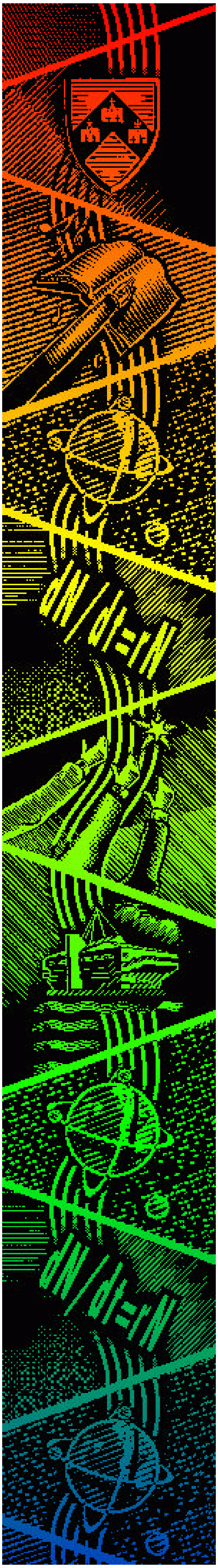

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ω - Homothetic Preferences: Theory and Applications

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ω-Homothetic Preferences: theory and applications.*

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Abstract

This paper develops a new class of homothetic preferences which generate Marshallian demand curves for individual goods which can be concave, convex or linear in own price under the assumption that agents treat aggregate price indices as given (as in Dixit-Stiglitz 1977). The preferences are represented by a cost function which has two parameters: one determining the curvature of the Marshallian demand; the other determining the elasticity of demand when all prices are equal. The elasticity of demand varies with relative prices. Illustrative examples are given of Cournot duopoly and exchange rate pass-through.

Keywords: homothetic, duality, pass-through, Cournot.

JEL Classification: D1, D2

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1 Introduction

This paper introduces a new class of homothetic preferences[1] which generalize *Linear homothetic* (LH) preferences introduced by Datta and Dixon (2000). We adopt the dual approach, specifying a unit cost-function for preferences, which has two parameters ω and γ. The general form of $\omega-$homothetic (ωH) preferences gives rise to a Marshallian demand curve for the individual product i of the form

$$x_i = A - Bp_i^{\omega-1}$$

where $\{A, B\}$ take positive values which depend on preference parameters, income and the aggregate price index. Following Dixit-Stiglitz (1977), we assume *aggregate price-taking* (APT): the aggregate price index is treated as a parameter by individual agents, as is natural in the cases of the monopolistic competition or general equilibrium with many sectors: this means that the individual agent in market i treats the parameters A, B as constants.

The new preferences result in the following useful properties of the Marshallian demand under APT:

- that the shape of the demand curve for an individual product can be chosen to be convex, concave or linear in own price. If $\omega > 2$ the demand curve is *strictly concave* in own price; if $2 > \omega > 1$ the demand curve is *strictly convex*. The special case of $\omega = 2$ (LH preferences) gives rise to linear demand.

- Given ω, a second parameter γ can be chosen to determine the *symmetric elasticity of demand* ε^**:* under APT when all prices are equal the elasticity of demand is equal to $\varepsilon^* = \gamma(\omega - 1)$. Hence we can specify *independently* both the shape and the symmetric elasticity of the Marshallian demand curve.

- There is a *variable elasticity of demand* ε along the Marshallian demand curve: ε is increasing and varies from 0 (when $p_i = 0$) to ∞ (when $x_i = 0$). This property of $\omega-$homothetic demands is particularly useful when

[1] Whilst we talk about preferences in this paper, the cost function can of course also represent a constant returns to scale technology.

we model an economy with two different sectors (e.g. traded and non-traded, unionized and non-unionized), where prices might not be equal in equilibrium).

- The Marshallian demands cut both axes: demand is zero for a finite *choke-off* price and demand is finite at zero price.

- Marginal revenue is strictly decreasing.

All but the second of these properties of ωH demands are fundamentally different from the family of CES and Cobb-Douglas (CD) demands. With CES/CD, we can only have strictly convex Marshallian demands; the elasticity of demand is constant along the demand curve; demands cut neither axis. These differences can have important consequences, as we illustrate in section 3. For example, concavity of demand is often assumed as a sufficient condition for important results, such as Kreps and Scheinkman's (1983) classic paper, or Mirman Samuelson and Schlee (1994) on the value of information and learning. Cutting the axes means that we can naturally restrict output or price to a compact convex interval which is assumed for existence and uniqueness results. Concave revenue (decreasing marginal revenue) is used as a sufficient condition for existence in Cournot oligopoly (Hart 1982). Lastly, in a strategic setting such as Cournot oligopoly, whilst CD preferences are tractable, CES preferences prove almost impossible except for symmetric equilibria.

Homotheticity is a widely used class of preferences in applied general equilibrium models, including trade theory, macroeconomics and CGE. All of the properties of ωH preferences can be obtained with non-homothetic preferences: e.g. quasi-linear demands[2]. However, with ωH preferences we can combine these properties with homotheticity.

The plan of the paper is as follows. In section 2, we outline the cost-function and derive the Marshallian demand and its properties. In section 3 we illustrate the practical use of these properties in applications. First, we consider a model of exchange rate pass through in a modified Obstfeld and Rogoff (1995) setting, where there are two sectors: industries which are foreign and industries which are domestic. This example is particularly useful in illustrating the variable

[2] For example, by specifying preferences for two goods x and y as quasi-linear $x+u(y)$, with the further assumption that $u(y) = y - y^2$. See also Singh and Vives (1984) for an explicit derivation of linear demand from quadratic preferences.

elasticity effect. Second, we consider Cournot duopoly, where each industry is the same but the within-industry equilibrium may be asymmetric. We show that with ωH preferences outputs are often strategic substitutes and that we can obtain simple explicit solutions for asymmetric equilibria with LH preferences.

