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Measure the Productive Efficiency in the Industrial Sector.  
A Comparative Study

by

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**THE USE OF PARAMETRIC AND NON PARAMETRIC FRONTIER METHODS TO  
MEASURE THE PRODUCTIVE EFFICIENCY IN THE INDUSTRIAL SECTOR.  
A COMPARATIVE STUDY**

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**Abstract**

Parametric frontier models and non-parametric methods have monopolised the recent literature on productive efficiency measurement. Empirical applications have usually dealt with either one or the other group of techniques. This paper applies a range of both types of approaches to an industrial organisation setup. The joint use can improve the accuracy of both, although some methodological difficulties can arise. The robustness of different methods in ranking productive units allows us to make a comparative analysis of them. Empirical results concern productive and market demand structure, returns-to-scale, and productive inefficiency sources. The techniques are illustrated using data from the US electric power industry.

*Keywords:* Productive efficiency, parametric frontiers, DEA, industrial sector

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## **1. Introduction**

Since such authors as Debreu (1951), Koopmans (1951) or Farrell (1957) introduced the analysis of efficiency in the economic literature, there has been a numerous and wide ranging collection of papers and articles devoted to the

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measurement of productive efficiency. There has always been a close link between the measurement of efficiency and the use of frontier functions. Different techniques have been utilised to either calculate or estimate these frontier functions. In this study we go through their joint use as well as their application to an industrial organisation framework.

Most of the papers related to the measurement of productive efficiency have based their analysis either on parametric or on non-parametric methods. The choice of estimation method has been an issue of debate, with some researchers preferring the parametric approach (e.g. Berger, 1993) and others the non-parametric approach (e.g., Seiford and Thrall, 1990). The main disadvantage of non-parametric approaches is their deterministic nature. Data Envelopment Analysis (DEA), for instance, does not distinguish between technical inefficiency and statistical noise effects. On the other hand, parametric frontier functions require the definition of a specific functional form for the technology and for the inefficiency error term. The functional form requirement causes both specification and estimation problems. Obviously, it would be desirable to introduce more flexibility into the parametric frontiers, as well as more thoroughly investigate the non-parametric and stochastic methodologies (e.g. Sengupta, 1987). In our opinion neither approach seems to be strictly preferable. Instead, we think that the joint use of the two groups of techniques can improve the accuracy with which they measure productive efficiency. Following recent literature (e.g., Sengupta, 1995), the aim of this paper is to provide the framework for the joint use of them. By doing so one hopes to avoid the weaknesses inherent, and benefit from the strong aspect of each to the two methods, although in general this is not a so easy job to be done.

The set of data utilised is partially taken from the one used in Lee (1995). The paper of Lee examines the issue of vertical integration in the US electricity industry in 1990. Three stages -- generation, transmission, and distribution -- are analysed in his study. Our study focuses just on the generation stage and therefore no comparative analysis with Lee's study is made.

We organise the paper as follows. Section 2 introduces the techniques used to measure the productive efficiency. Section 3 presents the data set and discusses the results. Finally, section 4 presents the conclusions.

## 2. Methods

### 2.1. *The parametric approach*

The parametric approach is naturally subdivided into deterministic and stochastic models. Deterministic models envelope all the observations, identifying the distance between the observed production and the maximum production, defined by the frontier and the available technology, as technical inefficiency. On the other hand, stochastic approaches permit one to distinguish between technical efficiency and statistical noise.

The measurement of productive efficiency by means of parametric techniques requires the specification of a particular frontier function. The Duality theory suggests the use of cost functions to define the production structure. Nerlove (1963) introduced the use of cost functions in the analysis of regulated industries with his application to electric sector. The output produced by firms under a regulated environment, as well as the prices they pay for factors in competitive markets, can be considered to be exogenous. This fact makes the choice of cost functions attractive.

