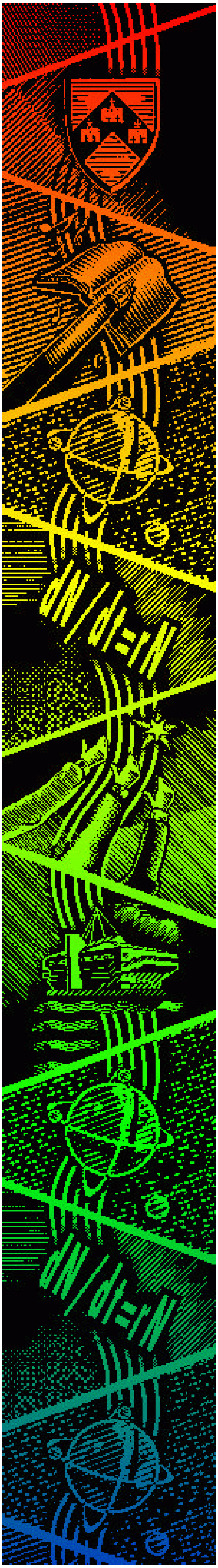

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Investment, Debt and Risk Management in a Context of Uncertain Returns to Investment

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Abstract

In a context of uncertain returns to investment, a firm may face increasing costs of borrowing and uncertain value of its internal finance. Froot, Scharfstein, and Stein (1993) develop a framework where the firm can hedge against the fluctuations of its cash flow, in order to better coordinate investment and financing decisions. This work moves within this framework and finds an approximated analytical solution that allows one to better understand the properties of the optimal hedging strategy, as well as the effects of hedging on firm's investing and financing behaviour. Numerical simulations of the non closed-form optimal solution are also obtained to validate the approximation, which is thus supported by numerical evidence.

JEL classification: G19; G31; G32.

Keywords: Hedging; Investment; Debt; Volatility; Internal funds.

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1 Introduction

A huge wave of innovation in technology and production has involved the whole global economy for a decade between 1991 and 2000. The most advanced features of this process are visible in the USA economy, which has attended to one of its most stable and highest growth patterns in the GDP, the productivity and the employment rate, without any relevant correlated growth in the core-inflation rate. Since 1991 the return on equity for American companies has risen by 108%. The so called new economy, which has leaded the process of general restructuring, has shown a worldwide rise of its share prices' index in the stock markets, up to much higher levels than the old economy share prices' index. Only over 1999, the Nasdaq gained the 85%. However, the face value of the high-tech shares has grown up together with an equally surprising increase in their volatility. As a consequence, the investments in the financial markets have become increasingly risky, so that, according to some commentators, the recent gap between new economy and old economy share prices indexes can be considered as the result of a frenzy of speculation, and the fall of the high tech share prices as the burst of the bubble. It is debateable if the gap between old and new economy corresponds to the proper (rational) evaluation of the firms priced in the stock markets. However, it is commonly agreed that the world's economy is involved in a radical technology-driven economic change.

The rise and the fall of a firm's share price provide, respectively, higher and lower internal financing to its project of investment. Therefore, the fluctuations of the stock markets can ultimately affect the investment choices of firms that face difficulties in raising financing from other sources.

To raise external finance (i.e. new equities or new debt) can be costly for several reasons, as the vast literature on corporate finance has shown. Jensen and Meckling (1976) argue that the external finance is costly as the investors cannot control the actions of the managers unless paying agency costs. Mayers and Majful (1984) show that raising new equities can be costly as the asymmetric information between the insiders and outsiders leads to underestimate the value of the better quality firms. Despite the banks, according to Diamond (1984), act as corporate monitors to mitigate the agency costs, the informational asymmetries between lenders and the borrowers may cause credit rationing, as Stiglitz and Weiss (1981) show. In general, it can be argued that firms give priority to the internal finance or cash flow to finance their projects of investment.

The more a firm is financially constrained, the more its investment choices will be affected by the availability of the internal finance. Jensen (1986) suggests that the investment choice of the firm can be strongly dependent on the cash flow available to the managers. More generally, the effects of the financial constraints on the investment choices of the firms are investigated in a number of works, for example Fazzari, Hubbard, and Petersen (1988), Greenwald and Stiglitz (1993), Gertler (1988).[1]

However, not only the pure availability, but also the volatility of the internal sources of finance may affect the investment choices and the firms performances. The issues related to the volatility are actually investigated, within the corporate finance theory, by relatively small number of contributions that deal with risk management. The risk management, i.e. trading derivatives on the financial markets, can be a useful tool available to any firm wishing to modify its own exposure to some hedgeable risk, for example, its market value's risk. Different models on risk management share the common consideration that hedging can affect the payoff of a risk-neutral firm as long as some market imperfections make the firm's payoff a concave function of some state contingent variable. The reasons for the concavity of the payoff function can be related to the firm's tax schedule (Smith and Stulz (1985)), to the costs of financial distress (Smith and Stulz (1985), Shapiro and Titman (1986)), to agency costs (Stulz (1990)), to asymmetric information problems (Rebello (1995), DeMarzo and Duffie (1995)), to costly external finance (Froot, Scharfstein, and Stein (1993)), or to a combination of some of these factors (Leland (1998)).

Most of the models on corporate hedging, however, do not pay attention to the investment decisions of the firms, as they assume the investment fixed and focus, instead, on the choices of the optimal capital structure. A valuable exception is the contribution of Froot, Scharfstein, and Stein (1993),

[1] See Schiantarelli (1996) for a short survey, where some methodological issues and empirical evidence are also discussed.

which describes the hedging strategy of a firm facing costly external finance and volatile internal finance. The framework of Froot, Scharfstein, and Stein (1993) seems reasonably adaptable to some aspect of the "new economy", where returns to investment are uncertain and share prices volatile (see section 2). However, as the system of equations in the paper of Froot, Scharfstein, and Stein (1993) remains unsolved, the implications of the model for the hedging strategy, the investment, and the financing behaviour of the firm are not explicitly shown, and the economical meaning of the optimal hedging strategy can be easily misinterpreted (see section 4). More exactly, the optimal hedging strategy would appear as depending on the *level* of the variable to be hedged.

This exercise aims at showing the effects of hedging availability on the firms' investment and debt decisions. An approximated analytical solution for the unsolved system of equations of the model is derived, allowing for a better understanding about the variables affecting the optimal hedging strategy. In particular, it shows that the optimal hedging strategy depends on the *variance* (and not on the *level*) of the variable to be hedged, consistently with the timing of the model (see 4.3). This approximated solution is evaluated in the light of numerical simulations of the non-closed-form optimal solution. The effects of some changes of the elasticity parameters are derived and discussed.

Section 2 is a brief description of the economic context related to the new economy events, which may justify the relevance of the risk management as it is modelled in this paper. In section 3, the model of risk management by Froot, Scharfstein, and Stein (1993) is presented. Section 4 shows how the relationships between variables implied by the model can be better understood after a second order approximation of the production and cost functions. In section 5, approximated solutions for optimal debt, investment and risk management are compared to numerically simulated ones. Some numerical experiments are attempted to show some possible effects of technology changes as well as cost of external finance function's changes on the hedging, investment and debt decisions of the firms. Section 6 concludes.

2 The relevance of risk management in the 'new economy' environment

2.1 New investments

The technological change can be considered as the main engine of the process of transformation of the economic landscape. According Goldman Sach estimations, in the five years between 1995 and 2000 the investment in information technology in USA has increased by an annual average of around 25% in real term. It is commonly known that the information technology has a shorter life than the more traditional capital equipments, such as buildings or industrial machinery. Because of its higher rate of depreciation, the fixed capital in the modern sectors has to be considered as more easily reversible and more volatile than the traditional sectors' fixed capital.

In a process of radical restructuring of the whole economy, investment in new technologies involves new and old firms. The new firms are building up their economic activity from the beginning and investing directly in the new high tech sectors. The old firms are motivated to dedicate new investments to restructuring themselves and adapt their production to the new competition mechanism. For example, investing in the e-commerce or information technology is both an opportunity and a necessity, as many firms would probably succumb if they did not do it by the right time. Therefore, by 'new investment' it can be meant either a direct investment in new technology and products, or an investment in restructuring the more traditional economic activities.

The investment spending is strictly related to the stock market performance: during the years of innovation in technology, the information technology investment has closely mirrored the Nasdaq share index. The casual relationship between share prices and investment spending can be considered twofold. Higher (lower) returns to investment and productivity may determine higher (lower) share prices of the companies that carry on the investments, as the share price reflects the greater (lower) profitability of the firm according to the shareholders' opinions. However, also the higher (lower) share price can determine higher (lower) investment spending, as a rise or fall of the share price corresponds to a rise or fall of shareholders' capital gain, which can be reflected to higher or lower profits or internal funds available at the corporate level.

