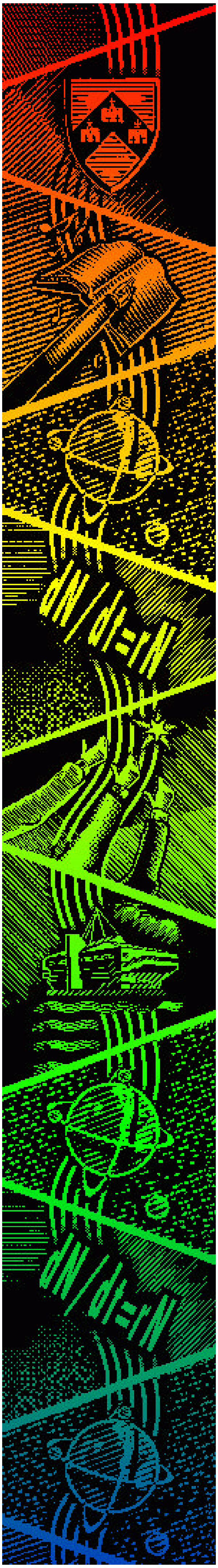

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The Form of Loan Contract with Risk Neutrality and No Commitment

by

Peter J Simmons and G Garino

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

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Abstract

Recent literature has considered the form of loan contract between two risk neutral parties where the revelation principle is inappropriate due to the lack of commitment to an auditing policy by the lender. The privately informed debtor has a stochastic return; once he knows its realisation, auditing and cheating are determined as a Nash equilibrium. The literature assumes that this leads to randomised cheating and auditing. In this paper we show that the optimal contract may involve this randomisation; or may involve truthtelling but random auditing; or may involve a single state independent repayment with no auditing. We define conditions on the state observation cost and the distribution of returns which determine which of these three forms of contract is optimal.

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Corresponding author

P Simmons
Department of Economics
University of York
York Y01 5DD
ps1@york.ac.uk

P Simmons & G Garino
University of York & Brunel University

1 Introduction

Loan contracts are generally signed in advance of knowledge of the income or wealth of the borrower at the time that repayment falls due. When the loan is for productive investment and is made between risk neutral parties, a varying loan size alters the distribution of returns to the borrower in different states. The first best loan size will then be set so as to equate the expected marginal product of the loan to the safe interest rate. If there is sufficient income to the borrower in all states to repay the fair return (the equivalent return to investing at the safe interest rate) then the contract can stipulate that the borrower should repay this constant amount in all states of the world. Alternatively if the borrower has insufficient resources in some states to repay this amount, the contract can allow the repayments of the borrower to vary by state so long as the realised income/wealth of the debtor at repayment time is commonly observed by both lender and borrower and the repayments can be enforced. However if either the realised debtor income is private information to the debtor; or if the state dependent repayments cannot be enforced, then the borrower has an incentive to cheat on repayments, declaring a low income state and so making a low repayment when in fact his realised income is high. The contract then has to incorporate the correct incentives for borrower revelation. The costly state verification literature (Townsend, 1979, 1988; Gale-Hellwig, 1985, 1989; Mookherjee and Png, 1989) allows for the lender to pay a cost to discover the true realised income of the debtor and finds that either the standard debt contract or a random monitoring contract is optimal depending on the instruments available. However, as several writers have pointed out (Moore, 1995; Khalil, 1998; Choe, 1998), these contracts have commitment problems in a one shot scenario: the contract imposes truthtelling on the debtor through punishments by the lender following a costly verification by the lender. But if the lender knows there is truthful reporting there is no incentive for the lender to incur the verification costs.

Jost(1996) overcomes this by imposing sequential rationality in the initial contract that the principal writes in the context of the standard principal-agent model where the principal pays the agent for an effort. Sequential rationality requires that the principal should be indifferent between monitoring and not monitoring the agent following the agents report and adds this as a constraint on the exante contract Since monitoring involves the state verification cost it means that the principal pays the agent more when the agent is not monitored than when he is monitored. From this Jost deduces that as the state verification cost rises, the optimal degree of monitoring rises and the spread of repayments to the agent between the not monitored and monitored cases rises. Fudenberg and Tirole(1990) show that imposing the sequential rationality (or renegotiation proofness condition) to the exante contract is equivalent to renegotiating the contract.

Khalil and Parigi (1998 - hereafter KP) consider a loan contract between two parties; the contract specifies the loan size and the repayments that should be made in various states at the date of repayment. A state is defined by the borrowers true income at repayment (which is privately observed by the borrower) and by the results of a costly audit that the lender can choose to make. Both parties to the contract are risk neutral and three scenarios are discussed:

(i) first best where the borrowers true income state at the repayment date is common information;

(ii) a commitment contract where the lender can precommit to carry out an auditing strategy at repayment;

(iii) a no commitment contract where at the repayment date cheating on the true state by the borrower and auditing by the lender are determined as a Nash equilibrium conditional on the contract. KP only consider Nash equilibria in which the probabilities of audit and of cheating are each strictly between zero and one. In contrast to Jost where it as if the lender is acting as a Stackleberg leader in writing the contract, KP determine monitoring and cheating noncooperatively.

At the repayment stage borrower income can have two values $y_H > y_L$ where $y_s = y_s(B)$ with $y_s^{'}(B) > 0$, $s = H, L$, $y_s^{''}(B) < 0$ and $B > 0$ is the amount of the loan. At this stage the borrower makes an income report to the lender; if this is H the repayment R_H is made; if it is L then the lender can choose whether or not to audit the borrower to discover his true state at a fixed cost of $\phi > 0$. If no audit is performed the repayment is R_L; if an audit is performed but the borrower's report is discovered to be truthful a repayment of R_{LL} is made and finally if an audit is performed and the borrower is found to have cheated a repayment of R_{HL} is made. The contract sets R_s and B; given these, the report the borrower makes and the decision to audit or not are determined to ensure truthtelling in the commitment contract and as a Nash equilibrium in the no commitment contract. KP's assumptions on revenues imply that it is not possible to avoid the incentive problems completely by setting $R_s = R$ for all s since they assume that for all B, $y_L(B) < (1+r)B$. Their full set of assumptions on technology are:

$$y_L(B) < (1+r)B < y_H(B) \quad \forall B \tag{1}$$

$$y_L^{'}(B) < (1+r) < y_H^{'}(B) \quad \forall B \tag{2}$$

$$py_H(B) + (1-p)y_L(B) - (1+r)B - \phi > 0 \tag{3}$$

where p is the probability of occurrence of the high state.

The first best B is determined to maximise $Ey_s(B) - (1+r)B$; since investment must be made prior to the state realisation there is still uncertainty but there is symmetric information at each stage between the two parties. Now if $y_L(B_1) \geq (1+r)B_1$ where B_1 is the first best level of investment, then the borrower has sufficient resources to repay the debt even in the low state. Hence setting $R_s = (1+r)B$ for all s removes any incentive to cheat and the first best can be implemented through a single repayment contract that is independent of state. Assumption (1) implies that the first best is unattainable and state contingent repayments with $R_L, R_{LL} \leq R_H, R_{HL}$ have to be suitably chosen to counteract the incentive problem.

KP show that the commitment contract satisfies:

(i) $R_{LL} = R_L = y_L$

(ii) $R_{HL} = y_H$

(iii) $y_L < R_H < y_H$

(iv) there is underborrowing compared with the first best.

Under the assumption that $py_H + (1-p)y_L - B - \phi > 0$ they also show that the contract without commitment has:

(i) $R_{LL} = R_L = y_L$

(ii) $R_{HL} = y_H$

(iii) $R_H < y_H$

(iv) there is more auditing than under commitment;

(iv) there is underborrowing compared with the first best but overborrowing compared with the commitment case.

In this paper we examine some of KP's no commitment results. Partly in the interest of clarity of exposition we divide this into two stages; first taking the case where the level of investment B is fixed exogenously. Whilst this is mainly a pedagogical device to highlight incentive effects, it also has some interest in its own right as describing the case in which the investment has the form of a set-up cost or an access/entry cost to the technology. To ensure that the first best is unattainable when B is fixed amounts to assuming that $y_L < (1+r)B$. In this scenario we show that the optimal repayments may not involve a subsequent interior mixed strategy Nash equilibrium of the reporting/auditing game, but, instead, a positive probability of auditing with truthtelling; and we identify the conditions for this to be true. When B is fixed we have two possible forms for the optimal state contingent pattern of repayments and for the type of equilibrium which will be established in the auditing/reporting game.

When B is variable we use slightly more general assumptions on the revenue functions than KP, which mean that the first best investment level B_1 is infeasible because $y_L(B_1) < (1+r)B_1$, but nevertheless a single repayment contract is a possibility because there is a range of values of B for which $y_L(B) > (1+r)B$. Our full set of technological assumptions are:

$$y_H(B) > y_L(B);\ y_H^{'}(B) > y_L^{'}(B) \geq 0;\ y_H^{''}(B) < y_L^{''}(B) < 0; \forall B \tag{4}$$

$$y_L(B_1) < (1+r)B_1 \quad where \quad py_H^{'}(B_1) + (1-p)y_L^{'}(B_1) = 1+r \tag{5}$$

$$py_H(B_1) + (1-p)y_L(B_1) - (1+r)B_1 - \phi > 0 \tag{6}$$

$$y_s(0) = 0, y_s^{'}(0) > 1+r, \lim_{B\to\infty} y_s^{'}(B) < 1+r \tag{7}$$

Here we find that there are three essential contract forms, each corresponding to a different sort of equilibrium of the audit/reporting game: the *single repayment* (SR) contract, where R_s is state independent and B is reduced sufficiently to ensure that the borrower can afford the repayment in the bad state; the *interior mixed strategy* (KP) contract, in which there is some cheating and some costly monitoring; and what we call a *hybrid* contract (H) in which there is truthtelling, state contingent repayments and some costly monitoring. We find sufficient conditions on the technology and observation cost for each of these forms to be the optimal contract.

Our results in the fixed B case indicate that for relatively high values of B, the H contract is optimal; for lower values of B the KP contract is optimal. When B is variable, for different configurations of the technological and observation cost parameters, the optimal contract may have any of the forms: SR, KP, H. We find that where the observation cost is independent of B, as this observation cost increases the optimal contract switches from KP to H and then to SR. As the observation cost increases there are two effects: firstly in regions where KP is the optimal form of contract the optimal investment level falls, the amount of cheating by the borrower rises and the amount of monitoring rises as well; while in regions where H is optimal the level of investment falls. Secondly the contract form itself changes at critical levels of ϕ from KP to H and then eventually to SR. This switch in the form of the optimal contract from KP to H leads to a jump in the investment level as the observation cost rises.

We provide various numerical examples of these effects. From these the dominant impression is that as the observation cost rises, the level of investment may rise to ensure that there are sufficient revenues especially in the high state of the world both to finance the monitoring and also to have a large enough scope for punishing the debtor for cheating to ensure truthtelling. However once the observation cost has become really very high, monitoring is pointless and investment is sharply reduced to a level which can sustain a SR contract. Thus in a somewhat counterintuitive way there is an increasing incentive for truthtelling as the observation cost rises.

2 The No Commitment Contract: the Second Period Game

Whether B is fixed exogenously or determined in the first period contract is immaterial for the solution of the second period game; however it was initially determined, by the time period 2 is reached, B is fixed from the past.

In period 2 the debtor chooses an income report. It will be clear that the debtor only has an incentive to underreport income (since $R_H > R_L, R_{LL}$) and will always make truthful reports of low income. But with true high income the debtor may make a false low income report to the lender with probability l; while in period 2 the lender will choose the probability m with which to monitor any low income report by the debtor. Each of these decisions are simultaneous best responses to the other party's choice, so that we are in a second period Nash equilibrium. Given this the monitoring is then implemented: truthful reports result in repayments that are set in the initial contract; false reports result in repayments that may incorporate some punishment. To solve this game note that the lender's expected profit is:

$$E\pi = p(1-l)R_H + plm(R_{HL} - \phi) + (1-p+pl)(1-m)R_L + (1-p)m(R_{LL} - \phi) - (1+r)B \tag{8}$$

So the lender's best response to l is:

$m = 0$ if $\partial E\pi / \partial m < 0$

$0 \leq m \leq 1$ if $\partial E\pi / \partial m = 0$

$m = 1$ if $\partial E\pi / \partial m > 0$ where:

$\partial E\pi/\partial m = pl(R_{HL} - \phi) + (1-p)(R_{LL} - \phi) - (1 - p + pl)R_L$

Note that $m \neq 0$ only if either $R_{HL} \geq \phi$ or $R_{LL} \geq \phi$; otherwise the costs of monitoring cannot be recouped.

On the other hand we know that the borrowers expected utility is:

$$EU = p(1-l)(y_H - R_H) + plm(y_H - R_{HL}) + pl(1-m)(y_H - R_L) \qquad (9)$$
$$+ (1-p)(1-m)(y_L - R_L) + (1-p)m(y_L - R_{LL})$$

So the borrowers best response to m is:

$l = 0$ if $\partial EU/\partial l < 0$

$0 \leq l \leq 1$ if $\partial EU/\partial l = 0$

$l = 1$ if $\partial EU/\partial l > 0$

where $\partial EU/\partial l = p[R_H - mR_{HL} - (1-m)R_L]$

2.1 Pure Strategy Equilibria

Depending on the values of the single repayment R there are several possibilities for a Nash equilibrium in pure strategies. For each candidate we use the definitions of the expected payoffs to the two parties, together with the fact that in any optimal contract written by the debtor, the repayments must be set so that the lender gets zero expected return (ie the lenders participation constraint always binds - if this were not so, it would be possible to reduce the repayments in each state and make the debtor better off), to derive expressions for the maximal payoff of the debtor. We can then use these expressions to rank the various forms of contract-game outcome.