Every cost function implies a set of derived demand equations. Christensen and Greene (1976) argued that the joint use of a cost function and a set of cost share equations as a multivariate regression system provides better estimates of the production structure than those derived from single equation procedures. The dual frontier econometric approach has also evolved from the estimation of single cost functions (e.g., Greene, 1990) to multiple equation systems (e.g., Ferrier and Lovell, 1990; Kumbhakar, 1991). However, some serious estimation and specification problems first noted by Greene (1980), and Nadiri and Schankerman (1981), still remain unsolved<sup>1</sup>. Because of this, the technology form finally adopted was a Cobb-Douglas production function and the frontier production function specified can be represented as

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<sup>1</sup> Panel data techniques can also improve the accuracy of the parametric approach to the measurement of productive efficiency. For a detailed comparative analysis of these techniques, see Kumbhakar (1997).

$$\log Y_i = \mathbf{a} + \sum_{k=1}^r \mathbf{b}_k \log X_{k,i} + v_i - u_i \quad (1)$$

where  $i=1, \dots, N$  indicates the units and  $k=1, \dots, r$  indicates the inputs,  $Y_i$  is output,  $X_{k,i}$  are productive factors. The term  $v_i - u_i$  is the composed error term where  $v_i$  represents randomness (or statistical noise) and  $u_i$  represents technical inefficiency. In the deterministic approach  $v_i$  will equal zero.

Several techniques have been developed in the econometric literature in order to estimate deterministic frontier models<sup>2</sup>. In Corrected Ordinary Least Squares (COLS)<sup>3</sup> methodology, the model's parameters, except the intercept term, can be consistently estimated by Ordinary Least Squares (OLS) since that estimation procedure is robust to non-normality<sup>4</sup>. If the estimated intercept term is corrected by shifting it upward until no residual is positive and at least one is zero, we also get a consistent estimator of the intercept term.

Let us assume the following model:

$$y_i = \mathbf{a} + \sum_j \mathbf{b}_j X_{ij} + \mathbf{e}_i \quad \text{where } \mathbf{e}_i \sim N(0, \sigma^2)$$

Thus,

$$\begin{aligned} \hat{\mathbf{b}}_{COLS} &= \hat{\mathbf{b}}_{OLS} \\ \hat{\mathbf{a}}_{COLS} &= \hat{\mathbf{a}}_{OLS} + \max \hat{\mathbf{e}}_i \\ \hat{\mathbf{m}}_{COLS} &= \hat{\mathbf{e}}_i - \max \hat{\mathbf{e}}_i \end{aligned} \quad (2)$$

and individual technical efficiency will be

$$TE_i = e^{-\hat{\mathbf{m}}_{COLS}}$$

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<sup>2</sup> As it is pointed out for one anonymous referee what is given in relations 1 to 7 is not new but it constitutes the theoretical framework used in the empirical application.

<sup>3</sup> Gabrielsen (1975).

<sup>4</sup> This was first noted by Richmond (1974).

Unlike the deterministic approach, the stochastic frontier models<sup>5</sup> capture the effects of exogenous shocks beyond the control of the analysed units. Errors in the observations and in the measurement of output are also taken into account in this kind of models.

For the Cobb-Douglas case, the stochastic frontier can be represented by eq. (1). The error representing statistical noise is assumed to be identical independent and identically distributed. With respect to the one-sided (inefficiency) error, a number of distributions have been assumed in the literature, being the most frequently used half-normal (SFN), truncated from below at zero (SFT) and exponential (SFE). If the two error terms are assumed independent of each other and of the input variables and some of the previous distributions is used, then the likelihood functions can be defined and maximum likelihood estimates can be determined.

Once the model has been estimated by using maximum likelihood techniques, we obtain a fitted value for the composed error term  $v_i - u_i$ . For efficiency measurement, we need to separate these two error terms. Jondrow, Lovell, Materov and Schmidt (1982) proposed one way to do it. They developed an explicit formula for the expected value of  $u_i$  conditional on the composed error term ( $E(u_i | v_i - u_i)$ ) in the half-normal and exponential cases.

Half-normal case:

$$E[u_i | e_i] = \frac{\mathbf{s} \mathbf{l}}{(1 + \mathbf{l}^2)} \left[ \frac{\mathbf{f}(e_i \mathbf{l} / \mathbf{s})}{\Phi(-e_i \mathbf{l} / \mathbf{s})} - \frac{e_i \mathbf{l}}{\mathbf{s}} \right] \quad (3)$$

where  $\mathbf{f}(\cdot)$  is the density of the standard normal distribution and  $\Phi(\cdot)$  the cumulative density function.