2.2 Share prices and expectations

During the period of technical change, the share prices generally reacted to any kind of investment in the new economy sectors in a positive way, sometimes even too euphorically. This positive reaction was a consequence of a positive shock to the market expectations about the value of the firm involved in the investment plan. As long as a new investment in some new economy sectors is seen as a very promising and highly profitable economic decision, the share price of the firm will raise together with its investment expenditures, providing to the firm an increased potential level of internal funding.

Whatever is the link between high tech investments and share prices through the expectations, the future outcomes of the investments are particularly uncertain during long periods of widespread innovations. This is a problem for the investing firms, as their financial assets can be the most important source of funding, and a sharp decrease in their share prices can have some consequences on their investment decisions.

2.3 Uncertainty

The rise in the new economy share prices reflects the positive expectations of future profitability of the current investment. The rise in the volatility is a consequence of the increased uncertainty about the real value of the firm and the outcome of its investment. In general, investors in the financial markets do not have clear guidance to their expectations and cannot reasonably be confident in a common evaluation of the value of the firms. During a period of radical restructuring of the production technologies and the final products offered, however, the financial investors' confidence in their knowledge about the firm is likely to be even more unstable.

Furthermore, the uncertainty about the new economy is not only a matter of asymmetric information between, say, managers and shareholders, as not even the managers can be confident on the outcome of their own projects of investment. In fact, the outcome of an investment in new technology sectors is uncertain by itself, as it strongly depends on the unknown results of the competition between firms in the markets of the new products. The typical businesses of the new economy firms involve huge costs for development but very low costs for every extra customer. In other words, there are not relevant barriers to the shifts of the customers between alternative competing products and, furthermore, the best products are not more costly than the worst ones. In this context, it is likely that the winner in the competition takes all the market. Economies of scale may be at the roots of natural monopolies, leading to a very hard competition where it is difficult to predict who will be the next Bill Gates. Consequently, the outcome of a typical investment in some new economy sectors is highly risky by itself.

2.4 Cost of borrowing

The level of the debt raised by the new economy firms is particularly low, compared to the old economy firms. The International Monetary Fund estimates that the share of telecommunication, media and technology sectors in new equity issues was 75% in the richer countries financial markets by the first half of 2000, whereas it was slightly over 30% in 1997 (The Economist, October 28th 2000, p.177). On the other hand, the average level of the debt-to-equity ratio of the American non financial firms has risen from 72% in 1997 to 83% in 2000 (The Economist, October 28th 2000, p.116). This new technology firms' preference for equities can be explained simply by considering that the new economy companies are relatively younger, but it would not be surprising if the reason of the lower debt levels was that the new firms are facing higher costs in the debt market. Four arguments can support this hypothesis. First, the high business risk is likely to boost the cost of borrowing because the lenders ask for a higher risk premium in order to finance riskier activities. Second, the cost of borrowing can be particularly high because the assets of the new economy firms are highly intangible and cannot be used as reliable collateral. Third, the equities are comparably much cheaper sources of finance, especially in a period of high enthusiasm for the new economy shares. Fourth, as the main source of finance is the stock market, the wealth of the firm is likely to be almost as volatile as its share price and the probability of default in case of a share price's fall can be particularly high.

2.5 Relevance of risk management

In brief, the firms operating in the new economy sectors in the earlier months of the year 2000 took advantage from an enthusiastic wave of financial investments in their shares, which explain their high prices. On the other hand, the same firms faced very costly access to the credit market and very high business risk. The latter, along with the contradictory news and the lack of information affecting the daily opinions of the shareholders, is the reason of the huge rise in the volatility of the share prices.

The increased uncertainty affecting the financial markets and the new investments brings up the relevance of the risk management as an important tool available to the firms to improve their performance by better coordinating the activity of raising funds and the activity of investment.

The financial wealth of a firm is hedgeable in the market of the derivatives, in such a way that it is technically possible to reduce the effects of the stock market fluctuations on the cash flow available to finance the investment. By accessing the market of derivatives, the firm can change the risk profile of its financial wealth. More exactly, the firm can choose its own hedging strategy and decide how big fluctuations of its financial wealth to allow. Theoretically, the firm can even decide to amplify such fluctuations over the level set by the stock market, as to fully hedge against the market fluctuations and set its wealth to a fixed level. Furthermore, the firm can over-hedge by changing the sign of the fluctuations, from positive to negative and viceversa.

The rational for hedging can vary according to different contexts. In the context of the new economy firms, it is sensible to think that the hedging strategy of a company is mainly driven by the twofold necessity of providing more stable funds to the investment, on one hand, and reducing the need to borrow money from a costly credit market, on the other hand.

2.6 Hedging strategies

The firm can technically control the volatility of its cash flow by using derivatives. By contrast, the volatility of the outcome of the investment is not so easy to control. There are plenty of idiosyncratic (i.e. non hedgeable) elements affecting the corporate investments in the new economy sectors. However, part of the risky outcome of the investment may be also correlated to some marketable risks, such as the foreign exchange rate or, notably, some equity prices, which are partially or wholly hedgeable.

As it has been pointed out so far, it might exist a correlation between the financial wealth of a firm and its investment's expected outcome. This suggests a possible key element in designing an optimal hedging strategy for a firm facing a costly debt market. In case of a positive correlation, it makes no sense to fully hedge the cash-flow fluctuations, as they may provide an extra amount of funds when the firm wishes to plan an extra amount of investment spending. In the case of a negative correlation, on the contrary, the extra amount of cash flow would be available when a lower amount of investment is required. Therefore, the hedging strategy chosen would reverse the sign of the correlation. Finally, in case of no correlation, i.e. completely idiosyncratic business risk, there would be no reason to let the cash flow fluctuate and the full-hedging strategy would be the best way to reduce the need (and the cost) of borrowing.

The framework provided by Froot, Scharfstein, and Stein (1993) allows for a description of such hedging behaviour without being committed to any specific assumption about agency costs or informational problems. All is needed is a concave payoff function, i.e. decreasing return of investment and increasing costs of external finance. To account for this section's description of the new economy types of firms, as well as to simplify the exposition, in the following pages the volatile internal funds (or cash flow) are identified with the firm's stock returns, whereas the external finance is identified with the debt. However, the model is general enough to include other sources of cash flow volatility, as well as other costly sources of external finance.

3 The model of risk management by Froot-Sharfstein-Stein 1993

3.1 The setup of the model

The model of Froot, Scharfstein, and Stein (1993) (FSS from now on) describes the behaviour of a firm that chooses its risk management program in order to better coordinate its investment and financing policies in a context where the investment opportunities are partially related to the fluctuations of the internal finance sources and the external finance is increasingly costly. In the FSS's example, "a company engaged in oil exploration and development will find that both its current cash flows (i.e., the net revenue from its already developed fields) and the marginal product of additional investments (i.e., expenditures on further exploration) decline when the price of oil falls"[2]. In another example, closer to the previous discussion about the new economy, a company deciding to develop some e-commerce may find that the revenue of its expenditure in computing structures, delivering system and marketing policy increases together with its share price in the stock market. By trading forward and future contracts in the derivative markets, the firm can avoid unnecessary fluctuations in the value of its existing assets. The model shows how the firm can calculate the optimal width of fluctuations and select accordingly its optimal hedging ratio. The latter is expressed as a function of the shock to the investment opportunities.

The FSS is a three-periods model where in the first two periods the firm takes its decisions on the base of the outcome expected in the third one. The firm chooses its hedging strategy at time 0 and the amount of investment and debt at time 1, when the uncertain variable to be hedged is realised. At time 2, the production is realised and sold and the debt is repaid.

[2]Froot, Scharfstein, and Stein (1993) 1993, pag. 1638.

Time structure		
time 0	time 1	time 2
Hedging strategy (against fluctuations of ε)	Investment, Debt	Output is sold
	ε is realised	

The key assumptions of the model are the following:

- *Assumption 1.* The marginal returns to investment are decreasing.

The net present value of investment expenditure is

$$F(I) = \theta f(I) - I \tag{1}$$

where I is the investment, $\theta f(I)$ is the expected revenue of the output, with $f'(I) = f_I > 0$ and $f''(I) = f_{II} < 0$. θ is a variable representing a shock to the expected outcome of the investment decision. The discount rate is assumed equal to zero for simplicity.

The assumption 1 is quite commonly used in the economic theory and it is usually justified by the characteristics of the production technology, such as decreasing returns.

The random variable θ represents a multiplicative shock to a given production function, and it is defined by FSS as a variable accounting for the firm's investment opportunities. There are two possible ways to interpret the economical meaning of the shock θ. The first is the usual idea of a neutral technical change, which does not modify the optimal proportions between the factors of production[3]. The second interpretation treats θ as the price of the product at the time when it will be sold. A further discussion about the meaning of θ is under Assumption 4's paragraph.

[3]In the FSS model there is only on input of the production function. However, the production function can be considered as a normalized one, where, say, the labor input, L, is considered equal to 1. A multiplicative shock affects in the same proportions both the inputs I and L.

- *Assumption 2.* The marginal costs of debt are increasing.

