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Dynamics of Output Growth, Consumption and Physical Capital in Two-Sector Models of Endogenous Growth

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Abstract. This paper considers transitional dynamics of a two-sector endogenous growth model in the Uzawa-Lucas framework. We find that when the ratio of physical to human capital is sufficiently high, it is optimal for both consumption and physical capital to fall for a finite period and then gradually rise along their transition path. The paper also shows that for high values of intertemporal elasticity of consumption, rate of growth of output is increasing in the ratio of physical to human capital, while when the elasticity is moderate or low, output growth is U-shaped.

Keywords : Endogenous growth; Uzawa-Lucas model; Transitional dynamics; Golden rule
JEL classification : O41

1 Introduction

Multi-sector endogenous growth models received great attention in recent years, though due to intractability, their dynamics are not yet well understood, see Turnovsky (2000, sec. 14.2) on this. This paper considers a two-sector endogenous growth model in the Uzawa-Lucas, hereafter UL, framework. We focus on the transitional dynamics and find that consumption responds in an asymmetric fashion to the relative scarcity of one type of capital with respect to the other. Whereas human capital always enhances consumption, this is not the case for injection of physical capital. An increase in the physical capital leads to fall of consumption and decumulation of physical capital along their off-balanced paths. They firstly fall for a finite period and then begin to rise on their transition toward the steady state.

Although the basic idea of two-sector endogenous growth model has been extended in some aspects, e.g. by Bond et. al (1996) and Ladron-de-Guevara et. al(1997), for the sake of tractability we rely on the seminal work of Lucas(1988) where physical capital is not included as an input of the education sector. We also assume that externality from accumulation of human capital is absent in the production process. In this regard our model is close to those

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of Mulligan and Sala-i-Martin(1993), Barro and Sala-i-Martin(1995, sec. 5.2.2) and Arnold(2000).

When the average productivity of physical capital is far short of its steady state, four episodes occurs. First, both consumption and physical capital fall. Then consumption rises but physical capital still decumulates. In third phase, both variables rise with different rate and finally, at the steady state they both grow with a common and constant rate. Numerical exercises suggest that the *falling phase* is likely to occur even in the vicinity of the steady state in a Lucas economy.

In their numerical investigation, Mulligan and Sala-i-Martin(1993) consider the possibility of falling of consumption and physical capital during transition path in the UL model; see panels (iii) and (iv) of their figure 1. They do not explain the problem however and go any further. We diagnose here through the falling symptom completely by means of analytical methods. Caballé and Santos(1993) also address the decline of both consumption and physical capital in an economy endowed in relative terms with a great amount of physical capital. They conclude then, physical capital and consumption in the two-sector endogenous growth model respond to the increment in physical capital in a similar qualitative way as in the Ramsey model. Our findings however show that the decline of consumption and decumulation of physical capital in the UL models are in some extents different from the basic Ramsey model where capital exceeds the golden rule. Here the cause of fall is attributed to the high ratios of physical to human capital which is richer than overaccumulation of capital. Moreover, in the Ramsey model consumption and capital decline monotonically toward their steady state, whereas in ours they firstly fall and then gradually rise at an increasing rate.

Transitional dynamics of output growth is also studied in this paper. We find that while for moderate and high degree of consumption smoothing, the rate of growth of output is U-shaped, this is not the case when the intertemporal elasticity of consumption is high enough. In this case, rate of growth of output is increasing in the ratio of physical to human capital . Our analysis partially supports the numerical findings of Mulligan and Sala-i-Martin(1993). The novelty here is that we fully characterize dynamics of the output growth by analytical methods and explore that U-shaped transition is valid only for limited subset of parameters.

In addition to some applications of the idea presented here, we show that our findings are valid for a wide range of multi-sector endogenous growth models. The model and main results will be presented in section 2 and 3 respectively. We conclude then in section 4.

2 The model

Basic concepts Consider a centrally planned economy with constant population normalized to one. The objective of the planner is to maximize welfare of the representative agent $W = \int_0^\infty e^{-\rho t} C^{1-\sigma}/(1-\sigma)dt$, where C denotes consumption, $\rho > 0$ is the discount factor and $1/\sigma > 0$ is the intertemporal elasticity of consumption. Output is determined by the technology $Y = AK^{\alpha}(uH)^{1-\alpha}$, where K is physical capital, H denotes human capital, u $(0 \leq u \leq 1)$ is the amount of effort devoted to production of final output, and $\alpha \in (0,1)$. The final good is consumed or invested to accumulate physical capital, hence $\dot{K} = Y - C - \delta K$, where $\delta \geq 0$ is the rate of depreciation of physical capital.

