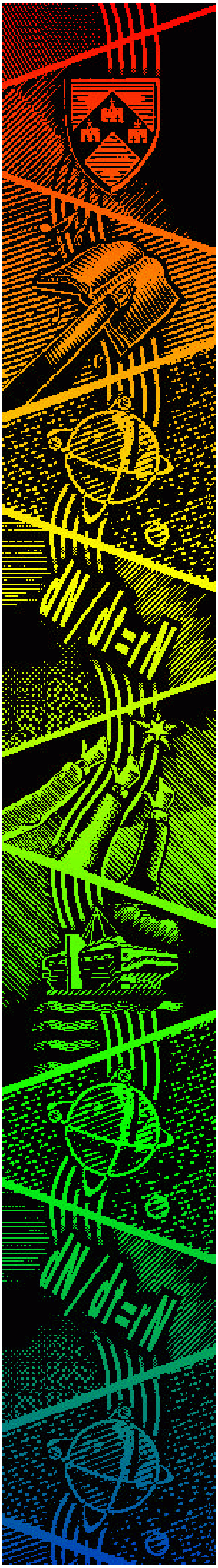

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Low Quality Leadership in Vertically Differentiated Duopoly

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Abstract

According to the business literature a firm's competitive position is determined by the nature of the market. In a 'premium' market, profit leadership falls to firms offering high quality, whereas in a 'value' market, it falls to low quality providers, which on grounds of a cost advantage capture a major market share. We incorporate this idea into a model of a vertically differentiated duopoly. In contrast to more conventional models, we assume that gross surplus from unit consumption consists of a benefit from quality and a baseline benefit. Consumers are heterogeneous (homogeneous) with regard to the former (latter). Marginal cost increases in quality. We derive the quality-then-price equilibrium in a non-covered market and show that a reduction in (maximum) willingness to pay relative to baseline benefit induces low quality leadership first in market share and subsequently in profit. Under low quality profit leadership, a minimum quality standard reduces consumer surplus if consumer's willingness to pay is sufficiently low and always reduces profits and welfare.

Keywords: Low quality leadership, vertical product differentiation, minimum quality standard.

JEL classification: L13, L15

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1 Introduction

Following the seminal work by Gabsewicz and Thisse (1979), models of vertically differentiated duopoly have been firmly established as a working horse to analyze firms' quality choices in a framework of strategic competition. However, the question of leadership – in terms of market share or profit – has been rarely commented upon. In fact, in the majority of the work, the high quality firm is the leader both in terms of market share and profit.

For the case of vertically differentiated firms competing in price and facing a zero marginal cost of production, Shaked and Sutton (1982) demonstrate the reason for high quality leadership. If any two qualities are offered at the same price, consumers unambiguously favour the higher one. Thus, a firm could always capture the complete market share of a low quality competitor by matching her price. Hence, for any price set by the low quality firm, the high quality firm can set a price giving it at the least an equal and usually a higher profit.[1] Lehmann-Grube (1997) extends their argument by showing that in the presence of a sunk cost of quality, the high quality firm still emerges as the profit leader. For a non-covered market, this result holds independently of whether firms choose qualities simultaneously or sequentially.

The notion of high quality leadership has been applied to innovation races and trade theory. Beath et al. (1997) put high quality leadership into a dynamic context by considering a sequence of patent races for product innovation. They ask under which conditions high quality leadership alternates and under which it persists. Herguera and Lutz (1996) incorporate the concept of high quality leadership into a trade model. They show that a minimum quality standard can be used to induce the domestic firm to leapfrog the foreign rival and attain a leadership position, which enhances profit and welfare.

The generality of high quality leadership in vertically differentiated markets is somewhat at odds with intuition. It should be noted at this point, that by the symmetry of these models, leadership in profit implies leadership in market share and vice versa. However, anecdotal evidence for many markets suggests that high quality firms are not leaders in market share.[2] While high quality profit leadership is based on the firm's capability of charging a premium price, the circle of customers of

[1] Shaked and Sutton extend this argument to demonstrate that for any bounded spread of consumers' preferences the market is limited to a certain number of variants (finiteness property). In the light of the above argument, the market is limited at the lower end of the quality spectrum. Their more general approach, which allows for variable costs of quality, however, includes the possibility that the finiteness property holds at the upper end of the quality spectrum (Shaked and Sutton 1983). The authors remain unclear about the implications for leadership.

[2] The markets for cars, consumer electronics, furniture or clothing are just a few of many examples, in which high quality firms are not leaders in terms of market share. Some authors model 'environmental friendliness' as a quality attribute (e.g, Arora and Gangopadhyay 1995, Cremer and Thisse 1999). The prediction of market share leadership in these models is then at odds with the factual market share of these products. (E.g., the market share of organic food in Germany hovers currently about 2 percent (reported in Sueddeutsche Zeitung, 24/01/2000); Brockmann et al. (1996) report a market share of low solvent paints in Western Germany of about 22.8 percent in 1993).

such firms is often quite narrow. If the provision of quality significantly raises variable cost of production, even profit leadership may lie with the low quality firm.

The business literature recognizes this greater richness in possibilities. Vishwanath and Mark (1997) try to explain a brand's profit in consumer goods markets. They argue that the explanation hinges on whether a market is 'premium' or 'value' based. 'Premium' markets are those, in which consumers have a considerable willingness to pay for brands, which they associate with quality. For these markets the authors recommend constant improvements of quality and service so as to guarantee a favourable brand image and eventually a high mark up. In contrast, 'value' based markets are characterized by price conscious consumers, where branded products are not able to capture any significant premium. Here, cost cutting is warranted so as to enable the producer of a brand to maintain or expand its market share. This, so the authors emphasize, should be followed even at the expense of a reduction of quality.

The notion of 'value' as opposed to 'premium' markets can be more precisely expressed by relating the price to the quality elasticity of market demand, $\eta := \left|\varepsilon(q,p)/\varepsilon(q,\theta)\right|$, where $\varepsilon(q,p)$ and $\varepsilon(q,\theta)$ are price and quality elasticity of demand, respectively. Thus, the greater η the more 'value' oriented a market. Then, there are two important determinants of the firm's strategies and their chances to acquire leadership. Variable cost is one and the measure η the other.

The purpose of this paper is to assess the role of preferences as expressed by η and variable cost in determining market share and profit leadership. We consider a vertically differentiated duopoly in a non-covered market, where firms' marginal cost is linear in quality. A consumer's relative evaluation of quality is captured by a gross surplus function, which consists of a quality dependent benefit and a quality independent baseline benefit. Consumers are heterogeneous with regard to the former and homogeneous with regard to the latter. The baseline benefit captures three aspects of vertically differentiated markets, which are not included in the standard model.[3]

Firstly, it recognizes that consumers may receive some benefit from the product, which is independent of quality. This provides a minimum quality level below which the product would not be functional. The baseline benefit captures the consumers' benefit at this quality. If 'quality' relates to some particular attribute of the product (e.g., product safety, a car's or microprocessor's speed, environmental friendliness) the baseline benefit measures the utility stream from all other attributes of the product. We use the baseline benefit as benchmark, against which we measure the weight consumers attach to quality in general or with regard to a particular dimension. A relatively low maximum (and average) willingness to pay for quality corresponds to a 'value' market. A high willingness to pay for quality corresponds to a 'premium' setting. Notice that the standard models of vertical differentiation,

[3] The introduction of a baseline benefit is no new. It is sometimes used to justify a covered market setting, where the baseline benefit is assumed to be large so that all consumers purchase the product (e.g., Cremer and Thisse 1994, Cremer and Thisse 1999). However, in a covered market the baseline benefit does not have any impact on firms' behaviour, market share, profit or welfare.

which do not take the baseline benefit into account, lack this benchmark. This is one of the reasons for the unambiguous finding of high quality leadership.

Secondly, the baseline benefit represents an alternative source of revenue for a firm besides the provision of quality. The relative importance of these two sources is crucial in determining a firm's choice of quality. Moreover, even if the marginal cost of quality exceeds the consumers' willingness to pay, a high quality firm can survive in the market if the baseline benefit creates a sufficient amount of revenue.

Finally, the baseline benefit captures the relative importance of heterogeneity, both with regard to products and consumers. As baseline benefit increases, the relative importance of product differentiation and heterogeneity of consumer tastes decreases. Both effects tend to enhance competition (Wauthy 1996) and, thus, imply a 'value' type of market.

Our main finding is that low quality profit and market share leadership is realized if and only if the low quality firm has a cost advantage and willingness to pay for quality is sufficiently small relative to the baseline benefit. A variable cost advantage is always necessary, while the presence of a positive baseline benefit is necessary if variable cost is concave in quality. The role of the baseline benefit then is to leverage an initial cost advantage. Leverage results since a high relative baseline benefit implies a relatively high degree of price sensitivity of consumers, i.e., a high η.

The switch from high to low quality leadership is sequential. For intermediate values of willingness to pay, a situation can arise, in which the low quality firm is leader in market share but not in profit. This is the empirically relevant case of high quality profit leadership under a niche strategy. If willingness to pay for quality is large relative to the baseline benefit, our model gives the conventional result of high quality leadership. To our knowledge, there exists no work, which raises the issue of low quality leadership.[4]

We also address the effect of a minimum quality standard under low quality leadership. Our findings qualify the result by Ronnen (1991) that, on the margin, the standard raises consumer surplus and welfare. We first show that under low quality leadership a minimum quality standard reduces aggregate consumer surplus if the willingness to pay is sufficiently small. To important extent, the result is driven by the (low) emphasis placed by consumers on quality as opposed to price. Even though the standard reduces product differentiation and enhances price competition, absolute prices increase if quality raises variable cost to a sufficient degree. Then, if consumer place little weight on quality, their surplus deteriorates.