2 The Cost Function and Marshallian demand.

There are n products, i=1...n, where n is presumed large. The $n-$vector of prices is $\mathbf{p}$. The expenditure function takes the homothetic form $E(\mathbf{p}, u) = b(\mathbf{p}).u$ where $b : \Re^n_+ \rightarrow \Re_+$ is the unit cost of utility.

The class of $\omega-$homothetic preferences is defined by the following unit cost function

$$b(\mathbf{p}) = [\mu + \gamma(\mu - \pi)]\,\phi \tag{1}$$

with parameters $\omega \geq 1$, $\gamma \geq 0$, and $\phi > 0$ where

$$\mu = \tfrac{1}{n}\textstyle\sum_{i=1}^{n} p_i \quad \pi = \left(\frac{\sum p_i^{\omega}}{n}\right)^{1/\omega}$$

The cost function is a composite of two price indices: the arithmetic mean price μ, and a CES price index which is in effect a measure of the variance of prices from zero. ϕ is a scaling parameter which can be set at $\phi = 1$ unless otherwise specified. Note that if all prices are equal ($p_i = p$) then $b = \mu = \pi = p$; otherwise $b < \mu < \pi$. In the special case where $\omega = 2$, the cost-function takes the special form of Linear Homothetic (LH) preferences (Datta and Dixon 2000); in the limiting case where $\gamma = 0$ or $\omega = 1$, then we have Leontief preferences with $b(\mathbf{p}) = \mu$. For the rest of this paper, we will assume that $\gamma > 0$ and $\omega > 1$. We will examine more closely the formal properties of the cost-function in the appendix. For now, we assume that b is a valid cost-function.

From Shepherd's Lemma the budget share of good i is the elasticity of the unit cost function (1) with respect to price:

$$\frac{p_i x_i}{Y} = \frac{p_i}{b} b_i \tag{2}$$

where Y is total expenditure on all commodities, and b_i is the derivative of b with respect to p_i. This can be rearranged to give the Marshallian demand

$$x_i = \left[(1+\gamma) - \gamma\left(\frac{p_i}{\pi}\right)^{\omega-1}\right]\frac{Y}{nb} \tag{3}$$

We now introduce the following assumption in the spirit of the Dixit-Stiglitz *(1977)* model of monopolistic competition:

Assumption: Aggregate Price Taking ($\boldsymbol{APT}$) The aggregate price indices π and b are treated as a parameters by individual agents.

In the context of macroeconomics, the APT assumption is very reasonable: a firm or firms in one market have no influence on the aggregate price level. In a single market, it would be justified if there are many producers as in the monopolistic case. Whilst agents might be price-makers in their own market, they are price-takers when they look at the whole economy. Clearly, in the case of oligopoly or n small, APT may not be appropriate. Note that since $b-(1+\gamma)\mu+\gamma\pi=0$, treating any two of $\{b,\pi,\mu\}$ as given means the third is also given. We will describe $\{\mu,\pi\}$ as price indices and refer to b as the cost-of-living index.

Under the assumption of APT, from the perspective of an individual price-setter, the Marshallian demand (3) has the form

$$x_i = A - Bp_i^{\omega-1}$$

where $A=\frac{(1+\gamma)Y}{nb}$ and $B=\frac{\gamma Y}{nb}\pi^{-(\omega-1)}$. Note that each good has the same *choke-off price* $\bar{p}$ such that $x_i=0$

$$\bar{p}=\pi.\left(\frac{1+\gamma}{\gamma}\right)^{1/(\omega-1)} \tag{4}$$

If $p_i>\bar{p}$, then the implied demands would be negative: it is to avoid this that we develop the restricted cost-function following the methodology of Datta and Dixon (2000) in the appendix. However, if we are considering price-making firms, then in general profit maximizing firms will set a price below the choke-off price $\bar{p}$. Also, when $p_i=0$, demand is bounded by A. Hence both price and quantity can naturally be restricted to compact convex sets, which is required for the use of some existence and uniqueness results. This contrasts with CD/CES where demand cuts neither axis.

Treating π and b as given we have the slope and change in slope of the demand curve

$$\left.\frac{dx_i}{dp_i}\right|_{b,\pi} = -\frac{\gamma(\omega-1)Y}{nb}\frac{p_i^{\omega-2}}{\pi^{\omega-1}} \tag{5}$$

$$\left.\frac{d^2x_i}{dp_i^2}\right|_{b,\pi} = -(\omega-2)\frac{\gamma(\omega-1)Y}{nb}\frac{p_i^{\omega-3}}{\pi^{\omega-1}} \tag{6}$$

From (6), we can see how the value of ω determines the shape of the demand curve in terms of the fundamental concavity/convexity: when $\omega > 2$ demand is strictly concave in own price; when $\omega < 2$ it is strictly convex; when $\omega = 2$ it is linear.

Under APT the elasticity of demand (in absolute value) is

$$\varepsilon_i(\mathbf{p}) = \frac{\gamma(\omega-1)\left(\frac{p_i}{\pi}\right)^{\omega-1}}{1+\gamma-\gamma\left(\frac{p_i}{\pi}\right)^{\omega-1}} \tag{7}$$

In the case of a symmetric equilibrium (i.e. each sector is the same), $p_i = \pi$, so that s*ymmetric elasticity* ε^* is

$$\varepsilon^* = \gamma(\omega-1) \tag{8}$$

This is very useful, it means in effect that for given ω, the one parameter γ can be chosen to determine the symmetric elasticity of demand, much as in Dixit-Stiglitz interpretation of the CES function[3].