The representative producer/consumer supplies one unit of labour inelastically and lives forever. She devotes $1-u$ fraction of her effort to the accumulation of human capital, referred to as education. Then $\dot{H} = B(1-u)H - \theta H$ $(\theta \geq 0)$ indicating that education does not requires final output and hence physical capital.

Optimal growth and the steady state The central planner chooses the level of C and u such that maximize W subject to the law of motion of K and H and given values for $K(0)$ and $H(0)$. The problem has been solved in Barro and Sala-i-Martin(1995, sec. 5.2). In particular the off-balanced rate of growth of the variables that we care about is determined by $g_C = [\alpha Y/K - (\rho+\delta)]/\sigma$, $g_U = -\lambda + Bu - C/K$ and $g_Y = -(\lambda+\delta) + \alpha Y/K - C/K$ where $\lambda \equiv (\alpha-1)(B-\theta+\delta)/\alpha$ and for any variable like y, $g_y \equiv \dot{y}/y$ denotes its exponential rate of growth.

In the steady state C, Y, K and H all grow with a common rate $\widetilde{g} = B(1-\widetilde{u}) - \theta$, and u remains constant, where tilde over a variable refers to its steady state. We transform the variables into a set of stationary variables $z \equiv Y/K$ and $\chi \equiv C/K$ which with u form *fundamentals* of the model. Steady state then can be defined as a situation where $\dot{z} = \dot{\chi} = \dot{u} = 0$. The dynamics of the economy is expressed now by a set of nonhomogeneous linear growth differential equations as

$$\begin{bmatrix} g_z \\ g_\chi \\ g_u \end{bmatrix} = \begin{bmatrix} \alpha-1 & 0 & 0 \\ \alpha/\sigma - 1 & 1 & 0 \\ 0 & -1 & B \end{bmatrix} \begin{bmatrix} z \\ \chi \\ u \end{bmatrix} - \begin{bmatrix} \lambda \\ \rho/\sigma \\ \lambda \end{bmatrix} \tag{1}$$

Since the matrix of coefficient, called M, is nonsingular there exists a unique solution for the above system. This results the existence and uniqueness of the steady state[2].

Solving recursively for the steady state value of the fundamental variables one obtains $\widetilde{z} = (B+\delta-\theta)/\alpha$, $\widetilde{\chi} = (B+\delta-\theta)/\alpha - [B-(\rho+\theta)/\sigma] - \delta$ and $\widetilde{u} = 1 - [B-(\rho+\theta)]/B\sigma - \theta/B$. This also gives the balanced rate of growth of the economy as $\widetilde{g} = (B-\rho-\theta)/\sigma$. Sustainability of growth in the long run requires $\widetilde{g} > 0$ and the transversality condition implies $\widetilde{g} < B - \theta$. We combine these two requirements by assuming

$$0 < B - (\rho+\theta) < \sigma(B-\sigma) \tag{2}$$

[2]See Bond et. al(1996) on the existence and uniqueness of the steady state in the general setting.

By simple manipulations one finds that the above inequality is also the sufficient condition for $\widetilde{z} > 0$ and $\widetilde{\chi} > 0$, and necessary and sufficient condition for $\widetilde{u} \in (0,1)$, i.e. for the steady state to be well defined.

Transitional dynamics By linearization of (1) around the steady state we obtain $\dot{X} \simeq \widetilde{M} \cdot (X - \widetilde{X})$ where $X = (z, \chi, u)^T$ is the vector of fundamentals and $\widetilde{M} = [\widetilde{m}_{ij}]$ is a 3×3 matrix for which we have $\widetilde{m}_{ij} = m_{ij} \cdot \widetilde{X}_i$ where m_{ij} is the (i,j) entry of M. Obviously $\widetilde{M}$ is lower triangular like M and its eigenvalues are on the main diagonal as $\lambda(\widetilde{M}) = \{\lambda, \widetilde{\chi}, B\widetilde{u}\}$.

Given (2) and range of parameters of the model, $\widetilde{M}$ has only one negative eigenvalue $\lambda = (\alpha - 1)(B - \theta + \delta)/\alpha$ whose magnitude determines the speed of convergence along the transition. It depends only on the technological parameters and is independent of the preference parameters.