The cost of debt is modelled by a generic function $C(D)$, where D is the amount of debt, $C'(D) = C_D > 0$ and $C''(D) = C_{DD} > 0$.

This cost function, as FSS point out, can arise from different sources, such as cost of bankruptcy and financial distress, informational asymmetry between lenders and borrowers, private benefits to the managers from limiting their dependence on external investors. Other sources of external finance are not considered.[4]

- *Assumption 3.* The value of the existing assets is random.

Without any hedging policy, the value of the existing assets is given by $V = V_0\varepsilon$, where V_0 is the initial value of the assets and ε is the primitive source of uncertainty, distributed as a Normal with mean 1 and variance σ^2. ε is realised at time 1.

The assumption 3 is quite specific, as it rules out possible non normal distributions and non stationary processes, which are often observed for the financial variables to be hedged.

The randomness can be theoretically referred to any firm's asset whose value is not known with certainty. However, it seems particularly appropriate to refer it to some financial assets, whose value is particularly volatile and intangible.

- *Assumption 4.* The returns on investment are related to the same random variable affecting the value of the assets.

The shock to the expected outcome of the investment is modelled as

[4] More exactly, Froot, Scharfstein, and Stein (1993) assume costly external finance, which includes also new equities (p.1633-4). Even though such more general assumption would not change the structure and the results of this work, in this paper the emphasis is on the debt as a more important source of external finance than new equity issues. This simplification is also carried on in the work of Whited (1992), on the ground of several empirical contributions showing that "share issues typically account for less than 5% of total new external finance" (p.1426). It can be also justified by an assumption of equity rationing, such as in Greenwald and Stiglitz (1993).

$$\theta = \alpha(\varepsilon - 1) + 1 \tag{2}$$

where α is a measure of the relation between such shock and the original source of uncertainty, ε.[5]

The assumption 4 is perhaps the strongest one and deserves some discussion. The parameter θ, which multiplies the production function's outcome, is related to the value of the existing assets according to the parameter α. Therefore, the variable θ incorporates the randomness of the investment opportunities. However, θ is a random variable at time 0, when the uncertainty is not solved, but it is fixed and treated as a parameter at time 1, when the investment and debt decisions are taken. In fact, in this model only one source of risk is taken into account and between time 1 and time 2 nothing changes by assumption.

If θ is meant as the price of the product, it can be argued that it is quite unrealistic that a price of a product to be sold at time 2 can be known with certainty at time 1. To simplify the ideas, the variable θ can be considered as a price of precommitted sales, i.e. known with certainty at time 1. Another way to interpret θ is as an expectation raised at time 1 about the price of the product at time 2. Whatever is the interpretation of θ, at time 1 it becomes a given parameter to be used to calculate the optimal investment and debt.

As the debt is increasingly costly (assumption 2), the firm prefers to raise the level of the debt only when it cannot provide enough internal funds to its project of investment. Its budget constraint at time 1 is given by

$$I = V + D \tag{3}$$

The necessity of hedging arises as the first period cash flow, V, is random (assumption 3). By trading derivatives at time 0, the firm can modify the distribution of its cash flow across possible values of the random variable ε.

[5] Froot, Scharfstein, and Stein (1993)consider α as a correlation parameter. This term, despite intuitive, does not seem appropriate, as 2 defines a deterministic link between the two variables, ε and θ. The two variables would be correlated if another source of uncertainty was introduced in the definition 2, in order to change the comovement from deterministic to stochastic.

FSS assume, for simplicity, that (i) the fluctuations of the cash flow, V, are completely hedgeable; (ii) hedging does not alter the expected value of the cash flow; (iii) hedging is linear, i.e. the sensitivity of the cash to the changes of the random variable is constant. The latter assumption concretely means that the usage of derivative described in this model is limited only to forward and future contracts. Options contracts are ruled out.

The internal funds after hedging are given by

$$V = V_0[h + (1 - h)\varepsilon] \tag{4}$$

where the value of h is determined as a solution of the maximisation problem of the firm at time 0.

Therefore, at time 0, cash flow is still a random variable, but its distribution across alternative values of ε can change according to the chosen value of h. In the special case of full hedging, where $h = 1$, the distribution collapses to the mean, and the value of the existing assets becomes non-stochastic:$V = V_0$.

The random variable ε, which is related to both the value of the existing assets and the expected returns on the investment, is realised at time 1. Hence, at time 1 the value of the available internal funds and the expected returns on investment are known with certainty. Therefore, at time 1 the firm solves a non-stochastic maximisation problem and chooses its optimal levels of investment and debt. At time 0, the firm solves the stochastic maximisation problem and chooses its optimal hedging strategy, h^*, by taking into account how the first best choice of I^* and D^* would change as the random variable ε changes.

The timing structure of the hedging firm, expressed in terms of the model's notation, is the following:

Time structure		
time 0	time 1	time 2
h^*	I^*, D^* $I = D + V_0[h^* + (1 - h^*)\varepsilon]$ ε, θ	$\pi^* = \theta f(I^*) - I^* - C(D^*)$

3.2 The optimal hedging strategy

The profit function is given by the difference between the returns on investment and the cost of debt.

$$\pi = F(I) - C(D) = \theta f(I) - I - C(D) \tag{5}$$

At time 1 the firm maximises its profit function with respect to the investment.

$$\frac{d\pi}{dI} = \frac{dF}{dI} - \frac{dC}{dD}\frac{dD}{dI}$$

As ε is realised, the values of V and θ are given, therefore, the derivative of the debt with respect to the investment is equal to 1, and the first order condition is the following:

$$\theta f_I = 1 + C_D, \tag{6}$$

showing that the marginal (expected) revenue of the investment must be equal to the marginal cost. The lower is the marginal cost of the debt, C_D, the higher is the optimal level of the investment. The FOC shows the inefficiency derived by a costly debt market. If it was $C_D = 0$, the level of the investment would be set at the first best optimum, where $\theta f_I = 1$.

Moving back to period 0, the profit function 5 becomes random, so does the FOC 6 through the random variable θ. The maximisation problem is:

$$\max_h E_0[\pi(V(\varepsilon, h)],$$

where the expectation is taken with respect to ε at time 0.

The FOC for this second problem is

$$E_0\left[\frac{d\pi}{dV}\frac{dV}{dh}\right]=0 \tag{7}$$

As $\frac{dV}{dh}=V_0(1-\varepsilon)$, equation 7 simplifies in the following way:

$$V_0E_0\left[\frac{d\pi}{dV}(1-\varepsilon)\right]=V_0Cov\left[\frac{d\pi}{dV},\varepsilon\right]=0, \tag{8}$$

where $Cov\left[\frac{d\pi}{dV},\varepsilon\right]=E_0[\varepsilon]E_0\left[\frac{d\pi}{dV}\right]-E_0\left[\frac{d\pi}{dV}\varepsilon\right]$ and $E_0[\varepsilon]=1$.

As shown in 8, the general result of the FSS model is that "the hedge ratio insulates marginal value of internal wealth from fluctuations in the variable to be hedged. Notice that this is not necessarily the same as insulating the total value of the firm from such fluctuations"[6].

Since ε is normally distributed, it is possible to rewrite the covariance as follows[7]:

$$E_0\left[\frac{d\left(\frac{d\pi}{dV}\right)}{d\varepsilon}\right]E_0\left[\frac{d\varepsilon}{d\varepsilon}\right]Cov\left[\varepsilon,\varepsilon\right]=0 \tag{9}$$

Since $E_0\left[\frac{d\varepsilon}{d\varepsilon}\right]=1$ and $Cov\left[\varepsilon,\varepsilon\right]=\sigma^2$, only the first factor is relevant for the maximisation problem.

To simplify the notation, $\frac{d\pi}{dV}=\pi_v$ and $\frac{d^2\pi}{dV^2}=\pi_{vv}$.It can be shown that

$$\frac{d\pi_v}{d\varepsilon}=V_0(1-h)\pi_{vv}+\alpha f_I\frac{dI}{dV}, \tag{10}$$

where $\pi_{vv}=-\frac{\theta f_IC_{DD}}{(\theta f_{II}-C_{DD})}$ and $\frac{dI}{dV}=-\frac{C_{DD}}{(\theta f_{II}-C_{DD})}$.

Taking the expected value and solving $E_0\left[\frac{d\pi_v}{d\varepsilon}\right]=0$ for h, the optimal hedging strategy turns out to be given by

[6] Froot, Scharfstein, and Stein (1993), pag 1639.