Crampes and Hollander (1995) find a similar potential for a minimum quality standard to lower consumer surplus in a covered market. In their model, too, it is the increase in variable cost and absolute price, which may deteriorate consumer surplus. However, this occurs only if the high quality producer reacts to the standard by raising her quality and thereby mitigates the increase in price competition to some

[4] The model by Crampes and Hollander (1995) shows a potential for purely cost-driven low quality leadership in a covered market. However, the authors do not comment on it.

extent. In our model, aggregate consumer surplus may be reduced even in the absence of any reaction of the high quality producer.

In contrast to Ronnen (1991) and Crampes and Hollander (1995), the standard always harms both firms. This is because under low quality leadership the low quality firm optimally produces the lowest feasible quality. Boom (1995) arrives at a similar result with regard to profits. Her analysis focuses on a covered market or a corner configuration, where in both configurations the low quality firm chooses the lowest feasible quality. Finally, we find that under low quality profit leadership a minimum quality standard always reduces welfare even if the minimum quality standard raises aggregate consumer surplus. Crampes and Hollander (1995) and Scarpa (1998) arrive at a similar result for a covered market and a triopoly, respectively.

The baseline benefit introduces a certain amount of homogeneity with respect to a non-quality benefit into a model of vertical differentiation. This mirrors the introduction of quality into a model of horizontal differentiation, where consumers are homogeneous with respect to their evaluation of quality. Economides (1989) considers a duopoly within such a setting. Due to the symmetry in his model, the question of leadership cannot be addressed. Economides (1993) extends this model to include consumer heterogeneity with respect to quality but again, a symmetry assumption does not allow for the analysis of leadership. Dos Santos Ferreira and Thisse (1996) model quality as the consumers' rate of transportation cost, which is chosen by the firm. They allow for asymmetric equilibria and here the high quality firm turns out to be the leader in profit and market share.

The remainder of the paper is organized as follows. The next section introduces the model. Section 3 derives the price and quality equilibria in a non-covered market and discusses the issue of leadership. Section 4 studies the impact of a minimum quality standard and section 6 concludes. The proofs are relegated to an appendix.

2 The model

Demand

Consumers buy either one or zero units of a vertically differentiated good. The gross surplus from consuming one unit of the differentiated good is given by $v\theta + u$.[5] Here, $v\theta$ is the benefit derived from quality, where θ denotes quality and v denotes the consumer's willingness to pay (WTP) for quality. Consumers are heterogeneous with regard to v. In particular, we assume v to be uniformly distributed along the interval $[0, \bar{v}]$ with unit density.[6] Heterogeneity may be due to differences in tastes or in income (Tirole, 1988, section 2.1). For the sake of exposition, we adopt the income interpretation in the following, where WTP for quality increases in income.

[5] A list of notation can be found in the appendix.

[6] The assumption that the lowest WTP equals zero has no influence on the generality of our results.

Let $\theta \in [\underline{\theta}, \bar{\theta}]$, where $\underline{\theta}$ is the minimum level of quality for which the product is functional. It is reasonable to assume that consumers would not purchase a product of a lower quality so that firms have no incentive to provide it. $\bar{\theta}$ is an exogenous upper bound on quality.

Let u denote the baseline benefit, which is independent of quality and which we assume to be the same across consumers. As the 'quality' level $\underline{\theta}$ is always realized, the benefit from quality improvements is given by $v(\theta - \underline{\theta})$. In this sense, $v\underline{\theta}$ is a component of the baseline benefit with regard to which consumers are heterogeneous. This captures the reasonable presumption that consumers who care more about a product itself (i.e., have a higher $v\underline{\theta}$) are also likely to value quality higher (i.e., have a higher $v(\theta - \underline{\theta})$. Similarly, high-income consumers are not only likely to value the differentiated product higher relative to income but also place a higher value on quality. While acknowledging that the relationships involved are somewhat more complicated, we facilitate our use of terminology by referring to v as 'WTP for quality', and u as 'baseline benefit'.

An individual consumer's net surplus is given by

$$U = v\theta + u - p(\theta) \tag{1}$$

if she purchases one unit at price $p(\theta)$ and zero otherwise. Suppose two variants of quality θ_H and θ_L, where $\bar{\theta} \geq \theta_H \geq \theta_L \geq \underline{\theta}$, are offered at prices p_H and p_L, respectively. Under use of (1), the demand functions can be derived and are given by

$$q_H = (\bar{v} - v_1)/\bar{v}$$

for the high quality (H) variant and

$$q_L = (v_1 - v_0)/\bar{v}$$

for the low quality (L) variant. Here, $v_1 \in [v_0, \bar{v}]$ corresponds to the marginal consumer between H and L variant and $v_0 \in [0, v_1]$ to the marginal consumer who just purchases the low quality variant. Assuming, that both firms face positive demand, v_1 and v_0 are given by

$$v_1 = (p_H - p_L)/(\theta_H - \theta_L),$$

$$v_0 = \begin{cases} (p_L - u)/\theta_L \\ 0 \end{cases} \quad \text{if} \quad u + v_0\theta_L - p_L \begin{cases} = 0 \\ > 0 \end{cases} \tag{2},$$

respectively. If $v_0 = (p_L - u)/\theta_L > 0$ some consumers refrain from buying and the market is non-covered. Only then does the baseline benefit have an impact and directly so only on the L-firm's demand.[7] If the baseline benefit is sufficiently large, $v_0 = 0$ and the market is covered.[8] The demand for each variant is independent of the baseline benefit then.

Market demand is given by $q = (\bar{v} - v_0)/\bar{v}$. Obviously, the price or quality elasticity of market demand is different from zero only in a non-covered market. Then, demand elasticities with respect to price and quality are $\varepsilon(q, p) = p/(\bar{v}\theta - p + u)$ and $\varepsilon(q,\theta) = (p - u)/(\bar{v}\theta - p + u)$. Hence, we find the measure

$$\eta := |\varepsilon(q, p)/\varepsilon(q,\theta)| = p/(p - u),$$

where an increasing level of η means increased 'value orientation' or relative price sensitivity of the market. As easily checked, $d\eta/du > 0$.

Proposition 1. Let a consumer's net surplus be given by $U = \max\{v\theta + u - p(\theta), 0\}$, where she purchases either one or zero units, and assume that price and quality are set such that the market is non-covered. Then, the market is the more value oriented, in the sense of a greater measure η, the greater the baseline benefit u.

While both price and quality elasticity of demand decrease in u, the reduction is more pronounced for the quality elasticity. The greater baseline benefit, the lower the valuation of quality of the marginal consumer who is indifferent between purchasing or not, and who, therefore, defines market size. Thus, for a greater baseline benefit, quality tends to become less effective than price as means to attract this consumer.

Supply

There are two firms in the market: the H-firm and the L-firm, each providing a single variant of quality θ_H and θ_L, respectively. Variable cost is given by,

$$C = c\theta q .$$

Marginal cost $c\theta$ increases in quality if more (labour or material) input per unit is required or if the input mix includes a greater share of more expensive high quality inputs. Notice that $\underline{\theta} > 0$ implies a quality independent component of marginal cost. For the sake of simplicity, we do not consider sunk costs of quality. Lehmann-Grube (1997) argues that variable costs are irrelevant, as they do not commit firms to quality. However, by choosing a quality, the firm may have to commit to a specific technology. For example, a firm could invest into a machine, which produces high quality output but can be operated only with high quality inputs (e.g., skilled labour). If the costs of switching technology in the market stage are high, the firm is

[7] For $u = 0$ the values v_1 and v_0 and the demand functions for the non-covered market correspond to the ones in Ronnen (1991), Choi and Shin (1992), Motta (1993) and Lehmann-Grube (1997).

[8] This is the case modelled by the work referred to in footnote 4.

committed even though quality only affects variable cost.[9] This not withstanding, quality dependent sunk costs can be easily incorporated into the model. We argue in the concluding section that this does not affect our main results as long as fixed costs of quality do not decrease in quality by too much.

The firms engage in the standard game of quality-then-price competition. In stage one, both firms simultaneously choose a quality. Observing their competitors' quality, they simultaneously choose prices in stage two. Sub-game perfection being the equilibrium concept, the game is solved backwards.

3 Non-covered market equilibria

In the following, we focus on equilibria, in which the market is not covered. This is guaranteed by assuming

$$u < c\underline{\theta} \qquad (3).$$

As the consumer with zero WTP for quality would purchase not even if the variant of the lowest quality were offered at marginal cost, the market is never covered.

Price competition

Using firms' profit functions

$$\pi_i = (p_i - c\theta_i) q_i (p_i, p_j, \theta_i, \theta_j) \qquad i, j = H, L \quad j \neq i,$$

it is straightforward to derive the pair of Bertrand-Nash equilibrium prices

$$p_H{}^* = \frac{(\theta_H - \theta_L)[2\bar{v}\theta_H + u] + c(2\theta_H + \theta_L)\theta_H}{4\theta_H - \theta_L},$$

$$p_L{}^* = \frac{(\theta_H - \theta_L)[\bar{v}\theta_L + 2u] + 3c\theta_H\theta_L}{4\theta_H - \theta_L} \qquad (4).[10]$$

Demand follows as

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[9] In general, quality commitment does not arise from the irreversibility of the investment but rather from the irreversibility of quality choice. Indeed, by analogy to Dixit's (1980) distinction between capacity and quantity it is easily seen that sinking a quality dependent cost does not imply commitment. Suppose a firm's (sunk) investment into R&D does not determine a single quality but rather an upper bound below which the firm may produce any quality. Obviously, the firm is not committed to produce at this upper bound, just as in Dixit's case it is not committed to use full capacity.

[10] The second order conditions hold.

$$q_H* = \frac{[2(\bar{v}-c)\theta_H + u]}{\bar{v}(4\theta_H - \theta_L)} \quad (5a) \qquad q_L* = \frac{[(\bar{v}-c)\theta_L + 2u]\theta_H}{\bar{v}(4\theta_H - \theta_L)\theta_L} \quad (5b).$$

The following relationships are easily established

$$q_H* > 0 \Leftrightarrow \bar{v} > c - u/2\theta_H \qquad (6),$$ [11]

$$q_H* > 0 \Rightarrow q_L* > 0 \qquad (7).$$

As the present analysis focuses on a duopoly, assume (6) to be satisfied. Condition (7) then implies that demand for the L variant is always positive in a non-covered market. It is easily checked that conditions (6) and (7) also guarantee non-negative price cost margins.

