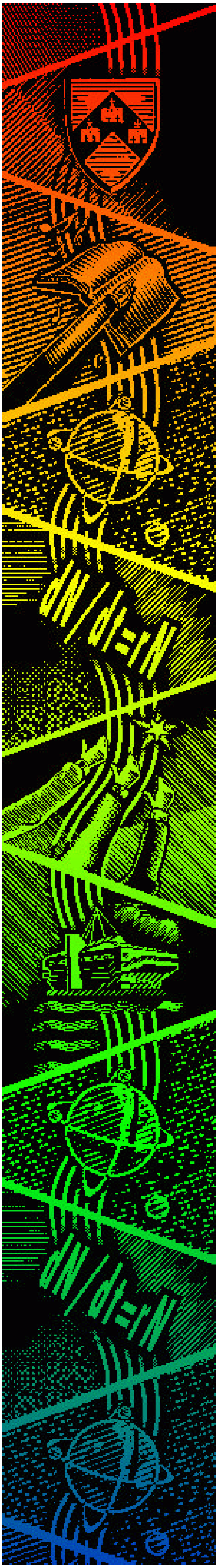

THE UNIVERSITY *of York*

Discussion Papers in Economics

No. 2001/08

Neutral and Non Neutral Shock Effects on Hedging, Investment and Debt

by

Marcello Spanò

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

Neutral and Non Neutral Shock Effects on Hedging, Investment, and Debt

Marcello Spanò*
Department of Economics and Related Studies. University of York (UK)

Abstract

By trading derivatives on the financial markets, a firm can hedge against the fluctuations of its internal funds, in order to better coordinate investment and financing decisions. This work shows how optimal investment, debt, and hedging strategy can be strongly dependent on the mechanism linking the firm's internal funds to its returns on investment. In particular, when internal funds react to a prospective price change (neutral shock), investment and debt would be positively related; when internal funds react to a non neutral productivity shock, investment and debt would be either negatively related (no hedging) or unrelated (hedging).

JEL classification: G19; G31; G32. Keywords: Hedging; Investment; Debt; Volatility; Productivity.

*Via Montecatini 14, 20144 Milano, Italy; e-mail: ms143@york.ac.uk; tel.:+44-7884298430, +39-3494975935; fax: +39-02471231.

1 Introduction

The technological change can be considered as the main engine of the recent process of transformation of the economic landscape. According to Goldman Sach estimations, in the five years between 1995 and 2000 the investment in information technology in USA has increased by an annual average of around 25% in real term. It is commonly known that the information technology has a shorter life than the more traditional capital equipments, such as buildings or industrial machinery. Because of its higher rate of depreciation, the fixed capital in the modern sectors has to be considered as more easily reversible and more volatile than the traditional sectors' fixed capital. Not only the investment is volatile: in a period of radical restructuring of the production technologies and the final products offered, also the financial investors' confidence in their knowledge about the real value of the firm is likely to be rather more unstable than in periods of ordinary business. As a result, corporations investing in new sectors may face greater uncertainty about the internal funds available to the investment, as well as about the outcome of the investment.

Firms carrying on investments with highly uncertain outcome are likely to face also high borrowing costs for the following reasons: first, the lenders ask for a higher risk premium in order to finance riskier activities; second, the assets of the high tech firms are highly intangible and cannot be used as reliable collateral; third, the equities are comparably much cheaper sources of finance, especially in a period of high enthusiasm for the 'new economy' shares; fourth, as the main source of finance is the stock market, the wealth of the firm is likely to be almost as volatile as its share price and the probability of default in case of a share price's fall can be particularly high. The literature on corporate finance has found several rationales for the external finance (i.e. new equities or new debt) to be costly. Jensen and Meckling (1976) argue that the external finance is costly as the investors cannot control the actions of the managers unless paying agency costs. Mayers and Majful (1984) show that raising new equities can be costly as the asymmetric information between the insiders and outsiders leads to underestimate the value of the better quality firms. Despite the banks, according to Diamond (1984), act as corporate monitors to mitigate the agency costs, the informational asymmetries between lenders and the borrowers may cause credit rationing, as Stiglitz and Weiss (1981) show. In general, it can be argued that firms give priority to the internal finance or cash flow to finance their projects of investment. The more a firm is financially constrained, the more its investment choices will be affected by the availability of the internal finance. Jensen (1986) suggests that the investment choice of the firm can be strongly dependent on the cash flow available to the managers. The effects of the financial constraints on the investment choices of the firms are investigated in a number of works, for example Fazzari, Hubbard, and Petersen (1988), Greenwald and Stiglitz (1993), Gertler (1988).[1]

However, an economic environment characterised by technological innovations, investment reversibility, and volatile share prices would suggests that not only the pure availability, but also the volatility of the internal sources of finance may affect the investment choice and the capital structure of the firms. The issues related to the volatility are investigated, within the corporate finance theory, by a relatively small number of contributions that deal with risk management. The risk management, i.e. trading derivatives on the financial markets, can be a useful tool available to any firm wishing to modify its own exposure to some hedgeable risk, for example, market value, currency, interest rate, commodity price risks. Different theoretical models on risk management share the common consideration that hedging can affect the payoff of a risk-neutral firm as long as some market imperfections make the firm's payoff a concave function of some state contingent variable. The rationales for the concavity of the payoff function can be related to the firm's tax schedule (Smith et al., 1985), to the costs of financial distress (Smith et al., 1985; Shapiro et al., 1998), to agency costs (Stulz, 1990), to asymmetric information problems (Rebello, 1995; DeMarzo et al., 1995), to costly external finance (Froot et al., 1993), or to a combination of some of these factors (Leland, 1998). Most of the models on corporate hedging, however, do not derive the investment decisions of the firms, as they assume given investment and focus, instead, on the choices of

[1]See Schiantarelli (1996) for a short survey, where some methodological issues and empirical evidence are also discussed.

the optimal capital structure. A valuable exception is the contribution of Froot, Scharfstein, and Stein (1993), where investment and external finance decisions are endogenously determined. Froot, Scharfstein, and Stein (1993) build up their model on the assumption that the returns on investment are partially related to the internal funds available. More exactly, they assume that a change of the internal funds is linked to a *neutral* (i.e. multiplicative) shock to the investment function. However, the neutral shock can be identified as a simple price change (or investment opportunity) effect, whereas in a context of technical change, the shock to the investment function is likely to be *non neutral*.

The present work centers around the comparison between a neutral 'investment opportunity' shock and a non neutral 'technical change' shock. The main questions is: How these different types of shock can affect hedging, investment and debt decisions of the firm? By working out the framework by Froot, Scharfstein, and Stein (1993) of hedging with costly external finance, two possible mechanisms linking the returns to investment to the shock to the internal finance are modelled and compared. In the first one (here called Investment Opportunity, IO), the shocks to cash flow are assumed to be related to multiplicative shocks to the investment function, such as shocks to the price of the final product. In the second one (here called Technical Change, TC), the shocks to cash flow are assumed to be linked to non neutral shocks to investment technology. The general solutions, expressed in non closed-form, do not allow for clear understanding about the determinants and the properties of hedging decision and the effect of hedging on investment and debt choices. To overcome this limit, the two models, IO and TC, are compared in the light of their empirical implications by using approximated analytical solutions.