Starting from a given initial value of the average productivity of physical capital $z(0) = z_0$, the stable adjustment path is described around the steady state by

$$X_t - \widetilde{X} = (z_0 - \widetilde{z}) \cdot V \exp(\lambda t)$$

where $V = (1, v_2, v_3)^T$ – the eigenvector corresponding to the stable eigenvalue λ – rules out the unstable paths.

Since there is only one negative eigenvalue, steady state is one-dimensional saddle stable and the stable arm can be denoted by the curve $(z, \chi(z), u(z))$ in $R_+^2 \times [0,1]$ that passes through the steady state, i.e. $\chi(\widetilde{z}) = \widetilde{\chi}$, and $u(\widetilde{z}) = \widetilde{u}$. Its slope at the steady state also satisfies the condition that distinguishes the stable path from nonstable ones, i.e. $\chi'(\widetilde{z}) = v_2$, and $u'(\widetilde{z}) = v_3$.

To derive the main result of the paper, we focus on the projection of the policy function on the (z, χ) plane. Regarding the relative size of α and σ two cases has been distinguished in the literature; Mulligan and Sala-i-Martin(1993). We follow the case where $\alpha < \sigma$ which is practically more interesting[3]. In this case the locus $\dot{\chi} = 0$ is upward sloping with slope $1 - \alpha/\sigma > 0$ and for $0 \leq z_0 < \widetilde{z}$ we have $0 < \chi'(z) < 1 - \alpha/\sigma$, and $\rho/\sigma + (1 - \alpha/\sigma)z < \chi(z) < \widetilde{\chi}$. Moreover solving the characteristic equation $(\widetilde{M} - \lambda I).V = 0$ for v_2 we obtain

$$v_2 = \frac{(1 - \alpha/\sigma)\widetilde{\chi}}{\widetilde{\chi} + (1 - \alpha)\widetilde{z}}$$

which as long as $\alpha < \sigma$ is positive.

Since the locus $\dot{z} = 0$ is vertical and stable, for $z_0 < \widetilde{z}$ (respectively $z_0 > \widetilde{z}$) , $z(t)$ converges monotonically to $\widetilde{z}$ from below (respective from above). Moreover

[3] $\alpha < \sigma$ refers to the *normal case* in the language of Caballé and Santos(1993) and Ladron-de-Guerara et. al (1997). Although according to empirical evidence, the Cobb-Douglas technology is in favour of the normal case, in a more general setting the numerical exrecise of Caballé and Santos do not rule out other possibilities.

since there is no restriction on the convergence of z to $\widetilde{z}$ and it attracts any $z > 0$, stability is global rather than local here. The phase diagram is depicted in the up panel of figure 1.

Figure 1 here

3 Results

Golden rule in the Lucas model The rate of growth of consumption $g_C = (\alpha z - \rho - \delta)/\sigma$ evolves with the same pattern as z along the transition. Since z is defined on the whole nonnegative real axis, it is possible for z_0 to be less than $(\rho + \delta)/\alpha$. In this case $g_C < 0$ and consumption falls along the transition. Since z adjusts monotonically to $\widetilde{z}$, $z(t)$ eventually passes its threshold and so consumption begins to rise. In this case consumption exhibits nonmonotonic time profile. It firstly declines and reaches its minimum at $z^{-1}((\rho + \delta)/\alpha)$,and then rises with an increasing rate. The time path of consumption is different when z_0 is higher than its threshold, $(\rho + \delta)/\alpha$. In this case, it monotonically grows with an increasing rate. Finally when $z_0 > \widetilde{z}$, consumption grows along the transition with a decreasing rate. In all cases due to sustainability of growth,i.e. condition (2), g_C eventually exceeds zero and approaches its long-run value on the balanced growth path, $\widetilde{g} > 0$.

On the other hand, according to law of motion of K we have $g_K < 0$ for $z < \chi(z) + \delta$. Since for $z < \widetilde{z}$, $\chi(z)$ lies above the $\dot{\chi} = 0$ locus, the condition for having a negative rate of growth for physical capital is less demanding with respect to C. Let us call $d(z)$ the discrepancy between the policy function $\chi(z)$ and the $\dot{\chi} = 0$ schedule. Then obviously from figure 1a, for $z < \widetilde{z}$ we will have $d > 0$, $d' < 0$, and $d(\widetilde{z}) = 0$. Accordingly we will have $g_K(z) = z - \chi(z) - \delta = g_C(z) - d(z)$. This implies $g_K < g_C$ for $z < \widetilde{z}$.