Exponential case:

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<sup>5</sup> Aigner, Lovell and Schmidt (1977), Meeusen and van den Broeck (1977), and Battese and Corra (1977).

$$E[u_i|e_i] = (e_i - \mathbf{q}\mathbf{s}_v^2) + \frac{\mathbf{s}_v \mathbf{f}[(e_i - \mathbf{q}\mathbf{s}_v^2)/\mathbf{s}_v]}{\Phi[(e_i - \mathbf{q}\mathbf{s}_v^2)/\mathbf{s}_v]} \quad (4)$$

where  $\mathbf{q} = 1/\mathbf{s}_u$ .

Truncated case:

Greene (1993) shows that the conditional technical inefficiencies for the truncated model are obtained by replacing  $e_i/\mathbf{s}$  in the expression for the half-normal case, with

$$u_i^* = \frac{e_i \mathbf{l}}{\mathbf{s}} + \frac{u_i}{\mathbf{s}\mathbf{l}} \quad (5)$$

Finally, individual (conditioned) technical efficiency scores will be

$$TE_i = e^{-E[u_i|e_i]}$$

## 2.2. The non-parametric approach

Non-parametric analysis (Charnes, Coopers and Rhodes, 1978) does not require the specification of any particular functional form to describe the efficient frontier or envelopment surface. The flexibility of non-parametric techniques allows for several alternative formulations. In this paper we analyse two versions of an output-oriented DEA model according to which returns hypothesis is assumed: namely, constant returns to scale (DEAc) and variable returns to scale (DEAv).

Consider a set of  $n$  homogenous Decision Making Units (DMU). There are  $m$  inputs and  $s$  outputs and each DMU is characterised by an input-output (X, Y) vector. In order to determine the efficiency score of each unit, these will be compared with a peer group consisting of a linear combination of efficient DMUs. For each unit not located on the efficient frontier we define a vector  $\bar{\mathbf{m}} = (m_1, \dots, m_n)$  where each  $m_j$  represents the weight of each DMU within that peer group. The DEA calculations are designed to maximise the relative efficiency score of each unit, subject to the constraint that the set

of weights obtained in this manner for each DMU must also be feasible for all the others included in the sample. That efficiency score can be calculated by means of the following mathematical programming formulation<sup>6</sup> where technical efficiency scores will be determined by the optimum  $\mathbf{y}$ . Constant (TEc) and variable returns to scale (TEv) formulations are described.

$$\begin{array}{ll}
 TE_C = \max \mathbf{m} \mathbf{y}^0 & TE_V = \max \mathbf{m} \mathbf{y}^0 \\
 \text{s.t.} & \text{s.t.} \\
 \sum_{j=1}^n \mathbf{m}_j Y_{ij} \geq Y_i^0 & \sum_{j=1}^n \mathbf{m}_j Y_{ij} \geq Y_i^0 \quad i=1, \dots, m \\
 \sum_{j=1}^n \mathbf{m}_j X_{rj} \leq X_r^0 & \sum_{j=1}^n \mathbf{m}_j X_{rj} \geq X_r^0 \quad r=1, \dots, s \\
 & \sum_{j=1}^n \mathbf{m}_j = 1
 \end{array} \tag{6}$$

Operation research techniques usually use the dual of the above problem in order to calculate the efficiency scores. Such a dual formulation can be obtained as the minimum of a ratio of weighted inputs to weighted outputs subject to the constraint that the similar ratios for every DMU be greater than or equal to unity. For an output-oriented model, the dual formulation is

$$\begin{array}{l}
 \text{Min}_{w_r, z_i} H_0 = \frac{\sum_i z_i X_{i0}}{\sum_r w_r Y_{r0}} \\
 \text{s.t.} \\
 \frac{\sum_i z_i X_{ij}}{\sum_r w_r Y_{rj}} \geq 1 \quad j=1, \dots, n \\
 w_r, z_i > 0 \quad r=1, \dots, s \quad i=1, \dots, m
 \end{array} \tag{7}$$

<sup>6</sup> See Charnes, Cooper and Rhodes (1978). A more detailed analysis of alternative formulations can be found in Ali and Seiford (1993), and Coelli, Rao and Battese (1998).



where  $w_r$  and  $z_i$  are the variable weights that solve this maximisation problem and  $Y_{ij}$  and  $X_{ij}$  the outputs and inputs attached to each DMU. A unit will be efficient if and only if this ratio equals one, otherwise it will be considered as relatively inefficient.