The two mechanisms are presented, first, in the general formulation ending up with non closed-form solutions (section 2), then in the approximated formulation, where analytical solutions for hedging, investment and debt are derived (section 3). Subsequently, the two mechanisms are compared, first, by demonstrating the properties of the optimal hedging strategies (section 4), then, by studying the effects of hedging on the investment and debt decisions (section 5), finally, by simulating how differently financing and investing behaviour react to the same productivity shock (section 6). Section 7 concludes.

2 The two models of hedging behaviour

This section works out the framework of Froot, Scharfstein, and Stein (1993) and presents two models, Investment Opportunity (IO) and Technical Change (TC), each one describing the behaviour of a firm that chooses its risk management program in order to better coordinate its investing and financing policies. The firm operates in a context where its returns on investment are partially related to the fluctuations of the sources of its internal finance and the external finance is increasingly costly. By trading forward and future contracts in the derivative market, the firm can avoid unnecessary fluctuations of the value of its existing assets. Both models show how the firm can calculate the optimal width of fluctuations and select accordingly its optimal hedging ratio. The two models are similar in everything but the mechanism linking cash flow fluctuations to the uncertain returns to investment.

2.1 The setup

A risk-neutral firm faces costly debt and uncertain returns to investment and solves two decisional problems: (i) a simultaneous choice about how much to invest and how much debt to raise, (ii) a choice of how much to hedge against its internal funds fluctuations. The time structure of both the models is the following (see also Table 1): at time 0, when both returns to investment and internal funds available are uncertain, it chooses its hedging strategy. At time 1, when the variable to be hedged is realised, it chooses the amount of investment and debt. At time 2, the production is realised and sold and the debt is repaid.

Table 1 - *Time structure*

time 0	time 1	time 2
Hedging strategy h^*	Investment, Debt	Output is sold
(against fluctuations of ε)	I^*, D^*	Debt is repaid
	ε, θ are realised	

The analytical structure of the firm's maximisation problem is built up on the following set of assumptions. Where it is not specified, the assumptions are common to both IO and TC models.

Marginal costs of debt:

The marginal cost of debt for both IO and TC mechanisms is an increasing function of the amount. It is modelled as a generic function $C(D)$, where D is the amount of debt, $C'(D) = C_D > 0$ and $C''(D) = C_{DD} > 0$. The cost of debt, as FSS point out, can arise from different sources, such as cost of bankruptcy and financial distress, informational asymmetry between lenders and borrowers, private benefits to the managers from limiting their dependence on external investors. Other sources of external finance are not considered. [2]

Random value of the internal funds

As the debt is increasingly costly, the firm prefers to raise the level of the debt only when it cannot provide enough internal funds to its project of investment. Its budget constraint at time 1 is given by $I = V + D$, where V is the value of the internal funds available at time 1. Without any hedging policy, the value of the internal funds is given by $V = V_0\varepsilon$, where V_0 is the initial value of the assets and ε is a hedgeable source of uncertainty,

[2] More exactly, Froot, Scharfstein, and Stein (1993) assume costly external finance, which includes also new equities (p.1633-4). Even though such more general assumption would not change the structure and the results of this work, in this paper the emphasis is on the debt as a more important source of external finance than new equity issues. This simplification is also carried on in the work of Whited (1992), on the ground of several empirical contributions showing that "share issues typically account for less than 5% of total new external finance" (p.1426). It can be also justified by an assumption of equity rationing, such as in Greenwald and Stiglitz (1993).

distributed as a Normal with mean 1 and variance σ^2. The budget constraint of a non-hedging firm would be given by

$$I = D + V_0 \varepsilon. \tag{1}$$

By trading derivatives at time 0, the firm can modify the distribution of its cash flow across possible values of the shock ε. The choice to hedge is modeled under the following simplifying assumptions: (i) the fluctuations of the cash flow, V, are completely hedgeable; (ii) hedging does not alter the expected value of the cash flow; (iii) hedging is linear, i.e. the sensitivity of the cash to the changes of the random variable is constant. The latter assumption concretely means that the usage of derivative described in this model is limited only to forward and future contracts, therefore options contracts are ruled out. The internal funds after hedging are given by $V = V_0[h + (1 - h)\varepsilon]$, and the budget constraint becomes

$$I = D + V_0[h + (1 - h)\varepsilon], \tag{2}$$

where the value of h is determined at time 0, as a solution of the maximisation problem. In the special case of full hedging, where $h = 1$, the distribution collapses to the mean, and the value of the existing assets becomes non-stochastic: $V = V_0$.

Returns on investment:

The two models, IO and TC, are different for the assumption about the shock to the investment function. In both models the marginal returns to investment are decreasing.

- **IO investment function.** The net present value of investment expenditures is given by

$$F(I) = \theta f(I) - I, \tag{3}$$

where I is the investment, $f(I)$ is the expected revenue of the output, with $f'(I) = f_I > 0$ and $f''(I) = f_{II} < 0$. θ is a multiplicative shock to the expected outcome of the investment decision.

- **TC investment function.** The net present value of investment expenditures is given by

$$F(I, \theta) = f(I, \theta) - I, \tag{4}$$

where, as before, $f_I > 0$ and $f_{II} < 0$. The shock to the investment function, θ, which is no longer multiplicative, represents a change of the investment technology.

The shock to the investment function, θ, is neutral for the IO model, non-neutral for the TC one. To simplify the interpretation, one can think that the IO company is hit by a shock to the price of its final product, whereas the TC company is hit by a shock to its investment function elasticity. Therefore, the variable θ incorporates the randomness of the investment opportunities in the first model (IO) and the randomness of the production technology in the second one (TC).

Link between returns on investment and internal funds.

The shock θ is related to the internal funds according to a parameter α.

- **IO shocks relation.** The neutral shock to the investment function, θ, is given by

$$\theta = \alpha(\varepsilon - 1) + 1. \tag{5}$$

- **TC shocks relation.** The non-neutral shock to the investment function, θ, is given by

$$\theta = \alpha(\varepsilon - 1) + \beta. \tag{6}$$

To illustrate the two different shocks, it can be thought about the links, through market expectations, between the firm's investment decisions and its market value. The IO company, producing information technology, will find that the market price of its product (say, a software) increases together with its market value on the stock market. The TC company deciding to introduce some new information technology into its old production process will find, instead, that its market value increases together with the expected profit from the new technology expenditure.

In the IO model, θ represents either a change in the price of the final product or a neutral technical change, hence its expected level is equal to 1 by construction. In the TC model, θ represents, instead, a non neutral shock to the investment function, hence, its expected value is equal to some expected parameter of the investment function. For example, if θ is the elasticity of the investment function, its expected value, β, will be equal to the expected value of the elasticity.

2.2 The optimal hedging strategies

The profit function is given by the difference between net revenues on investment expenditures and the full repayment of debt: $\pi = F(I) - C(D)$. Such function takes the following forms, respectively, for the IO and the TC models:

$$\textbf{IO profit:} \quad \pi = \theta f(I) - I - C(D), \tag{7}$$

$$\textbf{TC profit:} \quad \pi = f(I, \theta) - I - C(D). \tag{8}$$

The firm maximises the profit function with respect to the investment at time 1, when all ε, θ and V are realised. The discount rate is assumed equal to zero for simplicity. The first order conditions of time 1's maximisation problem are then given by

$$\textbf{IO f.o.c.:} \quad \theta f_I = 1 + C_D \tag{9}$$

for the IO model, and

$$\textbf{TC f.o.c.:} \quad f_I = 1 + C_D \tag{10}$$

for the TC model, where f_I in 10 is a function of both I and θ.[3]

Moving back to period 0, when the internal funds are still uncertain, the firm maximises its expected profit with respect to the hedging ratio, h:

$$\max_h E_0[\pi(V(\varepsilon, h)]. \tag{11}$$

[3]For both models, at time 1 V is given, hence $\frac{dD}{dI} = 1$.

