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Endogenous Fertility, Endogenous Growth and Public Pension System:
Should We Switch from a PAYG to a Fully-Funded System?

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Endogenous Fertility, Endogenous Growth and Public Pension System: Should We Switch from a PAYG to a Fully-Funded System?

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Abstract

This paper studies the implications of state pension plan reform on fertility and on growth. It extends the Grossman and Yanagawa (1993) endogenous growth framework by incorporating altruism, making fertility endogenous. We investigate the effect on long-run growth of a switch from a Pay-As-You-Go (PAYG) pension system to a fully-funded system. We show that a PAYG pension system is associated with a lower fertility rate than a fully-funded system. This lower fertility in turn increases the rate of growth. Hence, switching from a PAYG system to a fully-funded system may be harmful, especially for developing countries in which limited resources are heavily stressed by high fertility rates. In addition, we propose a hypothetical pension system, the Saving Subsidy Program (SSP), which would yield a higher growth rate than the PAYG system. The SSP consists of a minimum benefit level for each retiree and of a subsidy to private savings.

Keywords: Endogenous growth, Endogenous fertility, PAYG, Pension plan, Saving subsidy.
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1 Introduction

Most developed countries will face, over the next …fty years, an unprecedented ageing of their population, as the Baby Boom generation grows old and the mortality rates decrease. The most serious problem associated with the current population ageing is the enormous tax burden it imposes on the next generation. Sustaining the existing social security system, which is a Pay-as-You-Go (PAYG) system, requires substantial increases in the payroll tax rates. This tax increase will create a demographic politico-economic con‡ict between the active workforce and the retired [Von Weizsacker (1990)]. A proposal aimed at lessening these tensions is to switch from the current PAYG system to a fully-funded pension system; a closely related proposal is to privatize the social security system [see Altig and Gokhale(1997), Feldstein and Samwick (1998) and Kotliko¤ (1996)]. These authors focus on the tax burden to be accepted if the budget of the PAYG system is to be balanced, and on the work disincentive that it would create. However, there is at least one more factor to take into account when considering the long-run implications of such a profound structural change, namely the e¤ect of social security reform on fertility. We will see that taking explicitly into account fertility issues dramatically alter the assessment of the impact of the proposed reform.

In this paper, we extend the learning-by-doing model of Grossman and Yanagawa (1993) [GY] to allow for endogenous fertility. We show that, in this context, higher fertility leads to lower economic growth. Furthermore, we show that a PAYG system will be characterised by a lower fertility than a fully-funded system. Hence, switching from a PAYG to a fully-funded system might reduce economic growth.

Our basic setup is a three-period overlapping generations economy. Each generation lives three periods: (i) childhood when raised by one's parents, (ii) adulthood in which the individual works, pays taxes, raises children, saves, and provides for its parents because of pure altruism (ascending altruism),[2] (iii) retirement in which the individual lives off its social security benefits, its savings, and the gift provided by its children. This setting immediately provides for an endogenous fertility rationale as adults can think of children as a type of asset, whose initial price is the cost of raising a kid, and whose payoff is the monetary transfer (gift) that the child will provide when it reaches adulthood. Unlike GY, who assumed an exogenous fertility rate, the balanced growth rate in our model will depend not only savings but also on fertility.

Why does higher fertility lead to lower growth? Consider the problem of an adult living at time t. Optimization requires this agent to equate the marginal utility from altruism (the gift to one's parents) a time t to the discounted value of the marginal utility of consumption at $t + 1$. The higher the fertility rate n, the more gifts from the children to their parents, the higher the marginal utility from altruism. On the balanced growth equilibrium path, this agent will get $(1 + g)$ times the consumption as his parents in the current period where g is the rate of growth of the economy. The higher the growth rate, the higher consumption in the next period, the lower the marginal utility. Equating the two marginal utilities implies a negative relationship between n and g.

Why is a PAYG system associated with a lower fertility than a fully-funded system? Let us compare two economies which are identical except that the first one uses a PAYG system and the other a fully-funded system. Let us start this comparison by setting the contribution rate t_s to zero in both economies, then simultaneously increasing this rate. When $t_s = 0$, both economies are essentially in the same situation as if there was no social security system at all, hence their fertility rate (and all other endogenous variables) are identical. When t_s

[2] Voluntary transfer from children to parents commonly happens in Asia countries(Korea, Japan, and China etc) because those countries, steeped in benign Buddhist traditions, depend heavily on the family system to provide the bulk of support for the elderly. Andrews and others (1986), Martin (1988), and Kinsella (1992) argue that there are significant family support for the elderly in Asian countries.

increases in the fully-funded system, the increase in the contribution to the social security system is exactly offset by a decrease in private savings, an instance of the "Ricardian equivalence theorem", see Blanchard and Fisher (1989). The rebalancing of savings between the social security fund and private savings is the only effect of the increase in t_s. Hence, the economy with the fully-funded system experiences no change in its fertility rate. When t_s increases in the PAYG system, parents can count on higher public support in their old age, hence investing in children as a mean of support in old age becomes less attractive. Consequently, increasing t_s will decrease fertility in the PAYG system.