(i) $l = m = 0$ If $\phi - R_{LL} + R_L > 0$ and $R_H < R_L$ this is a Nash equilibrium. However it is of little interest since it gives incentives to cheat in the realised low income state.

(ii) $l = 0, m = 1$ If $\phi - R_{LL} + R_L < 0$ and $R_{HL} > R_H$ this is a Nash equilibrium; the lender knows that there is no lying but can make money by monitoring always since the observation cost is less than the gain in repayment. This is clearly very inefficient; observation costs are always paid although there is known to be no cheating by the lender. Effectively there are two states: the high income report state (which is never monitored) and the low income report state (which is always monitored). The expected observation cost with this game outcome is $(1-p)\phi$ and the expected returns to the two parties are:

- $E\pi = pR_H + (1-p)(R_{LL} - \phi) - (1+r)B$
- $EU = Ey - pR_H - (1-p)R_{LL} = Ey - (1+r)B - (1-p)\phi$

  Note that if $\phi = 0$ then this contract with a subsequent pure strategy equilibrium actually is identical to the first best.

(iii) $l = m = 1$ This is a Nash equilibrium if $p(R_{HL} - \phi - R_L) + (1-p)(R_{LL} - \phi - R_L) > 0$ and $R_{HL} < R_H$. Again this is very inefficient: since $R_{HL} < R_H$ the lender has an incentive to always cheat; but since observation costs are not too high, on average the lender will gain by monitoring every low income report. Again there are effectively two states in the second period: the high income state with a false low income report which is monitored and punishment is paid and the low income state which is truthfully reported but always monitored .

The expected observation cost is ϕ and the expected gains to the two parties are:

- $E\pi = p(R_{HL} - \phi) + (1-p)(R_{LL} - \phi) - (1+r)B$
- $EU = Ey - pR_{HL} - (1-p)R_{LL} = Ey - (1+r)B - \phi$

(iv) $l = 1, m = 0$ This is a Nash equilibrium if $p(R_{HL} - \phi - R_L) + (1-p)(R_{LL} - \phi - R_L) < 0$ but $R_H > R_L$ so that the borrower has an incentive to cheat but the lender has no incentive to monitor since ϕ is too high. Whenever $l = 1$ we have a pooling equilibrium with identical repayment offers R_L from both types of debtor. If ϕ is high enough the pooling equilibrium dominates the first pure strategy equilibrium.

The expected observation cost is zero and the returns to the two parties are:

- $E\pi = R_L - (1+r)B$
- $EU = Ey - R_L = Ey - (1+r)B$

2.2 Hybrid Equilibria

The second period game may have equilibria in which only one party plays a pure strategy and the other party randomises. For these equilibria to be mutual best responses the randomising party must be indifferent between different values of her choice probability whilst the deterministic party must strictly prefer the corner. The possibilities are:

(i) $m = 0; 0 < l < 1$ This requires $R_H = R_L$ and the expected observation cost is zero. The expected returns to the two parties (see appendix A.1) are:

- $E\pi = p(1-l)R_H + (1-p+pl)R_L - (1+r)B$
- $EU = Ey - p(1-l)R_H - (1-p+pl)R_L = Ey - (1+r)B$

(ii) $m = 1; 0 < l < 1$ This requires $R_{HL} = R_H$ and the expected observation cost is $(1-p+pl)\phi$. The expected returns to the two parties are:

- $E\pi = p(1-l)R_H + plR_{HL} + (1-p)R_{LL} - (1-p+pl)\phi - (1+r)B$
- $EU = Ey - p(1-l)R_H - plR_{HL} - (1-p)R_{LL} = Ey - (1+r)B - (1-p+pl)\phi$

(iii) $l = 0; 0 < m < 1$ This requires $(\phi + R_L - R_{LL}) = 0$ and the expected observation cost is $(1-p)m\phi$. The expected returns to the two parties are:

- $E\pi = pR_H + (1-p)(1-m)R_L + (1-p)m(R_{LL} - \phi) - (1+r)B$
- $EU = Ey - pR_H - (1-p)(1-m)R_L - (1-p)mR_{LL} = Ey - (1+r)B - (1-p)m\phi$

(iv) $l = 1; 0 < m < 1$ This requires $p(R_{HL} - R_L - \phi) - (1-p)(\phi + R_L - R_{LL}) = 0$. The expected observation cost is $(1-p)m\phi$ and the expected returns to the two parties are:

- $E\pi = pm(R_{HL} - \phi) + (1-m)R_L + (1-p)m(R_{LL} - \phi) - (1+r)B$
- $EU = Ey - pmR_{HL} - p(1-m)R_L - (1-p)(1-m)R_L - (1-p)mR_{LL} = Ey - (1+r)B - m\phi$

2.3 Interior Mixed Strategy Equilibrium

A Nash equilibrium in mixed strategies is defined by the equilibrium values of l and m:

$$l = (1-p)(\phi - R_{LL} + R_L)/[p(R_{HL} - \phi - R_L)] \tag{10}$$

$$m = (R_H - R_L)/(R_{HL} - R_L) \tag{11}$$

For an interior intersection of the reaction curves, (10)-(11) imply:

$R_{HL} > R_H$ to give $m < 1$ and either:

(A): $\phi - R_{LL} + R_L > 0$ $\quad R_{HL} - \phi - R_L > 0$ $\quad pR_{HL} + (1-p)R_{LL} - \phi - R_L > 0$ or

(B): $\phi - R_{LL} + R_L < 0$ $\quad R_{HL} - \phi - R_L < 0$ $\quad pR_{HL} + (1-p)R_{LL} - \phi - R_L < 0$

to give $0 \leq l \leq 1$

However, to be in an interior mixed strategy we have to rule out each of the pure strategies so that in particular to prevent the case $l = 0, m = 1$ we must be in *(A)*; $\phi - R_{LL} + R_L > 0$. We can write the expected payoffs to the two parties as:

- $E\pi = Ey - pR_H - (1-p)R_L - (1-p)(R_{LL} - R_L)(R_h - R_L)/(R_{HL} - R_L)$
- $EU = pR_H + (1-p)R_L - \frac{(1-p)(\phi + R_L - R_{LL})}{R_{HL} - R_L - \phi} \geq (1+r)B$

2.4 Dominance Relations between Game Forms

If B is fixed then the revenues of each state are also fixed and we can rank most of the pure and hybrid strategy outcomes. First, if $y_L \geq (1+r)B$ the pure strategy outcome $l = 1, m = 0$ is feasible and involves no incentive problems: it is the SR contract, which gives the first best and dominates all the others. But if $y_L < (1+r)B$ the SR contract is infeasible. In this case the H solution with $m = 0, 0 < l < 1$ is also infeasible since it too involves a single repayment R_L being made.

We can also rank the other alternatives. Consider the optimal contract problem conditional on a particular form of outcome of the monitoring/cheating game:

$$\max_{R_s, m, l} \quad E\pi \tag{12}$$

$$s.t. \quad EU \geq 0$$

$$l, m \quad constrained$$

The second constraint depicts the case being considered, e.g. the second pure strategy outcome $l = 0, m = 1$. The first constraint will always bind: we have already seen that this is so in the pure strategy cases. It is also true in the hybrid cases. Hence the optimal contract within any case conditional on the form of subsequent game equilibrium must always give a zero expected return to the lender. This means we can Pareto rank the contracts by the expected borrowers payoff. When B is fixed this amounts to comparing the expected observation cost in the different contracts, which gives:

- the hybrid case $l = 0, 0 < m < 1$ dominates the pure strategy case $l = 0, m = 1$ because it has lower observation cost; in turn the latter dominates the hybrid case $m = 1, 0 < l < 1$;
- the hybrid case $l = 0, 0 < m < 1$ also dominates the hybrid case $l = 1, 0 < m < 1$ for the same reasons;
- the pure strategy case $l = m = 1$ is dominated by $l = 0, m = 1$.

  So when B is fixed with $y_L < (1+r)B$ there are two possible candidates for an optimal contract-game form: the H case $l = 0, 0 < m < 1$ and the KP case.

Now suppose that B is variable: under our technological assumptions (5) and (7) whilst the first best cannot be implemented, nevertheless, either the hybrid form $m = 0, 0 < l < 1$ or the pure strategy case $m = 0, l = 1$ can be made feasible by reducing B to the level satisfying $y_L(B) = (1 + r)B$ and are then indifferent to each other.

Take any other pair of hybrid or pure strategy cases; at any fixed level of B we know the above rankings hold. So for example $E\{U(B)|l = 0, 0 < m < 1\} > E\{U(B)|l = 0, m = 1\}$ for any B. Letting $B_{01} = \arg\max E\{U(B)|l = 0, m = 1\}$, it follows that:

$$\max_B E\{U(B)|l = 0, 0 < m < 1\} \geq E\{U(B_{01})|l = 0, 0 < m < 1\} > E\{U(B_{01})|l = 0, m = 1\} \quad (13)$$

So also in the variable B case it follows that the optimal contract/game form must be one of the SR , KP or H cases with $l = 0, 0 < m < 1$.

3 The Optimal No Commitment Contract Repayments with Fixed B

We start with the case in which the size of the investment is exogenously fixed; this is primarily for presentation reasons so that we can highlight the differences between the forms of repayment contracts that may be optimal. It can also be interpreted as the case in which the investment has the form of a setup cost or entry fee to access the technology. Once we have characterised the optimal repayment structures as a function of B and the other exogenous parameters, we move on to examine the case in which B is endogenously determined.

The critical possibilities are that either the optimal contract involves a subsequent interior mixed strategy $(0 < l < 1; 0 < m < 1)$ or a hybrid equilibrium of type $l = 0; 0 < m < 1$. Looking at the reaction curves for the two parties at the repayment stage, as ϕ increases, the lenders reaction curve moves vertically upwards whilst the borrowers reaction curve is unchanged; the rise in the lenders reaction curve and the equilibrium tendency to cheat can be offset by

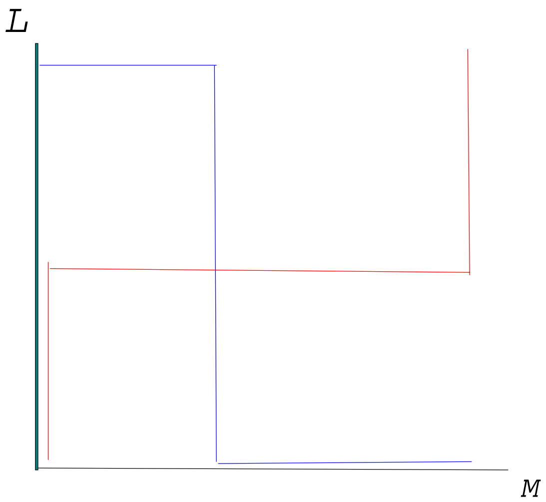


Figure 1:

raising $R_{LL} - R_L$ or raising $R_{HL} - R_L - \phi$ and both of these exert downward pressure on R_L (see Figure 1). These offsetting possibilities are limited by the conditions $R_{HL} \leq y_H$,$R_{LL} \leq y_L$; the possibility of reducing R_L is limited by the participation constraint of the lender (see below); so for high enough ϕ we would expect the optimal contract to involve pooling. At the other limit if the reaction curves have an interior intersection then by raising R_{LL} (when this is feasible) the reaction curve of the lender shifts vertically downwards whilst that of the borrower is unchanged. This will not change the equilibrium amount of auditing but will reduce the equilibrium amount of cheating and we might expect this to raise the efficiency of the contract. Hence there should be a range of low values of ϕ where the optimal contract has $l = 0; 0 < m < 1$.

We calculate the optimal repayments for each subsequent possible game equilibrium; then, still for an exogenously fixed level of B, we determine which of these repayment contract-game equilibrium pairs is optimal for given configurations of the exogenous parameters.

When B is fixed, revenues have a fixed relation to the loan cost: to match KP we assume that

$y_L < (1+r)B < y_H$ and also that

$$py_H(B) + (1-p)y_L(B) - (1+r)B - \phi > 0 \tag{14}$$

3.1 The Optimal Repayments Conditional on the Hybrid Solution

Knowing that $l = 0$, $\phi + R_L - R_{LL} = 0$ and $m \geq (R_H - R_L)/(R_{HL} - R_L)$, the optimal contract problem has the form:

$$\max Ey - pR_H - (1-p)R_L - (1-p)(R_{LL} - R_L)m \tag{15}$$

$$pR_H + (1-p)R_L \geq (1+r)B \tag{16}$$

$$m \geq (R_H - R_L)/(R_{HL} - R_L)$$

The zero profit constraint must bind; otherwise raising B will raise $Ey = py_H + (1-p)y_L$ without affecting feasibility, which then raises borrower expected returns. Similarly the constraint on m must bind; otherwise reducing m marginally reduces monitoring costs, directly raising debtor expected utility (since we know $R_{LL} = R_L + \phi > R_L$) without making $l > 0$. Substituting out m and then R_H from the constraints $m = (R_H - R_L)/(R_{HL} - R_L)$; $R_H = [(1+r)B - (1-p)R_L]/p$ the problem reduces to:

$$\max Ey - \frac{(1-p)\phi}{p} \frac{(1+r)B - R_L}{R_{HL} - R_L} \tag{17}$$

There are also feasibility constraints: $y_H \geq R_{HL}, R_H; y_L \geq R_L, R_{LL}$. We can replace the second of these by $R_L \leq y_L - \phi$ since we know that $R_{LL} = R_L + \phi$. Since (17) is increasing in R_{HL} (to avoid the first best being feasible $(1+r)B - R_L > 0$), there must be maximum punishment in a H contract i.e. $R^H_{HL} = y_H$. Similarly (17) is increasing in R_L since $y_H = R_{HL} > (1+r)B$; hence in the H contract R^H_L is set at its maximum value of $y_L - \phi$. Consequently $R^H_{LL} = y_L$. Knowing this we can also calculate the optimal levels of R_H and m in the H contract as:

$$R_H^H = [(1+r)B - (1-p)(y_L - \phi)]/p \tag{18}$$

$$m^H = \frac{(1+r)B - y_L + \phi}{p(y_H - y_L + \phi)} \tag{19}$$

which then gives maximal expected utility of:

$$EU_H = Ey - \frac{(p(y_H - y_L) + \phi)B(1+r) - \phi(y_L - \phi)(1-p)}{p(y_H - y_L + \phi)} \tag{20}$$

The H solution requires $0 < m^H < 1$ which automatically holds: to have sufficient expected revenue to pay for the investment requires $(1+r)B \leq y_H$ since $y_H > y_L$ and $(1+r)B > y_L$.