The general solutions to the problem 11 are different for the two models: for the IO model, the optimal hedging strategy is given by

$$h^*_{IO} = 1 + \frac{\alpha}{V_0} \frac{E_0\left[\frac{-f_I C_{DD}}{\theta f_{II} - C_{DD}}\right]}{E_0\left[\frac{-\theta f_I C_{DD}}{(\theta f_{II} - C_{DD})}\right]}, \tag{12}$$

whereas for the TC model it is given by

$$h^*_{TC} = 1 + \frac{E_0\left[\frac{-C_{DD}}{f_{II} - C_{DD}} \frac{\partial f_I}{\partial \varepsilon}\right]}{V_0 E_0\left[-\frac{f_I C_{DD}}{f_{II} - C_{DD}}\right]}. \tag{13}$$

The derivation of both formulas is in the Appendix.

In general, it can be observed that both optimal hedging strategies depend on the parameter α, expressing the relation between returns to investment and internal funds. In equation 12, a decreasing effect of α on the optimal hedging ratio is clearly visible, whereas in equation 13, such effect passes through the expression $\frac{\partial f_I}{\partial \varepsilon}$, where α is contained.[4]

A first empirical implication of both IO and TC mechanisms is that the best strategy is to fully hedge ($h = 1$) when there is no relation between returns on investment and internal funds fluctuations ($\alpha = 0$). In fact, there is no reason to let the firm's cash flow fluctuate if such fluctuations are independent from the firm's extra finance requirements.

From the hedging strategies expressed in general forms, as in 12 and 13, little more can be said about the determinants of hedging and the effect of hedging on investment and debt decisions, as well as about the difference and the empirical implications of both mechanisms, IO and TC, the reason being that expressions 12 and 13 are non closed-form solutions.[5] In the following sections, the different implications of both mechanisms are better investigated by comparing the analytical approximation of their solutions.

[4] In other words, the link between optimal hedging and the parameter α depends, in both solutions, on the sensitivity of the marginal return to investment to a change in the variable to be hedged. However, while in the IO model such sensitivity is constant and simplifies to αf_I, in the TC model $\frac{\partial f_I}{\partial \varepsilon}$ is not necessarily constant. See Appendix for technical details.

[5] The ratio between expected values on the rhs of both expressions includes, firstly, the levels of the investment and the debt, both depending on ε and h^*, secondly, a direct effect of ε on h^* through the shock to the investment function, θ.

3 The approximation

This section derives the approximated analytical solutions for optimal hedging strategies, investment and debt functions of both IO and TC mechanisms.[6] The results are commented in the subsequent sections.

The approximation method consists in carrying on a second order Taylor expansion of the investment and cost functions, respectively, around the expected levels of the investment, I_0, and the debt, D_0. The approximated expected revenue and cost functions defined above, i.e. $f(I)$ and $C(D)$, take the following quadratic forms:

$$f(I) \simeq \frac{a}{2}I^2 + bI + k, \tag{14}$$

with $a = f_{II}(I_0) < 0$, $b = f_I(I_0) - I_0 f_{II}(I_0) > 0$ and $k = f(I_0) - I_0 f_I(I_0) + \frac{1}{2}I_0^2 f_{II}(I_0)$, where $f_I = aI + b$, $f_{II} = a$;

$$C(D) \simeq \frac{c}{2}D^2 + rD + z, \tag{15}$$

with $c = C_{DD}(D_0) > 0$, $r = C_D(D_0) - D_0 C_{DD}(D_0) > 0$ and $z = C(D_0) - D_0 C_D(D_0) + \frac{1}{2}D_0^2 C_{DD}(D_0)$, where $C_D = r + cD$ and $C_{DD} = c$. This approximation simplifies the analysis of equations 12 and 13, as the second derivatives of both approximated functions, 14 and 15 are constant.

The multiplicative shock for the IO model is still given by expression 5, i.e. a neutral hit to the investment function related to a change of the internal funds available. The non neutral shock of the TC model has to be better specified. To simplify the analysis, let's assume that the shock to the investment function related to the internal funds is a hit to the marginal product of the investment function that leaves unchanged its concavity calculated at I_0. Therefore, starting from the second order Taylor approximation of the investment function, 14, the shock to the elasticity is defined as a shock to the parameter b:

[6] The three unknowns are h, I, D for both the IO and TC mechanisms; the systems of equations to be solved are 2, 9 and 12, for IO, and 2, 10 and 13 for TC.

$$b = \alpha(\varepsilon - 1) + \mu, \tag{16}$$

where μ is the expected level of b.

3.1 The IO solution

After substituting for the approximated functions 14 and 15, the first order condition of time 1 maximisation problem for the IO model is now given by

$$\theta(aI + b) = 1 + r + cD, \tag{17}$$

where θ is given with certainty at time 1, together with ε.

Investment and debt functions of a *non hedging firm* are derived by combining the first order condition 17 with the non hedger's budget constraint, 1:

$$I^*(\varepsilon) = \frac{\theta b - (1 + r) + cV_0\varepsilon}{c - \theta a}, \tag{18}$$

$$D^*(\varepsilon) = \frac{\theta b - (1 + r) + \theta a V_0\varepsilon}{c - \theta a}, \tag{19}$$

where θ is given by 5.

By using, instead, the hedger's budget constraint, 2, the investment and the debt functions of the *hedging firm* turn out to be the following:

$$I_h^{IO}(\varepsilon) = \frac{\theta b - (1 + r) + cV_0}{c - \theta a} + \frac{cV_0(1 - \varepsilon)}{c - \theta a}(h_{IO} - 1), \tag{20}$$

$$D_h^{IO}(\varepsilon) = \frac{\theta b - (1 + r) + \theta a V_0}{c - \theta a} + \frac{\theta a V_0(1 - \varepsilon)}{c - \theta a}(h_{IO} - 1), \tag{21}$$

The optimal hedging strategy is derived from a second order Taylor expansion of the two expected terms around $\varepsilon = 1$ of equation 12, after substituting for the approximated functions' derivatives, f_I, f_{II}, and C_{DD}, and for the optimal investment (equation 20) into the expression for f_I:

$$h_{IO} = 1 + \frac{\alpha}{V_0}\frac{\left(1 + r - cV_0 - \frac{bc}{a}\right)\left((a-c)^2 + 3a^2\alpha^2\sigma^2\right)}{(a-c)\left((a-c)^2 + 3ac\alpha^2\sigma^2\right)}. \tag{22}$$

Finally, equation 22 can be substituted into equations 20 an 21 to find the explicit solutions for the investment and the debt levels.

In both cases of hedging and no hedging, the investment and debt choices depend on the realisation of ε at time 1. However, for the hedger they depend *also* on the optimal hedging strategy h^*, which can regulate the effects of the random variable's fluctuations according to the firm's own necessities.

3.2 The TC solution

After substituting approximated functions 14 and 15, the first order condition of time 1 maximisation problem for the TC model becomes

$$aI + b(\varepsilon) = 1 + r + cD, \tag{23}$$

where b is given with certainty at time 1, together with ε.