[7] A proof is provided in Rubinstein (1976). See also Froot, Scharfstein, and Stein (1993), note 18 pag.1639.

$$h^* = 1 + \frac{\alpha}{V_0} \frac{E_0\left[\frac{-f_I C_{DD}}{\theta f_{II} - C_{DD}}\right]}{E_0\left[\pi_{vv}\right]}. \tag{11}$$

Equation 11 shows that the hedging strategy depends on the correlation parameter α. If $\alpha = 0$ the best strategy for the firm is to fully hedge ($h^* = 1$) against the fluctuations of its cash flow, as they are unrelated to the investment opportunities. If $\alpha > 0$, such a correlation is positive, and the firm can take advantage of a positive fluctuation in its cash flow to provide an extra amount of funds to a planned extra amount of investment. The optimal strategy is a hedge ratio lower than 1, i.e. to hedge in order to reduce the volatility of the internal cash flow without completely eliminating it. The higher is α, the lower is the hedge ratio, which is equal to zero when the firm does not hedge at all and lets its internal funds fluctuating according to the stock markets movements. If α is high enough, the best strategy can be the speculative one ($h^* < 0$), which amplifies the fluctuations of the cash flow and increases the firm's exposure to the risk. Finally, if $\alpha < 0$, the correlation between internal funds and investment opportunities is negative, therefore the best strategy is overhedging, in order to raise cash when ε is low.

4 Investment, debt and hedging strategy in a local approximation

Equations 3, 6 and 11 constitute an unsolved system of three equations with three unknowns: I, D, h. While this system of equations remains unsolved, the formula for h^* provided by FSS does not make perfectly clear what variables affect the hedging strategy. In fact, the ratio between expected values in the 11 includes the levels of the investment and the debt, both depending on ε and h^*. It includes also a direct effect of ε on h^* through the investment opportunities variable θ.[8]

[8] FSS affirm that in their model "optimal hedging ratios can be calculated as a function of shocks to investment ... opportunities" (Froot, Scharfstein, and Stein (1993) 1993, pag.1631). This is the result shown in the 11. However, it does not seem a convincing

It is theoretically possible to solve the system and calculate the levels of the investment, the debt and the hedging strategy as functions of the random variable ε. The following exercise aims at finding an analytical solution of this problem, in order to better understand the economic implications of the FSS general version.

The analytical solution makes clear that the optimal hedging strategy does not depend on the level of the variable to be hedged, as FSS state, but on its variance. Therefore, a concretely measurable variable can be substitute to expectations in the formula for hedging, as the variance is observable at time 0, whereas the value of the random variable is observable only at time 1, when the hedging decision is taken.

The FSS model is firstly specified by using Cobb-Douglas-like production and cost functions. The analytical solution, however, is calculated as a second-order local approximation of the Cobb-Douglas setup.

Once the analytical solution is provided and some properties of the hedging strategy are discussed, a comparison is carried on between the alternative behaviour of two firms equal to each other in everything but the possibility to enter the derivative market and hedge against its wealth fluctuations. Finally, alternative numerical simulations are tried in order to understand how the values of the parameters can affect the firm's behaviour.

4.1 A Cobb-Douglas version of the model

The production function $f(I)$ and the cost function $C(D)$ may be defined by familiar Cobb-Douglas functions, in the following way:

$$f(I) = \frac{\omega I^{1-\beta}}{1-\beta}, \qquad 0 < \beta < 1, \tag{12}$$

conclusion, for economical and technical reasons. Firstly, the economical meaning of this statement is not consistent with the assumptions about the timing of the model, as the shock to the investment opportunities, θ, is known one period after the firm chooses its hedging ratio, h^*. Secondly, such conclusion does not seem to take into account that the 11 is still an implicit function, which includes among its arguments the level of the investments, the level of the debt, the source of uncertainty to be hedged, ε, and the level of the hedging strategy itself, which should be the dependent variable.

where $f_I = \omega I^{-\beta} > 0$ and $f_{II} = -\omega\beta I^{-1-\beta} < 0$;

$$C(D) = \frac{\phi D^{1+\gamma}}{1+\gamma}, \qquad 0 < \gamma < 1, \tag{13}$$

where $C_D = \phi D^{\gamma} > 0$ and $C_{DD} = \phi\gamma D^{\gamma-1} > 0$. The production and the cost functions show decreasing returns to scale in the first and increasing marginal costs of borrowing in the second, consistently with the assumptions 1 and 2 previously outlined.

The parameter β determines the elasticity of the physical product with respect to the investment decision. Such elasticity is given by

$$e_I = \frac{df(I)}{dI}\frac{I}{f(I)} = 1 - \beta > 0.$$

The lower is β, the higher is the response of the production to a change in the investment.

The parameter γ determines the elasticity of the cost of debt with respect to the amount of money borrowed. Such elasticity is

$$e_D = \frac{dC(D)}{dD}\frac{D}{C(D)} = 1 + \gamma.$$

Given the functions 12 and 13, the profit function 5 of the general model becomes

$$\pi = \theta\omega\frac{I^{1-\beta}}{1-\beta} - I - \frac{\phi D^{1+\gamma}}{1+\gamma}.$$

The first order condition 6 becomes

$$\theta\omega I^{-\beta} = 1 + \phi D^{\gamma}. \tag{14}$$

The firm's budget constraint 3 becomes

$$I = D + V_0[h + (1-h)\varepsilon]. \tag{15}$$

Finally, the formula for the optimal hedging strategy, 11, is specified as follows:

$$h^* = 1 + \frac{\alpha}{V_0} \frac{E_0 \left[\frac{-\omega\phi I^{-\beta}\gamma D^{\gamma-1}}{-\theta\beta\omega I^{-1-\beta}-\gamma\phi D^{\gamma-1}} \right]}{E_0 \left[\frac{\theta\beta\omega I^{-1-\beta}\gamma\phi D^{\gamma-1}}{-\theta\beta\omega I^{-1-\beta}-\gamma\phi D^{\gamma-1}} \right]} \tag{16}$$

Equations 14, 15 and 16 form the system of three equations and three unknown, h, I and D, after the Cobb-Douglas specification of the general FSS framework.

Solving this system of equations would lead to express the three unknowns as function of the random variable, ε. However, it does not seem to be possible to find a closed solution. Instead, the system can be easily solved as a local approximation, after a second order Taylor expansion of the production and cost functions, respectively, around the expected levels of the investment, I_0, and the debt, D_0.

The production and the cost functions after the second order Taylor expansion of the generic production and cost functions, $f(I)$ and $C(D)$, are given by the following expressions:

$$f(I) \simeq \frac{a}{2} I^2 + bI + k, \tag{17}$$

with $a = f_{II}(I_0) < 0$, $b = f_I(I_0) - I_0 f_{II}(I_0) > 0$ and $k = f(I_0) - I_0 f_I(I_0) + \frac{1}{2} I_0^2 f_{II}(I_0)$, where $f_I = aI + b$, $f_{II} = a$;

$$C(D) \simeq \frac{c}{2} D^2 + rD + z, \tag{18}$$

with $c = C_{DD}(D_0) > 0$, $r = C_D(D_0) - D_0 C_{DD}(D_0) > 0$ and $z = C(D_0) - D_0 C_D(D_0) + \frac{1}{2} D_0^2 C_{DD}(D_0)$.

where $C_D = r + cD$ and $C_{DD} = c$.

The approximation introduced simplifies the analysis as the second derivatives of the approximated functions are constant. To model the quadratic functions setup as a local approximation of the Cobb-Douglas one, it is possible to choose appropriate values for the parameters a, b, r and c such that a correspondence with the elasticity parameters is maintained. The elasticities of the production and cost functions, e_I and e_D would be now respectively given by

$$\frac{aI^2 + bI}{\frac{a}{2}I^2 + bI + k} = 1 - \beta \tag{19}$$

and

$$\frac{cD^2 + rD}{\frac{c}{2}D^2 + rD + z} = 1 + \gamma.$$

The values of the parameters a and b of the local approximation can be calibrated to the value of β, as well as the parameters r and c to the value of γ. The calibration is described in section 5, and calculations are exposed in Appendix 2.

Notice that, in the cost function 18, the parameter r can be interpreted as a *risk-free interest rate* and the parameter c of the cost function's curvature as a risk premium parameter, showing that the risk premium grows more than proportionally as the level of debt grows. Therefore, the cost function expressed in quadratic terms is even more suitable for economic interpretation than the Cobb-Douglas version. The production function expressed in quadratic terms, on the other hand, does not lose information with respect to the Cobb-Douglas version, provided that its parameters a and b are linked to the elasticity of the product to the investment.

