partial s^\pi} = 0.$$

For Assumption 3, this is the condition for $s^\pi = s^*$.

The Lagrangian of the problem (15) of the public broadcaster is:

$$\begin{aligned}\mathcal{L}^w = &\sum_{i=1}^{n} H_i U(q_i^\pi + q_i^w) - N(s^w) - F \\ &- \gamma\Big[c\sum_{i=1}^{n} q_i^w - p\Big(\sum_{i=1}^{n} H_i(q_i^\pi + q_i^w), s^w\Big)s^w - (1-\alpha)F\Big] \\ &- \delta^w\Big(\sum_{i=1}^{n} q_i^w - (1-\beta)mT\Big) - \sum_{i=1}^{n}\lambda_i^w(q_i^w - T) + \sum_{i=1}^{n}\mu_i^w q_i^w + \rho^w s^w. \qquad (43)\end{aligned}$$

The first order conditions are:

$$\begin{aligned}\frac{\partial \mathcal{L}^w}{\partial q_i^w} =& H_i U'(q_i^\pi + q_i^w) - \gamma\left[c - H_i\frac{\partial p\big(\sum_{i=1}^{n} H_i(q_i^\pi + q_i^w), s^w\big)}{\partial q_i^w}s^w\right] \\ &- \delta^w - \lambda_i^w + \mu_i^w = 0, \qquad i = 1, \ldots, n; \qquad (44) \\ \frac{\partial \mathcal{L}^w}{\partial s^w} =& - N'(s^w) + \gamma\frac{\partial p\big(\sum_{i=1}^{n} H_i(q_i^\pi + q_i^w), s^w\big)}{\partial s^w}s^w \\ &+ \gamma p\Big(\sum_{i=1}^{n} H_i(q_i^\pi + q_i^w), s^w\Big) + \rho^w = 0. \qquad (45)\end{aligned}$$

We can write Eq. (45) as:

$$-N'(s^w) + \gamma\frac{\partial R(p, s^w)}{\partial s^w} + \rho^w = 0. \qquad (46)$$

If $\rho^w > 0$ we have $s^w = 0$, it follows: $-N'(0) = 0$, then Eq. (46) is rewritten as:

$$\gamma\frac{\partial R(p, s^w)}{\partial s^w} + \rho^w = 0. \qquad (47)$$

$\gamma \geq 0$; if $s^w = 0$, from Assumption 3, we have $\frac{\partial R(p,s^w)}{\partial s^w} > 0$. It follows that the LHS of Eq. (47) is positive, this contradicts the assumption $\rho^w > 0$, then $\rho^w = 0$ and $s^w > 0$. Therefore, we rewrite Eq. (46) as:

$$-N'(s^w) + \gamma\frac{\partial R(p, s^w)}{\partial s^w} = 0. \qquad (48)$$

Given that $s^w > 0$, we have $-N'(s^w) < 0$, this implies both $\gamma > 0$ and $\frac{\partial R(p,s^w)}{\partial s^w} > 0$. Therefore constraint (12) binds, and $0 < s^w < s^\pi = s^*$. The amount of advertising per extent of programmes chosen by the public broadcaster is smaller than the one chosen by the private broadcaster. Q.E.D.

Proof of Proposition 4. For Proposition 3, we have $0 < s^w < s^\pi = s^*$, then, consumers of type i, for $i = 1, \ldots, n$, watch:

$$q_i = \begin{cases} q_i^\pi + q_i^w & \text{if } q_i^\pi + q_i^w \leq T, \\ (T - q_i^w) + q_i^w & \text{if } q_i^\pi + q_i^w > T. \end{cases}$$

