APPENDIX

JOB REALLOCATION AND AVERAGE JOB TENURE

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January 2000

Modelling job flows

Following Caballero and Hammour (1994), firms combine labour and capital in fixed proportions to create a new productive unit (a new job) which they endow with the latest technology. The exogenous continuous technological progress is such that the productivity of new units grows at a positive rate γ throughout time t. Once created, however, technology is embodied and the productive unit will produce a constant flow of output $A(t_0)$ over its lifetime, from time period t_0. At any particular point in time t, there will be a distribution $f(a,t)$ of jobs of ages a, such that $0 \leq a \leq a_m(t)$ and $a_m(t)$ is the age of the oldest job still in existence. Aggregating across jobs at any time provides total industry employment (of labour or capital stock in operation) $N(t) = \int_0^{a_m(t)} f(a,t)da$ and total industry output is given by

$$Q(t) = \int_0^{a_m(t)} A(t-a)f(a,t)da. \tag{1}$$

There is a positive constant attrition rate δ which is exogenous. At any time t the number of jobs that have survived for a years is given by

$$f(a,t) = f(0,t-a)exp^{-\delta a}, \qquad 0<a\leq a_m(t). \tag{2}$$

Differentiating $N(t)$ over time, and allowing for (2), provides the fundamental equation for employment growth:

$$\dot{N}(t) = f(0,t) - (f(a_m(t),t)[1-\dot{a}_m(t)] + \delta N(t)) \tag{A}$$

The first term in equation (A) is the flow of creation of production units, $f(0,t)$. The second term is the total flow of destruction which consists of three parts: $f(a_m(t),t)$ units have reached their obsolescence age (a_m); changes in a_m over time lead to $-(f(a_m(t),t)\dot{a}_m(t))$ units being destroyed; and δN(t) units are retired due to attrition. The first two components of the destruction flow can be considered as endogenous flows. The third component $\delta N(t)$ is exogenous. Normalising the creation flow and the total destruction flow by $N(t)$ provides the job creation and job destruction rates, respectively. Average tenure in the firm across all units at a point in time t is:

$$\text{average tenure} = \frac{\int_{f(0,t)}^{f(a_m,t)} a(t).df(a,t)}{N(t)}$$

It will be positively affected by $a_m(t)$ and negatively by $f(0,t)$.

There may be a cost $c = c(f(0,t))$ involved in creating a job. If we assume free entry in the industry, the firm will equate the creation cost to the discounted value of the expected profit flow generated by the job over its lifetime. If the operating costs of the job are set at 1, then the profits π generated at time t by a production unit of age a are $\pi(a,t) = P(t)A(t-a)-1$ where $P(t)$ is the price of a unit of output and 1 denotes the operating costs of a production unit. Let $T(t)$ be the maximum life of a production unit created at time t, with perfect foresight

$$a_m[t+T(t)] = T(t) \tag{3}$$

The free entry condition at any time t is

$$c(f(0,t)) = \int_t^{t+T(t)} \pi(s-t,t)\exp^{-(r+\delta)(s-t)} ds \tag{4}$$

where $r>0$ is the interest rate (exogenously given). A production unit is destroyed when its profits reach zero. Thus, $a_m(t)$ satisfies

$$P(t)A(t-a_m(t)) = 1 \tag{5}$$

A unit elastic demand function is assumed with $\overline{D}(t)$ being total spending on industry output

$$\mathrm{P}(t)Q(t) = \overline{D}(t) \tag{6}$$

In steady state, the cost of creation is given by[1]

$$c(f^*(0)) = \frac{\exp^{\gamma a_m^*} - \exp^{-(r+\delta)a_m^*}}{\gamma + r + \delta} - \frac{1-\exp^{-(r+\delta)a_m^*}}{r+\delta} \tag{7}$$

and the creation flow is[2]

$$f^*(0) = \frac{(\gamma+\delta)\overline{D}}{\exp^{\gamma a_m^*} - \exp^{-\delta a_m^*}} \tag{8}$$

When the creation cost is constant (independent of the creation flow), a_m^* can be found from (7). This value for a_m^* can then be incorporated into (8) to find the creation flow given the level of demand. These analytical solutions are not very informative, however, we can easily

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[1]To find (7), solve (4) after substituting $P(s)A(t-a)$ -1 for $\pi(s,t)$ and expanding, remembering that in steady state $T(t)=a_m(t)=a_m^*$, that prices are falling at the rate γ and technology between a=0 and $a=a_m^*$ will have risen by $exp(\gamma a_m^*)$.