DEA can also be used to calculate scale efficiency. Total technical efficiency is defined<sup>7</sup> in terms of equiproportional increases in outputs that the firm could achieve while consuming the same quantities of its inputs if it were to operate on the constant returns to scale (CRS) production frontier. Pure technical efficiency measures the increase in outputs that the firm could achieve if it were to use the variable returns to scale (VRS) technology. Finally, scale efficiency would be calculated as the ratio of total technical efficiency to pure technical efficiency. If scale efficiency equals one, the firm is operating at CRS, otherwise it would be characterised by VRS.<sup>8</sup>

### 3. Data and results

A wide range of papers related to the treatment of the electric sector with frontier techniques is available in the empirical literature. Schmidt and Lovell (1979, 1980) and Greene (1990) introduced the analysis of electricity sector data sets into frontier functions literature. Fare, Grosskopf and Logan (1985) utilise mathematical programming techniques to calculate six different measures of efficiency and compare public versus private performance of electric utilities. Hjalmarsson and Veiderpass (1992) study the local retail distribution of electricity in Sweden in 1985. They apply different versions of the DEA model to 329 firms. Using DEA techniques and OLS analysis, Pollit (1994) examines the cost efficiency in 129 electricity transmission and 145 electricity distribution systems in 1990. Lastly, Ray and Mukerjee (1995) perform a comparative analysis of parametric frontier dual cost functions and non-parametric techniques applied to the data set used previously in Greene (1990).

The data set used in the present empirical application corresponds to a sample of 70 US (investor-owned) electric utility firms in 1990. These firms are approximately

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<sup>7</sup> According to an output-oriented model formulation.

<sup>8</sup> Whether those variable returns to scale represent increasing or decreasing returns to scale will depend on the relationships among technical efficiency scores calculated under constant, variable or non-increasing returns to scale.

evenly spread across the United States. Table 1 provides descriptive statistics for each of the variables used in this study.

<<< TABLE 1 >>>

The capital stock variable is constructed for four different asset classes: steam, nuclear, hydroelectric and other power-generating equipment. In any case, steam technology counts for most of the electricity generated by the companies analysed in this study. The labour variable indicates the number of workers of each firm. There are four main categories of fuel: coal, oil, natural gas, and nuclear. BTU equivalents are used to aggregate different types of fuels over all plants belonging to one firm. The fuel variable is measure in millions of BTUs used in generation of electricity. Finally, total output is indicated in megawatts hours (MWh).<sup>9</sup>

### 3.1 Efficiency scores

With respect to the parametric frontiers the estimated parameters of the deterministic and stochastic production functions are given in table 2.

<<< TABLE 2 >>>

These results come from estimating eq. (1) by means of COLS and MLE, where  $i=1,\dots,70$  indicates the firms,  $Y_i$  the output,  $X_{1,i} = K_i$  the Capital stock,  $X_{2,i} = L_i$  the number of workers, and  $X_{3,i} = F_i$  the fuel;  $\mathbf{b}_1$ ,  $\mathbf{b}_2$  and  $\mathbf{b}_3$  are the elasticities of output with respect to capital, labour and fuel. We infer the presence of constant returns to scale in all the specifications analysed<sup>10</sup>. We estimate a Cobb-Douglas production function. More flexible technologies, such as different versions of translog production functions, presented major problems in the significance of their estimated parameters. Without the factor share equations, estimation of full translog functions can be hampered by an important problem of multicollinearity.<sup>11</sup>

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<sup>9</sup> A major description of the set of data and variables used in this study can be found in Lee (1995).

<sup>10</sup> Actually, this hypothesis was strongly accepted when we imposed the constraint  $(\mathbf{b}_1) + (\mathbf{b}_2) + (\mathbf{b}_3) = 1$  to the initially unrestricted model. The estimation procedure was made using Limdep 7.0.