A very important property of ωH preferences which differentiates it from CES and CD preferences is that there is a *variable elasticity of demand: the elasticity is increasing in own price (decreasing in other prices).* From equation (7) ε_i is an increasing function of the ratio (p_i/π) and also of p_i

$$\frac{d\varepsilon_i(\mathbf{p})}{dp_i} = \frac{\gamma}{\pi}(1+\gamma(1-\frac{1}{n}))(\omega-1)^2 > 0$$

In particular, the elasticity of demand is 0 when $p_i = 0$ and ∞ when $p_i = \pi\left((1+\gamma)/\gamma\right)^{1/(\omega-1)}$. This of course mirrors the property of linear demand curves and is very useful in modelling equilibrium in sector i, since it means that MR (marginal revenue) is (strictly) decreasing in price[4], even when the demand curve is convex. This is a standard condition for existence and uniqueness in Cournot oligopoly models (see for example (Hart 1982, p.117, fn.7).

Note that if we do not adopt the assumption of APT (for example because n is small), the demand curve is still given by (3), but is less tractable. However,

[3] In view of this, an alternative parametrization of ωH preferences is

$$b = \mu + \frac{\varepsilon^*}{\omega-1}(\mu-\pi)$$

[4] Define $R = p_i x_i = Ap_i - Bp_i^{\omega}$. Then $MR = A - B\omega p_i^{\omega-1}$ and

$$\frac{\partial MR}{dp_i} = -(\omega-1)\omega B p_i^{\omega-2} < 0.$$

one can still compute the elasticity of demand allowing for the effect of p_i on π. In this case, we have the symmetric equilibrium elasticity of demand $\hat{\varepsilon} = \gamma(\omega - 1)(\frac{n-1}{n})$. The elasticity is still variable and increasing in own price. The assumption of APT is useful for tractability only: without it, we can still use ωH preferences in much the same way as we can use CES without it.

In Figure 1 we depict the Marshallian demand curves in $\{p_i, x_i\}$ space under APT for given $\{Y, b, \gamma\}$ as we vary ω

Figure 1.

There are two fixed points that are common to demand curves for all values of ω when $\pi > 0$: demand is equal to $x_i = Y/nb$ when $p_i = \pi$ (point C); $(1+\gamma)Y/nb$ when $p_i = 0$ (point D). For $\omega = 2$ we have a linear demand curve. When $\omega > 2$ the demand curve is strictly concave, being above the linear demand curve to the right of point C, and below to the left of C. When $2 > \omega > 1$ demand is strictly convex, with demand below the linear demand to the right of C, above to the left. The limiting case of $\omega = 1$ yields the vertical Leontief demand.

Figure 2.

Next consider the effect on the demand curve of an increase in total expenditure Y: this results in an anti-clockwise rotation of the Marshallian demand around the choke-off point. Put another way, for any given price p_i the demand increases proportionately with Y as shown in Fig 2a. A change in the general price level is a little more complex since $b - (1+\gamma)\mu + \gamma\pi = 0$: we can choose b and π independently to a limited extent. First, consider a change in b given π. This operates exactly like a decrease in Y: a clockwise rotation in demand around the choke-off point. Second, consider an increase in π holding b constant: In this case there is a clockwise rotation of the demand curve about point D as the choke-off price increases proportionately to π, as shown in Fig 2b. If both b and π change the previous two effects are combined.

Lastly, we consider the love of variety (LOV) or *Ethier effect.* This occurs when an increase in the number of products available causes a potential fall the unit cost of utility: if all goods have the same price, then with LOV unit cost falls with n. In (1), with $\phi = 1$ the cost of utility has been normalized so that if all prices are equal, the unit cost of utility equals this price irrespective of n.

However, LOV can be introduced to the cost function if we make ϕ a function of n, $\phi = \phi(n)$ where $\phi' < 0$. For example, we could have an isoelastic form $\phi = n^\delta$ with $\delta < 0$, as is standard in the treatment of CES preferences (see *e.g.* Benassy 1996).

3 Applications of $\omega-$homothetic preferences.

In this section, we will provide three examples of how it is possible to use ωH preferences in models with strategic behavior. Our aim is to show that our approach results in a demand system which is tractable and has desirable properties. In the special case of LH preferences, we are able to obtain simple analytic solutions. In each case we will contrast this to the properties of CES and CD preferences. Whilst these are not presented as fully specified general equilibrium models, each example can be easily integrated into such a setting.

3.1 Pass-through with price-setting.

The first example is chosen to illustrate the *variable elasticity property* of ωH preferences in situations where prices differ systematically between industries, which fundamentally differentiates it from the CES/CD case. As in Obstfeld and Rogoff (1995), a proportion α of industries are foreign setting a domestic currency price of p_f and $1-\alpha$ are domestic setting price p. Both foreign and domestic outputs are consumed by domestic consumers[5].

Hence we can define the following elasticities in terms of prices, with subscript f for foreign variables, none for domestic variables

$$\varepsilon(p_f, p) = \frac{\gamma(\omega-1)\left(\frac{p}{\pi}\right)^{\omega-1}}{1+\gamma-\gamma\left(\frac{p}{\pi}\right)^{\omega-1}} \quad \varepsilon_f(p_f, p) = \frac{\gamma(\omega-1)\left(\frac{p_f}{\pi}\right)^{\omega-1}}{1+\gamma-\gamma\left(\frac{p_f}{\pi}\right)^{\omega-1}}$$

where $\pi = \left[\alpha p_f^\omega + (1-\alpha)\, p^\omega\right]^{1/\omega}$.