[2]Substitute for Q_t in equation (6) making use of equation (1) and that in steady state $f(a,t) = f^*(a)$ for all t, so $f^*(a) = f^*(0)exp(-\delta a)$, prices and technology are treated similarly as when solving for (7).

substitute in values for the parameters[3] and obtain calibrated solutions. For example, if we assume that r=0.065, δ=0.15, γ=0.028 and c=0.5, we find a_m^* from (7) to be 7.3 years, substituting this value into (8) and assuming that $\bar{D}$=1 provides a creation flow of 20.1% per annum.[4] Outside of steady state, providing $c'(f(0,t))$=0, the system retains its recursive property so that if we double the cost of creation to c=1, *ceteris paribus*, we find a_m^* increases to 11 years and creation flow falls to 15.2%. If the creation cost is allowed to vary with the creation flow $c'(f(0,t))$>0, the system must be solved simultaneously. Nevertheless, the path $\{f(0,t),\ a_m(t),\ T(t),\ P(t),\ Q(t)\}_{t\geq 0}$ satisfying equations (2), (A) and (1) to (6) for all $t\geq 0$, given an initial density of $f(a,0)$, a>0, of production units provides an equilibrium for this industry and determines the right-hand side of equation (A) for employment change.

Demand shocks and creation costs

When industry demand $D(t)$ falls, the firm can either reduce the flow of creation of new jobs $f(0,t)$ or it can increase the endogenous destruction flow (by reducing the age at which redundancy occurs, $a_m(t)$). If the firm fully insulates incumbents by adjusting entirely through a fall in creation, the firm will have to undergo more rapid creation in future time periods to maintain a competitive level of productivity. If there is no association between the costs of creation and the extent of the creation flow, $c'(f(0,t)) = 0$, then the firm will indeed fully insulate[5] in the recession thereby temporarily saving itself the set up costs involved in creation, c. Thus the firm will lower employment by taking on fewer new hires. Since the retirement age is not changing, the expected job tenure of an individual remains the same, however, if the firm lowers the creation flow the average job tenure in the workplace will rise *ceteris paribus*.

It is quite possible, however, that there is a positive relationship between the creation rate and the costs of creation such that $c'(\cdot)>0$. For example, attracting new employees requires successful matching and there may be diminishing returns in the matching function, there may also limits to the resources available for the training of suitable applicants, similarly the availability of capital needed to bond with labour in the new job may be limited in any time period. Consider the case where creation costs are linear and of the form $c=c_0+c_1\,f(0,t)$, then in general, holding c_0

[3]By providing values for the interest rate (r), the attrition rate (δ), the rate of technological growth (γ) and the creation cost (c).

[4]The values for these parameters are set equal to the values chosen by Caballero and Hammour (1994) for comparisons sake.

[5]If $c'(f(0,t)) = 0$ and the parameters are set equal to that discussed above, a shift in demand from 0.5 to 1 to 2 has no impact on the redundancy age (it remains at 7.22 years) whilst the creation flow increases from 10% to 20.1% to 40.2%.

constant and increasing c_1 has the effect of raising a_m^* whilst lowering $f(0,t)$.[6] The stronger the relationship between the size of the creation flow and the costs of creation, the smaller the insulation effect will be. Firms will respond by trying to smooth the creation of jobs over time and business cycles, and falls in demand in a recession will be accommodated via an increase in destruction (by lowering the redundancy age) as well as lowering the creation flow[7]. In other words, firms will make adjustments on both margins leading to lower employment and contrary effects on average tenure. In the extreme, when the marginal creation cost is very high, firms will set a constant creation rate whilst accommodating the business cycle by varying redundancy age.

Workplace age and job reallocation

We have argued that newer jobs are more productive and less likely to be made redundant. Analogously, younger firms will have a greater proportion of new jobs and experience less adjustment through destruction (Caballero and Hammour, 1994; footnote 22). It would seem, however, that this outcome would depend on the nature of creation costs the firm is facing. Empirically, a negative relationship between the age of the firm and job reallocation has been established in the literature (Davis and Haltiwanger, 1999). Caballero and Hammour add that their prediction could be considered more formally by assuming that the exogenous destructive flow due to attrition $\delta N(t)$ is made a decreasing function of age, $\delta(a)$. We introduce the term δ/a for attrition as a simple example. Solving for the steady state provides:

$$c(f^*(0)) = \frac{\exp^{\gamma a_m^* - \delta} - \exp^{-ra_m^* - \delta}}{\gamma + r} - \frac{\exp^{-\delta} - \exp^{-ra_m^* - \delta}}{r} \tag{7a}$$

and

$$f^*(0) = \frac{\gamma \bar{D}}{\exp^{\gamma a_m^* - \delta} - \exp^{-\delta}} \tag{8a}$$