Our results provide a novel theoretical explanation for the empirical findings of Sala-I-Martin (1996) who found that the strongest explanatory variable with a positive effect on economic growth was the amount of public transfers. An other message from our analysis is that a switch from a PAYG system to a fully funded system may be harmful not only to a developed countries suffering from low rates of economic growth, but also to developing countries suffering from overpopulation, an interesting and important fodder for thoughts for most people interested in public finance.

Although a PAYG system might provide growth benefits, it does so by decreasing fertility rather than increasing total savings and capital accumulation. Indeed, low savings rate in the US have long been a source of concern. In recent years, favourable tax treatments of capital income, such as the IRA in the US, were meant to stimulate savings by low-income agents. However, these programs did not offer any incentives for additional savings for most high-income households as they were already saving more than the maximum eligible amount even before the introduction of the IRAs. In fact, there is little empirical evidence that the IRAs stimulated household savings between 1983 and 1986 (Gale and Scholz (1992)). We are proposing a new pension scheme, the Saving Subsidy Program (SSP), which can remedy this drawback. It combines a minimum social security benefit with a subsidy to individual's savings which stimulate savings and enhances the long-run balanced growth rate.

The organization of this paper is as follows. Section 2 sets up the model. In section 3 and 4, we …nd the balanced growth rate and present the concept of open-loop equilibrium . In section 5, we analyse the e¤ects of social security on the balanced growth rate. In section 6, we study the saving subsidy program and its comparative dynamics. Section 7 draws the conclusion.

2 Model

The economy which we are going to describe here is an extension of the endogenous growth model of GY. We assume a discrete time setting, $t = 0, 1, ...$, and a three-period overlapping generations (OLG) framework. All agents in the same generation are identical. In their …rst period of life, children take no economic decision and are cared-for by their adult parents. Upon becoming adults in their second period of life, agents work, decide on how many children to have and raise them, provide a private gift to their own old parents, and save. In their third period of life, old agents retire and live o¤ their savings and o¤ their adult o¤spring' gift. The size of the population N_t follows the dynamics $N_{t+1} = (1 + n_t)N_t$, where n_t is the fertility rate. In each period a single homogenous good is produced. This good can be consumed or stored as capital for next period.

2.1 Utility Function

We follow Nishmura and Zhang [NZ] (1992) characterisation of the agents' preferences with ascending altruism. Agents derive utility from their consumption during their lifetime and from the consumption of their old parents. In our framework, agents take no economic decision in their …rst period of life when they are children. The expenditure on rearing children is exogenous. In this paper, we do not consider decisions about education. Hence, consumption and all other variables relating to childhood are constants. For simplicity and without loss of generality, we will ignore them. We assume that the intertemporal utility function, V_t, takes the additively separable form of

$$V_t = u(c_t^t) + \theta u(c_{t+1}^t) + \tau u(c_t^{t-1}); \quad \theta, \tau > 0; \tag{1}$$

where c_t^t denotes the adult period consumption at time t, c_{t+1}^t the old age consumption at time $t + 1$, θ the discount factor, c_t^{t-1} the parents' consumption at time t and τ is the degree of altruism of children towards their parents. If $\tau = 0$, there is no ascending altruism. In this case, adults will not beget any children as kids are regarded as "capital goods" in our model and we will assume $\tau > 0$ for the reminder of the paper. We assume that the inter-temporal utility function (1) has the following properties.

- **Assumption 1.** The intertemporal utility function is additively separable and homothetic.
- **Assumption 2.** The utility function in each period, u is continuous and twice differentiable, $u' > 0$ and $u'' < 0$.
- **Assumption 3.** The utility function is strictly concave and increasing, defined on the positive orthan $\Re_+^2 \to \Re_+$, and, to avoid corner solutions, we assume that the indifference curves do not touch the axis, i.e. $\lim_{c \to \infty} u'(c) = 0$ and $\lim_{c \to 0} u'(c) = \infty$.

2.2 Budget Constraint

Following NZ's approach, we assume that the total cost of raising children, $h(1 + n_t)$, is an non-decreasing function of the number of children, $1 + n_t$, and is twice differentiable. Specifically, the total child-rearing cost is

$$h(1 + n_t) \equiv a(1 + n_t)^b; \quad a > 0; \quad 1 > b > 0;$$

where a is the exogenous cost of raising a single child (per parent) and b measures the concavity of the child-rearing cost. The condition $1 > b > 0$ means that there are economies of scale in raising children.