Profits can now be written as functions of quality alone

$$\pi_H*(\theta_H, \theta_L) = \frac{[2(\bar{v}-c)\theta_H + u]^2(\theta_H - \theta_L)}{\bar{v}(4\theta_H - \theta_L)^2} \qquad (8a),$$

$$\pi_L*(\theta_H, \theta_L) = \frac{[(\bar{v}-c)\theta_L + 2u]^2\theta_H(\theta_H - \theta_L)}{\bar{v}(4\theta_H - \theta_L)^2\theta_L} \qquad (8b).$$

Note that for $c = u = 0$, equilibrium profits are the same as in Choi and Shin (1992). For the case of $c > 0; \quad u > 0$, the following Proposition gives an important first result.

Proposition 2. In the presence of a baseline benefit, $u > 0$, the H firm and L firm make positive profit even if the cost rate of quality exceeds the maximum WTP, such that $\bar{v} < c$.

Proof: Straightforward from (6) and (7).

The presence of a positive baseline benefit allows firms to sell their product at a price in excess of marginal cost even if the latter exceeds the maximum WTP for quality. In the standard models of vertical differentiation, in which $u = 0$, a situation in which quality is provided under $\bar{v} < c$ cannot arise. This is because consumers WTP for the product relates to quality alone, which in this case is not sufficient for firms to recover marginal cost. So far, the result has been derived for exogenous qualities. A complete analysis, which endogenizes the firms' quality choices, follows below.

[11] From assumption (3) it follows that $\bar{v} > c/2$ is necessary.

Low quality leadership

We now derive the conditions under which the L firm attains market share and/or profit leadership. From (5a) and (5b) and (8a) and (8b), the following relations can be derived

$$q_L* > q_H* \Leftrightarrow \bar{v} < c + u\left(2\theta_H - \theta_L\right)/\left(\theta_H\theta_L\right) =: v_q \tag{9}$$

$$\pi_L* > \pi_H* \Leftrightarrow \bar{v} < c + u/\sqrt{\theta_H\theta_L} =: v_\pi \qquad (10).$$

From this, we obtain a characterisation of L firm leadership.

Proposition 3. (i) The L firm leads with regard to market share if $\bar{v} < v_q$. This is always the case if the cost rate exceeds maximum WTP, i.e., if $\bar{v} < c$. Otherwise, it is the more likely the greater ceteris paribus baseline benefit, cost rate of quality and degree of product differentiation and the lower the level of L quality. (ii) The L firm leads with regard to profit if $\bar{v} < v_\pi$. This is always the case if the cost rate exceeds maximum WTP. Otherwise, it is the more likely the greater ceteris paribus baseline benefit and cost rate of quality and the lower the degree of differentiation and L quality. (iii) Profit leadership implies market share leadership but not vice versa.

Proof: See Appendix.

It is hardly surprising that L market share and profit leadership arise if $\bar{v} < c$. In this case, the majority of consumers favour the L variant, which implies market share leadership. Operating at a higher price cost margin, the L firm acquires profit leadership. Given that condition (6) holds, this result is unaffected by the baseline benefit.

For $\bar{v} > c$ and given levels of quality, L leadership arises this is the case if the baseline benefit is sufficiently large. Here, the consumers' *relative* valuation of quality drives the result. If consumers value quality little relative to the baseline benefit, this corresponds to a value market and tends to favour L leadership. Notice that this holds independently of the absolute values of $\bar{v}$ and u.

The effect of product differentiation on L leadership is ambiguous. Differentiation enhances L market share leadership but delimits L profit leadership. The L firm is more likely to lead in market share in highly differentiated markets for the following reason. If products are close substitutes for small differences in quality levels, the H firm is unable to charge a high premium. But at a small price differential, more consumers tend to buy the H variant and it is more difficult for the L firm to gain leadership. A high level of differentiation is the outcome if the H firm implements a niche strategy and introduces a highly differentiated product. The H firm's target is then not to dominate the market in terms of market share but rather to use its quality leadership to extract consumer surplus. This is reflected in the L firm's dominant market share. To appreciate the negative impact of differentiation on L profit leadership, we have to take into account that the (absolute) demand faced by each firm tends to decrease in product differentiation. Under greater differentiation both firms are able to charge higher prices, which in turn reduces demand. Market share is then less important in generating revenue relative to price. As for $\bar{v} > c$ the H firm is able to charge a higher mark up, product differentiation favours H profit leadership.

The negative effect of the level of L quality both on market share and profit leadership is easily explained by referring to the sources of L leadership: cost advantage and baseline benefit. By raising quality the L firm gives away some of its cost advantage. Moreover, if the cost increase triggers an increase in absolute price, the L firm also loses demand from those consumers with a low WTP for quality, who have purchased the product mostly for the sake of the baseline benefit.[12] Both effects tend to weaken the L firm's position in terms of market share and profit. As expected, the cost rate of quality improves the L firm's competitive position and, thereby, favours L leadership.

According to part (iii) of Proposition 2, the L firm can acquire profit leadership only by becoming price leader and capturing the major share of the market. Clearly, this is in line with all anecdotal evidence and conventional marketing wisdom. The H firm can maintain profit leadership even if the L firm holds the dominant market share. Such a situation arises if the H firm can draw on its niche position and extract a sufficient amount of the high WTP consumers' surplus.

Before endogenizing quality choice to arrive at a complete picture of leadership, let us summarize the role of cost advantage and market type for L leadership.

Corollary 3. (i) L leadership arises (a) only if marginal cost increases in quality, and (b) for variable cost being linear in quantity and concave in quality, only in the presence of a baseline benefit. (ii) Baseline benefit and cost advantage are imperfect substitutes in generating L leadership. (iii) In the presence of a baseline benefit, the pattern of leadership is asymmetric.

The argument by Shaked and Sutton (1982) that under symmetric variable cost the H firm could always drive the L firm from the market applies completely unrelated to the baseline benefit. Thus, a cost advantage is a necessary condition for L leadership.[13]

For a linear specification of variable cost, L leadership in a non-covered market can never arise on grounds of the cost advantage alone. This is easily seen from (9) and (10), where for $u = 0$, L leadership implies $\bar{v} < c$. But this would violate the non-negative profit condition (6).

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[12] This can be easily verified by differentiating the RHS of (5b) with respect to θ_L.

[13] Notice that (9) and (10) are somewhat deceptive in that they suggest the possibility of L leadership for $c = 0$. It should be noted, however, that in this case the market switches to a corner or fully covered configuration, for which L leadership requires $c > 0$.

While part (ia) of the Proposition is general, it can be easily verified that part (ib) is limited to cost functions, which are not too convex in quality. For a function $C = c\theta^{\gamma} q$, this implies that a positive baseline benefit is a necessary condition for leadership if $\gamma \leq 1+\varepsilon$. We do not explore the particular value of ε in detail.[14] However, it can be checked for a quadratic specification $C = \left(\theta^2/2\right)q$, as in Motta (1993), that, in the absence of a baseline benefit, the H firm is leader in a non-covered market.[15],[16] This suggests that $\varepsilon \geq 2$.

The more general effect of the baseline benefit is captured in part (ii) of the Corollary and easily verified from (9) and (10). For a given cost advantage, the L firm is the more likely to attain leadership the larger the baseline benefit or, in other words, the more sensitive consumers are to price relative to quality.

Part (iii) of the Corollary is important in that in the presence of a baseline benefit an asymmetric pattern of leadership arises, which cannot be explained by recurring to a cost advantage alone.

Quality equilibrium

Consider now the quality stage of the game. From (8a) and (8b),

$$\frac{\partial \pi_H{}^*(\theta_H,\theta_L)}{\partial \theta_H} = \frac{\left[2(\bar{v}-c)\theta_H + u\right] r(\theta_H,\theta_L)}{(4\theta_H - \theta_L)^3}$$

$$\frac{\partial \pi_L{}^*(\theta_H,\theta_L)}{\partial \theta_L} = \frac{\left[(\bar{v}-c)\theta_L + 2u\right]\theta_H\, s(\theta_H,\theta_L)}{(4\theta_H - \theta_L)^3 \theta_L{}^2},$$

where

$$r(\theta_H,\theta_L) = 2(\bar{v}-c)\alpha(\theta_H,\theta_L) - u\beta(\theta_H,\theta_L) \qquad \text{(11a)},$$

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[14] If marginal cost is concave in quality there exists a critical level of $\gamma^* \in [0,1]$ such that the L firm has an effective cost-advantage only if $\gamma > \gamma^*$. The L firm is disadvantaged by the presence of quality dependent variable cost for lower levels of γ as it is more limited in its capability of passing cost on to consumers by raising price. Hence, if for low levels of γ, firms produce at almost equal marginal cost, the negative effect on the profit margin is more pronounced for the L than for the H firm. Only if γ is sufficiently high, does the difference in marginal cost become dominant so as to place the H firm at a disadvantage.

[15] This can be verified by using equations (20) and (21’) in Motta (1993) to derive a condition for H profit leadership and using the first order conditions (22) and (22’) to check that optimum choices of quality imply satisfaction of this condition. This generalizes to $C = c\theta^2 q$.