<sup>11</sup> According to *Klein's rule of thumb*, multicollinearity is a problem if  $\max R_j^2 > R^2$  where  $R_j^2$  is the  $R^2$  statistic from the OLS estimation of the auxiliary regression of the  $j^{\text{th}}$  regressor on the other regressor and

Each of the stochastic specifications yields similar estimates for the partial elasticities of output with respect to capital, labour and fuel. This result seems to confirm the robustness of the technology and distribution hypotheses assumed in the specification of the model.

Table 3 reports the average technical efficiency measures for each of the models explained in the Methods section.<sup>12</sup>

<<< TABLE 3 >>>

As the theory advances, the average efficiency scores of parametric deterministic techniques are lower than the ones estimated through stochastic frontier approaches. Given that COLS is a not stochastic procedure, noise is also reported as inefficiency.

COLS shifts all the residuals down to non-positive values and only one firm of the sample is estimated as efficient<sup>13</sup>. With respect to the DEA approaches, given that the constraint set is less restrictive under CRS than under VRS, lower efficiency scores are reported for the former case. In our example, DEAc presents an average level of technical efficiency of 73.32% while DEAv efficiency average is 78.71%. For the same reason, fewer units are found to be efficient under CRS than under VRS.

Within the stochastic approaches, no noticeable differences arise. The average efficiency is lower with normal/half-normal models than with the normal/exponential or normal/truncated models, but, in any case, the choice of distribution assumptions does not seem to have a significant effect on the values of the efficiency estimates.

Stochastic frontier models' estimates of  $\sigma_v^2$  and  $\sigma_u^2$  provide us with a measure for the relative importance of statistical noise and inefficiency in the estimation of frontier production functions. The variance of the composed error term  $\sigma_e^2$  is defined as

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the intercept term. Several auxiliary regressions were estimated and in all of them this condition was found. Moreover, when we checked the functional form specification of the model, applying a RESET-Test, the Cobb-Douglas technology turned out to be well specified.

<sup>12</sup> The individual efficiency scores generated by each method are available from the authors upon request.

<sup>13</sup> The one with the largest positive OLS residual.

the sum of the variance of the inefficiency error term  $\sigma_u^2$  and the variance of the statistical noise term  $\sigma_v^2$ . Therefore the (%) participation of each of these components - u and v - in the aggregated error term e can be determined by means of the relationships  $\%_u = \sigma_u^2 / (\sigma_u^2 + \sigma_v^2)$  and  $\%_v = \sigma_v^2 / (\sigma_u^2 + \sigma_v^2)$ . According to the information in table 2, noise represents 59.72% of total variance in the exponential model. In the half-normal and in the truncated cases, these proportions are lower, 25.18% and 17.08% respectively, but still broadly indicative of the importance of noise in the estimation of these models. Therefore, the fact that deterministic models do take noise into account seems to be quite important in our illustrative application. Especially noticeable is the COLS procedure where the average level of technical efficiency is around 60%. These models therefore suffer from both drawbacks: the problems of a rigid specification associated to their parametric nature, and the shortcoming of not distinguishing between inefficiency and noise given their deterministic structure.

### 3.2. Robustness

Having analysed the efficiency scores, we explore the consistency of the above models in ranking the 70 electric utilities that make up our sample. We are interested in the robustness of the relative position of each electric utility to the use of different methods, rather than in the average levels of technical efficiency found. Table 4 presents pairwise Spearman rank correlation coefficients of the efficiency scores yielded by the six methods used in our analysis.<sup>14</sup>

<<< TABLE 4 >>>

These results show that parametric models are extremely consistent in ranking the units. Their pairwise correlation coefficients are not less than 99%. The correlation is also high between parametric techniques and DEAc. On the other hand, correlation coefficients between DEAv and both the econometric approaches and DEAc are not so high. They are around 83% for the group of parametric techniques and 89% for the DEAc model. All parametric approaches were also estimated by imposing the CRS constraint. It seems that the choice of parametric or non-parametric techniques,

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<sup>14</sup> Spearman's correlation coefficients were calculated using the SPSS 8.0 package.

deterministic or stochastic approaches, or between different distribution assumptions within stochastic techniques is irrelevant if one is interested in ranking electric utilities according to their individual efficiency scores. Only the VRS specification leads to certain differences in those rankings, although such differences are not so large as to stop these rankings still being comparable with the others

Table 5 reports the returns to scale of the efficient units in the sample of firms analysed in our study.