In general when the average productivity of physical capital z is far short of its steady state $\widetilde{z}$, four episodes occurs. First, when $0 < z(t) < (\rho + \delta)/\alpha$ both consumption and physical capital fall, with $g_K(z) < g_C(z) < 0$. Then when $(\rho + \delta)/\alpha < z(t) < \widehat{z} \equiv \arg\{\chi(z) + \delta = z\}$ consumption rises but physical capital still decumulates, i.e. $g_K(z) < 0 < g_C(z)$. In third phase, when $\widehat{z} < z(t) < \widetilde{z}$, both variables rise with different rate, i.e. $0 < g_K(z) < g_C(z)$. Finally, at the steady state they both grow with a common and constant rate, $g_K(\widetilde{z}) = g_C(\widetilde{z}) = \widetilde{g}$.

The first and third phases are counterparts of those situations in the basic Ramsey model where the economy approaches the steady state from above and below respectively. The fourth phase arises because we work within an endogenous growth framework. Finally the second phase refers to the the existence of two types of capital with different technology of accumulation. What is important here is the sequence of the first and third episodes along the transition that generates the U-shaped path of both consumption and physical capital. Moreover due to the existence of the second episode in the middle, the minimum of C and K do not coincides. Evolution of the rates of growth of consumption and physical capital as functions of z along the left part of the stable arm is depicted in the down panel of figure 1. Both increase with z and approach gradually toward their common value at the steady state in the same direction. We sum up our findings as follows:

Proposition 1 *In the Uzawa-Lucas model, where $\alpha < \sigma$ and $z_0 < \widetilde{z}$, the following statements hold:*

i) Rate of growth of consumption and physical capital adjust monotonically toward their common steady state $\widetilde{g} = (B - \rho - \theta)/\sigma$. They change along the transition in the same direction and increase gradually with average productivity of physical capital. During transition $g_K(z) < g_C(z)$, though as z approaches $\widetilde{z}$ they become closer to each other.

ii) For sufficiently low value of z_0, consumption and physical capital display a U-shaped pattern along their transition paths. They firstly fall for a finite period and then rise. In spite of the Ramsey model, here there are two different threshold determining whether consumption and physical capital fall or rise during the adjustment process. Minimum of consumption always occurs sooner than physical capital, i.e. $\arg\min\{C(t)\} < \arg\min\{K(t)\}$.

The occurrence of negative rate of growth on the transition depends on the relative size of initial value of $z = A(uH/K)^{1-\alpha}$ which is a measure of imbalance in the economy; Mulligan and Sala-i-Martin(1993). The more scarce is the level of accumulated skills with respect to the equipments and plants, the lower is z and the more likely is for the economy to experience a declining trend in both consumption and physical capital. Hence falling of consumption and physical capital depends heavily on the extent of imbalance between two sectors. Furthermore since the speed of convergence along the transition is proportional with B, the less productive is the technology of human capital accumulation, the longer is the length of falling period.

The likelihood of the occurrence of the falling period for an economy that suffers from high ratio of physical to human capital depends on the magnitude of the effective discount factor $\rho + \delta$ relative to the net productivity of the education technology augmented by rate of depreciation of physical capital, $B - \theta + \delta$ on one hand and the shape of the policy function $\chi(z)$ on the other. The higher $(\rho + \delta)/(B - \theta + \delta)$, the more likely is for consumption to go down during transition. In addition, the flatter is the policy function, the wider is the region upon which the physical capital decumulates. Since the threshold for falling of consumption is short of those of physical capital, the higher $(\rho + \delta)/(B - \theta + \delta)$, the more likely is for physical capital to decumulate too. The range of benchmark values of parameters in the literatures suggest that the above mentioned threshold might be too close to $\widetilde{z}$. For example the ratio of $(\rho + \delta)/(B - \theta + \delta)$ in Mulligan and Sala-i-Martin(1993) and Barro and Sala-i-Martin(1995, sec. 5.2.2) is respectively 75 and 63.5 percent which states that consumption and physical capital might fall even in the vicinity of the steady state.