We assume a competitive labour market and an inelastic labour supply. An adult earns the wage (W_t) which he uses to consume in that period (c_t^t), to save (S_t), to raise children ($1 + n_t$), and to provide a gift (G_t) to his parents due to their ascending altruism.[3] In the old age, the elderly consume the proceeds from their savings, their social security benefit (b_{t+1}) and the gift from their children. The adult and old age budget constraints under the PAYG pension system are, respectively,

$$c_t^t = W_t(1 - t_s - G_t - a(1 + n_t)^b) - S_t; \tag{2}$$

$$c_{t+1}^t = (1 + r_{t+1})S_t + (1 + n_t)W_{t+1}G_{t+1} + b_{t+1}; \tag{3}$$

where t_s denotes the pension contribution rate and b_{t+1} the social security benefit. We impose a balanced budget constraint on the social security system, i.e.

$$b_{t+1} = (1 + n_t)W_{t+1}t_s:$$

The government adjusts the payroll tax rate to keep the social security system solvent.

Under the fully funded pension system, the budget constraints are

$$c_t^t = W_t(1 - t_s - G_t - a(1 + n_t)^b) - S_t; \tag{4}$$

$$c_{t+1}^t = (1 + r_{t+1})(S_t + W_t t_s) + (1 + n_t)W_{t+1}G_{t+1}: \tag{5}$$

In this case, the social security system budget is automatically balanced.

2.3 Production Function

The single homogenous good serves both as capital and as consumption good. As in GY, this good is produced in a competitive industry under constant returns to scale according to a concave production function. Hence the number of

[3] We exclude the possibility that the adult generation have heterogenous preference toward parents. That is, all adult generations like their parents.

firms is irrelevant and we will assume a single firm for simplicity. The aggregate production function F is

$$Y_t = F[K_t, A_t L_t], \tag{6}$$

where Y_t is aggregate output, K_t is aggregate capital stock, L_t is aggregate labour supply and A_t is the technological spillover, which is an increasing function of the capital per worker K_t/L_t

$$A_t \equiv \frac{K_t}{mL_t}, \tag{7}$$

where m is a scaling productivity parameter: the higher m, the lower labour productivity. (6) implies that there is a positive spillover from the size of the aggregate capital stock to the productivity of workers in individual firms (i.e. Romer type externality). In equilibrium, $L_t = N_t$. Let $k_t = K_t/A_t L_t$ be the capital-per-efficiency-unit ratio and let $f(k_t)$ be the per-efficiency-unit production function

$$f(k_t) \equiv F\left(\frac{K_t}{A_t L_t}, 1\right) \Rightarrow F[K_t, A_t L_t] = A_t L_t f(k_t).$$

The first-order conditions for profit maximization for the capital and the labour markets are

$$1 + r_t = f'(k_t), \tag{8}$$

$$W_t = [f(k_t) - k_t f'(k_t)] A_t, \tag{9}$$

where r_t is the interest rate and W_t is the wage rate. Let $w_t = W_t/A_t$ be the wage-per-efficiency-unit. Since the interest rate r_t is a function of capital per efficiency unit k_t, the implicit function theorem and (8) and (9) imply that the wage-per-efficiency-unit is a function ϕ (yet to be determined) of the interest rate

$$w_t = \frac{W_t}{A_t} = \phi(r_t). \tag{10}$$

By (7), the capital-per-efficiency-unit ratio is a constant over time

$$k_t = \frac{K_t}{A_t L_t} = m.$$

Substituting for k_t into (8), we find that the interest rate is constant over time, $r_t = r$, and so is the wage-per-efficiency-unit (10), $w_t = w$:

$$\forall t;\ 1 + r_t = f'(m) \Rightarrow \forall t;\ r_t = r; \tag{11}$$

$$\forall t;\ w = \frac{W_t}{A_t} = \phi(r). \tag{12}$$

Because the wage-per-efficiency-unit is constant, the wage rate W_t grows at the rate of growth of technological spillover (A_t). Let $\hat{k}_t$ be the capital-per-worker (capital-labour ratio)

$$\hat{k}_t \equiv \frac{K_t}{L_t} = A_t k_t.$$

Because the capital-per-efficiency-unit ratio is constant, the capital per worker ($\hat{k}$) also grows at the rate of growth of technological spillover (A_t). Let g be this common constant growth rate

$$1 + g \equiv \frac{A_{t+1}}{A_t} = \frac{W_{t+1}}{W_t} = \frac{\hat{k}_{t+1}}{\hat{k}_t}. \tag{13}$$

2.4 Capital Market

Physical capital is the only outlet for aggregate savings and accumulation. In the PAYG system, contributions to social security are used to pay for the benefits of the retirees and do not add to the aggregate savings. In the fully-funded system, individual contributions are invested by the social security system into physical capital and are part of aggregate savings. Hence

$$K_{t+1}^{PAYG} = N_t^{PAYG} S_t^{PAYG}, \tag{14}$$

$$K_{t+1}^{Funded} = N_t^{Funded}(S_t^{Funded} + W_t^{Funded} t_s). \tag{15}$$

Capital-per-efficiency-unit k, wage-per-efficiency-unit w, and the interest rate r are the same in both systems. However, savings S_t, fertility n_t, capital stock