Suppose to have programmes of type j for which $q_j^\pi + q_j^w = T$, then $q_j^\pi = T - q_j^w$, and consumers of type j watch: $q_j = (T - q_j^w) + q_j^w$. The price paid for advertising can written as:

$$p\Big(H_j T + \sum_{i \neq j} H_i q_i, s^\pi\Big),$$

Suppose now, for the same type of programmes, to have $q_j^\pi + q_j^w > T$, then $q_j^\pi > T - q_j^w$, anyway consumers will watch $q_j = (T - q_j^w) + q_j^w$, then the price paid for advertising is:

$$p\Big(H_j T + \sum_{i \neq j} H_i q_i, s^\pi\Big).$$

The price paid for advertising does not change if $q_j^\pi = T - q_j^w$ or if, for the same type of programmes, $q_j^\pi > T - q_j^w$, because consumers watch only $q_j = (T - q_j^w) + q_j^w$. Since the private broadcaster faces a positive marginal cost for broadcasting $q_j^\pi > T - q_j^w$ instead of $q_j^\pi = T - q_j^w$ and does not get any increase in the price paid for advertising and then in revenues, the private broadcaster can reallocate time in the broadcasting schedule offering $q_j^\pi = T - q_j^w$, for the programmes of type j, and more time of other types of programmes and get an increase in revenues. Then, it is not a best response to the public broadcaster's strategies, to supply $q_i^\pi > T - q_i^w$, for any i. This means that, at the optimum, $q_i^\pi + q_i^w \leq T$, for $i = 1, \ldots, n$.

Since at the optimum, we have: $q_i^\pi + q_i^w \leq T$, we rewrite the first order conditions (40), and (44):

$$\frac{\partial \mathcal{L}^\pi}{\partial q_i^\pi} = H_i \frac{\partial p\big(\sum_{i=1}^n H_i(q_i^\pi + q_i^w), s^\pi\big)}{\partial q_i^\pi} s^\pi - c - \delta^\pi - \lambda_i^\pi + \mu_i^\pi = 0, \quad i = 1, \ldots, n; \tag{49}$$

$$\frac{\partial \mathcal{L}^w}{\partial q_i^w} = H_i U'(q_i^\pi + q_i^w) - \gamma \left[c - H_i \frac{\partial p\big(\sum_{i=1}^n H_i(q_i^\pi + q_i^w), s^w\big)}{\partial q_i^w} s^w \right] - \delta^w - \lambda_i^w + \mu_i^w = 0, \qquad i = 1, \ldots, n. \tag{50}$$

First we verify if a pattern of solutions, similar to the ones of the public monopoly ($T = q_1 > q_2 > \cdots > q_n > 0$), with both broadcasters supplying all the types of programmes, is a Nash equilibrium for the duopoly competition game. For every i, $q_i^\pi + q_i^w \leq T$, if $q_i^w \in (0,T)$, as in the public monopoly, the best reply of the private broadcaster is: $q_i^\pi \leq T - q_i^w$; this means $q_i^\pi \in (0,T)$.

We write the first order conditions of the private broadcaster, for $i = 1, 2$, with $q_1^\pi + q_1^w = T$, and $q_2^\pi + q_2^w < T$. For $i = 1$, we have $q_1^w \in (0,T)$ and $q_1^\pi \in (0,T)$, then, for the private broadcaster, we have $\lambda_1^\pi = \mu_1^\pi = 0$. We write the first order condition (49) for $i = 1$:

$$\frac{\partial \mathcal{L}^\pi}{\partial q_1^\pi} = H_1 \frac{\partial p\big(\sum_{i=1}^n H_i(q_i^\pi + q_i^w), s^\pi\big)}{\partial q_1^\pi} s^\pi - c - \delta^\pi = 0. \tag{51}$$

For $i = 2$, we have $q_2^w \in (0,T)$ and $q_2^\pi \in (0,T)$, and $\lambda_2^\pi = \mu_2^\pi = 0$. We write the first order conditions (49) for $i = 2$

$$\frac{\partial \mathcal{L}^\pi}{\partial q_2^\pi} = H_2 \frac{\partial p\big(\sum_{i=1}^n H_i(q_i^\pi + q_i^w), s^\pi\big)}{\partial q_2^\pi} s^\pi - c - \delta^\pi = 0. \tag{52}$$

For Assumption 2, we have $\frac{\partial p}{\partial q_1} = \frac{\partial p}{\partial q_2} = \frac{\partial p}{\partial q_i}$, and subtracting (51) and (52) we get:

$$(H_1 - H_2)\frac{\partial p}{\partial q_i} = 0; \tag{53}$$

$H_1 > H_2$, and $\frac{\partial p}{\partial q_i} > 0$, so Eq. (53) is never equal to zero. This contradicts, the assumption $q_i^w \in (0,T)$ and $q_i^\pi \in (0,T)$, for every i, and $T = q_1 > q_2 > \cdots > q_n > 0$.