We assume that labor is the only factor of production and there is a constant marginal/average product of labor equal to unity. The domestic firms have a unit cost of w, the foreign firm a unit cost of $w_f = e.\bar{w}_f$, where $\bar{w}_f$ is the foreign currency wage and e the nominal exchange rate. We have the following

[5] In Obstfeld and Rogoff 1995), there are two countries: consumers in each country consume the output of monopolistic firms in both countries. Here we simplify to just one country.

equilibrium conditions for prices of domestic and foreign producers conditional on wages

$$\frac{p_f - w_f}{p_f} = \frac{1}{\varepsilon_f(p_f, p)} \quad \frac{p - w}{p} = \frac{1}{\varepsilon(p, p_f)} \tag{9}$$

To keep the algebra simple, we start from an initial position where all prices are equal, so that $w = w_f$, $p = p_f = \pi$, $\varepsilon_f(p_f, p) = \varepsilon(p, p_f) = \gamma(\omega - 1) > 1$. In this case we have

$$\begin{array}{ll} \frac{d\varepsilon_f}{dp_f} = \frac{\gamma}{\pi}(1 + \gamma(1-\alpha))(\omega-1)^2 > 0 & \frac{d\varepsilon_f}{dp} = -(1-\alpha)(\omega-1)^2\frac{\gamma^2}{\pi} < 0 \\ \frac{d\varepsilon}{dp} = \frac{\gamma}{\pi}(1 + \alpha\gamma)(\omega-1)^2 > 0 & \frac{d\varepsilon}{dp_f} = -\alpha(\omega-1)^2\frac{\gamma^2}{\pi} < 0 \end{array} \tag{10}$$

We now consider the effect of a change in the domestic currency value of the foreign wage, interpreted as a change in the nominal exchange rate (of course, it could just as well be a change in a subsidy, tariff or unit productivity). Hence, from (9) using (10) we obtain the following

$$\begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{(\omega-2)+\gamma(2-\alpha)(\omega-1)}{\gamma(\omega-1)} & (1-\alpha) \\ \alpha & -\frac{(1+\alpha)\gamma(\omega-1)+\omega-2}{\gamma(\omega-1)} \end{bmatrix} \begin{bmatrix} \frac{dp_f}{dw_f} \\ \frac{dp}{dw_f} \end{bmatrix}$$

The determinant of the matrix is

$$\Delta = \frac{2\left(\gamma(\omega-1)\right)^2 + (\omega-2)\left(1 + 3\gamma(\omega-1)\right)}{\left(\gamma(\omega-1)\right)^2}$$

For any γ, $\omega \geq 2$ implies $\Delta > 0$; also, for any $\gamma > 1$, there exists $\bar{\omega} \in (1, 2)$ such that $\omega > \bar{\omega}$ implies $\Delta > 0$[6]. We will restrict our attention to the first case

Proposition 1 *Let* $\omega \geq 2$ *(concavity). Then*
$1 > \frac{dp_f}{dw_f} > \frac{dp}{dw_f} > 0$

Proof.

$$\frac{dp_f}{dw_f} = \frac{(1+\alpha)\left(\gamma(\omega-1)\right)^2 + \gamma(\omega-2)(\omega-1)}{2\left(\gamma(\omega-1)\right)^2 + (\omega-2)\left(1+3\gamma(\omega-1)\right)}$$

$$\frac{dp}{dw_f} = \frac{\alpha\left(\gamma(\omega-1)\right)^2}{2\left(\gamma(\omega-1)\right)^2 + (\omega-2)\left(1+3\gamma(\omega-1)\right)}$$

[6] The numerator of Δ is zero at $\bar{\omega}$ where

$$\bar{\omega} = \frac{\left(4\gamma^2 + 9\gamma - 1 + \sqrt{(17\gamma^2 + 6\gamma + 1)}\right)}{2\left(2\gamma^2 + 3\gamma\right)}$$

For $\omega > \bar{\omega}$, $\Delta > 0$. For example, when $\gamma = 1$, we have $\bar{\omega} = 1.7$, when $\gamma = 3$, $\bar{\omega} = 1.391$.

By inspection, since $\omega \geq 2$, the numerator and denominators are all strictly positive for $\gamma > 0$. comparing the numerators shows that $\frac{dp_f}{dw_f} > \frac{dp}{dw_f}$. Comparing the numerator and denominator shows that $\frac{dp_f}{dw_f} < 1$. ■

In the special case of LH preferences, the expressions simplify further:

$$1 > \frac{dp_f}{dw_f} = \frac{1+\alpha}{2} > \frac{dp}{dw_f} = \frac{\alpha}{2} > 0$$

So, a revaluation leads to an increase in the price charged by foreign firms, which is less than the change in the value of foreign wages. The prices set by domestic firms also rise, but by less. Thus, the price of the foreign firms relative to domestic firms increases.

The explanation is that there is an *elasticity effect.* From (10) as the relative price of p_f rises, the elasticity ε_f rises, reducing the markup of the foreign firm, whilst the elasticity of domestic firms falls, so they are able to increase their prices even though costs are the same. It is simple to compute the expressions for the change in elasticity brought about by the change in the domestic value of foreign labor costs

$$\frac{d\varepsilon_f}{dw_f} = \left(\frac{\gamma}{\pi}\right)\frac{(1+\gamma)}{2}[(1-\alpha] > 0 > -\alpha\left(\frac{\gamma}{\pi}\right)\frac{(1+\gamma)}{2} = \frac{d\varepsilon}{dw_f}$$

The magnitude of these changes in elasticity can be large. Recall that in the initial equilibrium $\pi = \frac{\gamma}{\gamma-1}w_f$. Hence

$$\frac{w_f}{\gamma}\frac{d\varepsilon_f}{dw_f} = (1-\alpha)\left(\frac{\gamma^2-1}{2\gamma}\right) \quad \frac{w_f}{\gamma}\frac{d\varepsilon}{dw_f} = -\alpha\left(\frac{\gamma^2-1}{2\gamma}\right)$$

The terms on the LHS are the elasticities of the elasticities with respect to w_f. Thus if $\alpha = 0.5$ and $\gamma = 3$, a 12% appreciation of the foreign currency will cause an 8% increase in the elasticity of foreign firms, and an 8% decrease in the elasticity of domestic firms.