3.2 The Optimal Repayments Conditional on the Interior Mixed Strategy Solution

The contract problem has the form:

$$\max Ey - pR_H - (1-p)R_L - (1-p)(R_{LL} - R_L)(R_h - R_L)/(R_{HL} - R_L) \tag{21}$$

$$pR_H + (1-p)R_L - \frac{(1-p)(\phi + R_L - R_{LL})}{R_{HL} - R_L - \phi} \geq (1+r)B \tag{22}$$

where we have used $m = (R_H - R_L)/(R_{HL} - R_L)$; $l = (1-p)(\phi + R_L - R_{LL})/(p(R_{HL} - R_L - \phi))$ and know that $\phi + R_L - R_{LL} \geq 0$; $pR_{HL} + (1-p)R_{LL} - R_L - \phi \geq 0$ and $R_s \leq y_s$ for the relevant states.

KP show that the zero profit constraint must bind; $R_{LL} = y_L$ and $R_{HL} = y_H$ (see appendix A.2). Notice that the constraints $\phi + R_L - R_{LL} \geq 0$; $pR_{HL} + (1-p)R_{LL} - R_L - \phi \geq 0$ do not affect this solution and that the second requires that $p(y_H - y_L) - \phi + (y_L - R_L) \geq 0$.

Putting $R_{LL} = y_L$ and $R_{HL} = y_H$ in (58) yields:

$$EU = Ey - \frac{(Ey - R_L)(y_h - R_L - \phi)(1+r)B - \phi R_L(1-p)(y_H - y_L)}{(py_H + (1-p)y_L - \phi - R_L)(y_H - R_L)} \tag{23}$$

The problem is then to choose R_L to minimise:

$$\frac{(Ey - R_L)(y_H - R_L - \phi)(1+r)B - \phi R_L(1-p)(y_H - y_L)}{(py_H + (1-p)y_L - \phi - R_L)(y_H - R_L)} \tag{24}$$

subject to $R_L > y_L - \phi; R_L \leq \min\{Ey - \phi, y_L\}$.

The objective function (24) is concave in R_L (see appendix A.3).

Under the feasibility condition (14) and the assumption that the first best is unattainable so that $y_L - (1+r)B < 0$, $p(y_H - y_L) - \phi > 0$ which in turn implies $\min\{Ey - \phi, y_L\} = y_L$. Hence the problem is to minimise (24) subject to $y_L - \phi < R_L \leq y_L$. To satisfy the constraints $y_L - \phi < R_L \leq y_L$; but $EU(.)$ is convex in R_L so the optimum value of R_L must be either arbitrarily close to $R_L = y_L - \phi$ or at $R_L = y_L$. In the former case there is no optimal solution in KP form because of the open set restriction $y_L - \phi < R_L$; but as we have seen instead the H form involves $R_L = y_L - \phi$.

Putting each of these values of R_L in (61) in turn, we find that $R_L = y_L$ is optimal as in KP if $B(1+r)(1+p) < py_H + y_L - \phi$; otherwise $R_L = y_L - \phi$ is optimal with the H solution. This also gives us the switch point between the KP and H cases in terms of the value of B or ϕ.

Summarising, when $B(1+r)(1+p) < py_H + y_L - \phi$, maximal expected utility becomes:

$$EU_{KP} = Ey - \frac{p(y_H - y_L - \phi)(1+r)B - \phi y_L(1-p)}{p(y_H - y_L) - \phi} \tag{25}$$

Then the optimal repayment contract will have the KP form and involve:

$$l^{KP} = \frac{(1-p)\phi}{p(y_H - y_L - \phi)} \tag{26}$$

$$R_L^{KP} = y_L \tag{27}$$

$$R_{LL}^{KP} = y_L \tag{28}$$

$$R_{HL}^{KP} = y_H \tag{29}$$

$$R_H^{KP} = \frac{(1+r)B(y_H - y_L - \phi) - y_L(y_H - y_L)(1-p)}{p(y_H - y_L) - \phi} \tag{30}$$

$$m^{KP} = \frac{[(1+r)B - y_L][y_H - y_L - \phi]}{[p(y_H - y_L) - \phi][y_H - y_L]} \tag{31}$$

If B is fixed, (14) coincide with (1-9) so we have shown that there may be examples in which the revenues and debt satisfy all KP's assumptions but the optimal contract form under no commitment actually involves truthtelling. Now, KP assume $Ey - (1+r)B - \phi > 0$; however $(py_H + y_L - \phi)/(1+p) < Ey - \phi$ and so if $(py_H + y_L - \phi)/(1+p) < (1+r)B < Ey - \phi$ then KP's feasibility assumption is satisfied but the optimal no commitment contract involves $R_L = y_L - \phi < R_{LL} = y_L$. In this case there is a cost to truthtelling borne by the borrower; it is as if he is sharing in the observation cost.It is possible that the KP feasibility condition does not hold but the above analysis of the KP repayments is still valid, e.g. if $y_L < (1+r)B$ but $p(y_H - y_L) - \phi > 0$. In this case $\min\{Ey - \phi, y_L\} = y_L$ and so the above argument applies.

3.3 The Optimal Repayment Structure in a Fixed B World

We can rewrite the two possible maximal expected utility levels as:

$$EU_{KP} = Ey - (1+r)B - \frac{\phi(1-p)((1+r)B - y_L)}{p(y_H - y_L) - \phi} \tag{32}$$

$$EU_H = Ey - (1+r)B - \frac{\phi(1-p)((1+r)B + \phi - y_L)}{p(y_H - y_L + \phi)} \tag{33}$$

We know that $EU_H > EU_{KP}$ iff $\phi > py_H + y_L - B(1+r)(1+p)$. Given that the KP solution with fixed B is feasible only if $p(y_H - y_L) - \phi > 0$ we can write the following:

Proposition 1 *Given* $y_L < (1+r)B$:

(a) If either $p(y_H - y_L) - \phi > 0$ *or* $Ey - (1+r)B - \phi > 0$ *then the optimal contract has:*

the KP form if $\phi < py_H + y_L - B(1+r)(1+p)$;

the H form if $\phi > py_H + y_L - B(1+r)(1+p)$.

(b) If $p(y_H - y_L) - \phi < 0$ *then the optimal contract has the H form.*

Comparing the high state repayment in the two contract forms gives:

$$R_H^{KP} - R_H^H = -\frac{\phi(1-p)(Ey - (1+r)B - \phi)}{p(p(y_H - y_L) - \phi)} < 0 \tag{34}$$

but since $R_L^{KP} - R_L^H = \phi > 0$ we find that the repayments resulting when monitoring is not actually carried out are more spread out in the H contract than in the KP contract:

$$R_H^{KP} - R_L^{KP} < R_H^H - R_L^H \tag{35}$$

so that there is a higher incentive to cheat in the hybrid case. On the other hand this is negated by the monitoring strategies of the two contracts; we have:

$$m^{KP} = \frac{R_H^{KP} - y_L}{y_H - y_L} < \frac{R_H^{KP} - y_L + \phi}{y_H - y_L + \phi} < \frac{R_H^H - y_L + \phi}{y_H - y_L + \phi} = m^H \tag{36}$$

The interpretation is that for sufficiently high B the revenues in each state are high enough to be able to deter cheating through an aggressive auditing policy.

4 The Optimal Choice of B

Here we use the technological assumptions (5)-(7); the first best is still unattainable but there will typically exist a value B_{SR}, at which $y_L(B_{SR}) = (1+r)B_{SR}$ so that, so long as B is at or below B_{SR}, a SR contract is feasible in which $R_H = R_{HL} = R_L = R_{LL}$. So we start by outlining the form of the optimal SR contract and then move on to determine conditions under which the optimal contract is SR, KP or H in form.

4.1 The Single Repayment Contract

The only undominated pure strategy has $m = 0$ and $l = 1$; we call this a SR contract since there is always a low report which results in a single repayment of R_L. In this case the contract problem has the form:

$$\max Ey - R_L \tag{37}$$

$$R_L \geq (1+r)B$$

$$R_L \leq y_s$$

The constraint must bind since otherwise R_L could be reduced raising debtor expected utility. So:

$$\max Ey - R_L \tag{38}$$

$$(1+r)B \leq y_L$$

Since $y_s(.)$ is increasing and concave, the solution for B in the SR contract will be at B_{SR}, yielding a maximal expected utility of:

$$EU_{SR} = p(y_H - y_L) \tag{39}$$

We have to contrast this payoff with those arising from either a KP solution or a H solution in which $l = 0$.

4.2 The Optimal Level of B

Expected utility of each contract with optimal repayments will be given by:

$$EU_{SR}(B) = Ey(B) - (1+r)B \tag{40}$$

$$\begin{aligned} EU_H(B) &= Ey - \frac{(p(y_H - y_L) + \phi)B(1+r) - \phi(y_L - \phi)(1-p)}{p(y_H - y_L + \phi)} \\ &= EU_{SR}(B) - \frac{\phi(1-p)[(1+r)B - (y_L - \phi)]}{p(y_H - y_L + \phi)} \end{aligned} \tag{41}$$

$$
\begin{aligned}
EU_{KP}(B) &= Ey - \frac{p(y_H - y_L - \phi)B(1+r) - \phi y_L(1-p)}{p(y_H - y_L) - \phi} \\
&= EU_{SR}(B) - \frac{\phi(1-p)[(1+r)B - y_L]}{p(y_H - y_L) - \phi}
\end{aligned}
\tag{42}
$$

To determine the relationships between these contract forms, there are four questions: firstly for what range of values of B is each contract form defined? Secondly, for a given investment level, how do the welfare levels of the alternative contracts compare? Thirdly, how do optimal values of investment and monitoring compare within a given contract form? And, fourthly, which contract form is globally optimal, allowing for the optimal investment level to differ by contract?

4.2.1 Feasibility of the Contracts

(40) is feasible only for $B \leq B_{SR}$ defined by:

$$
y_L(B_{SR}) = (1+r)B_{SR} \tag{43}
$$

since only for this range of investments are there sufficient resources to repay the debt in the low state. Note that at $B_{SR}, EU_{KP} = EU_{SR} > EU_H$ so long as EU_{KP} is well defined. And for $B > B_{SR}$, $EU_{SR} > EU_{KP}$ again so long as EU_{KP} is well defined. We also know that by assumption $B_1 > B_{SR}$.

(42) is only valid for $0 < l^{KP} < 1$ and $0 < m^{KP} < 1$, which require $B > B_{KP}$ and $B > B_{SR}$ and $m^{KP} < 1$ where B_{KP} is defined by:

$$
p(y_H(B_{KP}) - y_L(B_{KP})) = \phi \tag{44}
$$

The condition $m^{KP} < 1$ can be expressed as:

$$
(y_H - y_L)[(1+r)B - py_H - (1-p)y_L + \phi] < [(1+r)B - y_L]\phi \tag{45}
$$

which we assume to hold at the optimum investment level in the KP contract[1] ; certainly it is valid in each numerical simulation below.

[1] Note that the KP feasibility condition (9) is a sufficient condition for this.

(41) is only well defined when $0 < m^H < 1$; this requires $(1+r)B - y_L + \phi > 0$ and $(1+r)B < y_H$. Generally there may exist two values of B, B_{HS1} and B_{HS2}, at which $(1+r)B + \phi = y_L$. For $B_{HS2} < B < B_{HS1}$, $(1+r)B + \phi < y_L$ although for sufficiently high ϕ there will be no solutions to this equation. Moreover, where the roots exist, $B_{HS_i} < B_{SR}$ $(i = 1, 2)$. Note that the H contract is feasible at any $B > B_{SR}$. From the form of (41) it follows that below B_{SR} the SR contract dominates the H contract in regions where the latter is defined. However since we are concerned with situations where the first best level of investment cannot be implemented by a SR contract, our second best optima will be above B_{SR}.

4.2.2 Welfare Levels of the Contracts at Common B

At any value of B_H defined by:

$$(1+p)(1+r)B_H = py_H(B_H) + y_L(B_H) - \phi \tag{46}$$

we have $EU_H = EU_{KP}$ so long as both expected utilities are well defined (i.e. $B > \max(B_{SR}, B_{KP})$). Let:

$$\begin{aligned} F(B,\phi) &= (1+p)(1+r)B - py_H(B) - y_L(B) + \phi \\ &= (1+p)[(1+r)B - y_L(B)] - [p(y_H(B) - y_L(B)) - \phi] \end{aligned} \tag{47}$$

So B_H is defined by $F(B_H, \phi) = 0$. Then $F''(B) > 0$ and, under our assumptions on revenues, $F(0) = \phi$; $F(\infty) = \infty$. When $F(B) > 0$ we have $EU_H > EU_{KP}$. Define (ϕ_f, B_f) by $F(B_f, \phi_f) = 0 = F^{'}(B_f, \phi_f)$ so that there is a single root B_f to the equation $F(B, \phi_f) = 0$ and for $\phi > \phi_f$ there are no roots.