4.2 Investment, debt, and hedging strategy: the approximated solution

The framework exposed until now seem to be adaptable to the some features of the so called new economy described so far (section 2). In fact, a company deciding to invest in a new economy sector may face a positive correlation between its share price and its expected returns to investment.[9] Following

[9] The causal relationship between variables is reversed, compared to the FSS example of the oil company: the future investment opportunities no longer depend on the current price of an asset, as in the FSS example, but it is more the current price of the financial asset (market value) that depends on the expected investment opportunities. In any case,

this example, let us assume for simplicity that the value of the internal funds, V, is related more specifically to the stock return of the firm. The internal funds available to the firm at time 1 is then equal to:

$$V_1 = V_0 \varepsilon, \tag{20}$$

where ε, which in the previous general version is a generic source of uncertainty, can now be interpreted as the stock return on the firms shares. Given the assumption that the stock return is distributed as a Normal with mean 1 and variance σ^2, taking the expectations at time 0, it turns out that the expected cash flow of the firm is equal to the current cash flow:

$$E_0(V_1) = V_0. \tag{21}$$

Equation 20 provides the definition of the internal finance available to a firm that does not enter the market of derivatives and does not hedge against its market value's fluctuations. The budget constraint of such a firm would be given by

$$I = D + V_0 \varepsilon, \tag{22}$$

whereas the budget constraint of a firm entering the derivative market would be given by

$$I = D + V_0[h + (1 - h)\varepsilon]. \tag{23}$$

Even though the firm's internal funds expected at time 0 (given by 21) is the same for both firms, their investment and debt choices at time 1 would be different as the realisation of ε moves away from its expected value.

By using the production and cost functions as defined by 17 and 18, the profit function of the firm is given by

the parameter α is still a measure of the relation between investment opportunities and risk to be hedged, which is the only relevant issue that matters for the structure of the model to be used.

$$\pi(V(\varepsilon)) = \theta \left(\frac{a}{2} I^2 + bI - k \right) - I - rD - \frac{c}{2} D^2$$

and the first order condition by

$$\theta(aI + b) = 1 + r + cD. \tag{24}$$

Solving the system of two equations 22 and 24 gives the optimal levels of investment and debt of a firm that does not hedge against the fluctuations of its share price:

$$I^*(\varepsilon) = \frac{\theta b - (1 + r) + cV_0\varepsilon}{c - \theta a}, \tag{25}$$

$$D^*(\varepsilon) = \frac{\theta b - (1 + r) + \theta a V_0 \varepsilon}{c - \theta a}, \tag{26}$$

where θ is given by 2.

Solving, instead, by using the budget constraint 23, the optimal investment and debt of a firm that hedges against the fluctuations of its share price turn out to be the following:

$$I_h^*(\varepsilon) = \frac{\theta b - (1 + r) + cV_0}{c - \theta a} + \frac{cV_0(1 - \varepsilon)}{c - \theta a}(h^* - 1), \tag{27}$$

$$D_h^*(\varepsilon) = \frac{\theta b - (1 + r) + \theta a V_0}{c - \theta a} + \frac{\theta a V_0(1 - \varepsilon)}{c - \theta a}(h^* - 1). \tag{28}$$

In both cases of hedging and no hedging, the investment and debt choices depend on the realization of ε at time 1. However, in the latter case, they depend *also* on the optimal hedging strategy h^*, which can regulate the effects of the random variable fluctuations according to the firm's own necessities. When is $h^* = 1$, i.e. full hedging, the optimal levels of I_h^* and D_h^* fluctuate as less as it is possible. More precisely, the investment and debt levels are only affected by the investment opportunities fluctuations, the internal funds available being fixed. In fact, the sources of fluctuations expressed by the terms $cV_0\varepsilon$ and $aV_0\varepsilon$, respectively in the equations 25 and 26, are fixed at their mean levels after hedging (see equations 27 and 28 with $h^* = 1$).

The implicit formula for the optimal hedging strategy is obtained by simply substituting the quadratic functions' derivatives in equation 11:

$$h^* = 1 + \frac{\alpha}{V_0} \frac{E_0\left[\frac{-c(aI(\varepsilon)+b)}{(\alpha\varepsilon-\alpha+1)a-c}\right]}{E_0\left[\frac{-ac(\alpha\varepsilon-\alpha+1)}{a(\alpha\varepsilon-\alpha+1)-c}\right]}. \tag{29}$$

Substituting equation 27 for the investment level $I(\varepsilon)$ in 29, after a second order Taylor expansion of the two expected terms around $\varepsilon = 1$ and solving for h, the explicit expression for the optimal hedging strategy is found:

$$h^* - 1 = \frac{\alpha}{V_0} \frac{\left(1 + r - cV_0 - \frac{bc}{a}\right)\left((a-c)^2 + 3a^2\alpha^2\sigma^2\right)}{(a-c)\left((a-c)^2 + 3ac\alpha^2\sigma^2\right)}. \tag{30}$$

Finally, equation 30 can be substituted in equations 27 an 28 to find the explicit expressions for the investment and the debt levels.

4.3 The properties of the optimal hedging strategy

Before analysing the effect of the hedging strategy on the investment and debt choices, some characteristics of the hedging strategy shown by the explicit formula 30 should be underlined.

A first relevant result, which is not visible in the implicit formulas 11 or 29, is that the optimal hedging ratio is affected by the variance and not by the level of the stock return. This is consistent with the idea that the variance can be known at time 0, when the hedging decision is taken, while the level of ε is only known at time 1 by assumption.

Proposition 1 *The optimal hedging ratio, h^*, is a function of the volatility of the stock return, σ^2. The level of the stock return does not have any effect on the optimal hedging strategy.*

This proposition does not need proof, as it states what is visible in the explicit expression for the hedging strategy (equation 30).

A second result concerns the slope of the hedging ratio as a function of the correlation parameter, α. The hedging ratio is a declining function of the parameter α, provided that the variance of the stock return is not too high.

Proposition 2 *The optimal hedging ratio, h^*, is a decreasing function of the parameter α for any σ^2 lower than the critical value $\sigma^{*2} = \frac{-(a-c)^2}{3\alpha^2 ac}$.*

- The sign of the hedging strategy as a function of α is given by the sign of factor multiplying the ratio $\frac{\alpha}{V_0}$ of the RHS in the equation 30. The expression $\left((a-c)^2 + 3a^2\alpha^2\sigma^2\right)$ on the numerator is always positive as it is a sum of squares. The expression $(a-c)$ on the denominator is always negative by the definitions of the parameters in 17 and 18. The expression $\left(1 + r - cV_0 - \frac{bc}{a}\right)$ on the numerator is positive for values of the parameters a, b, r and c consistent the elasticity of the product to the investment calculated in I_0, equation 19. In fact, taking the expectations at time 0 of the optimal investment level from the equation 27, the expected level of investment is $I_0^e = \frac{b-(1+r)+cV_0}{c-a}$; the expression $\left(1 + r - cV_0 - \frac{bc}{a}\right)$ is hence positive if $I_0^e(c-a) - b < -\frac{bc}{a}$, i.e. $I_0^e < -\frac{b}{a} = I_0^{e*}$. This upper bound condition to the expected investment is not binding for values of the parameters consistent with a positive elasticity of the product to the investment: substituting $I_0^{e*} = -\frac{b}{a}$ into the expression for e_I, it turns out that $e_I = 0$. Hence, the ratio that multiplies the parameter α is negative whenever the expression $\left((a-c)^2 + 3ac\alpha^2\sigma^2\right)$ on the denominator is positive, i.e. whenever $\sigma^2 < \sigma^{*2} = \frac{-(a-c)^2}{3\alpha^2 ac}$.

This proposition confirms the general result that the higher is the relation between investment opportunities and the share price of the firm, the lower is the amount of fluctuations hedged, but it adds also a limit: if the volatility of the market value fluctuations is too high, the hedging ratio is no longer a decreasing function of α. The maximum critical value of the variance depends on the concavity of the profit function, expressed by the parameters

a and c, and on the absolute value of the relation parameter α. Lower absolute values of the relation parameter allow for a greater volatility in the financial wealth to be hedged. In other words, the more idiosyncratic is the investment risk of the firm, the greater is the bearable market value volatility for a linear hedging strategy to be feasible.

A third property of the hedging strategy is about its dependence from the variance of the variable to be hedged.

Proposition 3 *Provided that $\sigma^2 < \sigma^{*2}$, the higher is the variance of the stock return of the firm, σ^2, the higher is the sensitivity of the hedging ratio, h^*, to the parameter α. Therefore, the optimal hedging ratio is a decreasing function of the variance if $\alpha > 0$ and an increasing function if $\alpha < 0$.*

- *Proof.* When the variance raises, the factor containing the variance in the numerator increases by the amount $3a^2\alpha^2\sigma^2$, while the factor in the denominator decreases by $3ac\alpha^2\sigma^2$. Hence, the value of $(h^* - 1)$ is farther from 0 (full hedging) as σ^2 is higher, for any value of α. In other words, h^* is more sensitive to the correlation parameter α.

This proposition can be illustrated by comparing the dotted and the continuous curves in Figure 1, the dotted line showing a hedging function with a greater variance. This property of the optimal hedging is a quite counterintuitive result: when the relation between investment opportunity and internal funds is positive, one would expect more hedging as the volatility of the stock return increase, whereas, according to this result, the optimal hedging strategy allows for greater fluctuations of internal funds as their volatility is higher.