K_t and all other endogenous variables may be different from one system to the other. Under both systems, savings S_t are a function S, constant over time, of the wage rate W_t and of the constant interest rate r[4]

$$j = PAYG, Funded; \quad S_t^j = S^j(W_t, r);$$

Since preferences are homothetic

$$\forall \lambda \geq 0; \quad S^j(\lambda W_t, r) = \lambda S^j(W_t, r). \qquad (16)$$

The saving function is an increasing function of both arguments: that is, $\partial S^j / \partial W_t > 0$ and $\partial S^j / \partial r_t > 0$.

3 Balanced Growth Analysis

The aim of this section is to characterise, if it exists, the equilibrium balanced growth path of our economy. As it turns out, the homotheticity of the utility function, assumption 1, is a necessary condition for the existence of a such a balanced growth path. We already know that, at an equilibrium, the wage rate and the capital per worker grow at the same rate as technological spillover. Furthermore, by the homotheticity assumption, consumption will also grow at the same rate ($c_{t+1}^{t+1}/c_t^t = c_{t+2}^{t+1}/c_{t+1}^t = W_{t+1}/W_t = 1 + g$). These considerations prompt us to define the balanced growth path as the path along which all of these variables grow at the same rate. In addition, balanced growth paths are often associated with constant fertility rates, see for instance Yip and Zhang (1996) or Blackburn and Cipriani (1998).

Definition 1 *The balanced growth path is an equilibrium path such that (i) per-capita consumption for the adult, and old age periods, technological spillovers, wage rate and capital-per-worker all grow at the same constant rate g: $g_{c_t^t} = g_{c_{t+1}^t} = g_A = g_W = g_k = g$; (ii) the rate of population growth is constant over time (denoted by n).*

[4] See Azariadis (1993, pp228 and excercise II-29)

As in GY, we are going to show that the growth rate of the economy g is inversely proportional to the rate of population growth n on a balanced growth path. However, unlike GY, for which the fertility rate was exogenous, n is an endogenous variable in our model. In section 5, we will study how the choice of the pension system affects n.

The balanced growth rate is given by

$$1+g=\frac{K_{t+1}}{K_t}\frac{N_t}{N_{t+1}}. \tag{17}$$

Substituting (14) and (15) into (17) yields, respectively,

$$1+g^{PAYG} = \frac{S^{PAYG}(W_t^{PAYG};r)}{(1+n^{PAYG})A_t^{PAYG}m}, \tag{18}$$

$$1+g^{Funded} = \frac{S^{Funded}(W_t^{Funded};r)+W_t^{Funded}t_s}{(1+n^{Funded})A_t^{Funded}m}. \tag{19}$$

Note that, although savings-per-worker $S^j\left(W_t^j;r\right)$ is growing at the common rate g, savings-per-efficiency-unit $S^j(w;r)$ is constant over time, thanks to the homotheticity of preferences (16)

$$\text{for } j=PAYG, Funded; \quad \frac{S^j(W_t^j;r)}{A_t^j}=S^j\left(\frac{W_t^j}{A_t^j};r\right)=S^j(w;r).$$

Also note that w and r are the same for both pension systems. Using this fact, we can rewrite (18) and (19) as

$$1+g^{PAYG} = \frac{S^{PAYG}(w;r)}{(1+n^{PAYG})m} \tag{20}$$

$$1+g^{Funded} = \frac{S^{Funded}(w;r)+wt_s}{(1+n^{Funded})m} \tag{21}$$

Both (20) and (21) imply that the balanced growth rate increases with savings, which is a standard result of the endogenous growth literature. In section 5, see (28), we will show that in fact on the balanced growth path, aggregate savings are the same under both systems: $S^{PAYG}(w;r)=S^{Funded}(w;r)+wt_s$. Hence, the regime of social security will only affect the balanced growth rate through its effect on fertility.

Before considering the impact of social security on economic growth, we first need to refine our concept of equilibrium by introducing the notion of open-loop equilibrium.

4 Open-Loop Equilibrium

The original papers on ascending altruism, NZ and O'Connell and Zeldes (1993), used a static concept of Nash equilibrium. Fudenberg and Tirole (1996) have argued that this static concept is inappropriate in the context of a dynamic OLG model. We are adopting their concept of "open-loop equilibrium" which deals with this deficiency.

In the overlapping generation model, the "Nash game", i.e. a game in which all moves are simultaneous, is inappropriate because there is only one active player in each period: the adult generation who does not face any opponent. The proper setting is a multi-stage game whose equilibrium concept has been labelled the "open-loop equilibrium". In the open-loop model, agents must precommit to an entire time path of actions without observing neither the past nor the future moves of other players. That is, each adult agent regards the moves of its parents and children as given.