Given expression 16 for the technical change, the sensitivity of the marginal return to investment to a change in the variable to be hedged, $\frac{\partial f_I}{\partial \varepsilon}$ (which is a determinant of the hedging strategy in 13) becomes:

$$\frac{\partial f_I}{\partial \varepsilon} = \alpha. \tag{24}$$

Substituting in 13 for the approximated functions' derivatives, f_I, f_{II}, and C_{DD}, and for $\frac{\partial f_I}{\partial \varepsilon}$, the optimal hedging strategy simplifies to

$$h_{TC} = 1 + \frac{\alpha}{V_0 a}, \tag{25}$$

all the terms included in equation 13's expected values being constant.

Solving the system of two equations 23 and 1 gives the optimal levels of investment and debt of the *firm that does not hedge* against the fluctuations of its internal funds:

$$I^{TC}(\varepsilon) = \frac{1}{c-a}\left[\alpha(\varepsilon - 1) + \mu - (1+r) + cV_0\varepsilon\right] \tag{26}$$

and

$$D^{TC}(\varepsilon) = \frac{1}{c-a}\left[\alpha(\varepsilon - 1) + \mu - (1+r) + aV_0\varepsilon\right]. \tag{27}$$

Solving the system of three equations 23, 2 and 25 gives the optimal investment and debt of a *firm that hedges* against its internal funds fluctuations:

$$I_h^{TC}(\varepsilon) = -\frac{\alpha}{a}(\varepsilon - 1) + \frac{1}{c-a}\left[\mu - (1+r) + cV_0\right] \tag{28}$$

and

$$D_h^{TC} = \frac{1}{c-a}\left[\mu - (1+r) + aV_0\right]. \tag{29}$$

4 Properties of hedging

This section derives the implications of the approximated analytical solutions for the optimal hedging strategies of both IO and TC models.

A first result, which is not visible in the implicit solutions 12 and 13, is that the optimal hedging ratio is not affected by the level of the variable to be hedged. This is consistent with the setup of the model, as the level of ε, by assumption, is only known at time 1, when the hedging decision is taken. In particular, from the two approximated analytical solutions the following propositions can be derived:

Proposition 1 *When internal funds are linked to a prospective price change (IO), the optimal hedging ratio, h_{IO}, is a function of the volatility of the shock to the internal funds, σ^2. The level of the shock, ε, does not have any effect on the optimal hedging strategy.*

Proposition 1 does not need proof, as it states what is visible from the explicit solution for IO hedging (22). Therefore, the analytical solution for the IO model allows one to substitute a virtually measurable variable, the standard deviation of the marketable shock, σ, to the unobservable expectation in the implicit formula for hedging (12). In the TC solution, on the other hand, the variance does not play any role:

Proposition 2 *When internal funds are linked to a productivity shock that does not affect the concavity of the investment function (TC), then the variance of the variable to be hedged does not affect the optimal hedging strategy.*

Also Proposition 2 does not need proof as it states what is visible from the explicit solution for TC hedging (25): the variance of the internal funds fluctuations in the TC model does not affect the hedging decision, which is affected only by the relation between internal funds and productivity shock, α, by the current cash flow of the firm, V_0, and by the concavity of the investment function, a.

The relationship between optimal hedging ratios and the parameter linking returns to investment and internal funds, α, is demonstrated by the following proposition and shown in Figure 1 for both the IO and TC solutions.

Proposition 3 *When internal funds are linked to a prospective price change (IO), the greater is the relation between internal funds and returns to investment, α, the lower is the hedging ratio, h, for any σ^2 lower than the critical value $\sigma^{*2} = \frac{-(a-c)^2}{3\alpha^2 ac}$. When internal funds are linked to a productivity shock that does not affect the concavity of the investment function (TC), this decreasing relation is linear.*

Proof of Proposition 3 is provided in the Appendix.

Figure 1 illustrates the shapes of the two hedging strategy as functions of the correlation parameter α.[7]

[7] The parameterisation of Figure 1 is the same as described in section 6.

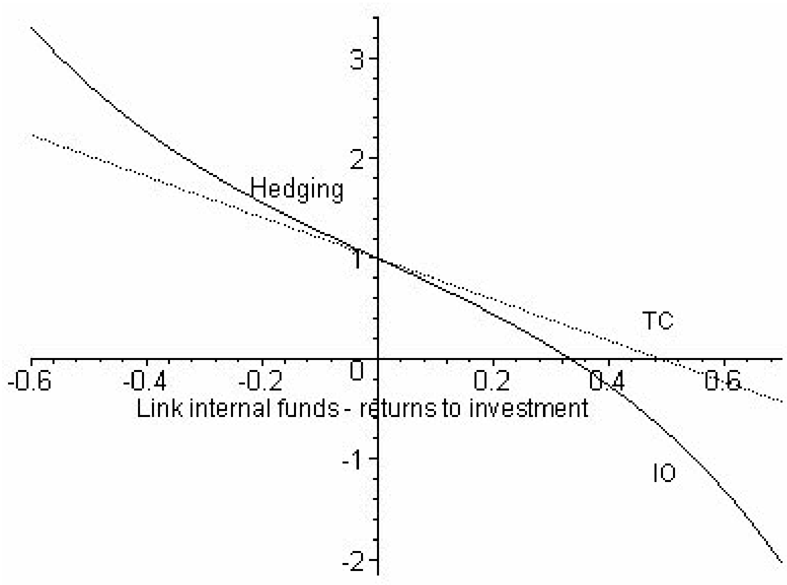

Figure 1

If $\alpha = 0$, the best strategy for the firm is to fully hedge ($h^* = 1$) against the fluctuations of its cash flow, as they are unrelated to the investment opportunities. If $\alpha > 0$, such correlation is positive, and the firm can take advantage of a positive fluctuation in its cash flow to provide an extra amount of funds to a planned extra amount of investment. The optimal strategy, in this case, is a hedge ratio lower than 1, i.e. the firm hedges in order to reduce the volatility of the internal cash flow without completely eliminating it. The higher is α, the lower is the hedge ratio, which is equal to zero when the firm does not hedge at all and lets its internal funds fluctuate according to the primary market movements. If α is high enough, the best strategy can be the speculative one ($h^* < 0$), which amplifies the fluctuations of the cash flow and increases the firm's exposure to the risk. Finally, if $\alpha < 0$, the relation between internal funds and returns on investment is negative, therefore the best strategy is overhedging, in order to raise cash when ε is low. Comparing the two hedging functions, h_{IO} (continuous line) and h_{TC} (dotted line), it can be observed that the sensitivity of the hedging ratio to the value of α is greater for the former than for the latter: greater relations between returns on investment and internal funds is needed in the TC model to move away from the full hedging ratio. The upper limit to the internal funds volatility for the IO model (σ^{*2}), for a hedging strategy to be feasible, depends on the concavity of the profit function, expressed by the parameters a and c, and on the absolute value of the relation parameter α.

5 Effects of hedging on debt and investment

This section derives the implications of the approximated analytical solutions for the optimal investment and debt.

Proposition 4 *If investment opportunity is related to the internal funds fluctuations (IO model), and if the marginal return on investment is more sensitive to the level of the investment than the marginal cost of debt to the level of the debt ($c < -a$), then the effect of the optimal hedging strategy is to stabilize more the debt than the investment.*

Proof of Proposition 4 is provided in the Appendix.