So, if we interpret the rise in w_f as a appreciation, then we can distinguish three elements of pass-through.

- *The pass-through to domestic prices of foreign firms p_f.* This is less than 100% due to the elasticity effect: higher prices relative to domestic producers mean that elasticities of demand increase, leading to a lower markup.

- *The pass through to domestic producer prices p.* As foreign firms raise their prices, domestic firms find that their demand becomes less elastic, and respond by raising their prices. This is a pure elasticity effect: domestic wages are assumed to be unaffected by the change in foreign wages, and the marginal cost of domestic producers is constant.

- *The pass through to the domestic cost of living b.* Under the assumption of symmetric wages in the initial position we have for LH preferences

$$\frac{db}{dw_f} = \alpha$$

  the combined pass-through is proportional to the share of foreign firms in terms of the number of products (which is also their initial market share).

The purpose of this analysis of pass through has been purely illustrative. We would of course want to go on to analyze further general equilibrium feedbacks in terms of the entry of firms, labor supply and wages. However, we would like to stress the following. First, with CD and CES preferences, there is a 100% pass-through to foreign prices in this model. There would be no immediate pass-through to domestic firms. This all stems form the fact that both CES and CD preferences have a constant elasticity. Secondly, this analysis has been interpreted as a revaluation of the currency: clearly, it could be interpreted as a tariff or subsidy, a change in technology and so on - all we require is that w_f is the unit cost of output of the foreign firms.

3.2 Cournot Duopoly: strategic substitutes and asymmetric equilibria.

In the first example, we took the case where there was only one firm in each industry, but industries were different due to differences in firm type (i.e. foreign and domestic). In the next examples, we will look at an economy in which each industry is the same and has two Cournot firms, but where *within the industry* the equilibrium may be asymmetric when firms differ. We will show how the strategic properties arising from ωH preferences are much more standard than with CD/CES and asymmetric equilibria are much more tractable.

With ωH preferences, the inverse demand for the industry takes the following form

$$p_i = (a_0 - a_1 X)^{\frac{1}{\omega - 1}}$$

Where X is total output and the coefficients are

$$a_0 = \frac{1+\gamma}{\gamma}\pi^{\omega-1} \quad ; \quad a_1 = \frac{nb}{\gamma Y}\pi^{\omega-1}$$

Y is total nominal expenditure which for this example we will take as given, although it would be simple to embed in a general equilibrium model. As we shall see, when using ωH preferences, under APT we are able to treat the coefficients $\{a_0, a_1\}$ as given. We can then substitute in the macroeconomic variables to obtain the solution.

Each firm uses a unit of labor to produce a unit of output, with wage w. Hence the payoff function for firm 1 is

$$\pi_1(x_1, x_2) = x_1 \left(a_0 - a_1(x_1 + x_2)\right)^{\frac{1}{\omega-1}} - w x_1$$

and analogously for firm 2. Hence we can say that

Proposition 2 *(a) For $\omega \geq 2$ (concave demand), outputs are strategic substitutes.*

(b) For $1 < \omega < 2$ (convex demand), outputs can be strategic complements or substitutes.

Proof. (a) For $\omega \geq 2$

$$\begin{aligned} \frac{\partial \pi_1}{\partial x_1} &= p_i - \frac{x_1 a_1}{\omega - 1}\left(a_0 - a_1\left(x_1 + x_2\right)\right)^{\frac{2-\omega}{\omega-1}} - w \\ \frac{\partial^2 \pi_1}{\partial x_1 \partial x_2} &= \frac{a_1\left(a_0 - a_1\left(x_1 + x_2\right)\right)^{\frac{2-\omega}{\omega-1}}}{\omega - 1}\left[\frac{(2-\omega)}{(\omega-1)}\frac{a_1 x_1}{\left(a_0 - a_1\left(x_1 + x_2\right)\right)} - 1\right] < 0 \end{aligned}$$

(b) For $\omega < 2$, we have

$$sign \ \frac{\partial^2 \pi_1}{\partial x_1 \partial x_2} = sign \ \left[\frac{a_1 x_1}{\left(a_0 - a_1\left(x_1 + x_2\right)\right)} - \frac{(\omega-1)}{(2-\omega)}\right]$$

For any ω and $x_2 > 0$,if x_1 is small enough, then we have strategic substitutes; if it is large enough we have complements. On the other hand, for any (x_1, x_2) if $\omega = 1$ then we have strategic complements; if $\omega = 2$ then we have strategic substitutes; hence, there exists ω^* such that there is complementarity when $\omega < \omega^*$, and substitutability when $\omega > \omega^*$. ■

Clearly, the strategic set-up here is very different from the CD case. With CD preferences, the reaction function is non-monotonic: in a symmetric equilibrium, the reaction functions have zero slope. This means that the Cournot-Nash and Stackelberg equilibrium coincide: there is no first or second mover advantage. With CES preferences, the reaction functions are also non-monotonic. With ωH preferences, at least in the non-convex case $\omega \geq 2$, the fact that outputs in the Cournot model are strategic substitutes conforms to the standard properties found in partial equilibrium models (see for example Tirole 1988, chapter 5). Hence we have

Observation. Let $\omega \geq 2$. Then there exists a unique Nash equilibrium. In the Stackelberg equilibrium, the leader has a higher output and earns higher profits than the follower. The output of the leader exceeds the Nash output, whilst the follower's is lower.