For $\phi < \phi_f$ there are two roots B_{Hi} satisfying $F(B_{Hi}, \phi) = 0$; we label them as $B_{H2} < B_{H1}$. Then for $B_{H1} < B < B_{H2}$, $EU_H < EU_{KP}$. Also if there are two roots then the lower root satisfies $B_{H2} < B_1$ since we have $F^{'}(B_{H2}) < 0$ but:

$$F^{'}(B_1) = p(1 + r - y_L^{'}(B_1)) > 0 \tag{48}$$

while B_{H1} can be either $><$ B_1.

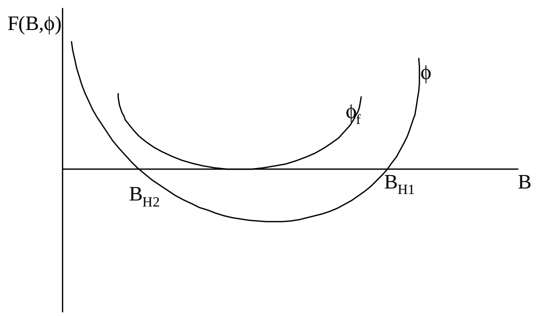


Figure 2:

4.2.3 Optimal Investment and Monitoring under Hybrid and Mixed Strategy Contracts

Let B_H^* solve $EU_H^{'}(B) = 0$; and B_{KP}^* solve $EU_{KP}^{'}(B) = 0$. If each expected utility is strictly concave these conditions define the relevant optimal investment levels. If we can show that $EU_H^{'}(B_{KP}^*) > 0$, so that EU_H is still rising when EU_{KP} attains a maximum, then we know that $B_H^* > B_{KP}^*$. Now, since $F(.)$ is strictly convex with at most one turning point, we can divide up the range of B into three subregions in which B_{KP}^* may exist. It may be that at B_{KP}^*, $F'(B_{KP}^*) < 0$; or $F'(B_{KP}^*) > 0$ but $F(B_{KP}^*) < 0$; or it may be that $F'(B_{KP}^*) > 0$ and $F(B_{KP}^*) > 0$. In appendices A.4-A.5-A.6 we show that:

Proposition 2 *If EU_H and EU_K are strictly concave then the optimal investment level in each case is below the first best B_1. Moreover in each case optimally investment is higher in the H contract than in the KP contract.*

One way of understanding this is that in the H contract there is truthtelling; this requires a relatively high level of monitoring to police the debtor. But then since the monitoring costs are high, the contract has to ensure that there is sufficient revenue to cover these costs. This is achieved by having a high investment level.

From this as expected it follows that there is more monitoring in the H than in the KP contract, as shown in appendix A.7:

Proposition 3 *If EU_H and EU_K are strictly concave, the optimal level of monitoring in the H contract is higher than the optimal level of monitoring in the KP contract.*

4.2.4 The Globally Optimal Contract Form

We can use the structure of the function $F(B, \phi)$ together with knowledge of the interval in which the optimal investment level for a given contract lies to characterise which form of contract is globally optimal.

To proceed we first have to determine the relationship between various critical levels of B. Given $y_s(B)$,r, ϕ, p, recall that B_{SR} solves $(1+r)B = y_L(B)$. Note that B_{SR} depends on r, but is independent of p and ϕ.

We can use the expression for marginal utility:

$$\begin{aligned} EU_{KP}^{'} &= Ey' - (1+r) - \frac{(1-p)\phi[(1+r) - y_L^{'}]}{p(y_H - y_L) - \phi} + \frac{(1-p)\phi[(1+r)B - y_L]p[y_H^{'} - y_L^{'}]}{[p(y_H - y_L) - \phi]^2} \\ &= Ey' - (1+r) - \frac{(1-p)\phi[(1+r) - y_L^{'}]}{p(y_H - y_L) - \phi} \end{aligned} \tag{49}$$

to see how the sign of $EU_{KP}^{'}(B_{SR})$ varies with ϕ : when $\phi = 0, EU_{KP}^{'}(B_{SR}) > 0$ since $B_{SR} < B_1$ and at $\phi = \phi^*, EU_{KP}^{'}(B_{SR}) < 0$. Also: $\partial EU_{KP}^{'}(B_{SR})/\partial\phi = -(1-p)[1+r-y_L^{'}]p(y_H - y_L)/[p(y_H - y_L 0 - \phi]^2 < 0$, hence there is a ϕ^{**} at which $EU_{KP}^{'}(B_{SR}) = 0$. At ϕ^{**} :

$$[p(y_H - y_L) - \phi^{**}][Ey^{'} - (1+r)] = (1-p)\phi^{**}[1 + r - y_L^{'}] > 0 \tag{50}$$

which implies $p(y_H - y_L) - \phi^{**} > 0$.

Then recall that $B_{KP}(\phi)$ solves $p(y_H - y_L) = \phi$ and ϕ^* solve $p(y_H(B_{SR}) - y_L(B_{SR})) = \phi^*$. Hence $\phi^{**} < \phi^*$.

Any two of the three conditions $F(.) = 0, p(y_H - y_L) = \phi, (1+r)B = y_L$ imply the third. At ϕ^* all three conditions hold and $B_{KP}(\phi^*) = B_{SR}$ and, writing $F'(.) = \partial F/\partial B$, if $F'(B_{SR}, \phi^*) < 0$ then $B_{H2}(\phi^*) = B_{SR}$ whilst if $F'(B_{SR}, \phi^*) > 0$ then $B_{H1}(\phi^*) = B_{SR}$.

We can use the relationships between the expected utilities of the different contracts at these critical levels of B together with knowledge of where the optimal investment level corresponding to a particular contract lies to determine the globally optimal form of contract. The detailed proof is in appendix A.8.

Proposition 4 *(i) If $\phi^* > \phi$ and $B_{SR} < B_H^* < B_{H1}$ the globally optimal contract cannot be H;*
(ii) If $\phi^* > \phi$ and if $B_{H_1} < B_{KP}^* < B_1$ the globally optimal contract cannot be KP;
(iii) If $\phi > \phi^*$ and if $F^{'}(B_{SR}, \phi^*) > 0$ the globally optimal contract cannot be KP;
(iv) If $\phi > \phi^*$ and if $F^{'}(B_{SR}, \phi^*) < 0$ and if $B_{H2} < B_H^* < B_{H1}$ the globally optimal contract cannot be H;
(v) If $\phi > \phi^*$ and if $F^{'}(B_{SR}, \phi^*) < 0$ and if either $B_{KP} < B_{KP}^* < B_{H2}$ or, when $B_{H1} < B_1$, $B_{H1} < B_{KP}^* < B_1$ the globally optimal contract cannot be KP.

However this requires knowledge of the position of the optimal investment level within a contract and in most cases there are still two possible candidates for optimality: the SR contract and either H or KP; in general to compare these requires a global comparison of utilities.

If the expected utility of the H and KP contracts is strictly concave in B (when feasible) then we can identify the range in which the optimal investment level of the different contracts lies by knowing the sign of $EU^{'}(B)$ at the roots of $F(B, \phi) = 0$. The details of the argument differ by case but the essential intuition is that between the roots B_{H_i} we know that $EU_{KP}(B) > EU_H(B)$ whereas outside this region, $EU_{KP}(B) < EU_H(B)$ wherever both roots and both contracts are well defined. Then if $EU^{'}_{KP}(B_{H_1}) > 0$, it must be that $B_{KP}^* > B_{H_1}$ and so $EU_{KP}(B_K^*) < EU_H(B_H^*)$. Similarly if $EU^{'}_H(B_{H_2}) > 0$ but $EU^{'}_H(B_{H_1}) < 0$, then $B_{H_2} < B_H^* < B_{H_1}$ and so $EU_{KP}(B_K^*) > EU_H(B_H^*)$. This forms the basis for comparing the welfare levels of the optimal KP and H contracts.

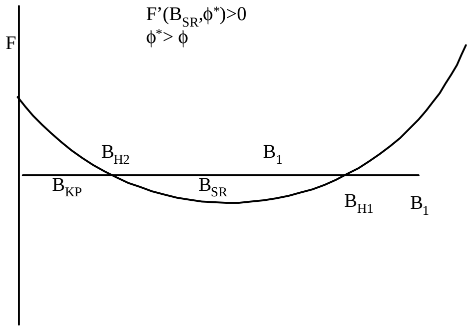


Figure 3

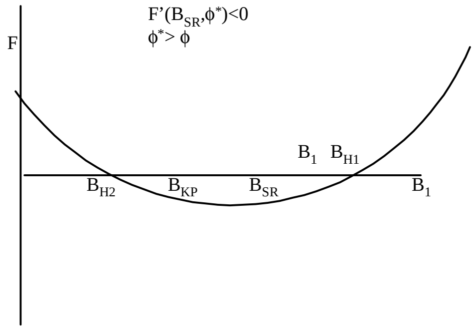


Figure 4

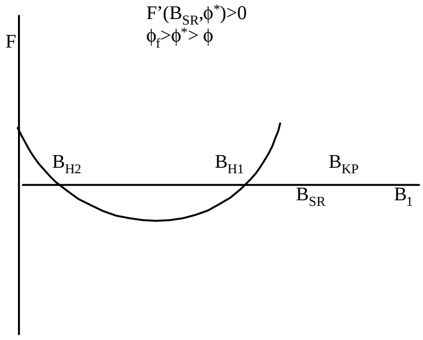


Figure 5

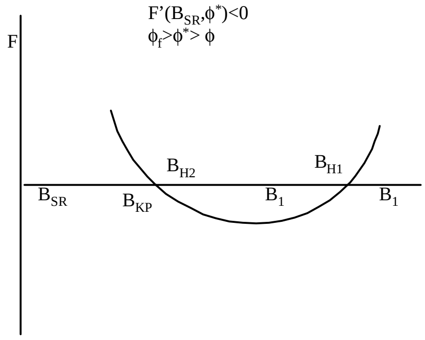


Figure 6

To determine the relationship between these welfare levels and that of the optimal SR contract we use the facts that, at $B_{SR}, EU_{SR}(B_{SR}) > EU_H(B_{SR})$ and $EU_H^{'}(B_{SR}) > 0$, and, if $0 < \phi < \phi^*$ so that $B_{KP} < B_{SR}$ and the KP contract is defined at B_{SR}, $EU_{SR}(B_{SR}) = EU_{KP}(B_{SR})$.

Proposition 5 *If EU_H and EU_{KP} are each strictly concave then:*

(a) If $F^{'}(B_{SR}, \phi^*) < 0$

and (a.1) $\phi_f > \phi > \phi^*$ and $B_{H1} > B_1$ the globally optimal contract depends on a global comparison of KP and SR;

and (a.2) $\phi_f > \phi > \phi^*$ and $B_{H1} < B_1$ and $EU_H^{'}(B_{H1}) < 0$;the globally optimal contract depends on a global comparison of KP and SR;

and (a.3) $\phi_f > \phi > \phi^*$ and $B_{H1} < B_1$ and $EU_{KP}^{'}(B_{H1}) > 0$;the globally optimal contract depends on a global comparison of H and SR;

and (a.4) $\phi_f > \phi > \phi^*$ and $B_{H1} < B_1$ and $EU_H^{'}(B_{H1}) > 0$ and $EU_{KP}^{'}(B_{H1}) < 0$;the globally optimal contract is single repayment if depends on a global comparison of KP, H and SR.

(b) If $F^{'}(B_{SR}, \phi^*) > 0$ and $\phi_f > \phi > \phi^*$ the globally optimal contract is single repayment if $EU_H^{'}(B_{SR}) < 0$ depends on a global comparison of H and SR if $EU_H^{'}(B_{SR}) > 0$.

(c) If $\phi^* > \phi > \phi^{**}$ the globally optimal contract is single repayment if $EU_H^{'}(B_{SR}) < 0$ depends on a global comparison of H and SR if $EU_H^{'}(B_{SR}) > 0$.

(d) $\phi^{**} > \phi$ *and* $B_1 < B_{H1}$ *the globally optimal contract is KP.*

(e) $\phi^{**} > \phi$ *and* $B_1 > B_{H1}$ *the globally optimal contract is KP if* $EU_H'(B_{H1}) < 0$.

(f) $\phi^{**} > \phi$ *and* $B_1 > B_{H1}$ *the globally optimal contract is H if* $EU_{KP}'(B_{H1}) > 0$;

(g) $\phi^{**} > \phi$ *and* $B_1 > B_{H1}$ *the globally optimal contract depends on a global comparison of KP and H if* $EU_H'(B_{H1}) > 0$ *and* $EU_{KP}'(B_{H1}) < 0$.