Proposition 4 states that in the IO model, under the condition that the second derivative of the investment function ($-a = f_{II}$) is greater than the second derivative of the debt function ($c = C_{DD}$), the debt functions after hedging are much flatter than the investment functions after hedging. In other words, provided that the revenue of an extra unit of investment is more sensitive to a shock than the cost of an extra unit of debt, the negative events are weighted by the firm more than the positive ones, therefore, the firm prefers, when it is possible, to slightly sacrifice the probability of higher investment in order to drastically decrease the probability of a higher debt. The fluctuations of the debt function, however, are not fully eliminable (the first term on the RHS of the debt function, 21, still fluctuates with θ independently from the value of h_{IO}).

In the TC model, conversely, hedging *even partially* has the effect of *fully eliminate* the fluctuations of the debt.

Proposition 5 *When internal funds are linked to a productivity shock that does not affect the concavity of the investment function (TC), then the effect of the optimal hedging strategy is to fully stabilize the level of the debt.*

Proof of proposition 5 is provided in the Appendix.

Investment and debt, without hedging, depend on the realisation of ε through two factors: the change in the marginal product of investment, $\alpha(\varepsilon - 1)$, and the change in the internal funds available, $V_0\varepsilon$ (see expressions 26 and 27). The effect of hedging for the TC model is to set the debt at a constant level, whereas fluctuations of the investment and the internal funds are maintained to some extent after hedging, according to the value of the parameter α.

The case of idiosyncratic return on investment, i.e. a return to investment completely unrelated to movements of the internal funds available (i.e. $\alpha = 0$), is a special case for both the IO and TC models. As already seen from Figure 1, as well as from the general solutions 12 and 13, the optimal hedging strategy will be to fully hedge. The impact of the full hedging strategy in case of idiosyncratic return to investment, is to completely eliminate the randomness of the investment and the debt functions.

Proposition 6 *If returns of investment are unrelated to the internal funds fluctuations ($\alpha = 0$ for both IO and TC), then the full hedging optimal strategy will fully stabilize both the investment and debt decisions.*

The proof follows from substituting for $\alpha = 0$ in equations 20, 21, 28, and 29.

Table 2 shows that when the returns on investment are not related to internal funds fluctuations, the fluctuations in the desired investment and debt levels (first column) would depend only on the internal finance component, $V_0\varepsilon$, and not on the shocks to the investment function parameters, (given by $\alpha(\varepsilon - 1)$ in equations 26 and 27, and by θ in equations 18 and 19). Hence, the full hedging strategy would fix the investment and debt levels at their expected values, independently from realisation of the hedgeable shock, ε.

Table 2 - *Idiosyncratic technological change*

$\alpha = 0$	no hedging	hedging
I_{IO}	$\frac{b-(1+r)+cV_0\varepsilon}{c-a}$	$\frac{b-(1+r)+cV_0}{c-a}$
D_{IO}	$\frac{b-(1+r)+aV_0\varepsilon}{c-a}$	$\frac{b-(1+r)+aV_0}{c-a}$
I_{TC}	$\frac{1}{c-a}\left[\mu-(1+r)+cV_0\varepsilon\right]$	$\frac{1}{c-a}\left[\mu-(1+r)+cV_0\right]$
D_{TC}	$\frac{1}{c-a}\left[\mu-(1+r)+aV_0\varepsilon\right]$	$\frac{1}{c-a}\left[\mu-(1+r)+aV_0\right]$

6 Effects of a productivity shock

To better understand the difference between the two models introduced in the previous sections, in this section the Investment Opportunities (IO) and the Technological Change (TC) mechanisms are compared by observing the effects of a common shock starting from a common initial equilibrium.

A shock to the parameter b, i.e. a shock to the marginal product of investment, is exogenous in the IO model and endogenous in the TC model. In the latter, a technological shock is represented by a shift along both the investment and debt curves, whereas in the former it is represented by an upwards shift of the whole curves.

The following figures show the differences between the two alternative behaviour. Respectively, they show the effects of the same technological shock: (i) on the investment functions in case of no hedging (Figure 2); (ii) on the investment functions in case of hedging (Figure 3); (iii) on the debt functions in case of no hedging (Figure 4); (iv) on the debt functions in case of hedging (Figure 5).

The parameters of the investment and cost functions (a, b, k, c, r, and z from equations 14 and 15) are calibrated to respect the following criteria: (i) the expected value of the product elasticity to the investment is equal to 0.25, a value typically used in the Cobb-Douglas production function for the elasticity of the capital; (ii) the expected investment is greater than the expected internal funds available ($I_0 > V_0$).[8] The expected cash flow, V_0, is

[8]The parameters of the approximated analytical solution are obtained by carrying on the second order Taylor expansion of Cobb-Douglas functions around the expected equilibrium (I_0,V_0,D_0, $\varepsilon = 1$).

equal to 10; the expected investment, I_0, is equal to 20 and, consequently, the expected debt, D_0, is equal to 10. The standard deviation of the shock to the internal funds is $\sigma = 0.7$. The parameter α is set equal to 0.2. The shock to b is a shift from 2.27 to 2.48, which in the TC model is related to a shock to the internal funds, ε, from 1 to 1.2 (x axis), whereas in the IO model, ε remains unchanged. The continuous lines are the TC curves, the dotted lines are the IO ones.

The quantitative effects of the same marginal productivity shock on the two models' investment choices are slightly greater in case of no hedging (Figure 2) than in case of hedging (Figure 3). In both figures, the IO mechanism shows an upwards shift of the curve and a rise of the new optimal investment, corresponding to an unchanged level of the internal funds, $\varepsilon = 1$, whereas the TC mechanism show relatively smaller rise of the optimal investment, corresponding to a new position on the curve at $\varepsilon = 1.2$. After hedging (Figure 3), the investment curves are slightly flatter, but the reactions of the investment decisions to the shock do not change in a relevant way with respect to the non hedging firm.