[16] For the covered market model by Crampes and Hollander (1995) it can be shown that for a cost function $C = c\theta^{\gamma} q$, L leadership arises if and only if $\gamma > 2$.

$$s(\theta_H,\theta_L)=(\bar{v}-c)\theta_H\theta_L\beta(\theta_H,\theta_L)-2u\alpha(\theta_H,\theta_L) \tag{11b}$$

with

$$\alpha(\theta_H,\theta_L):=4\theta_H^2-3\theta_H\theta_L+2\theta_L^2>0 \quad \text{(12a)}; \qquad \beta(\theta_H,\theta_L):=4\theta_H-7\theta_L \quad \text{(12b)}.$$

Observe that

$$\alpha(\cdot)>\max\{-\theta_H\beta(\cdot),\theta_H\beta(\cdot)\}\ \ \forall\theta_H>\theta_L \tag{13}$$

Obviously, $\text{sgn}(\partial\pi_H*/\partial\theta_H)=\text{sgn}\, r$ and $\text{sgn}(\partial\pi_L*/\partial\theta_L)=\text{sgn}\, s$. We can therefore focus on the expressions in (11a) and (11b) alone. The following can be shown

Lemma 1. An interior equilibrium, in which the firms choose qualities θ_L* and θ_H* such that $\underline{\theta}<\theta_L*\leq\theta_H*<\bar{\theta}$, does not exist.

Proof: See Appendix.

This leaves the following three candidate equilibria.

- lower boundary equilibrium, where $\underline{\theta}=\theta_L*<\theta_H*<\bar{\theta}$;
- maximum differentiation equilibrium, where $\underline{\theta}=\theta_L*<\theta_H*=\bar{\theta}$;
- upper boundary equilibrium, where $\underline{\theta}<\theta_L*<\theta_H*=\bar{\theta}$.

In the following, we establish the three types of equilibria as depending on the maximum WTP, $\bar{v}$. As a prerequisite assume that

$$(7/4)\underline{\theta}<\bar{\theta}<(13/4)\underline{\theta} \tag{14}$$

This assumption guarantees the existence of a unique equilibrium. Moreover, define

$$v_l:=c+\left[\beta(\bar{\theta},\underline{\theta})/2\alpha(\bar{\theta},\underline{\theta})\right]u \tag{15a}$$

$$v_h:=c+\left[2\alpha(\bar{\theta},\underline{\theta})/\bar{\theta}\underline{\theta}\beta(\bar{\theta},\underline{\theta})\right]u \tag{15b}$$

From (13), it is easy to verify that $v_l<v_h$. We can now establish the equilibria.

Proposition 4. (i) A unique lower boundary equilibrium is realized if and only if $\bar{v}<v_l$. (ii) A unique maximum differentiation equilibrium is realized if and only if $\bar{v}\in[v_l,v_h]$. (iii) A unique upper boundary equilibrium with $\theta_L*\in\left[\underline{\theta},(4/7)\bar{\theta}\right]$ and $\theta_H*=\bar{\theta}$ is realized if and only if $\bar{v}>v_h$. (iv) The comparative static properties are as follows.

	$d\theta_H$ *	$d\theta_L$ *
du	*UB and MD:* $=0$ *LB:* $>0 \Leftrightarrow \bar{v} < c$ $<0 \Leftrightarrow \bar{v} > c$	*UB:* <0 *LB and MD:* $=0$
$d(\bar{v} - c))$	*UB and MD:* $=0$ *LB:* >0	*UB:* >0 *LB and MD:* $=0$

Table 1: Comparative statics. (UB: Upper bound, MD: Maximum Differentiation, LB: Lower bound)

Proof: See Appendix.

Corollary 4.1. The lower (upper) boundary equilibrium involves L (H) market share and profit leadership.

Proof: See Appendix.

We have, thus, established a sequence of three equilibria, which for a given cost rate and a given range of feasible qualities depend on the (maximum) WTP for quality relative to baseline benefit. Figure 1 gives an overview of the equilibria and the associated pattern of leadership.

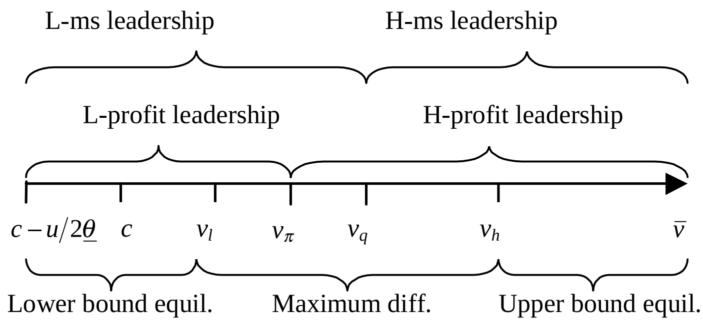

Figure 1: Equilibria and leadership.

The lower bound and upper bound equilibria roughly correspond to the value and premium market. Generally, firms have an incentive to maintain product differentiation in order to stifle price competition. This incentive is curbed if the market is either very value or very premium oriented. In the former case, consumers are not much concern for quality and perceive products as relatively good substitutes. Thus, price competition is intense even under product differentiation. In order to remain competitive in price, the H firm has to reduce variable cost by cutting quality. In the latter case, consumers are willing to pay a substantial premium for quality. Here, the L firm has an incentive to raise quality in order to capture some of the surplus, consumers receive from quality, even if this is at the expense of stiffer competition. If the market configuration is indeterminate, the incentive to stifle competition dominates and maximum product differentiation arises.

The pattern of leadership follows immediately from the arguments given in the previous section. Notice that the lower and upper boundary equilibria are fully associated with L and H leadership, respectively. The switch of leadership occurs under maximum differentiation, where for a decreasing level of WTP, the L firm acquires first leadership in market share and then in profit. The following is easily checked from (9) and (10).

Corollary 4.2. (i) The range of L profit leadership increase in the cost rate and baseline benefit. (ii) The range of L market share leadership in baseline benefit.

We finally consider the impact of the market configuration on average quality. Determining average quality as

$$\theta_a = \left(\theta_H * q_H * + \theta_L * q_L *\right) / \left(q_H * + q_L *\right)$$

and inserting (5a) and (5b) to derive an explicit expression, the following is easily shown.

Corollary 4.3. The average quality in the market (i) decreases in the cost rate of quality, and (ii) decreases in baseline benefit if and only if $\bar{v} > c$.

The exogenous variables affect average quality directly by determining the firms' quality choices and indirectly by determining market share. By strengthening the cost advantage of the L firm, an increase in the cost rate shifts market share towards the L quality variant. For given quality levels, this reduces average quality. In the upper and lower boundary equilibrium, average quality tends to be further reduced by reductions in L and H quality, respectively.[17]

For $\bar{v} > c$, average quality decreases in baseline benefit. Clearly, as consumers become relatively more price-sensitive, this enhances the market share of the L firm. Likewise, both firms' incentives to provide quality are reduced and L and H quality falls in the upper and lower boundary regime, respectively.

For $\bar{v} < c$, baseline benefit enhances average quality. As WTP for quality falls short of the cost rate, the H firm chooses a very low quality in order to remain profitable. At the resulting low degree of product differentiation price competition is intense. An increase in baseline benefit induces the L firm to raise its price by more than the H firm does. One consequence, is that the L firm looses market share relative to the H firm, which has a positive impact on average quality. With the immediate

[17] The reduction in quality has an additional impact through market share, which does not change the direction of the effect.

competitive pressure on the H firm, thus, reduced, it has some scope to raise quality in order to stifle competition a little further. Hence, an additional positive stimulus on average quality.

4 Welfare effects of a Minimum Quality Standard

Suppose a minimum quality standard (MQS) is introduced. In the following, we assess the welfare impact of this. We constrain our analysis in two ways. Firstly, we only consider the marginal case, where the MQS is set at the laissez-faire level of L quality and is then raised marginally. Secondly, we focus on the case of L (profit) leadership alone. The case of H leadership follows as an extension to our model and has been extensively treated elsewhere (Ronnen 1991).

Welfare is measured by gross surplus

$$W = S + \pi_H + \pi_L,$$

where

$$S = \frac{1}{\bar{v}}\left[\int_{v_0}^{v_1}(v\theta_L + u - p_L)dv + \int_{v_1}^{\bar{v}}(v\theta_H + u - p_H)dv\right]$$

$$= q_L\left[\frac{(v_0 + v_1)\theta_L}{2} + u - p_L\right] + q_H\left[\frac{(v_1 + \bar{v})\theta_H}{2} + u - p_H\right] \qquad (16)$$

is consumer surplus. By (16) this is conveniently expressed as the weighted sum of average surplus from each variant, where quantities are used as weights.

We begin by considering the effect of the MQS on the firms' quality choices. Afterwards, we study the impact on consumer surplus and profits separately and finally we consider the welfare impact.

Considering the case of L leadership only, we can focus on the lower bound and maximum differentiation equilibria. Assume that the MQS is set such that $\theta_{\min} = \theta_L *$. Obviously, then $d\theta_L */d\theta_{\min} = 1$. The impact of the MQS on H quality depends on the type of equilibrium. For the lower boundary equilibrium, where $\theta_H * < \bar{\theta}$ we have to consider the reaction

$$\left.\frac{d\theta_H *}{d\theta_{\min}}\right|_{\theta_H * < \bar{\theta}} = \frac{d\theta_H *}{d\theta_L} = -\frac{\partial r/\partial\theta_L}{\partial r/\partial\theta_H} = \frac{2(\bar{v} - c)(3\theta_H * - 4\underline{\theta}) - 7u}{2(\bar{v} - c)(8\theta_H * - 3\underline{\theta}) - 4u},$$

where the derivatives of r are easily found from (11a). By using the fact that $\bar{v} < v_l$ and the definition in (15a) one can easily establish that

$$\left.\frac{d\theta_H *}{d\theta_{\min}}\right|_{\theta_H *<\bar{\theta}} \in \left[1, \frac{\theta_H *}{\underline{\theta}}\right].$$

The H producer reacts to a MQS in the same way in our model as she does in models that involve a convex cost of quality (Ronnen 1991, Crampes and Hollander 1995).[18] The firm raises quality in order to alleviate the tighter competition, but only imperfectly so. Thus, the relative degree of differentiation $\theta_H */\theta_{\min}$ decreases. In contrast to previous work, where this is due to the convexity of cost, in our model, the H firm is reluctant to raise quality as for the lower boundary regime it wishes to remain competitive in price. Obviously, for the maximum differentiation equilibrium $d\theta_H */d\theta_{\min} = 0$.[19]