An explicit solution of the reaction function is not in general possible[7] except in the LH case when $\omega = 2$, when we have the best response function

$$x_1 = \frac{a_0 - w}{2a_1} - \frac{x_2}{2} = \frac{Y}{2nb}\left[1 + \gamma - \frac{\gamma w}{\pi}\right] - \frac{x_2}{2}$$

The symmetric Nash equilibrium output is then

$$x^N = \frac{a_0 - w}{3a_1} = \frac{Y}{3nb}\left[1 + \gamma - \gamma\frac{w}{\pi}\right]$$

The stackelberg outputs for the leader x_L and follower x_F are as follows

$$x_L = \tfrac{3}{2}x^N \quad x_F = \tfrac{1}{2}x^N$$

Notice that the analysis of the individual industry follows directly from the partial equilibrium analysis. In order to go to the general equilibrium, we have simply to substitute in for the parameters a_0 and a_1. We can clearly see the impact of aggregate expenditure Y: it shifts the intercept term of the reaction function and determines equilibrium outputs.

We can solve for price in the Cournot and Stackelberg cases conditional on w and choke-off price $\bar{p} = (1+\gamma)\pi/\gamma$

$$p^N = \frac{\bar{p}}{3} + \frac{2}{3}w > p^S = \frac{\bar{p}}{4} + \frac{3}{4}w$$

[7] Whilst an explicit solution might not be possible, the conditions implicitly defining the reaction functions can be analysed and are tractable, at least for $\omega \geq 2$.

In an equilibrium that is symmetric across sectors we have $p_i = \pi = b = 1$ (taking the consumption good as numeraire), so that the equilibrium real wages are

$$w^S = \frac{3\gamma - 1}{3\gamma} > w^N = \frac{2\gamma - 1}{2\gamma}$$

Clearly $1 > w^S > w^N > 0$: the Walrasian real wage is 1, the Stackelberg wage is below one but above the Cournot-Nash, reflecting the positive but lower markup. Hence, so long as the labor supply is upward sloping, we have a higher level of employment in the Stackelberg case.

3.3 Exchange rate Pass-through.

Now let us apply the Cournot duopoly framework to the issue of exchange-rate pass-through. As in the Stackelberg case, we will be looking at an economy which is *symmetric across industries* (i.e. each industry is the same), but which is potentially *asymmetric within industries* (i.e. the equilibrium in the industry can be asymmetric).

We suppose that in each industry, there are two firms: one is a domestic producer, the other is a foreign firm (subscript f). As in Dixon (1994) and Santoni (1996), they produce a homogeneous good and there is Cournot competition. For simplicity (so that we can obtain an explicit solution) we adopt the LH case. The two reaction functions for the firms form a system of equations

$$2a_1 x + x_f a_1 = a_0 - w$$
$$2a_1 x_f + x_f a_1 = a_0 - e.\bar{w}_f$$

The solution is

$$x = \left[(a_0 - w) - \tfrac{1}{2}(a_0 - e.\bar{w}_f)\right]$$
$$x_f = \left[(a_0 - e.\bar{w}_f) - \tfrac{1}{2}(a_0 - w)\right]$$

with total output $X = x + x_f = \left[a_0 - \frac{1}{2}[w + e.\bar{w}_f]\right]$. The resultant market shares are:

$$s = \frac{\frac{a_0}{2} - w + \frac{1}{2}e.\bar{w}_f}{a_0 + \frac{1}{2}[w + e.\bar{w}_f]}; \quad s_f = (1 - s)$$

Now, consider the effect of a revaluation, represented by a rise in the domestic currency value of the foreign wage: $w_f = e.\bar{w}_f$.

$$\frac{dX}{dw_f} = -\frac{1}{2} \qquad \frac{dx}{dw_f} = \frac{1}{2}$$
$$\frac{dx_f}{dw_f} = -1 \qquad \frac{ds}{dw_f} = \frac{a_0 + 3w}{4a_0 - 2[w + w_f]} > 0$$

Since this equilibrium is symmetric across markets, there is one price in each market, which we can take as the numeraire $p_i = \pi = b = 1$. Thus,the equilibrium elasticity is the symmetric elasticity $\varepsilon^* = \gamma$. The markups are given by

$$\mu = 1 - w = s/\gamma \quad \mu_f = 1 - w_f = (1 - s)/\gamma$$

Note that in any Cournot equilibrium, $\mu + \mu_f = 1/\gamma$ (the markups of the two firms add up to the reciprocal of the industry elasticity).

How do we measure the extent of exchange-rate pass through? Since output is the numeraire, we do not have a nominal price as such. Hence, a natural measure of pass-through is based on the change in markups μ and μ_f. Since the industry elasticity is constant ($\varepsilon^* = \gamma$), markups change because firm market shares change. Since the two market shares add up to unity, what is not passed through to the domestic firm is absorbed by the foreign firm.

Assuming that the economy is initially in a symmetric Cournot equilibrium with $w = w_f = \frac{2\gamma-1}{2\gamma}$, so that $s = s_f = \frac{1}{2}$ we have the effect of the change in w_f on the domestic firm's markup (equal and opposite in sign to the effect on the foreign firm's markup):

$$\frac{d\mu}{dw_f} = \frac{1}{\gamma}\frac{ds}{dw_f} = \frac{1 + 4\gamma - 0.5}{5\gamma + 2\gamma^2}$$

The pass through to the domestic firm is positive but less than 100%, and depends on γ: for $\gamma = 1$ we have 64% pass through to the domestic firm (36% is absorbed by the foreign firm); for $\gamma = 2$, we have 47% pass through (53% absorbed by the foreign firm).

4 Conclusion

In this paper we have developed a new class of homothetic preferences that generate a class of Marshallian demand curves with different properties to the commonly used *CES* and Cobb-Douglas preferences. In particular, with aggregate price-taking the Marshallian demands can have a concave, convex or linear shape in price/quantity space, cutting both axes. Furthermore, the demands have a variable elasticity.

The preferences are represented by a cost function with two parameters which enable us to choose separately the *shape* of the demand curve (concavity/convexity) and the own-price elasticity when all prices are equal.