4.1 Construction of Game

At time t, the only players are the adult generation which is composed of identical agents. We concentrate on the problem of the representative agent. At time t, given the actions of the other agents (past and future generations), the representative agent takes an action concerning, simultaneously, his savings, his fertility, his gift to his parents, and his consumption when adult and old, $a_t = \left(S_t; 1 + n_t; G_t; c_t^t; c_{t+1}^t\right) \in X_t = \Re_+^5$, where X_t is the set of all possible actions and $\Re_+$ is the positive half-line.

In the open-loop game, the representative agent t will maximize his utility function subject to the budget constraints, taking a_{t-1} and a_{t+1}, and the government pension policy (P) as given. The pension policy P consists of the choice of a pension system, PAYG or fully-funded, and of the contribution rate and benefit. When this maximization problem admits a solution, then, for each t, there exists a sequence of actions a_t^* such that the utility generated by a_t^* is higher than the utility generated by any alternative admissible action $\tilde{a}_t$. Thus we postulate the following definition for the open-loop equilibrium.

Definition 2 Let us consider a sequence of actions for all generations: $a \equiv \left\{ a_t = \left(S_t; 1+n_t; G_t; c_t^t; c_{t+1}^t \right) \right\}_{t=0}^{\infty}$. A sequence of actions $a^* \equiv \{a_t^*\}_{t=0}^{\infty}$ is an open-loop equilibrium if, and only if,

$$\forall t,\ a_t^* \in \underset{\{S_t; G_t; 1+n_t; c_t^t; c_{t+1}^t\}}{\arg\max} V_t(S_t; 1+n_t; G_t; c_t^t; c_{t+1}^t \mid a_{t-1}^*; a_{t+1}^*, \text{ and } P);$$

where a_{t-1}^* and a_{t+1}^* are actions of generation $t-1$ and $t+1$, respectively and P denotes the government's pension policy.

4.2 Consumer optimum when the utility is logarithmic

As an example, suppose that the instantaneous utility function, u, is logarithmic. The maximization problem under the PAYG system is

$$\left\{ S_t; 1+n_t; G_t; c_t^t; c_{t+1}^t \right\} \in \arg\max \left\{ \ln c_t^t + \theta \ln c_{t+1}^t + \tau \ln c_t^{t-1} \right\} \quad \text{st: (2) and (3)}:$$

Under the fully-funded system, the maximization problem is identical, except that the budget constraints (2) and (3) are replaced by (4) and (5). In both case, after substituting the current and future consumption into the objective function, the first-order conditions can be written explicitly as

$$\frac{1}{c_t^t} = \frac{\theta(1+r)}{c_{t+1}^t}; \tag{22}$$

$$\frac{1}{c_t^t} = \frac{\tau(1+n_{t-1})}{c_t^{t-1}}; \tag{23}$$

$$\frac{aW_t(1+n_t)^{b-1}}{c_t^t} = \frac{\theta W_{t+1} G_{t+1}}{c_{t+1}^t}: \tag{24}$$

An adult today can increase his current consumption by decreasing his savings (with adverse consequences for his consumption when old), by decreasing his gift to his parents (which hurts him because of his ascending altruism) or by decreasing his expenditure on child-rearing (with adverse consequences on the total gift he will be receiving from his children when old). At an optimum, the marginal utility from adult consumption must be equal to (according to the first equation) the discounted marginal utility of the corresponding increase in consumption when old, to (according to the second equation) the marginal utility of the corresponding increase in the gift to one's parents. The third equation states that the marginal utility lost through child-rearing must equal the discounted marginal benefit received through the increased gift from his children.

5 Balanced Growth Fertility under Each System

Many authors in the public finance literature have reported that population ageing threatens to unbalance the budget of the pension plan system. These authors, notably Feldstein, have argued that attempts at balancing the pension plan budget by increasing payroll tax rate would reduce the working-age population's incentive to save. Capital accumulation would be reduced and the growth rate would fall. Instead, they have suggested that the PAYG system should be replaced by a fully-funded or privatised pension system. However, this line of research has, so far, ignored the effect of such a switch on fertility rates, which in turn would have implications for long-run growth. The purpose of this section is to investigate this question.