The impact of the MQS on consumer surplus is given by the total derivative of (16). For $\theta_L * = \theta_{\min}$

$$\frac{dS}{d\theta_{\min}} = \frac{dq_L *}{d\theta_{\min}}\left[\frac{(v_0+v_1)\theta_{\min}}{2} + u - p_L\right] + q_L *\left[\left(\frac{dv_0}{d\theta_{\min}} + \frac{dv_1}{d\theta_{\min}}\right)\frac{\theta_{\min}}{2} + \frac{v_0+v_1}{2} - \frac{dp_L}{d\theta_{\min}}\right]$$
$$+ \frac{dq_H *}{d\theta_{\min}}\left[\frac{(v_1+\bar{v})\theta_H *}{2} + u - p_H\right] + q_H *\left[\frac{dv_1}{d\theta_{\min}}\frac{\theta_H *}{2} + \frac{v_1+\bar{v}}{2}\frac{d\theta_H *}{d\theta_{\min}} - \frac{dp_H}{d\theta_{\min}}\right]$$

and after cancelling terms

$$\frac{dS}{d\theta_{\min}} = q_L *\left(\frac{v_0+v_1}{2} - \frac{dp_L}{d\theta_{\min}}\right) + q_H *\left(\frac{v_1+\bar{v}}{2}\frac{d\theta_H *}{d\theta_{\min}} - \frac{dp_H}{d\theta_{\min}}\right).$$

Thus, the marginal impact of the MQS on aggregate consumer surplus is given by the change of average consumer surplus from each variant at given levels of quantities. Notice that the changes in market share are implicit in the above expression. Writing the above expression explicitly gives

$$\frac{dS}{d\theta_{\min}} = q_L * \rho + q_H * \psi \qquad (17),$$

where

$$\rho := \frac{3[(\bar{v}-c)\theta_{\min} + 2u]}{(4\theta_H * - \theta_{\min})^2}\left(\theta_H * - \frac{d\theta_H *}{d\theta_{\min}}\theta_{\min}\right) + \frac{(\bar{v}-c)\theta_H * \theta_{\min} - 2(\theta_H * + \theta_{\min})u}{2\theta_{\min}(4\theta_H * - \theta_{\min})} \qquad (18a)$$

---

[18] The reaction is quite different in a triopoly (Scarpa 1998).

[19] Notice that the MQS lowers the boundary level v_l implying that the range of maximum differentiation equilibria increases. We do not formally analyze the switch of regime as this would not give further insights.

$$\psi := \frac{3\left[2(\bar{v}-c)\theta_H * + u\right]}{(4\theta_H * - \theta_{\min})^2}\left(\theta_H * - \frac{d\theta_H *}{d\theta_{\min}}\theta_{\min}\right) + \frac{2(\bar{v}-c)(\theta_H * + \theta_{\min}) - u}{2(4\theta_H * - \theta_{\min})}\frac{d\theta_H *}{d\theta_{\min}} \quad \text{(18b).}$$

The first terms on the RHS of (18a) and (18b), respectively, are strictly positive, which follows from (6) and $\theta_H / \underline{\theta} > d\theta_H * / d\theta_{\min}$. These terms capture the positive impact on average consumer surplus from increased price competition, as the variants become closer substitutes.

The second terms on the RHS of (18a) and (18b), respectively, give the direct impact of quality changes on a variant's average surplus. For $u = c = 0$, both terms are strictly positive, implying an under-provision of quality (Ronnen 1991). This is due to the well-known fact that the firms target their quality choice on the marginal rather than the average consumer. However, the direct impact of quality on surplus is negative if $\bar{v} < c$ or if u is sufficiently large. Here, we have to observe that in the absence of competition, a firm would raise its price with an increase in quality. In a value market, with a high level of u, consumers are more concerned about the price rather than the quality increase. The following Lemma establishes an important intermediate result, which follows from direct comparison of ρ and ψ .

Lemma 2. $\rho < \psi$*. Hence, for a small increase in the MQS, the net change in average surplus is always greater for the H variant.*

On average the consumers of the H variant are more prone to benefit from a MQS policy. For $\rho < 0 < \psi$, the MQS deteriorates average surplus from consumption of the L variant and raises that of the H variant.[20] Even though average consumer surplus may decrease in the model of Crampes and Hollander (1995), an asymmetry cannot arise. The asymmetry helps to explain the realistic situation in which (on average) consumers of H quality are in favour of quality regulation whereas consumers of L quality are opposed to it.

Proposition 5. Consider a small increase in the MQS over the laissez-faire level of L quality. (i) In the lower boundary regime, where $\bar{v} < v_l$*, this unambiguously reduces aggregate consumer surplus. (ii) In the maximum differentiation regime with L profit leadership, where* $\bar{v} \in (v_l, v_\pi)$*, this reduces (increases) consumer surplus if* $\bar{v}$ *is sufficiently close to* v_l *(*v_π*).*

Proof: See Appendix.

In the lower boundary regime the consumers' relative concern for quality is so limited that the increase in absolute price outweighs the increase in quality.

[20] It should be noted that the argument relates to average consumer surplus. While it is straightforward to show that the net effect of the MQS on individual consumer's surplus from a given variant decreases in income, the effects are not unambiguously ordered across variants. The lowest income purchaser of the H variant obtains a lesser net-benefit from the MQS than the highest income purchaser of the L variant.

Aggregate consumer surplus, thus, decreases in the MQS. Market coverage is also reduced in this instance. These findings are in contrast to Ronnen (1991), Boom (1995) and Scarpa (1998), where, in the absence of a baseline benefit, consumers always benefit from the MQS. Crampes and Hollander (1995) identify a potentially negative impact of the MQS on consumer surplus in a covered market. Their result, however, is driven by variable cost alone. They find that consumer surplus is reduced if and only if the increase in competition is softened by a sufficiently strong reaction $d\theta_H{}^*/d\theta_{\min} > 0$, i.e., if the cost function is not too convex. In our model, the reduction of consumer surplus through the MQS cannot be explained on grounds of cost alone. Indeed, Ronnen shows that the MQS unambiguously raises welfare if marginal cost is linear in quality. In our model, the negative impact of a MQS on consumer surplus, therefore, reflects the low relative preference for quality.

For the maximum differentiation regime with L profit leadership, the impact of a MQS is ambiguous. If baseline benefit is sufficiently large the MQS still lowers consumer surplus. Remarkably, this is the case even though for $d\theta_H{}^*/d\theta_{\min}\big|_{\bar{v}\in[v_l,v_\pi]} = 0$ the increase in competition between the variants is uncushioned. The contrast of this to Crampes and Hollander's finding emphasises the fact that our result is driven by preferences rather than by cost.

In the maximum differentiation regime, H consumers' surplus is unambiguously raised.[21] Obviously, for a constant H quality, the decrease in price under tighter competition raises surplus. While for a relatively low WTP for quality the increase in H consumers' surplus is still overcompensated by the decrease in L consumers' surplus, this is reversed as WTP becomes sufficiently large. Then, the MQS increases aggregate consumer surplus even in a market with L leadership.[22]

Now, consider the impact of the MQS on profits. From (8a)

$$\frac{d\pi_H}{d\theta_{\min}} = \frac{\partial \pi_H}{\partial \theta_L} = -\frac{(2\theta_H{}^* + \theta_{\min})[2(\bar{v}-c)\theta_H{}^* + u]q_H{}^*}{(4\theta_H{}^* - \theta_{\min})^2} < 0 \qquad (19),$$

which for the case of $d\theta_H{}^*/d\theta_{\min} > 0$, follows under use of the envelope theorem. As in the standard model, the MQS reduces H profit, as products become less differentiated and competition increases. From (8b), we obtain

$$\frac{d\pi_L}{d\theta_{\min}} = \frac{\partial \pi_L}{\partial \theta_{\min}} + \frac{\partial \pi_L}{\partial \theta_H}\frac{d\theta_H{}^*}{d\theta_{\min}} = \frac{\lambda q_L{}^*}{(4\theta_H{}^* - \theta_{\min})^2 \theta_H{}^* \theta_{\min}} \qquad (20),$$

$$\lambda := \theta_H{}^* s(\theta_H{}^*, \theta_{\min}) + (2\theta_H{}^* + \theta_{\min})\theta_{\min}^2 [(\bar{v}-c)\theta_{\min} + 2u] d\theta_H{}^*/d\theta_{\min} \qquad (21).$$

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[21] This is easily checked from (18b), where $d\theta_H{}^*/d\theta_{\min} = 0$.

[22] It can be shown that for $\bar{\theta} \to (7/4)\theta_{\min}$ even the L consumers' average surplus increases for $\bar{v} = v_\pi$.

Obviously, $\text{sgn}(d\pi_L/d\theta_{\min}) = \text{sgn}\,\lambda$. A priori, λ cannot be signed unambiguously. Intuitively, this is because a reaction $d\theta_H{}^*/d\theta_{\min} > 0$ tends to increase L profit, while the direct impact of the MQS is negative in the lower boundary and maximum differentiation regimes. The following applies.

Lemma 3. $d\pi_L/d\theta_{\min} < 0 \quad \forall\bar{v} < v_h$.

Proof: See Appendix.

Thus, the MQS reduces L profit under both forms of L leadership. This occurs due to a reduction in the price cost margin and a loss of demand at the bottom end, as consumers with a low WTP refrain from buying the product at the higher absolute price.[23]

We now establish the impact of the MQS on welfare as

$$\frac{dW}{d\theta_{\min}} = \frac{dS}{d\theta_{\min}} + \frac{d\pi_H}{d\theta_{\min}} + \frac{d\pi_L}{d\theta_{\min}} \qquad (22).$$

Proposition 6. An increase in the MQS always lowers welfare under L profit leadership, i.e., for $\bar{v} \le v_\pi$.

Proof: See Appendix.