Comparison with the Ramsey model One can compare our findings about the U-shaped path of consumption and physical capital with the basic Ramsey model where capital exceeds the modified golden rule. In that case the economy approaches the steady state from above and both consumption and capital decline monotonically toward their steady state. We briefly mention here the differences between these two cases. Firstly, here the cause of decline of C and K is attributed to the high ratio of K to H which happens either due to abundance of physical capital or shortage of human capital. This measure of inter-sectoral imbalance is richer and more intuitive than overaccumulation of capital in the Ramsey model, though obviously the marginal productivity of physical capital exceeds the discount factor in both cases. Second, in the UL model there are two different thresholds for consumption and physical capital to fall. Consumption falls when $z < (\rho+\delta)/\alpha$, but physical capital falls when $z < \widehat{z}$, where $(\rho+\delta)/\alpha < \widehat{z}$. Third, here productivity of human capital accumulation is the key factor that determines how long the economy should stay in the falling period and pays the price of intersectoral imbalances. Finally, time profile of consumption and physical capital are nonmonotonic here. They firstly fall and then gradually rise at an increasing rate. This happens in the presence of sustainability of growth in the steady state while there is no endogenous growth in the Ramsey model.

Dynamics of the rate of growth of output The rate of growth of output on the transition is $g_Y(z) = -(\lambda+\delta) + \alpha z - \chi(z)$. This implies that the *isogrowth* lines of Y in (z, χ) plane are those with slope α. Moreover, the higher their intercepts the lower is the rate of growth of output along the isogrowth lines. The position of isogrowth lines relative to the policy function $\chi(z)$, determines the characteristics of g_Y along the transition and hence the dynamics of output. Our information about the slope of χ is however limited to its lower and upper bound. In other words we have

$$\begin{aligned} 0 &< \chi'(z) = \frac{\dot{\chi}}{\dot{z}} = \frac{g_C(z) - g_K(z)}{(\alpha-1)(z-\widetilde{z})} . \frac{\chi}{z} < 1 - \alpha/\sigma, \qquad z \neq \widetilde{z} \\ 0 &< \chi'(\widetilde{z}) = \frac{1-\alpha/\sigma}{1+(1-\alpha)(1-\widetilde{s})^{-1}} < 1 - \alpha/\sigma \end{aligned}$$

where $\widetilde{s} = (\widetilde{g}+\delta)/\widetilde{z}$ is the saving rate at the steady state.

We distinguish four cases, concerning the slope of isogrowth lines relative to $\chi(z)$:

Case 1 $\alpha \geq 1 - \alpha/\sigma$.

In this case, the isogrowth lines are steeper than the $\dot{\chi} = 0$ locus and hence $\chi(z)$ itself. They cross the policy function only once. The lower values of χ correspond to the isogrowth lines that lie farther to the left. Since $\chi(z)$ is increasing, this introduces a one to one relationship between z and g_Y, implying that $g_Y(z)$ is increasing. The position of isogrowth lines in the phase diagram in this case and also $g_Y(z)$ is depicted in panels i and ii of figure 2.

Figure 2 here

Case 2 $v_2 < \alpha < 1 - \alpha/\sigma$.

The isogrowth lines are steeper than χ in the vicinity of the steady state here, but are flatter than the $\dot{\chi} = 0$ locus. A smooth enough policy function, is crossed only twice by an isogrowth line with low enough intercept. In particular, correspond to the set of parameters, there is a unique $\overline{z} < \widetilde{z}$ such that $\alpha = \chi'(\overline{z})$ and $g_Y(\overline{z}) = \min\{g_Y(z) : z > 0\}$. This case is depicted in panels iii and iv of figure 2.

Case 3 $v_2 = \alpha$.

The isogrowth line passing through $(\widetilde{z}, \widetilde{\chi})$ point, in this case is tangent of the policy function. This implies that the rate of growth of output is minimized at the steady state, i.e. $g_Y(\widetilde{z}) = \min\{g_Y(z) : z > 0\}$. See panels v and vi of figure 2.

Case 4 $v_2 > \alpha$.

In this case the isogrowth lines are flatter than $\chi(z)$ in a neighborhood of the steady state. Since χ is concave for $z > \widetilde{z}$, the isogrowth lines with sufficiently low intercepts cross it twice. In particular there exists $\overline{z} > \widetilde{z}$ such that $\alpha = \chi'(\overline{z})$ and $g_Y(\overline{z}) = \min\{g_Y(z) : z > 0\}$. This case is depicted in panels vii and viii of figure 2.