5.1 Impact of Social Security on fertility

From the first order conditions (22) and (23), we find that the balanced growth rate is negatively related with fertility rate and that this relationship is the same for both pension system

$$j = PAYG, Funded; \quad (1+n^j)(1+g^j) = \frac{(1+r)\theta}{\tau}. \tag{25}$$

Substituting (25) in (20) and (21), respectively, we obtain

$$S^{PAYG}(w,r) = \frac{(1+r)m\theta}{\tau}, \tag{26}$$

$$S^{Funded}(w,r) = \frac{(1+r)m\theta}{\tau} - wt_s. \tag{27}$$

Under both regimes, an increase of altruism decreases savings-per-efficiency units because it leads to a higher gift. (26) indicates that savings-per-efficiency units are independent of the contribution rate t_s. This effect is very similar to Barro's "Ricardian equivalence theorem" (Barro (1974)), except that the public "undoes" the government taxation policy by adjusting their gift to their parents rather than by adjusting their savings. Suppose the government increases the contribution rate t_s, thereby increasing the benefits of the retirees. The adult generation experiences a loss of after-tax income, but they also notice that their parents are receiving a higher entitlement. Consider the case in which the adults decrease their gift to their parents one-for-one relative to their higher payroll contribution. Their income after payroll tax and gift to the elderly is the same as before; the retirees are also getting the same net income as before, more from social security but less from their children. Hence, this allocation is identical to the one prior to the increase in t_s and is an optimum. By contrast, an increase of the payroll tax in the funded pension system is exactly offset by a decrease in savings-per-efficiency-unit. Combining (26) and (27), we verify that, as stated in section 3,

$$S^{PAYG}(w,r) = S^{Funded}(w,r) + wt_s. \tag{28}$$

Next, we derive fertility in each case. For the reminder of the paper, we assume $\theta = 1$. First, we derive the relationship between gift and fertility. Assuming, combining (22) and (24) yields for both pension systems

$$j = PAYG; Funded; \quad G^j_{t+1} = ab\left(\frac{W^j_t}{W^j_{t+1}}\right)(1+r)(1+n^j)^{b-1}. \tag{29}$$

In the case of the PAYG [resp. fully-funded] system, substituting(2) and (3) [resp. (4) and (5)] into (22) yields, respectively

$$\begin{aligned} 2S^{PAYG}(w;r) &= W^{PAYG}_t\left[1 - t_s - a(1+n^{PAYG})^b - G^{PAYG}_t\right] \\ &\quad - \frac{W^{PAYG}_{t+1}G^{PAYG}_{t+1}(1+n^{PAYG})}{1+r} - \frac{b_{t+1}}{1+r}; \end{aligned} \tag{30}$$

$$\begin{aligned} 2(S^{Funded}(w;r) + W^{Funded}_t t_s) &= W^{Funded}_t\left[1 - a(1+n^{Funded})^b - G^{Funded}_t\right] \\ &\quad - \frac{W^{Funded}_{t+1}G^{Funded}_{t+1}(1+n^{Funded})}{1+r}. \end{aligned} \tag{31}$$

Note that $b_{t+1} = (1+n^{PAYG})W^{PAYG}_{t+1}t_s$. Dividing (30) and (31) by A^j_t and plugging (29) into (30) and (31), we find, respectively

$$1 + n^{PAYG} = \left[\frac{w\left(1 - t_s(1+\frac{1}{\tau})\right) - 2S^{PAYG}(w;r)}{wa(1+b(1+\tau))}\right]^{\frac{1}{b}}; \tag{32}$$

$$1 + n^{Funded} = \left(\frac{w - 2(S^{Funded}(w;r) + wt_s)}{wa(1+b(1+\tau))}\right)^{\frac{1}{b}}. \tag{33}$$

Substituting (28) into (33) yields

$$1 + n^{Funded} = \left(\frac{w - 2S^{PAYG}(w;r)}{wa(1+b(1+\tau))}\right)^{\frac{1}{b}}. \tag{34}$$

Recall that S^{Funded} may depend on the contribution rate t_s, but that S^{PAYG} does not. (32) implies that, under a PAYG system, fertility decreases with the contribution rate. By contrast, (34) shows that, under a fully-funded system, the contribution rate does not affect fertility. This result is summarised in the next proposition.

Proposition 1 Assume $\mu = 1$ (no discounting of future consumption). In a PAYG pension system, an increase in the social security payroll tax decreases fertility. In a fully-funded pension system, the payroll tax has no effect on fertility.

In a PAYG system, an increase in the payroll tax (and in its corresponding benefits) replaces, for the retirees, private support from their children with public support. As children become less useful to parents, adults will reduce their fertility. Under the funded pension system, social security does not affect fertility because the payroll tax only affects private savings and no other endogenous variables: private savings are exactly offset by the payroll tax.

Note that, if $t_s > 0$, nominator in (32) is less than the one in (34). Therefore, fertility in the funded pension system is greater than one in the PAYG pension system. This result is summarised in the next proposition.

Proposition 2 Assume $\theta = 1$ and $t_s > 0$. The PAYG pension system is characterised by a higher fertility rate than the funded pension system.