<<<TABLE 5>>>

There is detect an almost perfect correlation between the size of the efficient firms and their returns to scale, in the sense that the bigger firms have decreasing returns to scale and vice versa. It seems that economies of scale are exhausted at the greatest levels of production while they are still available at lower levels. This result agrees with the low value found for the average scale inefficiency and is supporting evidence that the units in our sample are operating at the correct scale. Some studies as Cummins and Zi (1998), for example, have found a direct relationship between the size of units and their inefficiency levels. In our case, no such relationship seems to appear.

So far, we have analysed different methods and their robustness in the measurement of productive efficiency. The next step in this empirical application will provide some possible explanations for the efficiency scores described above.

### *3.3. Inefficiency sources*

One common practice in the literature is to regress the efficiency scores against a vector of explanatory variables. Disaggregated data for different types of capital and output are used as proxies for the productive structure and market demand structure faced by each electric utility. Capital stock levels attached to steam, nuclear and hydroelectric assets are used to evaluate the influence of each of those technologies on higher or lower efficiency scores. Similarly, the allocation of total megawatt-hours to three different demand categories -- commercial, industrial and residential -- is also considered on the basis of explaining individual efficiency scores.

The high degree of correlation between those proxies for productive and market structure and the original variables specified in our model is a handicap for two-stage models. However, the choice of a one stage model, as Lovell (1993) points out does not solve this problem of correlation between the variables used in the initial specification of the model and those used in the subsequent analysis of the efficiency sources: it just replaces a problem of omitted (two stages model) with one of multicollinearity.<sup>15</sup>

For the series of inefficiency scores to take into account as the dependent variable, we have used that generated by the DEAc model<sup>16</sup>. The DEA-based efficiency scores are truncated from below at one. OLS regression would produce biased and inconsistent parameter estimates, so we use a truncated regression model (Tobit model). The estimated parameters are given in table 5.

<<< TABLE 6 >>>

Given the statistical significance of the three parameters used as proxies, it seems that the productive structure affects the efficiency scores attained by the different electric utilities. The market demand structure, on the other hand, seems not to have any influence.

The variables used to measure the effects of market demand structure on the inefficiency of each unit are characterised by a high degree of homogeneity across observations (see table 1). Therefore it is not surprising to find that they are not significant explanations for the inefficiency of units.

Within productive structure factors, steam and nuclear technologies are found to be directly related to inefficient behaviour of the units in the sample, while the use of hydroelectric technology seems to have positive effects on their efficiency. Nuclear and

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<sup>15</sup> Some functional forms with disaggregated levels of capital and output used as regressors were also estimated. However, such a large list of variables, especially in the translog version, and the high degree of correlation among them requires a very high order in the convergence criteria of the maximum likelihood algorithms of stochastic frontier models. This precluded the estimation of these stochastic models.

<sup>16</sup> The results with the COLS, SFN, SFE and SFT efficiency series were almost identical.

even more so steam technologies seem to be exhausting their particular economies of scale.

The main problem of “two-stage” models, such as that used in this paper, is to know which regressors must be included in the estimation of efficiency levels and which in their explanation. In the light of our results, besides their not being highly correlated with the variables utilised in the frontier estimation procedure, a necessary although not sufficient condition for regressors to be considered as proxies for inefficiency sources is that they must be able to introduce heterogeneity in the analysis. Thus, a necessary extension to the empirical analysis that we have so far presented would be the introduction of additional information through variables properly representative of the industrial organisation, such as market structure, regulatory environment, ownership or internal organisation of the firm.

#### **4. Conclusions**

The joint use of parametric and non-parametric techniques devoted to the measurement of efficiency in the industrial sector is a novel issue in the recent empirical literature. However, this is not always feasible. Our paper has focused on the definitions of a framework for the joint use of these techniques.