(h) If $\phi > \phi_f$ *the globally optimal contract depends on a global comparison of SR and H.*

The assumption that expected utilities are concave is somewhat delicate. Assuming that each revenue function $y_s(B)$ is increasing and concave in B and that $y_H''(B) < y_L''(B)$, a sufficient condition for $EU_{KP}(B)$ to be concave (see appendix A.4) is that:

$$[p(y_H - y_L) - \phi][1 + r - y_L'] < p(y_H' - y_L')[(1 + r)B - y_L] \tag{51}$$

and for $EU_H(B)$ to be concave that:

$$[y_H - y_L + \phi][1 + r - y_L'] < (y_H' - y_L')[(1 + r)B - y_L + \phi] \tag{52}$$

These conditions cannot hold globally; for example at any stationary point of the H contract:

$$[y_H - y_L + \phi][1 + r - y_L'] - (y_H' - y_L')[(1 + r)B - y_L + \phi] \tag{53}$$

$$= \quad [Ey' - (1 + r)]p(y_H - y_L + \phi)^2/(\phi(1 - p)) > 0 \tag{54}$$

if there is underinvestment at the stationary point (see below). Nevertheless the simulations that we present do have concave expected utilities within the whole range where the respective contracts are feasible.

It would be possible to assume that just one of the two expected utilities is concave and combine this with information about the location of the optimal investment level in the nonconcave contract to get similar characterisations of the possible global comparisons. Generally what comes out of these comparisons is that the higher the spread between y_H and y_L the less likely is the SR contract to be optimal. On the other hand the higher is ϕ the more likely is the SR contract to be optimal. For a given high spread between y_H and y_L, the higher is ϕ the more likely is the optimum to be H rather than KP - at first sight this is counterintuitive.

5 Numerical illustrations

To illustrate the various possibilities we provide some numerical solutions for three basic scenarios that have varying levels of the observation cost. In both scenarios we take isoelastic revenue functions with multiplicative uncertainty:

$$y_s = a_s\sqrt{B} \tag{55}$$

In every case the expected utility of each type of game/contract turns out to be concave in B where it is defined; and the feasibility condition is satisfied everywhere above $max(B_{KP}, B_{SR})$.

In our first scenario we take the parameters to have values:

$$p = 0.5; r = 0.1, a_L = 1, a_H = 20 \tag{56}$$

and then let ϕ vary. In this case the first best level of investment is $B_1 = 91.12$ and the optimal pooling repayment contract satisfying $y_L(B) = (1 + r)B$ has investment of $B_{SR} = 3.31$ which generates expected utility for the debtor of $EU(B_{SR}) = 34.54$. Then for varying ϕ we get the following:

ϕ	B	l	m	EU	R_H	B_K	B_{H2}	B_{H1}
10						.277	.222	165.4
KP	88.60	.0288	.453	97.879	180.75			
H	88.87		.482	97.80	186.671			
20						1.11	.96	152.
KP	86.102	.060	.459	95.277	180.612			
H	88.875		.518	94.991	193.040			
40						4.43	4.72	124.57
KP	81.465	.132	.481	89.046	182.940			
H	84.930		.590	88.320	208.414			
50						6.92	8.4	108.
KP	79.622	.173	.479	85.216	186.492			
H	84.336		.624	84.479	217.171			
60						9.97	14.6	90.4
KP	78.477	.217	.520	80.706	192.937			
H	84.009		.656	80.365	226.489			
65						11.7	19.5	79.4
KP	78.311	.240	.535	78.127	197.720			
H	83.932		.672	78.195	231.327			
70						13.57	27.5	65.4
KP	78.512	.262	.553	75.284	203.859			
H	83.904		.688	75.960	236.270			
80						17.7		
KP	80.325	.307	.600	68.649	221.200			
H	83.980		.720	71.289	246.429			

Our second scenario has the same parameter values except that p rises to 0.7. In this case the first best level of investment is $B_1 = 169.00$ and the first best utility level is $EU(B_1) = 185.90$, whilst the optimal investment in the SR contract is still 3.305 but the corresponding expected utility is $EU(B_{SR}) = 48.36$. For varying ϕ we get:

ϕ	B	l	m	EU	R_H	B_{KP}	B_{H2}	B_{H1}
100						14.133	22.325	128.94
KP	156	.114	.498	166.67	261.498			
H	160.854		.617	167.262	284.757			
80						9.045	11.410	160.399
KP	157.918	.086	.486	171.669	257.419			
H	161.676		.590	171.661	277.450			
60						5.09	5.48	187.7
KP	160.395	.061	.478	175.953	255.228			
H	162.824		.560	175.757	270.643			
40						2.261	2.154	212.436
KP	163.165	.0385	.472	179.682	254.256			
H	164.368		.529	179.518	264.446			
150						31.799	-	-
KP	157.180	.197	.552	149.410				
H	159.849		.681	155.113	304.640			

Our third scenario has $p = 0.5$; $r = 0.1$; $a_L = 1$; $a_H = 5$ with a first best investment level of $B_1 = 7.438$ and $EU(B_1) = 8.182$; $B_{SR} = 3.306$; $EU(B_{SR}) = 7.273$ and gives the results in the table below:

ϕ	B	l	m	EU	R_H	B_{KP}	B_{H2}	B_{H1}
6.25						2.441	1.632	8.790
KP	3.305	.753	.192	7.270	3.636			
H	6.76		.627	6.203	15.923			
7.0						3.062	2.605	6.908
KP	3.306	.928	0	7.273	3.636			
H	6.785	0	.665	5.838	16.718			

6 Conclusions

Because of the lack of devices to ensure that contracted auditing will actually be carried out in a two party, one shot loan contract scenario (Hart, 1995), there is recognition that analysing contracts without commitment is a valuable contribution. The existing wisdom in this area is that such contracts which fix loan size and the state dependent pattern of repayments must recognise that, at the repayment stage, there will be a noncooperative game to determine the auditing and cheating strategies. The literature also generally claims that the outcome of this game is an interior mixed strategy (Khalil and Parigi, 1998; Choe, 1998); the effect of this is that under risk neutrality, the optimal contract involves repayments following a low report of the state that are independent of the act of auditing (ie $R_L = R_{LL}$). That is, there is no reward/punishment on the debtor for truthfully declaring a low state. In this paper we show that the optimal contract may not induce an interior mixed strategy of the reporting/auditing game but that for particular properties of the revenue functions, the best contract may either lead to certain cheating and zero auditing (a single repayment/pooling contract); or to zero cheating but positive auditing; or to the interior mixed strategy. We also delineate the regions within which each of these contract forms is the optimum. When the no committment contract involves truthtelling, it also involves a higher investment level and a higher probability of monitoring than the mixed strategy solution that allows for some randomised cheating. In the hybrid truthtelling but random monitoring scenario there is a "punishment" on the debtor for truthfully reporting a low state in the sense that $R_{LL} > R_L$; an alternative way of interpreting this is that it is optimal to require the borrower to take on some share of the monitoring costs to encourage sufficient monitoring to keep the debtor truthful in his reports. Yet another way of looking at the possibility of having truthtelling in the Nash equilibrium is to regard it as a case in which the second best sequentially rational contract of Jost (1996) emerges as a non-cooperative outcome.

All of this no committment analysis is predicated on the assumption that the endogenous probabilities of cheating and monitoring are determined in a noncooperative Nash equilibrium. Most of the renegotiation literature instead sees the renegotiation stage as being in the form of a leadership game: one party suggests a renegotiation to the other that both can accept. If the debtor actually writes the original contract (which has some efficiency features to recommend it, Choe), then, as an alternative at the repayment stage, we could have a Stackleberg setup with either the lender or the debtor as the leader. Suppose the lender is the leader; they can choose any point on the debtors reaction curve and presumably will want to ensure low monitoring (to avoid monitoring costs) and low incidence of cheating (to extract rent from the debtor). It is then quite plausible that what we have called the hybrid equilibrium - which emerges as a noncooperative Nash equilibrium in our setting - is actually also the Stackleberg equilibrium of the game when the lender is the leader. Or if the monitoring costs are very high then the lender as leader may actually prefer the single repayment scenario where $l = 1, m = 0$. Thus either the single repayment contract investment and repayment levels may emerge from a contract in which the second renegotiation stage has the lender as Stackleberg leader. If instead at the renegotiation stage the debtor is the leader then they can choose any point on the lenders reaction curve and presumably prefer outcomes with a low probability of monitoring and a high probability of cheating. This is likely to lead to the outcome in which $m = 0, 0 < l < 1$. In the noncooperative Nash scenario we rule this out because it requires $R_H = R_L$ to give debtor indifference between truthtelling and cheating;

but in a Stackleberg scenario it is quite possible since it gives the highest level of cheating that is consistent with no monitoring.

A Appendix

A.1 Conditions for Hybrid Equilibria

A.1.1 $l = 0, m = 1$

This requires $\partial E\pi/\partial m|l = pl(R_{HL} - R_L - \phi) - (1-p)(\phi + R_L - R_{LL}) < 0$ and $\partial EU/\partial l|m = (R_H - R_L) - m(R_{HL} - R_L) = (R_H - R_L) = 0$ at $m = 0$. Taken together these yield $R_H = R_L$ and (so long as $R_{HL} - R_L - \phi > 0$) any nonnegative value of $l < (1-p)(\phi + R_L - R_{LL})/p(R_{HL} - R_L - \phi)$. Alternatively if $R_{HL} - R_L - \phi < 0$ any value of $l > (1-p)(\phi + R_L - R_{LL})/p(R_{HL} - R_L - \phi)$ but less than unity will suffice.

A.1.2 $m = 1; 0 < l < 1$

This requires $\partial E\pi/\partial m|l = pl(R_{HL} - R_L - \phi) - (1-p)(\phi + R_L - R_{LL}) > 0$ and $\partial EU/\partial l|m = (R_H - R_L) - (R_{HL} - R_L) = 0$. Taken together these yield $R_{HL} = R_H$ and (so long as $R_{HL} - R_L - \phi > 0$) any value of $l > (1-p)(\phi + R_L - R_{LL})/p(R_{HL} - R_L - \phi)$ which does not exceed unity. Alternatively if $R_{HL} - R_L - \phi < 0$ any nonnegative value of $l < (1-p)(\phi + R_L - R_{LL})/p(R_{HL} - R_L - \phi)$ but less than unity will suffice (note this then requires $\phi + R_L - R_{LL} < 0$).

A.1.3 $l = 0; 0 < m < 1$

This requires $\partial EU/\partial l|m = (R_H - R_L) - m(R_{HL} - R_L) < 0$ and $\partial E\pi/\partial m|l = (1-p)(\phi + R_L - R_{LL}) = 0$ when $l = 0$. Taken together these yield $(\phi + R_L - R_{LL}) = 0$ and (so long as $R_{HL.} > R_L$ which is an innocuous assumption) any value of $m > (R_H - R_L)/(R_{HL} - R_L)$ which does not exceed unity.

A.1.4 $l = 1; 0 < m < 1$

This requires $\partial EU/\partial l|m = (R_H - R_L) - m(R_{HL} - R_L) > 0$ and $\partial E\pi/\partial m|l = p(R_{HL} - R_L - \phi) - (1-p)(\phi + R_L - R_{LL}) = 0$ when $l = 1$. This requires any nonnegative value of $m < (R_H - R_L)/(R_{HL} - R_L)$ which does not exceed unity.

A.2 Repayments Conditional on an Interior Mixed Strategy

The zero profit constraint must bind; otherwise raising B will raise Ey without affecting feasibility which then raises borrower expected returns. But since it binds we can follow KP and solve out R_H :

$$R_H = \frac{(1+r)B(R_{HL}-R_L-\phi)-R_L(R_{HL}-R_{LL})(1-p)}{pR_{HL}+(1-p)R_{LL}-\phi-R_L} \tag{A.57}$$

and replacing this in the borrowers expected return gives:

$$\begin{aligned} EU &= Ey \\ &\quad -\frac{[(1+r)B(R_{HL}-R_L-\phi)-R_L(R_{HL}-R_{LL})(1-p)][pR_{HL}+(1-p)R_{LL}-R_L]}{(pR_{HL}+(1-p)R_{LL}-\phi-R_L)(R_{HL}-R_L)} \\ &\quad -(1-p)R_L\frac{(R_{HL}-R_{LL})}{(R_{HL}-R_L)} \end{aligned} \tag{A.58}$$

Then exactly as in KP:

$$\frac{\partial EU}{\partial R_{LL}} = \frac{(1-p)\phi(R_{HL}-R_L-\phi)((1+r)B-R_L)}{(pR_{HL}+(1-p)R_{LL}-\phi-R_L)^2(R_{HL}-R_L)} > 0 \tag{A.59}$$

so that $R_{LL}=y_L$. Using this:

$$\begin{aligned} \frac{\partial EU}{\partial R_{HL}} &= \frac{\phi(1-p)((1+r)B-R_L)[p(R_{HL}-y_L)^2-(y_L-R_L)^2+(y_L-R_L)\phi]}{(pR_{HL}+(1-p)y_L-\phi-R_L)^2(R_{HL}-R_L)^2} \\ &= \frac{\phi(1-p)((1+r)B-R_L)[p(R_{HL}-y_L)^2-(y_L-R_L)\{y_L-R_L-\phi\}]}{(pR_{HL}+(1-p)y_L-\phi-R_L)^2(R_{HL}-R_L)^2} \\ &> 0 \end{aligned} \tag{A.60}$$

so $R_{HL}=y_H$. Notice that the constraints $\phi+R_L-R_{LL}\geq 0; pR_{HL}+(1-p)R_{LL}-R_L-\phi\geq 0$ do not affect this solution.