Given parameters of the model (except σ), we can characterize the above cases according to the size of intertemporal elasticity of consumption. Considering that for $\chi(z)$ to be upward sloping we have assumed $\alpha < \sigma$, then case 1 corresponds to $\alpha < \sigma \leq \alpha/(1-\alpha)$. The condition for case 3, i.e. $\alpha\widetilde{\chi} = \sigma(1-\alpha)[(1-\alpha)\widetilde{z} - \delta + \widetilde{g}]$ defines a quadratic polynomial in σ that has only one root - called $\widetilde{\sigma}$ - greater than $\alpha/(1-\alpha)$. Hence for $\sigma > \alpha/(1-\alpha)$, g_Y is U-shaped in z. Its minimum occurs at the left or right of $\widetilde{z}$, depends on whether $\sigma < \widetilde{\sigma}$ or $\sigma > \widetilde{\sigma}$ respectively. We summarize our findings in the following statement.

Proposition 2 *Dynamics of the rate of growth of output along the transition depends on the size of intertemporal elasticity of consumption. When* $\alpha < \sigma \leq \alpha/(1-\alpha)$*,* g_Y *is increasing in* z*, while for* $\sigma > \alpha/(1-\alpha)$*, it is U-shaped. The larger* σ*, the minimum of* $g_Y(z)$ *occurs at the higher level of* z*. In particular there is a unique* $\widetilde{\sigma} \in (\frac{\alpha}{1-\alpha}, \infty)$ *such that*

$$\arg\min g_Y(z) \lesseqgtr \widetilde{z} \Longleftrightarrow \sigma \lesseqgtr \widetilde{\sigma}$$

To give an idea of how the size of σ classifies the dynamics of g_Y, we consult the baseline setting of Mulligan and Sala-i-Martin(1993) where $\alpha = 0.5$, $\delta = \theta = 0.05$, $A = 1$, $B = 0.12$ and $\rho = 0.04$. Solving the equation $\alpha = v_2$ for σ, one obtains $\widetilde{\sigma} = 3$. Hence for $0.5 < \sigma \leq 1$, $g_Y(z)$ is increasing, while for $\sigma > 1$, It is U-shaped. For $\sigma = 3$, the minimum of g_Y occurs at $\widetilde{z} = 0.24$, while for $\sigma \lessgtr 3$, we have $\arg\min g_Y(z) \lessgtr 0.24$.

In Barro and Sala-i-Martin(1995, p.191), the only difference in the baseline parameters are $\rho = 0.02$ and $B = 0.11$. This gives $\widetilde{z} = 0.22$ and $\widetilde{\sigma} = 3.3$. Hence still for $\sigma > 1$, g_Y is U-shaped but we have $\arg\min g_Y(z) \lesseqqgtr 0.22 \Leftrightarrow \sigma \lesseqqgtr 3.3$.

The above analysis clarifies the ambiguity of the dynamics of g_Y, stated by Barro and Sala-i-Martin. It identifies the case where $g_Y(z)$ is increasing as a new regime that has not been addressed in the literature yet. It also delivers the U-shaped feature of g_Y in a clear way by means of analytical methods.

Applications There are at least two immediate application for the ideas presented in this paper. Regarding the U-shaped path of consumption and physical capital, if one augments the production structure with an essential flow of nonrenewable natural resource, our findings sheds light on the debate on the compatibility of endogenous growth and sustainable development; Aghion and Howitt(1998, ch.5). In our model sustainability, in both *flow-based* and *stock-based* measures[4], violated on the transition path while growth is sustained in the steady state.

In addition, our findings about the dynamics of output growth suggests that for an economy whose inhabitants have not a high degree of consumption smoothing, relative abundance of physical capital coincides with low rate of growth of the economy. In that situation, the idea presented here provides an intuitive framework for explanation of poor growth performance of economies that are affluence in terms of their physical assets but have not accumulated required skill enough.

4 Concluding remarks

This study finds that in a Lucas economy where the ratio of physical to human capital is high enough, the time profile of both consumption and physical capital is U-shaped. It highlights the distinctions of our model from overaccumulation of capital in the Ramsey model too. This paper also characterizes dynamics of the growth rate of output, based on the size of the intertemporal elasticity of consumption. Our findings shows that the U-shaped pattern of output growth - that has been found so far by numerical methods - is only valid for the low and moderate degree of intertemporal elasticity of substitution. When current consumption is a good substitute for future consumption however, output growth is increasing in the ratio of physical to human capital.