This result, however, is not surprising if one thinks that the increased risk in the stock return, being related to the volatility of the returns on investment, does not have a big effect on the risk of raising debt, as any wider extra investment needed will be financed by a proportionally wider extra amount of internal finance. Therefore, *the firm is not concerned about the market value fluctuations by themselves, but it is rather concerned about the risk of borrowing.*

A fourth property of the hedging function is about its dependence on the firm's current market value. The following proposition apply:

Proposition 4 *The higher is the current cash flow of the firm, the lower is the sensitivity of the hedging ratio, h^*, to the parameter α. Therefore, the optimal hedging ratio is an increasing function of the current cash flow if $\alpha > 0$ and a decreasing function if $\alpha < 0$.*

- *Proof.* From equation 30 the factors containing V_0 can be insulated: $\frac{\left(1+r-cV_0-\frac{bc}{a}\right)}{V_0} = \frac{\left(1+r-\frac{bc}{a}\right)}{V_0} - c$. This expression is positive (see proof of Proposition 4.1) and is lower as V_0 is higher. Hence, the value of $(h^* - 1)$ is closer to 0 (full hedging) as V_0 is higher, for any value of α. In other words, h^* is less sensitive to the relation parameter α.

The current cash flow of the firm, according to the 21, is equal to the expected next period cash flow. In fact, the hedging strategy is taken under the assumption that the current market value always incorporates the best prediction about the future value, given the amount of information available. Therefore, any variation in the current cash flow corresponds to a variation in its expected future value.

The consequence of a positive shock in the expected market value is a hedging ratio closer to the full hedging level for any value of the relation parameter α. This can be due to the fact that, after such positive shock, the gap between investment and internal funds is reduced, so is the marginal cost of the expected debt needed. Hence, given a value of the relation parameter, α, less fluctuations of the internal funds are needed as the cost of an extra amount of debt is now lower.

The relation already described between the optimal hedging strategy and the parameter α is shown in the next figure. The dotted line corresponds to a hedging curve with greater variance than the continuous line. Notice that when α (x axis) is big enough, i.e. when the relation between investment opportunities and stock return is positive and higher than a certain critical value, the hedging ratio, h^* (y axis), becomes negative, i.e. the firm adopt a speculative strategy to amplify, by trading derivatives, the fluctuations of its internal funds to higher levels than its stock price's fluctuations. When $\alpha < 0$, the hedging ratio is greater than 1, i.e. the firm adopt an overhedging strategy and trades derivatives in order to reverse the sign of its internal funds fluctuations.

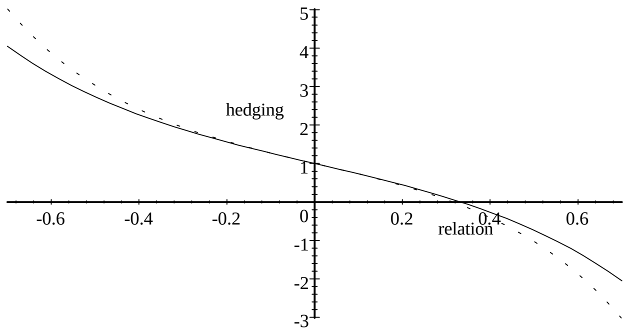


Figure 1 - Optimal hedging ratio h^* as function of α.

5 The effect of the risk management on the investment and debt choices

It is possible to graphically visualise the impact of the hedging possibility on the firm's choices of investment and debt levels.

In this section some results from numerical experiments are shown and discussed. Firstly, a graphical comparison is made between hedging and non hedging possibilities, and the effects of hedging on the firm's financing and investing behaviour are discussed. Secondly, some effects of changes in concavity parameters on the optimal values are obtained and discussed. Such effects are computed with both a numerical simulation and the approximated solution of the Cobb-Douglas version of the model.

The investment, debt and hedging functions are calibrated to respect the following criteria: (i) the expected value of the physical product elasticity to the investment (β) is equal to 0.25, a value typically used in the Cobb-Douglas production function for the elasticity of the capital; (ii) the expected investment is greater than the expected internal funds available ($I_0 > V_0$). As the mean of ε is 1, all the other parameters are set accordingly. The expected cash flow, V_0, is equal to 10; the expected investment, I_0, is equal to 20 and, consequently, the expected debt, D_0, is equal to 10. The standard deviation of the stock return is $\sigma = 0.7$. The parameter, α, is set equal to 0.2 in case of positive correlation and to -0.2 in case of negative correlation.

5.1 Hedging vs. non hedging

Figures 2 and 3 represent, respectively, the investment and the debt choices as functions of the stock return, ε, when the latter is positively related to the investment opportunities. The functions are those taken from the approximated solution, i.e. equations 25 and 27 for Figure 2, and 26 and 28 for Figure 3. As the approximated solution is derived around the expected value of ε, the figures show the investment and debt choices only around values of ε close to its mean. The dotted lines show the functions in case of no hedging (equations 25 and 26), the continuous lines the functions after adopting the optimal hedging strategy h^* (equations 27 and 28).

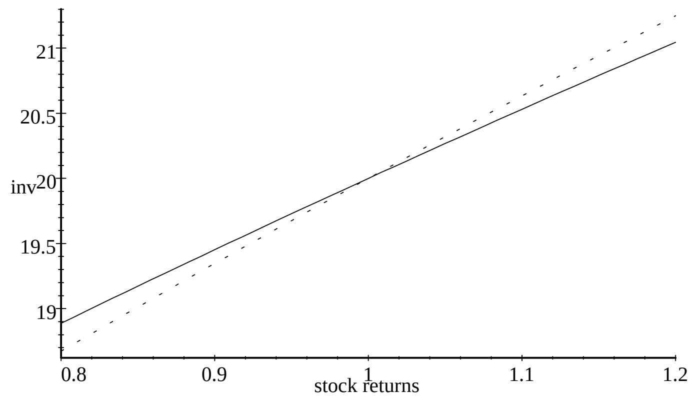


Figure 2 - Investment when $\alpha > 0$

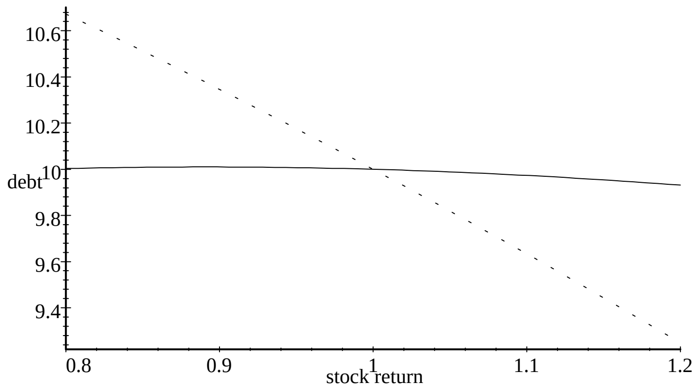


Figure 3 - Debt when $\alpha > 0$

The graphs show that the risk management effect is to smooth the fluctuations in the investment and the debt without changing the sign of the slopes. An unexpected rise in the stock return, if positively related to a rise in the expected returns on investment, would push the firm to use the larger internal funds available to finance a new extra investment and to borrow less money than initially planned. The introduction of a risk strategy does not change this mechanism, but simply reduces the fluctuations of the investment, the debt and the internal funds to more suitable levels for a firm with a concave profit function. In other words, given decreasing returns to scale and increasing costs of borrowing, the negative events are weighted by the firm more than the positive ones, therefore, the firm prefers, when it is possible, to slightly sacrifice the probability of higher investment in order to drastically decrease the probability of a higher debt. The debt functions after hedging is much flatter than the investment function after hedging. This result confirms that the main purpose of hedging, according to this model, is to lower the risk of raising costly debt.

To compare the approximated solution with the original setup of the model, Figures 4 and 5 show the investment and debt choices of the firms, computed as numerical simulation of the Cobb-Douglas setup, using the same values of the parameters (equations 14, 15 and 16 for the hedging firm, 14 and 22 for the non-hedging firm).[10] The range of possible values of ε is wider than in Figures 2 and 3 and includes values of ε corresponding the whole simulated distribution. To compare the simulated solutions with the approximated ones, one should focus on the values of ε around the mean (i.e. around 1). The numerical solution confirms the effects previously discussed about the stabilisation of hedging on both investment and debt, but particularly on debt.

[10] The method of calculation is discussed in 5.2 and illustrated in details in the Appendix. The values of the parameters are those shown in the first columns of tables 1 and 2.