Putting $R_{LL}=y_L$ and $R_{HL}=y_H$ in (58) yields:

$$EU = Ey-\frac{(Ey-R_L)(y_h-R_L-\phi)(1+r)B-\phi R_L(1-p)(y_H-y_L)}{(py_H+(1-p)y_L-\phi-R_L)(y_H-R_L)} \tag{A.61}$$

The objective function (24) is concave in R_L

$$\frac{\partial EU}{\partial R_L} = \frac{\phi(1-p)(y_H-y_L)[(Ey-\phi)(y_H-(1+r)B)-(1+r)By_H+2R_L(1+r)B-R_L^2]}{(py_H+(1-p)y_L-\phi-R_L)^2(y_H-R_L)^2} \tag{A.62}$$

From this it follows that EU is convex in R_L: since $(1+r)B > R_L$ the numerator of (62) is increasing in R_L; on the other hand each term in the denominator is decreasing in R_L since $py_H+(1-p)y_L-\phi-R_L > 0$ and $y_H - R_L > 0$. Hence $\partial^2 EU/\partial R_L^2 > 0$. But (24) has a derivative of $-\frac{\partial EU}{\partial R_L}$.

A.3 Concavity of Expected Utilities in B

Second derivatives of expected utilities are given by:

$$\begin{aligned} EU_H^{''} &= py_H^{''}+(1-p)y_L^{''}+\frac{y_L^{''}(1-p)\phi}{p(y_H-y_L+\phi)}+2\frac{(1-p)\phi}{p}\frac{(1+r-y_L^{'})(y_H^{'}-y_L^{'})}{(y_H-y_L+\phi)^2} \\ &\quad -2\frac{(1-p)\phi}{p}\frac{[(1+r)B-y_L+\phi](y_H^{'}-y_L^{'})^2}{(y_H-y_L+\phi)^3}+\frac{(1-p)\phi}{p}\frac{[(1+r)B-y_L+\phi](y_H^{''}-y_L^{''})}{(y_H-y_L+\phi)^2} \end{aligned} \tag{A.63}$$

$$\begin{aligned} EU_{KP}^{''} &= py_H^{''}+(1-p)y_L^{''}+\frac{y_L^{''}(1-p)\phi}{p(y_H-y_L)-\phi}+2(1-p)p\frac{(1+r-y_L^{'})(y_H^{'}-y_L^{'})}{[p(y_H-y_L)-\phi]^2} \\ &\quad -2(1-p)p\frac{[(1+r)B-y_L](y_H^{'}-y_L^{'})^2}{[p(y_H-y_L)-\phi]^3}+(1-p)\frac{[(1+r)B-y_L]p(y_H^{''}-y_L^{''})}{(p(y_H-y_L)-\phi)^2} \end{aligned} \tag{A.64}$$

Since $y_s^{''} < 0$, $y_H^{''}-y_L^{''} < 0$ and $(1+r)B > y_L$ since we are above B_{SR}; the only terms of ambiguous sign in $EU_s^{''}$ are:

$$\frac{(1+r-y_L^{'})(y_H^{'}-y_L^{'})}{(y_H-y_L+\phi)^2}-\frac{[(1+r)B-y_L+\phi](y_H^{'}-y_L^{'})^2}{(y_H-y_L+\phi)^3} \tag{A.65}$$

and:

$$\frac{(1+r-y_L^{'})(y_H^{'}-y_L^{'})}{[p(y_H-y_L)-\phi]^2}-\frac{[(1+r)B-y_L](y_H^{'}-y_L^{'})^2}{[p(y_H-y_L)-\phi]^3} \tag{A.66}$$

So if $(1+r-y_L^{'})(y_H-y_L+\phi) < [(1+r)B-y_L+\phi](y_H^{'}-y_L^{'})$ and $(1+r-y_L^{'})[p(y_H-y_L)-\phi] <$ $[(1+r)B-y_L](y_H^{'}-y_L^{'})$ then we would know expected utilities are globally concave. However these conditions cannot hold globally: consider for example $EU_H^{''}$ at B_H^*. At $EU_H^{'}=0$ we have:

$$\frac{(1-p)\phi[(1+r)B-y_L+\phi][y_H^{'}-y_L^{'}]}{p[y_H-y_L+\phi]^2} = -Ey'+(1+r)+\frac{(1-p)\phi[(1+r)-y_L^{'}]}{p(y_H-y_L+\phi)} \tag{A.67}$$

$$\begin{aligned}(1+r-y_L^{'})-\frac{[(1+r)B-y_L+\phi](y_H^{'}-y_L^{'})}{(y_H-y_L+\phi)} &= \\ (1+r-y_L^{'})+\left(\frac{p}{(1-p)\phi}[y_H-y_L+\phi][Ey'-(1+r)]-[(1+r)-y_L^{'}]\right) & \\ =\left(\frac{p}{(1-p)\phi}[y_H-y_L+\phi][Ey'-(1+r)]\right) &> 0\end{aligned}$$

if $B_H^* < B_1$ so that optimal H investment is below the first best level.

A.4 The Optimal H and KP Investment is below the First Best: $B_H^*, B_{KP}^* < B_1$

For the H contract since $EU_H(B)$ is concave, this is true if $EU_H^{'}(B_1) < 0$. However:

$$EU_H^{'}(B_1) = \frac{(1-p)\phi(1+r-y_L^{'}]}{p^2(y_H-y_L+\phi)^2}[(1+r)B-py_H-(1-p)y_L+\phi(1-p)] \tag{A.68}$$

which under the feasibility condition is negative.

Similarly for the KP contract:

$$\begin{aligned}EU_{KP}^{'}(B_1) &= \frac{-[(1+r)-y_L^{'}][p(y_H-y_L)-\phi]+p(y_H^{'}-y_L^{'})[(1+r)B_1-y_L]}{[p(y_H-y_L)-\phi]^2} \quad \text{(A.69)} \\ &= \frac{-[(1+r)-y_L^{'}]\{Ey-\phi]-(1+r)B_1\}}{[p(y_H-y_L)-\phi]^2} < 0\end{aligned}$$

because of the first best condition that $p(y_H^{'}-y_L^{'})=(1+r)-y_L^{'}$ and the feasibility condition which gives the sign.

A.5 Optimally $B_{KP}^* < B_H^*$

First suppose that the optimal investment in KP occurs when $F'(.) < 0$:

(i) $F'(B^*_{KP}) < 0$ We can rewrite the derivatives of expected utility with respect to B as:

$$EU^{'}_{KP}(B) = Ey' - (1+r) - P_K(1-p)\phi \quad \text{(A.70)}$$

$$EU^{'}_{H}(B) = Ey' - (1+r) - P_H(1-p)\phi \quad \text{(A.71)}$$

where:

$$P_{KP} = \frac{(1+r) - y^{'}_L}{p(y_H - y_L) - \phi} - \frac{p(y^{'}_H - y^{'}_L)[(1+r)B - y_L]}{[p(y_H - y_L) - \phi]^2} \quad \text{(A.72)}$$

$$P_H = \frac{(1+r) - y^{'}_L}{p(y_H - y_L + \phi)} - \frac{p(y^{'}_H - y^{'}_L)[(1+r)B - y_L + \phi]}{[p(y_H - y_L + \phi)]^2} \quad \text{(A.73)}$$

So for any B :

$$\begin{aligned} EU^{'}_H - EU^{'}_{KP} &= (1-p)\phi(P_{KP} - P_H) \quad \text{(A.74)} \\ &= \frac{(1-p)\phi}{[p(y_H - y_L) - \phi]^2[p(y_H - y_L + \phi)]^2} \cdot \\ &\quad \{[(1+r) - y^{'}_L][p(y_h - y_L) - \phi][p(y_H - y_L + \phi]^2 \\ &\quad -p(y^{'}_H - y^{'}_L)[(1+r)B - y_L] \cdot [p(y_H - y_L + \phi)]^2 - \\ &\quad [[(1+r) - y^{'}_L][p(y_H - y_L) - \phi]^2[p(y_H - y_L + \phi)]] \\ &\quad +p(y^{'}_H - y^{'}_L)[(1+r)B - y_L][p(y_H - y_L) - \phi]^2 \\ &\quad +p(y^{'}_H - y^{'}_L)\phi[p(y_H - y_L) - \phi]^2\} \end{aligned}$$

Define $Q = \{.\}$; then using $p(y_H - y_L + \phi) = [p(y_H - y_L) - \phi] + (1+p)\phi$:

$$\begin{aligned} Q &= \{[(1+r) - y_L^{'}][p(y_H - y_L) - \phi] - p(y_H^{'} - y_L^{'})[(1+r)B - y_L]\} \cdot \\ &\quad \{(1+p)^2\phi^2 + 2(p(y_H - y_L) - \phi)(1+p)\phi\} - \\ &\quad [p(y_H - y_L) - \phi]^2\phi[(1+p)[(1+r) - y_L^{'}] - p(y_H^{'} - y_L^{'})] \\ &= P_{KP}[p(y_H - y_L) - \phi]^2[(1+p)^2\phi^2 + 2(p(y_H - y_L) - \phi)(1+p)\phi] - \\ &\quad p(y_H^{'} - y_L^{'})\phi[p(y_H - y_L) - \phi]^2 F' \end{aligned} \tag{A.75}$$

Hence:

$$\begin{aligned} EU_H^{'} - EU_{KP}^{'} &= \frac{(1-p)\phi}{[p(y_H - y_L) - \phi]^2[p(y_H - y_L + \phi)]^2} \cdot \\ &\quad \{P_{KP}[p(y_H - y_L) - \phi]^2[(1+p)^2\phi^2 + 2(p(y_H - y_L) - \phi)(1+p)\phi] \\ &\quad -p(y_H^{'} - y_L^{'})\phi[p(y_H - y_L) - \phi]^2 F'\} \end{aligned} \tag{A.76}$$

and:

$$EU_H^{'} - EU_K^{'} \quad > \quad 0 \quad if \quad F' < 0 \quad and \quad P_{KP} > 0 \tag{A.77}$$

Now when $EU_{KP}^{'} = 0$ $P_{KP} > 0$ since then P_{KP} is equal to the net benefit $Ey' - (1+r) > 0$; it follows that at B such that $EU_K^{'} = 0$ (i.e. at B_{KP}^*) if $F' < 0$ then $EU_H^{'} > 0$. From this we know that whenever $B_{KP}^* < B_{H2}$ then $E\acute{U}_H^{'}(B_{KP}^*) > 0$.

If the optimal investment in KP occurs above the upper root of $F(.)$, then:

(ii) $F'(B^*_{KP}) > 0$ **and** $F(B^*_{KP}) > 0$ In this case we can derive an alternative expression for Q :

$$
\begin{aligned}
Q &= [(1+r)-y_L^{'}][p(y_H-y_L)-\phi][(1+p)^2\phi^2+(p(y_H-y_L)-\phi)(1+p)\phi]- \quad \text{(A.78)}\\
&\quad p(y_H^{'}-y_L^{'})[(1+r)B-y_L][(1+p)^2\phi^2+2(p(y_H-y_L)-\phi)(1+p)\phi]+\\
&\quad p(y_H^{'}-y_L^{'})\phi[p(y_H-y_L)-\phi]^2\\
&= \{[(1+r)-y_L^{'}][p(y_H-y_L)-\phi]-p(y_H^{'}-y_L^{'})[(1+r)B-y_L]\}\cdot\\
&\quad [(1+p)^2\phi^2+(p(y_H-y_L)-\phi)(1+p)\phi]+\\
&\quad p(y_H^{'}-y_L^{'})\phi[p(y_H-y_L)-\phi]^2-\\
&\quad -p(y_H^{'}-y_L^{'})[(1+r)B-y_L](p(y_H-y_L)-\phi)(1+p)\phi\\
&= P_{KP}[p(y_H-y_L)-\phi]^2[(1+p)^2\phi^2+(p(y_H-y_L)-\phi)(1+p)\phi]+\\
&\quad p(y_H^{'}-y_L^{'})\phi[p(y_H-y_L)-\phi]\{p(y_H-y_L)-\phi-(1+p)[(1+r)B-y_L]\}\\
&= P_{KP}[p(y_H-y_L)-\phi]^2[(1+p)^2\phi^2+(p(y_H-y_L)-\phi)(1+p)\phi]+\\
&\quad p(y_H^{'}-y_L^{'})\phi[p(y_H-y_L)-\phi]F
\end{aligned}
$$

Then since $EU_H^{'} - EU_{KP}^{'} = \frac{(1-p)\phi}{[p(y_H-y_L)-\phi]^2[p(y_H-y_L+\phi)]^2}Q$, when $EU_{KP}^{'} = 0$ we know that $P_{KP} > 0$; hence if $F(B^*_{KP}) > 0$ then $EU_H^{'}(B^*_{KP}) - EU_K^{'}(B^*_{KP}) > 0$.