Hence, a MQS is strictly welfare reducing under L profit leadership. Even at the switching point from L to H leadership the reduction in profits overcompensates the increase in consumer surplus. It is easily verified in our model that a MQS raises welfare in the case of H market share leadership, i.e., if $\bar{v} \ge v_q$. This is the case depicted by Ronnen (1991). However, the conclusion drawn by Ronnen that a marginal MQS unambiguously raises welfare in a non-covered market must be qualified with regard to the question of leadership.

Usually, quality regulation relates to some particular feature of the product, e.g., environmental friendliness (Aurora and Gangopadhyay 1995) or safety (Ronnen 1991). Now suppose that this feature alone is measured by θ. Assuming homogeneity with regard to all other characteristics it is easy to see that u captures all benefits flowing from the product apart from the benefit relating to θ. Given that the regulated feature is of relatively minor importance for the consumption decision, i.e., given a low value of $\bar{v}$, quality regulation triggers a reduction in welfare. Such a constellation is likely, for instance, in the case of environmental regulation. While the presence of externalities associated with the product may warrant regulation, one should be careful to avoid a justification on grounds of its effects on aggregate surplus alone.

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[23] It is easily checked that $\left[dv_0(\theta_H{}^*, \theta_{\min})/d\theta_{\min}\right]\Big|_{\bar{v} \le v_\pi} > 0$.

5 Conclusions

This paper has formally established the case for low quality market share and profit leadership in a non-covered market. We have shown that low quality leadership arises first in market share and then in profit if the low quality firm has an advantage in the variable cost of production and if the consumers' willingness to pay for a product decreases relative to a quality independent baseline benefit.

These findings give a formal underpinning to the business literature, which recommends a strategy to acquire or catch up with quality leadership only in markets in which consumers are willing to pay a premium for quality. For markets, in which consumers are more concerned about price, the message is to focus on a lowering of production cost in order to acquire market share leadership.

The case of low quality leadership has implications for advanced analyses. For instance, the race for product innovation considered by Beath et al. (1997) would turn into a race for process innovation, much as in a homogeneous goods market. Additionally, a case may arise, in which some firms race for product innovation in order to be leaders in a market niche, while others push process innovation to compete on grounds of low cost.

We have shown that a MQS reduces welfare if the low quality firm leads in profit. Profits are unambiguously reduced and aggregate consumers' surplus is reduced if the preference for quality is sufficiently weak. While this result underlines concerns about quality regulation, it would be wrong to dismiss quality regulation in the presence of low quality leadership on the grounds of our narrow concept of welfare alone. The presence of externalities, imperfect information or other market distortions may still justify quality regulation. However, there is no unambiguous case for quality standards anymore as suggested by Ronnen (1991). In the context of trade theory, the role of a minimum quality standard as a device to trigger leapfrogging towards high quality leadership (Herguera and Lutz 1996) is turned around in the presence of low quality leadership. Now an incentive arises to deregulate in order to allow domestic firms to become low quality leaders.

Let us now briefly argue, why we do not believe that our qualitative results are substantially changed in the presence of sunk costs of quality. Consider the case of a convex cost of quality, which has to be sunk up front. This would have three main effects on our results.

Firstly, it would reinforce the case of profit leadership for the low quality firm as it always faces a lower sunk cost. Secondly, the convexity would generate a tendency towards interior solutions, where the high quality firm always chooses a quality $\theta_H{*} < \overline{\theta}$. This would blur our distinction between the lower boundary and maximum differentiation regime, where in both regimes the firms would choose $\underline{\theta} = \theta_L{*} < \theta_H{*} < \overline{\theta}$. For the lower boundary regime the tendency towards reducing quality as generated by preferences would only be reinforced. For the former upper boundary regime, we would obtain a fully interior configuration $\underline{\theta} < \theta_L{*} < \theta_H{*} < \overline{\theta}$. Thirdly, and this seems to be the only substantial difference, the impact of u on high quality choice would become ambiguous: On the one hand, a higher u implies a relatively greater emphasis on price, which induces the high quality firm to lower quality in order to become more competitive. This is the only effect present in our variable cost setting. On the other hand, in a non-covered market a greater u tends to increase both firms' market share. The high quality firm would then be able to spread the sunk cost over a greater quantity and could use the ensuing economies to scale to raise quality.

Real differences would arise in the following two cases. (i) Suppose by sinking a cost, a firm cannot only improve its quality but also reduce its variable cost of production (e.g., by replacing labour by a machine, which operates at higher precision). In this case, we have high-quality– high-fixed-cost firms competing against low-quality-high-variable cost firms. Obviously, in such a setting high quality leadership arises. (ii) Suppose sunk cost decreases in quality (e.g., if highly specific technology is replaced by a flexible technology, which can easily be rented out or sold if not used; if production tasks are out-sourced to specialists, which offer superior quality). It then remains unclear if leadership of the L firm with respect to operating profit carries over to leadership with respect to overall profit.

Let us briefly comment on assumption (14), which limits the range of quality. The lower bound is necessary for the feasibility of an interior quality choice by one of the firms if $\bar{v} > c$. For $\bar{\theta} < (7/4)\underline{\theta}$ we would always obtain maximum differentiation. The upper bound guarantees uniqueness. Suppose $\bar{\theta} >> (13/4)\underline{\theta}$ holds for the lower boundary regime. In this case, the second order condition for a maximum fails for all $\theta_H > (13/4)\underline{\theta}$. Maximum differentiation is then an equilibrium if $r(\bar{\theta}, \underline{\theta}) > 0$. It is the unique equilibrium if $r[(13/4)\underline{\theta}, \underline{\theta}] \geq 0$. It is one of two equilibria if $r[(13/4)\underline{\theta}, \underline{\theta}] < 0$. In the latter case we would have to apply a selection criterion.[24] For a maximum differentiation equilibrium at $\bar{\theta} > (13/4)\underline{\theta}$ our results with regard to the minimum quality standard are only reinforced. Consider Proposition 5. Here, part (ii) of the proof now applies for both $\bar{v} < v_l$ and $\bar{v} \in [v_l, v_\pi]$. According to this, the MQS unambiguously lowers consumer surplus for $\bar{v} \leq v_l$. However, for $\bar{v} = v_\pi$, the MQS now lowers consumer surplus if $\bar{\theta} >> (13/4)\underline{\theta}$. Lemma 3 and Proposition 6 remain unaffected.

We have not addressed low quality leadership in a covered market. In a companion paper, we show that our main results carry over.[25] Low quality leadership requires the presence of a cost advantage and is then the more likely the lower WTP. However the role of baseline benefit is now less clear-cut. For the interior configuration of a covered market, demand is independent of baseline benefit, which is, thus, irrelevant for leadership. Under the corner configuration, the low quality firm sets a price $\hat{p}_L = u$, which extracts the full surplus of the consumer with the lowest, i.e., zero, WTP for quality. With the price being fixed at $\hat{p}_L = u$, the low

[24] A similar argument applies to the upper boundary regime.

[25] This paper will be available from the author upon request.

quality firm's market share can be shown to fall in u, so that its profit is concave in u. In contrast, the high quality firm's price, market share and profit increase in u. This implies an ambiguous impact of the baseline benefit on low quality leadership under the corner configuration.

Even though we expect our results to carry over to the case of Cournot competition, there is scope for confirming this. A more interesting extension, however, may lie in modelling an issue raised by Vishwanath and Mark (1997). They argue that the strength of consumers' willingness to pay for quality relative to the baseline benefit (i.e., whether a market is 'value' or 'premium'), is to some extent determined by the past behaviour of firms. They cite an example, in which an aggressive pricing strategy on the part of one firm turned a market from a 'premium' into a 'value' configuration. It may prove interesting to follow this idea and endogenize the market configuration.

6 Appendix

Proof of Proposition 3. Part (i) follows directly from (9). The role of differentiation and L quality can be seen by rewriting $v_q = c + \frac{u}{\theta_L}\left(2 - \frac{1}{\zeta}\right)$, where $\zeta := \frac{\theta_H}{\theta_L} \geq 1$ is a measure of product differentiation. Obviously, v_q decreases in θ_L and increases in ζ, implying the statement in the Proposition. Part (ii) follows from (10), where $v_\pi = c + u/\left(\theta_L\sqrt{\zeta}\right)$. Part (iii): By direct comparison of the definitions in (9) and (10), it follows that $v_q > v_\pi$. Profit leadership implies $\bar{v} < v_\pi < v_q$ and thus market-share leadership. As seen for $v_\pi < \bar{v} < v_q$, the reverse is not true. QED.

Proof of Lemma 1. We prove by contradiction. Suppose an interior equilibrium exists. This requires $r(\theta_H*, \theta_L*) = s(\theta_H*, \theta_L*) = 0$. According to (11a), $r(\theta_H*, \theta_L*) = 0$ requires $u = 2(\bar{v} - c)\alpha(\cdot)/\beta(\cdot)$. Inserting the RHS into (11b) gives $s(\theta_H*, \theta_L*) = (\bar{v} - c)\left[\theta_H\theta_L\beta(\cdot)^2 - 2\alpha(\cdot)^2\right]/\beta(\cdot) < 0$. The inequality follows as $\operatorname{sgn}[(\bar{v} - c)/\beta(\cdot)] = \operatorname{sgn}[u]/\operatorname{sgn}[\alpha(\cdot)] = 1$ from (11a) and $\operatorname{sgn}\left[\theta_H\theta_L\beta(\cdot)^2 - 2\alpha(\cdot)^2\right] = -1$ from (13). Hence, $r(\theta_H*, \theta_L*) = 0 \Rightarrow s(\theta_H*, \theta_L*) < 0$, a contradiction. QED.

Proof of Proposition 4. We first prove part (i) and then part (iii) of the Proposition, while presuming the absence of leapfrogging. Part (ii) follows as a 'residual'. The comparative static properties in table 1 of part (iv) follow along the way. Finally, we establish that leapfrogging does not occur.