The paper has provided three illustrative examples of how these properties can be used in standard models in a tractable manner. We very much hope that this new class of preferences will be of use to economists who wish to model strategic agents in situations where the assumption of APT is appropriate. As a final word, we have not mentioned the primal utility function: indeed, to our best knowledge there is no explicit functional form for primal utility (although the indifference curves can be plotted using the dual). The approach we have developed is only possible if you start form the dual: the dual approach here actually tells us something new, rather than being merely a simpler or more elegant alternative to the primal.

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5 Appendix: The Restricted Cost Function.

In this section we consider the formal properties of the proposed ωH cost function. b is a unit cost function if it satisfies the following properties[8]: (i) b is non-negative and non-decreasing in prices; (ii) b is homogenous of degree 1 and concave in $\mathbf{p}$; (iii) b is continuously differentiable[9]. The function b clearly satisfies properties (ii) and (iii) for any $\mathbf{p} \in \Re_+^n$. However, property (i) will not be satisfied for all $\mathbf{p} \in \Re_+^n$ unless $\{\gamma, \omega, n\}$ satisfy the condition

$$\gamma \leq \frac{1}{\left(n^{(\omega-1)/\omega}\right) - 1} \tag{11}$$

This condition may be overly restrictive in some cases. For example with monopolistic firms it is often necessary to assume that the industry demand is elastic. To justify APT, a large number of industries is needed: but (11) then implies that for large n the symmetric elasticity (8) has to be very small, tending to zero as $n \to \infty$. To get over this problem, we employ the methodology developed in Datta and Dixon (2000), where we developed a *restricted cost function* that will allow any combinations $\gamma, \omega > 0$.

Our solution is to restrict the cost function b in (1), so that when some "raw" demands are negative we *cap* prices. If all prices are equal (and strictly positive), then demands are all positive. However demands may become negative if prices

[8] For a fuller listing of necessary and sufficient conditions, see (Diewert 1982, pp.537-547).

[9] Continuous differentiability is not required for b to be a cost-function: however this enables us to apply Shephard's Lemma.

become too dispersed and the higher prices exceed the choke-off price (4). To avoid this, we introduce a *price-capping function* $g : \Re_+^{n+1} \to \Re_+^n$, which places a price-cap (upper bound) of λ on prices, yielding the n-vector of capped prices $\hat{\mathbf{p}} = g(\lambda, \mathbf{p}) = (\min[\lambda, p_i])_{i=1..n}$ We set the price-cap to be as large as possible (but no larger than the highest price $p^{\max}$) so that all capped prices are below the choke-off price (evaluated at the capped prices)

$$\lambda(\mathbf{p}) = \max_{p^{\max} \geq \lambda \geq 0} \lambda$$
$$s.t. \quad \hat{p} \geq \hat{\pi}(1+\gamma)/\gamma \qquad i = 1...n$$

We can now define the *restricted cost function* $B(\mathbf{p}) = b(g(\lambda(\mathbf{p}), \mathbf{p}))$. In Datta and Dixon (2000) we show that in the case of LH preferences, the restricted cost function $B(\mathbf{p})$ satisfies the required properties: the proof is easily extended to the case of ωH preferences.[10].

[10] The Lemma (Datta and Dixon 2000, page 59) requires only the appropriate formula corresponding to (9) to be generalized; Proposition 2 does not depend on the specific functional form and applies directly.