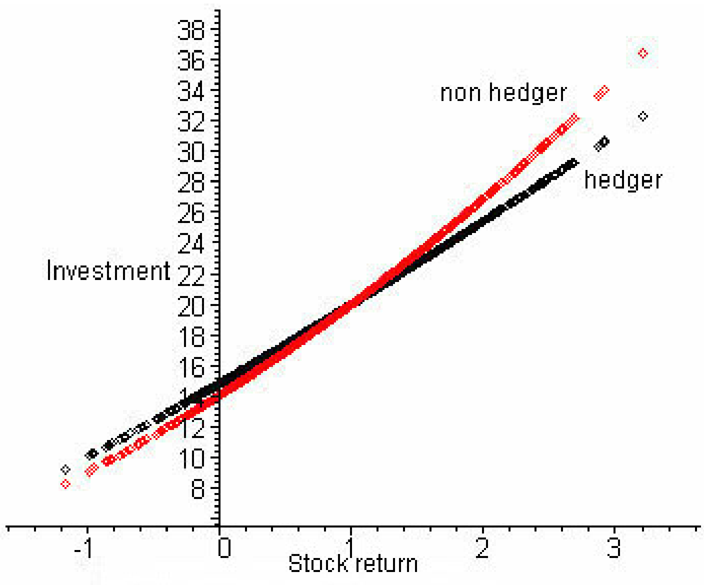
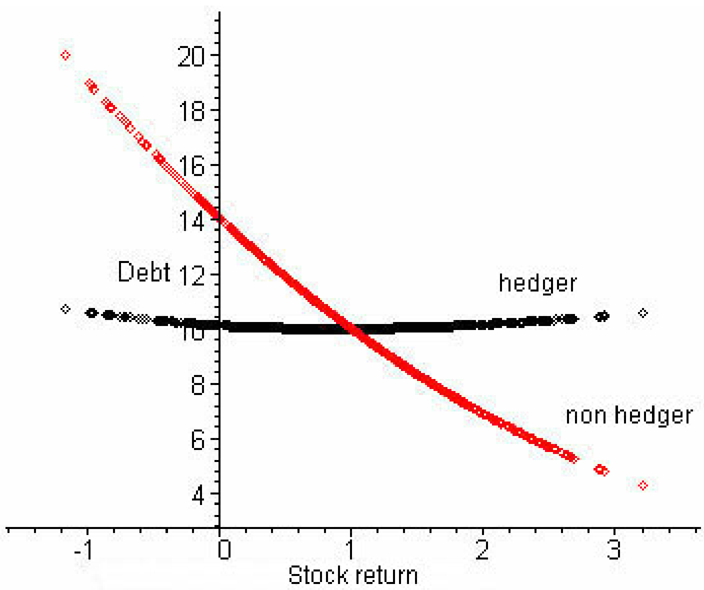


Figure 4 - Investment when $\alpha > 0$. Numerical simulation



Debt when $\alpha > 0$. Numerical simulation

The case where $\alpha < 0$ is partially different: the slopes of the investment and the debt curves change sign after hedging. The next graphs, Figures 6 and 7, show the behaviour of the investment and debt functions in the approximated version of the model (same equations of Figures 2 and 3, with $\alpha = -0.2$). Without hedging possibility (dotted lines), an unexpected decrease in the stock return would frustrate the better investment opportunities by providing less internal funds and pushing the firm to borrow extra money and slow down the investment. As the hedging strategy reverses the sign of the market value's fluctuations, a rise in the investment opportunities can be now supported by an extra amount of internal finance allowing for a reduction in the debt level (continuous lines).

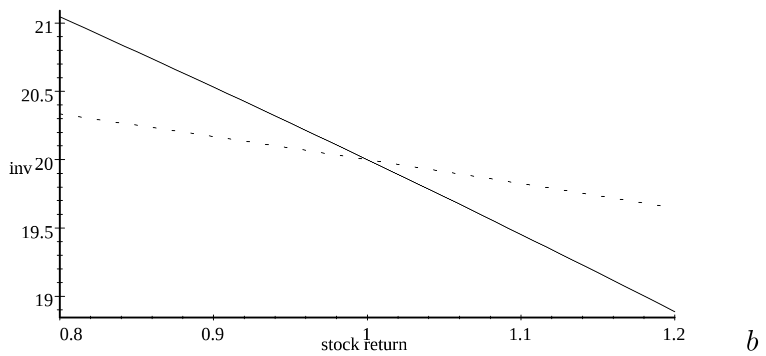


Figure 6 - Investment when $\alpha < 0$

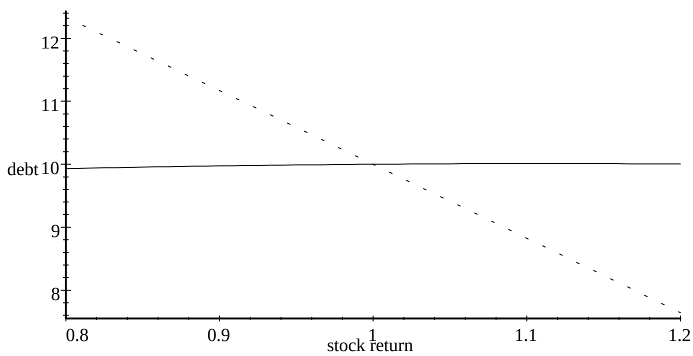


Figure 7 - Debt when $\alpha < 0$

5.2 Changes in the concavity parameters.

Until now, the production and cost functions parameters have been held constant. The following numerical experiments aim at finding how the behaviour of the hedging firm is sensitive to different values of the concavity parameters.

Tables 1 and 3 show the results of the numerical simulation of the Cobb-Douglas version of the models (equations 14, 16 and 15 for the hedging firm, 14 and 22 for the non-hedging firm). Tables 2 and 4 replicate the numerical experiment using the approximated analytical solution around the mean of ε (equations 25 and 26 for the non hedging firm, equations 27, 28 and 30 for the hedging firm).

The tables show the effects of changes in elasticity parameters, respectively β and γ, on the expected levels and the volatility of the investment and the debt, as well as on the hedging strategy. The change in β (Tables 1 and 2) corresponds to a rise in the elasticity parameter of the production function, while the change in γ (Tables 3 and 4) to a rise in the elasticity parameter of the cost function.

The volatilities of investment and debt are calculated in the approximated solution differently then in the simulated one. In the simulated solution, investment and debt volatility are measured by their standard deviations obtained from the randomly generated sample (σ_k with $k = I, D, I_h, D_h$, in Tables 1 and 3). In the approximated analytical solution, investment and debt volatilities are measured by an approximated slope coefficient, λ, given by:

$$\lambda_k = \frac{k\,(1.2) - k(0.8)}{0.4} = \frac{\Delta k}{\Delta \varepsilon}$$

with $k = I, D, I_h, D_h$. The values of 1.2 and 0.8 are respectively the maximum and the minimum values of ε considered in the approximated solution, and the value at the denominator is the difference between them. Such values correspond to the extreme values of the x axis shown in 5.1. The coefficient λ can be positive or negative according to the slopes of the functions considered. The closer to zero is its absolute value, the lower is the measured volatility of the variable k.

Therefore, differently than the optimal hedging ratios and the expected levels, the volatilities calculated in the approximated solutions are not numerically comparable to the standard deviations of the simulated solutions. However, they can be compared in qualitative terms: a rising or falling volatility is easily recognisable with both measures. It should be noticed that the approximated measures of the volatilities add a further information about the sign of the relation between the stock price, ε, and the investment or debt decisions: in fact, the computed values of λ_k can be positive or negative, whereas the standard deviations σ_k, by construction, can only be positive.

Another difference between approximation and simulation is that, in the former, the equilibrium levels (I_k, D_k) of a hedging firm are, by construction, the same as the non hedging firm, whereas, in the latter, they may differ. The reason is that, in the approximated analytical solution, the equilibrium is given as the starting point of the computation, whereas in the simulated non-closed-form solution, such equilibrium is found as a solution, the starting point being the randomly generated sample of ε.

A change in the parameter β from 0.75 to 0.35, corresponds to an exogenous change in the production function elasticity e_I from 0.25 to 0.45. The effects of a positive change of the investment elasticity on a hedging firm are the following (see Tables 1 and 2):

- increasing expected levels of investment and debt;
- increasing variability of the investment decision;
- decreasing value of the hedging ratio, h^*.
- slightly increasing variability of the debt decision.

Table 1 - *Investment elasticity effect*
Simulated solution

e_I	0.25	0.35	0.45
e_D	1.50		
Expected I_h	19.92	28.04	43.21
Expected D_h	10.06	18.32	33.72
h*	0.470	0.144	−0.543
σ_{Ih}	3.76	6.04	10.91
σ_{Dh}	0.09	0.25	0.64
Expected I	20.06	28.07	43.09
Expected D	10.33	18.41	33.43
σ_I	4.57	6.29	9.82
σ_D	2.56	0.82	2.79

Table 2 - *Investment elasticity effect*
Approximated analytical solution

e_I	0.25	0.35	0.45
e_D	0.50		
ExpectedI	20	28.15	43.29
ExpectedD	10	18.15	33.29
h*	0.442	0.095	−0.64
λ_{Ih}	5.39	8.77	15.95
λ_{Dh}	−0.18	−0.28	−0.45
λ_I	6.43	9.01	14.06
λ_D	−3.56	−0.98	4.06

A rise in the production function elasticity leads to greater returns to investment, therefore to increase the investment level. The expected cash flow is not changed, and the level of the debt raises up to the level where the marginal return to investment equals the marginal cost of debt. Given the parameter α, the hedging strategy becomes less tight, as greater cash flow fluctuations are needed to finance greater fluctuations of the investment.