To understand this second proposition, let us first consider both pension systems when the payroll tax $t_s = 0$. It is clear that, in this case, the economic allocation is the same as if there were no pension system at all. An immediate consequence is that the fertility rate will also be the same, as can be checked from (32) and (34)

$$n^{Funded}(t_s = 0) = n^{PAYG}(t_s = 0) = \left(\frac{w - 2S^{PAYG}(w; r)}{wa(1 + b(1 + \tau))} \right)^{\frac{1}{b}} - 1.$$

Let the payroll tax increase. From proposition 1, we know that $n^{Funded}(t_s)$ does not change, and that $n^{PAYG}(t_s)$ decreases, hence the proposition.

A consequence of proposition 2 and of the fact that a higher fertility is associated with a lower growth rate is that a fully-funded pension system will under-perform a PAYG system in terms of economic growth, as stated in the next proposition.

Proposition 3 Assume $\theta = 1$. A PAYG pension system generates a higher balanced growth rate than a fully-funded pension system. The rate of growth under the PAYG system increases with the payroll tax t_s.

Proof. We know from (28) that the savings-per-efficiency-unit (including contributions to the social security system in the case of the fully-funded system) are the same under both regimes. Hence, the numerators in (20) and (21), the equations characterising the balance growth rates, are equal. Therefore, whether the growth rate of the PAYG system is higher or lower than the growth rate of the fully-funded system depends solely on whether the fertility rate is lower or higher in the PAYG system. Since we know from proposition 2 that the former is true, first part of the proposition is established. The second part is a direct consequence of the relationship between fertility and growth (20). Q.E.D. ■

The engine of growth in our model is the technological spillover. A lower fertility rate implies a higher rate of growth of our technological spillover, as can be seen from our production function (7). Since a PAYG system reduces fertility, proposition 2, it outperforms a fully-funded system in terms of growth.

5.2 Empirical evidence

Our result is very much in line and can provide an explanation with the empirical result in Sala-I-Martin (1996). Estimating an equation that explains economic growth rate using initial GDP, public investment, public consumption, and public transfers, he found that the only significant variable positively correlated with growth is public transfers. He concluded that social security is conducive to growth. His explanation was that social security buys the elderly out of the labour force, which is conducive to the economic growth because output per capita is higher if the elderly do not work.

Our results suggests an alternative and complimentary explanation, and adds a word of caution to his result. In our model, under a PAYG, an increase in the payroll tax t_s will increase the rate of growth of the economy (proposition 3). It also qualifies as an increase in public transfers, hence validates Sala-I-Martin's empirical results. However, we found that an increase in t_s would not have any effect under a fully-funded system. Essentially, in a fully-funded system, pension plan contributions are perfect substitutes for private savings. Although an increase in the payroll tax would be recorded as an increase in public transfers, in practice it has no redistributional effects. Hence, one should be careful when applying Sala-I-Martin's findings to public policy as what appears to be a public transfer may turn out to be something quite different.

6 Saving Subsidy Program

6.1 Description of the SSP

Social Security reform has often meant little more than switching from a PAYG to a fully-funded system. Our analysis suggests that such a switch may actually prove harmful to growth. In this section, we propose a new system for social security with the purpose of enhancing economic growth.

In the two systems considered so far, PAYG and fully-funded, fiscal policy has no impact on savings in the following sense: the savings-per-efficiency-unit (including contribution to social security in the fully-funded system) is unaffected by the payroll tax rate, see (28). The scheme we propose, the Saving Subsidy Program (SSP), is designed to raise the said savings which will deliver higher rates of capital accumulation and of economic growth. The SSP will also reduce fertility rates, which we have seen is helpful for growth in our model.

The SSP programme is a two-part programme: a minimum benefit, $\bar{b}$, to each retiree, and a subsidy to savings. More precisely, if an household saves at time t (when adult) $S^{SSP}(W_t^{SSP}; r)$, under this programme it will receive the benefit b_{t+1}^{SSP} at time t + 1 given by

$$b_{t+1}^{SSP} = \bar{b} + qS^{SSP}(W_t^{SSP}; r); \quad 0 < q < 1: \tag{35}$$

The payroll tax rate is set at a level that will balance the system's budget

$$b_{t+1}^{SSP} = (1 + n^{SSP})W_{t+1}^{SSP} t_s:$$

Substituting this identity into (35) leads to

$$\frac{(1+n^{SSP})W_{t+1}^{SSP}t_s}{A_t^{SSP}} = \frac{\bar{b}}{A_t^{SSP}} + \frac{qS^{SSP}(W_t^{SSP};r)}{A_t^{SSP}} = \frac{\bar{b}}{A_t^{SSP}} + qS^{SSP}(w;r). \quad (36)$$