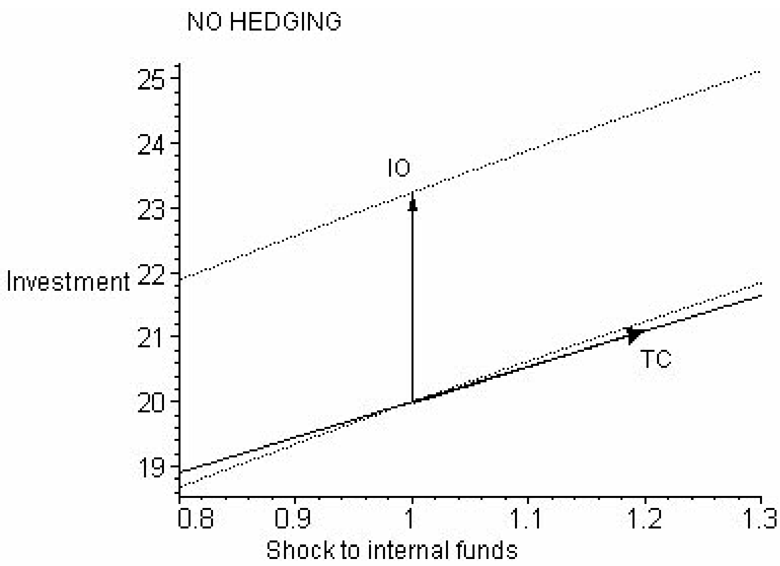

Figure 2

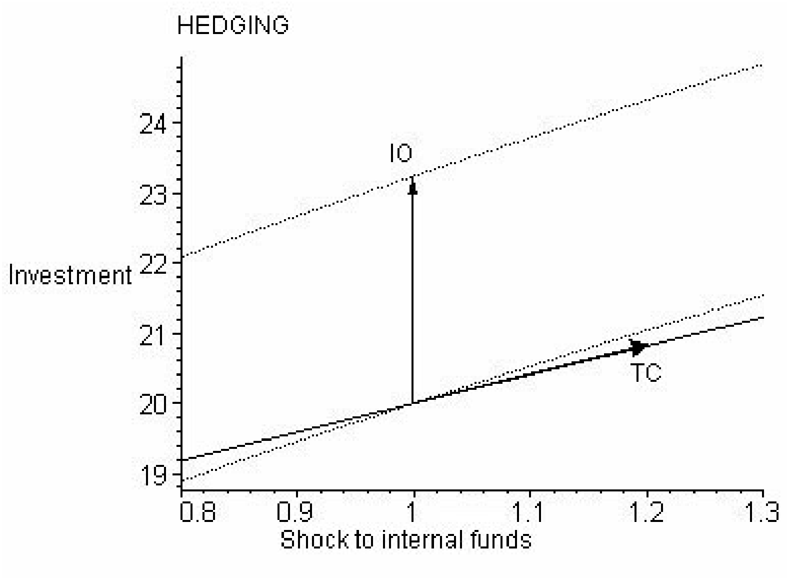

Figure 3

The two models are much more different for debt than for investment behaviour. After the technological shock, the IO mechanism shows a rise in the debt level, the level of the internal funds being unchanged ($\varepsilon = 1$), whereas the TC mechanism shows a reduction of the optimal debt in case of no hedging (Figure 4), and a fixed level of debt in case of hedging (Figure 5).

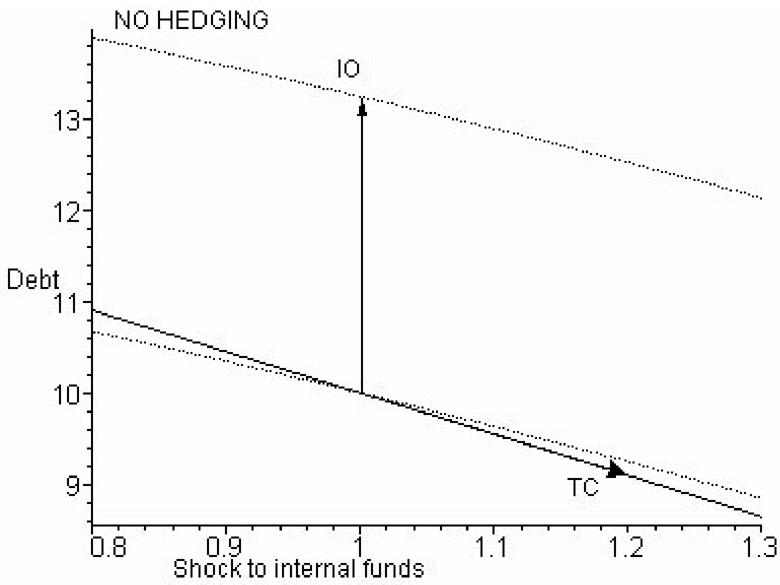

Figure 4

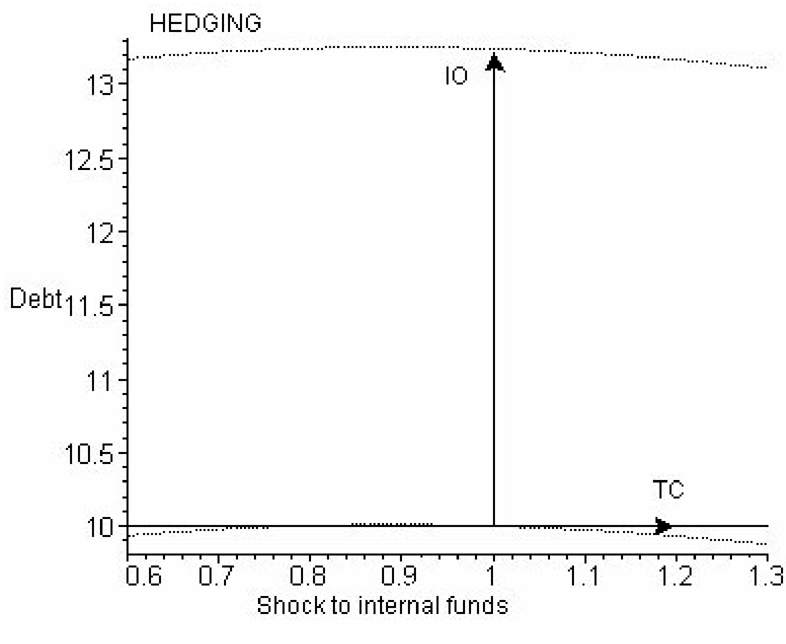

Figure 5

The different reactions to the same productivity shock rely on the different behaviour of the internal funds available to the firm. While in the TC model the internal funds rise along with the extra investment needed, in the IO model a rise of the investment productivity is not related to any change in the internal funds, therefore, the firm needs to raise debt.

The empirical implications of the previous graphs are summarised in Table 3, which shows the different comovements of the variables following a positive non neutral productivity shock.

Table 3 - *Change in marginal productivity of investment*
IO=investment opportunities, TC=technological change

	ε, b unrelated	ε, b positively related
I, D positively related	IO	
I, D negatively related		TC - no hedging
I, D unrelated		TC - hedging

The two alternative mechanisms can virtually be distinguished by observing the behaviour of the same variables after a productivity shock. In the IO mechanism, where the shock is not linked to any change in the internal cash flow, investment and debt would be positively related; in the TC mechanism, where such shock is linked to a change in the internal cash flow, investment and debt would be either negatively related, in case of no hedging, or unrelated, in case of hedging, the level of debt being fixed in the latter case.

The described different behaviour after a non neutral productivity shock should make clear that financing and investment decisions may be strongly affected by the different mechanisms linking internal funds to returns on investment.

7 Conclusion

The aim of this work was to show the effects of risk management on the firms' investment and debt decisions in two alternative models of hedging. The models share the same setup, provided by Froot, Scharfstein, and Stein (1993), where the decision to hedge by using financial derivatives is taken in a context where the returns to investment are partially related to the fluctuations of the internal finance sources and the external finance is increasingly costly. The models are different from each other for the mechanism linking the internal cash flow's fluctuations to the returns to investment. In the first model (Investment Opportunity, IO), cash flow and investment returns are linked through a *neutral shock*, whereas, in the second one (Technical Change, TC), through a *non neutral shock* to the production technology.

Approximated analytical solutions are found for both mechanisms (section 3), allowing for a better understanding than the non closed-form solutions about the variables involved in the optimal hedging decisions. In particular, they show that optimal hedging strategies do not depend on the level of the variable to be hedged. Some propositions are derived about the determinants and the properties of the optimal hedging strategies (section 4), and about the effect of hedging on investment and debt decisions (section 5).

The approximated solution of the TC model has been derived under the simplifying assumption that the shock to the investment function only affects the marginal product of investment without changing the concavity of the investment function. Under such assumption, the effect of the optimal hedging strategy turns out to be to completely fix the debt level, therefore, to completely eliminate the risk of borrowing extra money in case of negative events, whereas, in the IO model, the optimal hedging strategy does not fully eliminate debt fluctuations. It also turns out that, in the TC model, the optimal hedging strategy is determined by the correlation between internal funds and productivity shock, and it is independent from the internal funds volatility, whereas in the IO model the internal funds volatility is among the determinants of hedging.