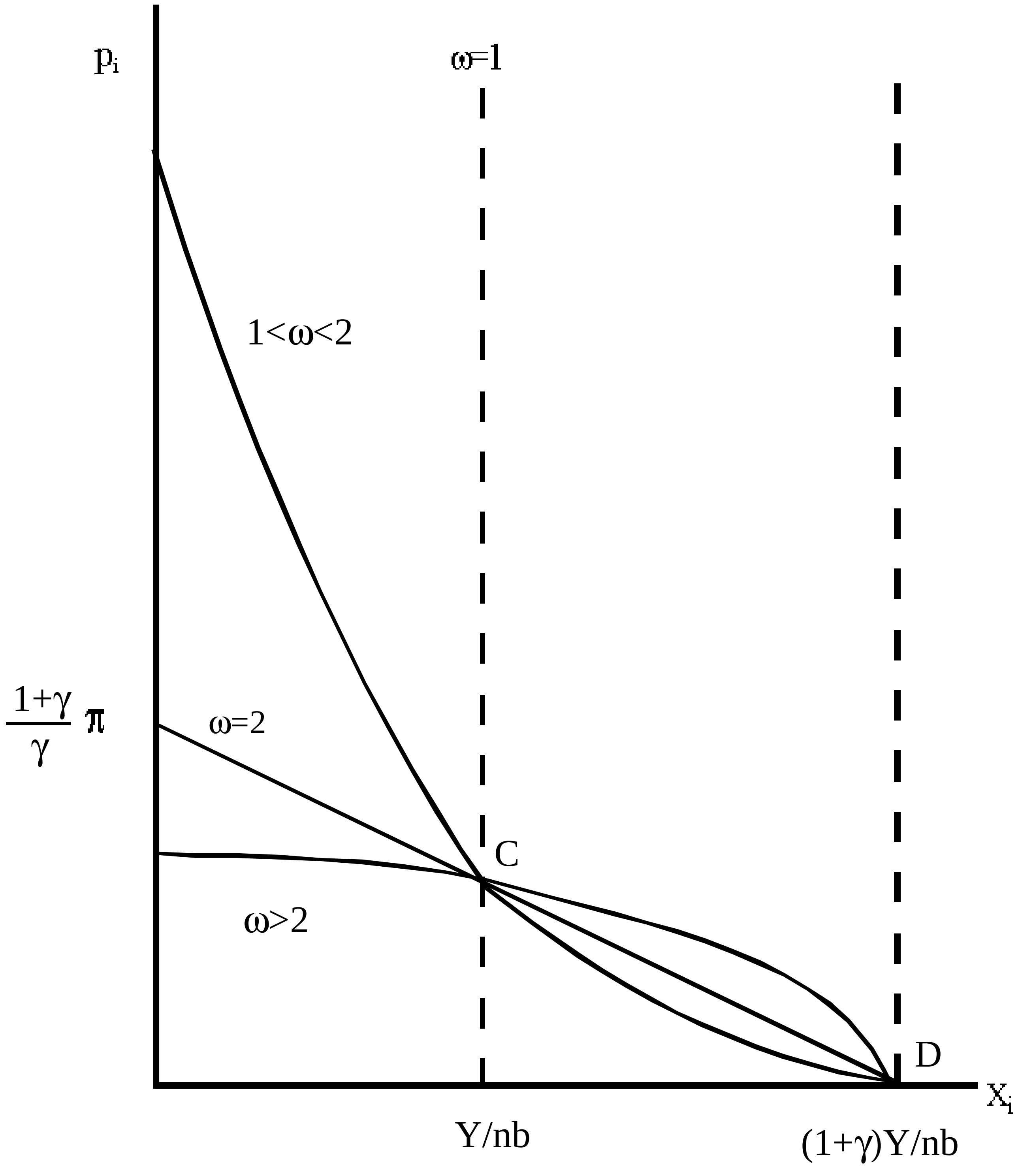
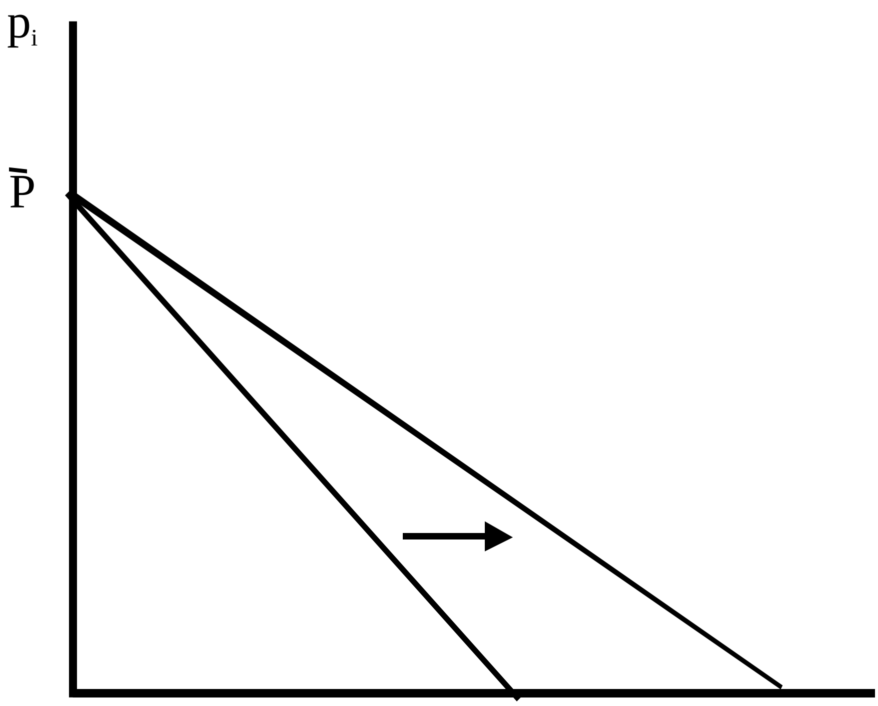
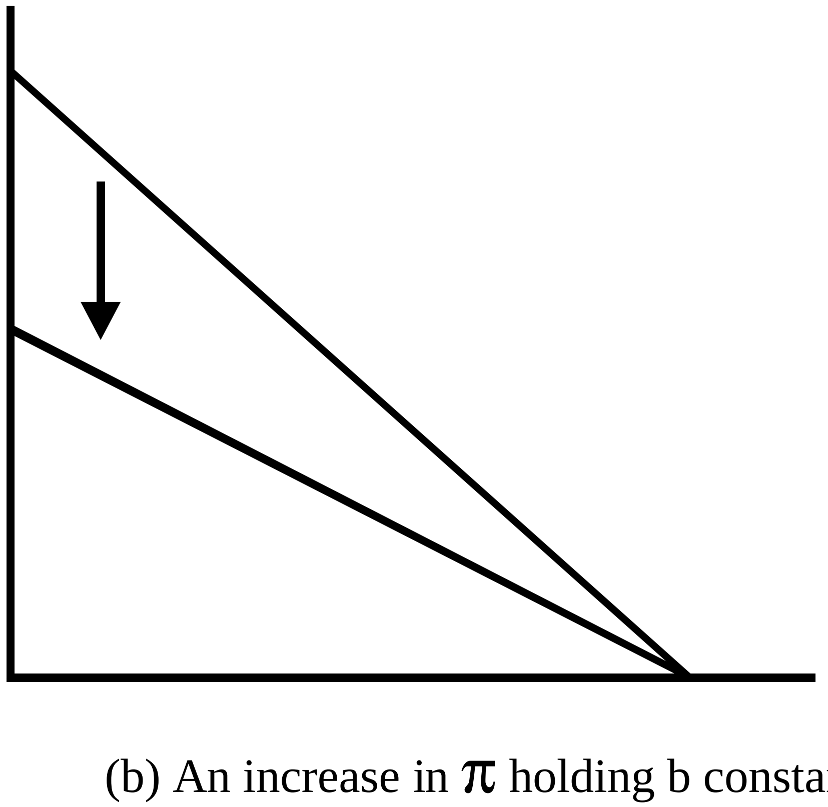


Fig 1: ω-homothetic Preferences:
Marshallian demands.



(a) An increase in Y



(b) An increase in π holding b constant.

Figure 2: Shifts in the Marshallian demand.