The current result also covers two-sector endogenous growth models where knowledge plays the role of human capital. For example in the expanding variety

[4]See Hanley(2000) on these measures.

model of Romer and in the creative destruction model of Aghion and Howitt, the stock of human capital can be interpreted as the variety of brands and the stock of social knowledge respectively. In addition the fraction of effort devoted to education can be interpreted as the amount of research employment in both models. Therefore the result obtained here can be easily extended to these R&D based endogenous growth models.

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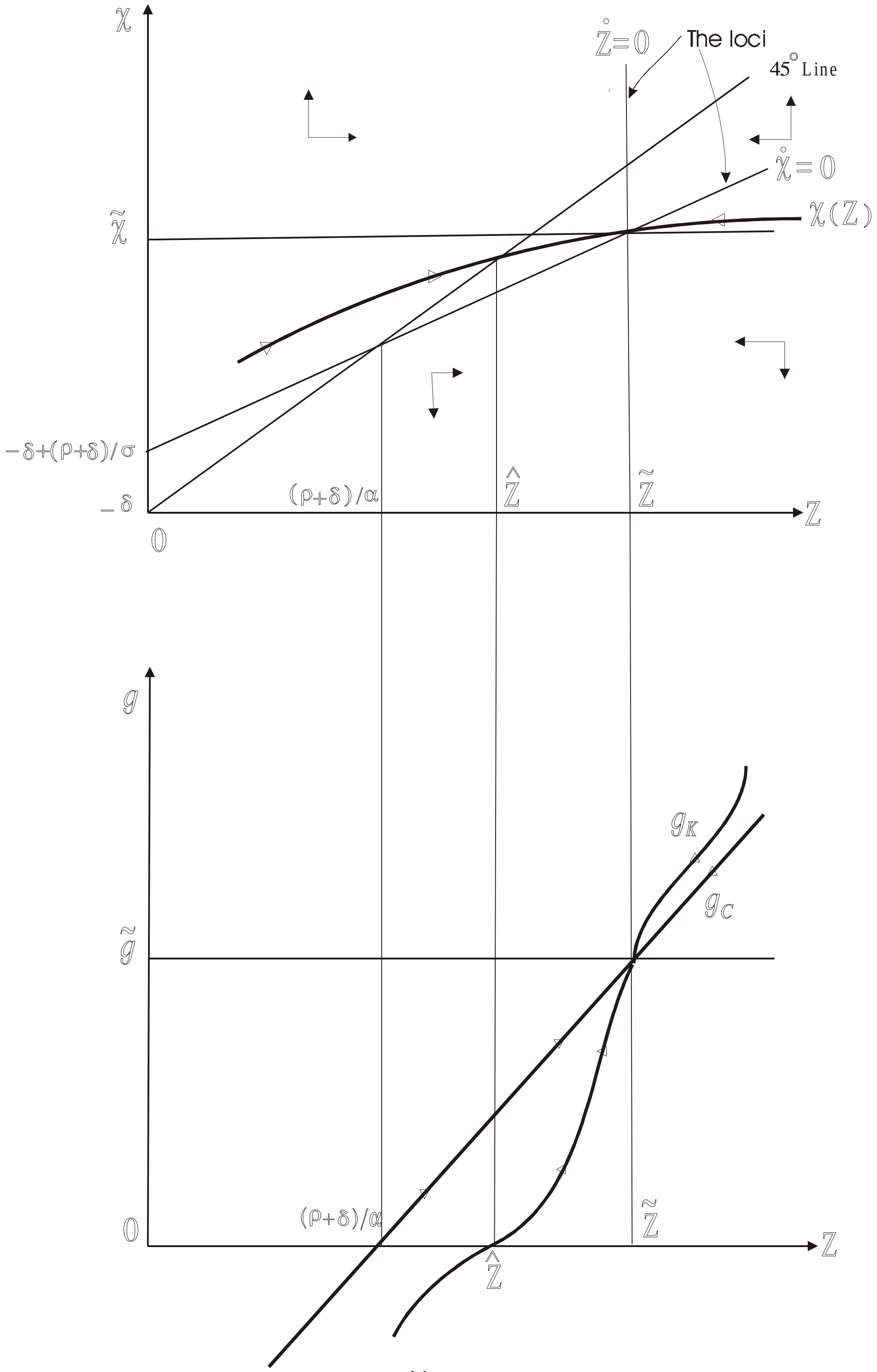

Figure 1: Convergence of z and χ (up panel), and convergence of g_C and g_K (down panel) along the transition path.