The main disadvantage of non-parametric approaches is their deterministic nature. DEA techniques, for instance, make no accommodation for noise. Parametric techniques, as we have seen, require specification of a particular technology for the frontier function as well as the definition of a specific statistical distribution for the inefficiency term. The functional form requirement causes both specification and estimation problems. Hence, the parametric-deterministic approaches for the measurement of productive efficiency does not seem to be suitable for this kind of analysis. As our results suggest, they suffer from the disadvantages of both methods.

With respect to parametric-stochastic approaches, in so far as the disturbances about the frontier estimator tend to be symmetrically distributed, the frontier approach can be interpreted as a neutral transformation of the “average” technology. Then only Timmer’s “Holy Grail” (Timmer, 1971) i.e. the necessity of placing the frontier in order

to give numerical values to efficiency performances of each analysed unit, would justify a frontier approach instead of the traditional OLS-average approach. However, the presence of skewness in the disturbances is another reason why frontier functions might be taken into account: the underlying technology assumed under the average and the frontier specification can describe structural dissimilarities between the two techniques, such as different returns to scale or elasticities of substitution.

On the basis of the robustness of different techniques in ranking productive units, DEA can improve the accuracy of parametric techniques. DEA flexibility permits the introduction of relevant issues such as non-discretionary variables (Banker and Morey, 1986a), categorical variables (Banker and Morey, 1986b), or constrained multipliers (Charnes, Cooper, Wey and Huang, 1989). Moreover, a recent paper (Sengupta, 1999) extends the use of DEA to a dynamic framework by incorporating changes in productivity due to technological progress or regress. These aspects may correct some of the specification problems associated with parametric methods.

The versatility of DEA techniques also provides a simple way of analysing the scale efficiency. In our study, no relationship between the size of firms and their inefficiencies seems to exist. On the basis of the aforementioned robustness it is also possible to analyse the sources of productive inefficiency by using two-stage models. These models will only be meaningful if the variables used as regressors introduce heterogeneity into the analysis.

We have here described some methodological considerations based on the data set used for this study. Much work remains to be done. For instance, additional information on prices and a larger sample of observations might improve the measurement of economic efficiency in an industrial sector by taking into account technical and allocative efficiencies as well as cost and revenue efficiencies. As the literature shows, serious problems arise when applying duality theory to parametric frontier models. However, Data Envelopment Analysis provides a suitable way of treating the measurement of economic efficiency. This approach has been used in a number of empirical applications related to nonprofit, regulated and private sectors. In conclusion, the present results provide encouragement for the continued development of the collaboration between parametric and non-parametric methods.



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## Tables

Table 1. Main descriptive statistics of variables used in the study.

Variable	Mean	Max.	Min.	Standard Deviation
<b>Total Output</b>	15.582	70.517	1.678	1.568E+10
<b>Total Capital</b>	94.914	409.673	9.367	91.747
<b>Total Labour</b>	4.993	24.607	440	5.198
<b>Total Fuel</b>	1.324E+10	4.750E+11	7.001E+09	1.111E+11
<b>% Ksteam (1)</b>	0.7674	0.9999	0.084	0.2192
<b>% Knuclear (1)</b>	0.1120	0.6754	0	0.1762
<b>% Khydroelectric(1)</b>	0.0422	0.3256	0	0.0757
<b>% Kother GE(1)</b>	0.0783	0.9150	0	0.1280
<b>% Ocommercial (2)</b>	0.2664	0.6421	0.037	0.0987
<b>% Oindustrial(2)</b>	0.3485	0.5533	0.1052	0.0774
<b>% Oresidential(2)</b>	0.3850	0.8113	0.063	0.1272

(1) Represents the percentage of capital stock levels attached to steam, nuclear, hydroelectric and other power- generating equipment assets.

(2) Allocation of total MWh to commercial, industrial and residential demand categories.

Table 2. Estimated parameters of deterministic and stochastic production frontiers.