We verify if a pattern of solutions similar to the ones of the private monopoly ($q_i = T$ for some types of programmes and $q_i = 0$ for the others) are solutions in the duopoly competition game:

$$\begin{array}{llll} q_i^\pi = T, & \text{and} & q_i^w = 0, & i = 1, \ldots, \beta m \\ q_i^\pi = 0, & \text{and} & q_i^w \in (0,T), & i = \beta m + 1, \ldots, n \end{array}$$

We write the first order conditions of the private broadcaster, for $i = 1, 2$. Applying $\frac{\partial p}{\partial q_1} = \frac{\partial p}{\partial q_2} = \frac{\partial p}{\partial q_i}$ and considering: $q_1^\pi = T$, $q_2^\pi = T$, $\lambda_1^\pi > 0$, $\lambda_2^\pi > 0$, $\mu_1^\pi = \mu_2^\pi = 0$. We have:

$$\frac{\partial \mathcal{L}^\pi}{\partial q_1^\pi} = H_1 \frac{\partial p}{\partial q_i} s^\pi - c - \delta^\pi - \lambda_1^\pi = 0,$$
$$\frac{\partial \mathcal{L}^\pi}{\partial q_2^\pi} = H_2 \frac{\partial p}{\partial q_i} s^\pi - c - \delta^\pi - \lambda_2^\pi = 0.$$

Subtracting them we get:

$$(H_1 - H_2)\frac{\partial p}{\partial q_i}s^\pi - \lambda_1^\pi + \lambda_2^\pi = 0.$$

There is no contradiction for the private broadcaster.

We write the first order condition of the public broadcaster with: $q_1^\pi + q_1^w = T$ and $q_2^\pi + q_2^w = T$; $\lambda_1^w = \lambda_2^w = 0$; $\mu_1^w > 0$, and $\mu_2^w > 0$.

$$\frac{\partial \mathcal{L}^w}{\partial q_1^w} = H_1 U'(T) - \gamma\Big(c - H_1\frac{\partial p}{\partial q_i}s^w\Big) - \delta^w + \mu_1^w = 0,$$
$$\frac{\partial \mathcal{L}^w}{\partial q_2^w} = H_2 U'(T) - \gamma\Big(c - H_2\frac{\partial p}{\partial q_i}s^w\Big) - \delta^w + \mu_2^w = 0.$$

Since $U'(T) = 0$, subtracting them we get:

$$\gamma(H_1 - H_2)\frac{\partial p}{\partial q_i}s^w + \mu_1^w - \mu_2^w = 0;$$

with $\gamma > 0$; $H_1 > H_2$; $\frac{\partial p}{\partial q_i} > 0$, $s^w > 0$, there is no contradiction for the public broadcaster too.

We write now the first order conditions (40) and (44) for $i = \beta m$, and $i = \beta m + 1$. Since the group βm gets T hours of programmes they like, we suppose the public broadcaster starts broadcasting $q_{\beta m+1}^w < T$. We have $\mu_{\beta m}^\pi = \mu_{\beta m+1}^w = \lambda_{\beta m+1}^\pi = \lambda_{\beta m}^w = \lambda_{\beta m+1}^w = 0$, $\mu_{\beta m+1}^\pi > 0$, $\mu_{\beta m}^w > 0$, and $\lambda_{\beta m}^\pi > 0$.

The first order conditions of the private broadcaster are:

$$\frac{\partial \mathcal{L}^\pi}{\partial q_{\beta m}^\pi} = H_{\beta m}\frac{\partial p}{\partial q_i}s^\pi - c - \delta^\pi - \lambda_{\beta m}^\pi = 0,$$
$$\frac{\partial \mathcal{L}^\pi}{\partial q_{\beta m+1}^\pi} = H_{\beta m+1}\frac{\partial p}{\partial q_i}s^\pi - c - \delta^\pi + \mu_{\beta m+1}^\pi = 0.$$

Subtracting them we get:

$$(H_{\beta m} - H_{\beta m+1})\frac{\partial p}{\partial q_i}s^\pi - \lambda_{\beta m}^\pi - \mu_{\beta m+1}^\pi = 0.$$

Since $H_{\beta m} > H_{\beta m+1}$, and $\frac{\partial p}{\partial q_i} > 0$, there is no contradiction for the private broadcaster.