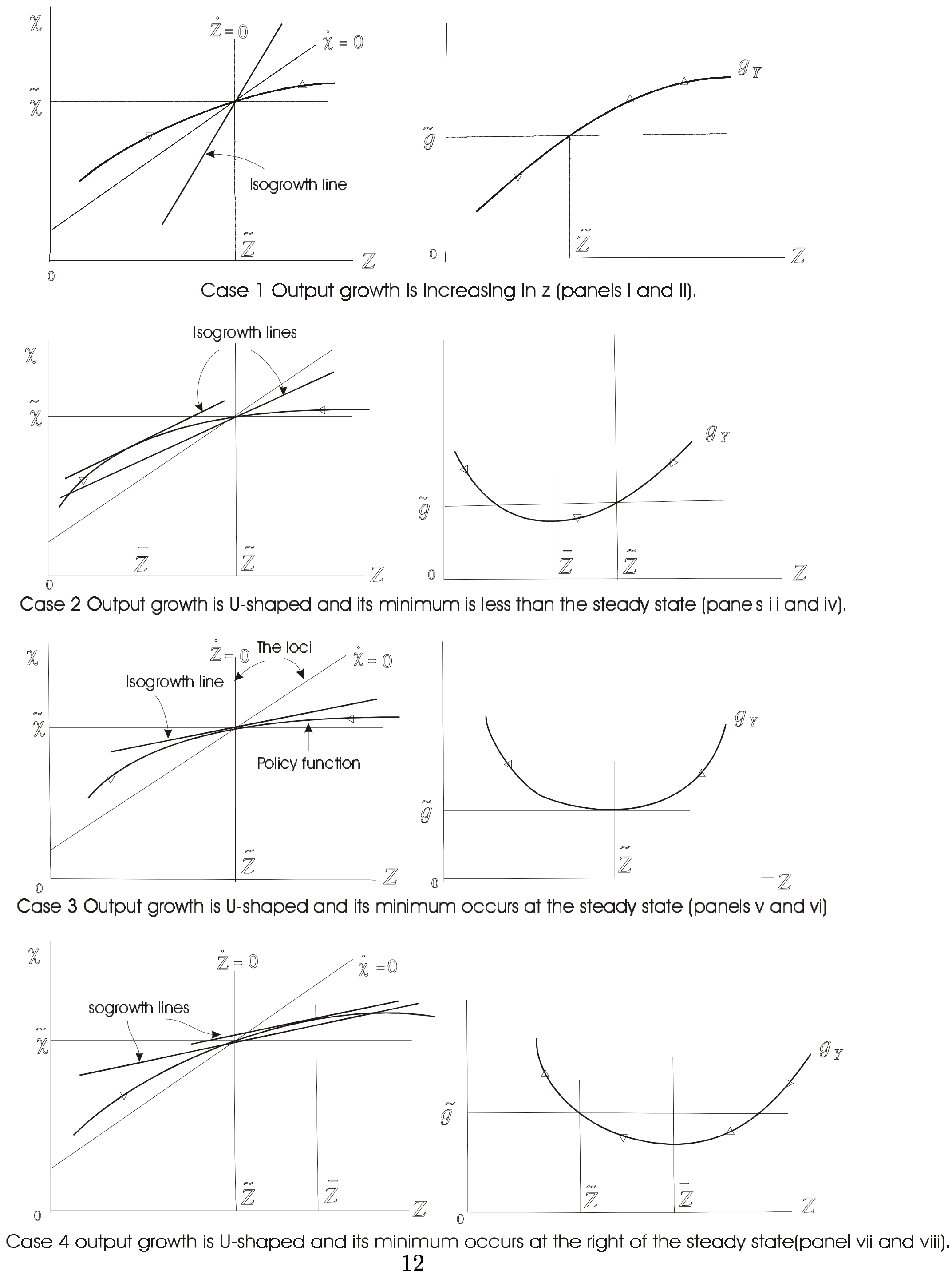

Figure 2: Dynamics of output growth changes according to the position of isogrowth lines relative to the policy function.