Finally if the optimal investment in KP is between these levels then:

(iii) $F(B^*_{KP}) < 0$ **and** $F'(B^*_{KP}) > 0$

$$\begin{aligned} EU^{'}_H - EU^{'}_{KP} &= (1-p)\phi\{[1+r-y^{'}_L]\frac{\phi(1+p)}{p(y_H-y_L+\phi)[p(y_H-y_L)-\phi} - \\ &\quad \frac{[(1+r)B-y_L](y^{'}_H-y^{'}_L)\phi(1+p)[2p(y_H-y_L)-(1-p)\phi]}{p(y_H-y_L+\phi)^2(p(y_H-y_L)-\phi)^2} \\ &\quad +\frac{\phi(y^{'}_H-y^{'}_L)}{p(y_H-y_L+\phi)^2}\} \\ &= (1-p)\phi^2(1+p)\{\frac{1+r-y^{'}_L}{p(y_H-y_L+\phi)[p(y_H-y_L)-\phi} - \\ &\quad \frac{[(1+r)B-y_L](y^{'}_H-y^{'}_L)[2p(y_H-y_L)-(1-p)\phi]}{p(y_H-y_L+\phi)^2(p(y_H-y_L)-\phi)^2} \\ &\quad +\frac{(y^{'}_H-y^{'}_L)}{(1+p)p(y_H-y_L+\phi)^2}\} \\ &= \frac{(1-p)\phi^2}{p(y_H-y_L+\phi)^2(p(y_H-y_L)-\phi)^2(1+p)}\{ \\ &\quad [1+r-y^{'}_L](1+p)(y_H-y_L+\phi)(p(y_H-y_L)-\phi) \\ &\quad -(1+p)[(1+r)B-y_L](y^{'}_H-y^{'}_L)[2p(y_H-y_L)-(1-p)\phi] \\ &\quad +(y^{'}_H-y^{'}_L)p(y_H-y_L)-\phi)^2\} \end{aligned} \tag{A.79}$$

But:

$$F = (1+p)(1+r)B - py_H - y_L + \phi$$

So:

$$F' = (1+p)(1+r) - py^{'}_H - y^{'}_L$$

$$[(1+r)-y^{'}_L](1+p) = F^{'} + p(y^{'}_H - y^{'}_L) \tag{A.80}$$

$$
\begin{aligned}
EU_H^{'} - EU_{KP}^{'} &= \frac{(1-p)\phi^2}{p(y_H - y_L + \phi)^2 (p(y_H - y_L) - \phi)^2 (1+p)} \{ \\
&\quad (y_H - y_L + \phi)(p(y_H - y_L) - \phi)(F^{'} + p(y_H^{'} - y_L^{'})) \\
&\quad -(1+p)[(1+r)B - y_L](y_H^{'} - y_L^{'})[2p(y_H - y_L) - (1-p)\phi] \\
&\quad +(y_H^{'} - y_L^{'})(p(y_H - y_L) - \phi)^2 \} \\
&= \frac{(1-p)\phi^2 (y_H^{'} - y_L^{'})}{p(y_H - y_L + \phi)^2 (p(y_H - y_L) - \phi)^2 (1+p)} \{ \\
&\quad p(y_H - y_L + \phi)(p(y_H - y_L) - \phi) + (p(y_H - y_L) - \phi)^2 \\
&\quad -(1+p)[(1+r)B - y_L][2p(y_H - y_L) - (1-p)\phi] \} \\
&\quad + \frac{(1-p)\phi^2 F^{'}}{p(y_H - y_L + \phi)(p(y_H - y_L) - \phi)(1+p)} \\
&= \frac{(1-p)\phi^2 (y_H^{'} - y_L^{'})}{p(y_H - y_L + \phi)^2 (p(y_H - y_L) - \phi)^2 (1+p)} \{ \\
&\quad (p(y_H - y_L) - \phi)[2p(y_H - y_L) - (1-p)\phi] - \\
&\quad -(1+p)[(1+r)B - y_L][2p(y_H - y_L) - (1-p)\phi] \} \\
&\quad + \frac{(1-p)\phi^2 F^{'}}{p(y_H - y_L + \phi)(p(y_H - y_L) - \phi)(1+p)} \\
&= \frac{(1-p)\phi^2 (y_H^{'} - y_L^{'})[2p(y_H - y_L) - (1-p)\phi]}{p(y_H - y_L + \phi)^2 (p(y_H - y_L) - \phi)^2 (1+p)} \{-F\} \\
&\quad + \frac{(1-p)\phi^2 F^{'}}{p(y_H - y_L + \phi)(p(y_H - y_L) - \phi)(1+p)} \\
&= \frac{(1-p)\phi^2}{p(y_H - y_L + \phi)(p(y_H - y_L) - \phi)(1+p)} \\
&\quad * \{ F^{'} - \frac{(y_H^{'} - y_L^{'})[2p(y_H - y_L) - (1-p)\phi]}{(y_H - y_L + \phi)(p(y_H - y_L) - \phi)} F \}
\end{aligned} \tag{A.81}
$$

In this case $F < 0$ and $F' > 0$; hence $EU_H^{'} - EU_K^{'} > 0$.

A.6 There is less monitoring in KP than in H

Now $m_{KP}^{'} > 0$ for $B > B_{KP}^{*}$.

We have:

$$m'_{KP} = \frac{(1+r-y'_L)(y_H-y_L-\phi)}{(p(y_H-y_L)-\phi)(y_H-y_L)} + \frac{((1+r)B-y_L)(y'_H-y'_L)}{(p(y_H-y_L)-\phi)(y_H-y_L)} - \frac{((1+r)B-y_L)(y_H-y_L-\phi)p(y'_H-y'_L)}{(p(y_H-y_L)-\phi)^2(y_H-y_L)} - \frac{((1+r)B-y_L)(y_H-y_L-\phi)(y'_H-y'_L)}{(p(y_H-y_L)-\phi)(y_H-y_L)^2} \tag{A.82}$$

$$m'_{KP} = \frac{(1+r-y'_L)(y_H-y_L-\phi)}{(p(y_H-y_L)-\phi)(y_H-y_L)} - \frac{((1+r)B-y_L)(y'_H-y'_L)[p(y_H-y_L)^2-\phi\{2p(y_H-y_L)-\phi\}]}{[p(y_H-y_L)-\phi]^2[y_h-y_L]^2} \tag{A.83}$$

On the other hand using $Ey' = p(y'_h - y'_L) + y'_L$:

$$EU'_{KP} = \frac{p(y'_H-y'_L)}{[p(y_H-y_L)-\phi]^2}[p(y_H-y_L)^2-\phi\{2p(y_H-y_L)-\phi\} + (1-p)\phi\{(1+r)B-y_L\}] - \frac{[(1+r)-y'_L]}{p(y_H-y_L)-\phi}p[y_H-y_L-\phi] \tag{A.84}$$

Substituting the second into the first:

$$m'_{KP} = -\frac{EU'_K}{p(y_H-y_L)} + \frac{(y'_H-y'_L)[p(y_H-y_L)^2-\phi\{2p(y_H-y_L)-\phi\}][y_H-(1+r)B]}{[p(y_H-y_L)-\phi]^2[y_H-y_L]^2} + \frac{(y'_H-y'_L)(1-p)\phi((1+r)B-y_L)}{[p(y_H-y_L)-\phi]^2[y_H-y_L]} \tag{A.85}$$

Now since $EU'_{KP}(B^*_K) = 0$ and B^*_{KP} is a maximum then for $B > B^*_{KP}$, $EU'_K(B) < 0$. The other two terms in m'_{KP} are positive so long as $p(y_H-y_L)^2-\phi\{2p(y_H-y_L)-\phi\} > 0$. But this expression is:

$$[p(y_H-y_L)-\phi]^2 + p(1-p)(y_H-y_L)^2 > 0 \tag{A.86}$$

Since $B^*_{KP} < B^*_H$ we have $m^{KP}(B^*_{KP}) < m^{KP}(B^*_H) < m^H(B^*_H)$ as $m^{KP} < m^H$ in the fixed B case (see section 2.3).

A.7 Proof of Proposition 4

(i) $F'(B_{SR}, \phi^*) < 0$

Here we know that $B_{H2}(\phi^*) = B_{SR} = B_{KP}(\phi^*) < B_{H1}(\phi^*)$ and that $B_{SR} < B_1$. If $\phi < \phi^*$ then $B_{H2}(\phi) < B_{H2}(\phi^*)$ and $B_{KP}(\phi) < B_{KP}(\phi^*)$. But $F(B_{KP}(\phi), \phi) = (1+p)((1+r)B_{KP}(\phi) - y_L) < 0$ since $B_{KP}(\phi) < B_{SR}$. As $F'(B_{SR}, \phi) < 0$ and $F(.)$ is convex in B it follows that $B_{H2}(\phi) < B_{KP}(\phi)$. So for $\phi < \phi^*$ we have $B_{H2}(\phi) < B_{KP}(\phi) < B_{SR} < B_1$.

If $\phi > \phi^*$ then by an analogous argument we have $B_{H1}(\phi) > B_{H2}(\phi) > B_{KP}(\phi) > B_{SR}$ and again $B_1 > B_{H2}(\phi)$.

As ϕ continues to increase above ϕ^* the two roots of $F(.)$ converge at ϕ_f. At a suitably high ϕ_f, $B_{H1}(\phi_f) = B_{H2}(\phi_f)$ and for $\phi > \phi_f$ there are no roots to $F(B, \phi) = 0$.

(ii) $F'(B_s, \phi^*) > 0$

If $\phi < \phi^*$ then $B_K(\phi) < B_{H2} < B_{SR} < B_{H1}(\phi)$; the first of these follows since $F(B_{H2}, \phi) = 0$ whilst since $B_{KP} < B_{SR}$ we know that $F(B_{KP}, \phi) = (1+p)((1+r)B_{KP} - y_L) > 0$ but because $F(.)$ is linear in ϕ, $F'(B_{KP}, \phi) > 0$. The second follows since $B_{KP} < B_{H2}$ implies that $p(y_H(B_{H2}) - y_L(B_{H2})) > \phi$ which implies (since $F(B_{H2}, \phi) = 0$), $(1+r)B_{H2} - y_L > 0$ meaning that $B_{H2} < B_{SR}$.

If $\phi > \phi^*$ then $B_{H2}(\phi) > B_{H1}(\phi) < B_{SR} < B_{KP}(\phi) < B_1$ At a suitably high ϕ_f, $B_{H1}(\phi_f) = B_{H2}(\phi_f)$ and for $\phi > \phi_f$ there are again no roots to $F(B, \phi) = 0$.

We can put together the above knowledge of the regions in which each contract form is feasible with the location of B_H^* and B_{KP}^* to derive:

A.8 Proof of Proposition 5

Recall the expressions for the expected marginal utility:

$$EU_S' = Ey' - (1+r) \tag{87}$$

$$
\begin{aligned}
EU'_{KP} &= Ey' - (1+r) - \frac{(1-p)\phi[(1+r) - y'_L]}{p(y_H - y_L) - \phi} + \frac{(1-p)\phi[(1+r)B - y_L]p[y'_H - y'_L]}{[p(y_H - y_L) - \phi]^2} \\
&= Ey' - (1+r) - \frac{(1-p)\phi[(1+r) - y'_L]}{p(y_H - y_L) - \phi} \qquad \text{(A.88)}
\end{aligned}
$$

$$
EU'_H = Ey' - (1+r) - \frac{(1-p)\phi[(1+r) - y'_L]}{p(y_H - y_L + \phi)} + \frac{(1-p)\phi[(1+r)B - y_L + \phi][y'_H - y'_L]}{p[y_H - y_L + \phi]^2} \qquad (89)
$$

Case (a): $F'(B_{SR}, \phi^*) < 0$ and $\phi_f > \phi > \phi^*$

If $B_{H1} > B_1$ then $EU'_H(B_{H1}) < 0$ and $EU'_{KP}(B_{H1}) < 0$; and we also know that $EU'_H(B_{H2}) > 0$. Hence the optimal contract cannot be H. Since $B_{KP} > B_{SR}$, the globally optimal contract involves a comparison between Sr contract and KP.

If $B_{H1} < B_1$ the outcome depends on the signs of $EU'_H(B_{H1})$ and $EU'_{KP}(B_{H1})$. If $EU'_H(B_{H1}) < 0$ the global optimum cannot be H and so will be the result of a global comparison of KP and SR. If $EU'_{KP}(B_{H1}) > 0$ the global optimum cannot be KP and so will be the result of a global comparison of H and SR. Finally if $EU'_H(B_{H1}) > 0$ and $EU'_{KP}(B_{H1}) < 0$ the global optimum is the result of a comparison of all three contracts SR, H, KP. This covers case (a).

Case (b): $F'(B_{SR}, \phi^*) > 0$ and $\phi_f > \phi > \phi^*$

Here $B_{H1} < B_{KP} < B_1$ and so $B^*_{KP} > B_{H1}$ so the globally optimal contract cannot be KP. Then if $EU'_H(B_{SR}) < 0$ the global optimum is the SR contract; whilst if $EU'_H(B_{SR}) > 0$, the globally optimal form depends on a comparison of H and SR.

Case (c): $\phi^* > \phi > \phi^{**}$

For $\phi > \phi^{**}$ we know $EU'_{KP}(B_{SR}) < 0$ and so $EU'_{KP}(B_{KP}) < 0$. which implies that the solution cannot be KP. On the other hand $EU_{SR}(B_{SR}) > EU_H(B_{SR})$ so that if $EU'_H(B_{SR}) < 0$ the solution cannot be hybrid either and so must be SR. Instead if $EU'_H(B_{SR}) > 0$ a global comparison of the hybrid and SR contract are necessary.

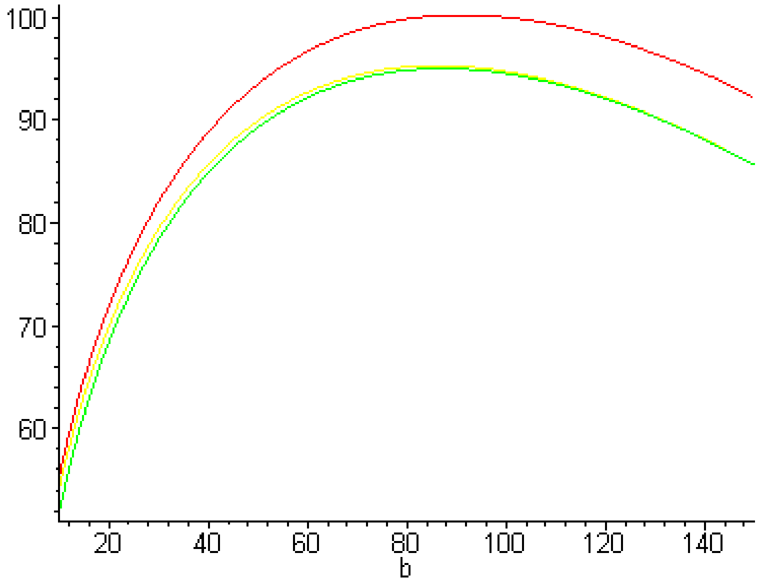
Cases (d)-(g): $\phi < \phi^{**}$

If $\phi < \phi^{**}$ then the outcome depends on whether $B_1 <> B_{H1}$. If $B_1 < B_{H1}$ we know that $EU'_{KP}(B_{H1}) < 0$ and $EU'_H(B_{H1}) < 0$ since there is underinvestment relative to the first best.