In the third example shown in Tables 1 and 2, the change in the production function elasticity is so great (e_I=0.45) that the hedging strategy changes sign and becomes speculative. This can be a possible description of some speculative behaviour recently observed in the financial markets on the high tech share prices. The shock in the production technology can push the firm to raise dramatically the optimal investment, and consequently the level of debt by the same amount, the expected internal cash flow being unchanged. In front of a raised proportion of debt over internal cash flow, the firm is motivated to raise the fluctuations of the internal funds in order to reduce the fluctuation of the debt around its expected level.

The effects of a positive change of the cost function elasticity, i.e. the parameter γ, on a hedging firm are the following (see Tables 3 and 4):

- slightly decreasing expected levels of investment and debt;
- slightly decreasing variability of the investment decision;
- slightly decreasing variability of the debt decision;
- slightly increasing hedging ratio, h^*.

Table 3 - *Cost of debt effect*
Simulated solution

e_I	0.25		
e_D	1.50	1.60	1.70
Expected I_h	19.92	18.48	17.89
Expected D_h	10.06	9.01	8.05
h^*	0.470	0.498	0.524
σ_{Ih}	3.76	3.53	3.35
σ_{Dh}	0.09	0.08	0.07
Expected I	20.06	19.00	18.08
Expected D	10.33	9.34	8.41
σ_I	4.57	4.61	4.73
σ_D	2.56	2.48	2.37

Table 4 - *Cost of debt effect*
Approximated analytical solution

e_I	0.25		
e_D	0.50	0.60	0.70
ExpectedI	20	18.95	18.00
ExpectedD	10	8.95	8.00
h^*	0.442	0.474	0.502
λ_{Ih}	5.39	5.12	4.87
λ_{Dh}	-0.18	-0.14	-0.11
λ_I	6.43	6.58	6.77
λ_D	-3.56	-3.42	-3.23

A rise of the cost of debt leads to lower profits per unit of debt, therefore, it induce the firm to reduce the scale of production, i.e. the investment and the debt level. As the expected cash flow is not changed, a greater proportion of internal cash flow is expected to finance the lower investment. As less external finance is needed for a lower investment, the hedging strategy becomes slightly tighter, given the parameter α. In other words, lower fluctuations of the cash flow are needed to reduce the risk of raising debt.

With regard to all tables, it can be observed that the pure effect of the hedging possibility, given the value of the concavity parameters, can be seen by looking through each column. Reducing the variability of the internal cash flow ($h^* > 0$) has always a reduction effect on both investment and debt volatilities. The debt volatility, however, is always reduced much more in proportion. In case of speculative behaviour ($e_I = 0.45$), the volatility of the internal cash flow increases as well as the investment volatility, whereas the debt volatility decreases. This result confirms the interpretation already suggested (see 4.3) that the firm is concerned about the risk of borrowing rather than about its share price volatility. Therefore, when the investment opportunities are particularly high, it prefers to increase the shares volatility in order to reduce the risk of borrowing.

Comparing the impacts of both elasticity parameters, e_I and e_D, it can be observed that the impact of a change in the production function elasticity on the firm's optimal decisions is much greater than the impact of a change in the cost function elasticity. This consideration can be used, again, to describe the behaviour recently observed of the 'new economy' companies. As the business risk of the high tech companies is considerably higher than the average, they may face an increased marginal cost of debt that might discourage the production activity. However, their investments' profitability is so high that the incentive to raise the levels of the investment and the debt prevails on the disincentive to lower them. Furthermore, the access to the derivative market makes possible for such companies to amplify the fluctuations of their internal cash flow in order to reduce to a certain extent the fluctuations of the debt level.

Tables 2 and 4 replicate the results shown in Tables 1 and 3 quite accurately. The hedging ratio and the expected values of both investment and debt are close to those computed by the numerical simulation. The approximated hedging ratio tend to slightly undervalue the simulated one. The error slightly raises when the investment elasticity rises, presumably because of the greater levels and volatilities involved in the computation.

Even if volatilities are not numerically comparable, it can be observed that the effects of hedging (columns) and the effects of changing concavity (lines) in the simulated solutions are the same as in the approximated ones. The approximated solutions add information about the sign of the relation between the stock price, ε, and the investment or debt decisions. It can be noticed that, when the optimal hedging strategy becomes negative, i.e. speculative ($e_I = 0.45$), the hedging strategy not only reduces the volatility of the debt function, but also reverse it slope from positive to negative.

Overall, the numerically simulated solution has successfully validated the approximated analytical solution. The properties and the implications of the approximated analytical solution, therefore, can be used with reasonable confidence, at least for values of the variables around their expected levels.

6 Conclusion

The aim of this work was to show the effects of hedging availability on the firms' investment and debt decisions. A framework by Froot, Scharfstein, and Stein (1993) has been chosen, where the decisions of invest and raise external funds are coordinated through (and affected by) the firm's risk management. The hedging decision is taken in a context where the investment opportunities are partially related to the fluctuations of the internal finance sources and the external finance is increasingly costly.

The general formula derived by Froot, Scharfstein, and Stein (1993) does not show explicitly which variables affect the hedging strategy of the firm. In this work, an approximated analytical solution is found, which allows a better understanding about the variables involved in the optimal hedging decision. In particular, it shows that the optimal hedging strategy depends on the *variance* (and not on the *level*) of the variable to be hedged. This is consistent with the timing of the model, as the level of the random variable cannot be known at the time when the hedging decision is taken (otherwise, by definition, such variable would not be random). Some propositions about the sensitivity of the optimal hedging ratio to its determinants are derived.

The implications of the hedging strategy on both investment and debt (or external finance) decisions are then derived in this work by using both the analytical approximation and a numerical simulation of the non-closed-form optimal hedging strategy. For reasonable values of the parameters, the hedging decision reduces the variability of both investment and debt decision. However, it stabilises the debt decisions much more than the investment ones. Therefore, the firm hedge against the internal funds fluctuations in order to reduce the external funds fluctuations.

Finally, using again numerical and approximated solutions, the effects of changing elasticities are shown for both hedging and non hedging firms. In particular, some comparative static experiments are attempted by changing the values of either production or cost functions elasticities. The values of the optimal hedging ratios, as well as the levels and the variabilities of investment and debt are computed with different elasticity parameters. The effect of a rise of the investment profitability on both levels and volatilities turns out to be much stronger than the opposite effect of a rise of the cost of debt. This result may show how a high tech firm, which is also high risk, can be motivated to raise debt (external funds) despite its higher cost. It may also show that, when investment is highly profitable, the firm can even adopt a speculative strategy, which amplify its internal funds fluctuations, in order to reduce the probability of raising extra debt.

7 Appendix - Non closed-form simulation procedure

Non closed-form solutions for the hedging firm are simulated as follows:

(i) a random sample of 1000 realisations of ε_i i.i.d $N(1, \sigma^2)$ is generated, with $i = 1...1000$;

(ii) a starting value is assigned to h_j, taken arbitrarily from the approximated analytical solution, say, $h_j = h_0$;

(iii) given h_j, for each realisation ε_i, a realisation of the investment, I_i, is derived by solving numerically the following implicit function:

$$I_i = \left(\frac{\theta \omega I_i^{-\beta} - 1}{\phi} \right)^{1/\gamma} + V_0[h_j + (1 - h_j)\varepsilon_i],$$

which is simply derived by substituting the first order condition, 14, into the hedger's budget constraint, 23;

(iv) given h_j, for each realisation ε_i, a numerical value for D_i is also found by substituting the computed I_i into the first order condition, 14;

(v) given h_j, for each realisation ε_i, the numerator, $n_i = \frac{-f_I C_{DD}}{\theta f_{II} - C_{DD}}$, and the denominator, $d_i = \frac{-\theta f_I C_{DD}}{(\theta f_{II} - C_{DD})}$, of the hedging function, 16, are computed by substituting the numerical values of ε_i, I_i, D_i and h_j into θ, f_I, f_{II}, C_{DD} (see expressions 12, 13, 2);

(vi) given 1000 realisations of both n_i and d_i, their mean values are calculated: n_j and d_j;

(vii) Substituting n_j and d_j into the formula for hedging, a new value for h is computed:

$$h_{j+1} = 1 + \frac{\alpha}{V_0} \frac{n_j}{d_j};$$

(vii) if $h_{j+1} - h_j \geq 10^{-5}$, the computation restart from (iii) by using h_{j+1}; if otherwise, the calculation ends.

For the non hedging solution, the same generated sample *sub* (i) is used to compute numerical values for I_i and D_i by combining the f.o.c., 14, and the non hedger budget constraint, 22.

The computed optimal hedging ratios, h^*, have always been obtained after three recursions. The computation for both hedging and non hedging equilibria have been repeated for different values of the parameters β and γ as shown in Tables 1 and 3.

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