The same non neutral productivity shock has been used to compare the two models, IO and TC (section 6). The comparison has shown that the firm would react much differently: in the first mechanism (IO), where the shock is not related to any change of the internal funds, investment and debt would be positively related; in the second one (TC), where such shock is related to a change of the internal funds, investment and debt would be either negatively related, in case of no hedging, or unrelated, in case of hedging, the level of debt being fixed in the latter case. Therefore, the different mechanisms linking internal funds and returns to investment imply also different empirical predictions.

8 Appendix

8.1 Proof of expressions 12 and 13

In this appendix, the optimal hedging general solutions (expressions 12 and 13) of the two alternative models, IO and TC, are proven. [9]

The first order condition of the time 0 problem (11) for both the IO and TC models is the following

$$E_0\left[\frac{d\pi}{dV}\frac{dV}{dh}\right] = 0 \tag{30}$$

which is equivalent to solve:

$$E_0\left[\frac{d\pi_v}{d\varepsilon}\right] = 0. \tag{31}$$

In fact, as $\frac{dV}{dh} = V_0(1-\varepsilon)$, equation 30 simplifies in the following way: $V_0 E_0\left[\frac{d\pi}{dV}(1-\varepsilon)\right] = V_0 Cov\left[\frac{d\pi}{dV},\varepsilon\right] = 0$, where $Cov\left[\frac{d\pi}{dV},\varepsilon\right] = E_0[\varepsilon]E_0\left[\frac{d\pi}{dV}\right] - E_0\left[\frac{d\pi}{dV}\varepsilon\right]$ and $E_0[\varepsilon] = 1$. Applying a result by and Rubinstein (1976) to the expression for the covariance, one obtains $E_0\left[\frac{d\left(\frac{d\pi}{dV}\right)}{d\varepsilon}\right] E_0\left[\frac{d\varepsilon}{d\varepsilon}\right] Cov\left[\varepsilon,\varepsilon\right] = 0$, which simplifies to 31.[10]

The two models are different in their expressions for $\frac{d\pi_v}{d\varepsilon}$. This expression is given, in the IO model, by

$$\frac{d\pi_v}{d\varepsilon} = V_0(1-h)\pi_{vv} + \alpha f_I \frac{-C_{DD}}{\theta f_{II} - C_{DD}}, \tag{32}$$

and in the TC model by

$$\frac{d\pi_v}{d\varepsilon} = V_0(1-h)\pi_{vv} + \frac{\partial f_I}{\partial \varepsilon}\frac{-C_{DD}}{f_{II} - C_{DD}}. \tag{33}$$

Taking the expected value and solving each expression for h give the two general formulas for the optimal hedging strategy, respectively, 12 and 13. In this subsection, expressions 32 and 33 are derived.

[9] Froot, Scharfstein, and Stein (1993) show the general solution for 12, but they do not show the derivation.

[10] See Also Froot, Scharfstein, and Stein (1993), note 18 p.1639.

IO and TC models share the next steps of the proofs. Whatever are the profit functions, either 7 or 8, the first derivative of the profit function with respect to the internal funds, V, is given by

$$\pi_v = \frac{dF}{dI}\frac{dI}{dV} - C_D\left(\frac{dI}{dV} - 1\right), \tag{34}$$

where $\frac{dI}{dV} - 1 = \frac{dD}{dV}$. The second derivative of the profit function with respect to V is given by

$$\frac{d\pi_v}{dV} = \frac{d^2F}{dI^2}\left(\frac{dI}{dV}\right)^2 + \frac{d^2I}{dV^2}\frac{dF}{dI} - \frac{d^2C}{dD^2}\left(\frac{dI}{dV} - 1\right)^2 - \frac{d^2I}{dV^2}\frac{dC}{dD}. \tag{35}$$

The time 1 maximisation problem first order condition (either 9 or 10) implies that $\frac{d^2I}{dV^2}\frac{dF}{dI} = \frac{d^2I}{dV^2}\frac{dC}{dD}$, hence,

$$\frac{d\pi_v}{dV} = \frac{d^2F}{dI^2}\left(\frac{dI}{dV}\right)^2 - \frac{d^2C}{dD^2}\left(\frac{dI}{dV} - 1\right)^2. \tag{36}$$

The general expression for the derivative of 34 with respect to ε is:

$$\frac{d\pi_v}{d\varepsilon} = \frac{dI}{dV}\frac{d}{d\varepsilon}\left(\frac{dF}{dI}\right) + \frac{dF}{dI}\frac{d}{d\varepsilon}\left(\frac{dI}{dV}\right) - \left(\frac{dI}{dV} - 1\right)\frac{d}{d\varepsilon}\left(\frac{dC}{dD}\right) - \frac{dC}{dD}\frac{d}{d\varepsilon}\left(\frac{dI}{dV}\right).$$

After substituting time 1's first order condition, $\frac{d\pi_v}{d\varepsilon}$ simplifies as follows:

$$\frac{d\pi_v}{d\varepsilon} = \frac{dI}{dV}\frac{d}{d\varepsilon}\left(\frac{dF}{dI}\right) - \left(\frac{dI}{dV} - 1\right)\frac{d}{d\varepsilon}\left(\frac{dC}{dD}\right). \tag{37}$$

From now on the two models will differentiate from each other for their expressions for $\frac{d}{d\varepsilon}\left(\frac{dF}{dI}\right)$ and $\frac{dI}{dV}$, which imply different expressions for the profit function second derivative $\frac{d\pi_v}{dV}$.

Proof of expression 32:

In the IO model, the expression for $\frac{d}{d\varepsilon}\left(\frac{dF}{dI}\right)$ is given by

$$\frac{d}{d\varepsilon}\left(\theta(\varepsilon) f_I\left(I\left(V(\varepsilon)\right)\right)\right) = \frac{d^2F}{dI^2}\frac{dI}{dV}\frac{dV}{d\varepsilon} + \alpha f_I. \tag{38}$$

The expression for $\frac{dI}{dV}$ is found by applying the implicit function theorem to the first order condition 9 evaluated at the optimal level of the investment, $I = I^*$:

$$\frac{dI^*}{dV} = -\frac{\frac{\partial \pi_I}{\partial V}}{\frac{\partial \pi_I}{\partial I}} = \frac{-C_{DD}}{\theta f_{II} - C_{DD}} > 0. \tag{39}$$

Substituting into 35, the following expression for the second derivative of the profit function 7 with respect to V evaluated at $I = I^*$ is given by

$$\pi_{vv} = \theta f_{II} \left(\frac{dI^*}{dV} \right)^2 - C_{DD} \left(\frac{dI^*}{dV} - 1 \right)^2, \tag{40}$$

After substituting 39, the latter becomes

$$\pi_{vv} = \frac{-\theta f_{II} C_{DD}}{(\theta f_{II} - C_{DD})} < 0, \tag{41}$$

where f_{II} and C_{DD} are evaluated at $I = I^*$. Expression 41 shows that the profit function is concave, i.e. the firm improves the expected profit by reducing the profit riskiness.

Finally, substituting expressions 38, 39, and $\frac{dV}{d\varepsilon} = V_0(1-h)$, from 2, into 37, expression 32 is found.