From $EU^{'}_{H}(B_{H1}) < 0$ it follows that KP dominates H. But since $\phi < \phi^{**}$ $EU^{'}_{KP}(B_{SR}) > 0$ and so KP dominates the SR contract. If instead $B_1 > B_{H1}$ it is more complex. If $EU^{'}_{KP}(B_{H1}) > 0$ then $B^{*}_{KP} > B_{H1}$ and so H dominates KP. On the other hand if $EU^{'}_{H}(B_{H1}) < 0$ then $B^{*}_{H} < B_{H1}$which implies that KP dominates H. Finally if $EU^{'}_{KP}(B_{H1}) < 0$ but $EU^{'}_{H}(B_{H1}) > 0$ then $B^{*}_{KP} < B_{H1}$ but $B^{*}_{H} > B_{H1}$ so that a global comparison of utilities is necessary.

For $\phi > \phi_f$ the optimum again involves a comparison between the SR contract at B_{SR} and the maximum value of the H contract.

B Graphical Illustrations



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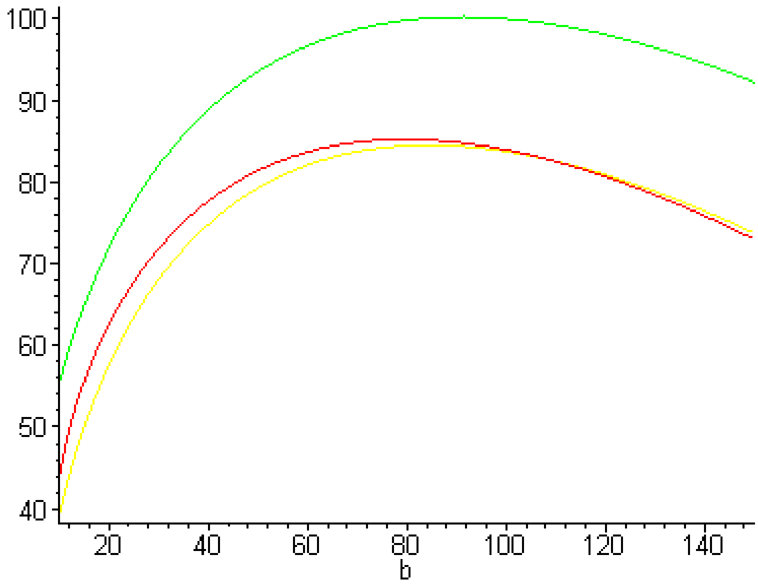

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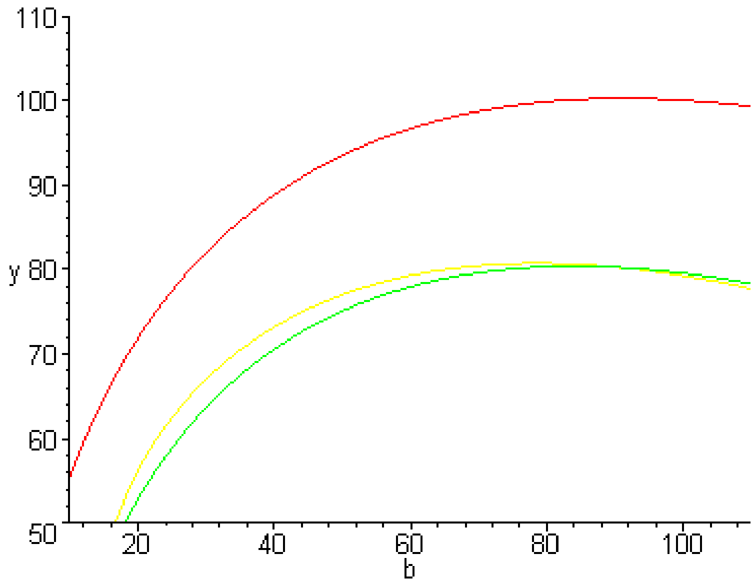
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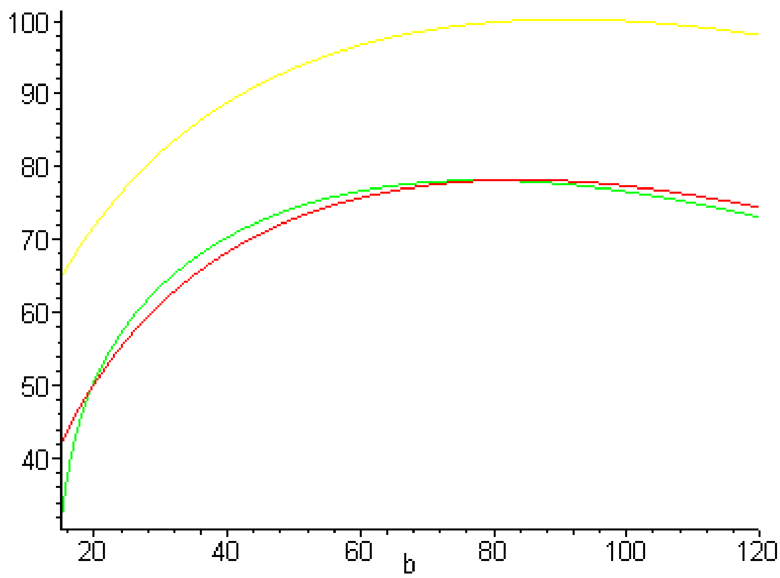
100
90
y 80
70
60
50
20
40
60
80
100
b

phi=60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80
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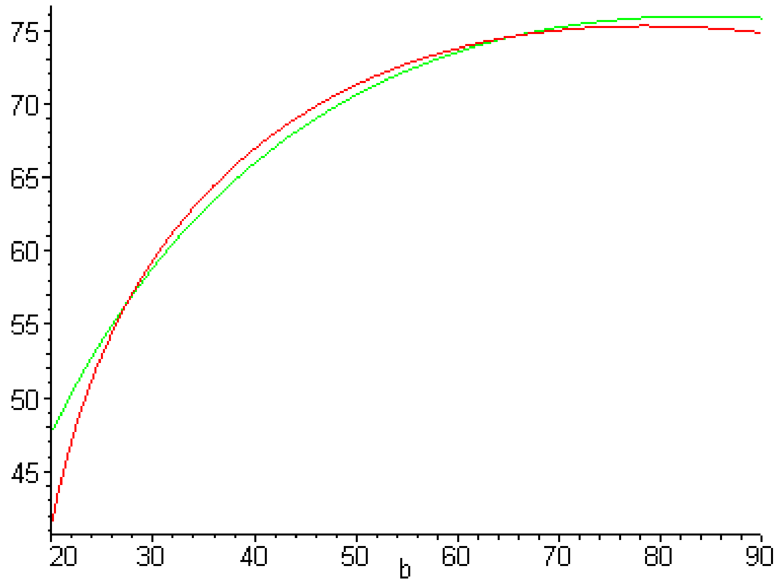
60
50
40
20
40
60
b
80
100
120

phi=65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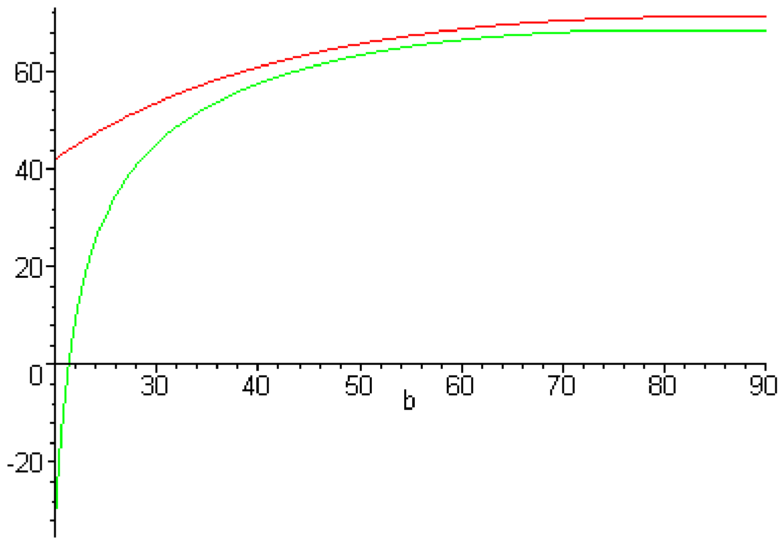
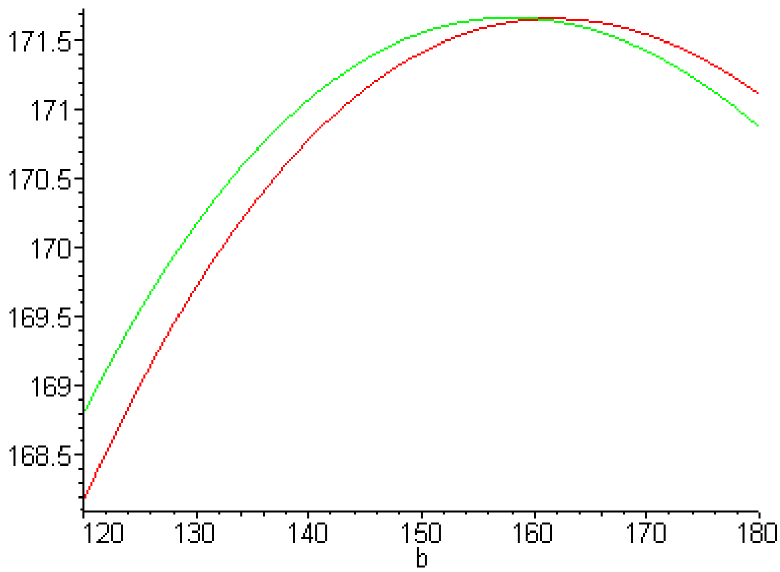
70
65
60
55
50
45
20
30
40
50
b
60
70
80
90

phi=70


60
40
20
0
-20
30
40
50
b
60
70
80
90

phi=80



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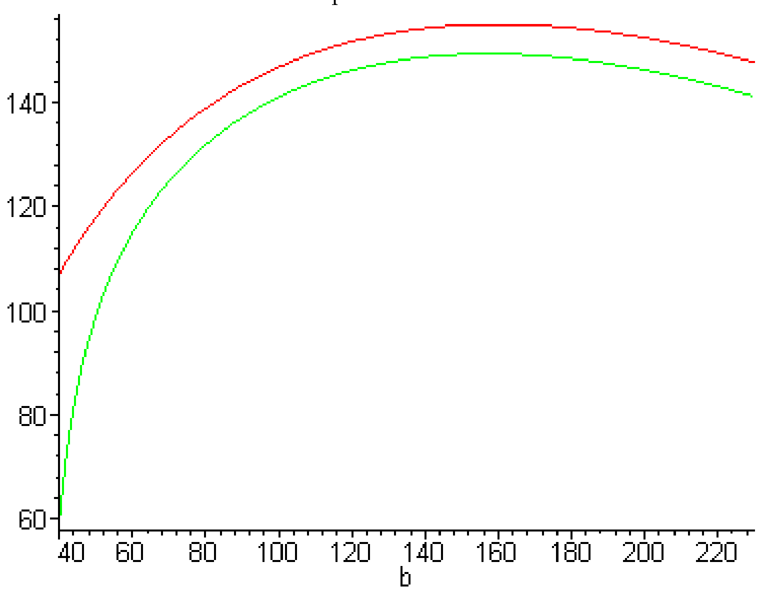
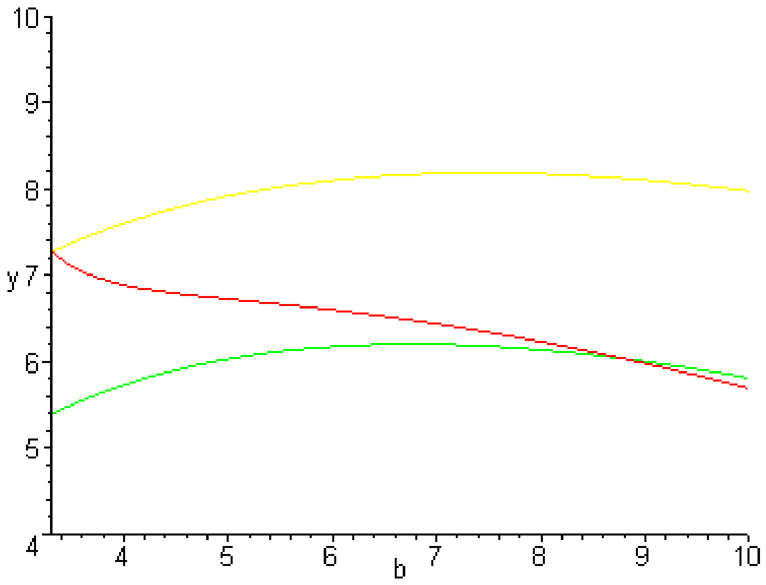


phi=150



$\phi = 6.25$

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