The first order conditions of the public broadcaster are:

$$\frac{\partial \mathcal{L}^w}{\partial q^w_{\beta m}} = H_{\beta m} U'(T) - \gamma\Big(c + H_{\beta m}\frac{\partial p}{\partial q_i} s^w\Big) - \delta^w + \mu^w_{\beta m} = 0,$$
$$\frac{\partial \mathcal{L}^w}{\partial q^w_{\beta m+1}} = H_{\beta m} U'(q^w_{\beta m+1}) - \gamma\Big(c + H_{\beta m}\frac{\partial p}{\partial q_i} s^w\Big) - \delta^w = 0.$$

Since $U'(T) = 0$, subtracting them we get:

$$-H_{\beta m} U'(q^w_{\beta m+1}) + \gamma(H_{\beta m} - H_{\beta m+1})\frac{\partial p}{q_i} s^w + \mu^w_{\beta m} = 0.$$

$U'(q^w_{\beta m+1}) > 0$; $\gamma > 0$; $H_{\beta m} > H_{\beta m+1}$; $s^w > 0$; and $\mu^w_{\beta m} > 0$, then there is no contradiction for the public broadcaster too.

The Nash equilibrium of the duopoly competition game is: $q^\pi_i = T$, and $q^w_i = 0$, for $i = 1, \ldots, \beta m$ and $q^\pi_i = 0$, and $q^w_i \in (0, T)$, for $i = \beta m + 1, \ldots, n$. Q.E.D.

Proof of Lemma 1 Constraint (17) of the public broadcaster in duopoly can be written as:

$$\sum_{i=1}^{\beta m} q_i + \sum_{i=\beta m+1}^{n} q_i \leq (1-\beta) m T. \tag{54}$$

Where $\sum_{i=1}^{\beta m} q_i = 0$, because these are the types of programmes offered by the private broadcaster. The public broadcaster is actually a monopolist for programmes of types $i = \beta m + 1, \ldots, n$. Then the problem of the public broadcaster in duopoly can me written as the problem of a public monopolist that chooses the amount to broadcast of all the n types of programmes, with the constraint $q_i = 0$, for $i = 1, \ldots, \beta m$ (because these types of programmes are broadcast by the private broadcaster).

To write this problem we need to rewrite constraint (54), such that we take out the amount of time broadcast by the private broadcaster on its βm channels, $\beta m T$, and give back the time it would have used on its βm channels if it were a public monopolist, $\sum_{i=1}^{\beta m} q^B_i$. Then constraint (54) becomes:

$$\sum_{i=\beta m+1}^{n} q_i \leq (1-\beta) m T + \sum_{i=1}^{\beta m} q^B_i - \beta m T, \tag{55}$$

using constraint (55) and $q_i = 0$ for $i = 1, \ldots, \beta m$, the problem of the public broadcaster in duopoly is equivalent to problem (18), for $i = \beta m + 1, \ldots, n$. Q.E.D.

Proof of Proposition 5 For Lemma 1, in duopoly the public broadcaster's problem is equivalent to problem (18), for $i = \beta m + 1, \ldots, n$. The first order conditions of this problem, for $i = \beta m + 1, \ldots, n$, are:

$$H_i U'(q_i) + \gamma H_i \frac{\partial p}{\partial q_i} s = \overline{\delta} + \gamma c, \tag{56}$$

where $\overline{\delta}$ is the multiplier of constraint (20). The LHS of Eq. (56) is a decreasing function of q_i ($U'(q_i) < 0$ and $U''(q_i) \leq 0$; $\frac{\partial p}{\partial q_i}$ is constant since $\frac{\partial^2 p}{\partial q_i^2} = 0$) and the RHS is constant. As constraint (20) of problem (18) leaves the public broadcaster with less time available than constraint (6) of the public monopolist's problem, it follows that the multiplier $\overline{\delta}$ of problem (18) is larger than the same multiplier of the public monopolist's problem. This results in a lower level of q_i, for $i = \beta m + 1, \ldots, n$. Q.E.D.