Proof of expression 33:

In the TC model, the expression for $\frac{d}{d\varepsilon}\left(\frac{dF}{dI}\right)$ is given by

$$\frac{d}{d\varepsilon}\left(f_I(I\left(V(\varepsilon)\right), \varepsilon)\right) = \frac{\partial f_I}{\partial I}\frac{dI}{dV}\frac{dV}{d\varepsilon} + \frac{\partial f_I}{\partial \varepsilon}. \tag{42}$$

The second derivative of the profit function with respect to V evaluated at $I = I^*$ is given by

$$\pi_{vv} = f_{II} \left(\frac{dI^*}{dV} \right)^2 - C_{DD} \left(\frac{dI^*}{dV} - 1 \right)^2,$$

which is equivalent to

$$\pi_{vv} = \frac{-f_{II} C_{DD}}{(f_{II} - C_{DD})} < 0,$$

after substituting for

$$\frac{dI^*}{dV} = -\frac{\frac{\partial \pi_I}{\partial V}}{\frac{\partial \pi_I}{\partial I}} = \frac{-C_{DD}}{f_{II} - C_{DD}} > 0. \tag{43}$$

Substituting expressions 42, 43, and $\frac{dV}{d\varepsilon} = V_0(1-h)$, from 2, into 37, expression 33 is found.

8.2 Proof of Proposition 3

The proof is divided in two parts: part a demonstrates the decreasing relation between h_{IO} and α in the IO model; part b demonstrates a decreasing linear relation between h_{TC} and α in the TC model.

(a) In solution 22 for h_{IO}, the sign of the hedging strategy as a function of α is given by the sign of factor multiplying the ratio $\frac{\alpha}{V_0}$ of the RHS in the equation 22. The expression $\left((a-c)^2 + 3a^2\alpha^2\sigma^2\right)$ at the numerator is always positive as it is a sum of squares. The expression $(a-c)$ at the denominator is always negative by the definitions of the parameters in 14 and 15. The expression $\left(1 + r - cV_0 - \frac{bc}{a}\right)$ at the numerator is positive for values of the parameters a, b, r and c consistent the elasticity of the product to the investment calculated in I_0, $e_I = \frac{aI_0^2 + bI_0}{\frac{a}{2}I_0^2 + bI_0 + k}$. In fact, taking the expectations at time 0 of the optimal investment level from the equation 20, the expected level of investment is $I_0^e = \frac{b-(1+r)+cV_0}{c-a}$; the expression $\left(1 + r - cV_0 - \frac{bc}{a}\right)$ is hence positive if $I_0^e(c-a)-b < -\frac{bc}{a}$, i.e. $I_0^e < -\frac{b}{a} = I_0^{e*}$. This upper bound condition to the expected investment is not binding for values of the parameters consistent with a positive elasticity of the product to the investment: substituting $I_0^{e*} = -\frac{b}{a}$ into the expression for e_I, it turns out that $e_I = 0$. Hence, the ratio that multiplies the parameter α is negative whenever the expression $\left((a-c)^2 + 3ac\alpha^2\sigma^2\right)$ at the denominator is positive, i.e. whenever $\sigma^2 < \sigma^{*2} = \frac{-(a-c)^2}{3\alpha^2 ac}$.

(b) In solution 25 for h_{TC}, the optimal hedging strategy depends linearly on the value of the parameter α. The denominator, $V_0 a$, is negative by the assumptions of the model (concavity of investment function).

8.3 Proof of Proposition 4

The impact of the hedging possibility on the investment and debt choice can be measured, respectively, by the difference between 18 and 20, $I^{IO}(\varepsilon) - I_h^{IO}(\varepsilon) = h_{IO}cV_0(\varepsilon - 1)$, and by the difference between 19 and 21, $D^{IO}(\varepsilon) - D_h^{IO}(\varepsilon) = h_{IO}\theta a V_0(\varepsilon - 1)$. The latter impact is grater than the former if the absolute value of the debt functions difference is greater than the absolute value of the investment functions difference, i.e. $|\theta a| > |c|$. By construction, $c > 0$ and $a < 0$, hence the previous condition becomes $|\theta| > -\frac{c}{a}$. This constraint is not binding if $c < -a$: it can be rewritten as $|\alpha(\varepsilon - 1)| > -\frac{c}{a} - 1$, the LHS being always greater or equal to zero, and the RHS being lower than zero for $c < -a$.

8.4 Proof of Proposition 5

The sensitivity of investment and debt to the cash flow fluctuations, for the two cases of hedging and no hedging, are given by the functions' first derivatives. The investment function's first derivatives, $\frac{dI}{d\varepsilon}$, are equal to $\alpha + cV_0$ for the non-hedger (equation 26), to $-\frac{\alpha}{a}$ for the hedger (equation 28). The debt function's first derivative, $\frac{dD}{d\varepsilon}$, are equal to $\alpha + aV_0$ for the non-hedger (equation 27), to 0 for the hedger (equation 29).

References

De Marzo, P.M., D. Duffie, 1995. Corporate incentives for hedging and hedge accounting. Review of Financial Studies 8, 743–771.

Diamond, D., 1984. Financial intermediation and delegated monitoring. Review of Economic Studies 51, 393–414.

Fazzari, S., Hubbard, R., Petersen, B., 1988. Financing constraints and corporate investment. Brooking Papers on Economic Activity 1, 141–195.

Froot, K.A., Scharfstein, D.S., and Stein, J.C., 1993. Risk management: coordinating corporate investment and financing policies. Journal of Finance 48, 1629–1658.

Gertler, M., 1988. Financial structure and real economic activity. Journal of Money, Credit and Banking 20, 559-598.

Greenwald, B., Stiglitz, J., 1993. Financial market imperfections and business cycles. Quarterly Journal of Economics 108, 77–104.

Jensen, M., 1986. Agency costs of free cash flow, corporate finance and takeovers. American Economic Review 76, 323–339.

Jensen, M., Meckling, W., 1976. Theory of the firm: managerial behaviour, agency costs, and ownership structure. Journal of Financial Economics 4, 305–360.

Leland, H.E.,1998. Agency costs, risk management, and capital structure. Journal of Finance 53, 1213–1243.

Myers, S.C., Majluf, S., 1984. Corporate financing and investment decisions when firms have information that investors do not have. Journal of Financial Economics 13, 187–221.

Rebello, M.J., 1995. Adverse selection costs and the firms financing and insurance decisions. Journal of Financial Intermediation 4, 21–47.

Rubinstein, M., 1976. The valuation of uncertain income streams and the pricing of options. Bell Journal of Economics 7, 407–426.

Schiantarelli, F., 1996. Financial constraints and investment: methodological issues and international evidence. Oxford Review of Economic Policy 12, 70–89.

Shapiro, A., Titman, S., 1998. An integrated approach to corporate risk management, in: Stern, J.M., Chew, D.H., jr. (Eds.), The revolution in corporate finance. Blackwell Business, Malden, Massachusset and Oxford.

Smith, C., Stulz, R., 1985. Managerial discretion and optimal financing policies. Journal of Financial Economics 26, 3–27.

Stiglitz, J., Weiss, A.,1981. Credit rationing in markets with imperfect information. American Economic Review 71, 393–410.

Stulz, R., 1990. Managerial discretion and optimal financing policy. Journal of Financial Economics 26, 3–27.

Whited, T.M., 1992. Debt, liquidity constraints, and corporate investment: evidence from panel data. Journal of Finance 47, 1425–1960.