It is perhaps not obvious from the above what difference this modification has made. As a comparison, we solve (7) and (7a) for a_m assuming in both cases that r=0.065, δ=0.15, γ=0.028 and c=0.5. We find that the obsolescence age falls from 7.2 years to 6.7 years if attrition falls with age according to δ/a. Substituting these values for a_m into (8) and (8a) respectively, and assuming that D=1, we find creation flows accordingly fall from 20% to 15.8% in the steady state. If the obsolescence age and the creation flow have both fallen, employment will fall. We expect, therefore to find a negative relationship between workplace age and net employment change, the

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[6]If we compare the cases where c_0=0.4 and c_1=0.5 with c_0=0.4 and c_1=0.95, we find values for a_m^* and $f(0,t)$ of 7.2, 20.9% and 7.9, 18.9% respectively.

[7]If we consider the case where c_0=0.4 and c_1=0.5, with other parameters remaining constant except for D, we find D=0.5 associated with a_m^* = 6.8 and $f(0,t)$=10.5%, at D=1 a_m^* = 7.2 and $f(0,t)$=20.9% and at D=2 a_m^* = 7.95 and $f(0,t)$=37.6%.

impact on gross employment change is not clear.[8] Similarly, a fall in the creation flow will increase average job tenure whilst a fall in the redundancy age will lower average tenure: the impact on average tenure is also not clear.

Training

Caballero and Hammour (1994) also briefly (in the conclusion) consider the possibility of a range of productivity within a cohort (perhaps reflecting differences in ability) and/or the existence of a learning curve so that units become wiser with age. These additions are, however, somewhat *ad hoc* to their model. Aghion and Howitt (1994; 489) explicitly consider the possibility of production units steadily increasing their output throughout their lifetime if they engage in a process of learning-by-doing. They argue that a production unit could increase its productivity according to learning-by-doing by some rate (say at some constant proportional rate γ_1). In terms of the Caballero and Hammour framework, the technology of a productive unit once created is no longer constant over its lifetime, rather, productivity will be related to the age of the unit $A(t_0, a)$, where $A(t_0,a) = A(t_0)exp(\gamma_0+\gamma_1 a)$. If the overall growth in technology incorporates this learning-by-doing effect then $\gamma=\gamma_0+\gamma_1 a$ where $\gamma_0, \gamma_1, \gamma_1' > 0$. This will impact on the steady state condition:

$$c(f^*(0)) = \frac{\exp^{\gamma_0 a_m^*} - \exp^{-(r+\delta)a_m^*}}{\gamma + \gamma_1 + r + \delta} - \frac{1 - \exp^{-(r+\delta)a_m^*}}{r+\delta} \tag{7b}$$

and

$$f^*(0) = \frac{(-\gamma_0 + \gamma + \delta)\overline{D}}{\exp^{\gamma_0 a_m^*} - \exp^{-\delta a_m^*}}. \tag{8b}$$

If we consider the case where training adds to total growth, for example γ_0=0.028 and γ_1=0.003 *ceteris paribus*, and compare our results with the standard model, (7) and (8), we find that the obsolescence age has risen (from 7.2 to 7.7 years) and the creation flow has fallen (from 20.1 to 18.96%).[9] If the obsolescence age has risen, destruction has fallen and net employment will rise. A fall in the creation flow will, however, lead to a fall in net employment. It is not clear which of these effects will dominate. The fall in creation flow and increase in the redundancy age will both lead to longer average job tenure, however.

An alternative view of the impact of training is presented in Mortensen and Pissarides (1997), they argue that firms can take an alternative option to destroying an unprofitable job and

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[8]We could expect this to be a sizable impact considering the multiple impacts on the destruction flow of the fall in a_m and the substitution of δ/a for δ in the employment equation (A).

[9]If we consider the case where γ_0=0.025 and γ_1=0.003, so that the total growth rate γ remains at 0.028 *ceteris paribus*, and compare our results with the standard model, (7) and (8), we find larger effects of the same direction: the obsolescence age rises (from 7.2 to 8.8 years) and the creation flow falls (from 20.1 to 17.9%).