(t-test statistics appear in parentheses)

	COLS	SFN	SFT	SFE
<b>Intercept (a)</b>	10.819(*) (10.014)	11.786 (15.870)	11.145 (13.886)	10.951 (14.453)
<b>Capital (b<sub>1</sub>)</b>	0.1392 (2.414)	0.1340 (1.893)	0.1066 (1.330)	0.1391 (2.270)
<b>Labour (b<sub>2</sub>)</b>	0.6441 (10.539)	0.6745 (10.865)	0.6713 (10.084)	0.6441 (11.485)
<b>Fuel (b<sub>3</sub>)</b>	0.2174 (3.474)	0.1794 (3.954)	0.2170 (4.705)	0.2174 (4.847)
<b>R<sup>2</sup></b>	0.9506			
<b>F</b>	423.529			
<b>Log-Lik.</b>	10.1631	11.3880	11.1224	11.8625
<b>S<sub>u</sub>/S<sub>v</sub></b>		1.7239 (1.897)	2.2007 (1.405)	
<b>S<sub>u</sub><sup>2</sup></b>		0.0621	0.0995	0.0176
<b>S<sub>v</sub><sup>2</sup></b>		0.0209	0.0205	0.0261
<b>(b<sub>1</sub>) + (b<sub>2</sub>) + (b<sub>3</sub>)**</b>	1.0007 {0.9756}	0.9879 [0.3570]	0.9949 [0.1421]	1.0006 [0.0230]

(\*) If the estimated intercept term is corrected by shifting it upward until no residual is positive and at least one is zero, we will get a consistent estimator of the intercept term. In our case this consistent intercept is 11.349.

(\*\*) CRS hypothesis test.: { \_ } : Probability associated with an F-Test (1.66). [ \_ ] : Significance level in a Wald Test- $\chi^2$  (1).

Table 3. Technical efficiency averages.

Method	Average Efficiency	Max.	Min.	Standard Deviation	Number of efficient units
<b>COLS</b>	60.09	1	28.95	0.123	1
<b>SFN</b>	82.61	94.86	49.72	0.086	0
<b>SFT</b>	87.77	96.31	54.33	0.073	0
<b>SFE</b>	87.64	95.93	49.66	0.080	0
<b>DEAc</b>	73.32	100	33.3	14.77	6
<b>DEAv</b>	78.71	100	6.9	19.39	16

(\*) The average efficiency measures of COLS, SFN, SFT, and SFE were estimated under the null hypothesis of Constant Returns to Scale.

Table 4. Spearman correlation coefficients among alternative efficiency measures(\*).

	<b>COLS</b>	<b>SFN</b>	<b>SFT</b>	<b>SFE</b>	<b>DEAc</b>	<b>DEAv</b>
<b>COLS</b>	1.000					
<b>SFN</b>	0.994	1.000				
<b>SFT</b>	0.995	0.994	1.000			
<b>SFE</b>	0.991	0.998	0.994	1.000		
<b>DEAc</b>	0.909	0.907	0.918	0.915	1.000	
<b>DEAv</b>	0.833	0.829	0.843	0.835	0.890	1.000

(\*). All the correlation coefficients among different methods are significant at the .01 level (2-tailed).

Table 5. Returns to scale of efficient units

Observations (1)	Total output (2)	Returns (3)
2	1.731	IRS
3	1.823	IRS
4	2.382	IRS
8	2.683	IRS
9	3.240	CRS
15	4.473	CRS
17	4.620	CRS
28	7.149	DRS
30	7.721	CRS
46	15.539	CRS
52	19.678	CRS
64	36.309	DRS
67	51.776	DRS
68	63.558	DRS
69	64.410	DRS
70	70.517	DRS

(1) Ordered by output produced.

(2) MWh.

(3) IRS: Increasing Returns to Scale, CRS: Constant Returns to Scale, DRS: Decreasing Returns to Scale.



Table 6. Tobit model estimated parameters

<u>Variable (%)</u>	Parameter Estimate	t-student	Mean	Max.	min.	Standard Deviation
Ksteam	0.2975	2.346**	0.7674	0.9999	0.084	0.2192
Knuclear	0.2848	1.856 *	0.1120	0.6754	0	0.1762
Khydro.	-0.4295	-1.820 *	0.0422	0.3256	0	0.0757
Ocommercial	0.1049	0.530	0.2664	0.6421	0.037	0.0987
Oindustrial	0.2526	1.596	0.3485	0.5533	0.1052	0.0774
Oresidential	-0.2848	-1.484	0.3850	0.8113	0.063	0.1272

\*\* Significant coefficients at the 5% level (2-tailed).

\* Significant coefficients at the 10% level (2-tailed).