6.2 Equilibrium

The household maximization problem is (suppressing the superscript SSP for clarity)

$$\left\{S_t; 1+n_t; G_t; c_t^t; c_{t+1}^t\right\} \in \arg\max \left\{\ln c_t^t + \theta \ln c_{t+1}^t + \tau \ln c_t^{t-1}\right\}$$

$$\text{st} \quad : \quad c_t^t = W_t(1 - t_s - G_t - a(1+n_t)^b) - S_t;$$

$$c_{t+1}^t = (1+r)S_t - (1+n_t)W_{t+1}G_{t+1} + b_{t+1}(S_t).$$

The first-order conditions with respect to fertility (23) and gift (24) are identical to those in the PAYG and fully-funded systems. However, the first-order condition with respect to savings (22) is replaced by

$$\frac{1}{c_t^t} = \frac{(1+r+q)\theta}{c_{t+1}^t}. \quad (37)$$

Equation (37) reflects the higher rate of return on savings due to the subsidy. Combining (23) and (37) yields

$$(1+g^{SSP})(1+n^{SSP}) = \frac{(1+r+q)\theta}{\tau}. \quad (38)$$

Capital accumulation is identical to the one in the PAYG system. Thus, the growth rate of capital per worker is given by (20). Savings-per-efficiency-unit in the SSP system is given by

$$S^{SSP}(w;r) = S^{PAYG}(w;r) + \frac{qm\theta}{\tau}. \quad (39)$$

Note that these savings are higher than those in the PAYG and fully-funded systems. Now, plugging $W_{t+1}^{SSP} = W_t^{SSP}(1+g)$ and (38) into (36) results in

$$\frac{wt_s}{\tau} = \frac{\bar{b}}{A_t^{SSP}(1+r+q)} + \frac{qS^{SSP}(w;r)}{1+r+q}. \quad (40)$$

Substituting both periods' budget constraints, (29), and (40) into (37) yields

$$1 + n^{SSP} = \left[\frac{w\left(1 - t_s\left(1 + \frac{1}{\tau}\right)\right) - \left\{ S^{SSP}(w; r)\left(2 + \frac{q}{1+r+q}\right)\right\}}{wa(1 + b(1 + \tau))} \right]^{\frac{1}{b}}. \tag{41}$$

Proposition 4 An increase in the rate of savings subsidy, q, leads to an increase in savings-per-efficiency-unit and a decrease in fertility.

Proof. From (39) and (41), we have

$$\frac{dS^{SSP}(w; r)}{dq} = \frac{m}{\tau} > 0;$$

$$\frac{dn^{SSP}}{dq} = -\frac{1}{b}\left(\frac{1}{1 + n^{SSP}}\right)\left(\frac{S^{SSP}(w; r)(1 + r)}{wa(1 + b(1 + \tau))(1 + r + q)^2}\right) < 0.$$

■

Intuition is simple. As the government increases saving subsidy, the adult generations tend to increase saving by the substitution effect. The higher return on savings means that investment into children is less attractive reducing fertility.

Since the balanced growth rate depends positively on saving and negatively on fertility, we have the following propositions:

Proposition 5 The SSP is growth-enhancing.

Proof. From (20), we know

$$1 + g^{SSP} = \frac{S^{SSP}(w; r)}{(1 + n^{SSP})m}. \tag{42}$$

Then, differentiating (42) with respect to q yields

$$\frac{dg^{SSP}}{dq} = \frac{1}{(1 + n^{SSP})m}\frac{\partial S^{SSP}(w; r)}{\partial q} - \frac{S^{SSP}(w; r)}{(1 + n^{SSP})^2 m}\frac{\partial n^{SSP}}{\partial q} > 0;$$

because of $\frac{\partial S^{SSP}(w;r)}{\partial q} > 0$ and $\frac{\partial n^{SSP}}{\partial q} < 0$. Q.E.D. ■

Proposition 6 $1+g^{SSP} > 1+g^{PAYG} > 1+g^{Funded}$

Proof. According to (39), $S^{SSP} > S^{PAYG}$. (41) can be rewritten as follows.

$$1 + n^{SSP} = \left[\frac{\left(1 + n^{PAYG}\right)^{\beta} - S^{PAYG}(w; r)\frac{\theta}{1+r+\theta}}{wa(1 + \beta(1 + \tau))} \right]^{\frac{1}{\beta}}.$$

This implies $1 + n^{SSP} < 1 + n^{PAYG}$. Therefore, $S^{SSP}(w; r) > S^{PAYG}(w; r)$ and $1 + n^{SSP} < 1 + n^{PAYG}$ yields the first inequality. The second inequality comes from proposition 3. Q.E.D. ■

7 Conclusion

A number of studies have over the years argued that the current social security financed by the PAYG system should be replaced by the fully-funded pension system or the privately managed system in order to alleviate the enormous tax burden on the oncoming young generations and keep the level of benefit intact for the elderly. It is argued that while generations alive during the transition face higher fiscal burdens, privatisation can offer substantial long-run economic gains[see Feldstein and Samwick (1998) and Kotlikoff (1996)].

This paper shows that there are reasons to believe that these proposed reforms may adversely affect the long-run growth prospects. We propose a pension scheme that would offer better growth prospects than current and proposed schemes.

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