creating an entirely new job. Firms can keep their otherwise obsolete worker by retraining and combining with current capital to form a new production unit. In so doing, the firm can save itself the costs and uncertainties involved in the hiring process. (It may also gain by keeping the learning-by-doing productivity bonus inherent in longer tenured employees discussed above.) It could be argued that this is a simple result that merely arises from the definition of job creation used in the model. It is true that in both the Caballero and Hammour (1994) and Aghion and Howitt (1994) models the possibility of combining an old worker with new technology is not allowed for, and it would be a very simple process to incorporate this possibility within the Caballero and Hammour framework. Indeed, the retrained worker is actually occupying a new job with a new capital endowment and state-of-the-art technology and his/her previous job has been destroyed. Since this process of reallocation occurred within the same firm, however, employment levels have not changed. In the longer run, these firms can extract a larger return from an initial hire outside of the firm: they can create further jobs at a lower cost because they can retrain and make use of their incumbent workforce. As discussed previously, lowering the cost of creation will lead to a fall in the obsolescence age (lowering net employment) but an increase in job creation (raising net employment). Note, the impact of training is slightly different if it acts to increase the productivity of the production unit (discussed above) than if it acts via lowering the creation cost. With an increase in productivity the redundancy age will rise and creation flow will fall (increasing average tenure), the opposite happens with a fall in creation costs (decreasing job average tenure), whilst both result in offsetting effects on total employment.

It might seem that it would always be cheaper for a firm to retrain an incumbent worker than to seek and train a new employee from outside. This is not necessarily true, for example, if the job required skills of a general nature, the employer may prefer an outside candidate who has recently finished an education programme with more current skills. Alternatively, if the job required high levels of job-specific training (such as may occur if the firm used a capital intensive production process) the value of a successful match would be more important and the firm would be less willing to part with an incumbent (we also argue that a young workforce, *ceteris paribus*, will have less job-specific training). A third scenario may occur if the human capital component of the job is very little, in which case the firm could again save hiring costs by keeping an employee providing they had a similar opening to slot them in to. We could also expect large firms (with more openings at any point in time) to be more able to accommodate in this way and have less creation costs.

Wage changes

Caballero and Hammour (1994) assume a constant consumption wage, implying that newer workers do not receive higher wages and that workers do not have bargaining strength over wages. The first is an intuitively unappealing assumption given that all workers from the same cohort have equal productivity and see their relative productivity fall at the same rate, γ, compared to newer cohorts. Indeed, both Aghion and Howitt (1994) and Mortensen and Pissarides (1997) allow for the wages of more recent units to rise over time. Relative wage changes play an important part in the reallocation process in the general equilibrium models: workers are aware that they can earn more by being hired in a new production unit, they are therefore constantly seeking a new appointment. If firms do not raise the wages of their incumbent employees, they will lose these members of their workforce. This means that the wages of incumbent workers increase even though their productivity is set at the time of creation. The increase in wages will eat into the operating surplus causing the value of the job to depreciate at a faster rate and decreasing the obsolescence age. Whilst Caballero and Hammour do not discuss wage increases, their exogenous destruction rate δ can encompass this effect. We can consider this impact in (4') below:

$$c(f(0,t)) = \int_t^{t+T(t)} \pi(s-t,t)e^{-(r+\delta)(s-t)+bs}ds \qquad (4')$$

The integrand has changed between 4 and 4' due to the introduction of the factor e^{bs} which captures the depreciation due to wage movements. Solving for the steady state provides:

$$c(f^*(0)) = \frac{\exp^{(\gamma_0+b)a_m^*} - \exp^{-(r+\delta-b)a_m^*}}{\gamma_0+r+\delta} - \frac{1-\exp^{-(r+\delta-b)a_m^*}}{r+\delta-b} \qquad (7c)$$

and

$$f^*(0) = \frac{(\gamma_0+b+\delta)\overline{D}}{\exp^{(\gamma_0+b)a_m^*} - \exp^{-\delta a_m^*}} \qquad (8c).$$

If we assume that wages increase by 1% per annum (b=0.01) and we maintain all the assumptions we have previously made about the remaining parameters, we find that (compared to the standard model as expressed in (7) and (8)) $a_m(t)$ falls from 7.22 years to 5.9 years whilst $f(0,t)$ rises from 20.1% to 22.4%. These will once again have offsetting effects on employment change, an increase in the wage will lower the obsolescence age, increasing destruction and decreasing net employment whilst an increase in the creation rate increases net employment. Both of these effects will lead to lower levels of average job tenure.

Bargaining strength

Workers and firms may also share the operating surplus according to their relative bargaining strength β:

$$c(f(0,t)) = \beta \int_t^{t+T(t)} \pi(s-t,t)e^{-(r+\delta)(s-t)}ds \qquad (4'')$$

If workers have no bargaining strength, β = 1 and equations 4 and 4" are equivalent. As the worker's bargaining strength increases, however, the period of profitable employment needs to rise in order to justify the original expenditure on creation. As β falls from 1 to 0.75 to 0.5, obsolescence age rises from 7.2 to 8.6 to 11 years and the creation flow falls from 20.1% to 17.9% to 15.2%. This is again an offsetting effect on employment change, union bargaining strength is positively related to a rise in obsolescence age (more employment and longer tenure) whilst negatively related to job creation (less employment but greater average tenure).

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