To prove part (i) we have to show that $\underline{\theta} = \theta_L* < \theta_H* < \bar{\theta}$ is implied by $\bar{v} < v_l$. $\underline{\theta} < \theta_H* < \bar{\theta}$ requires

$$r(\theta_H*, \theta_L) = 2(\bar{v} - c)\alpha(\cdot) - u\beta(\cdot) = 0 \qquad \text{(A1)},$$

or

$$\bar{v} = c + [\beta(\cdot)/2\alpha(\cdot)]u \tag{A1'}$$

By Lemma 1, $r(\theta_H *, \theta_L) = 0 \Rightarrow s(\theta_H *, \theta_L) < 0$ and, therefore, $\theta_L * = \underline{\theta}$. The second order condition for a profit maximum is given by

$$r_{\theta_H} := \partial\, r(\theta_H *, \underline{\theta})/\partial\theta_H = 2[(\bar{v} - c)(8\theta_H * - 3\underline{\theta}) - 2u] < 0 \qquad \text{(A2).}$$

This holds for all $\theta_H * \leq \bar{\theta}$ if

$$\bar{v} < c + [2/(8\bar{\theta} - 3\underline{\theta})]u =: \tilde{v} \qquad \text{(A2').}$$

First, consider $\bar{v} > c$. Satisfaction of (A1) implies $\beta(\theta_H *, \underline{\theta}) = 4\theta_H * - 7\underline{\theta} > 0$. Satisfaction of the second order condition requires

$$\max_{\theta_H \in ((7/4)\underline{\theta}, \bar{\theta})} c + [\beta(\cdot)/2\alpha(\cdot)]u < \tilde{v} \qquad \text{(A2'').}$$

Using (12a) and (12b), it is easily verified that the LHS is maximized by $\theta_H = \bar{\theta}$ for all $\bar{\theta} \leq (13/4)\underline{\theta}$. Observing the definition of $\tilde{v}$ in (A2') it is then straightforward to show that (A2'') holds for all $\theta_H \leq \bar{\theta} < (13/4)\underline{\theta}$, where the second inequality is satisfied by assumption (14).

Comparative statics, summarized in part (iv), follow from (A1) and (A2) as

$$d\theta_H */du = \beta(\cdot)/r_{\theta_H} \quad \text{(A3a)} \qquad d\theta_H */d(\bar{v} - c) = -2\alpha(\cdot)/r_{\theta_H} > 0 \quad \text{(A3b).}$$

Notice that $d\theta_H */du < 0$ as $\beta(\cdot) > 0$ for $\bar{v} > c$. From (A3b), there follows the existence of a boundary value v_l such that $\theta_H * = \bar{\theta} \quad \forall \bar{v} \geq v_l$. From (A1'), it follows that this boundary value is given by $v_l = c + [\beta(\bar{\theta}, \underline{\theta})/2\alpha(\bar{\theta}, \underline{\theta})]u$ as defined in (15a) in the main text. Consequently, $\theta_H * \in \,](7/4)\underline{\theta}; \bar{\theta}[$ if and only if $v_l > \bar{v} > c$ as indicated in the Proposition.

Now, consider $\bar{v} \leq c$. Satisfaction of (A1') implies $\beta(\theta_H *, \underline{\theta}) = 4\theta_H * - 7\underline{\theta} < 0$. Inspection of (A2) shows that the second order condition is always satisfied. From (A3a), it follows that $d\theta_H */du > 0$. We are left to prove that $\theta_H * > \underline{\theta}$. Recall from (6) that $\bar{v} > c - u/2\underline{\theta}$ guarantees a non-negative profit for some $\theta_H > \underline{\theta}$. From (A1'), it follows that an interior equilibrium with $\theta_H * > \underline{\theta}$ then exists if $c - u/2\underline{\theta} < u = c + [\beta(\cdot)/2\alpha(\cdot)]u$ or, equivalently, $\underline{\theta}\beta(\cdot) > -\alpha(\cdot)$. From (12a) and (12b), this is satisfied for $\theta_H * > \underline{\theta}$. Therefore, condition (6) is sufficient to guarantee the existence of an equilibrium with $\theta_H * > \underline{\theta}$. Consequently, $\theta_H * \in \,]\underline{\theta}, (7/4)\underline{\theta}]$ if $\bar{v} \in \,]c - u/2\underline{\theta}; c]$ as indicated in the Proposition.

For part (iii) we have to show that $\underline{\theta} < \theta_L * < \theta_H * = \overline{\theta}$ is implied by $\overline{v} > v_h$. $\underline{\theta} < \theta_L * < \overline{\theta}$ requires

$$s(\theta_H, \theta_L *) = (\overline{v} - c)\theta_H \theta_L * \beta(\cdot) - 2u\alpha(\cdot) = 0 \qquad \text{(A4)},$$

or

$$\overline{v} = c + [2\alpha(\cdot)/\theta_H \theta_L * \beta(\cdot)]u \qquad \text{(A4')}$$

By Lemma 1, $s(\theta_H, \theta_L *) = 0 \Rightarrow r(\theta_H, \theta_L *) > 0$ and, therefore, $\theta_H * = \overline{\theta}$. Notice that (6) and (A4') cannot be simultaneously satisfied for $\overline{v} < c$. This would require $4\alpha(\cdot) < -\theta_L * \beta(\cdot)$, which contradicts (13). Similarly, (A4') cannot be satisfied for $\overline{v} = c$. Therefore, $\overline{v} > c$ is necessary for $\underline{\theta} < \theta_L * < \overline{\theta}$ and we can focus on this case alone. Satisfaction of (A4') then implies $\beta(\overline{\theta}, \theta_L *) = 4\overline{\theta} - 7\theta_L * \geq 0$. The second order condition for L quality choice is given by

$$s_{\theta_L} := \partial s(\overline{\theta}, \theta_L *)/\partial \theta_L = 2[(\overline{v} - c)(2\overline{\theta} - 7\theta_L *)\overline{\theta} + u(3\overline{\theta} - 4\theta_L *)] < 0 \qquad \text{(A5)}.$$

This holds for all $\theta_L * \in [\underline{\theta}, (4/7)\overline{\theta}]$ if

$$\overline{v} > c + u\,(3\overline{\theta} - 4\underline{\theta})/[\overline{\theta}\,(7\underline{\theta} - 2\overline{\theta})] =: \tilde{\tilde{v}} \qquad \text{(A5')},$$

where numerator and denominator of the fraction in the middle expression are positive by virtue of (14). Satisfaction of the second order condition requires

$$\min_{\theta_L \in (\underline{\theta}, (4/7)\overline{\theta})} c + [2\alpha(\cdot)/\overline{\theta}\theta_L \beta(\cdot)]u < \tilde{\tilde{v}} \qquad \text{(A5'')}.$$

Using (12a) and (12b) it is easily verified that the LHS is minimized by $\theta_L = \underline{\theta}$ for all $\underline{\theta} \geq (4/13)\overline{\theta}$. Observing the definition of $\tilde{\tilde{v}}$ in (A5') it is immediate to show that the inequality in (A5'') holds for all $\theta_L \geq \underline{\theta} > (4/13)\overline{\theta}$, where the second inequality is satisfied by assumption (14).

Comparative static properties, as given in part (iv), follow from (A4) and (A5) as

$$d\theta_L */du = 2\alpha(\cdot)/s_{\theta_L} < 0 \qquad d\theta_L */d(\overline{v} - c) = -\overline{\theta}\theta_L * \beta(\cdot)/s_{\theta_L} > 0 \qquad \text{(A6)}.$$

From (A6), there follows the existence of a boundary value v_h such that $\theta_L * = \underline{\theta} \quad \forall \overline{v} \leq v_h$. From (A4'), it follows that this boundary value is given by $v_h = c + [2\alpha(\overline{\theta}, \underline{\theta})/\overline{\theta}\underline{\theta}\beta(\overline{\theta}, \underline{\theta})]u$ as defined in (15b) in the main text. Consequently, $\theta_L * \in]\underline{\theta}, (4/7)\overline{\theta}]$ and $\theta_H * = \overline{\theta}$ if and only if $\overline{v} > v_h$ as indicated in the Proposition.

Part (ii) of the Proposition follows immediately from the previous proofs and $v_l < v_h$.

Finally, we have to show for the boundary equilibria that firms do not have an incentive to leapfrog. In the lower (upper) boundary equilibrium only the L (H) firm can leapfrog. Then, it has to be guaranteed that the equilibrium profit for the respective firm exceeds the maximum profit it could attain by leapfrogging.

This is easily proved in two steps. Firstly, we show that the L (H) firm is profit leader in the lower (upper) boundary equilibrium. As both, the quality dependent part of preferences and variable cost are linear in quality, we can then refer to the standard proof that in a lower (upper) boundary equilibrium, the H (L) firm can never improve its profit by leapfrogging (Shaked and Sutton 1982, Proof of Proposition 1).

L (H) profit leadership is given for the upper (lower) boundary equilibrium if $v_\pi \in [v_l, v_h]$. Using the definitions in (10) and (15a), $v_\pi > v_l \Leftrightarrow \beta(\bar{\theta}, \underline{\theta})\sqrt{\theta_H \theta_L} < 2\alpha(\bar{\theta}, \underline{\theta})$. This condition is always satisfied as $\beta(\bar{\theta}, \underline{\theta})\sqrt{\theta_H \theta_L} < \beta(\bar{\theta}, \underline{\theta})\bar{\theta} < 2\alpha(\bar{\theta}, \underline{\theta})$, where the second inequality follows from (13). Similarly, using (15b), $v_\pi < v_h \Leftrightarrow 2\alpha(\bar{\theta}, \underline{\theta})\sqrt{\theta_H \theta_L} > \bar{\theta}\underline{\theta}\beta(\bar{\theta}, \underline{\theta})$. Again, this is always satisfied since by virtue of (13), $2\alpha(\bar{\theta}, \underline{\theta})\sqrt{\theta_H \theta_L} > 2\alpha(\bar{\theta}, \underline{\theta})\underline{\theta} > \bar{\theta}\underline{\theta}\beta(\bar{\theta}, \underline{\theta})$. Thus, $v_\pi \in [v_l, v_h]$.

The proof a la Shaked and Sutton applies directly to the case of upper boundary equilibrium and extends by symmetry to the lower boundary equilibrium. Thus, leapfrogging cannot arise, which completes the proof of the Proposition. QED.

Proof of Corollary 4.1. The proof relating to profit leadership is part of the previous proof. Recall that $v_\pi \in [v_l, v_h]$. From $v_\pi < v_q$, it follows that lower bound equilibrium implies L market share leadership. The upper bound equilibrium involves H market share leadership if $v_q < v_h$. Using (9) and (15b), this holds if and only if $2\theta_H \theta_L \alpha(\bar{\theta}, \underline{\theta}) > (2\theta_H - \theta_L)\bar{\theta}\underline{\theta}\beta(\bar{\theta}, \underline{\theta})$. This is always satisfied as $2\theta_H \theta_L \alpha(\bar{\theta}, \underline{\theta})\theta_H \theta_L > 2\theta_H \bar{\theta}\underline{\theta}\beta(\bar{\theta}, \underline{\theta}) > (2\theta_H - \theta_L)\bar{\theta}\underline{\theta}\beta(\bar{\theta}, \underline{\theta})$, where the first inequality follows from (13). QED.

Proof of Proposition 5. Part (i): We wish to show that $dS/d\theta_{\min}\big|_{\bar{v}<v_l} = (q_L{*}\rho + q_H{*}\psi)\big|_{\bar{v}<v_l} < 0$. Recall $q_L{*} > q_H{*}$ for $\bar{v} < v_l$. By Lemma 2, $\rho < \psi$. Then $(\rho + \psi)\big|_{\bar{v}<v_l} < 0$ is sufficient for $dS/d\theta_{\min}\big|_{\bar{v}<v_l} < 0$. Observing the first order condition for H quality choice we can substitute the RHS of (A1') for $\bar{v}$ in (18a) and (18b). After collection of terms this yields

$$(\rho + \psi)\big|_{\bar{v}<v_l} = \frac{u}{4(4\theta_H{*} - \theta_{\min})^2 \theta_{\min}\alpha(\theta_H{*}, \theta_{\min})}\left\{\phi - 2\theta_{\min}\kappa \frac{d\theta_H{*}}{d\theta_{\min}}\bigg|_{\theta_H{*}<\bar{\theta}}\right\} < 0,$$

where,

$$\phi := \theta_H{}^* \theta_{\min}\left(16\theta_H{}^* + 5\theta_{\min}\right)\beta(\cdot) - 4\left[4(\theta_H{}^*)^2 - 6\theta_H{}^*\theta_{\min} - \theta_{\min}^2\right]\alpha(\cdot) < 0,$$

$$\kappa := \left(4\theta_H{}^* + 17\theta_{\min}\right)\alpha(\cdot) - \left[4(\theta_H{}^*)^2 - 3\theta_H{}^*\theta_{\min} - 4\theta_{\min}^2\right]\beta(\cdot) > 0$$

are easily verified under use of the definitions of $\alpha(\cdot)$ and $\beta(\cdot)$ in (12a) and (12b).

Part (ii): For $\bar{v} \in [v_l, v_\pi]$, we know that $q_L{}^* > q_H{}^*$ and $d\theta_H{}^*/d\theta_{\min}\big|_{\bar{v}\in[v_l,v_\pi]} = 0$. We first prove that $dS/d\theta_{\min}\big|_{\bar{v}=v_l} < 0$. Recall $v_l = c + \left[\beta(\bar{\theta},\underline{\theta})/2\alpha(\bar{\theta},\underline{\theta})\right]u$. We can, thus, write $(\rho+\psi)\big|_{\bar{v}=v_l} = \dfrac{u\bar{\phi}}{4\left(4\bar{\theta}-\theta_{\min}\right)^2\theta_{\min}\alpha(\bar{\theta},\theta_{\min})} < 0$, where $\bar{\phi} = \phi\big|_{\theta_H{}^*=\bar{\theta}} < 0$. By continuity, this holds for a range of $\bar{v} > v_l$. To prove $dS/d\theta_{\min}\big|_{\bar{v}=v_\pi} > 0$ we have to evaluate

$$\left(q_L{}^*\rho + q_H{}^*\psi\right)\Big|_{\bar{v}=v_\pi=c+u/\sqrt{\bar{\theta}\theta_{\min}}}$$

$$= \frac{u^2\left[-8\bar{\theta}^2\left(2\bar{\theta}-3\theta_{\min}\right) + 11\bar{\theta}\theta_{\min}^2 + 2\theta_{\min}\left(28\bar{\theta}+\theta_{\min}\right)\sqrt{\bar{\theta}\theta_{\min}}\right]}{2\left(c + u/\sqrt{\bar{\theta}\theta_{\min}}\right)\left(4\bar{\theta}-\theta_{\min}\right)^3\theta_{\min}^2}$$

where the expression in the second line follows from straightforward calculation. The expression in square brackets is positive for $\bar{\theta} < (13/4)\theta_{\min}$. QED.

Proof of Lemma 3. We have to show that $\operatorname{sgn}\lambda = \operatorname{sgn}(d\pi_L/d\theta_{\min}) = -1 \quad \forall \bar{v} < v_h$. Recall $s(\theta_H{}^*, \theta_{\min}) < 0 \quad \forall \bar{v} < v_h$. For $\bar{v} \in (v_l, v_h)$, where $d\theta_H{}^*/d\theta_{\min} = 0$, it follows that $\lambda = \theta_H{}^* s(\theta_H{}^*, \theta_{\min}) < 0$. For $\bar{v} < v_l$, we can rewrite (21) by substituting from (11b) for $s(\cdot)$ and from (A1') for $\bar{v}$. After simplification this yields

$$\lambda\big|_{\bar{v}<v_l} = \frac{u}{2\alpha(\cdot)}\left[\theta_{\min}\beta(\cdot) - 4\alpha(\cdot)\right]\left[4(\theta_H{}^*)^2 - 5\theta_H{}^*\theta_{\min} + \theta_{\min}^2\right] < 0.$$

The inequality follows since the first expression in square brackets is negative from (13) and the second expression in square brackets is positive. Hence, $\operatorname{sgn}\lambda = \operatorname{sgn}(d\pi_L/d\theta_{\min}) = -1 \quad \forall \bar{v} < v_h$. QED.

Proof of Proposition 6. Firstly, consider $\bar{v} < v_l$. From part (i) of Proposition 5, Lemma 3 and (19), we have $dW/d\theta_{\min}\big|_{\bar{v}<v_l} < 0$. Now, consider $\bar{v} \in [v_l, v_h]$. We rewrite (22) conveniently by using the RHS expressions of (17), (19) and (20) to get

$$\frac{dW}{d\theta_{\min}} = \left\{\rho + \frac{\lambda}{(4\theta_H{}^* - \theta_{\min})^2\theta_H{}^*\theta_{\min}}\right\}q_L{}^* + \left\{\psi - \frac{(2\theta_H{}^* + \theta_{\min})\left[2(\bar{v}-c)\theta_H{}^* + u\right]}{(4\theta_H{}^* - \theta_{\min})^2}\right\}q_H{}^*.$$

Inserting the definitions (18a), (18b) and (21) of ρ, ψ and λ, respectively, and observing $\theta_H * = \bar{\theta}$ and $d\theta_H */d\theta_{\min}\big|_{\bar{v}\in[v_l,v_\pi]} = 0$, we obtain

$$\frac{dW}{d\theta_{\min}}\bigg|_{\bar{v}\in[v_l,v_\pi]} = \frac{\varpi q_L * + 2\bar{\theta}\theta_{\min}\left(\bar{\theta}-\theta_{\min}\right)\left[2(\bar{v}-c)\bar{\theta}+u\right]q_H *}{2\bar{\theta}\theta_{\min}\left(4\bar{\theta}-\theta_{\min}\right)^2};$$

$$\varpi := 3(\bar{v}-c)\bar{\theta}^2\theta_{\min}\left(4\bar{\theta}-3\theta_{\min}\right) - 2u\bar{\theta}\left(8\bar{\theta}^2 - 6\bar{\theta}\theta_{\min} + \theta_{\min}^2\right).$$

Recalling that $q_L * > q_H *$ for $\bar{v} \in [v_l, v_h]$, we conclude that

$$\operatorname{sgn} dW/d\theta_{\min}\big|_{\bar{v}\in[v_j,v_\pi]} = \operatorname{sgn} \vartheta(\bar{v}),$$

where

$$\begin{aligned}\vartheta(\bar{v}) &:= \varpi + 2\bar{\theta}\theta_{\min}\left(\bar{\theta}-\theta_{\min}\right)\left[2(\bar{v}-c)\bar{\theta}+u\right]\\ &= \bar{\theta}\left[(\bar{v}-c)\bar{\theta}\theta_{\min}\left(16\bar{\theta}-13\theta_{\min}\right) - 2u\left(8\bar{\theta}^2 - 7\bar{\theta}\theta_{\min} + 2\theta_{\min}^2\right)\right]\end{aligned}$$

From $\partial\vartheta/\partial\bar{v} > 0$, it follows that

$$\vartheta(\bar{v})\big|_{\bar{v}<v_\pi} < \vartheta(v_\pi) = \frac{u\bar{\theta}}{\sqrt{\bar{\theta}\theta_{\min}}}\left[\bar{\theta}\theta_{\min}\left(16\bar{\theta}-13\theta_{\min}\right) - 2\left(8\bar{\theta}^2 - 7\bar{\theta}\theta_{\min} + 2\theta_{\min}^2\right)\sqrt{\bar{\theta}\theta_{\min}}\right].$$

It is easily verified that the RHS expression is negative for $\bar{\theta} > (7/4)\theta_{\min}$. Thus, $\operatorname{sgn} dW/d\theta_{\min}\big|_{\bar{v}\in[v_l,v_\pi]} = \operatorname{sgn}\vartheta(\bar{v}) = -1$. QED.

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